

Lattice design and dynamics studies of a low-alpha storage ring with tens of nanometer bunch length

Zhilong Pan^{1,*}, Weishi Wan,² Alexander Chao,³ Xiujie Deng,³ Yao Zhang⁴,
Wenhui Huang,¹ and Chuanxiang Tang^{1,†}

¹*Department of Engineering Physics, Tsinghua University, Beijing, 100084, China*

²*Quantum Science Center of Guangdong-Hong Kong-Macao Greater Bay Area, Shenzhen, Guangdong, China*

³*Institute for Advanced Study, Tsinghua University, Beijing, China*

⁴*Laboratory for Ultrafast Transient Facility, Chongqing University, Chongqing 401331, China*



(Received 11 June 2026; accepted 11 September 2026; published 1 October 2026)

A novel concept is introduced to reduce the longitudinal emittance in a low momentum compaction factor (low-alpha) storage ring, enabling the stable storage of electron bunches shorter than 100 nm. This design strategy can be applied to any quasi-isochronous storage ring to achieve very high radiation power through the longitudinal coherence of the emitted radiation. This paper addresses the nonlinear dynamics and aperture optimization for this type of storage ring. An optimal design example based on our analysis is presented. Single-particle tracking results demonstrate that an electron beam with an equilibrium root-mean-square bunch length of approximately 80 nm can be maintained in this ring, featuring a dynamic aperture larger than 1 mm. Additionally, preliminary considerations of intrabeam scattering effects in this ring are discussed.

DOI: [10.1103/nwjr-p5st](https://doi.org/10.1103/nwjr-p5st)

I. INTRODUCTION

Operating storage rings in a low-alpha mode is highly significant within the accelerator-based light source field. Over recent decades, low-alpha storage rings have enabled the production of coherent THz and far-infrared radiation [1–3]. To date, the shortest bunch lengths achieved in storage rings are on the order of picoseconds. For example, the Diamond storage ring has demonstrated bunch lengths between 1 and 5 picoseconds [1], while the Metrology Light Source (MLS) ring has achieved bunch lengths ranging from 1 to 10 picoseconds [2,4]. Attaining ultra-short bunch lengths at the sub-femtosecond scale or shorter in electron storage rings could open up opportunities to generate very high-power coherent radiation at deep ultraviolet or extreme ultraviolet (EUV) wavelengths, thereby broadening potential applications. The challenge of reducing bunch lengths below the picosecond or even femtosecond range lies not only in technological capabilities but also in understanding longitudinal beam dynamics and lattice design. From the perspective of single-particle

dynamics, the primary difficulties in designing and maintaining low-alpha storage rings involve managing high-order momentum compaction factors and the local momentum compaction factor, known as the partial alpha effect. This partial alpha effect results from longitudinal excitation occurring within bending magnets [5]. High-order momentum compaction factors can be controlled using sextupole and octupole magnets, as discussed in Refs. [6,7]. Although recent theoretical progress has been made in beam dynamics for low-alpha storage rings [8,9], especially concerning their complex nonlinear behavior, design methodologies for these rings are still not widely comprehended within the community.

In the meanwhile, a possible application to maintain the sub-femtosecond length electron bunch in storage ring is the steady-state microbunching (SSMB) scheme. Ratner and Chao [10] proposed a storage-ring-based SSMB approach designed to operate at MHz repetition rates or continuous wave (cw) mode, with the potential to generate clean EUV radiation at 13.5 nm wavelength and average power levels in the kilowatt range for lithography applications. To realize SSMB, various methods have been suggested, including longitudinal weak focusing [9,11,12], longitudinal strong focusing [13,14], a generalized longitudinal strong focusing scheme [15], reversible seeding techniques, and hybrid approaches [14,16–19]. Among these, the fundamental concept involves using a low-alpha storage ring where the radio-frequency (rf) system is

*Contact author: panzl@mail.tsinghua.edu.cn

†Contact author: tang.xuh@mail.tsinghua.edu.cn

Published by the American Physical Society under the terms of the *Creative Commons Attribution 4.0 International license*. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

replaced by a laser modulator to sustain steady-state microbunching, enabling the production of very high-power radiation at short wavelengths. In such storage rings, the electron beam typically has a bunch length shorter than 100 nm.

It cannot be initially guaranteed that in a low-alpha ring the bunch lengthening effect caused by partial alpha is sufficiently small, because this effect depends on the variance of the local momentum compaction factor throughout the ring. Typically, low-alpha storage ring lattices are designed so that the overall momentum compaction factor is nearly zero, while the local momentum compaction factor within one cell is relatively large and positive, and in another cell it has the same magnitude but a negative sign. A good example of this is the MLS electron storage ring [4]. Our new approach, aimed at storing sub-femtosecond electron bunches, is to simultaneously minimize both the global and local momentum compaction factors during lattice design. Another important aspect to consider is the nonlinear dynamics in this type of storage ring when the bunching system is replaced by a laser modulator with a wavelength on the order of 1 μm , as opposed to the conventional rf wavelength of about 1 m. This change imposes more stringent requirements on the optimization of nonlinear dynamics compared to conventional rings, especially the nonlinear transverse-longitudinal (T-L) coupling dynamics.

The paper is structured as follows. In Sec. II, we introduce the partial alpha effect and provide an intuitive explanation of a method to minimize it. Section III presents a detailed discussion on nonlinear dynamics. In Sec. IV, we discuss a lattice design example based on this method. The results of single-particle dynamics tracking are shown in Sec. V, confirming that microbunching at the 100 nm scale can be maintained in this ring. Section VI presents the dynamic aperture simulation. We also briefly address intrabeam scattering (IBS) effects in Sec. VII and some preliminary discussions on the operation of this kind of ring in Sec. VIII. A brief summary is provided in the final section.

II. MINIMIZING PARTIAL ALPHA EFFECT

A. Introduction to partial alpha effect

The well-known $\sigma_z \propto \sqrt{\eta}$ scaling law for the zero-current bunch length in storage rings, originally proposed by Sands [20], fails when η becomes very small. Here, σ_z is zero-current equilibrium bunch length, $\eta = \alpha_c - \frac{1}{\gamma_0^2}$ represents the global phase slippage factor of the ring, where α_c is the global momentum compaction factor and γ_0 is the Lorentz factor of the reference particles. The breakdown of this scaling law was first theoretically studied by Shoji *et al.* [5] and later experimentally verified by Deng *et al.* [21]. As Shoji *et al.* [5] explained, the primary cause of the failure of Sands' scaling law is the

longitudinal quantum radiation excitation. Since synchrotron radiation is a stochastic process, photons can be emitted at any point within the bending magnets. When a particle emits radiation at different longitudinal positions around the ring, it results in varying one-turn path lengths to the observation point. This effect can be analytically described by introducing the concept of partial alpha.

The partial alpha is defined as the momentum compaction factor between the radiation source point and the observation point, represented as follows:

$$\tilde{\alpha}(s_j) = \frac{1}{C_0} \int_{s_j}^{s_0} \frac{\eta(s)}{\rho(s)} ds. \quad (1)$$

Here, s_j represents an arbitrary radiating point within the bending magnets, s_0 denotes the observation point, and C_0 is the circumference of the storage ring. The functions $\eta(s)$ and $\rho(s)$ correspond to the dispersion function and bending radius at position s , respectively. Taking into account the partial alpha effect, the electron beam's bunch length can be expressed as shown in Ref. [5]

$$\sigma_s = \sigma_{s0} C_0 \sqrt{\left(\frac{\alpha_c}{2\pi\nu_s}\right)^2 + I_{\tilde{\alpha}}}. \quad (2)$$

Here, σ_{s0} denotes the equilibrium root-mean-square relative energy spread as given by Sands [20], ν_s is the synchrotron tune, and $I_{\tilde{\alpha}}$ corresponds to the variance of the partial alpha $\tilde{\alpha}(s_j)$ along the ring. (Strictly speaking, the local phase slippage factor should be used; however, since this does not affect the underlying physics, we adopt the formula in Shoji *et al.* [5].) This effect is inherently accounted for by the SLIM formalism [22] and can also be calculated analytically by defining longitudinal Courant-Snyder parameters [23]. In this work, we aim to present it in a more intuitive manner below to provide better insight into lattice design.

Because of the partial alpha effect, maintaining an ultrashort bunch length (such as femtoseconds or sub-femtoseconds) in a storage ring using the conventional dispersion cancellation method will not be effective. In other words, the cancellation needs to be applied not only on a global scale (α_c) but also as locally as possible [$\tilde{\alpha}(s_j)$].

In a previous paper [9], we presented an analytical approach to minimize the longitudinal emittance in storage rings; however, that work provided limited insight into how to arrange the magnetic elements to reach the design objectives. In this paper, we demonstrate how to reduce the longitudinal emittance by considering the partial alpha effect, offering greater intuition for lattice design. Additionally, this paper addresses nonlinear dynamics and the optimization of dynamic aperture for this kind of ring.

Let us now examine the periodic structure illustrated in Fig. 1. The dispersion function curve within the dipoles, as

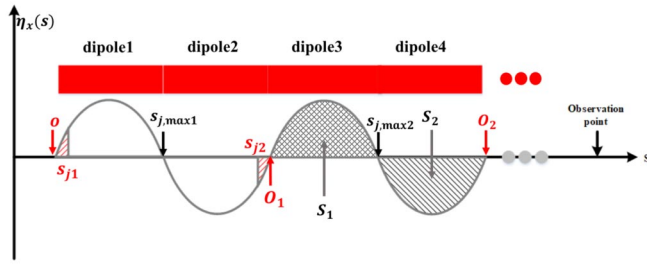


FIG. 1. Sketch for the partial alpha effect.

depicted in Fig. 1, is a rough sketch but sufficient to convey the idea. Each red box in Fig. 1 indicates the position of a dipole. We define S_1 as the integral of the dispersion function over a dipole, expressed as $S_1 = \frac{1}{C_0} \int_{O_1}^{s_{j,max2}} \frac{\eta_x(s)}{\rho(s)} ds$. Similarly, S_2 is defined as $S_2 = \frac{1}{C_0} \int_{s_{j,max2}}^{O_2} \frac{\eta_x(s)}{\rho(s)} ds$. By adjusting the lattice to maintain the condition $S_1 + S_2 = 0$, the lattice can be made into a low-alpha ring through periodic repetition of this structure.

For any arbitrary point inside a dipole, such as point s_{j1} (or s_{j2}), the partial alpha $\tilde{\alpha}(s_{j1})$ [or $\tilde{\alpha}(s_{j2})$] is defined as the integral of the dispersion function from that point to the observation point. This corresponds to the integration area bounded by the dispersion curve from point o to s_{j1} (or from s_{j2} to O_1). Specifically, it can be expressed as

$$\begin{aligned} \tilde{\alpha}(s_{j1}) &= \frac{1}{C_0} \int_{s_{j1}}^{s_0} \frac{\eta_x(s)}{\rho(s)} ds \\ &= -\frac{1}{C_0} \int_o^{s_{j1}} \frac{\eta_x(s)}{\rho(s)} ds + N(S_1 + S_2) \\ &= -\frac{1}{C_0} \int_o^{s_{j1}} \frac{\eta_x(s)}{\rho(s)} ds \end{aligned} \quad (3)$$

or

$$\begin{aligned} \tilde{\alpha}(s_{j2}) &= \frac{1}{C_0} \int_{s_{j2}}^{s_0} \frac{\eta_x(s)}{\rho(s)} ds \\ &= \frac{1}{C_0} \int_{s_{j2}}^{O_1} \frac{\eta_x(s)}{\rho(s)} ds + (N-1)(S_1 + S_2) \\ &= \frac{1}{C_0} \int_{s_{j2}}^{O_1} \frac{\eta_x(s)}{\rho(s)} ds. \end{aligned} \quad (4)$$

Here, N represents the number of periods, and s_0 is the observation point. Generally, the inequality $|\tilde{\alpha}(s_{j1})| < S_1$ holds at any point. Therefore, the value of $I_{\tilde{\alpha}}$ is determined by S_1 , which is the maximum value of the dispersion function within the dipoles. Consequently, minimizing the partial alpha effect is equivalent to minimizing the maximum dispersion function value inside the dipoles.

B. Minimizing partial alpha in any single dipole

A simple approach is to set the integral $\int_0^{l_b} \frac{\eta_x(s)}{\rho(s)} ds$ equal to zero for each individual dipole, while maintaining a small bending angle for each dipole, where l_b represents the length of a single dipole. This method naturally ensures a low-alpha ring. The concept is demonstrated below to effectively reduce the maximum dispersion value within the dipoles.

As an example, we consider a dipole magnet without higher-order magnetic fields (such as quadrupole fields, although the approach is also applicable if they are present), with a half bending angle denoted by θ_0 and a radius of curvature ρ (assuming constant curvature here, and that θ_0 is much less than 1). Given the dispersion functions (η_0, η'_0) at the center of this dipole, as illustrated in Fig. 2, the dispersion at any point inside the dipole can be expressed as $\eta(\theta) = \eta_0 \cos \theta + \eta'_0 \rho \sin \theta + \rho(1 - \cos \theta)$, where θ ranges from 0 to θ_0 . The condition $\int_0^{\theta_0} \eta(\theta) d\theta = 0$ must be satisfied, from which the following relation can be derived:

$$\eta'_0 = \frac{\sin \theta_0 - \theta_0 - \frac{\eta_0}{\rho} \sin \theta_0}{1 - \cos \theta_0}. \quad (5)$$

The dispersion distribution in Fig. 2 indicates that the optimal condition to minimize the partial alpha effect is when $\eta'_0 = 0$. Under this condition, η_0 can be determined from Eq. (5) as $\eta_0 = -\frac{1}{6}\rho\theta_0^2$, and consequently, $\eta(\theta_0) = \frac{1}{3}\rho\theta_0^2$. Therefore, $I_{\tilde{\alpha}}$ can be directly calculated as follows:

$$\sqrt{I_{\tilde{\alpha}}} = \frac{\sqrt{210}}{315C_0} \rho \theta_0^3. \quad (6)$$

If the bunch length contribution from partial alpha, given by $\sigma_s = \sigma_{\delta} C_0 \sqrt{I_{\tilde{\alpha}}}$, is restricted to 10 nm, and using the values $C_0 = 100$ m, $\sigma_{\delta} = 2.4 \times 10^{-4}$, and $\rho = 2$ m, the half bending angle must satisfy $\theta_0 < 4.40^\circ$. The beam

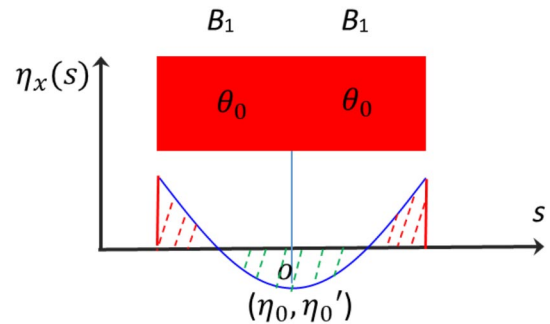


FIG. 2. The dispersion distribution in the dipole to minimize the partial alpha effect.

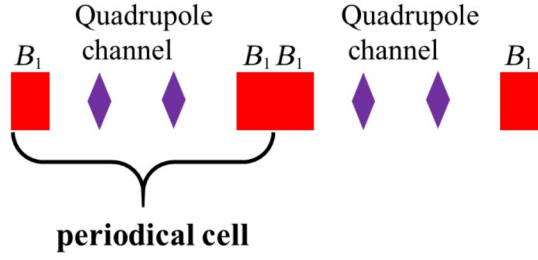


FIG. 3. The sketch of periodic cell to achieve the dispersion distribution in Fig. 2.

energy discussed herein is around a few hundred MeV. For energies above 1 GeV, achieving a bunch length below 100 nm becomes harder due to longitudinal emittance increasing with energy. A storage ring for such beams has a circumference of about 100 m, a bending radius that is kept as small as possible to enhance damping, and an energy spread that adopts the typical value corresponding to the beam energy and bending radius. To achieve the dispersion distribution shown in Fig. 2, a schematic of the periodic cell lattice is presented in Fig. 3.

Because of the lattice's mirror symmetry, we can readily determine that the α function and the dispersion angle are zero at both the beginning and the end of the periodic cell. Therefore, the transfer matrix of the periodic cell is expressed as follows:

$$M_t = \begin{bmatrix} \cos \mu_x & \beta_x \sin \mu_x & 0 & m_{16} \\ -\gamma_x \sin \mu_x & \cos \mu_x & 0 & m_{26} \\ -m_{26} & -m_{16} & 1 & -\alpha_c L_0 + m_{26} \eta_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (7)$$

with $m_{16} = \eta_0 - \cos \mu_x \eta_0$, $m_{26} = \gamma_x \sin \mu_x \eta_0$, and $\mu_x = 2\pi\nu_x$, where ν_x is the horizontal tune, η_0 is the dispersion function at both the entrance and exit, α_c is the momentum compaction factor, and L_0 is the length of the main cell. The horizontal Twiss parameters are γ_x and β_x . In this analysis, we focus solely on the dynamics within the x - z plane for horizontal bending dipoles. The coordinate vector associated with the transfer matrix is $X = (x, x', z, \delta)$.

To minimize the partial alpha effect, the condition is given by $\eta_0 = -\frac{1}{6}\rho\theta^2$, where ρ is the bending radius and θ is the bending angle of dipole B_1 , as shown in Fig. 3.

We divide the periodic cell into three sections: dipole B_1 , the quadrupole channel between the two dipoles, and another dipole B_1 . By applying the thin-lens approximation for the dipoles, the transfer matrices for the dipole and the quadrupole channel can be expressed as follows:

$$M_{B_1} = \begin{bmatrix} 1 & \rho\theta & 0 & \rho\theta^2/2 \\ 0 & 1 & 0 & \theta \\ -\theta & -\rho\theta^2/2 & 1 & -\rho\theta^3/6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

$$M_{QC} = \begin{bmatrix} M_{11} & M_{12} & 0 & 0 \\ M_{21} & M_{22} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The total map should be

$$M_t = M_{B_1} M_{QC} M_{B_1} = \begin{bmatrix} M_{11} + M_{21}\rho\theta & R_{12} & 0 & R_{16} \\ M_{21} & M_{22} + M_{21}\rho\theta & 0 & R_{26} \\ -R_{26} & -R_{16} & 1 & R_{56} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (9)$$

with

$$\begin{aligned} R_{12} &= M_{12} + \rho\theta(M_{11} + M_{22} + M_{21}\rho\theta) \\ R_{16} &= \frac{1}{2}\theta[2M_{12} + \rho\theta(1 + M_{11} + 2M_{22} + M_{21}\rho\theta)] \\ R_{26} &= \frac{1}{2}\theta(2 + 2M_{22} + M_{21}\rho\theta) \\ R_{56} &= -\frac{1}{12}\theta^2[12M_{12} + \rho\theta(4 + 6M_{11} + 6M_{22} + 3M_{21}\rho\theta)]. \end{aligned} \quad (10)$$

In the lattice with mirror symmetry, the condition $M_{11} = M_{22}$ is satisfied. By comparing Eqs. (7) and (9), we find that

$$\begin{aligned} M_{11} + M_{21}\rho\theta &= \cos \mu_x \\ M_{21} &= -\gamma_x \sin \mu_x \\ \frac{1}{2}\theta(2 + 2M_{22} + M_{21}\rho\theta) &= \gamma_x \sin \mu_x \eta_0 \\ \frac{1}{2}\theta[2M_{12} + \rho\theta(1 + M_{11} + 2M_{22} + M_{21}\rho\theta)] &= \eta_0 - \cos \mu_x \eta_0. \end{aligned} \quad (11)$$

According to Eq. (11) and using the condition $\eta_0 = -\frac{1}{6}\rho\theta^2$, we can get

$$\beta_x = -\frac{2\rho\theta}{3} \frac{\sin \mu_x}{1 + \cos \mu_x}. \quad (12)$$

Equation (12) provides some insights into the optics of this periodic cell. First, the parameter β_x must always be

positive, which means that $\mu_x > \pi$ should be satisfied in the design of the periodic cell lattice. In other words, the tune of the periodic cell must be greater than 0.5. Using Eq. (12), one can easily derive the transverse emittance for this type of storage ring, as shown in Eq. (2.80) of Ref. [8]. Second, when the tune of the periodic cell is fixed, β_x is also fixed. During the design of the periodic cell, certain specific tunes are selected to optimize nonlinear dynamics. For example, if there are six periodic cells in a superperiod, it is preferable to set the tune at $\nu_x = \frac{i}{6}$. To satisfy the condition in Eq. (12), the tune should be chosen as $\nu_x = \frac{4}{6}$ or $\nu_x = \frac{5}{6}$.

C. Minimizing partial alpha effect by TBA cell

The lattice design described in Sec. II B is the best approach to minimize the partial alpha effect, but it lacks flexibility for nonlinear optimization. This is because the dispersion within any periodic cell cannot be large due to the small bending angle of the magnets, while the tune of a single periodic cell is relatively high (it must be greater than π). As a result, numerous strong sextupoles are required to correct chromaticities and nonlinear momentum compaction factors, which restricts the nonlinear performance of the T-L coupling path length deviation discussed in the following section. To alleviate these constraints, we propose a triple-bend-achromat (TBA) lattice that includes a small reverse bending dipole, as illustrated in Fig. 4.

In the TBA cell, we aim for a larger dispersion function in the dipole B_2 . Therefore, the bending radius of B_2 shown in Fig. 4 will be significantly greater than that of B_1 . Since the longitudinal emittance contribution is proportional to $\frac{1}{|\rho^3|}$, the effect of B_2 on the bunch length can be considered negligible.

In a similar manner, we can analyze the theoretical bunch length when the partial alpha effect is taken into account for the TBA cell. After some calculations, the

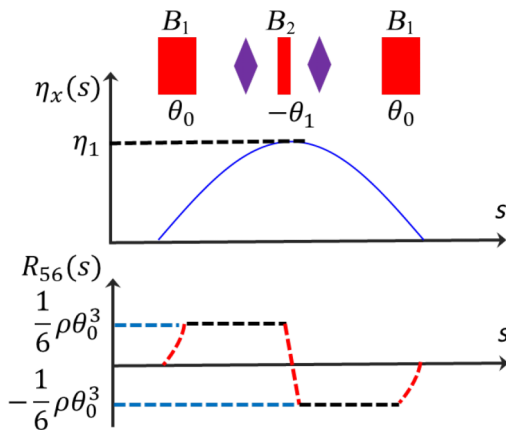


FIG. 4. The sketch of TBA cell to achieve the isochronous condition. Dipole B_2 is a reverse bending dipole.

contribution from the two B_1 dipoles can be expressed as follows:

$$\sqrt{I_a} = \frac{\sqrt{14}}{42C_0} \rho \theta_0^3. \quad (13)$$

Here, ρ represents the bending radius of B_1 . By comparing Eqs. (13) and (6), we find that the partial alpha effect in the TBA cell is approximately two times greater than that shown in Fig. 3. Additionally, the horizontal betatron phase advance in the TBA cell is about π , and the dispersion within the TBA cell can be larger than that in Fig. 3 for the same drift space. Therefore, in the subsequent design, we choose the TBA cell as our fundamental structure.

III. NONLINEAR DYNAMICS DISCUSSION

A. High-order momentum compaction factors

In the SSMB storage ring discussed in this paper, the overall momentum compaction factor is extremely small, typically on the order of 10^{-7} . Therefore, in addition to correcting chromaticities, it is also necessary to address higher-order momentum compaction factors. We will first focus on the second-order momentum compaction factor, which is defined by the relation $\Delta z/C_0 = \alpha_c \delta + \alpha_{c2} \delta^2 + \alpha_{c3} \delta^3$. Here, Δz represents the path length deviation per turn for a particle with a relative momentum deviation δ , C_0 is the ring circumference, α_c is the global linear momentum compaction factor as defined earlier, and α_{c2} and α_{c3} denote the second- and third-order momentum compaction factors, respectively. In storage rings, α_{c2} can be expressed as follows:

$$\alpha_{c2} = \frac{1}{C_0} \int_0^{C_0} \left(\frac{\eta_2}{\rho} + \frac{\eta_1'^2}{2} \right) ds. \quad (14)$$

Here, ρ represents the bending radius, η_2 denotes the second-order dispersion, and η_1' corresponds to the derivative of the dispersion function. Additionally, η_2 can be expressed as shown in Ref. [6].

$$\begin{aligned} \eta_2(s) = & -\eta_1(s) + \frac{1}{2 \sin(\pi Q_x)} \int_s^{s+C_0} \sqrt{\beta_x(s) \beta_x(\sigma)} \\ & \times \cos(\pi Q_x - \mu_{\sigma s}) \left[K_1(\sigma) \eta_1(\sigma) - \frac{1}{2} K_2(\sigma) \eta_1^2(\sigma) \right] d\sigma. \end{aligned} \quad (15)$$

In Eqs. (14) and (15), we have neglected the contribution from the Lorentz factor. Here, η_1 represents the dispersion function, Q_x denotes the total betatron tune of the ring in the horizontal direction, β_x is the horizontal beta function, $\mu_{\sigma s}$ is the horizontal phase advance between points s and σ , K_1 stands for the quadrupole strength, and K_2 is the sextupole strength.

When the second-order momentum compaction factor α_{c2} becomes very large—for instance, under the condition $\alpha_{c2}\delta_{rf} > \alpha_c/10$ —the rf bucket shape becomes distorted, transforming into what is called an α -bucket. Here, δ_{rf} refers to the bucket height calculated considering only the linear momentum compaction factor. According to Eqs. (14) and (15), sextupoles can be placed at locations with large dispersion and beta functions to correct α_{c2} . When sextupoles are used to address nonlinear dynamics, α_{c2} should be adjusted so that the condition $\alpha_{c2}\delta_{rf} < \alpha_c/10$ is satisfied. Under this condition, the path length deviation caused by α_{c2} is significantly smaller than that caused by α_c .

Another point to highlight is that the Taylor map coefficient T_{566} , commonly used in linac lattice design, is comparable in magnitude to α_{c2} . At a dispersion-free location in a storage ring, the one-turn map evaluated at that point satisfies the relation $T_{566} = C_0\alpha_{c2}$.

The third-order momentum compaction factor α_{c3} , which corresponds to the Q_{5666} term in the Taylor map, is associated with the coefficient h_{00004} in the storage ring's Hamiltonian [24]. The approach to optimizing these higher-order terms will be discussed later.

B. T-L coupling

In storage rings, particles undergo transverse oscillations called betatron oscillations. Particles with different betatron amplitudes travel different longitudinal path lengths compared to a reference particle. A key stability criterion is that if the path length deviation caused by T-L coupling and high-order momentum compaction factor exceeds the rf wavelength, the particle becomes unstable. Consequently, the dynamic aperture will be smaller than the corresponding amplitude.

In a conventional storage ring with a 500 MHz rf cavity, the rf wavelength is approximately 0.6 m. Therefore, the path length difference between any particle and the reference particle can probably not exceed this rf wavelength. In our SSMB design, the traditional bunching system is replaced by a laser modulator to achieve extremely short electron bunch length in the storage ring. Since the laser wavelength is typically around 1 μm , a crucial optimization goal for this type of ring is to minimize path length deviations caused by nonlinear T-L coupling and higher-order momentum compaction factors.

Reference [24] provides a detailed discussion of the nonlinear T-L coupling effects. Based on this, we will employ the high-order-achromat (HOA) method to design the lattice and use sextupoles to correct both the chromaticities and the second-order momentum compaction factor, α_{c2} . This approach allows us to eliminate the beam's path length deviations arising from third-order terms (specifically, the Taylor map terms T_{5xx}) in the Hamiltonian expressions presented in Ref. [24]. At the

same time, the path length deviations caused by fourth-order terms in the Hamiltonian will be minimized through an optimization algorithm during the lattice design process. In our optimization, we define the root-mean-square deviation of the path length arising from the fourth-order terms in the Hamiltonian as [24]

$$\begin{aligned} \sigma_{z,4}^2 = & 15h_{00004}^2\sigma_\delta^6 + 6|h_{10003}|^2\epsilon_{x0}\sigma_\delta^4 \\ & + 12h_{00004}(h_{11002}\epsilon_{x0} + h_{00112}\epsilon_{y0})\sigma_\delta^4 \\ & + 8\text{Re}(h_{10003}h_{21001}^*)\epsilon_{x0}^2\sigma_\delta^2 \\ & + 4\text{Re}(h_{10003}h_{10111}^*)\epsilon_{x0}\epsilon_{y0}\sigma_\delta^2 \\ & + (8h_{11002}^2 + 4|h_{20002}|^2)\epsilon_{x0}^2\sigma_\delta^2 \\ & + (8h_{00112}^2 + 4|h_{00202}|^2)\epsilon_{y0}^2\sigma_\delta^2 \\ & + 8h_{11002}h_{00112}\epsilon_{x0}\epsilon_{y0}\sigma_\delta^2 \\ & + 24(|h_{21001}|^2 + |h_{30001}|^2)\epsilon_{x0}^3 + 8|h_{10111}|^2\epsilon_{x0}\epsilon_{y0}^2 \\ & + 8|h_{10201}|^2\epsilon_{x0}\epsilon_{y0}^2 + 8|h_{10021}|^2\epsilon_{x0}\epsilon_{y0}^2 \\ & + 16\text{Re}(h_{21001}h_{10111}^*)\epsilon_{x0}^2\epsilon_{y0}. \end{aligned} \quad (16)$$

In this context, the term h_{ijklm} represents fourth-order driving terms. The symbol ϵ_{x0} denotes the horizontal emittance of the beam, while ϵ_{y0} stands for the vertical emittance. The parameter σ_δ indicates the relative energy spread of the beam. Additionally, $\text{Re}(Z)$ refers to the real part of the complex number Z . For more detailed information, please refer to Ref. [24].

IV. AN EXAMPLE OF LATTICE DESIGN

Figure 5 illustrates the layout of the designed storage ring. The ring is composed of two superperiods, with each superperiod containing 2 “matching cells” and 12 “TBA cells,” as depicted in Fig. 5. The number of “TBA cells” is chosen based on the target equilibrium bunch length of the electron beam, $\sigma_s < 100$ nm, as explained in Eq. (13).

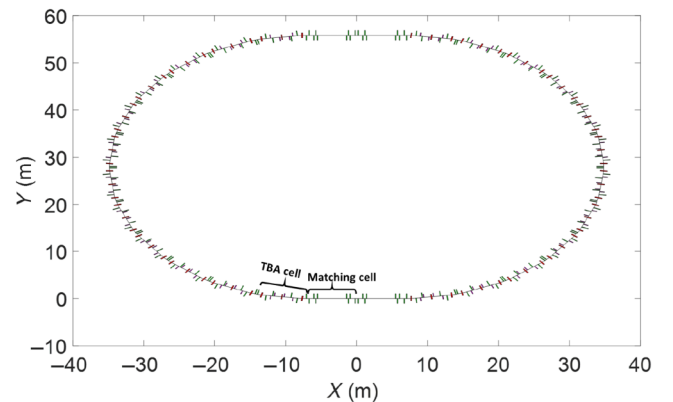


FIG. 5. Layout of SSMB storage ring.

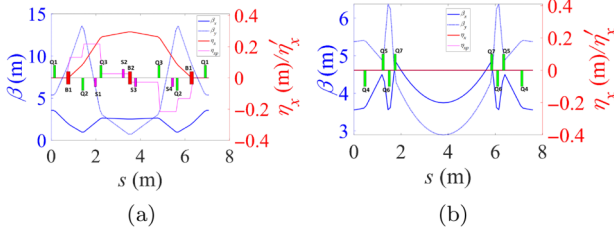


FIG. 6. Lattice arrangement of (a) TBA cell and (b) matching cell.

When optimizing the lattice using an algorithm, the parameters of dipoles and quadrupoles within the “TBA cell” are adjusted to satisfy constraints related to tune, achromat, and quasi-isochronous conditions. The final results indicate tunes of $\nu_x = 7/12$ and $\nu_y = 5/12$, with the momentum compaction for each “TBA cell” being $R_{56,t} = 2.93 \times 10^{-7}$ m. The Twiss functions for the “TBA cell” are presented in Fig. 6(a). The purpose of the “matching cell” in Fig. 6(b) is to fine-tune the overall ring tune and to preserve straight sections for injection and bunching systems (noting that a laser modulator is employed in the SSMB storage ring). The total R_{56} of the ring is the sum of 24 “TBA cells” and 4 “matching cells.” The momentum compaction $R_{56,m}$ of each “matching cell” is coming from the Lorentz factor by $\frac{L_0}{\gamma^2} = 1.24 \times 10^{-5}$ m, with L_0 represents the total length of “matching cell.” So the phase slippage factor of the ring is $\eta_c = -\frac{24 \cdot R_{56,t} + 4 \cdot R_{56,m}}{C_0} = -2.83 \times 10^{-7}$, with C_0 is the ring circumference.

For the entire ring, several key constraints are imposed, such as maintaining the equilibrium bunch length at the straight section $\sigma_s < 100$ nm, and setting the tune near 0.1 or 0.2 to stay as far away from the resonance line as possible. Four sextupole families, labeled “S1” through “S4” within the “TBA cell” in Fig. 6(a), are used to correct the chromaticities ξ_x and ξ_y , as well as the second-order momentum compaction factor α_{c2} of the ring.

Two optimization objectives are established for nonlinear dynamics during the ring design. The first objective targets the nonlinear path length contributions from third-order terms, defined as $obj_1 = |h_{20001}| + |h_{00201}|$ (contributed from “matching cell”). The terms h_{20001} and h_{00201} , along with their expressions, can be found in Eqs. (A8) and (A10) of Ref. [24]. Since chromaticities and α_{c2} are corrected by sextupoles, and h_{10002} is minimized through the HOA design of the “TBA cell,” the path length variation due to third-order terms can be expressed as $\sigma_z^2 = 4|h_{20001}|^2 \epsilon_x^2 + 4|h_{00201}|^2 \epsilon_y^2$ (comes from the “matching cell”), according to Eq. (A11) in Ref. [24]. The second objective concerns the nonlinear path length contributions from fourth-order terms, defined as $obj_2 = \sigma_{z,4}$. The emittance used for calculating $\sigma_{z,4}$ corresponds to a beam size of target aperture. For instance, if the beta function at

the injection point is $\beta_x = 2$ m and with a target aperture of 1 mm, the emittance applied is $\epsilon_x = \frac{(1 \text{ mm})^2}{2 \text{ m}} = 5 \times 10^{-7}$ m rad. The energy spread used in the $\sigma_{z,4}$ calculation is chosen to be on the same order as δ_{rf} , approximately $1 \sim 2 \times 10^{-3}$ in our case.

In the design, the chromaticities are corrected to zero. This is because we need to control the nonlinear transverse-longitudinal coupling to keep the path length deviation small enough for injecting particles with 2 mm amplitude. The rms path length deviation caused by chromaticities is given by $\sigma_{\delta z} = \sqrt{(2\pi\xi_x\epsilon_x)^2 + (2\pi\xi_y\epsilon_y)^2}$. For particles injected with a transverse displacement of about 2 mm and a beta function β_x of approximately 4 m, the emittance is 1×10^{-6} m rad. If the chromaticities are kept at 1, the resulting path length deviation would be several micrometers, causing instability during injection. To maintain a dynamic aperture of 1 mm, keeping chromaticities around 0.1 is acceptable. However, regarding collective instability, it is not yet clearly understood whether operating at chromaticities of 0.1 is stable. Therefore, to clarify the design, we set the chromaticities to zero in this paper.

The optimization is carried out using the multiobjective genetic algorithm [25], and an optimal solution is selected from the final results. The Twiss functions for one super-period are shown in Fig. 7, and the ring parameters are summarized in Table I. The simulation results are discussed in the following section.

The parameters listed in Table I present an excellent configuration for SSMB, featuring an extremely small phase slippage factor of $\eta_c = -2.83 \times 10^{-7}$ and a very small variance of local momentum compaction factor of $\sqrt{T_{\alpha}} = 8.85 \times 10^{-7}$. The equilibrium bunch length of the electron beam due to the partial alpha effect is $\sigma_{s,p} = \sigma_{\delta 0} C_0 \sqrt{T_{\alpha}} = 55.36$ nm, while according to Sand’s formula, it is $\sigma_{s,s} = \sigma_{\delta 0} C_0 \frac{\eta_c}{2\pi\nu_s} = 53.57$ nm contributed by the global momentum compaction factor for this lattice. The overall bunch length is therefore $\sigma_s = 77.31$ nm, which aligns well with the SLIM formalism calculation result of $\sigma_s = 77.94$ nm.

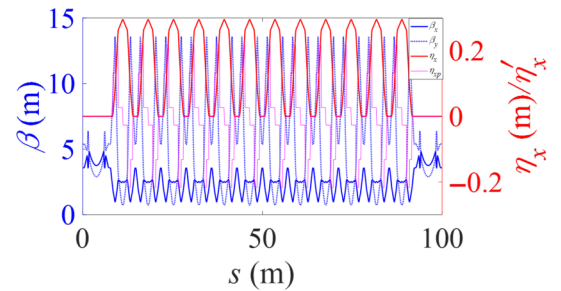


FIG. 7. The Twiss functions for one superperiod of the storage ring.

TABLE I. Parameters of SSMB ring.

Parameters	Value	Units
Circumference C_0	199.86	m
Beam energy E_0	400	MeV
Tunes (ν_x/ν_y)	15.210/11.204	...
Natural chromaticities (ξ_{x0}/ξ_{y0})	-12.85/-32.48	...
Corrected chromaticities (ξ_x/ξ_y)	0/0	...
Phase slippage factor η_c	-2.83×10^{-7}	...
Corrected second order phase slippage factor η_{c2}	-1.98×10^{-6}	...
Partial alpha ($\sqrt{T_\alpha}$)	8.85×10^{-7}	...
Relative energy spread ($\sigma_{\delta 0}$) by Sand's formula	3.13×10^{-4}	...
Relative energy spread (σ_δ) by SLIM formalism	4.44×10^{-4}	...
Energy loss per turn (U_0)	1.91	keV
Natural emittance (ϵ_0)	2.29	nm
Hor./Ver. damping times	279.21/276.61	ms
Long. damping time	140.27	ms

The equilibrium relative energy spread can likewise be determined as [8]:

$$\begin{aligned}
 \sigma_\delta &= \sigma_{\delta 0} \sqrt{1 + \left(\frac{\sigma_{s,p}}{\sigma_{s,s}} \right)^2} \\
 &= 3.13 \times 10^{-4} \sqrt{1 + \left(\frac{55.36}{53.57} \right)^2} \\
 &= 4.48 \times 10^{-4}.
 \end{aligned} \tag{17}$$

Here, $\sigma_{\delta 0}$ and $\sigma_{s,s}$ represent the energy spread and bunch length as defined by Sands [20], while $\sigma_{s,p}$ denotes the bunch length contribution due to the partial alpha effect. The outcome of Eq. (17) matches closely with the values obtained using the SLIM formalism, as demonstrated in Table I.

V. SINGLE PARTICLE SIMULATION

A macroparticle simulation using ELEGANT [26] was conducted for this lattice, taking into account only single-particle effects and excluding collective effects. In the simulation, the laser modulator is substituted with a standard rf cavity having an equivalent wavelength of 1 μm , with its parameters detailed in Table II. The

TABLE II. rf setting for tracking.

$\lambda = 1 \mu\text{m}$	
Voltage	Half bucket height δ_{rf}
120 kV	1.81×10^{-3}
Synchrotron tune ν_s	Bunch length σ_s
0.052	77.94 nm

corresponding laser modulation power can be estimated according to [19,27,28]

$$\begin{aligned}
 V_0[V] &= \sqrt{2} E_{f0} R_z [JJ] k_u \omega_0 \int_0^{L_u} \frac{A}{\sqrt{1+z^2}} e^{-\frac{A^2}{1+z^2}} dz, \\
 P[W] &= \frac{\pi \omega_0^2}{4} E_{f0}^2 \sqrt{\frac{\epsilon_0}{\mu_0}}.
 \end{aligned} \tag{18}$$

Here, $[JJ] = J_0(\frac{K^2}{4+2K^2}) - J_1(\frac{K^2}{4+2K^2})$, where J_0 and J_1 are Bessel functions. The parameter A is defined as $A = \frac{K}{\sqrt{2} \gamma k_u \omega_0}$. R_z represents the Rayleigh length of the laser, ω_0 is the laser beam waist, k_u is the undulator wave number, L_u is the undulator length, and K is the normalized undulator strength. V_0 denotes the equivalent modulation voltage, and E_{f0} is the peak electric field strength. For the values $L_u = 2.5$ m, $\omega_0 = 450 \mu\text{m}$, wavelength $\lambda = 1 \mu\text{m}$, $k_u = \frac{2\pi}{5 \times 10^{-2}} \text{m}^{-1}$, and $K = 6.86$, the required laser power to achieve an equivalent modulation voltage of 120 kV is 193 kW. This power requirement is within the capabilities of current state-of-the-art optical enhancement cavities [29,30].

A. Obtain the “rf” bucket by tracking

To ensure that higher-order momentum compaction factors have been corrected to a negligible level, we conduct a simple tracking simulation to obtain the “rf” bucket. The term “rf” is in quotation marks because our bunching system has been replaced by a laser modulator with a wavelength of μm . The input bunch is characterized by a transverse Gaussian distribution and a longitudinal line bunch, with parameters $\epsilon_x = 1$ nm, $\epsilon_y = 0.1$ nm, $\sigma_s = 160$ nm, and $\sigma_p = 0$. We use 100 macroparticles and track them for approximately 400 turns. Particle coordinates are recorded each turn and plotted together in a single figure, as shown in Fig. 8.

Figure 8 displays a very well-defined “rf” bucket, indicating that the higher-order momentum compaction factors have been corrected to an insignificant level.

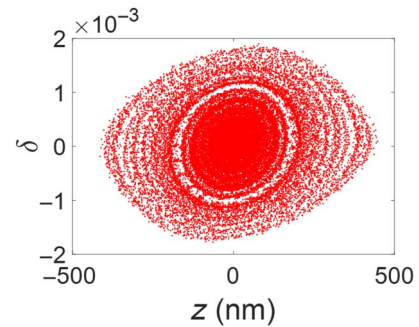


FIG. 8. The graph of the “rf” bucket. The particle coordinates are recorded at each turn during tracking. The image displays the well-known “rf” bucket of this ring.

Additionally, the bucket height matches the calculated values presented in Table II.

B. Long-term tracking

The long-term tracking begins from a bunch with a Gaussian distribution in all dimensions for 3000 macroparticles. We set the initial rms bunch length $\sigma_s = 20$ nm, initial horizontal emittance $\epsilon_x = 1$ nm, vertical emittance $\epsilon_y = 0.1$ nm, and initial energy spread $\sigma_\delta = 2 \times 10^{-4}$. In the simulation, radiation damping and quantum excitation are included. After tracking 800,000 turns in the ring (about 3.8 longitudinal damping times), only 16 particles were lost. The final horizontal, vertical, and longitudinal distributions shown in Fig. 9 provide the equilibrium parameters of the electron bunch as $\sigma_s = 80.97$ nm, $\sigma_\delta = 4.19 \times 10^{-4}$, $\epsilon_x = 2.22$ nm, and $\epsilon_y = 2.26$ pm. The error between simulation result and numerical calculation for bunch length is $\text{err}_{\sigma_s} = \frac{80.97 - 77.94}{77.94} = 3.89\%$, for relative energy spread $\text{err}_{\sigma_\delta} = \frac{4.19 \times 10^{-4} - 4.44 \times 10^{-4}}{4.44 \times 10^{-4}} = -5.63\%$, for horizontal emittance $\text{err}_{\epsilon_x} = \frac{2.22 \times 10^{-9} - 2.29 \times 10^{-9}}{2.29 \times 10^{-9}} = -3.06\%$. The numerical error of rms value for a Gaussian distribution sample should be $\text{err}_\sigma = \frac{1}{\sqrt{N-1}} = \frac{1}{\sqrt{3000-1}} = 1.83\%$, where N is the sample number. So the simulation error is within the error acceptance range. Meanwhile, we record the evolution of beam emittance, bunch length, and relative energy spread every 10,000 turns, as shown in Fig. 10.

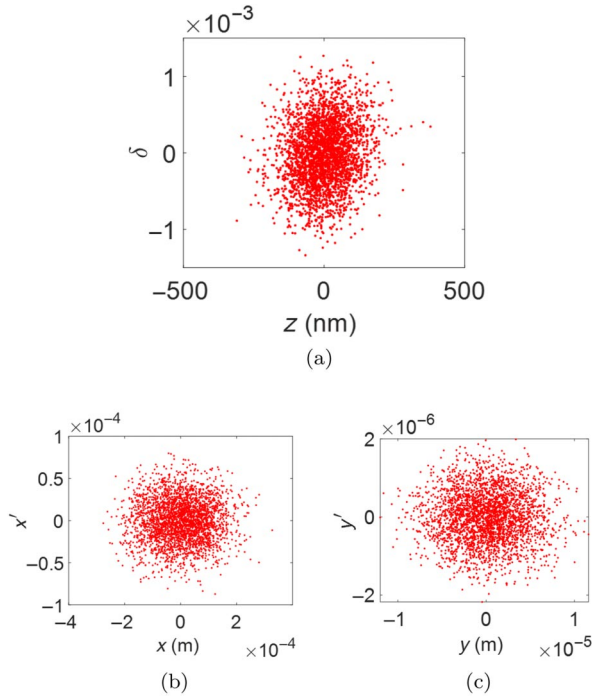


FIG. 9. Final distribution for macroparticle simulation. (a) Final longitudinal distribution of macroparticles, (b) final horizontal distribution of macroparticles, and (c) final vertical distribution of macroparticles.

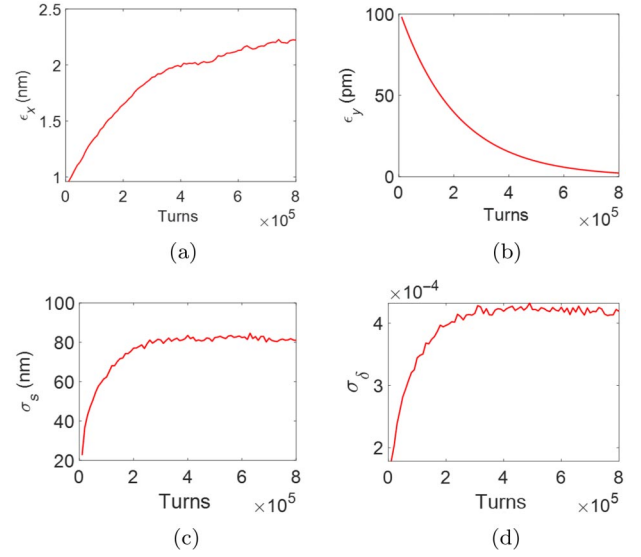


FIG. 10. Beam parameters evolution during tracking, while one data point is recorded per 10,000 turns. In order to decrease the impact on emittance from the unstable particle, we plot the beam parameters by dropping the particles jumped out of the bucket but not lost at the observation turn. (a) Beam horizontal emittance evolution during tracking, (b) beam vertical emittance evolution during tracking, (c) bunch length evolution during tracking, and (d) beam relative energy spread evolution during tracking.

To obtain a more precise estimate of the quantum lifetime, we conduct an additional simulation using the initial beam parameters at equilibrium: $\epsilon_x = 2.3$ nm, $\epsilon_y = 2$ pm, $\sigma_\delta = 4.4 \times 10^{-4}$, and $\sigma_s = 80$ nm. We track the beam with 3000 macroparticles over 800,000 turns in ELEGANT, recording the particles that are lost. Figure 11 displays the number of surviving particles as a function of tracking turns, alongside the exponential fit displayed in the figure. The fitting equation used in Fig. 11 is $N = N_0 \exp(-t/t_0)$, where t_0 represents a lifetime of about 55 s. This lifetime is primarily limited by the

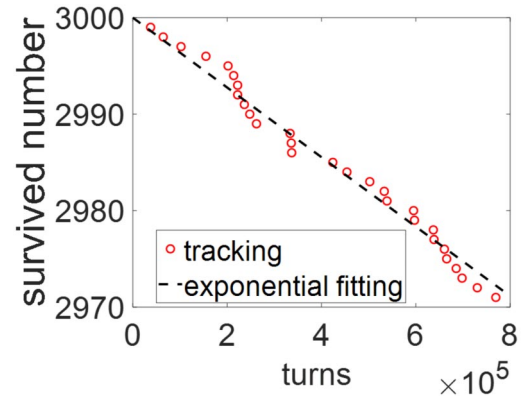


FIG. 11. Survived particle number evolution. The exponential fitting line represents the quantum lifetime of 55 s.

longitudinal quantum lifetime. The longitudinal bucket half height is about 4.32 times the equilibrium energy spread, and the dynamic aperture is roughly 10 times the equilibrium bunch size in the transverse directions, as discussed in the next section. Therefore, the quantum lifetime formula is given by [31]

$$\tau_q = \tau_d \frac{\sigma_\delta^2}{\delta_{rf}^2} \exp\left(\frac{\delta_{rf}^2}{2\sigma_\delta^2}\right). \quad (19)$$

The parameter τ_d represents the longitudinal damping time, σ_δ denotes the equilibrium energy spread of the bunch, and δ_{rf} is the half height of the longitudinal bucket. Using the formula, the quantum lifetime τ_q is approximately 85 s while the bucket height is 4.32 times of equilibrium energy spread. The tracking result of 55 s lifetime corresponds to the case that bucket height is 4.2 times of equilibrium energy spread. That is foreseeable as the bucket height in Table II is calculated based on the linear dynamics. The results demonstrate a good agreement between the simulation and the analytical calculations.

VI. DYNAMIC APERTURE ISSUES

As discussed in Sec. III, the dynamic aperture behavior in this SSMB ring differs from that of a conventional storage ring. This is because the bunching system's wavelength is extremely small, on the order of micrometers. Additionally, as shown in Table I, the damping time is quite long. Therefore, in this section, we present results that offer a different perspective on the dynamic aperture compared to previous analyses.

When optimizing the dynamic aperture of this ring, we set two objectives to minimize. Both objectives represent the beam's path length deviation per turn as it travels through the ring. If the standard deviation of the beam's path length in the lattice exceeds the bunching system's wavelength, particles will become unstable, jumping between different bunching buckets and violating the principle of phase stability. For the solution presented in Table I, we selected the one with objectives $\text{obj}_1 = 0.367$ and $\text{obj}_2 = 215$ nm, where $\text{obj}_1 = 2|h_{20001}| + 2|h_{00201}|$, with $h_{20001} = 0.0065$ and $|h_{00201}| = 0.18$. These objectives were calculated using beam parameters of $\epsilon_x = 2.81 \times 10^{-7}$ m rad, $\epsilon_y = 4.65 \times 10^{-8}$ m rad, and $\sigma_\delta = 1.81 \times 10^{-3}$, corresponding to rms beam sizes of $\sigma_x = 1$ mm and $\sigma_y = 0.5$ mm. The bunch length contribution from obj_1 is approximately 17 nm, leading to a total bunch length deviation on the order of 220 nm with the beam parameters considered here. From this, we conclude that the dynamic aperture of this lattice can exceed 1 mm in the horizontal (x) direction and 0.5 mm in the vertical (y) direction. That can be verified by following simulation results. And the dynamics aperture

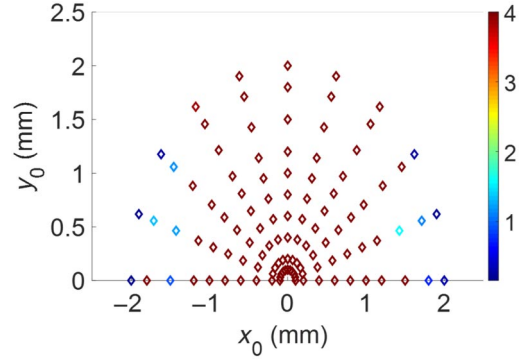


FIG. 12. The dynamics aperture of this ring. The value of color bar in this figure represents the surviving turn divided by 100,000.

in the y direction is much larger than 0.5 mm, as shown in Fig. 12 with 2 mm in the y direction. The reason is that the contribution in the $\sigma_{z,4}$ from vertical emittance is small in this lattice.

We verified this optimization result through ELEGANT tracking simulations. The tracking was performed on several tens of particles with varying initial coordinates $(x_0, 0, y_0, 0, 0, 0)$. Each particle was tracked for 400,000 turns (approximately the transverse damping time), and the turn number at which each particle was lost was recorded. The results are shown in Fig. 12.

As shown in Fig. 12, the majority of particles remain after tracking for 400,000 turns, indicating that the dynamic aperture of this ring exceeds 1.2 mm horizontally and 2 mm vertically. Additionally, the dynamic aperture in the transverse direction is more than ten times the size of the equilibrium beam.

VII. INTRABEAM SCATTERING

Since the beam operates at a low energy of 400 MeV and has a relatively small emittance, IBS in this ring could be significant. We have designed the ring's working point to approach the difference resonance region, enabling round beam operation to help mitigate the IBS effect. The bunch length at a dispersion-free location is approximately 80 nm, but it becomes longer due to transverse-longitudinal coupling at dispersive locations, as discussed in Ref. [32]. Using the method described in Ref. [32], we calculated the equilibrium emittance in this storage ring for various beam currents. In the calculation, a uniform beam-filling pattern is assumed, whereby each laser bucket holds one bunch. For example, setting the average beam current to 0.1 A yields a bunch charge of about $Q = I_0 T_b = (0.1 \text{ A}) * (1 \text{ } \mu\text{s}) / (299792458 \text{ m} * \text{s}^{-1}) \approx 0.33 \text{ fC}$. The results are presented in Table III. In these calculations, the coupling between the horizontal and vertical directions was set to 100%.

Based on the results shown in Table III, we observe that even at a beam current of 0.1 A, the equilibrium emittance

TABLE III. IBS equilibrium emittance of SSMB ring for different beam current.

Equilibrium parameters	Average current $I_0 = 0.1$ A	Average current $I_0 = 1$ A
Bunch length σ_s	88.41 nm	114.93 nm
Energy spread σ_δ	5.04×10^{-4}	6.55×10^{-4}
Hor. emittance ϵ_x	1.35 nm	2.02 nm
Ver. emittance ϵ_y	1.35 nm	2.02 nm

increases by approximately 20%. When the average beam current reaches 1 A, the equilibrium emittance rises by about 100%. One effective method to reduce the impact of IBS is to add damping wigglers, which enhance the ring's damping. We do not recommend increasing the beam energy because the zero-current equilibrium beam energy spread is sensitive to beam energy but not significantly affected by adding damping wigglers. Maintaining the requirement of $\sigma_s < 100$ nm becomes challenging at higher beam energies.

VIII. PRELIMINARY DISCUSSION ON THE OPERATION OF THIS KIND OF LIGHT SOURCE

In the design example illustrated in Fig. 5, we have not allocated excessive space for the radiation beamline. In fact, additional drift space can be added at the entrance and exit of the TBA cell to accommodate the undulator radiator. This would allow the construction of several tens of beamlines for versatile research purposes. An additional drawback of this approach is that it will increase the ring's circumference and also require matching quadrupole magnets for focusing. A more flexible alternative is to design enough matching cells, which can alleviate the difficulties encountered in the previous scheme.

To achieve a more realistic operation of the light source, the initial proposal is to incorporate an additional rf cavity that creates a second bucket. This allows particles that leave the laser bucket due to quantum lifetime effects to be stored for a longer period and potentially return to the laser bucket through radiation damping. As a result, the overall lifetime could be larger than the 55 s when there is only laser modulator. Since this topic extends beyond the scope of this paper, we do not explore it in detail here; further research is necessary to accurately determine the beam lifetime when using an additional rf cavity.

The injection method could be on-axis injection, which is supported by the dynamic aperture of 2 mm level. However, the longitudinal injection parameters are quite restricted. We conducted additional simulations to verify the injection efficiency, using two different tracking scenarios. In the first scenario, the initial beam parameters were set to $\epsilon_x = 2.3$ nm, $\epsilon_y = 2$ pm, $\sigma_s = 3$ mm,

$\sigma_\delta = 6 \times 10^{-4}$. Here, the transverse beam size is significantly smaller than the dynamic aperture, and the chosen bunch length can represent a typical injection bunch. After tracking for 400,000 turns, 1656 particles were lost among 5000 initial particles, indicating an injection efficiency of approximately 66.9%. In the second scenario, the initial beam parameters were $\epsilon_x = 30.6$ nm, $\epsilon_y = 20.8$ nm, $\sigma_s = 3$ mm, $\sigma_\delta = 6 \times 10^{-4}$, where the maximum transverse size can reach 1 mm. This case represents a more realistic injection beam. After tracking for 400,000 turns, the injection efficiency was found to be 64.5%. These two cases demonstrate that the injection efficiency is primarily governed by the longitudinal bucket. The millimeter-scale bunch length used in above simulation is much larger than the width of “rf” bucket, it is hard to find more information by zooming in the bunch to μm scale, as only a few particles reside with a bucket averagely. To illustrate this point more clear, we conduct another simulation with initial bunch parameters of $\epsilon_x = 2.3$ nm, $\epsilon_y = 2$ pm, $\sigma_s = 3$ μm , $\sigma_\delta = 6 \times 10^{-4}$. The results can be found in Fig. 13. The lost particles, predominantly located outside the bucket or in close proximity to its separatrix, are marked with red circles. This presentation clearly shows that injection efficiency is mainly limited by the “rf” bucket.

The transverse emittance of this storage ring is intended to be as large as possible to reduce the IBS effect, yet remain below the diffraction limit relative to the radiation wavelength to ensure higher brightness of the light source.

Regarding collective effects in this type of storage rings, numerous corrections to the traditional formulas are necessary. These adjustments arise because the beam

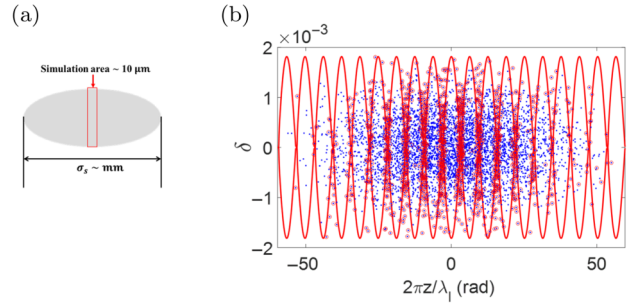


FIG. 13. A simulation was conducted to verify the injection efficiency. In (a), the red curve indicates the “rf” bucket, the blue dots show the longitudinal positions of the injected particles, and the red circles mark the particles lost after tracking for 400,000 turns. Out of a total of 5000 macro-particles, 1646 were lost, resulting in an injection efficiency of about 67.0%. The most lost particles are initially located at the outer edge or boundary of the bucket area. The practical injection bunch length would be about mm level. We use a 10 μm long bunch to illustrate that the injection loss is mostly due to the initial particle outside the longitudinal bucket, as sketched in (b).

parameters exceed the assumptions made in earlier models. For instance, the beam energy is relatively low, making the ultrarelativistic approximation invalid for calculating resistive wall wakefields [33]. Additionally, the bunch length in the straight sections is much shorter than the transverse beam size, which differs significantly from conventional beam characteristics. The bunch length varies around the ring, with the length in the arc sections reaching the order of 10 μm (comparing to the bunch length of tens of nanometer in the straight sections) due to transverse-longitudinal coupling. This variation necessitates corrections in intrabeam scattering calculations [32] and space charge computations [34,35]. Moreover, studies of coherent synchrotron radiation in these storage rings differ from those in conventional rings; further details can be found in the literature [36]. Additional research is also required on the topic of bunch length variation caused by potential well distortion, as reported in previous studies [1].

IX. CONCLUSIONS

In this paper, we have deeply discussed the dynamics and design strategy of the low-alpha storage ring with electron bunch length shorter than 100 nm. We propose a new method to reduce both the global and local momentum compaction factors in storage rings, which can be applied to the design of any quasi-isochronous storage ring. Additionally, the second and third-order momentum compaction factors have been optimized. The dynamic aperture has been improved to approximately 1.2 mm in the horizontal direction and 2 mm in the vertical direction. It is important to highlight that the dynamic aperture optimization approach presented here can also be applied to conventional storage rings, particularly in situations where the aperture shrinks due to the presence of an rf cavity. Simulations considering only single-particle effects indicate that the root-mean-square electron bunch length stored in the ring is about 80 nm, while the quantum lifetime is primarily limited by the bucket height, as demonstrated by the simulations.

ACKNOWLEDGMENTS

The authors thank Zhenhe Bai from University of Science and Technology of China for the helpful discussions. This work was supported by the Beijing Natural Science Foundation (Grant No. 1254058), the National Key R&D Program of China (Grant No. 2022YFA1603402), and the Tsinghua University Initiative Scientific Research Program.

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

- [1] I. P. S. Martin, G. Rehm, C. Thomas, and R. Bartolini, Experience with low-alpha lattices at the diamond light source, *Phys. Rev. ST Accel. Beams* **14**, 040705 (2011).
- [2] J. Feikes, M. von Hartrott, M. Ries, P. Schmid, G. Wüstefeld, A. Hoehl, R. Klein, R. Müller, and G. Ulm, Metrology light source: The first electron storage ring optimized for generating coherent thz radiation, *Phys. Rev. ST Accel. Beams* **14**, 030705 (2011).
- [3] D. Robin, E. Forest, C. Pellegrini, and A. Amiry, Quasi-isochronous storage rings, *Phys. Rev. E* **48**, 2149 (1993).
- [4] J. Feikes, in Proceedings of Partical Accelerator Conference (JACOW, Vancouver, BC, Canada, 2009).
- [5] Y. Shoji, H. Tanaka, M. Takao, and K. Soutome, *Phys. Rev. E* **54**, R4556 (1996).
- [6] P. Gladkikh, [arXiv:hep-lat/0406007](https://arxiv.org/abs/hep-lat/0406007).
- [7] E.-S. Kim, *Jpn. J. Appl. Phys.* **46**, 7952 (2007).
- [8] X. Deng, *Theoretical and Experimental Studies on Steady-State Microbunching* (Springer Nature, Berlin, Heidelberg, 2024).
- [9] Y. Zhang, X. J. Deng, Z. L. Pan, Z. Z. Li, K. S. Zhou, W. H. Huang, R. K. Li, C. X. Tang, and A. W. Chao, Ultralow longitudinal emittance storage rings, *Phys. Rev. Accel. Beams* **24**, 090701 (2021).
- [10] D. F. Ratner and A. W. Chao, *Phys. Rev. Lett.* **105**, 154801 (2010).
- [11] X. J. Deng, A. W. Chao, J. Feikes, W. H. Huang, M. Ries, and C. X. Tang, Single-particle dynamics of microbunching, *Phys. Rev. Accel. Beams* **23**, 044002 (2020).
- [12] Z. Pan, A. Chao, X. Deng, W.-H. Huang, T. Rui, C. Tang, W. Wan, and Y. Zhang, A storage ring design for steady-state microbunching to generate coherent EUV light source, SLAC National Accelerator Laboratory (SLAC), Menlo Park, CA (United States), Tech. Rep., 2019.
- [13] Y. Zhang, Research on longitudinal strong focusing SSMB ring, Ph.D. thesis, Tsinghua University, Beijing, China, 2022.
- [14] A. W. Chao, E. Granados, X. Huang, H. W. Luo, and D. F. Ratner, in Proc. of Int. Part. Accel. Conf., Busan, Korea (2016).
- [15] Z. Li, X. Deng, Z. Pan, C. Tang, and A. Chao, Generalized longitudinal strong focusing in a steady-state microbunching storage ring, *Phys. Rev. Accel. Beams* **26**, 110701 (2023).
- [16] A. W. Chao and D. F. Ratner, in Proceedings of FEL 2012 Nara Japan (JACOW, Nara, Japan, 2012).
- [17] D. F. Ratner and A. W. Chao, in Proceedings, 33rd International Free Electron Laser Conference, Shanghai, China (JACOW, Shanghai, China, 2011).
- [18] Y. Jiao, D. F. Ratner, and A. W. Chao, *Phys. Rev. ST Accel. Beams* **14**, 110702 (2011).
- [19] X.-J. Deng, A. W. Chao, W.-H. Huang, Z.-Z. Li, Z.-L. Pan, and C.-X. Tang, Steady-state microbunching based on transverse-longitudinal coupling, *Nucl. Sci. Tech.* **37**, 2 (2026).
- [20] M. Sands, SLAC Technical Report No. SLAC-121, 1970, 1970.
- [21] X. J. Deng, A. W. Chao, J. Feikes, A. Hoehl, W. H. Huang, R. Klein, A. Kruschinski, J. Li, M. Ries, and C. X. Tang, Breakdown of classical bunch length and energy spread

- formula in a quasi-isochronous electron storage ring, *Phys. Rev. Accel. Beams* **26**, 054001 (2023).
- [22] A. Chao, Slim—a formalism for linear coupled systems, *Chin. Phys. C* **33**, 115 (2009).
- [23] X. J. Deng, A. W. Chao, W. H. Huang, and C. X. Tang, Courant-Snyder formalism of longitudinal dynamics, *Phys. Rev. Accel. Beams* **24**, 094001 (2021).
- [24] Z. Pan, J. Zhao, C. Tang, X. Deng, and A. W. Chao, Isochronous lattice design with time-of-flight fluctuation reaching subattoseconds based on high-order achromat, *Phys. Rev. Accel. Beams* **28**, 124001 (2025).
- [25] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, A fast and elitist multiobjective genetic algorithm: Nsga-ii, *IEEE Trans. Evol. Comput.* **6**, 182 (2002).
- [26] M. Borland, ANL Advanced Photon Source Report No. LS-287, 2000.
- [27] E. L. Saldin, E. A. Schneidmiller, and M. V. Yurkov, *Nucl. Instrum. Methods Phys. Res. Sect. A* **539**, 499 (2005).
- [28] J. P. Duris, P. Musumeci, and R. K. Li, *Phys. Rev. ST Accel. Beams* **15**, 061301 (2012).
- [29] X.-Y. Lu, R. Chiche, K. Dupraz, A. Martens, D. Nutarelli, V. Soskov, F. Zomer, X. Liu, L.-X. Yan, W.-H. Huang *et al.*, 710 kW stable average power in a 45,000 finesse two-mirror optical cavity, *Opt. Lett.* **49**, 6884 (2024).
- [30] X.-Y. Lu, R. Chiche, K. Dupraz, F. Johora, A. Martens, D. Nutarelli, Y. Peinaud, V. Soskov, A. Stocchi, F. Zomer *et al.*, Stable 500 kW average power of infrared light in a finesse 35 000 enhancement cavity, *Appl. Phys. Lett.* **124**, 912 (2024).
- [31] A. W. Chao, *Lectures on Accelerator Physics* (World Scientific, Singapore, 2020).10.1142/12004
- [32] Z. Pan, W. Huang, C. Tang, X. Deng, and A. W. Chao, Intrabeam scattering study for longitudinal strong focusing storage rings and lattice optimization, *Phys. Rev. Accel. Beams* **28**, 114201 (2025).
- [33] J. Tang, X. Deng, Z. Pan, C. Tang, and A. Chao, Resistive wall wake for nonultrarelativistic beam in ring model, *Phys. Rev. Accel. Beams* **29**, 024202 (2026).
- [34] Z. Zhao, N. Wang, H. Xu, and Z. Pan, Calculations of space-charge tune shifts in storage rings with extremely short bunches and small bunch spacing, *Phys. Rev. Accel. Beams* **27**, 094201 (2024).
- [35] Z. Zhao, H. Xu, N. Wang, and Z. Pan, Method for calculating the longitudinal space-charge tune shift suitable for arbitrary bunch lengths, *Phys. Rev. Accel. Beams* **28**, 104201 (2025).
- [36] J. H. Bian, X. J. Deng, W. H. Huang, and C. X. Tang, Threshold studies of coherent synchrotron radiation induced microwave instability beyond adiabatic approximation, *Phys. Rev. Accel. Beams* **28**, 074401 (2025).