

Generation and manipulation of MeV subfemtosecond or attosecond electron microbunch train via THz modulation

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(Received 10 November 2025; accepted 24 June 2026; published 29 July 2026)

Femtosecond science has opened up new opportunities for probing atomic and molecular dynamics across multiple disciplines, while attosecond science further extends such capabilities to investigate the dynamical processes of electron motion in matter with unprecedented temporal resolution. In such situations, ultrashort electron beams serve as effective tools, providing complementary probing capabilities to ultrashort laser beams. In this paper, we propose a scheme that employs THz modulation to generate a mega-electron-volt (MeV) relativistic electron microbunch train. The scheme integrates the well-established undulator technology commonly used in the accelerator community with a tailored sawtooth THz field to manipulate the electron-beam phase space. To assess the feasibility of this scheme, we have developed an efficient and effective particle tracking model that incorporates the relevant physical effects, including the transport dynamics of relativistic MeV electrons interacting with the modulation field inside the undulator, as well as the beam longitudinal space charge effects. The simulation results show that, based on the proposed scheme, a microbunch train can be generated with a beam kinetic energy of approximately 3 MeV, a microbunch length of about 1.9 fs full width at half maximum, a microbunch charge of up to 2 fC, and a tunable microbunch spacing of around 0.2 mm. The output microbunch length can be controlled by tuning the downstream longitudinal dispersion, while adjustment of the upstream dispersion manipulates the microbunch structure into a periodic comblike train, each accompanied by one or more pairs of temporal satellite pulses. We believe that the proposed scheme and the analysis shall provide useful insights for generating few-femtosecond or even attosecond electron bunches.

DOI: [10.1103/mgbs-y8cd](https://doi.org/10.1103/mgbs-y8cd)

I. INTRODUCTION

The pump-probe technique enables the investigation of structural dynamics by exciting a sample with a laser pulse and probing it at controlled time delays using ultrashort electron or laser pulses. It has become a standard tool for studying the temporal evolution of matter, with mega-electron-volt (MeV) ultrafast electron diffraction (UED) playing an indispensable role [1–4]. Beyond static imaging, time-resolved UED enables direct observation of ultrafast dynamics and, with continuously improving temporal resolution, has enabled progress from femtosecond-scale molecular and atomic dynamics to attosecond-scale electron dynamics [5]. The performance of MeV

UED experiments is critically determined by the electron-beam quality, with the transverse size, energy spread, and bunch length, respectively, defining the spatial, spectral, and temporal resolutions.

To improve the temporal resolution, considerable effort has been devoted to compressing the electron bunch to achieve a shorter bunch length while minimizing the arrival-time jitter. A typical approach involves passing the electron beam through an rf structure to impart an energy modulation, which is then converted into temporal compression via velocity bunching, leading to the shortest bunch length at the sample position. Nonlinear distortions in the longitudinal phase space, arising from space charge effects and/or rf curvature, limit the minimum achievable bunch length featuring the S- or C-shaped distributions. Such nonlinearities may be mitigated by introducing higher-harmonic rf components to linearize the phase space (see, e.g., Refs. [6–8]). Beyond rf-based methods, recent advances in strong-field THz generation have motivated alternative approaches in which a THz field, produced by the same laser source that generates the photoelectrons, is used to drive subwavelength resonant or nonresonant structures for bunch compression [9–18]. The

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operating principle is analogous to that of a high-harmonic rf cavity, but with the advantage that the driving source is optically synchronized with the photocathode laser, thereby avoiding the engineering challenges and potential mismatches associated with rf-laser synchronization. With a properly designed downstream transport line, it is even possible to suppress or eliminate arrival-time jitter, further enhancing the temporal resolution. The above methods mainly target the generation of a single, isolated ultrashort electron bunch.

In addition to producing a single ultrashort bunch, the generation of a train of periodic ultrashort electron pulses with fixed intervals also offers potential applications, for example, in studying picosecond-reversible processes. Representative examples include coherent lattice oscillations in bismuth [19,20], electron-phonon coupling in thin films [21,22], order-parameter dynamics in charge-density-wave materials [23], and recovery of polarization or magnetic order in ferroelectric and antiferromagnetic materials [24]. To date, most demonstrations of ultrashort electron bunch trains have been realized in the keV energy range [25–28], while higher-energy implementations targeting coherent radiation generation have reached 14 MeV [29] and 32 MeV [30], and those aimed at compact accelerator schemes have reached 60 MeV (see, e.g., Ref. [31]). For relativistic electron diffraction applications, a typical choice of energy is around 3 to 5 MeV to effectively balance the high bunch charge, the suppression of space charge effects, and the technical demands of high-gradient acceleration. In this paper, we propose a scheme that employs THz modulation to generate a kinetic energy of 3 MeV relativistic electron microbunch train. The scheme integrates a well-established undulator technology with a tailored sawtooth THz field to manipulate the electron-beam phase space longitudinally. To quantitatively assess the feasibility of the proposed scheme, we have developed an efficient and effective particle tracking model that incorporates the relevant physical effects, including the transport dynamics of MeV relativistic electrons interacting with the modulation field inside the undulator and the longitudinal space charge (LSC) effects from the bunch itself. Our simulation results demonstrate that this scheme can produce a microbunch train with a beam kinetic energy of approximately 3 MeV, a microbunch length of about 1.9 fs (FWHM) with microbunch charge of up to 2 fC, and a tunable microbunch spacing of around 200 μm . Although these results do not represent optimized performance, the proposed scheme and analysis may provide valuable insights into the generation of few-femtosecond or even attosecond electron bunches.

While our work focuses on generating ultrashort microbunch trains in the MeV energy range, it is useful to briefly summarize some typical schemes employed for the generation of attosecond electron pulses in the keV regime. A well-established approach relies on coherent inelastic

electron-light scattering at a thin dielectric membrane, where a laser field imparts a sinusoidal longitudinal modulation of momentum onto an electron beam, and, after dispersive propagation, the beam becomes a periodic train of attosecond pulses synchronized to the optical cycle (see, e.g., Ref. [25]). This scheme has been demonstrated both with femtosecond laser sources in the ultrafast transmission electron microscope (TEM) community, e.g., yielding pulse durations of 350 as (rms) at 70 keV [28], and with continuous-wave laser illumination of a conventional 120 keV TEM, achieving about 0.9 fs (rms) pulses at near-full source brightness [32]. For a recent review, the interested reader may be referred to Ref. [33] and references therein. Here we note that all the above-mentioned keV attosecond electron schemes operate in the single-electron regime (in fact, fewer than one electron per microbunch), so there will be no space charge effect. While the single-electron regime is well-suited for quantum-coherent measurements and near-field imaging, high-charge operation at the fC level as targeted in MeV-range schemes is indispensable for single-shot structural dynamics studies and applications demanding high signal-to-noise ratio per pulse.

The scheme proposed in the present work draws conceptual inspiration from high-gain harmonic generation (HG) developed in the free-electron laser (FEL) community, yet addresses a fundamentally different physical regime. To the best of our knowledge, the strategy of first longitudinally stretching the bunch via a chicane and subsequently compressing only a localized portion of the beam core through THz-induced modulation (more detailed explanation in Sec. II or see Fig. 1) has not been previously proposed for low-energy electron beams in the context of UED applications. This “first stretch, then compress” idea offers a distinct advantage over conventional monotonic bunch compression schemes, in which the bunch length undergoes continuous progressive compression along the beamline and space charge effects accumulate accordingly throughout transport. By maintaining the bunch in a stretched state until the final compression stage, the present scheme substantially mitigates space charge effects, albeit at the cost of sacrificing particles in the head and tail regions. A further distinguishing feature is the targeted operation in the single-MeV energy regime, where space charge effects during beam transport are non-negligible and represent a significant perturbation to the longitudinal phase space dynamics. Moreover, unlike high-energy FELs, where conventional optical lasers can be used directly for beam modulation and space charge effects are largely negligible, the present scheme requires THz modulation fields, consideration of the space charge repulsion, and THz phase space manipulation, an aspect that has not been studied in detail using currently available simulation tools.

This paper is organized as follows: In Sec. II, we provide a qualitative description of the proposed scheme,

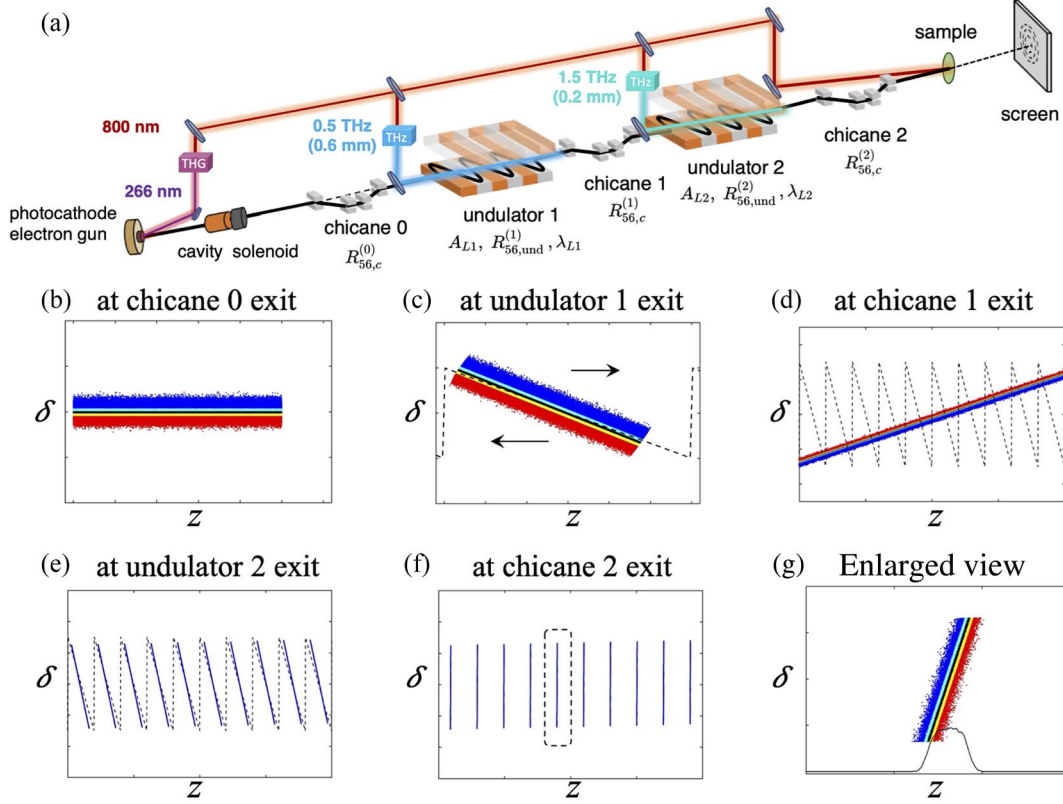


FIG. 1. (a) Schematic layout of a MeV UED facility incorporating the proposed scheme for generating an ultrashort microbunch train (see main text for further description); (b)–(f) show the bunch phase space distributions at different locations along the beamline; (g) presents an enlarged view of panel (f). To better illustrate the phase space distributions, particles are color-coded according to their initial energy deviation: the core region is marked black, intermediate particles are yellow or green, and those farther from the core are red or blue. In panels (c)–(e), the dashed line represents the sawtooth waveform of THz modulation. Note that in panel (a), the schematic is not drawn to scale: the distance from the photocathode to chicane 0 is approximately 0.5 m; the lengths of the three chicanes are roughly 0.3, 0.45, and 0.2 m; and the lengths of the first and second undulators are approximately 0.18 and 0.09 m, respectively. In panel (c), the two arrows indicate that the bunch phase space distribution will shear to both sides due to $R_{56,c}^{(1)} > 0$ in chicane 1. The THz waveform synthesizer (sinusoidal-to-sawtooth converter) is not shown in the figure.

including an overview of the system layout. Section III describes the electron-beam transport at the entrance and exit of several key components of the setup, upon which a particle tracking model is constructed. The theoretical model incorporates the beam space charge field, enabling the study of charge-dependent effects. Because the proposed scheme relies on THz modulation, the feasibility of the required THz field and the undulator design parameters is briefly discussed in Sec. IV. In Sec. V, we present an example, including preliminary design parameters, and analyze the phase space dynamics of the microbunch train during modulation and compression. We also demonstrate that the proposed scheme can be used to manipulate the phase space distribution. By adjusting the upstream longitudinal dispersion, the generated microbunch structure can be tailored to exhibit a periodic comb structure consisting of a train of microbunches, each accompanied by one or more pairs of temporal satellite pulses. Further tuning of the downstream longitudinal dispersion allows control

over the output microbunch length. The conclusions of this work are summarized in Sec. VI.

II. DESCRIPTION OF THE SCHEME

The schematic layout for generating a microbunch train is illustrated in Fig. 1. First, an 800 nm laser pulse is frequency-tripled to 266 nm and directed onto a photocathode to generate photoelectrons. Using a commercially available 800-nm Ti:sapphire laser with millijoule-level pulse energy, it is straightforward to produce electron bunches with a charge on the order of tens of picocoulombs. Taking the HUST MeV UED facility as an example [34], the electron beam is accelerated by a 1.4-cell, 2856-MHz rf gun to a kinetic energy of approximately 3 MeV. After exiting the photocathode gun, a bunch length of $0.5 \sim 1$ ps is typically obtained. In the following discussion, unless otherwise specified, we assume an initial bunch charge of 0.1 pC and a full bunch duration of 1 ps at

the entrance of the chicane 0. For simplicity, we assume a uniform flat-top charge-density distribution, which can be realized by appropriately shaping the laser temporal profile on the photocathode. The chicane 0 (stretcher) shown in Fig. 1(a) can be considered optional: if the bunch length at the entrance of the chicane 0 is insufficient or if an energy-position correlation exists, the chicane 0 can provide additional longitudinal dispersion to stretch the bunch; otherwise, it can be bypassed for straight-through beam transport. The stretched longitudinal phase space distribution at the exit of chicane 0 is shown in Fig. 1(b).

Next, the bunch enters the first undulator, where a copropagating THz field is used to modulate the electron beam. To achieve a stronger and more efficient modulation than that produced by a sinusoidal waveform, a sawtooth-shaped THz field is adopted in this scheme [see the dashed lines in panels (c) and (e)]. Such a waveform can be realized using techniques such as chirped-pulse shaping, multicolor pumping, or waveform synthesis. Although the detailed generation of this waveform is beyond the scope of this paper, we assume here that the waveform synthesis technique is used; its feasibility and requirements on the seed laser will be discussed in Sec. IV. The longitudinal phase space distribution after modulation by the sawtooth THz waveform is shown in Fig. 1(c), where an energy chirp across the bunch can be clearly observed.

After exiting the undulator 1, the electron beam passes through the chicane 1, which provides a longitudinal dispersion to further stretch the bunch, as shown in Fig. 1(d). It should be noted that after this stretching process, the charge density of the bunch is significantly reduced, by approximately one order of magnitude, depending on the stretching factor, which helps mitigate the space charge effects during beam transport. A quantitative discussion of this effect will be given in Sec. V. The electron bunch then continues to the undulator 2, where it is modulated by a second THz field. Since the bunch has already been longitudinally extended, it will now be modulated or “chopped” into multiple segments (beamlets), as illustrated in Fig. 1(e). The wavelength of the second THz modulation directly determines the spacing between the resulting microbunches. Finally, by properly adjusting the dispersion strength of the chicane 2, the modulated beamlets can be compressed downstream to form the desired microbunch train, as shown in Fig. 1(f). Figure 1(g) presents an enlarged view of the dashed rectangular region in Fig. 1(f), together with the corresponding line density profile.

The above qualitative description neglects the intrinsic dispersion of the undulator. For the few-MeV electrons, the undulator itself introduces non-negligible dispersion that must be taken into account. In the next section, we will quantitatively analyze the beam transport dynamics of this scheme, including both the undulator and chicane sections, as well as the possible effects arising from the bunch self-field.

III. PARTICLE TRACKING EQUATIONS AND SPACE CHARGE FIELD CALCULATION

A. Chicane 0

Since the proposed scheme involves only the longitudinal dimension, our analysis focuses solely on the longitudinal dynamics. Starting from the entrance of chicane 0, we write the longitudinal phase space state vector as $[z \ \delta]^T$, where z denotes the bunch coordinate ($z > 0$ for the bunch head and $z < 0$ for the bunch tail), and $\delta = (E - E_0)/E_0$ represents the fractional energy deviation. According to the standard accelerator theory (see, e.g., Refs. [35,36]), under linear transport, the particle phase space coordinates at the exit of the chicane $[z_1 \ \delta_1]^T$ can be expressed in terms of those at the entrance $[z_0 \ \delta_0]^T$ as follows

$$\begin{aligned} z_m &= z_0 + \frac{1}{2} R_{56,c}^{(0)} \delta_0 \\ \delta_1 &= \delta_0 + \delta_{\text{LSC}}(z_m, \delta_0), \\ z_1 &= z_m + \frac{1}{2} R_{56,c}^{(0)} \delta_1 \end{aligned} \quad (1)$$

where the subscript 0 refers to quantities at the entrance of the chicane 0 and the subscript 1 corresponds to those at the exit. Here, $R_{56,c}^{(0)}$ denotes the longitudinal dispersion strength of the chicane 0. To account for the effect of the LSC field, we adopt the drift-kick-drift algorithm commonly used in particle tracking simulations. In this method, the particles first drift through half the chicane length, after which the LSC-induced energy change is calculated for each particle, denoted by δ_{LSC} . Here, z_m represents the intermediate longitudinal position coordinate. Finally, using the updated energy coordinate, the local bunch coordinate at the exit of chicane 0 is obtained. The LSC-induced energy deviation δ_{LSC} is given by

$$\delta_{\text{LSC}} = -\frac{eE_z(z_m)L_{c,0}}{\gamma_0 mc^2}, \quad (2)$$

where $L_{c,0}$ is the total length of the chicane 0, E_z is the longitudinal electric field exerted at the position z_m , e is the elementary charge, $\gamma_0 = E_0/(mc^2)$ is the Lorentz relativistic factor in units of the electron rest energy ($mc^2 \approx 0.511$ MeV), m is the electron rest mass, and c is the speed of light in vacuum. The detailed calculation of the longitudinal space charge field E_z will be discussed in Sec. III F.

B. Undulator 1

After exiting the chicane 0, the electrons enter the transport line of our proposed scheme. First, the electron beam copropagates with an externally injected THz wave through a short undulator section. For a few-MeV electron beam, the intrinsic dispersion of the undulator is

non-negligible. According to Refs. [35,36], the longitudinal dispersion of an undulator can be expressed as $R_{56,\text{und}} = 2N_u\lambda_L$, where N_u is the number of undulator periods and λ_L is the resonant wavelength that satisfies the undulator resonance condition, corresponding to the THz modulation wavelength (to be discussed in detail later). To account for the space charge effect during transport, rather than treating the undulator as a single component as is commonly done in accelerator textbooks, we divide it and perform particle tracking on a period-by-period basis. In this way, each undulator period contributes a longitudinal dispersion of $R_{56,p} = \frac{R_{56,\text{und}}}{N_u}$. Similarly, if we denote the total modulation amplitude imparted to the electron beam by the THz wave as A_L , then the modulation amplitude within each undulator period is $A_{L,p} = \frac{A_L}{N_u}$.

We can now describe the particle transport through the undulator 1. Assigning the phase space coordinates at the exit of the chicane 0 as the undulator entrance coordinates at $p = 0$ here, we have $[z_{p=0} \ \delta_{p=0}] = [z_1 \ \delta_1]$, with p referring to the index of undulator periods. The phase space coordinates at the entrance and exit of the p th period are then given by

$$\begin{aligned} z_{p+1/2} &= z_p + \frac{1}{2}R_{56,p}^{(1)}\delta_p \\ \delta_{p+1} &= \delta_p - A_{L1,p}\text{saw}(k_{L1}z_{p+1/2}) + \delta_{\text{LSC}}(z_{p+1/2}, \delta_p), \\ z_{p+1} &= z_{p+1/2} + \frac{1}{2}R_{56,p}^{(1)}\delta_{p+1} \end{aligned} \quad (3)$$

where $A_{L1,p}$ denotes the modulation amplitude experienced by the electrons within each undulator period. The modulation wave number is $k_{L1} = 2\pi/\lambda_{L1}$, with λ_{L1} the THz modulation wavelength in the first undulator. The modulation waveform is taken to be a sawtooth function, whose functional form can be written as

$$\text{saw}(kz) = \frac{2}{\pi} \sum_{h=1}^{\infty} (-1)^h \frac{\sin(hkz)}{h}. \quad (4)$$

The sawtooth function is similar to a sine wave but generates a waveform with peaks of -1 and 1 . In Eq. (3), $R_{56,p}^{(1)}$ represents the longitudinal dispersion strength per period in undulator 1. The subscript p refers to the phase space coordinates at the entrance of the p th period, while the subscript $p + 1/2$ denotes intermediate quantities (analogous to the previously defined z_m), and the subscript $p + 1$ refers to the coordinates at the exit of the p th period or the entrance of the $(p + 1)$ th period.

According to Ref. [37], the total THz modulation amplitude in a modulator can be written as

$$A_L = \frac{eK_u[\text{JJ}]}{\gamma_0^2 mc^2} \sqrt{\frac{4P_L Z_0}{\pi w_0^2}} \frac{L_u}{2}, \quad (5)$$

where $K_u \approx 0.934B_0[\text{T}]\lambda_u[\text{cm}]$ is the dimensionless undulator parameter with B_0 the peak magnetic field in units of Tesla and λ_u the undulator period in units of cm, P_L is the THz modulation power, $Z_0 \approx 377 \ \Omega$ is the free-space impedance, w_0 is the THz beam waist, and $L_u = N_u\lambda_u$ is the length of the undulator. Here we assume a planar undulator, for which the coupling coefficient is

$$[\text{JJ}] = J_0\left(\frac{K_u^2}{4 + 2K_u^2}\right) - J_1\left(\frac{K_u^2}{4 + 2K_u^2}\right), \quad (6)$$

with $J_{0,1}$ the zeroth- and first-order Bessel functions of the first kind. In general, for the cases considered below, $[\text{JJ}] \approx 0.99$. For a helical undulator, $[\text{JJ}] = 1$, although such undulators are not considered here. We note that Eq. (5) employs the plane-wave approximation, assuming that diffraction effects of the THz field during its copropagation with the electron beam in the undulator are negligible. This approximation is reasonably valid when the undulator is sufficiently short. Similar to Eq. (2), it can be expected that in Eq. (3) the longitudinal space charge contribution accumulated in each undulator period is

$$\delta_{\text{LSC}} = -\frac{eE_z(z_{p+1/2})\lambda_{u1}}{\gamma_0 mc^2}. \quad (7)$$

Following the mapping defined in Eq. (3), the phase space coordinates at the exit of the undulator 1 can be written as $[z_2 \ \delta_2] = [z_{p=N_{u1}} \ \delta_{p=N_{u1}}]$, with N_{u1} denoting the total number of periods in the first undulator. The corresponding beam phase space distribution is schematically illustrated in Fig. 1(c).

C. Chicane 1

After exiting the undulator 1, the particles will enter chicane 1, at which the entrance phase space coordinates are expressed as $[z_2 \ \delta_2]$. The beam transport through chicane 1 can be described as

$$\begin{aligned} z_m &= z_2 + \frac{1}{2}R_{56,c}^{(1)}\delta_2 \\ \delta_3 &= \delta_2 + \delta_{\text{LSC}}(z_m, \delta_2). \\ z_3 &= z_m + \frac{1}{2}R_{56,c}^{(1)}\delta_3 \end{aligned} \quad (8)$$

Here, the subscript m denotes intermediate quantities, and the phase space coordinates at the exit of chicane 1 are written as $[z_3 \ \delta_3]$. The expression of the LSC term is similar to that in Eq. (2), except that $L_{c,0}$ is now replaced by $L_{c,1}$. According to the mapping relation in Eq. (8), the phase space distribution of the bunch at the exit of the chicane 1 is similar to that shown in Fig. 1(d). When $R_{56,c}^{(1)} > 0$ increases, the bunch becomes more stretched longitudinally.

D. Undulator 2

Stretched by chicane 1, the beam then enters the undulator 2, where the entrance coordinates are expressed as $[z_{p=0} \ \delta_{p=0}] = [z_3 \ \delta_3]$. The period-by-period mapping equations can be written as

$$\begin{aligned} z_{p+1/2} &= z_p + \frac{1}{2} R_{56,p}^{(2)} \delta_p \\ \delta_{p+1} &= \delta_p - A_{L2,p} \text{saw}(k_{L2} z_{p+1/2}) + \delta_{\text{LSC}}(z_{p+1/2}, \delta_p), \\ z_{p+1} &= z_{p+1/2} + \frac{1}{2} R_{56,p}^{(2)} \delta_{p+1} \end{aligned} \quad (9)$$

with $k_{L2} = 2\pi/\lambda_{L2}$, λ_{L2} being the THz modulation wavelength in the second undulator, $R_{56,p}^{(2)} = R_{56,\text{und}}^{(2)}/N_{u2}$, and $A_{L2,p} = A_{L2}/N_{u2}$. The form of the LSC term here is similar to that in Eq. (7), except that λ_{u1} is replaced by λ_{u2} . According to the mapping in Eq. (9), the phase space coordinates at the exit of the undulator 2 can be expressed as $[z_4 \ \delta_4] = [z_{p=N_{u2}} \ \delta_{p=N_{u2}}]$, with N_{u2} the total number of periods in the second undulator. The phase space distribution of the bunch at this stage is similar to that shown in Fig. 1(e). After modulation, the bunch has been chopped into discrete beamlets, exhibiting a distribution pattern approximately following the sawtooth waveform.

E. Chicane 2

After being modulated by undulator 2, the segmented beamlets enter chicane 2. Assuming the entrance phase space coordinates are $[z_4 \ \delta_4]$, the transport through the chicane 2 can be described by

$$\begin{aligned} z_m &= z_4 + \frac{1}{2} R_{56,c}^{(2)} \delta_4 \\ \delta_5 &= \delta_4 + \delta_{\text{LSC}}(z_m, \delta_4), \\ z_5 &= z_m + \frac{1}{2} R_{56,c}^{(2)} \delta_5 \end{aligned} \quad (10)$$

resulting in the final phase space coordinates at the end of the transport line, $[z_5 \ \delta_5]$. The LSC term is similar to that in Eq. (2), except that $L_{c,0}$ is replaced by $L_{c,2}$. According to the mapping in Eq. (10), the phase space distribution of the beamlets at the exit of the chicane 2 is similar to that shown in Fig. 1(f). When $R_{56,c}^{(2)} > 0$ is appropriately chosen, the beamlets become vertically aligned in the phase space, resulting in a series of sufficiently short microbunches.

From the above subsections, we have derived the mapping relations for particle transport through each beamline element. These relations will be used as the particle tracking equations in the example presented in Sec. V. Before that, however, we will describe how to numerically evaluate the LSC term δ_{LSC} .

F. Calculation of the longitudinal space charge field

We now describe how to evaluate the LSC field within the bunch. According to the Lorentz transformation, the static Coulomb field in the beam rest frame can be converted into its corresponding form in the laboratory frame [38]

$$E_z(z) = \frac{Q}{4\pi\epsilon_0\gamma_0^2} \int_{-\infty}^{\infty} \rho(\zeta) \frac{z - \zeta}{|z - \zeta|^3} d\zeta, \quad (11)$$

where Q is the total bunch charge and $\rho(z)$ denotes the line charge density, satisfying the normalization condition $\int_{-\infty}^{\infty} \rho(\zeta) d\zeta = 1$. It is evident that Eq. (11) diverges as $\zeta \rightarrow z$, which is an inherent consequence of the one-dimensional line-charge model. To avoid this issue, we introduce a finite transverse beam size by assuming a uniform transverse charge distribution with the radius a , so that Eq. (11) can be rewritten as

$$\begin{aligned} E_z(z) &= -\frac{Q}{4\pi\epsilon_0} \frac{2}{a^2} \int_{-\infty}^{\infty} \rho(\zeta) (z - \zeta) \\ &\quad \times \left(\frac{1}{\sqrt{\frac{a^2}{\gamma_0^2} + (z - \zeta)^2}} - \frac{1}{|z - \zeta|} \right) d\zeta. \end{aligned} \quad (12)$$

In this form, when ζ is close to z , the integrand no longer exhibits a singularity. The numerical computation of the integral Eq. (12) is briefly described as follows. First, the randomly generated macroparticles are tracked through the transport governed by Eqs. (1), (3), (8), (9), and (10). Before calculating the line charge density $\rho(z)$ required for computing E_z , all simulation particles are binned into a histogram with a sufficiently large number of bins (while converging the results). The obtained $\rho(z)$ must also satisfy the normalization condition. In our simulations, all particles within the same bin z_j experience the same longitudinal field $E_z(z_j)$ contributed from all other neighboring bins. We note that for cases such as the region between chicane 1 and undulator 2, where the bunch becomes significantly stretched, a large number of bins may be required, leading to substantial computational load. To handle this, we set a minimum resolved bin size, e.g., 0.1 fs, which is approximately one-tenth of the shortest microbunch length in our demonstration example, to ensure adequate resolution of the space charge calculation. However, if the number of bins becomes excessively large that the average number of particles per bin drops below a threshold (e.g., fewer than 50 particles), we cap the total number of bins at a fixed upper limit. Generally, when the bunch is highly stretched, the LSC effect becomes less significant, and this situation does not raise physical concern.

Having now established the particle tracking equations incorporating the LSC effect, we shall proceed in Sec. V to

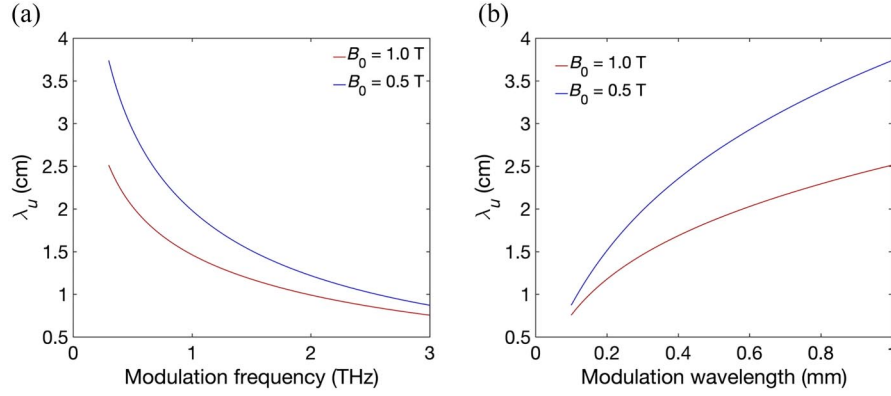


FIG. 2. Dependence of the undulator period on the resonant modulation frequency in units of THz (a) or wavelength in units of mm (b), determined by the resonance condition. Here the electron kinetic energy is assumed to be 3 MeV.

perform numerical simulations. Starting from an initial beam phase space distribution, we track its phase space evolution through the sequence of undulators and chicanes and investigate how various physical parameters influence the performance of microbunch generation and compression.

IV. DISCUSSION OF UNDULATOR PARAMETERS AND LASER REQUIREMENTS

In this section, we discuss the choice of undulator parameters and laser requirements. First, to ensure effective interaction between the electron beam and the THz modulation, several relevant physical parameters of the two undulators must satisfy the resonance condition with the THz modulation wavelength, i.e.,

$$\lambda_L = \lambda_{\text{THz}} = \frac{\lambda_u}{2\gamma_0^2} \left(1 + \frac{K_u^2}{2} \right), \quad (13)$$

where $K_u \approx 0.934 B_0 [\text{T}] \lambda_u [\text{cm}]$. Assuming the electron total energy of 3.511 MeV (or kinetic energy of 3 MeV) and an undulator peak magnetic field of $B_0 = 0.5$ T, the relationship between the undulator period and the modulation frequency or wavelength is shown by the blue curves in Fig. 2; the red curves correspond to $B_0 = 1$ T. In general, the undulator periods on the order of 1 ~ 5 cm are technologically mature, for example, using permanent magnets with on-axis fields of approximately 0.5 ~ 1 T, with typical magnet gaps of 5 ~ 10 mm and field uniformity reaching the 10^{-3} level.

Beyond the undulator parameters, we briefly discuss the requirements and potential challenges for generation of THz fields. We consider THz generation from a Ti:sapphire 800-nm femtosecond laser via optical rectification in LiNbO₃ employing the tilted-pulse-front technique [39,40]. In this configuration, the pulse-front tilt compensates for the velocity mismatch between the optical and THz waves in the crystal, enabling efficient energy

transfer in the 0.3 ~ 2 THz spectral range. According to the example that we will demonstrate in Sec. V, the required THz pulse energy is approximately 2.5 μJ for the 0.5-THz case and 160 μJ for the 1.5-THz case. Based on our estimates, the former is readily achievable, while the latter is at the edge of feasibility. The spectral and temporal characteristics of the emitted THz field are primarily determined by the pump pulse duration, crystal temperature, and angular tilt precision. To extend the waveform to multiple oscillation cycles, one must tailor the pump pulse train or adjust the phase matching geometry to produce a spectrally narrower THz output. For the 0.5-THz modulation, a typical conversion efficiency on the order of 0.5% can be expected for multicycle emission [41–43], yielding a THz pulse energy of a few μJ from an incident 800-nm optical pulse of several mJ. The waveform-shaping stage, using nonlinear crystals, interferometric delay stacks, or programmable phase modulators, plays a critical role in defining the final sawtooth profile but inevitably limits overall efficiency. The sawtooth waveform can be approximated by introducing controlled phase shifts among successive THz frequency components. When waveform shaping is applied to synthesize a sawtoothlike temporal profile, e.g., via programmable phase modulation or dielectric multilayer filters, an additional ~30% loss may occur [44,45], leading to an estimated input laser requirement of 1.5 mJ.

For the 1.5-THz modulation, the higher frequency favors a shorter optical-to-THz interaction length but suffers more from phonon absorption in LiNbO₃ [43]. Compared to the 0.5-THz case, the achievable spectral bandwidth is broader, and the waveform-shaping stage must compensate by combining several narrowband components or by imposing a controlled phase chirp on the THz pulse. The optical-to-THz conversion efficiency in this frequency range is lower than that for the 0.5-THz case, assumed to be ~0.1% [43]. Assuming similar waveform-shaping losses, if the required final pulse energy is 160 μJ , sufficient for our application, an input laser energy

of 0.54 J would be needed. This may be commercially attainable but close to the limit of current Ti:sapphire laser technology, typically requiring multistage chirped-pulse amplification and power amplification to achieve such performance. In summary, these THz requirements are physically achievable for both 0.5 and 1.5 THz.

The requirements for THz modulation power can also be mitigated in other ways. From Eq. (5), the modulation amplitude is proportional to the undulator length and to the square root of the THz modulation power. Therefore, to achieve the same modulation amplitude, a longer undulator can reduce the required THz power. However, longer undulators, or a larger number of undulator periods, also imply stronger longitudinal dispersion ($R_{56,\text{und}} \propto N_u$). As will be seen in the demonstrated example, there exists an upper limit for the longitudinal dispersion of the chicane 2 to achieve a minimum microbunch length. Therefore, to achieve an ultrashort microbunch train, one must appropriately select the number of undulator periods (hence the undulator length), while evaluating whether the required modulation power is compatible with current engineering capabilities. Here we do not perform a full multiparameter optimization of the overall design but rather provide rough estimates of relevant physical quantities and demonstrate the feasibility of the proposed scheme.

V. NUMERICAL SIMULATIONS

A. Example

In this subsection, we apply our developed particle tracking model to demonstrate a scheme for generating a microbunch train and to explore its phase space dynamics. Table I lists the preliminary design parameters. Using the HUST MeV UED [34,46,47] as an example, a 1.4-cell 2856-MHz rf gun is driven by a 266-nm UV laser illuminating a copper cathode to generate photoelectrons. The rf cavity provides an accelerating gradient of 76.6 MV/m, and by choosing an appropriate rf phase, the electron beam is accelerated to a kinetic energy of 3 MeV. At the entrance of the chicane 0 [see Fig. 1(a)], we assume that the electron beam has a full duration of approximately 1 ps, a charge of 0.1 pC, and a transverse size of 0.5 mm. It is worth briefly discussing the feasibility of some relevant parameters. According to Table I, with an undulator parameter $K_{u1} \approx 1.37$, the corresponding transverse wiggling amplitude of the electron beam is approximately $\pm K_{u1}/\gamma k_{u1} \approx \pm 1$ mm, with $k_{u1} = 2\pi/\lambda_{u1}$. Considering the effective transverse beam size, taken as $\pm 3\sigma_{\perp} \sim \pm 1.5$ mm and combining the two contributions in quadrature, the total transverse extent is about 1.8 mm (half-range), smaller than the THz beam waist, indicating that the effect from transverse variations of the modulation field can be neglected. The line density and phase space distributions of the beam after successive undulators and chicanes are shown in Figs. 3 and 4, respectively.

TABLE I. The initial electron-beam parameters and the undulator and chicane parameters relevant to the THz modulation used in the simulations.

Name	Value	Unit
<i>Initial electron-beam parameters</i>		
Electron beam energy	3.511	MeV
Initial bunch charge	0.1 (may vary)	pC
Initial bunch length (uniform, full duration)	1, 0.3	ps, mm
Initial rms energy spread	10^{-3}	
Transverse beam size	0.5	mm
<i>THz modulator parameters</i>		
Resonant wavelength (1, 2)	0.6, 0.2	mm
Resonant frequency (1, 2)	0.5, 1.5	THz
Modulation power (1, 2)	0.2, 27	MW
Required THz pulse energy (1, 2)	2.6, 108	μJ
THz waist size	0.5	cm
Undulator period (1, 2)	2.9, 1.5	cm
Undulator peak magnetic field	0.5	T
Undulator parameter (1, 2)	1.37, 0.7	
Number of undulator periods	6	
Chicane 0 dispersion strength $R_{56,c}^{(0)}$	15	cm
Chicane 1 dispersion strength $R_{56,c}^{(1)}$	42.3	cm
Chicane 2 dispersion strength $R_{56,c}^{(2)}$	0.25	cm

Before performing particle tracking, we randomly generate 6.25×10^5 electrons according to the initial beam parameters in Table I, corresponding to a 0.1 pC bunch charge. Figures 3(a) and 3(c) show the line density and beam phase space distributions at the exit of chicane 0 (gray) and at the exit of undulator 1 (black or color dots). The initial bunch length of 0.3 mm (1 ps) is slightly stretched to about 0.5 mm due to the nonzero $R_{56,c}^{(0)}$. To better illustrate the phase space distributions, particles in the beam at the undulator exit are color-coded according to their energy deviation: the core region is black, intermediate particles are yellow or green, and those farther from the core are red or blue. After the undulator 1, the beam is modulated by the THz field. The first THz modulation wavelength is chosen to be relatively long, and a sawtooth waveform is employed to make the energy-position correlation of electrons more linear than with the conventional sinusoidal wave, which facilitates downstream phase space manipulation. The beam then enters chicane 1, where a larger $R_{56,c}^{(1)}$ stretches it into a longitudinally extended bunch, reducing the bunch charge density. The corresponding line density and beam phase space distributions are shown in Figs. 3(b) and 3(d) as gray lines and dots. Besides the about tenfold reduction in line density, the slice energy spread is also reduced due to conservation of the phase space area (Liouville theorem), further aiding subsequent phase space manipulation. The

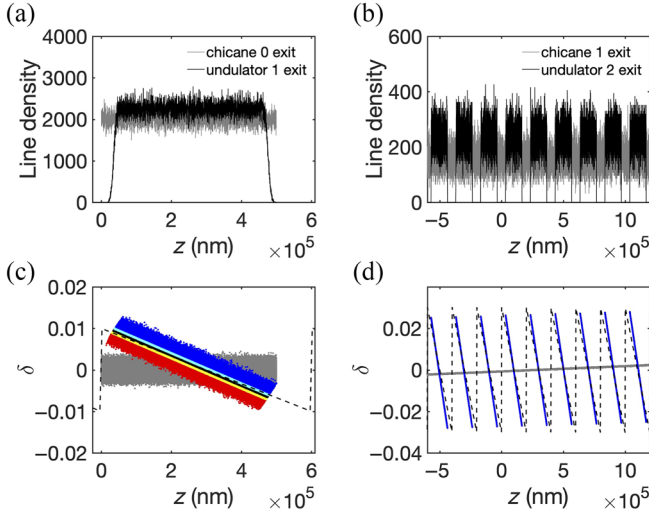


FIG. 3. (a),(c) Line density and phase space distributions at the exit of the chicane 0 (gray) and of the undulator 1 (black), with initial bunch charge of 0.1 pC and initial bunch length set as 1 ps (uniform flattop); (b),(d) line density and phase space distributions at the exit of the chicane 1 (gray) and of the undulator 2 (black). The dashed lines in (c),(d) indicate the sawtooth waveform of the THz modulation in the undulator 1 and undulator 2 with different wavelengths. Longitudinal space charge fields have been included in the simulation.

stretched beam then enters the undulator 2 for the second THz modulation, as shown by the black line in Fig. 3(b) and colored dots in Fig. 3(d) for the line density and the phase space distribution, respectively. Although it may visually appear predominantly blue, close-up inspection reveals that particles of other colors are still present, i.e., they are closely spaced, like Fig. 1(g). At this stage, the beam is effectively chopped into nearly separated beamlets.

After leaving the undulator 2, the beamlets enter the final chicane 2. Due to $R_{56,c}^{(2)}$, the beamlets experience slight phase space shear. With a proper choice of $R_{56,c}^{(2)}$, the beamlets become vertically aligned, and their projected line density becomes very sharp, as shown in Figs. 4(a) and 4(c). Here we note that these simulations have included LSC effects. For the case of the initial 0.1 pC bunch charge, turning off LSC produces results only slightly different from those in Figs. 3 and 4, since the charge density is relatively low during most of the transport, and weak space charge fields induce only minor distortions. Our analysis shows that a single microbunch contains the electron charge of roughly 2 fC, about 2% of the total charge (for more details, see later discussion). Here the microbunch charge is quantified as the charge contained within the FWHM region. Figures 4(b) and 4(d) display the line density profile and phase space distribution of a single compressed microbunch. The beamlet's phase space is nearly vertical, with a compressed bunch length of about 600 nm or 2 fs (FWHM) or 850 as (rms) by a factor of $2\sqrt{2 \ln 2} \approx 2.354$.

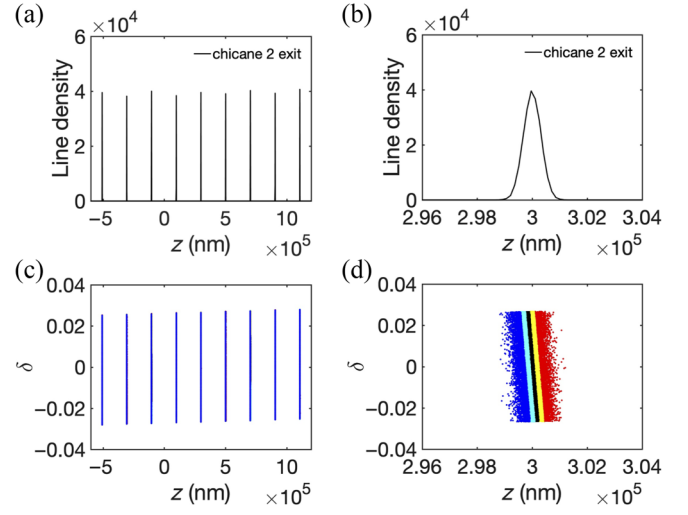


FIG. 4. (a),(c) Line density and phase space distributions at the exit of the chicane 2; (b),(d) enlarged views of panels (a) and (c). Longitudinal space charge fields have been included in the simulation.

It can be seen from Fig. 4(d) that the maximum energy deviation for the microbunch may reach $\sim 2\%$. Although prior work has indicated that energy spreads of 0.1% produce negligible diffraction degradation [1], recent successful UED experiments have been demonstrated using beams from a laser-wakefield accelerator carrying energy spreads of several percent [48]. An energy spread of $\sim 2\%$ may therefore be acceptable for some UED experiments, though applications requiring stringent energy resolution may benefit from additional beam manipulation techniques, e.g., excess energy spread may be reduced through collimation in an upstream dispersive section [37].

Although the theoretical model quantifies the dispersion strengths of the three chicanes using only $R_{56,c}^{(0)}$, $R_{56,c}^{(1)}$, and $R_{56,c}^{(2)}$, one may wonder how to implement the specific beamline of these chicanes. We consider constructing the chicanes using three or four dipole magnets to form a transverse achromatic transport line. Such chicanes are commonly used in the FEL community and are typically used to generate kilo-ampere electron bunches. We adopt and extend the results of Ref. [49] to construct the design parameters for the chicanes. First, using the standard transfer matrix method in accelerator physics, the optics functions of a symmetric three- or four-dipole chicane can be analytically obtained (see Ref. [49] for details). At the chicane exit, one finds $R_{16,c} = 0$, $R_{51,c} = 0$, $R_{52,c} = 0$, and

$$\begin{aligned}
 R_{56,c} &= \frac{2L_b^2}{\rho_0^2} \Delta L + \frac{2(\Delta L + 2L_b)}{\gamma_0^2} \\
 &= 2\Delta L \left(\frac{L_b^2}{\rho_0^2} + \frac{1}{\gamma_0^2} \right) + \frac{4L_b}{\gamma_0^2}, \quad (14)
 \end{aligned}$$

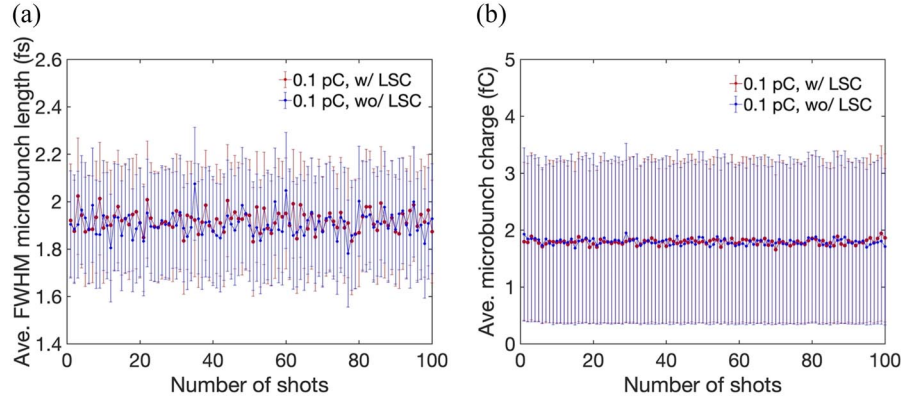


FIG. 5. (a) The average (dot) and standard deviation (error bar) of the final microbunch lengths and (b) the average (dot) and standard deviation (error bar) of the individual microbunch charges at the exit of chicane 2 for a total of 100 shots with $\pm 3\%$ (rms) fluctuating amplitude of the THz power around 0.2 and 27 MW and $\pm 3\%$ (rms) charge fluctuation around 0.1 pC. In the simulation, blue dots and curves refer to the case without space charge; red dots and curves include the longitudinal space charge fields. The two cases do not deviate too much. For the case with the longitudinal space charge, the statistically averaged results over 100 shots: the microbunch length is approximately 1.92 ± 0.233 fs (FWHM); the microbunch charge is about 1.79 ± 1.42 fC.

where ρ_0 and L_b are the bending radius and length of a single dipole magnet, respectively, and ΔL is the distance between the first and second (middle) dipoles, the same as the distance between the middle and third (last) dipoles. This expression applies to symmetric three-dipole chicanes, assuming the bending angle of the middle dipole is twice that of the first and last, or to symmetric four-dipole chicanes, assuming that the distance between the middle two dipoles is much smaller than ΔL . Once the required $R_{56,c}$ is known, ΔL can be calculated

$$\Delta L = \frac{R_{56,c} - \frac{4L_b}{\gamma_0^2}}{2\left(\frac{L_b^2}{\rho_0^2} + \frac{1}{\gamma_0^2}\right)}. \quad (15)$$

Therefore, the total length of the chicane is $2\Delta L + 4L_b$. For this preliminary design, we assume all bending magnets have $L_b = 5$ cm, with $\rho_{c0} = \rho_{c1} = 4$ cm for the chicane 0 and the chicane 1, and $\rho_{c2} = 7.8$ cm for the chicane 2. The lengths of the three chicanes are therefore 0.28, 0.45, and 0.2 m, respectively. Up to this point, we have demonstrated the proposed scheme using two undulators and three chicanes to generate a microbunch train, compressing an initial 1-ps, 0.1-pC bunch into ~ 2 -fs, 2-fC beamlets, containing about 56 microbunches with 0.2 mm microbunch spacing. The microbunch spacing is determined by the wavelength of the second THz modulation.

We then investigate the robustness of the scheme and the shot-to-shot fluctuations of the microbunch train when certain systematic errors are present. Using a Monte Carlo simulation, we model 100 independent shots. For each shot, we assume that the modulation powers P_{L1} and P_{L2} fluctuate with $\pm 3\%$ rms relative to their nominal values of 0.2 and 27 MW (see Table I). In addition, the initial bunch

charge fluctuates $\pm 3\%$ rms around 0.1 pC. Figure 5 shows the simulation results. The dots represent the average of the microbunch lengths and the microbunch charges among the generated microbunches, and the error bars indicate the standard deviations among the individual microbunches. Statistical analysis of the simulation shows that the single microbunch length is approximately 1.92 ± 0.233 fs (FWHM), and the single microbunch charge is about 1.79 ± 1.42 fC. These results indicate that the proposed scheme provides a reliable and effective method for generating a microbunch train with sub-fs or atto-second microbunch length and fC microbunch charge.

Although our focus here is on the generation of a microbunch train rather than a single isolated pulse, it is important to note that the challenge of reducing arrival-time jitter is another critical issue for UED applications. In the present case, such a periodic bunch structure, e.g., with microbunch spacing about 0.2 mm or 0.6 ps, offers unique advantages for studying frequency-domain dynamics, as the intratrain pulse spacing, governed by the modulation laser wavelength, can remain highly stable even in the presence of global timing drifts. To mitigate the arrival-time jitter for future UED applications, several typical and promising approaches may be considered, e.g., (i) all-optical synchronization: by employing THz or optical fields derived from the same driving laser source for acceleration and bunch compression, the inherent synchronization can bypass the phase noise limitations of conventional rf systems, potentially pushing the jitter into the subfemtosecond regime; see, e.g., Ref. [50]. (ii) shot-by-shot time-stamping: implementing high-resolution timing diagnostics, such as an rf deflector, would allow for postexposure data binning. This technique effectively decouples the experimental time resolution from the global jitter, provided the duration of individual microbunches remains in the few-fs range (see, e.g., Refs. [51,16]).

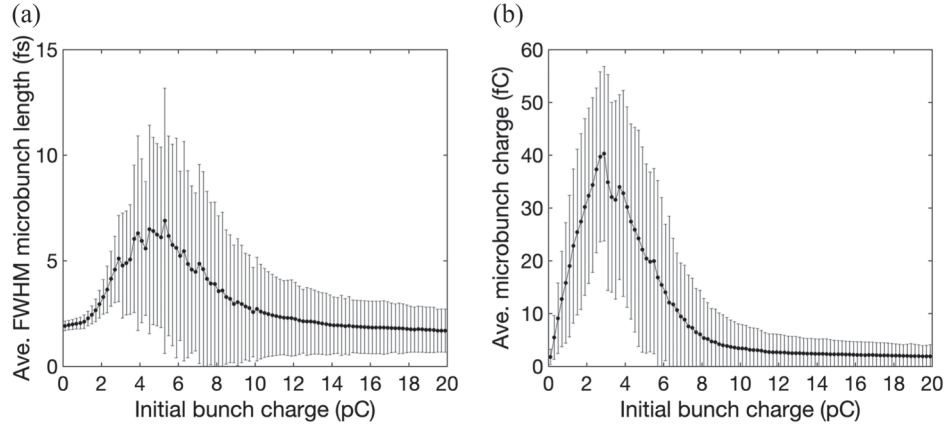


FIG. 6. Dependence of the microbunch characteristics on the initial bunch charge at the beamline exit: (a) the average microbunch length (dots) and its standard deviation (error bars); (b) the average individual microbunch charge (dots) and its standard deviation (error bars).

The above simulations assume an initial bunch charge of 0.1 pC. Due to space charge effects, it is of interest to explore how different initial bunch charges affect characteristics of the generated microbunch train. Figure 6 shows the dependence of microbunch length and microbunch charge on the initial bunch charge, assuming all other parameters except for the bunch charge in Table I remain the same. As the initial bunch charge increases, the enhanced space charge fields tend to redistribute electron energy among the head and tail particles during transport, modifying the energy-position correlation. The previously optimal THz modulation powers and $R_{56,c}^{(1)}$, $R_{56,c}^{(2)}$ now may become mismatched, resulting in a less perfectly vertically aligned phase space distribution of the beamlets and an

increase in the averaged microbunch length, say, in the range of 0 ~ 3 pC [Fig. 6(a)]. Simultaneously, the deviation of individual microbunch durations also increases, i.e., shown as the error bars in panel (a). For the 0.1-pC example in Table I, the results are close to those obtained neglecting space charge effect. Figure 6(b) further shows that, as the initial bunch charge increases in the range of 0 ~ 3 pC, the charge captured by each microbunch also increases.

There is an interesting observation from Fig. 6: in panel (a), as the initial bunch charge continues to increase, the average microbunch length first increases noticeably, but then starts to decrease once the initial charge exceeds 5 pC. Meanwhile, in panel (b), when the initial charge exceeds

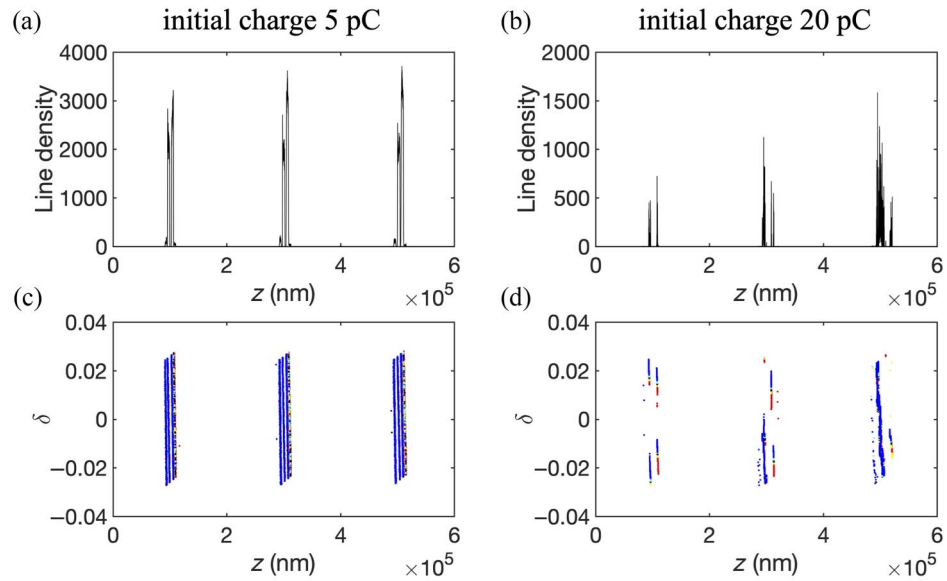


FIG. 7. Line density and phase space distributions at the exit of the chicane 2 for the initial bunch charge 5 pC (a),(c) and 20 pC (b),(d). Other parameters except for the bunch charge remain the same as in Table I. Longitudinal space charge fields have been included in the simulation.

3 pC, the individual microbunch charge obtained through numerical postprocessing begins to decrease. Here we attempt to provide an interpretation of this dependence. Figure 7 illustrates the phase space distributions and line density profiles of selected microbunches at the exit of the chicane 2 for the cases of the two initial bunch charges 5 pC [panels (a) and (c)] and 20 pC [panels (b) and (d)]. As the initial bunch charge increases, the space charge field becomes stronger, causing the energy modulation between the head and tail particles within the bunch to deviate from the ideal/original design. Consequently, the modulation and compression no longer remain ideal. This is manifested in the phase space distributions, where different energy strips become less well organized, and the line density profile projected onto the z axis not only broadens but also exhibits multiple peaks within an originally single microbunch [see Figs. 7(a) and 7(c)]. In Fig. 6(a), the fact that the average microbunch length becomes shorter with further increasing initial bunch charge is primarily a numerical artifact: since a single microbunch now splits into multiple peaks, the numerical postprocessing effectively measures the width of each *peak* rather than that of the original microbunch as a whole. As the initial bunch charge even further increases, this effect becomes more pronounced, as shown in Figs. 7(b) and 7(d) for the 20-pC case.

As a side note, it is instructive to distinguish the present scheme from high-repetition-rate electron guns, as the two approaches serve different scientific purposes. The high-repetition-rate guns operating at, e.g., 1 MHz deliver pulses at $\sim 1 \mu\text{s}$ intervals, far exceeding the characteristic timescales of most ultrafast collective phenomena; by contrast, the ~ 0.66 ps microbunch spacing in our scheme is naturally matched to THz-regime lattice and collective

dynamics, enabling multiframe sampling of a single transient event within a sub-ps temporal window. Furthermore, whereas the time resolution of high-repetition-rate guns is ultimately limited by shot-to-shot arrival-time jitter, the microbunch spacing in our scheme is phase-locked to the modulating laser wavelength, implying intrinsic intratrain stability.

B. Fine tuning of the microbunch length

In our proposed scheme, if we need to tune the microbunch length within the train, it can be achieved by tuning the dispersion strength of the chicane 2. Figure 8 shows the dependence of the final microbunch length on $R_{56,c}^{(2)}$. It can be seen that for the parameters set in Table I, corresponding to the minimum averaged microbunch length of about 2 fs (FWHM), any deviation from this design $R_{56,c}^{(2)}$ will result in a stretched microbunch length. Moreover, due to the slight differences in energy-position correlation among individual microbunches, a larger deviation among the microbunches from this design can be expected.

C. Phase space manipulation of the microbunch train

In addition to generating a microbunch train with microbunch spacing determined by the second THz wavelength, our scheme also allows the creation of different

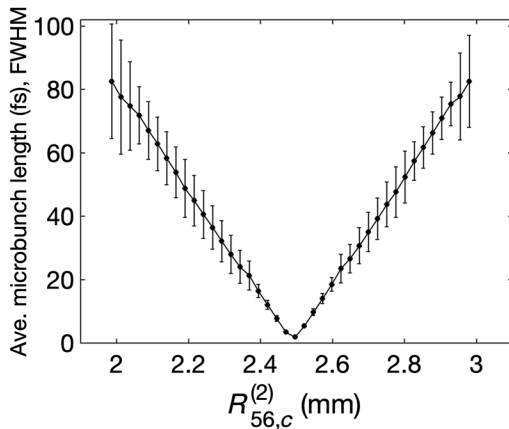


FIG. 8. The average (dot) and standard deviation (error bar) of the final microbunch lengths as a function of the longitudinal dispersion strength $R_{56,c}^{(2)}$. The optimal value $R_{56,c}^{(2)} = 2.5$ mm is chosen to achieve the shortest average microbunch length. Deviations from this setting result in elongated microbunches, which can be used to fine-tune the microbunch length of the train.

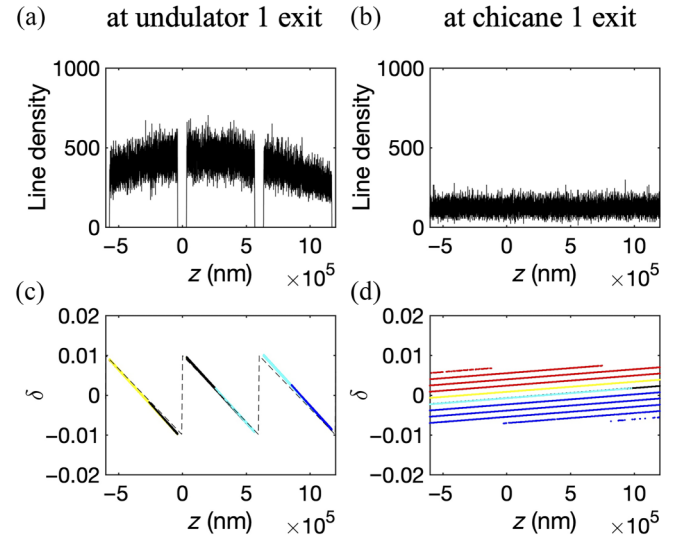


FIG. 9. Line density and phase space distributions at the exit of the undulator 1 (a),(c) and at the exit of the chicane 1 (b),(d). The dashed lines in (c) indicate the sawtooth waveform of the THz modulation in the undulator 1. This is the case with $R_{56,c}^{(0)} = 100$ cm, illustrating one example of beam phase space manipulation based on the proposed scheme. At the entrance of the undulator 1, the bunch has been substantially stretched, leading to the formation of multiple longitudinal stripes at the exit of the chicane 1, as a result of the dispersion $R_{56,c}^{(1)}$. Longitudinal space charge fields have been included in the simulation.

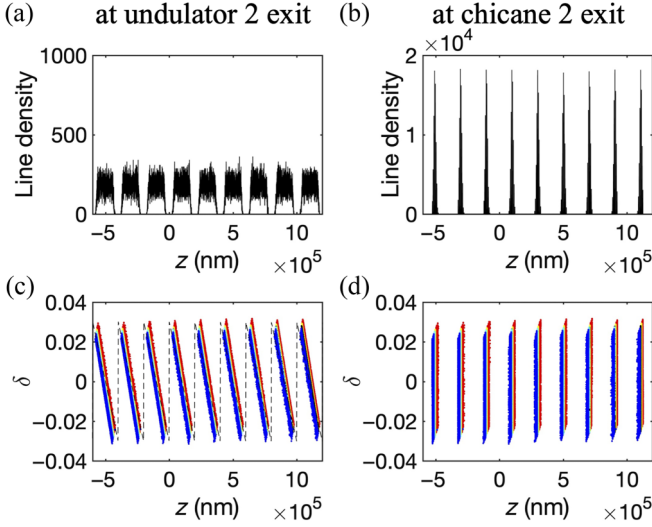


FIG. 10. Line density and phase space distributions at the exit of the undulator 2 (a),(c) and at the exit of the chicane 2 (b),(d). The dashed lines in (c) indicate the sawtooth waveform of the THz modulation in the undulator 2 with the wavelength three times shorter than that of the undulator 1. Due to the presence of multiple longitudinal stripes, together with the dispersion $R_{56,c}^{(2)}$, the final output shows a periodic comb structure consisting of a train of microbunches, each accompanied by one or more pairs of temporal satellite pulses (see Fig. 11 below). Longitudinal space charge fields have been included in the simulation.

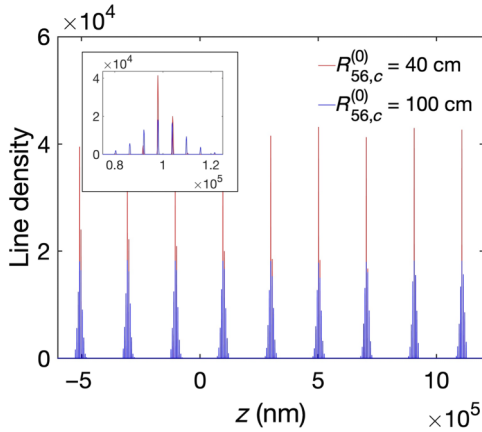


FIG. 11. Line density distributions at the exit of chicane 2 for two different $R_{56,c}^{(2)}$. Different longitudinal dispersion strengths lead to distinct microbunch profiles. The enlarged plot shows a magnified view of a selected region.

train structures through tailored phase space manipulations. For example, if the incoming bunch length before the undulator 1 is further stretched by increasing $R_{56,c}^{(0)}$, e.g., from 15 cm in Table I to 100 cm, the stretched bunch length now becomes much longer than the first THz wavelength. Consequently, the sawtooth waveform in the undulator 1 segments the electron bunch into discrete sub-bunches, as shown in Fig. 9(c). These segmented

sub-bunches are further sheared by $R_{56,c}^{(1)}$, forming striplike structures in the phase space [see Fig. 9(d)].

After these striplike structures pass through the undulator 2, they are modulated into more complicated patterns, as illustrated in Fig. 10(c). Note the difference between Figs. 3(d) and 10(c). Finally, when applying an appropriate longitudinal dispersion from the chicane 2, these complicated phase space structures are “uprighted.” Unlike the generated regular microbunch train in the original example, each microbunch now carries associated submicrobunch pulses, with their spacing and number varying depending on $R_{56,c}^{(0)}$. This type of microbunch structure generation is commonly referred to as an echo, similar to the mechanism proposed in FEL for producing high-order harmonic radiation [52,53]. Figure 11 illustrates the resulting microbunch structures obtained by varying $R_{56,c}^{(0)}$.

VI. SUMMARY AND DISCUSSION

In summary, we have developed and verified our proposed scheme that combines two undulators with dual THz modulations to generate few-femtosecond to attosecond electron microbunch trains. To study the dynamics of low-energy relativistic electrons under external electromagnetic modulation in the undulators, we construct period-by-period tracking equations that incorporate the longitudinal space charge effects, exploring the resulting phase space dynamics. Simulation results indicate that, given the current capabilities of laser, THz, and undulator technology, this scheme can produce an electron microbunch train with kinetic energies around 3 MeV, final microbunch lengths of approximately 1.9 fs (FWHM), microbunch charges on the order of 2 fC, and microbunch spacing of ~ 0.2 mm (0.66 ps). Such an ultrafast microbunch train may have potential applications in probing reversible processes of matter on the picosecond timescale.

Several key limiting factors and future directions are worth discussing. The primary performance-limiting factors identified in the present scheme are space charge effects and timing jitter. Regarding the space charge effects, both the longitudinal and transverse fields are present; however, given the focus on longitudinal microbunching dynamics, LSC plays the dominant role at this stage. The transverse beam size, e.g., ~ 1.5 mm ($3\sigma_x$) corresponds to a temporal equivalent of ~ 5 ps, at least one order of magnitude larger than the microbunch length, so the transverse space charge is expected not to be the dominant factor. For shot-to-shot fluctuations, variations from the modulation power and bunch charge (assuming each $\pm 3\%$) have been considered in the present analysis. However, arrival-time jitter of the microbunches, driven by upstream rf cavity phase and amplitude fluctuations, may represent an additional critical factor that requires further investigation. A comprehensive start-to-end simulation, including the above fluctuations and full three-dimensional

space charge effects, will be essential to finalize the performance evaluation.

It deserves here to comment that the mechanism for generating a microbunch train described here is analogous to the HGHG scheme [54] widely used in the FEL community for producing high-order harmonic radiation. The echo mechanism in Sec. V C, on the other hand, resembles the echo-enabled harmonic generation approach [52,53]. In light of our recent work [18], it could be concluded that the integration of undulators with THz modulation allows, under present engineering limitations, the production of subfemtosecond or attosecond single isolated electron bunches (sub-pC charge) as well as microbunch trains with individual charges up to the fC level, at an electron kinetic energy of 3 MeV.

ACKNOWLEDGMENTS

This work is supported by the Fundamental Research Funds for the Central Universities (HUST) under Project No. 2021GCRC006 and the National Natural Science Foundation of China under Project No. 12275094. The author acknowledges J. Li for helpful discussions and D. Liu for help in the preparation of this manuscript.

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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