

Influence of an external static magnetic field on prebreakdown electron emission and heating

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High magnetic fields can increase the occurrence of vacuum arcing, suggesting that both electric and magnetic fields can play a role in the vacuum arcing process. The mechanism of vacuum arcing in high magnetic fields is believed to involve both the cathode and the anode, with the cathode serving as the originator of field-emitting nanoprotusions or tips, while the anode serves a secondary role. Significant heating of the anode surface can be achieved by magnetic focusing of the emitted electron beam, leading to an increased heat flux due to greater current density. We simulated the emitted electron beam in different configurations of the electric and magnetic fields using the particle-in-cell and finite-element methods. The heating caused by the impacting electron beam was simulated for magnetic fields ranging from 0 to 30 T. The directions of the electric and magnetic fields were found to play a major role in the focusing of the electron beam. We found that a sufficient temperature increase on the anode surface for evaporation can be reached at magnetic fields on the order of 10–30 T, suggesting the possibility of plasma initiation on the anode side.

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I. INTRODUCTION

Vacuum arcing presents technical challenges in applications where high electric fields are used, such as electron sources, particle accelerators, and vacuum interrupters. The origin and mechanics of these breakdowns have been the subject of numerous studies, but a complete picture of the process is yet to be obtained.

While both electric and magnetic fields are present during the vacuum arcing process, it has been determined that the electric field dominates over the magnetic field as the main arc initiation factor. The cathode is typically considered the main originator of dc vacuum arcs, while the anode plays a lesser role [1–4]. This cathodic arc initiation has been proposed to be due to the self-heating and thermal runaway of microscopic protrusions on the

cathode as a result of field-induced electron emission, which provide the influx of neutrals for plasma formation.

The roles of the cathode and anode could be reversed under certain conditions [5,6], especially in the presence of a strong static or quasistatic magnetic field. In that case, a different arc initiation mechanism might provide the initial source of neutrals for the plasma onset. Such conditions may be present in particle accelerators, such as the proposed muon collider [7]. While accelerators use rf cavities where there is no fixed cathode and anode, the dc plate-to-plate electrode case serves as a useful simplified model where vacuum breakdown mechanisms can be studied and distinguished [8]. As a basis for the current arcing simulation studies, we use the Large Electrode System experiments at CERN, which use a pulsed dc plate-to-plate configuration [9].

At high magnetic fields, vacuum arcing can become a major design consideration. Muon collider design studies foresee peak fields of 17 T [10] in the rectilinear cooling channel and up to 30 T [11,12] in the final-cooling stage. However, it should be noted that the focusing solenoids are situated such that these peak fields do not directly impinge on rf accelerating cavities [12]. The proposed muon collider would operate at roughly 10 TeV center-of-mass energy. However, delivering and sustaining such intense beams presents significant technical challenges. Ionization

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cooling is indispensable for a multi-TeV muon collider because muon beams emerge from the target with high six-dimensional emittance and must be shrunk by $\geq 10^5$ before most muons decay ($\tau_\mu = 2.2 \mu\text{s}$) [7]. Each cooling cell combines three collinear elements [10]: a low- Z absorber, normal conducting rf cavities, and superconducting solenoids. The rf cavities are typically pillbox-type single-cell structures with large apertures (e.g., ~ 28 cm diameter iris at 352 MHz) that operate at high vacuum levels ($\sim 10^{-7}$ Torr) [10,13]. These cavities sit inside the solenoid bore, such that the magnetic field has both parallel and perpendicular components relative to the rf electric field [10]. That overlap is vital for cooling but drives vacuum breakdown. Field-emitted electrons are forced by the solenoid into narrow beamlets that strike and locally melt the opposite wall. Measurements at the MuCool Test Area showed that an 805 MHz copper cavity that held 40 MV/m at 0 T was limited to ~ 12 MV/m at 3 T [14,15]. Muon ionization cooling experiments 201 MHz cavities achieved 16 MV/m at 3 T only after extensive conditioning [16]. Because later cooling stages demand both higher gradients (≥ 25 MV/m) and higher magnetic fields (> 10 T), the probability of vacuum breakdown grows precisely where maximum performance is required.

Current theories on the origin of vacuum arcs give the appearance of nanoprotusions or tips on the metal surface as the origin of local field enhancement and field emission [17–19]. Although the mechanism of tip growth remains unclear despite extensive study [20–23], we assume the presence of these tips on the cathode surface. Strength of field emission is dependent on factors such as geometric field enhancement. This electron emission can initiate an arc via two distinct mechanisms, which we shall call “anodic” and “cathodic” following Utsumi’s nomenclature [5]. Although rf structures have neither anodes nor cathodes, both distinct mechanisms might appear depending on the conditions. In the cathodic mechanism, which has been extensively studied elsewhere [18,19,24], field emission causes self-heating of the protrusion, leading to evaporation of neutral atoms from the cathode surface, which ionize to form the arc plasma. In the anodic mechanism, the electrons are accelerated by the field and cause intense heating at the electrode that captures the electrons (the anode in dc), initiating the arc there. In this study, we aim to investigate the possibility of evaporation of the anode material as a result of focused emitted electron beams. If a sufficiently dense neutral vapor can accumulate above the anode surface, this may provide the preconditions for anode-initiated breakdown. If these conditions are met and ions can form on the anode side, they would be accelerated from the anode to the cathode, potentially causing heating, evaporation, and sputtering.

High axial magnetic fields have been observed to increase arc occurrence [25]. In the presence of an

external magnetic field, the emitted electron beam can be significantly focused, as supported by simulations and experiments [14,15,26,27]. This focusing depends on the characteristics of electron emission (tip geometry and other factors), as well as the magnitude and direction of the electric and magnetic fields. This focusing of the electron beam could lead to significant heating and potential arc initiation on the anode side. While in previous studies electron emission and heating have been solved in an approximate way, we use multiphysics simulations of each stage of the electron thermal-field emission, electron beam focusing characteristics, and the anode heating to study this process, using the dc case as a simplified model to investigate the underlying physical mechanisms. This approach aims to provide a complementary view to the studies focused on specific rf accelerating structure designs.

II. METHODS

The motion of the emitted electron beam is studied both theoretically and computationally. Solutions to specific cases can be obtained analytically, while the study of more complicated dynamics may require simulation. These two approaches are compared to verify the validity of our results.

A. Theoretical solution of electron beam motion

The geometry of the system consists of parallel cathode and anode planes separated by a distance d (Fig. 1). A nanoscale tip of height h and radius r_{tip} is assumed to be present on the cathode surface (Fig. 1). Near the tip, the

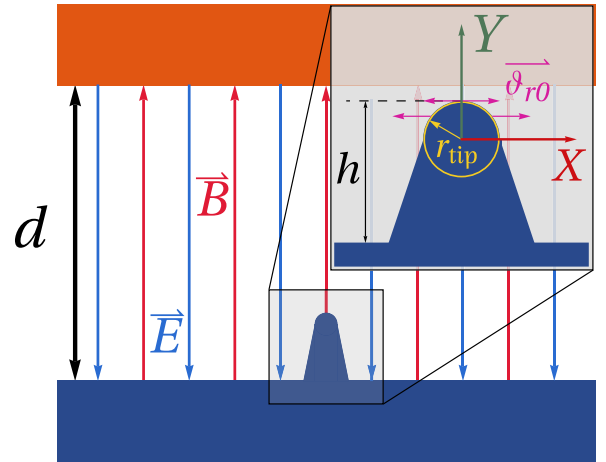


FIG. 1. Geometry of the system. The parallel electrodes (the cathode is shown in blue and the anode in red) are separated by the gap length d . The blue and red arrows show the directions of electric $\vec{E}$ and magnetic $\vec{B}$ fields. A nanotip is placed on the surface of the cathode. The inset shows the height h and the curvature radius r_{tip} of the nanotip, X and Y indicate the Cartesian coordinates used in the calculations. The initial velocity of the electrons v_{r0} is due to the field enhancement around the nanotip.

electric field is significantly distorted, resulting in localized field enhancement. This field enhancement can lead to vacuum breakdown due to electron emission from the tip surface [5]. The electrons experience radial acceleration near the tip due to the presence of the radial component of the field. At the same time, calculating the electrostatic potential near the tip can be challenging [28]. Instead, following Utsumi's [5] approach, we assume in the analytical solution that the electrons emitted from the tip possess nonzero initial velocities v_{r0} . These velocities are sufficiently large that the electrons follow the same parabolic trajectories as they would if field enhancements were explicitly accounted for.

Assuming that the emitted electron beam is moving in static and uniform electric and magnetic fields, we can solve the motion analytically. The electrons experience acceleration due to the Lorentz force $\mathbf{F} = q_e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$, where $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields, respectively; $\mathbf{v}$ is the velocity; and q_e is the charge of the electron. The acceleration of the charged particle depends on the charge-to-mass ratio q_e/m_e , so the equations are easily applied to other types of charged particles, such as ions.

We solve these equations and consider two special cases: one where the electric and magnetic fields are parallel, $\mathbf{E} = (0 \ E_y \ 0)$ and $\mathbf{B} = (0 \ B_y \ 0)$, and one where they are perpendicular $\mathbf{E} = (0 \ E_y \ 0)$ and $\mathbf{B} = (0 \ 0 \ B_z)$. These cases are illustrated in Fig. 2.

In the parallel case, we obtain the solution

$$\begin{aligned} x(t) &= x_0 + a_m v_{z0} (\cos(t/a_m) - 1) + a_m v_{x0} \sin(t/a_m), \\ y(t) &= y_0 + v_{y0} t + \frac{q_e E_y}{2m_e} t^2, \\ z(t) &= z_0 - a_m v_{x0} (\cos(t/a_m) - 1) + a_m v_{z0} \sin(t/a_m), \end{aligned} \quad (1)$$

where $a_m = \frac{m_e}{q_e B_y}$, $(x_0 \ y_0 \ z_0)$ is the initial position of the particle, and $(v_{x0} \ v_{y0} \ v_{z0})$ is the initial velocity. The constant a_m describes the influence of the magnetic field strength on the electron beam, which affects the amplitude of the beam oscillation.

By assuming that the electrons follow a parabolic trajectory, we can get an estimate for the beam radius $r_0(y) = \sqrt{4\eta h y}$, where h is the height of the emitter and η is the spreading factor, which depends on the tip shape. For a hemispherical tip, we assume $\eta = 0.5$ [5]. The ratio of the beam radius r and beam radius without magnetic field r_0 can be obtained from theory by calculating the radial velocity

$$v_{r0} = \frac{dr_0[y(t)]}{dt} = \sqrt{\frac{2\eta h q_e}{m_e} E_y}, \quad (2)$$

and assuming $x_0 = y_0 = z_0 = 0$, $v_{y0} = 0$. This leads to (e.g., given by Utsumi [5])

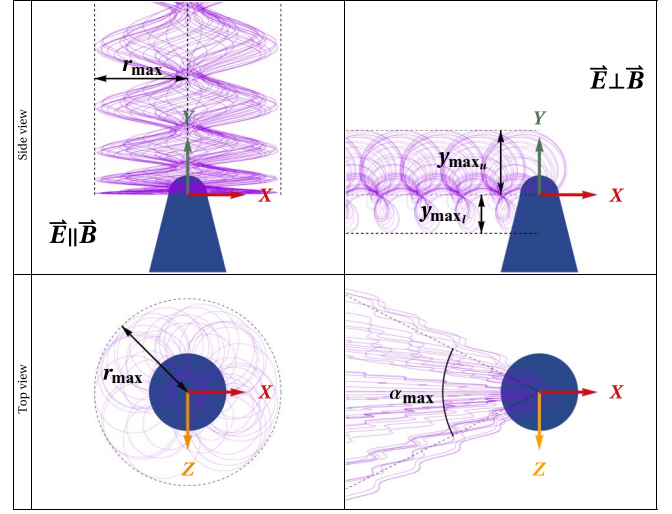


FIG. 2. Top and side views: schematic representation of the electron beam dynamics ejected from the cathode tip (blue) for both cases: $\mathbf{E} \parallel \mathbf{B}$ and $\mathbf{E} \perp \mathbf{B}$, with theoretical parameter illustrations. The purple lines represent the trajectories of individual electrons. The electric field component E_y and the charge-to-mass ratio q_e/m_e are negative, while the magnetic field components B_y (for $\mathbf{E} \parallel \mathbf{B}$) and B_z (for $\mathbf{E} \perp \mathbf{B}$) are positive. The initial velocity vectors $\mathbf{v}_{r0}$ exhibit radial symmetry around the center of the tip in both cases. $r_{\max}$ is the radius of the amplitude of the helix trajectory of the electrons or the maximal deviation of the electron trajectory from the axis of the tip for $\mathbf{E} \parallel \mathbf{B}$; $y_{\max_u}$ and $y_{\max_l}$ define the maximum thickness of the spiral trajectory for $\mathbf{E} \perp \mathbf{B}$ and $\alpha_{\max}$ is the beam's opening angle for $\mathbf{E} \perp \mathbf{B}$.

$$\frac{r(y)}{r_0(y)} = \rho |\sin(\rho^{-1})|, \quad \rho = \frac{a}{B} \sqrt{\frac{E}{y}}, \quad (3)$$

where $a = \sqrt{\frac{2m_e}{q_e}} = 3.37 \times 10^3 \text{ nm T V}^{-1/2}$, E is electric field, B is magnetic field, and y is position. Based on this equation, we expect the radius of the beam to oscillate with a constant amplitude, depending on the magnitudes of the electric and magnetic fields as given by the parameter ρ .

The beam radius depends on distance and magnetic field as shown in Fig. 3(a), where parameters from Sec. III B are used. We can see that both the amplitude and period of the beam radius depend on the magnetic field B . For a larger magnetic field, the beam produced will be narrower and the frequency of oscillation higher. The maximum beam radius can be calculated as $r_{\max} = \sqrt{r_{\text{tip}}^2 + (a_m v_{r0})^2} + |a_m v_{r0}|$ (Fig. 2).

The beam radius can be used to calculate the heat flux at the anode. The power deposited by the electron beam at the anode is $V_0 I_e$, where V_0 is the applied voltage at the anode and I_e is the emission current. The heat flux depends on the area of the impacting beam, i.e., the beam radius r . The heat flux is

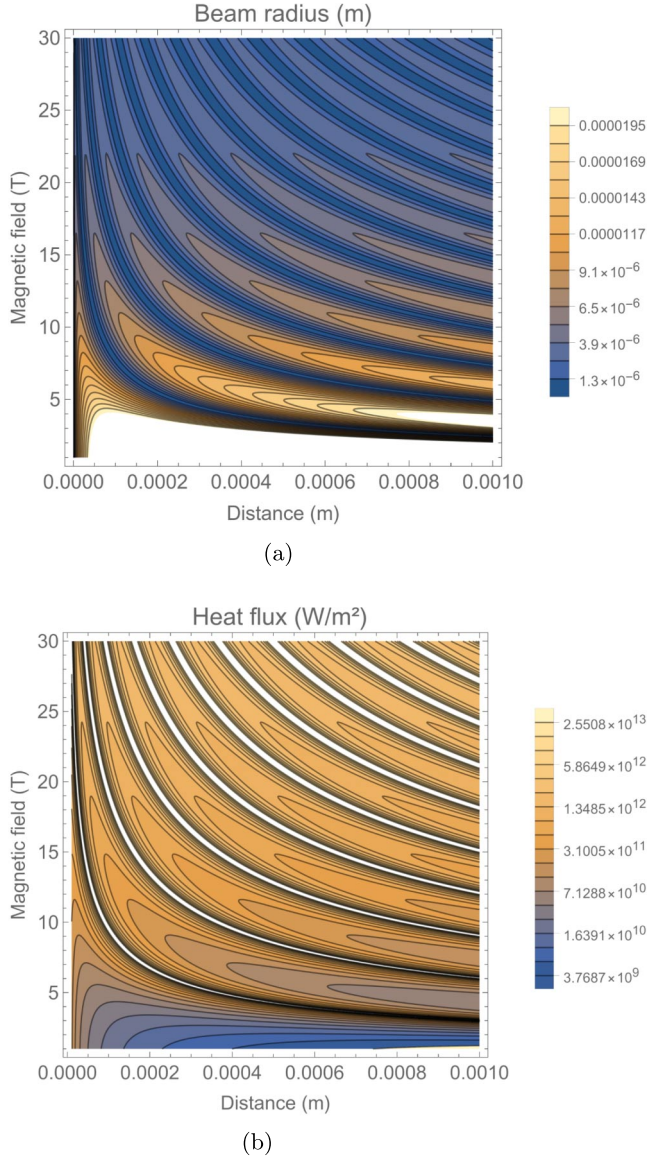


FIG. 3. Electron beam quantities in the parallel case ($\mathbf{E} \parallel \mathbf{B}$) on the anode surface, calculated based on theory by assuming an emitted current of $I_e = 1$ mA. (a) Beam radius as a function of distance y and magnetic field B . (b) Heat flux as a function of distance y and magnetic field B .

$$P = \frac{V_0 I_e}{\pi r^2} = V_0 j = E_0 d j, \quad (4)$$

where j is current density, E_0 is the applied macroscopic electric field, and d is gap distance. We can see that heat flux increases not only with current density but also with voltage. This shows that if we keep the electric field E_0 constant, increasing the gap size will also affect the heat flux. Heat flux at different magnetic fields and distances is shown in Fig. 3(b) for a beam with a current of $I_e = 1$ mA. At locations where the beam radius reaches 0, singularities are present in the heat flux. Although in reality the electron

beam always has some finite diameter, this suggests very large heat fluxes can be achieved on the anode surface.

In the perpendicular case, we have the solution

$$\begin{aligned} x(t) &= x_0 + a_m v_{y0} + \frac{E_y}{B_z} t \\ &\quad - a_m v_{y0} \cos(t/a_m) \\ &\quad + a_m \left(v_{x0} - \frac{E_y}{B_z} \right) \sin(t/a_m), \\ y(t) &= y_0 - a_m v_{x0} + a_m \frac{E_y}{B_z} \\ &\quad + a_m \left(v_{x0} - \frac{E_y}{B_z} \right) \cos(t/a_m) \\ &\quad + a_m v_{y0} \sin(t/a_m), \\ z(t) &= z_0 + v_{z0} t, \end{aligned} \quad (5)$$

where $a_m = \frac{m_e}{q_e B_z}$, $(x_0 \ y_0 \ z_0)$ is the initial position of the particle, and $(v_{x0} \ v_{y0} \ v_{z0})$ is the initial velocity. The electron beam has rotational motion in the xy plane, while no force acts in the z direction. The propagation of particles in y direction will be limited by $y_{\max} = a_m b (1 \pm \sqrt{1 + (\frac{v_{y0}}{b})^2}) + y_0$, where $b = \frac{E_y}{B_z} - v_{r0}$. There is a dependency on the ratio of the electric and magnetic fields, expressed as E_y/B_z . This ratio influences the beam dynamics, causing the beam to expand outward with an opening angle $\alpha_{\max}$, which is defined as follows: $\alpha_{\max} = \pi - 2 \arctan \left| \frac{E_y}{B_z v_{r0}} \right|$ (Fig. 2).

As the ratio $\frac{E_y}{B_z v_{r0}}$ decreases, the angle distribution of the particles shows a distinct pattern of shifting density. Angles closer to 0° , which coincide with the x direction, become sparsely distributed, whereas those near $\pm 90^\circ$ exhibit a higher concentration. In the limiting case, where $\frac{E_y}{B_z v_{r0}} \rightarrow 0$, $\alpha = \pi$ and $v_x = 0$, and consequently, the beam propagates only in the $\pm z$ directions, and then the radius of the beam is given by $r = 2a_m v_{r0} + r_{\text{tip}}$.

B. Simulation methods

Simulation of electron beams under electric and magnetic fields is well suited for the particle-in-cell (PIC) method [29], which we use to calculate trajectories of emitted electrons while including electron-electron interactions using Monte Carlo collisions [30]. We use the FEMOCS code [19] to perform these simulations in 3D, with electron emission calculations performed using GETELEC [31]. Electric field, current, and heat calculations are performed using the finite-element method using the deal.II library [32], with meshes generated using Gmsh [33] and TetGen [34]. The electric field with space charge is solved from Poisson's equation. The current and heat distributions in the cathode are solved from the continuity and heat equations, respectively. A more detailed description of our current simulation model is given in [24].

On the cathode surface (Cu), a nanotip is assumed to be present, causing local field enhancement and emission of

electrons. These electrons travel across the cathode-anode gap and impact the anode. PIC simulation was performed for electrons with a time step of $\Delta t_{\text{PIC}} = 0.1$ fs and superparticle weight of $w_{\text{SP}} = 10$. In the electron beam, we perform elastic and Coulomb collisions between electrons.

We can calculate the distribution of electrons impacting the anode (Cu) and determine the current density on the anode surface to get the temperature distribution due to electron bombardment [1]. The current density is used for heating calculations using the finite-element method (FEM), performed using COMSOL (version 6.2). In this model, we calculate the heat rate in the anode as $P = V_0 j / z_d$, where j is current density, V_0 is the applied voltage, and z_d is the electron beam CSDA (continuous-slowing-down approximation) range in Cu. When $z > z_d$, $P = 0$. The base of the anode is assumed to be at a temperature of $T_a = 300$ K. The simulation model includes solid heat transfer inside the anode, electric current calculations, electromagnetic heating (resistive heating), and radiative cooling of the anode surface.

III. RESULTS

We performed two types of simulations: PIC simulations to study the influence of the direction and magnitude of the magnetic field on the dynamics of the emitted electron beam and FEM heating simulations to calculate the heating of the anode surface due to electron bombardment.

A. Direction of magnetic field

To investigate the electron beam dynamics, we simulated two cases: one where the magnetic field is parallel with the electric field [Figs. 4(a)–4(c)], and one where

it is perpendicular to it [Figs. 5(a)–5(f)]. Depending on the direction of the magnetic field, the electron beam can be either focused or defocused. In both parallel and perpendicular cases, the simulated electron beam can be seen to produce a shape similar to our theoretical solutions [Eqs. (1) and (5)]. This verifies that the simulation results roughly follow the theoretical solution while showing that the beam dynamics is not fully captured by theory alone.

In the parallel case [Figs. 4(a)–4(c)], the electrons are effectively focused by the magnetic field, i.e., their cyclotron trajectories are transversely confined while they are accelerated by the electric field from the cathode toward the anode. Here, “focusing” is used in an effective sense to denote the magnetic-field-induced confinement of electron trajectories and the resulting reduction of the beam radius on the anode surface, rather than focusing produced by magnetic-field gradients in the strict optical sense. The magnetic field causes the electron beam to rotate around the y axis and thus confines its transverse motion. Consequently, the beam radius oscillates during propagation, and the impact area on the anode is significantly smaller than in the zero-field case.

In the perpendicular case [Figs. 5(a)–5(f)], the electron beam expands in the direction of the magnetic field and rotates around the magnetic field axis. The beam propagates in a direction perpendicular to the emitting tip and does not impact the anode. The amplitude and frequency of rotation depend on the magnitude of the applied magnetic field, while the opening angle of the beam depends on the ratio of the electric and magnetic fields as calculated in Sec. II A. For the theoretical solutions (orange line), the path of the electrons depends on their initial velocities, which is assumed to be given by Eq. (2). The initial velocity in the xy plots is assumed to be such that each of the x and y components are $v_{x0} = v_{y0} = v_{r0}/\sqrt{2}$, while in

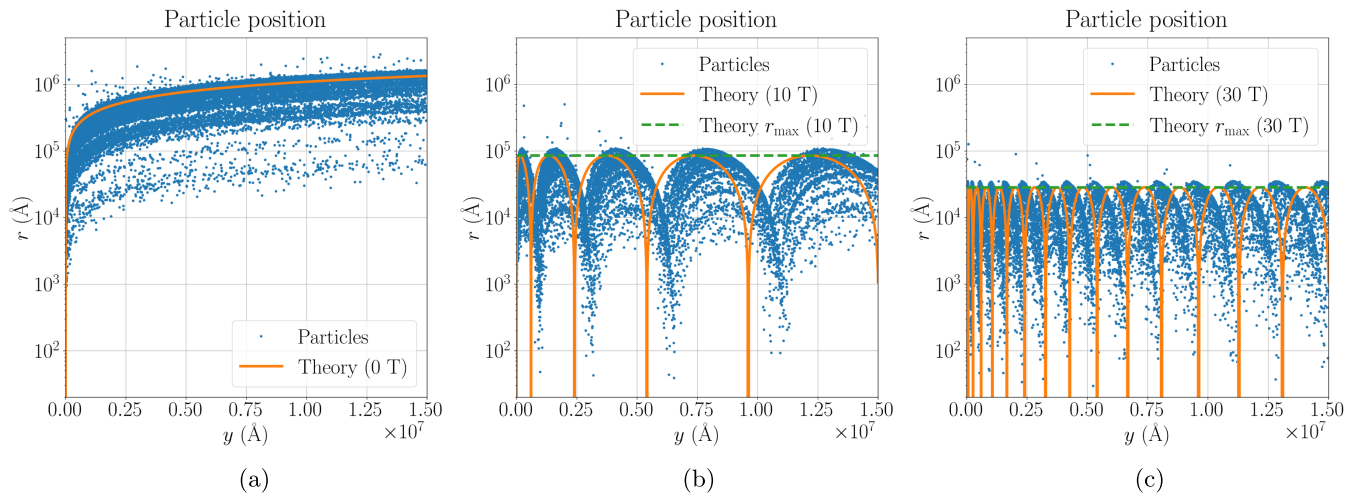


FIG. 4. Modulation of the electron beam shape at different magnetic fields in the case of parallel electric and magnetic fields ($\mathbf{E} \parallel \mathbf{B}$). In the legend, “Theory” refers to the results of Eq. (1) and r_{max} is the radius of maximal radial deviation from the tip axis for the $\mathbf{E} \parallel \mathbf{B}$ geometry (Fig. 2). (a) $B_y = 0$ T. (b) $B_y = 10$ T. (c) $B_y = 30$ T.

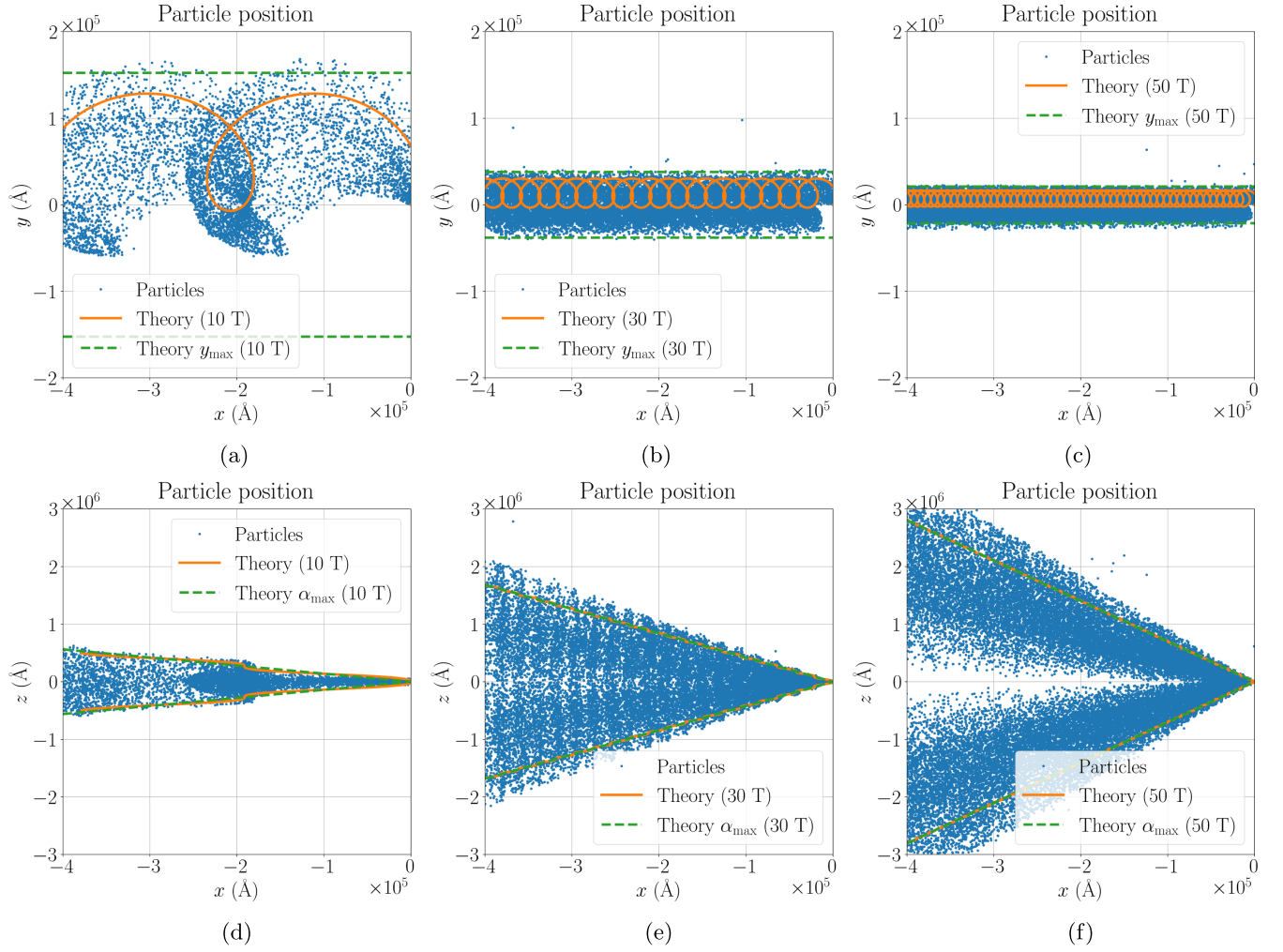


FIG. 5. Modulation of the electron beam shape at different magnetic fields in the case of perpendicular electric and magnetic fields ($\mathbf{E} \perp \mathbf{B}$). The top and bottom rows show the side and top views of the beam shapes for the corresponding values of the magnetic fields. In the legend, “Theory” refers to the results of Eq. (5) and $y_{\max}$ is the maximal vertical deviation of the electron trajectory for the $\mathbf{E} \perp \mathbf{B}$ geometry (Fig. 2). (a) $B_z = 10$ T (side). (b) $B_z = 30$ T (side). (c) $B_z = 50$ T (side). (d) $B_z = 10$ T (top). (e) $B_z = 30$ T (top). (f) $B_z = 50$ T (top).

the xz plots the initial velocity is $v_{z0} = v_{r0}$. In the simulations, the initial velocity depends on the exact geometry, which results in differences observed in the results. These results show that the direction of the magnetic field can significantly influence the beam, which can change the magnitude of heating on the anode side.

B. Heating due to magnetic field

Electron beam simulations are performed using PIC with a parallel magnetic field at 0, 10 and 30 T to determine how the magnetic field strength influences heating on the anode side. The distance between the cathode and anode (gap size) is $d = 1.5$ mm, while the diameter of the cathode and anode is about $300 \mu\text{m}$. On the cathode surface, there is a tip with a height of $6 \mu\text{m}$ and an apex radius of 15 nm. The applied voltage is $V_0 = 80$ kV,

giving a macroscopic field of 53 MV/m and a local field of $E_0 = 10$ GV/m at the tip apex. This results in an electron emission current of about $I_e = 2.5$ mA.

The simulated electron beam shape is plotted with the theoretical prediction [Eq. (3)] in Figs. 4(a)–4(c). We can see that the general agreement is good, with some variation in the electron paths depending on where they were emitted on the cathode surface and their initial velocities. There may be some influence due to Coulomb repulsion of the electrons, which could defocus the beam. In the theoretical predictions, the spreading factor is assumed to be $\eta = 0.5$, which can also lead to differences compared to the simulation results. In the 0 T case, the electrons can be seen following a parabolic trajectory, as expected. In the 10 and 30 T cases, the electron beam is confined by the magnetic field so that its diameter stays within constant limits.

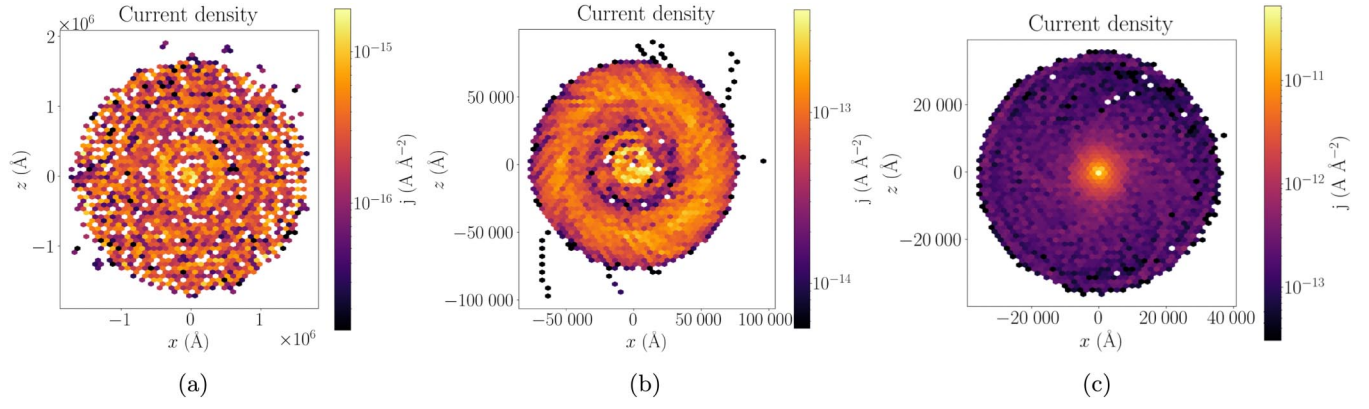


FIG. 6. Spatial electron current density on the anode surface in the parallel case ($\mathbf{E} \parallel \mathbf{B}$) for the different magnitudes of the magnetic field, obtained by PIC simulation. The color of each hexagonal pixel represents the 2.5 ps time-averaged electron current density on the anode surface. Increasing the magnetic field from 0 to 10 T reduces the spot diameter to about 1/20 of its size, reducing the area to just 1/400 of its original size. Increasing the magnetic field from 10 to 30 T reduces the diameter further to about 1/2 or to 1/4 of the previous area. (a) $B_y = 0$ T. (b) $B_y = 10$ T. (c) $B_y = 30$ T.

The current density of the impacting electrons on the anode surface is calculated based on the PIC simulations [Figs. 6(a)–6(c)]. This is done by calculating the density and velocity of electrons above the anode surface ($j = \rho v$) and time averaging over 100 frames. It can be seen that increasing the magnetic field from 0 to 10 T reduces the spot diameter to about 1/20 of its size, reducing the area to just 1/400 of its original size. Increasing the magnetic field from 10 to 30 T reduces the diameter further to about 1/2 or to 1/4 of the previous area. As the emitted current stays roughly constant, this reduction in area results in a corresponding increase in current density. The rotational motion of the electron beam in the magnetic field can be seen in the current density distributions as spiraling streaks.

Based on the calculated current densities [Figs. 6(a)–6(c)], we calculate the steady-state temperature distribution in

the anode using COMSOL as described in Sec. II B. This is done for the 0 T [Fig. 7(a)], 10 T [Fig. 7(b)] and 30 T [Fig. 7(c)] cases. According to the NIST ESTAR database [35], the CSDA range for electrons at 80 keV is $1.535 \times 10^{-2} \text{ g cm}^{-2}$ or $z_d = 17.1 \text{ } \mu\text{m}$ for Cu, which we use as the penetration depth of the beam. The maximum temperatures observed were 1440 K at 0 T, 7010 K at 10 T, and 11700 K at 30 T. Increasing the magnetic field from 0 to 10 T results in a temperature increase of over 5000 K, from the melting point to well above the boiling point. This would result in the evaporation of neutral atoms from the surface, which could start plasma formation. A further increase to 30 T leads to even more extreme temperatures, demonstrating how the magnetic field can lead to significant heating. However, these temperatures are localized to a very small point and would likely rapidly diffuse

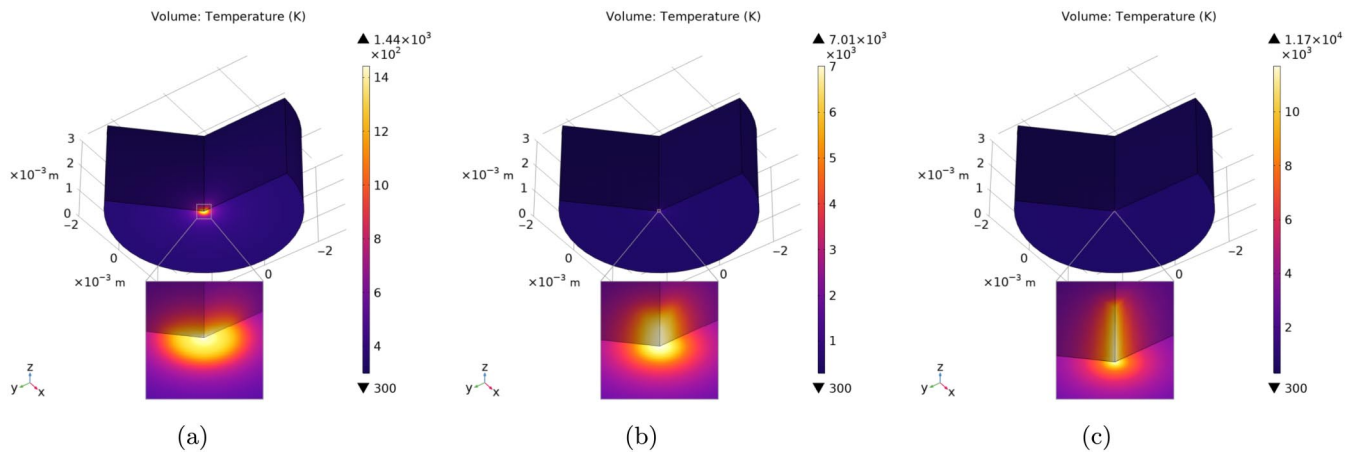


FIG. 7. Spatial plot of anode steady-state temperature in the parallel alignment of electric and magnetic fields ($\mathbf{E} \parallel \mathbf{B}$) for different values of the magnetic field, obtained by COMSOL (based on the current densities from Figs. 6(a)–6(c)). The color represents the anode temperature. The maximum temperatures observed were 1440 K at 0 T, 7010 K at 10 T, and 11700 K at 30 T. (a) $B_y = 0$ T. (b) $B_y = 10$ T. (c) $B_y = 30$ T.

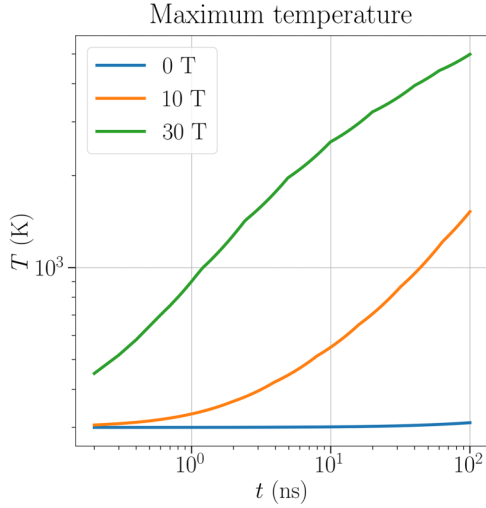


FIG. 8. Time-dependent maximum temperature on anode surface (produced by a pulse of width t) for the 0 T [Fig. 7(a)], 10 T [Fig. 7(b)], and 30 T [Fig. 7(c)] cases.

away unless continuous heating is applied, as instabilities could lead to shifting of the beam. Furthermore, at such temperatures, a phase change would occur, limiting the temperature.

The effect of pulsed heating on the maximum temperature is illustrated in Fig. 8. A time-dependent simulation was run for the cases in Figs. 7(a)–7(c) up to 100 ns. The time for an electron to cross the gap was determined to be much shorter than the period corresponding to the expected operational frequency, even for a larger cavity of up to 10 cm, for which the crossing time is about 150 ps. The maximum temperature reached at time t in Fig. 8 is the instantaneous local temperature resulting from a pulse of width t . For example, a rectangular wave with 1 MHz frequency and 10^{-3} duty cycle would have a pulse width of 1 ns. A 1 ns pulse resulted in a maximum temperature of 300 K at 0 T, 333 K at 10 T, and 959 K at 30 T, showing a major reduction in heating of the anode surface.

The heating of the anode surface was additionally studied by varying the spot radius r_s and power density Q_s and calculating the steady-state temperature on the anode surface. The heat flux within the spot is assumed to be uniformly distributed, and the maximum temperature on the anode surface is evaluated. A smaller spot radius and higher power density are the factors that contribute to greater heating of the anode surface. The results of these simulations are shown in Fig. 9, which agree with theoretical and experimental values for critical anode power density for Cu as defined in [5]. The semiempirical prediction for Cu in [5] is given as a red line for reference. In combination with Eqs. (3) and (4), the results of Fig. 9 can be used to estimate the expected heating of the anode surface for different parameters such as electric/magnetic field, voltage, emission current, and gap size.

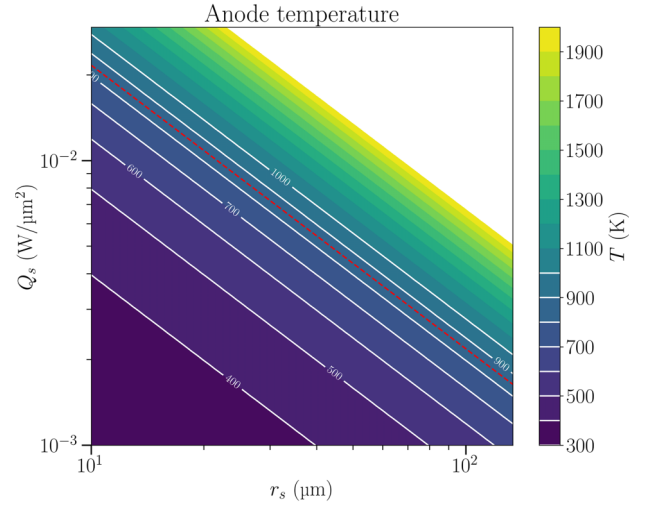


FIG. 9. Dependence of the maximum steady-state temperature (up to 2000 K) on the Cu anode surface as a function of heat spot radius r_s and surface power density Q_s (energy transfer rate per unit area).

IV. DISCUSSION

The presence of high magnetic fields has been shown to decrease the performance of devices operating at high electric fields [25]. There is support for the theory that magnetic fields can significantly focus emitted electron beams and lead to the heating of metal surfaces at impact. Our simulations have demonstrated that the increase in temperature can be very significant, depending on voltage, magnetic field, or geometry. Localized temperatures can be high enough for metal evaporation to start, which is an important precursor process to vacuum arc formation. Sufficient evaporation can lead to ionization and the formation of a conducting pathway between the anode and cathode. These results suggest that arc formation may begin on the anode side when high magnetic fields are present.

Particle simulations suggest that the geometry of the metal surfaces, as well as the shape of the electric and magnetic fields, can significantly impact the motion of the emitted electron beam. Simulation of these processes can give insight beyond theoretical solutions, since actual accelerating structure geometries can be studied in detail. Depending on the situation, emitted electron beams can be either focused or defocused, which changes the heating dynamics of the anode surface. A more focused beam leads to higher current density and greater heat flux on the anode surface, leading to higher temperatures and possible evaporation of neutral atoms from the anode surface. The strength of the magnetic field is a major factor influencing heating of the anode surface and arc occurrence, since greater magnetic fields have the potential to lead to more tightly focused electron beams.

Previous simulations have considered fatigue damage and plastic deformation as the origin of arc formation in rf

accelerating structures [14,15,26]. In the rf case, the roles of the cathode and anode cannot be easily distinguished, which makes simplified dc simulations useful for complementary understanding of the underlying phenomena. We found that at local electric fields of 10 GV/m, emitted electron current can lead to very high temperatures on the anode side at magnetic fields on the order of 10–30 T. At such temperatures, metal evaporation flux on the anode surface would be high, suggesting an anodic arc formation mechanism. This is a distinctly different mechanism, since it does not arise from the cathodic self-heating that has been proposed as the main mechanism of vacuum arc initiation. Surface temperatures will depend on the energy deposited by the electron beam, which results from the accelerating electric field, i.e., applied voltage and gap size. In pulsed mode, the heating is greatly reduced compared to the dc case.

We have focused on electron dynamics and heating, ignoring the formation of plasma that has been studied in cathodic arcing [24,36]. The role of the magnetic field for plasma formation is still unclear, but magnetic fields could lead to plasma confinement. This would lead to higher number densities, higher collision rates, and possibly faster plasma development. Bombardment of the cathode by ions originating from the anode could lead to further plasma development on the cathode side. While the dynamics of nanotips has been studied previously [19], the presence of an external magnetic field could impact this process. An external magnetic field could lead to additional forces and torque on an emitting tip. Additionally, the role of the material used can have an impact. The performance of materials such as Be, Cu, and Al has been compared previously [14], which suggests significant differences. To study this material dependence in more detail, a model considering both their thermal and mechanical properties would be needed.

V. CONCLUSIONS

High magnetic fields can change the nature of vacuum arc occurrence and reduce the performance of devices operating at high electric fields. Therefore, understanding the vacuum arcing phenomenon in the presence of magnetic fields is important for the design of such devices. We have studied the behavior of emitted electron beams in high electric and magnetic fields to better understand the underlying processes that contribute to arc occurrence. To distinguish between the mechanisms of cathodic self-heating and anodic electron heating, we performed dc simulations in simplified geometries. The direction of the electric and magnetic fields was shown to be an important factor for electron beam focusing and heating of the anode surface, which can contribute to plasma formation. The case where the electric and magnetic fields are in the parallel configuration was found to be most detrimental for

performance, leading to the greatest anode heating, while the opposite was observed in the perpendicular case. Similarly, the magnitude of the magnetic field was found to be a major contributor to the heating on the anode surface. The temperatures reached on the anode surface suggest that evaporation of neutrals from the anode side is a possibility in high magnetic fields on the order of 10 T, which is a different mechanism compared to the much lower heating seen at magnetic fields on the order of 1 T. Our results support the role of focused electron beams as the origin of the anodic vacuum arcing mechanism. A direction for future work is the simulation of full anode-initiated plasma formation in the cathode-anode gap.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not

technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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