

**Tunable, unmountable, permanent-magnet-based accelerator magnet**Dániel Barna<sup>\*</sup>*HUN-REN Wigner Research Centre for Physics, 1121 Budapest, Konkoly-Thege Miklós út 29-33, Hungary*Gábor Anda<sup>†</sup>*HEN-REN Centre for Energy Research, 1121 Budapest, Konkoly-Thege Miklós út 29-33, Hungary*

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This paper presents the concept and design of a novel accelerator magnet consisting of two concentric rings of permanent magnets arranged in a Halbach layout. By replacing one component, the device can be converted to any transverse multipole configuration. By the individual rotation of the two rings, both the strength and the direction of the field pattern can be continuously tuned in a wide range, including the special case of polarity reversal. The tuning range can be changed by a parametric change of the design. The device is split into two halves and can be easily opened to allow for its installation and removal around an existing beam pipe. The performance of a quadrupole demonstrator prototype is presented, including measurement results of field quality, integrated gradient and hysteresis, and discussion of sensitivity to the remanent magnetization of the permanent magnets.

DOI: [10.1103/m58n-mjcb](https://doi.org/10.1103/m58n-mjcb)**I. INTRODUCTION**

In the pursuit of advancing particle accelerator technology, the quest for more efficient and versatile magnet systems has been relentless. Traditional accelerator magnets, relying heavily on electromagnets, have long been the backbone of particle accelerators worldwide. These electromagnets, while effective, often come with drawbacks such as high power consumption and complex cooling requirements. With the increase of the cost of energy, the quest for energy-efficient systems is more vivid than ever before.

Normal-conducting electromagnets are limited to about 2 T due to the saturation of their iron yokes. In the high-field domain, superconducting magnets have been employed to either decrease the energy consumption or to make realization of the desired magnetic field possible at all. These magnets require expensive cryogenic cooling systems using liquid helium.

However, amid these challenges, a promising solution lies in the realm of permanent magnet-based tunable accelerator magnets. These innovative systems harness the inherent properties of permanent magnets (PMs) while incorporating mechanisms for tunability, offering a

compelling blend of stability, control, and energy efficiency. There are numerous examples for their use in particle accelerators. The two primary concepts to overcome their “permanent” nature and make tunable systems are either the movement of the permanent magnets or other ferromagnetic components, or the superposition of electromagnets, with the first option being more widespread. Practical realizations of these concepts show a very large variety, often having limitations. It is interesting that while Halbach proposed a tunable quadrupole by two nested, rotatable Halbach rings already in his original paper in 1980 [1], and the pure concept of an openable nested Halbach multipole was patented already in 2009 [2] and a single, openable Halbach ring was even constructed recently [3] that can be mounted around a long sample, no practical designs of the combined, extremely versatile and powerful concept have been realized to date, as far as we know. In the following, we give an overview of permanent magnet-based accelerator magnets trying to cite examples using different construction and tuning concepts. This overview is far from being complete.

Halbach and coworkers constructed a hybrid quadrupole consisting of four iron pole pieces sandwiching permanent magnets [4]. The gradient strength could be changed by a factor of 1.5 by rotating an outer ring of permanent magnets, which either added or subtracted flux from the pole pieces. Calculations indicated that a tunability by a factor of 3 is achievable with the same concept.

Gupta [5] proposed dipole magnets utilizing Strontium Ferrite bricks with remanent magnetization of 0.38 T, an iron yoke and iron pole plates. In different variations of the same principle, simulations predicted a field up to 0.42 T in

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a gap of 31.75 mm. A tunability exceeding a factor of 5 is achieved through movable shunts that divert the magnetic flux away from the gap. We are not aware of this concept having been realized.

The Sirius project at Brazil's LNS facility uses 20 "Superbend" magnets [6,7] generating a maximum field of 3.2 T in a gap of 11 mm at the center and feature long combined function sections on either side with a 0.5 T field and a 9.5 T/m gradient. Adjustable "floating poles" and a control gap in the return yoke allow for a  $\pm 4\%$  tuning range in both the magnetic field and gradient. A prototype magnet has been constructed and tested.

The 128 longitudinal gradient dipoles of the "Extremely Brilliant Source" upgrade at the ESRF facility are constructed using  $\text{Sm}_2\text{Co}_{17}$  PMs [8] and produce a magnetic field varying between 0.17 and 0.67 T in a vertical gap of 26 mm along the 1.85 m length of the magnet. The magnets are not tunable but include Fe-Ni shunts for temperature compensation.

The Cornell Electron Storage Ring (CESR) used non-tunable PM quadrupoles consisting of wedge-shaped PMs tightly arranged in a circular, thick Halbach layout [9]. Three units of a length of 9.2 cm were bolted together, producing a field gradient of about 29 T/m in an aperture with a diameter of 67 mm.

For the SPring-8-II upgrade a PM-based dipole was proposed, which consists of three independently tunable units with a field of 0.13, 0.23, and 0.45 T and a length of 100 mm in a gap of 25 mm. The field can be tuned to 30% of its maximum value by a movable outer plate which diverts the flux away from the gap. Fe-Ni magnetic shunts reduce temperature-related field variations [10].

The CBETA 4-pass energy recovery linac has a non-scaling FFAG lattice, entirely based on 214 PM-based magnets [11]. The two different types of quadrupoles and three different types of combined function magnets are realized by wedge-shaped NdFeB PMs arranged in a Halbach ring pattern. For the combined function magnets, the PM blocks vary in size azimuthally around the beam to create the dipole field component. The length and aperture of the magnets vary between 61–133 mm and 80–100 mm, respectively. The dipole and quadrupole strengths are 0.1–0.3 T and 11.5 T/m, respectively. The magnets are temperature stabilized by water chillers to decrease temperature related field variations and are not tunable. Average relative field errors of a few times  $10^{-4}$  were achieved by passive shimming using iron wires.

At the COXINEL laser-plasma experiment (QUAPEVA project [12,13]), strong small-aperture quadrupoles are required to focus a highly divergent plasma-accelerated electron beam. SOLEIL has developed a highly tunable PM quadrupole. It features a central hybrid Halbach ring combined with rotating PM cylinders driven by four independent motors. The magnet aperture is 12 mm, with an adjustment range of 100–200 T/m. Initial prototypes had

issues with magnetic center movement during adjustment, but this has been reduced to around  $\pm 10 \mu\text{m}$  in later models. A triplet of QUAPEVA magnets has been installed on the COXINEL beamline.

For the Next Linear Collider, a hybrid tunable quadrupole was designed and constructed [14]. It uses four sets of PMs which are retracted by four motors, between four steel pole pieces. It produced an integrated gradient of 68.7 T over a length of 42 cm, in an aperture of 13 mm. Magnetic center stability was better than  $0.2 \mu\text{m}$  during a strength adjustment of 20%.

The ZEPTO (ZEro-Power Tunable Optics) collaboration developed two designs with different strengths for a highly tunable (by about a factor of 4) PM-based quadrupole for the Compact Linear Collider (CLIC) Drive Beam Decelerator [15,16]. Both designs are based on moving a pair of PMs vertically in opposite directions by a single motor and several gearboxes to vary the flux going through four steel poles. The high/low-strength design produces a gradient of 60.4/43.8 T/m (integrated gradient: 14.6/8.5 T) with a pole length of 230/180 mm, respectively. The high-strength design creates a variable air gap between the PMs and the poles and requires a force of 16.4 kN to move the magnets. The low-strength design moves the PMs perpendicular to their magnetization to redirect the flux into an external steel yoke, requiring a maximum force of 0.7 kN.

At Bhabha Atomic Research Centre, a PM quadrupole was constructed in a Halbach ring arrangement [17]. The paper's title describes this device as tunable. However, it only means that the integrated gradient of the individual units can be varied during manufacturing by adjusting the length of the attached iron poles.

At Lawrence Livermore National Laboratory, an adjustable final focus system was constructed from Halbach rings containing wedge-shaped permanent magnets with a varying direction of magnetization [18,19]. For the 2 MeV  $\text{H}^-$  RFQ linac of the University of Bern, an adjustable focusing system was realized using Halbach rings consisting of square permanent magnets [20]. It is important to note that "adjustability" in these two cases does not refer to the individual quadrupoles but to the entire system by varying the drift space between the nonadjustable magnets.

The nested Halbach ring presented in [21] contains a single, fixed inner ring, enveloped by an outer ring, which is segmented along the beamline into 4 parts of lengths 1, 2, 4, and 8 cm. These segments can individually be rotated by  $90^\circ$  (so as to be either parallel or antiparallel to the field of the inner ring), producing 16 different discrete values of the integrated gradient between 3.47 and 24.2 T in an aperture with a diameter of 20 mm.

A final focus permanent-magnet quadrupole was designed and constructed for the International Linear Collider [22]. The device uses Gluckstern's configuration [23,24] consisting of five axially stacked, rotatable Halbach

rings with specific lengths. Tuning is achieved by rotating the individual rings in a way that the coupling between the two transverse planes, introduced by the rotated individual rings, cancels from their integrated effect.

The same paper [22] describes a nested double Halbach ring sextupole to focus very cold neutrons. It consists of a fixed inner ring and a rotating outer ring to modulate the field strength with a frequency up to 30 Hz.

Further examples with small apertures and consequently quite strong gradients can be found in the extensive overview of PM-based quadrupoles [25] with a table comparing their parameters.

The review paper by Blümner and Soltner [3] presents a very clear illustration of the concept of Halbach arrays or rings and an extensive summary of the theory and practical aspects. The design of a Halbach ring for NMR application (named Mandhala—Magnet Arrangements for Novel Discrete Halbach Layout) is presented in [26]. Blümner published an extensive review [27] on the magnetic guiding of paramagnetic objects (mentioning medical devices, miniature robots, nanoparticles in microfluids, etc.) with permanent magnets, in particular with Halbach rings. The author briefly mentions the magnetic guiding of charged particles via the Lorentz force, specifically the application of Halbach rings in particle accelerators. A patent [2] filed by Blümner and Soltner even describes the pure concept of a split and openable, rotating nested Halbach ring device, applicable to an “elongated sample whose ends are not accessible,” without associating this “sample” with a beamline.

Several of the above-mentioned devices were designed to comply with very strict requirements on field quality, magnetic center stability, etc. The aim of the current work was to develop a general, versatile device directly applicable in various accelerators or beamlines, which can be easily converted between dipole, quadrupole, or higher multipoles, easily scaled or adapted to different specifications, and therefore commercialized.

## II. CONCEPT

Using the complex notation for a 2D transverse magnetic field (“hat” over a symbol denoting a complex number), an arbitrary pattern can be described as a complex Taylor series (also known as multipole expansion)

$$\hat{B} \equiv B_y + iB_x = \sum_{m=1}^{\infty} \hat{A}_m \left( \frac{\hat{z}}{r_0} \right)^{m-1}, \quad (1)$$

where  $B_x$  and  $B_y$  are the horizontal and vertical field components, respectively,  $\hat{z} = x + iy$  is the complex coordinate in the 2D transverse plane,  $r_0$  is the radius of a reference (sampling) circle, and  $\hat{A}_m$  are complex amplitudes: their magnitude describes the strength of the given multipole, and their phase describes the rotation of the field pattern around the origin compared to the *normal* field

pattern (characterized by having  $B_x = 0$  along the  $x$  axis). In accelerator magnets, typically a single multipole component is desired (called the *main multipole*,  $m_0$ ), and upper limits are set for the other (unwanted) multipole amplitudes.

Let us superimpose two pure multipole fields with the same multipolarity (i.e., each containing a single term in this expansion with the same  $m_0$ ), where coefficients  $\hat{A}_{m_0}^{(1)}$  and  $\hat{A}_{m_0}^{(2)}$  are real and positive, so that the hat will be removed from their symbols in the following discussion. The upper index in parentheses will differentiate the two field patterns. In this case, both fields are normal and they interfere constructively. Let us now rotate the two field patterns by the angles  $\vartheta^{(1)}$  and  $\vartheta^{(2)}$ , respectively. The rotation of the magnetic field vector  $\hat{B}$  by an angle  $\vartheta$  can be described as a multiplication by  $e^{-i\vartheta}$ . The rotation of the entire field pattern can be described by applying this operation to the original field pattern evaluated at the coordinates rotated by the angle  $-\vartheta$  (i.e., multiplied by  $e^{-i\vartheta}$ ). The superimposed field is therefore

$$\begin{aligned} \hat{B} &= \sum_{j=1,2} \hat{B}^{(j)} = \sum_{j=1,2} e^{-i\vartheta^{(j)}} A_{m_0}^{(j)} \left( \frac{e^{-i\vartheta^{(j)}} \hat{z}}{r_0} \right)^{m_0-1} \\ &= \sum_{j=1,2} A_{m_0}^{(j)} e^{-im_0\vartheta^{(j)}} \left( \frac{\hat{z}}{r_0} \right)^{m_0-1}. \end{aligned} \quad (2)$$

This is still a pure multipole field with the same multipolarity, which is normal if the complex magnetic field is real:

$$\text{Im}(\hat{B}^{(1)} + \hat{B}^{(2)}) = 0. \quad (3)$$

Assuming that  $A_{m_0}^{(2)} \leq A_{m_0}^{(1)}$ , we can express  $\vartheta^{(1)}$  from  $\vartheta^{(2)}$  at any value of the latter:

$$\vartheta^{(1)} = -\frac{1}{m_0} \text{asin} \left[ \frac{A_{m_0}^{(2)}}{A_{m_0}^{(1)}} \sin(m_0\vartheta^{(2)}) \right]. \quad (4)$$

In case the inequality relation between  $A^{(1)}$  and  $A^{(2)}$  is reversed, the two indices in this equation must be swapped so that the angle of the ring with the stronger field is expressed from the angle of the weaker ring. In case  $A^{(1)} = A^{(2)}$ , this equation simplifies to  $\vartheta^{(1)} = -\vartheta^{(2)}$ . This means that by a proper, synchronized rotation of the two fields, described by Eq. (4), the field pattern can be kept normal. Its strength, given by

$$\left| A_{m_0}^{(1)} e^{-im_0\vartheta^{(1)}} + A_{m_0}^{(2)} e^{-im_0\vartheta^{(2)}} \right|, \quad (5)$$

reaches a maximum of  $(A_{m_0}^{(1)} + A_{m_0}^{(2)})$  at  $\vartheta^{(1)} = \vartheta^{(2)} = 0$ , and a minimum of  $(A_{m_0}^{(1)} - A_{m_0}^{(2)})$  at  $\vartheta^{(2)} = \pi/m_0$  and  $\vartheta^{(1)} = 0$ , where the two fields are aligned in opposite directions.

The entire tuning range (from  $A_{m_0}^{(1)} - A_{m_0}^{(2)}$  to  $A_{m_0}^{(1)} + A_{m_0}^{(2)}$ ) can be covered by scanning  $\vartheta^{(2)}$  in the range

$$[k\pi/m_0; (k+1)\pi/m_0], \quad (6)$$

where  $k$  is an integer, and  $\vartheta^{(1)}$  is determined by Eq. (4). This has an important consequence for the requirements on the driving mechanism, as we will show later.

By a common rotation applied to both fields, the resulting pattern's angle can be arbitrarily changed—as special cases, it can be rotated into a skew pattern ( $B_y = 0$  along the  $x$  axis) or its sign can be reversed.

Halbach rings have long been known [1] as an effective arrangement of permanent magnets to generate transverse multipole fields. Our device realizes the above concept with independently rotatable nested Halbach rings with the same multipolarity, proposed in that paper [1]. A graphical illustration for the underlying principle is given in [3]. Referring to two specific examples: (1) In the case of a dipole (deflecting) field, the direction of the magnetic field (deflection) is freely adjustable, and its magnitude (the degree of deflection) can be varied between the extreme values determined by the rings. This would be the equivalent of a corrector magnet with independently tunable  $B_x$  and  $B_y$  fields or a 2D scanning magnet. However, the device is of course also applicable as a bending magnet without utilizing the feature of the adjustable field direction. (2) In the case of a quadrupole (focusing) field pattern, the direction of the focus planes can be arbitrarily adjusted to introduce or cancel a coupling between the two transverse planes or to swap polarity. The magnitude of the field pattern (i.e., the strength of the focusing) can be varied between the extreme values determined by the rings.

The length of the magnetic volume created by this unit along the geometric axis in both cases is limited by the length of the permanent magnets, but similarly to the NMR Mandhala [26], any length of magnetized volume can be created by axially stacking several units.

### III. PROTOTYPE DESIGN

A prototype device with two nested quadrupole Halbach rings was designed and constructed by Rubiclin Ltd. [28]. Both rings are split into two halves, as illustrated in Fig. 1. The PMs (1) are installed in rectangular slots machined by wire EDM into the aluminum body of the rings and are fixed by two cover plates (2). The PMs have a square cross section with dimensions of  $10 \times 10 \times 50$  mm ( $\pm 0.1$  mm) and were purchased from Neomagnet Ltd. [29]. They have a relative error on the remanent magnetization specified as  $\pm 2\%$ . The inner/outer rings contain 16/24 PMs, respectively. The slots had an increased size of  $10.15 \times 10.15$  mm to accommodate the size error of the PMs. In case of the inner ring, there are extra spacers at both ends of the PM bars, since the ring body has a larger axial extent

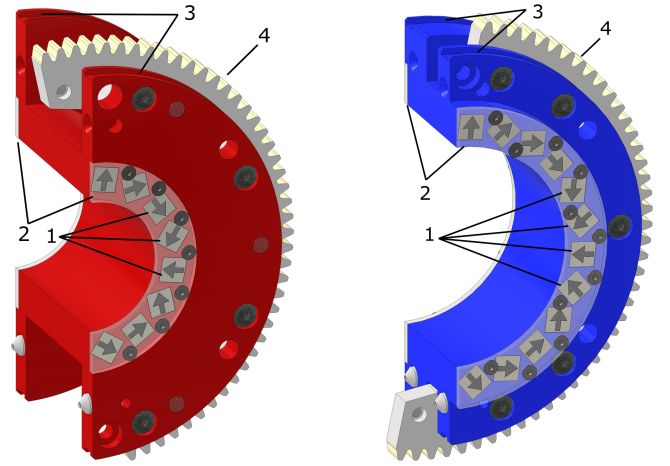


FIG. 1. Inner (left) and outer (right) half rings. (1) permanent magnets, (2) magnet covers, (3) grooves for split bearings, and (4) gears. The arrows indicate the direction of the magnetization. Axial spacers are added to the PM bars at both of their ends in the inner ring's slots (not shown).

than the permanent magnets. Each ring features two circular grooves (3) for slim split bearings (KMF PBXA 075 HS) allowing their independent support and rotation in an external housing (see later). Split gears (4) are mounted to the body of the rings with an azimuthal offset to allow their mechanical closure after installation around the beamline. This proved to be unnecessary in practice. A pair of two identical half rings constitutes a full ring.

During assembly, one inner and one outer half ring are first mounted together using temporary fixation pins (not shown) and installed and temporarily fixed into a half support block (2 and 3 in Fig. 2) by bolts (not shown). The fixed half unit is installed first onto the adjustable base plate (1) via a dove-tail interface which allows accurate repositioning in all degrees of freedom without realignment. The openable half unit (3) is then placed to its top, upside down, and the hinge axis (4) is installed. The openable half unit can then be easily folded down by hand.

The interface plane between the two half rings must naturally lie between the slots of the permanent magnets. Blümli and Soltner propose [2,3] to choose this plane, and the relative azimuthal alignment of the two rings in the assembly position such that the force between the two halves is minimal. In the current design, we chose the assembly position and interface planes of the rings as shown in Fig. 5. The two rings are aligned with nearly parallel field patterns, and are bisected directly next to the PM at one of their poles. In this configuration, there is a repulsive force between the two halves, which helps to avoid bouncing them together causing damage. The openable half unit, when hanging only at the top hinge by its own weight, leaves a gap of about 2 cm to the fixed half unit at the bottom. A simple threaded closing mechanism (5 and 6 in Fig. 2) and special features make it then very simple to

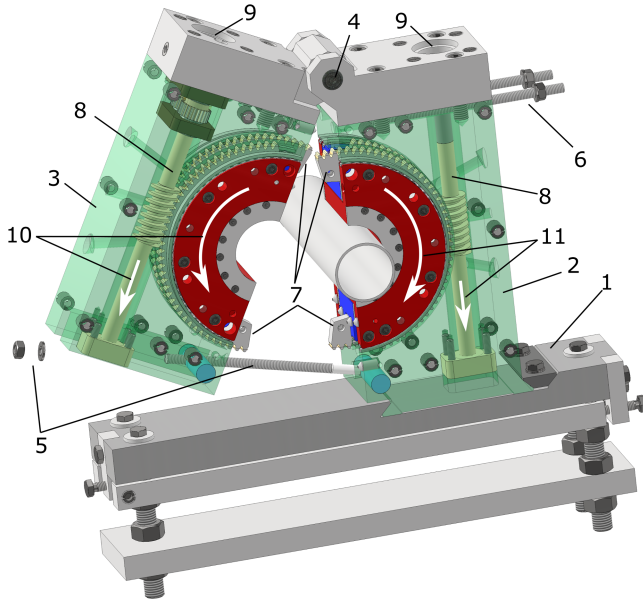


FIG. 2. Illustration of the device during installation around the beamline. (1) Adjustable base plate, (2) fixed half unit, (3) openable half unit, (4) hinge, (5,6) closing bolts, (7) azimuthally offset gears, (8) worm gear shafts, (9) stepper motor mounting interface, (10) torque on the inner ring and force on its worm gear shaft in the operational range, and (11) torque on the outer ring and force on its worm gear shaft in the operational range. The two stepper motors are not shown.

close this gap with precise alignment of the two halves in all degrees of freedom. As described before, the azimuthally offset gears (7 in Fig. 2) were planned to allow the mechanical fixation of the two half rings with extra bolts. However, the play-free clamping by the two half support blocks with very precisely machined surfaces and alignment features at all interfaces seems to make it unnecessary, rendering the assembly process around the beamline even simpler.

Two worm gear shafts (8 in Fig. 2) drive the two rings separately. Two stepper motors with a maximum rated torque of 1.3 Nm (mounted at the top of the device at the interface marked with 9, not shown) were sufficient to rotate the two rings.

In a series of initial tests, clear clicking sounds were audible while the rings were rotated, which were associated with the permanent magnets snapping between two extreme alignments within their slots, which was loose in both transverse directions by about 0.15 mm (see earlier). Having gained experience in manipulating and installing the permanent magnets, the magnets were removed and shimmed with a commercial, self-adhesive kapton tape (about 0.1 mm thick) at their north-eastern corners. This introduced a systematic displacement of the magnets but eliminated the clicking sound and rendered the measurement results more comprehensible and predictable.

The built-in drivers of the stepper motors were directly driven by a Raspberry Pi 4 computer without any intermediate hardware, using a custom C++ application

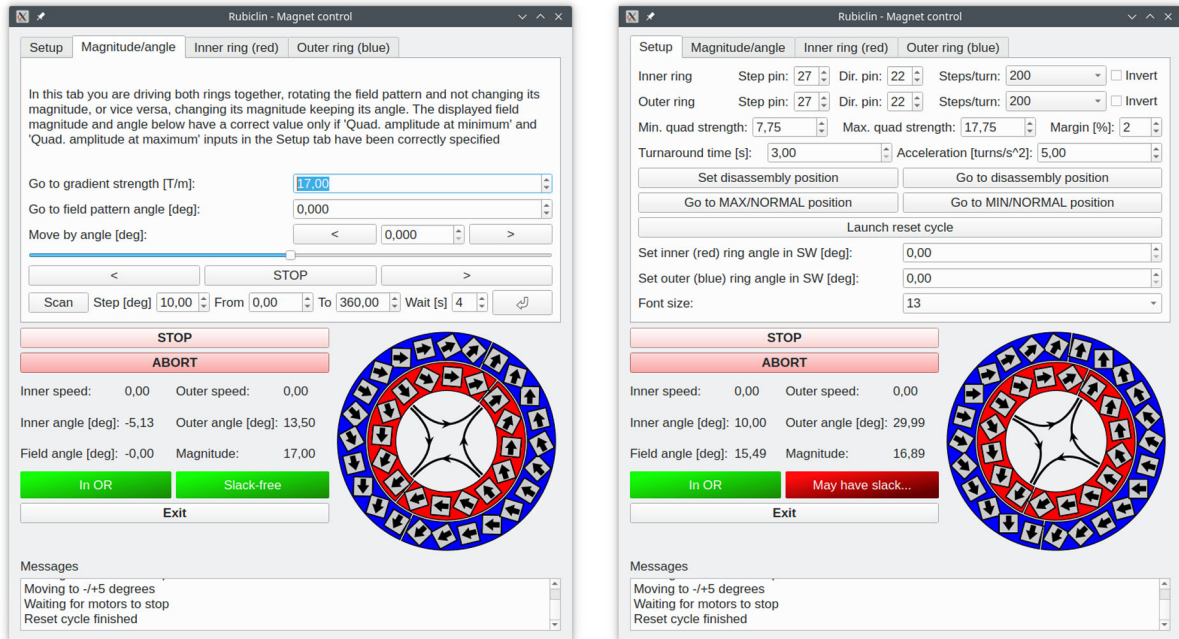


FIG. 3. Screenshots of the control GUI. Left: user-friendly interface to set focusing strength and field pattern orientation. Right: setup interface to set parameters and to launch standard service protocols.

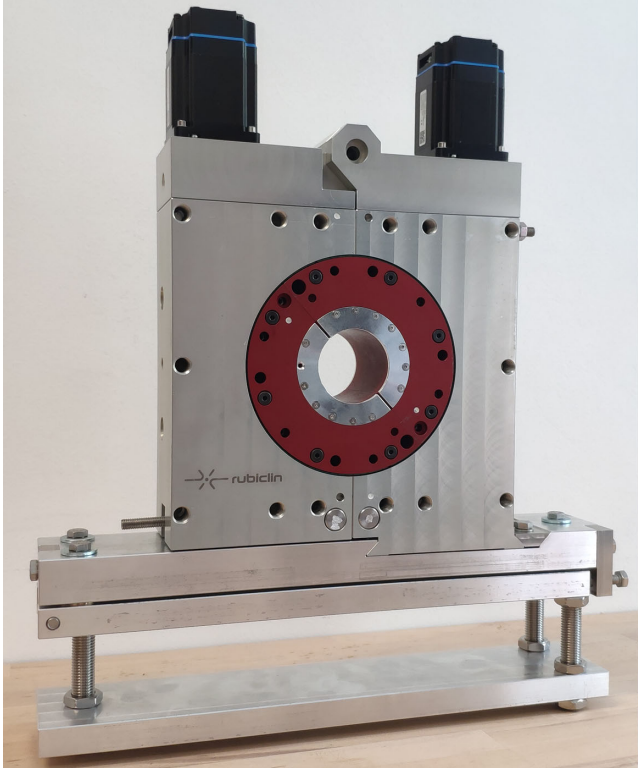


FIG. 4. Photograph of the constructed quadrupole demonstrator prototype.

developed for this device, with a graphical user interface. The software provides setup and expert interfaces where parameters can be defined, standard service protocols can be launched and the two rings can be driven independently, and an application-friendly interface where—once the field strengths of the two rings have been determined—the user can directly set the quadrupole strength or the angle of the field pattern. Two indicators give feedback whether the device is currently in the operational range or if it might be affected by mechanical slack (see later). The orientations of the rings and the field pattern are displayed graphically, following their actual movement. Figure 3 shows screenshots of the software.

The device is capable of switching between its minimum and maximum field configurations within approximately

1 s, thanks to the low-friction bearing and gear design. This rapid transition is particularly valuable in experimental setups requiring dynamic changes in the optics.

The concept, design, and the first photos of the assembled prototype were presented at the 3rd I.FAST Annual Meeting [30], but the slides have been removed from the public domain for IP protection purposes. A photograph of the constructed device is shown in Fig. 4. The parameters of the device are summarized in Table I.

#### IV. MEASUREMENT PRINCIPLE

A standard way to measure the field quality of accelerator magnets is the rotating coil method. A coil is mounted on a rotating shaft inside the magnet's aperture, and samples the radial field, producing an inductive voltage. The multipole amplitudes are directly derived by a Fourier analysis of the signal. The precise alignment of the rotating shaft with the magnet axis is important to avoid harmonic feed-down. The present construction offers to invert this approach and rotate the magnet around a fixed measurement coil or a Hall sensor. It must be emphasized, nevertheless, that this is not perfectly equivalent to the field quality measurement of the stationary device by a rotating sensor. The interplay of the magnetic forces with the split mechanical structure can eventually create varying deformations during the rotation of the rings, which are absent in the second case. We will assume nevertheless that these effects do not play a role and interpret our results accordingly.

In a quick first set of measurements, a Hall sensor (Rinich Industrial Co. Ltd., type HT208 [31]) was mounted in the magnet aperture at about  $2/3$  of the aperture ( $x = 18 \pm 1$  mm,  $y = 0 \pm 1$  mm). The sensor's sensitive axis was aligned by eye to be horizontal. As we will argue later, these kinds of misalignments are relevant only for the measurement of absolute multipole amplitudes (introducing an uncertainty in their magnitude, and a common angle error related to the common rotation of the two rings) but cancel in field quality measurements deriving normalized, relative multipole amplitudes.

The angles  $\vartheta^{(1)}$  and  $\vartheta^{(2)}$  of the two rings are measured from the theoretical normal maximum position, where their northern poles are at  $45^\circ$  with respect to the horizontal axis (Fig. 5). A positive angle means anticlockwise rotation in this figure and in Fig. 2. In the assembly position, the interface planes of both rings are aligned with the vertical interface plane between the two halves of the support blocks, and the two rings are at the angles  $\vartheta_1 = 34.67^\circ$ ,  $\vartheta_2 = 38.33^\circ$ . The device is naturally in this position after the initial assembly. If the position record of the stepper motors is lost for any reasons, this initial position can be reset by first carefully tuning the inner (red) ring to have its bisection plane aligned with the vertical midplane, then carefully tuning the outer (blue) ring until the four alignment pins between the two rings can be inserted.

TABLE I. Prototype parameters.

| Parameter                                                        | Value      | Unit |
|------------------------------------------------------------------|------------|------|
| Aperture diameter                                                | 54         | mm   |
| Device length                                                    | 80         | mm   |
| Permanent magnet length                                          | 50         | mm   |
| Number of magnets, inner/outer ring                              | 16/24      |      |
| Permanent magnet grade                                           | NdFeB N52  |      |
| Remanent magnetization $B_r$                                     | 1.47       | T    |
| Gradient (device center)                                         | 7.75–17.75 | T/m  |
| Integrated gradient (simulation scaled to measurement at center) | 0.36–1.01  | T    |

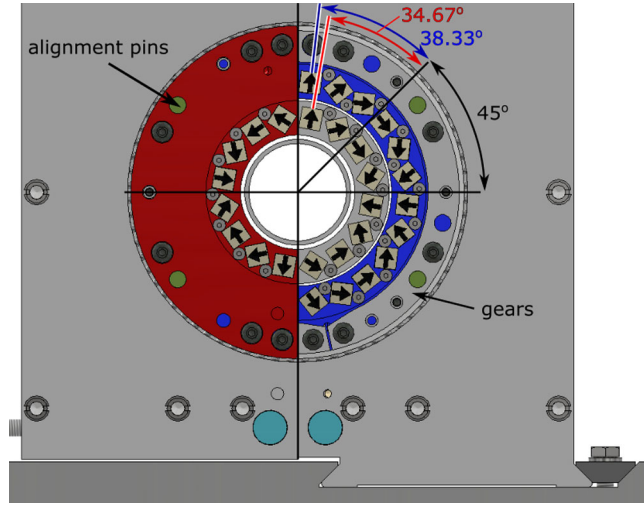


FIG. 5. Explanation of the ring orientations—the device in assembly position. The right half of the inner (red) ring is invisible, revealing the right half of the outer (blue) ring and the permanent magnets. The blue (red) lines indicate the north poles of the outer (inner) rings, respectively. The black line indicates the nominal orientation of the rings' north poles in the maximum-strength setting with normal field alignment.

In future versions, a rotary encoder is planned to provide absolute readings of the azimuthal alignments of the rings.

Mechanical slack in a system can spoil accuracy and introduce hysteretic behavior. Eliminating slack typically requires a good design, high precision of the components, or an adjusting mechanism. The former increases the manufacturing costs, while the latter adds extra work. What is more, wearing of the components (re)introduces slack with time.

In the current device, very precise features were inevitable to accurately close and match the two half rings and the two half support blocks, and to support the two rings coaxially, without play. Operational movement of the components (the rotation of the rings) is realized via ball bearings with negligible friction and wear. However, the driving system, a worm gear, is another source of possible slack and positioning inaccuracies. The device was designed such that at least this subsystem does not require overengineered, very precise components. Using a proper operating protocol, the rotation of the rings can be made slack-free, with very high repositioning accuracy, as explained below.

The magnetic torque between the two rings tries to align their field patterns to the same direction, i.e., to the maximum strength alignment (MAX), which is therefore a stable equilibrium. The minimum strength alignment (MIN, the field patterns having opposite directions) is an unstable equilibrium. The relative angle between the two rings is restricted to the operational range (OR): between a MIN and its neighboring MAX position with some margin. In this range, the magnetic torque on both rings remains always finite without changing sign. Note that this is a restriction only on the relative angle between the two rings

and still allows their common, arbitrary rotation. We have chosen a margin of  $10^\circ$ . The OR is chosen such that the torques on the two rings, and the corresponding forces on their driving worm gear shafts, are as shown in Fig. 2 (white arrows marked by 10 and 11), pointing in the same direction as gravity. Note that every second scanning range in Eq. (6) represents a possible OR. This ensures that these two forces never compete, and the worm gears will always be positioned at the lowest extremity of their axial (vertical) slack. Similarly, the rings will always be aligned to one extremity of their rotational slack, regardless of the driving direction. If an eventual wearing of the worm gear or the gear of the rings during long-term operation would cause measurable deviations, this could be compensated by the driver software, introducing some offset for the stepper motor positions.

After the initial assembly or installation around a beam-line, or each time the device quits the operational range, a reset cycle must be carried out to ensure that all components are at the desired extremity of their slack. This cycle consists of scanning the device through the operational range in a way that both worm gears are rotating when the maximum torque is crossed. The rotational movement of the worm gear shafts ensures that the magnetic torque will drive them to the desired extremity of their axial slack. In practice, we have chosen the following protocol: (1) drive the device to normal MAX position, (2) drive the inner ring to  $-80^\circ$  and back to  $-5^\circ$ , and (3) drive the outer ring to  $80^\circ$  and back to  $5^\circ$ .

The goal of the field quality measurement of accelerator magnets is to characterize their field pattern in terms of multipole amplitudes. In our case, this characterization needs to be done at different relative alignments of the two rings (different field strengths), or equivalently, both rings need to be characterized separately, *in situ*. Precisely knowing the amplitude of the main multipole of each ring is also important for the synchronous rotation of the two rings [Eq. (4)]. The following paragraphs describe this measurement method.

This device has been designed and manufactured with an inherent quadrupole symmetry. In case of an ideal device with no manufacturing, assembly or remanent magnetization errors, only the multipoles  $m = 2 + 4 \cdot k$  would be allowed, where  $k$  is an integer number (the higher order multipoles are caused by the discrete nature of the PMs). What is more, all amplitudes  $A_m^{(i)}$  ( $i = 1, 2$ ) would be real. If one wants to measure the effect of possible imperfections, these assumptions need to be given up. In the following, we describe the mathematical framework that can account for the uncertainties of the Hall sensor's position and alignment, described earlier, and the field errors caused by hardware imperfections.

Let the Hall sensor's sensitive axis have an angle  $\eta$  with the  $x$  axis. The value measured by the sensor is the projection of the magnetic field onto this axis:

$$B_H = B_x \cos(\eta) + B_y \sin(\eta) = \text{Im}\{\hat{B} \cdot e^{i\eta}\}. \quad (7)$$

Let the measurement point be at the complex coordinate  $r_0 \cdot e^{i\mu}$ . Taking into account the unwanted multipole components as well, the magnetic field at this point, at the rotation angles  $\vartheta^{(1)}$  and  $\vartheta^{(2)}$  of the two rings is

$$\hat{B} = \sum_m \exp[i(m-1)\mu] \left( A_m^{(1)} \exp[-im\vartheta^{(1)}] + A_m^{(2)} \exp[-im\vartheta^{(2)}] \right), \quad (8)$$

therefore the value measured by the Hall sensor is

$$B_H = \sum_m \text{Im} \left\{ \hat{F}_m (A_m^{(1)} \exp[-im\vartheta^{(1)}] + A_m^{(2)} \exp[-im\vartheta^{(2)}]) \right\}, \quad (9)$$

$$\hat{F}_m = \exp[i(m-1)\mu + i\eta] = \exp[im\mu] \exp[i(\eta - \mu)]. \quad (10)$$

Here, the phase factors  $\hat{F}_m$ , common to both rings, contain the uncertainties of the alignment of the Hall sensor's sensitive axis ( $\eta$ ) and the uncertainties of its position ( $\mu$ ).

If one of the rings is rotated around by a full turn in discrete steps and the measured field is Fourier analyzed, the field of the (static) other ring only constitutes a steady background field. The multipole amplitudes of the rotated ring could be determined from the Fourier spectrum, up to the unknown phase factors  $\hat{F}_m$ , apart from the (normally absent)  $m = 0$  cusp component, which is shadowed by the steady field of the other ring. In this way, both rings could be characterized separately. A problem with this approach is that the device quits the operational range, the relative torque between the two rings changes sign during the scan several times, and the mechanical slack introduces perturbations in the measurement data which would be very hard to interpret.

In order to avoid this problem, we propose a different measurement strategy. Let us introduce the average and difference angles

$$\sigma = [\vartheta^{(1)} + \vartheta^{(2)}]/2, \quad (11)$$

$$\delta = \vartheta^{(1)} - \vartheta^{(2)}. \quad (12)$$

With these variables, the Hall sensor's reading can be described as

$$B_H = \sum_m \text{Im} \left\{ \hat{F}_m (A_m^{(1)} \exp[-im\delta/2] + A_m^{(2)} \exp[im\delta/2]) \exp[-im\sigma] \right\}. \quad (13)$$

Now, fixing  $\delta$  within the operational range,  $\sigma$  can be scanned by a full turn (meaning a common rotation of the two rings) without swapping the sign of the relative torque between the rings. The  $-m$ th component of the Fourier spectrum of these samples is then

$$\frac{\hat{F}_m}{2i} \left( A_m^{(1)} \exp[-im\delta/2] + A_m^{(2)} \exp[im\delta/2] \right). \quad (14)$$

This analysis can be made at different values of  $\delta$  within the operational range, as a function of which these values trace a 2D curve in the complex plane. We note that this trace is the superposition of two circular motions with the same, opposite frequency, although we could not find a graphic and intuitive interpretation of this fact. If the measurement is made at the values  $\delta_k$  ( $k = 1 \dots K$ ), one gets, for each  $m$ , a linear system of  $K$  equations for the 2 unknowns  $\hat{F}_m A_m^{(1)}$  and  $\hat{F}_m A_m^{(2)}$ :

$$\hat{S}_{m,k} = \sum_{r=1,2} M_{m,k,r} (\hat{F}_m A_m^{(r)}), \quad (15)$$

$$M_{m,k,r} = \frac{1}{2i} \exp[(-1)^r i m \delta_k/2], \quad (16)$$

where  $\hat{S}_{m,k}$  is the  $m$ th component of the measured Fourier spectrum at the relative angle  $\delta_k$ , and  $r = 1, 2$  indexes the two rings. These systems of equations are overdetermined if  $K > 2$ . They were solved for each  $m$  using the pseudoinverse of  $M_{m,\cdot,\cdot}$  calculated by singular value decomposition, where the two dots in the subscript indicate the two free indices.

Note that the unknowns contain the phase factors  $\hat{F}_m$  common to both rings. This means that the misalignment of the field patterns of the individual rings from their theoretical normal alignment cannot be determined, and consequently, cannot be compensated, and the device cannot be rotated accurately into the normal position. That would require a precisely positioned and aligned Hall sensor. However, the relative phase offset and magnitudes can be calculated from the measured data, which are sufficient to use Eq. (4) for the synchronous rotation of the two rings.

The derivation of the formulas in this section was made for a 2D magnetic field. In practice, integrated quantities through the entire magnet and its fringe field should be used.

## V. SIMULATION

The device was simulated in two and three dimensions in COMSOL. The 3D simulation utilized the mirror symmetry

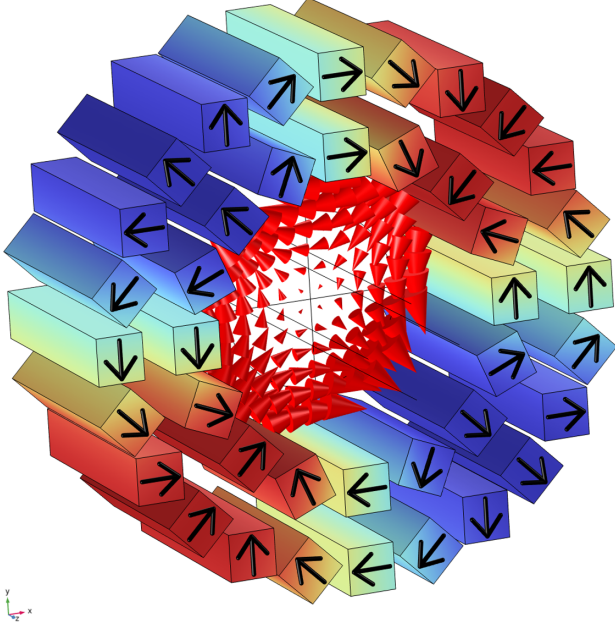


FIG. 6. The 3D COMSOL model in the maximum-field configuration. The red arrows indicate the magnetic field in the midplane and the black arrows indicate the remanent magnetization of the permanent magnets. The color on the permanent magnets is a visual representation of the magnetic poles around each ring.

across the  $x - y$  plane of the device and was restricted to the  $z > 0$  domain. Mirror symmetries across the  $x - z$  and  $y - z$  planes only hold in the MIN and MAX configurations, with no perturbations of the PMs' positions. These

symmetries were therefore not used. For illustration, Fig. 6 shows the full 3D model and the magnetic field pattern in the MAX configuration. The nominal remanent magnetization of 1.47 T was assumed for all permanent magnets.

## VI. MEASUREMENT AND SIMULATION RESULTS

### A. Measurement of the tuning range

The main (quadrupole) amplitudes of the two rings were determined as described in Sec. IV, at the relative angles  $\delta_k = -10^\circ, 20^\circ, \dots, 80^\circ$  (the choice of the sign, i.e., the operational range, ensures that both worm gear shafts are pushed downward, as described earlier). Figure 7(a) shows the 2nd component of the Fourier spectrum as a function of  $\delta$  (symbols), the derived values of  $\hat{F}_2 A_2^{(1,2)}$ , and the expression Eq. (14) with these calculated values (solid line). The difference between the two phases, 6.6 mrad corresponds to a simultaneous, opposite circumferential motion of about 0.3 mm of the gears of both rings. This can be attributed to the two rings being driven to the opposite ends of their slacks and can be compensated by the driver software. The deviation of the phases from  $\pi$  (about  $7^\circ$ ) is dominated by the unprecise alignment of the Hall sensor's position and angle. The field magnitudes 229.6 and 90 mT measured in the axial midplane of the device, at a radial position of  $18 \pm 1$  mm correspond to gradients of  $12.75 \pm 0.7$  and  $5 \pm 0.26$  T/m. The tuning range is therefore 7.75–17.75 T/m at the center of the device. Figure 7(b) shows the central quadrupole strength of the device as a function of the angle of the outer ring, when the synchronous rotation condition, Eq. (4) is respected.

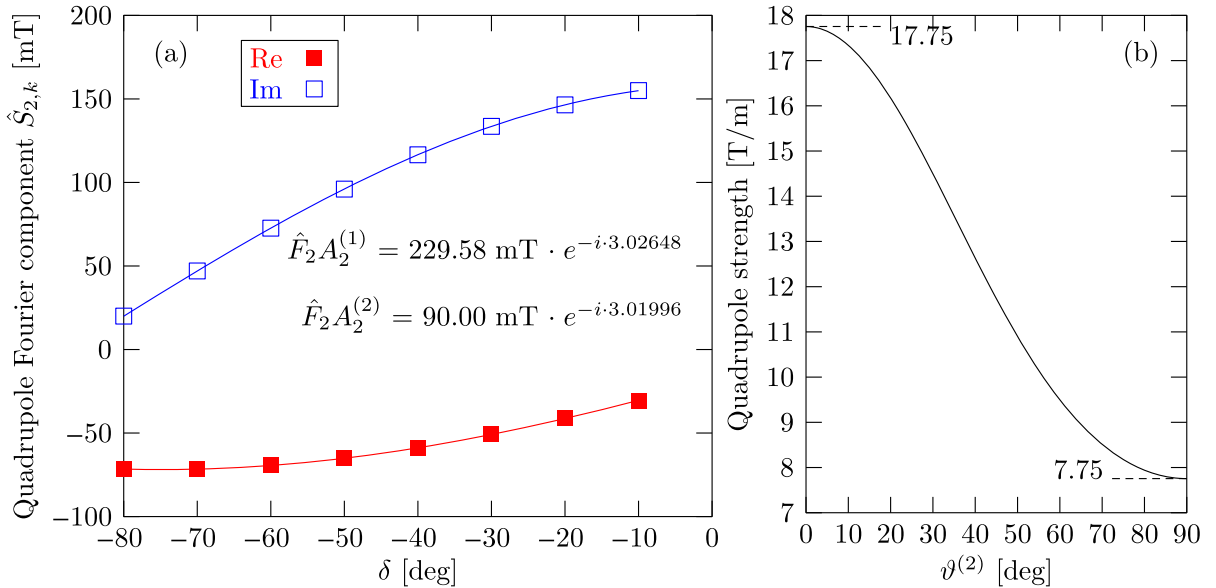


FIG. 7. (a) Symbols: quadrupole (2nd) Fourier component [see Eqs. (14) and (15)] of the Hall sensor's reading during a full, common rotation of both rings, as a function of relative angle  $\delta$  between the rings (measurement). Solid lines: Eq. (14) with the calculated (and displayed) values of  $\hat{F}_2 A_2^{(1,2)}$ . (b) The normal quadrupole strength as a function of  $\vartheta^{(2)}$  with the synchronous rotation condition [Eq. (4)], using the measured amplitudes shown in (a).

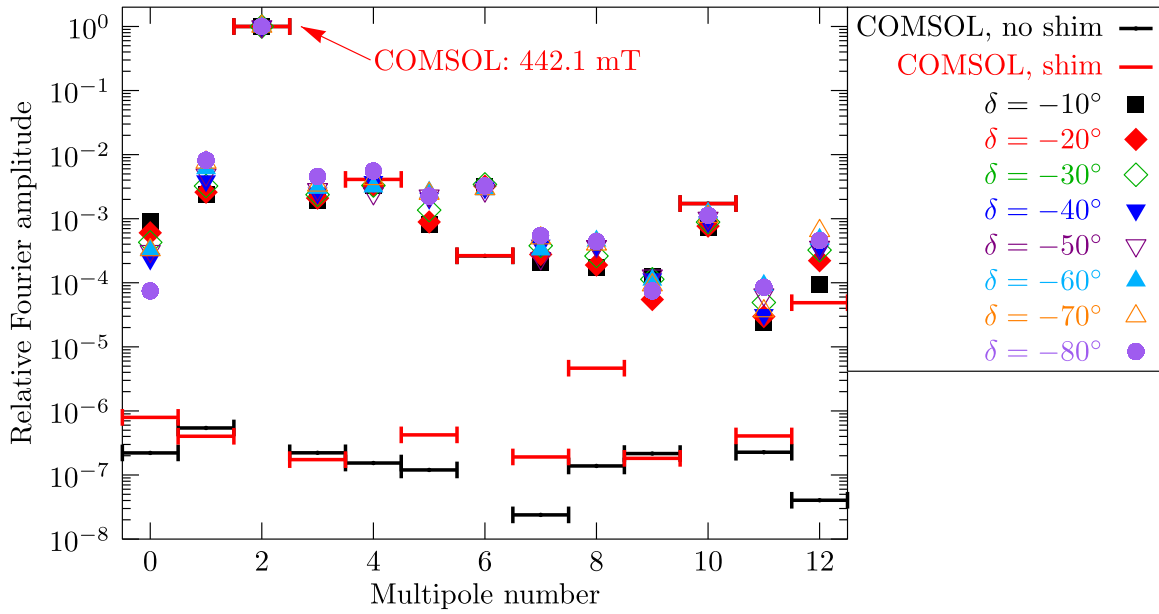


FIG. 8. Relative magnitudes of the Fourier components. Symbols: measurements at different relative angles  $\delta$  of the rings (see legend). Horizontal lines: 2D COMSOL simulation at  $\delta = 0^\circ$  relative angle (MAX position), without (black) and with (red) shimming (displacement by  $50 \mu\text{m}$  toward their south-west directions) of the PMs. At  $m = 6, 10$ , the black lines are perfectly covered by the red ones.

### B. Field quality

Figure 8 shows the magnitudes of the Fourier components normalized to the main (quadrupole) component at different relative angles  $\delta$  of the rings (symbols). In general, all unwanted multipoles are suppressed at least by 2 orders of magnitude. The measurement precision is limited by the Hall sensor. The horizontal lines show the same values obtained from a 2D COMSOL simulation at zero relative angle between the two rings (MAX position) and assuming no error on the magnetization of the PMs, without (black) and with (red) shimming (displacement by  $50 \mu\text{m}$  toward their south-west directions) of the PMs. A 3D COMSOL simulation gave qualitatively similar results but with a worse contrast between the nonallowed and allowed multipoles due to the coarser mesh. This is why the much clearer 2D simulation is presented here as a comparison.

The first observation is that the value obtained from COMSOL (442.1 mT) is 38% higher than the value obtained by the measurements ( $229.6 \pm 90 \text{ mT} = 319.6 \text{ mT}$ ). This is partly due to the fact that the magnet is short, its field is dominated by end effects and it does not reach a plateau at the center. As we will see later, the device also underperforms with respect to a 3D simulation.

For the multipoles  $m = 6$  and  $m = 10$  (the only ones allowed by symmetry in the displayed  $m$  range), the shimmed and nonshimmed simulated values are in perfect agreement and are several orders of magnitude above the apparent simulation precision ( $10^{-6}$  range). The measured values of these multipoles are also clear outliers from the general trend, although with a much worse contrast due to

the limited precision of the measurement and the presence of unwanted multipoles.

The multipoles  $m = 4, 8$ , and  $12$  are introduced by the shimming, as demonstrated by the clear difference between the black (nonshimmed) and red (shimmed) lines. The same phenomenon can also be seen in the measured values, although a little overshadowed by the overall measurement uncertainties.

The relative magnitudes of the dipole component in the nearly minimum (maximum) settings are approximately  $A_1/A_2 = 10^{-2} (2 \times 10^{-3})$ , respectively (see Fig. 8). This can be attributed to harmonic feed-down and used to estimate the displacement of the magnetic center with respect to the rotation center as  $d = A_1/A_2 \cdot R_0 = 180(36) \mu\text{m}$ , respectively, where  $R_0$  is the sampling radius. As a comparison, the final-focus tunable quadrupole [22] reached a magnetic center displacement as low as  $10 \mu\text{m}$ , and relative magnitudes of the unwanted multipoles below  $10^{-3}$  by the radial adjustment of the individual PMs. A similar field quality was reached by the nested Halbach quadrupole [21]. In light of having an uncontrolled and unsorted set of permanent magnets without tight tolerances on their magnetization, having made no efforts for any corrections and the limited precision of the measurements, the achieved field quality of the current device is judged to be satisfactory.

In order to study the dependence of field quality on the magnetization error of the PMs ( $\pm 2\%$  specified by the vendor and no data on the angle error), 500 different 2D COMSOL simulations were carried out with normally distributed random errors generated either for the angle

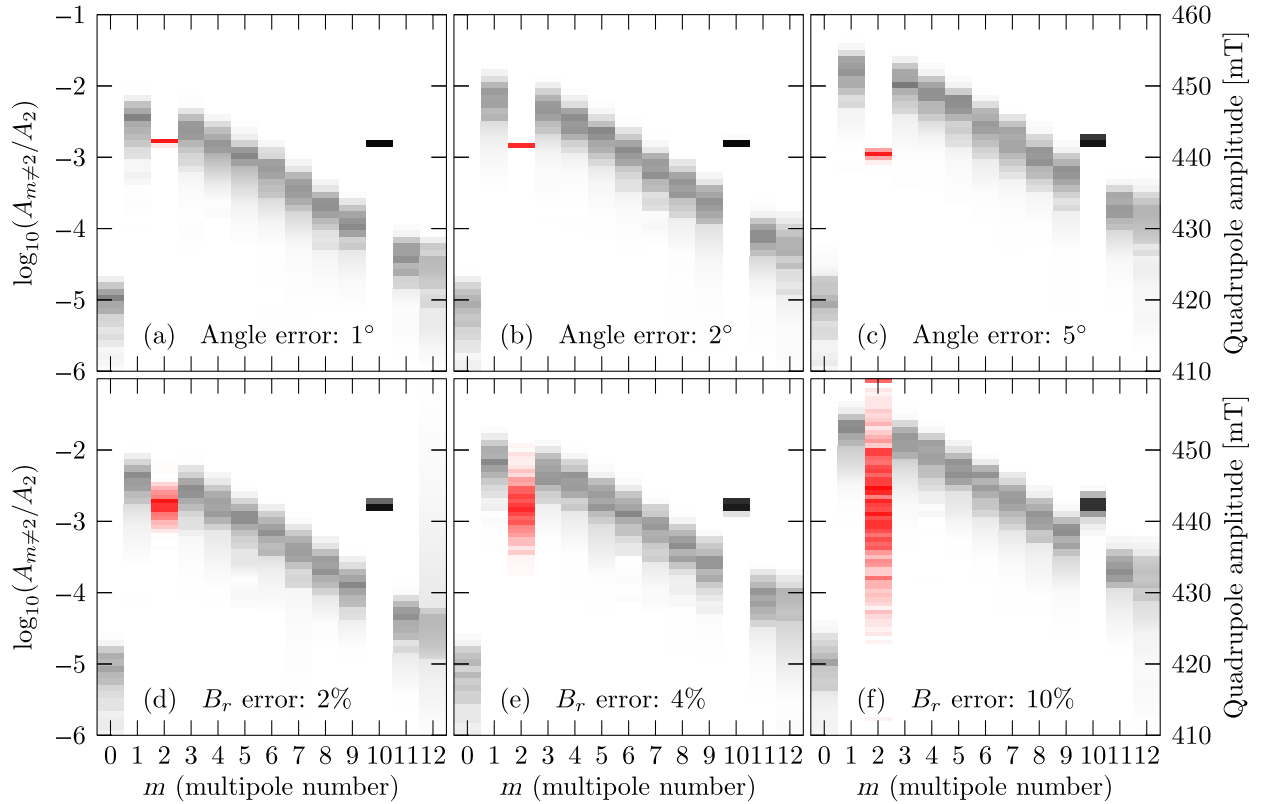


FIG. 9. Distribution of the normalized multipole amplitudes (for  $m \neq 2$ ) obtained from 500 2D COMSOL simulations with normally distributed, random errors generated for the direction (a)–(c), and magnitude (d)–(f) of the magnetization of the PMs (black, left axis). For  $m = 2$  (red), the absolute quadrupole amplitude is shown on a linear scale on the right axis. (a) Angle error:  $1^\circ$ . (b) Angle error:  $2^\circ$ . (c) Angle error:  $5^\circ$ . (d)  $B_r$  error: 2%. (e)  $B_r$  error: 4%. (f)  $B_r$  error: 10%.

or for the magnitude of the magnetization of each individual PM with  $1^\circ$ ,  $2^\circ$ ,  $5^\circ$  and 2%, 4%, 10% standard deviation, respectively. As it is visible in Fig. 9, angle and magnitude errors between  $1^\circ$ – $2^\circ$  and 2%–4%, respectively, reproduce the experimentally observed field quality. Further improvements may be achieved by specifying more stringent criteria on the PMs or by the introduction of a sorting algorithm.

The preceding analysis studied the Fourier components of the radial field sampled on a circle in the device’s midplane. However, “good field quality” required by beam optics is related to the linearity of the field with transverse coordinates. These two only have a 1:1 correspondence in infinitely long magnets. Having a physical aperture of 54 mm and PM length of 50 mm, the current device is short in the sense that it has no flattop of its field in its interior and its field pattern is dominated by end effects. In such magnets, the correspondence between linearity and absence of nonquadrupolar Fourier components is valid for the longitudinal integral of the radial field component across the magnet [32]. During the first tests, no instrumentation was available to measure the longitudinal integral of the field, therefore only 3D simulation results are shown. Figure 10(a) shows the multipole magnitudes of the radial

field component as a function of the axial position for one half of the device. Only the multipoles allowed by the quadrupole symmetry are indicated in the legend. Figure 10(b) shows the quadrupole magnitude on a linear scale in the MAX and MIN settings. The curve has an undershoot in the fringe field region in the MIN configuration due to the two rings having opposite polarities, different strengths, and axial fall-off distances. Figure 10(c) shows the integrated multipole amplitudes. All unwanted multipoles are suppressed by at least 3 orders of magnitude, except  $m = 10$  which is a little higher.

The central quadrupole values (389 and 172 mT in the MAX and MIN positions, respectively) are about 22% higher than the measured values (319.6 and 139.6 mT). We can only explain this by the PMs having a lower-than-nominal magnetization. Characterization of the PMs and a much more accurate measurement of the field pattern of the device is in preparation. If we assume that this discrepancy can be described by a global scaling by the ratio of the measured and simulated gradients at the magnet’s center, and the axial field dependence is untouched, the true tuning range of the device is 0.36–1.01 T for the integrated gradient. The measurement and simulation results are compared in Table II.

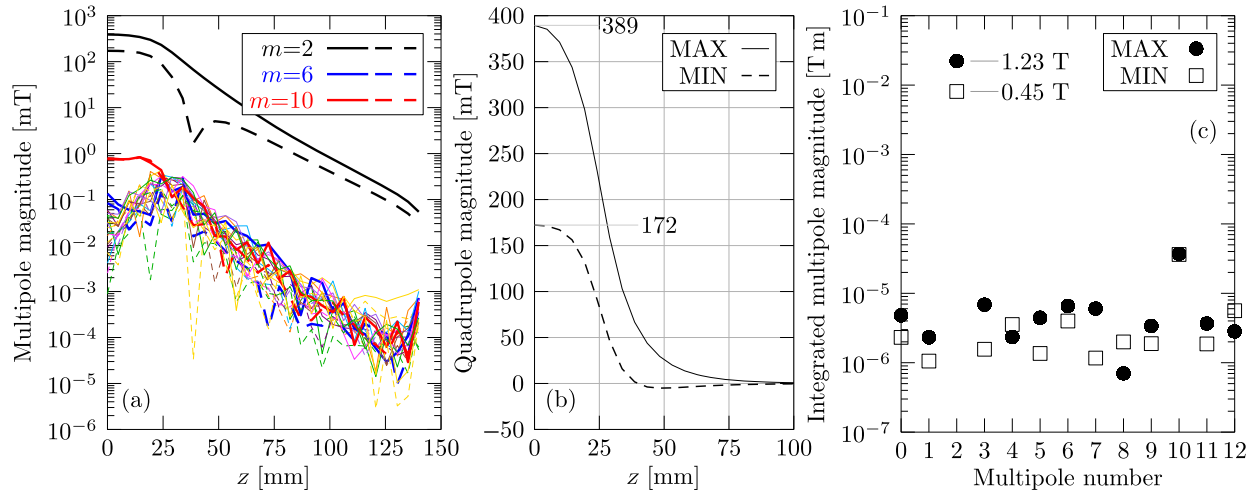


FIG. 10. 3D COMSOL simulation results. (a) Multipole magnitudes of the radial field as a function of axial position. Only the multipoles allowed by the quadrupole symmetry are indicated by legends, the others (up to  $m = 12$ ) are shown by thinner lines only. Solid lines: MAX position, dashed lines: MIN position. (b) Quadrupole magnitudes as a function of axial position in the MIN/MAX settings on linear scale. (c) Integrated  $B_r$  multipole magnitudes (symbols) and the values of the integrated quadrupole gradient (labels). The sampling radius was  $R_0 = 18$  mm (2/3 of the physical aperture) in all cases.

### C. Hysteresis

Hysteresis of the PMs during the relative rotation of the two rings is an important question. It was studied by an extended measurement cycle, recording the Hall sensor's reading during full, common rotations of both rings at relative angles  $\delta = -10^\circ, \dots, -70^\circ, -80^\circ, -70^\circ, \dots, -10^\circ$  and comparing the measured values at the same  $\delta$ . Figure 11 shows the differences as a function of the common rotation angle  $\sigma$ . The deviations exhibit a quadrupolelike behavior and decrease from about  $\pm 1.5$  mT at  $\delta = -10^\circ$  (where the largest hysteresis is expected) down to a few tenths of a mT at  $\delta = -60^\circ$ . The results at  $\delta = -40^\circ$  and  $-70^\circ$  exhibit a strange behavior which we can not explain with certainty. A possible explanation is that the PMs still have a little mechanical play within their slots, despite the shimming by the kapton tape, and are in an unstable position, making an unpredictable little movement.

The hysteresis is only about 1% or less over the entire tuning range (140–320 mT at the sampling radius). This can be explained as follows. Since the Halbach rings have negligible stray field outside, the inner ring's field does not introduce any hysteresis effects on the outer ring. Considering only the inner ring, its own, strong field (0.58, 0.73, or 0.85 T according to the 2D simulation, depending on the position of the PM within the quadrupolar PM

pattern) is practically parallel to the magnetization of the individual PMs. This field is strong enough to ensure that the PMs remain safely in the first quadrant, on the flat part of their B-H curve even after the superposition of the rotating 0.175 T field of the outer ring, producing only a minor hysteresis loop.

The measured hysteresis shows a quadrupolar pattern. It is expected in general that the hysteresis effects keep the multipolar symmetry of the device. Inside each PM of the inner ring, the field of the outer ring can be described as a rotating field with the same magnitude. Its angle with respect to, and hence its component along the easy axis of each PM is the same, so even if one considers the minor hysteresis loop of the PMs during this process, they all experience the same demagnetization force and change their magnetization similarly, keeping the multipolar pattern. Nevertheless, if a precisely known field strength is required, the device can be first driven to one end of the operational range and then to the desired position in order to provide reproducible results.

### D. Ultimate tuning range

A stronger magnetic field can be produced by increasing the filling factor of the rings by the PMs. This can be achieved by wedge-shaped PMs with custom field

TABLE II. Measured and simulated parameters.

| Parameter                           | Measurement | 3D simulation | Unit |
|-------------------------------------|-------------|---------------|------|
| Gradient (device center)            | 7.75–17.75  | 9.5–21.6      | T/m  |
| Quadrupole magnitude (MAX position) | 319.6       | 389           | mT   |
| Quadrupole magnitude (MIN position) | 139.6       | 172           | mT   |

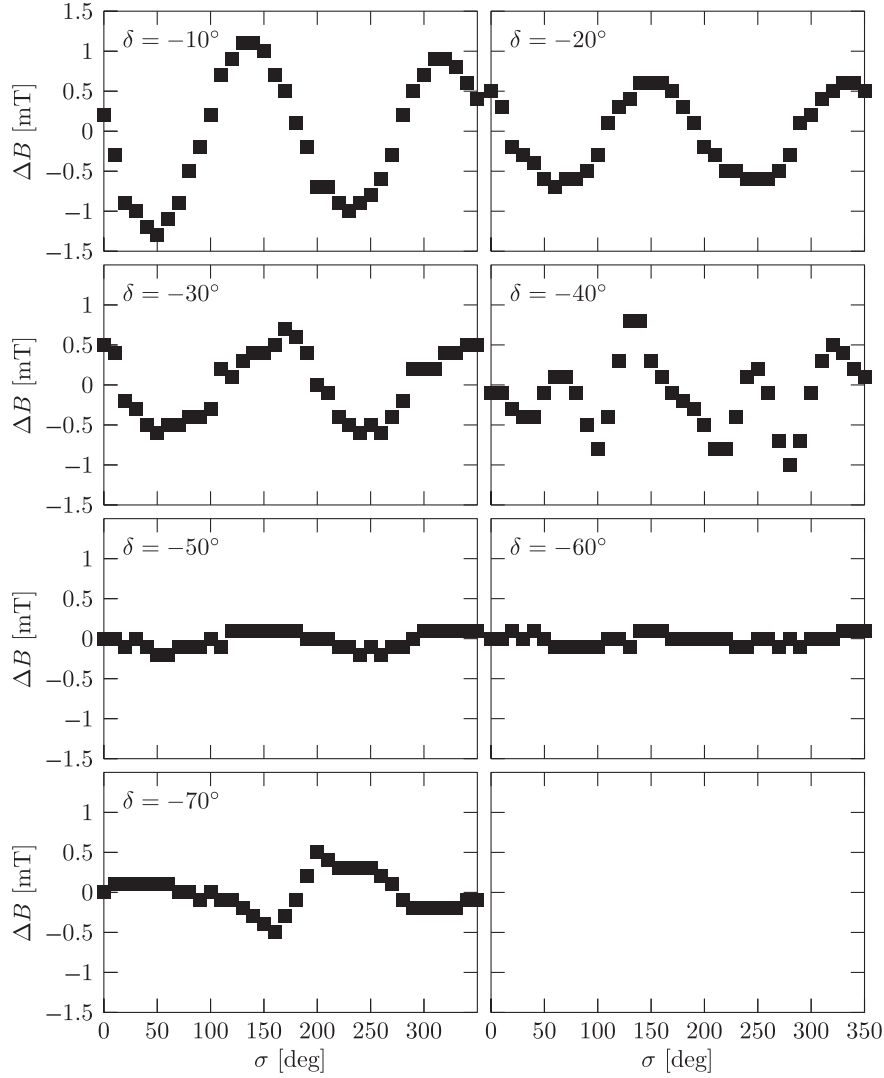


FIG. 11. Measurement of hysteresis: see text for details.

orientation as used for example in [18]. Although this would increase the price and the complexity of spare part management for the PMs, it would have practically no effect on the design concept and manufacturing procedure. In order to simulate the ultimate limit with a geometry similar to the current demonstrator for different apertures, two continuous rings (i.e., with a 100% fill factor) enveloping the two rings of discrete PM bars were assumed, with a continuous magnetization of

$$B_x = -\sin[(m+1)\vartheta], \quad (17)$$

$$B_y = \cos[(m+1)\vartheta], \quad (18)$$

where  $m = 2$  results in a quadrupolar field. The inner and outer radii of the two rings were  $R^{(i)} \pm S/\sqrt{2}$ , where  $i = 1, 2$  indexes the two rings, and  $S = 10$  mm is the

transverse size of the present PM bars. The radial distance between the aperture's perimeter and the two rings was kept constant, when varying the aperture. Figure 12(a) shows the tuning range of the integrated gradient of a device with 50 mm long permanent magnets obtained by a 3D simulation, compared to our demonstrator with discrete magnets. Figure 12(b) shows the gradient strength for an infinitely long magnet (i.e., 2D case) based on the formula [3]:

$$\frac{d^{m-1}|B|}{dr} = \begin{cases} (m-1)! B_R \ln(R_o/R_i) & \text{for } m = 1 \\ (m-1)! B_R \frac{m}{m-1} \left( \frac{1}{R_i^{m-1}} - \frac{1}{R_o^{m-1}} \right) & \text{for } m > 1 \end{cases} \quad (19)$$

where  $R_i(R_o)$  are the inner(outer) radii of the given ring.

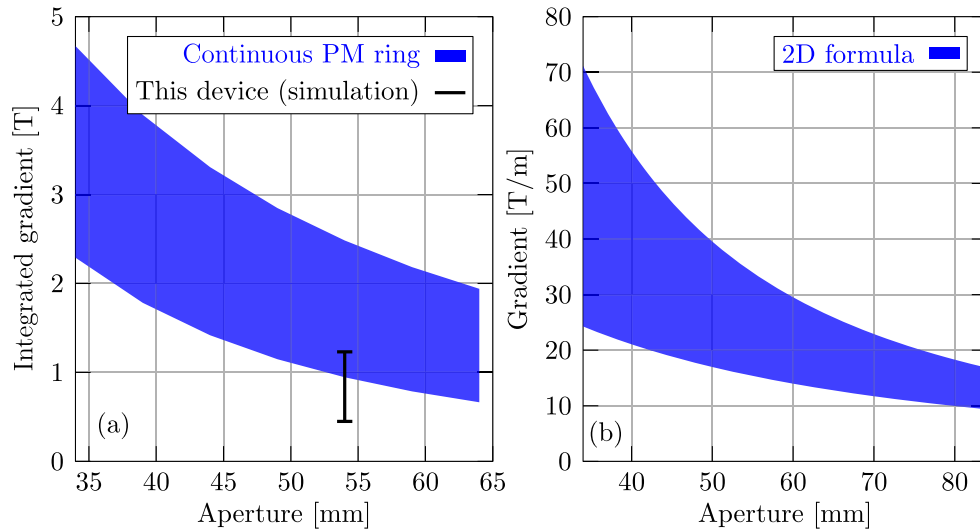


FIG. 12. Theoretical limits of the quadrupole strength assuming continuous rings with varying orientation of permanent magnetization: (a) 3D COMSOL simulation, integrated gradient with 50 mm long magnets as a function of aperture (shaded area), with the current device consisting of discrete PM bars overlaid (black line), (b) field gradient in a 2D geometry (analytical formula, see text for details) as a function of aperture.

## VII. SUMMARY AND OUTLOOK

A permanent magnet-based, tunable quadrupole magnet prototype has been designed and constructed by Rubiclin Ltd. The device consists of two nested, concentric Halbach rings of 16 and 24 square NdFeB magnets of grade N52 and size  $10 \times 10 \times 50$  mm. The primary goal of this first prototype was the demonstration of the feasibility of the mechanical concept: the split structure and the separate driving mechanism for the two rings. Cheap, commercial magnets, without tight tolerances were used in the first tests. Grade N52 has a low temperature limit and coercivity, making it unsuitable for radiation environments [33]; however, replacing these magnets for later, more ambitious tests is quick and straightforward. The length and aperture of the device are 80 and 54 mm, respectively. The two rings can be rotated independently by  $360^\circ$ , allowing the continuous tuning of its strength in a wide range, and the adjustment of the orientation of the field pattern, including polarity reversal as a special case.

The magnetic field pattern of the device has been measured by rotating the magnet around a single, fixed Hall sensor mounted at  $2/3$  of the aperture. The tuning range at the center of the device was measured to be 7.75–17.75 T/m. Comparing to 3D simulations, the range of the integrated gradient is estimated as 0.36–1.01 T. All unwanted multipole components are suppressed at least by 2 orders of magnitude. The observed field quality is compatible with the prediction of simulations with realistic errors of the magnetization of the permanent magnets. In simulations aimed at estimating the ultimate performance limits of such an arrangement, continuous ring magnets with a 100% fill factor and the same length (50 mm) were used. These simulations predict a significantly extended

tuning range: at an aperture size of 35 mm and same radial extents of the rings, the integrated gradient can be tuned between approximately 2.3 and 4.5 T, while at 65 mm, the range is about 0.8–2 T. Similarly, 2D simulations show that the corresponding field gradient varies from roughly 25 to 70 T/m at 35 mm aperture and from 10 to 16 T/m at 65 mm.

The device can switch between its minimum and maximum field configurations in about 1 s, which is valuable when dynamic changes in the optics are required.

The device has a split, openable construction, making its installation and removal (during bake-out, for example) around an existing beamline easy. The mounting mechanism to its adjustable base plate guarantees a high repositioning accuracy without realignment. By a proper choice of the operational range and protocol, the mechanical slack can be completely eliminated, relaxing the tolerance requirements (and consequently the price) of the components of the driving mechanism. The device can be stacked face-to-face realizing a dense magnet arrangement for beam optics or an extended length with the same type of unit. This arrangement, however, may need extra mechanical features and an assembly strategy to overcome the difficulties due to the repulsive force between similar magnets. The design allows an easy axial up-scaling as well if needed. The device is modular, and it can be easily converted into a dipole or higher multipole magnet by replacing its two rotating rings, while keeping all other components. A rotatable dipole is especially adapted for the role of a corrector magnet. These features makes it a very versatile device, offering flexibility and cost savings in the design of beamlines, and facilitating spare parts and inventory management in large facilities using a large number of magnets. The device has practically zero energy

consumption, requires no cooling water and is maintenance-free. While there are several other designs targeting very strict field quality and stability requirements, the presented device has a great advantage of being general, scalable and very easily applicable, allowing easy installation around existing beampipes.

Homogeneous effects (temperature, for example) only cause an overall change of the field strength and are expected not to cause field quality degradation. Radiation effects, however, are often times inhomogeneous, being concentrated in the bending plane of the accelerator. If these effects are of concern, the *in situ* monitoring of the field strength and quality is possible by magnetic field sensors permanently installed between the vacuum pipe and the wall of the magnet's bore. This allows the detection of temperature dependence or radiation damage and eventually the compensation of the former. The field strength setting can be adjusted without intervention at the device, or, the permanent magnets can be replaced or shuffled in a specific way when necessary. To address these issues, a printed circuit board with a linear array of 16 Hall sensors covering an axial range of 150 mm, and their driver and readout electronics has been recently designed and constructed by Rubiclin Ltd., which can be mounted to the device, inside the gap between the vacuum pipe and the wall of the magnet's bore. Calibration of the Hall sensors is underway.

Concerning further possibilities to improve field quality, using permanent magnets with tighter tolerances on their magnetization, and the measurement and sorting of the “as received” magnets are the first, trivial options. The design of the rotating rings can easily be modified to include slots for thin passive shimming rods—either between the PMs or in a few mm extra space radially inward from the PMs when wedge-shaped PMs are used in a tight arrangement. Temperature stabilization by water chillers can easily be implemented in the nonrotating support blocks of the device, however, its effectiveness is questionable due to the weak thermal link to the PMs via only the balls of the split bearings.

A precise measurement campaign of the prototype demonstrator with more accurate equipment is in preparation. Its results will be presented in a separate paper.

## ACKNOWLEDGMENTS

The authors are grateful to Rubiclin Ltd for the provision of the prototype device for these measurements.

## DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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