

Permanent magnet electron optics for low-energy electron systems: The art of extraordinary performance from ordinary components

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Permanent magnet electron optics offer many advantages over electromagnets and are being increasingly used in high-energy (GeV) electron accelerator designs. Here, we identify the advantages of permanent magnet electron optics for low-energy (keV) electron accelerators. We explore the applications of a class of designs based on axially magnetized permanent magnets, which offer a variety of advantages such as short focal lengths (few mm), while also preventing apparent emittance growth resulting from starting particles in a magnetic field. The proposed design philosophy is applied to an accelerator based on the ultracold electron source. The design is shown to be “emittance preserving” even for very short focal lengths (approximately 5 mm) at an emittance level better than 1 nm rad, while the short beamline (≈ 12 cm) limits space-charge effects. Two remedies for the mitigation of typical manufacturing and alignment challenges are considered. The performance of the design (related to parasitic aberrations) is enhanced by the proposed techniques. Applications of this design philosophy can improve the performance of ultrafast electron diffraction setups with minimal manufacturing effort.

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I. INTRODUCTION

The use of permanent magnets instead of electromagnets is being increasingly explored in the accelerator community for reasons of compactness, energy efficiency, and the absence of noise. While permanent magnets have routinely been used in the past for undulators, increasingly accelerator designs have included permanent magnet dipoles such as the Swiss Light Source upgrade project [1,2].

In addition to these radiation sources, permanent magnets are also increasingly employed to control electron-beam dynamics, such as the tunable permanent-magnet-based quadrupole [3–6].

In this article, we explore the use of permanent magnets for low-energy (keV) electron beamlines. We consider the combination of a permanent magnet focusing system with a dc accelerator, making our findings relevant to, e.g., ultrafast electron diffraction setups [7,8], dc-rf accelerators [9] and systems for electron microscopy [10] and lithography [11]. Our specific design case is the development of

a microfocus beamline based on an ultracold (or cold-atom-based) electron source [12–14].

II. MOTIVATION FOR USING PERMANENT MAGNETS: HIGH-PERFORMANCE SYSTEM DESIGN

For low-energy electrons, it may not be immediately clear why it would be desirable to use permanent magnets, since relatively low power is needed for electromagnets. The motivation can be derived from two challenging aspects of electromagnets. The first concerns the stability and noise of power supplies. Noise and ripple on electromagnet current effectively degrade instrument performance, particularly relevant for low-emittance beams, where they can for example limit resolution in electron microscopy and lithography. While the high-frequency noise can be damped by suitable filters, ensuring low drift is challenging.

The second aspect is mechanical integration. In the aforementioned applications, it is desirable to ensure a small longitudinal spread of the magnetic fields to ensure that a complex “column” can be constructed in a limited length. In order to achieve this goal, the electromagnets need to be placed in vacuum, which poses constraints on the materials and methods of construction. In addition, the electromagnet must now necessarily be cooled (as opposed to possibly passive cooling outside of vacuum), which further complicates the mechanical design.

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Permanent magnets, in contrast, are practically noise-free, are only affected by negligible long-term drift (typically ppm per year [15]) and do not require cooling in vacuum, allowing more compact, simple designs.

While radiation damage to permanent magnets is of concern for large accelerator facilities [16], the radiation levels present in keV electron setups are well below the levels at which the onset of radiation damage has been reported [17]. The challenges of permanent magnets in ultra-high vacuum environments have been reported in the literature on account of undulator designs. Either Ni or TiN coatings are suggested methods to ensure ultra-high vacuum compatibility of NdFeB magnets [18,19].

But, as we consider the combination of a magnetic focusing system with a dc accelerator, another question comes into play: the alignment between the two. As we will show in Sec. VIII, using permanent magnets allows for a compact, “self-aligned” solution that is not possible with electromagnets.

III. APPARENT EMITTANCE

A high-brightness/low-emittance electron-beam setup almost always consists of some magnetic focusing element for the purpose of focusing the beam onto the sample/detector/next accelerator element. Typically, for nonrelativistic electron-beam systems (or photoinjectors), solenoids are used as the focusing element. In practice, the magnetic field of a solenoid extends beyond its geometric limits, creating a nonzero axial magnetic field at the source and focus. The interaction between the electron beam and this axial fringe field causes apparent emittance growth, which is undesirable for low-emittance beams. This apparent emittance growth is related to the “angular momentum” of the magnetic field. The apparent (normalized rms) emittance due to nonzero axial magnetic field (B_z) can be expressed as [9,20]

$$\epsilon_n = \frac{eB_z r_0^2}{8mc}, \quad (1)$$

where r_0 is the radius of the beam (r_0 defined as $\sqrt{\sigma_x^2 + \sigma_y^2}$). Therefore, in applications where beam emittance is critical, a “bucking solenoid” is often used behind the source, which counteracts the magnetic field of downstream solenoids to produce a zero axial magnetic field at the source. In the context of ultrashort pulsed beams and/or large bunch charges, the source size can be approximately 1 mm to reduce space-charge effects and in such a case, it is important to ensure the zero of the magnetic field. Alternatively, appropriate shielding and large distances can be used to minimize the magnetic field at source and focus, but this is not always possible,

especially for compact injectors and ultrafast electron diffraction setups where propagation at subrelativistic energies is kept at a minimum to prevent longitudinal expansion of the bunch as well as to limit space-charge-related emittance growth. It is also clear from Eq. (1) that for best performance with low-emittance beams it is desirable to have the magnetic field be zero at the focus as well (for a formal analysis see [21]). Electron microscopy or diffraction studies on magnetic samples may also be a reason to require zero magnetic field at the focus [24–27]. In the next section, we present a magnet design that ensures presence of two magnetic zeros.

IV. AXIALLY MAGNETIZED MAGNETS

While it is possible to achieve these zero-field points by adding bucking solenoids for both the source and focus point, we propose that the same can be achieved with a single axially magnetized ring-shaped permanent magnet. The field lines of such a permanent magnet geometry are shown in Fig. 1. The magnetic fields outside a homogeneously magnetized permanent magnet can be described by an effective surface current $J = M \times \hat{n}$, where M is the magnetization of the magnet and $\hat{n}$ the surface normal. In the case of an axially magnetized ring, this approach yields two surface currents with equal current density but opposite sign: One flowing on the outer radius (R_o) in azimuthal direction, and one on the inner radius (R_i) with opposite sign, both over the full z -extent of the magnet ranging from z_1 to z_2 . Consequently, the on-axis field can be decomposed in two contributions $B_o(z)$ and $B_i(z)$ due to the outer and inner effective surface currents, respectively. This yields, for the on-axis field:

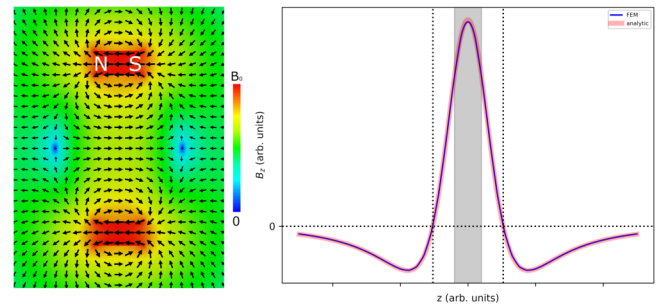


FIG. 1. Field of an axially magnetized ring-shaped permanent magnet. Left: The field lines are plotted on a cross section of the magnet through the symmetry axis. Right: The plot shows the axial magnetic field B_z as a function of axial coordinate z . The two zeros of the magnetic field are indicated at the intersection of dashed lines. The bounding region of the magnet is shown in gray. The analytical expression from Eq. (2) is shown in red and compared against a finite element simulation in blue.

$$\begin{aligned}
B_z(z) &= B_o(z) - B_i(z) \\
B_o(z) &= \frac{\mu_0 M}{2} \left(\frac{z - z_2}{\sqrt{(z - z_2)^2 + R_o^2}} - \frac{z - z_1}{\sqrt{(z - z_1)^2 + R_o^2}} \right) \\
B_i(z) &= \frac{\mu_0 M}{2} \left(\frac{z - z_2}{\sqrt{(z - z_2)^2 + R_i^2}} - \frac{z - z_1}{\sqrt{(z - z_1)^2 + R_i^2}} \right),
\end{aligned} \tag{2}$$

where $\mu_0 M$ is the field strength of the magnet expressed in Tesla. While the inner and outer effective surface currents are nominally equal, the field of the inner “solenoid” is confined to a smaller region and thus has a higher peak strength. The field of the outer “solenoid” is spread out over a larger region and thus a lower peak strength. This higher localized field opposed to the lower delocalized field is the origin of the two zeros of the magnetic field.

Although such a magnet geometry is already known [28,29], to the best knowledge of the authors, its use in preventing apparent emittance growth has not been explored. A convenient feature of this configuration is that the shape of the magnetic field, and therefore the location of the zeros, depends only on the magnet geometry, and not on the exact value of the magnetization. This is extremely useful because the magnetization process has a typical tolerance in the range of 3%–5%, which will thus only provide a scaling to the field, leaving the position of the zeros unaffected. From the preceding discussion, it follows that the magnet should be constructed such that the first zero coincides with the source, and the second coincides with the focus, possibly respecting additional constraints. Next, we introduce an optimization strategy based on genetic optimization to solve the constraint problem.

V. OPTIMIZATION STRATEGY

We first discuss some constraints on the magnetic fields. Although the representative schematic of the magnetic field as shown in Fig. 1 has mirror symmetry in z , it may be desirable to achieve an asymmetrical configuration for reasons stated below. Accelerating the electrons between the source and the focus makes the beam dynamics asymmetrical, irrespective of the symmetry of the magnetic field. From the beam-dynamics perspective, it may be desirable to have a specific magnetic field gradient at the source to compensate the transverse blow-up of the beam due to space-charge effects or for emittance compensation of rf photoinjectors. Additionally, the shape and magnetization of the magnet (and possibly the acceleration gradient and length) can be varied in order to achieve the optimal configuration.

In practice, instead of machining a single magnet to a specific shape and accurately controlling its magnetization, it is more practical to approach the ideal shape and magnetization by appropriate tiling with smaller magnets.

By adopting an appropriate packing fraction, the field profile can be fine tuned. Although such a tiled arrangement is contrary to the guaranteed location of the zeros mentioned above, in practice it allows a higher degree of control on the field profile. As stated above, the challenge is to find a magnet configuration that optimizes the quantities of interest subject to various constraints on the magnetic field as well as any other constraints of the beamline.

We choose genetic optimization as a suitable optimization strategy because of its ability to explore complete parameter spaces without getting stuck at a local minimum. More formally, a set of constraints for the genetic optimization consist of: (i) Magnetic field $B_z|_{z=\text{source}} \Rightarrow 0$ (ii) Magnetic field $B_z|_{z=\text{focus}} \Rightarrow 0$ (iii) Magnetic field gradient $\frac{\partial B_z}{\partial z}|_{z=\text{source}} \Rightarrow B'_0$ (iv) Any geometric constraints on the location of the magnet(s) along the beamline axis.

A set of optimization criteria consist of: (i) Minimize beam radius $r|_{z=\text{focus}}$ (ii) Maximize charge $Q|_{z=\text{focus}}$.

A set of optimization variables consists of: (i) Geometric properties of the magnet(s) (ii) Properties of the source.

As the problem has more than one optimization criterion, it is a “multiobjective optimization problem,” while the set of optimized (“Pareto-efficient”) solutions forms a Pareto front (see, e.g., [30]). This Pareto front is the outcome of the optimization procedure, showing the trade-off between the two optimization criteria. Based on this Pareto front, an optimally informed design choice can be made. We will now use this scheme to design a microfocus beamline for a high-brightness electron injector based on the ultracold electron source. This application is especially demanding because it requires the shortest focal lengths and thus requires the highest magnetic fields, while still respecting the constraints stated above.

VI. THE ULTRACOLD ELECTRON SOURCE

The ultracold electron source was proposed in [31] and realized in subsequent versions in Eindhoven [12,14,32,33] and Melbourne [34]. It is based on the photo-ionization of laser-cooled Rubidium atoms to produce an electron bunch with a record-low bunch temperature on the order of 10 K—i.e., a mean transverse energy on the order of 1 meV. Uniquely, the electrons are generated from a three-dimensional “cathode” (the ionizing fraction of the laser-cooled atoms), the shape of which can be controlled in 3D. This in turn allows shaping of the initial phase space distribution of the electron beam [35]. Also, each shot is generated from a “fresh” cathode of neutral atoms. The generated electron bunches can have sub-picosecond duration [36,37], but bunch charge has so far been limited to the order of fC. Finally, the use of laser-cooled atoms introduces two requirements that are relevant to the present design. First, the laser-cooled atoms are trapped in the center of a magneto-optical trap, which

needs to be a quadrupolar zero of the magnetic field with a certain magnetic field gradient (see, e.g., [38]). Second, the dc electric field used to accelerate the electron beam imparts Stark shifts on the different atomic levels used in laser cooling. For the laser-cooling to be efficient, the accelerating field is limited to about about 1 MV/m [39,40].

VII. MICROFOCUS BEAMLINE DESIGN BASED ON AN ULTRACOLD ELECTRON SOURCE

At a generic level, the accelerator design consists of a dc accelerator assembly and a focusing assembly. The features and designs of both are highlighted below. A representative schematic of the accelerator and magnet assembly is shown in Fig. 2, alongside electrostatic potential and magnetic field-maps illustrating the zeros of the magnetic field. A simplified CAD drawing of the combined assembly is shown in Fig. 3.

A. DC accelerator

The dc accelerator design of the ultracold electron source is heavily influenced by the requirements of atom trapping mentioned above. Furthermore, like our current ultracold electron source ([14]), the proposed injector employs the grating magneto-optical trap [41–43], which necessitates on-axis optical access to the dc accelerator. While in existing designs, this is achieved through an indium-tin-oxide-coated window as the negative electrode, this introduces issues related to charging of the window [37], which we overcome in this new design through the use of a cylindrical electrode with a central bore (“E” in Fig. 2) to enable optical access of the trapping laser. The design requires a nominal operating voltage of -40.6 kV for the electrode in order to produce a 16.6 keV beam. This is because the Rb atoms are situated in between the negative electrode and ground electrode (“S” in Fig. 2).

B. Magnet assembly

As introduced above, the atom trapping occurs at a point of zero magnetic field, with a region of quadrupolar magnetic field around the zero point. The quadrupolar nature of the zero can include rotational symmetry, i.e., the quadrupolar symmetry can be in the x - z plane while allowing rotational symmetry around the z axis. This coincides with the requirement introduced in Sec. III: to minimize apparent emittance growth, the magnetic field should be zero at the source. While in particle accelerators, the optimal value of the magnetic field gradient at the source is set by space-charge effects and beam dynamics, for atom trapping the optimal gradient is related to the trap size and trap density required. The requirement for atom-trapping supersedes any optimality conditions from the beam dynamics perspective. In the case of Rb atoms, a

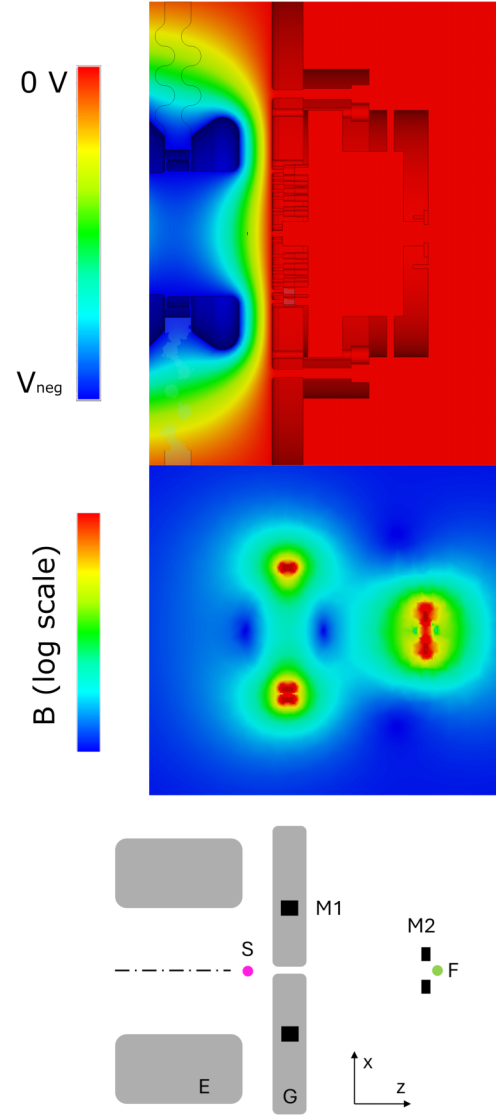


FIG. 2. Electromagnetic design of the accelerator and magnet assembly. Top: Electrostatic Potential: The color scheme represents the electric potential, with red representing ground and blue representing the highest negative potential. Center: The magnetic field strength in the same structure in a section passing through the symmetry axis. The color scheme is chosen to be logarithmic to illustrate the zero-field points. The discretization causes some loss to the symmetry of the field in the immediate vicinity of the magnet (M1), but this asymmetry is averaged out close to the symmetry axis. Bottom: Basic schematic of the same structure, ground electrodes (G), and the negative electrode (E) in gray and magnets (M1 and M2) in black. The source (S) is illustrated as a pink dot and the focal point (F) as a green dot. The origin of the coordinate system is at the centroid of the source, with the z axis corresponding to the symmetry axis of the accelerator.

suitable value for the magnetic field gradient is between 0.1 and 0.2 T/m [40,44]. Geometric constraints were also placed to ensure that the magnets were placed downstream of the ground electrode and at least 5 mm upstream of the

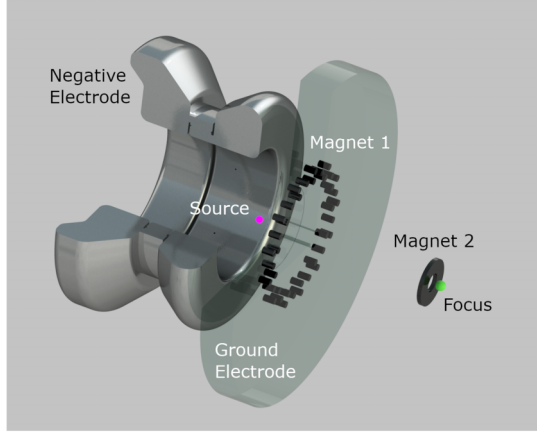


FIG. 3. Illustration of the proposed design based on the ultra-cold electron source. The pink dot represents the laser-cooled Rb atom cloud. The first “magnet” is discretized in a symmetrically patterned array of axially magnetized magnets. The second magnet is ring shaped with a rectangular cross section. The green dot represents the focal point of the electron beam.

“focal” point to ensure mechanical access for probes/diagnostics at the focus.

To test the limits of this design methodology and of the magnet design, it is desirable to have as small a focal spot as possible while maximizing the bunch charge within the performance limits of the source. The magnet optimization process involves varying the parametrized geometrical cross section of the magnet, as well as the (manual) tiling of multiple small magnets as introduced above. Additionally, the source shape and volume was allowed to be varied, with minimum and maximum extents of 30 and 100 μm for both transverse and longitudinal dimensions. This additional degree of freedom, not present in conventional electron sources, allows for an optimal trade-off between transverse emittance, longitudinal emittance, and bunch charge.

In line with the design principles stated above, the magnet design was optimized with genetic optimization as built into the general particle tracer [45]. The optimization process was first tried out with a single magnet (similar to the one shown in Fig. 1), though with a cross section that was allowed to be nonorthogonal. The optimal magnet configuration produced a focal spot size of approximately $\sigma_r = 2.5 \mu\text{m}$. In the view of the authors, the geometric structure of the magnet was considered to be difficult to realize in practice. Consequently, a solution space consisting of two magnets was explored. This enables a sub-micron focal spot size. This magnet configuration is shown in Fig. 3. The optimal solution consists of a larger magnet that generates the necessary magnetic field for the magneto-optical trapping. The smaller magnet is mainly the focusing element with a short focal length and is placed very close to the focal point. It is convenient to tile the

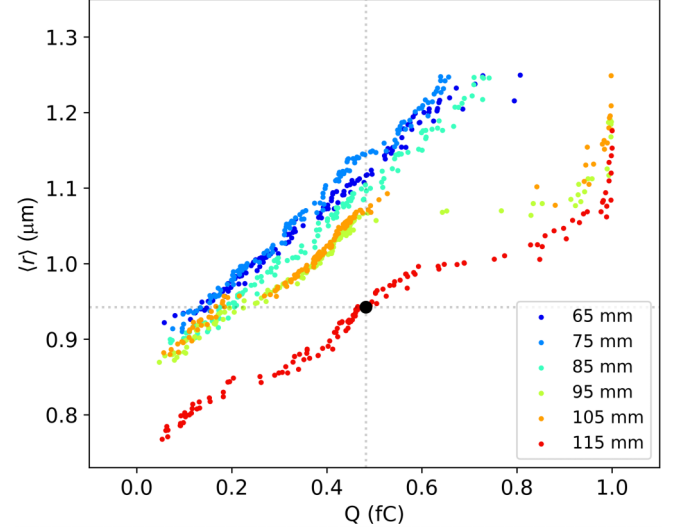


FIG. 4. Pareto fronts for different focal positions: Each Pareto front shows the trade-off between the two optimization criteria, i.e., focal spot size $\langle r \rangle$ ($r = \sqrt{x^2 + y^2}$) and bunch charge Q . The black dot indicates the selected design.

larger magnet because comparable performance can be achieved using off the shelf magnets. The resulting magnetic field error when compared to the “optimal” solution is about 1% with a 4% increase to the focal spot size, which is an acceptable compromise. Finally, we allow tuning of the accelerating potential to accommodate a 10% error margin in the strength of the magnets, exceeding typical manufacturing tolerances of magnets.

The optimization process was carried out for six different positions of the focus, thus producing six Pareto fronts, shown in Fig. 4. Each point in the graph corresponds to a different magnet geometry. Each Pareto front shows the trade-off between the two optimization criteria, i.e., the focal spot size and the bunch charge. Clearly, moving the focus further downstream reduces the focal spot size, which is because the beam is slightly diverging before the focusing magnet (Magnet 2). Moving the focus downstream also moves Magnet 2 downstream, increasing the beam size at the magnet thus causing a tighter focus. We limit the focal position to 115 mm to keep the design compact and limit pointing instabilities. Since this limit offers the smallest focal spot size, we select a design from this Pareto front. For this, we choose a bunch charge of 0.5 fC, a conservative value readily achieved with our current ultracold electron source. A reasonable operating regime of this design in terms of space-charge effects is described in [21].

C. Particle tracking simulations

The particle tracking simulations were performed using the general particle tracer with field-maps generated by CST, Traceon (now Voltrace), and custom elements

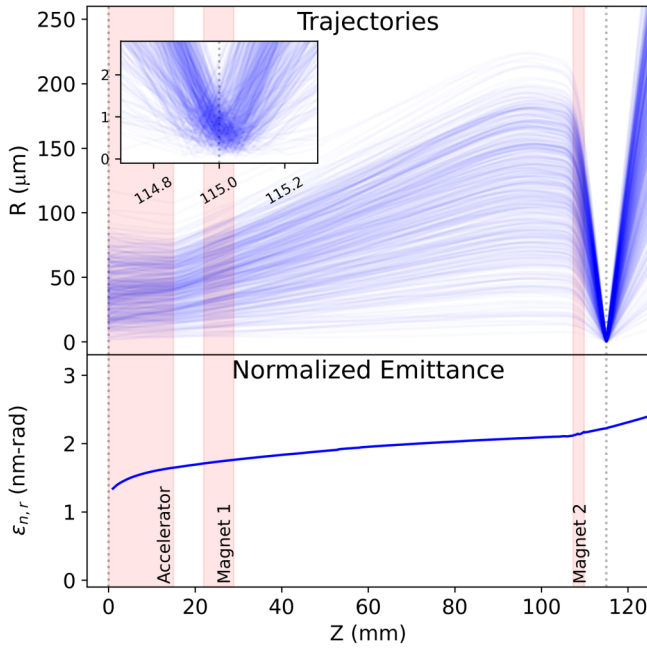


FIG. 5. Particle tracking results: Top: The trajectories of a random sample of the electrons are shown. The inset shows a smaller portion of the trajectories near the focal point. Bottom: The normalized emittance as calculated by the general particle tracer nemirrms routine is shown.

[22,45,46]. (These custom elements are available upon request.) Using multiple “field-maps” provides a sanity check against suboptimal meshing or convergence settings. The plots in Fig. 5 show the results of beam dynamics simulations. The trajectories show a region of acceleration (from 0 to 15 mm) where the beam becomes divergent due to the negative lens effect of the ground electrode. Magnet 1 only contributes weakly to beam focussing due to nominally “low” field strength, which is in turn related to the gradient requirement at the source point. The beam continues to diverge until the second magnet is encountered, which creates a tight focus. The slow emittance growth is attributed to space-charge effects up to the second magnet ($z \approx 105$ mm). Beyond this, the emittance growth can be attributed to misaligned slices in the transverse phase-space, i.e., chromatic aberration (Fig. 2 in [21]).

While typically space-charge effects are dominant at a focus, in our case the focus is so tight that the pulse length exceeds the Rayleigh length, i.e., at any given moment only a fraction of the bunch is tightly focused. As a result, emittance growth at the focus is negligible. A focal spot size of $\sigma_r = 920$ nm ($\langle r \rangle = 780$ nm) is obtained for a beam energy of 16.6 keV and a bunch charge of 0.5 fC. The energy spread of the beam is 0.2% (σ_E/E), with a bunch length of $\sigma_t = 1.4$ ps. While the divergence angle of the beam at the focus is relatively large ($\sigma_\theta = 18.8$ mrad), the

Rayleigh length is in the order of 100 μm . The robustness of the design in terms of the focal spot size and the effects of apparent emittance growth are quantified in the [21].

VIII. SELF-ALIGNED MAGNETIC FIELD: MAGNETS EMBEDDED INSIDE A HIGH-VOLTAGE ELECTRODE

The preceding section tacitly assumes that the dc accelerator and the magnets are aligned to one another exactly, which cannot be the case in practice. This alignment concerns both the offset and the tilt of the axes with respect to each other.

In the case of the dc high-voltage assemblies, the challenge of alignment is amplified due to the necessity of using an insulator as a mechanical support. It is important to note that the symmetry axes of a dc accelerator are actually two separate datums, the datum of the cathode and the datum of the anode (or ground). These datums are necessarily misaligned, and the misalignment is amplified due to the large creep distance (and thus large dimension) of the high-voltage insulator. Aligning a magnet to the ground electrode is simple because anything placed upstream of the anode is automatically grounded and shielded. However, this ultimately increases the beamline length. In the case of ultrafast electron diffraction, it is desirable to limit the beamline to the smallest possible dimension to reduce bunch length. In the case of dc-rf photoinjectors [47], it may also be desirable to separate the magnetic field from the rf cavity for example to prevent multipacting. For both these situations, placing the magnet inside the high-voltage electrode can enable a compact design that is self-aligned. By self-alignment, we refer to the idea that the magnets (and hence the magnetic field) are mechanically referenced directly to the high-voltage electrode without intermediate “parts” that need to transfer the accuracy of the reference features.

Embedding magnets inside a high-voltage electrode can be achieved with relative ease, with care being needed to ensure that no magnet feature has a high electric field gradient; a challenge which can be addressed by recessing the magnets with respect to an outward surface of the electrode. Note that achieving a similar result with electromagnets requires floating a high current supply ($I \approx 10$ A and $P \approx 100$ W) at high voltages ($V \approx 1$ –100 keV), in addition to, e.g., galvanic isolation of the cooling circuit. In contrast, a permanent magnet needs no floating high current supply and no cooling with the added benefit of low drift.

We present a variant of the presented dc accelerator design with embedded permanent magnets. The electrode was designed for use in a compact 100 keV dc-rf photoinjector for time-resolved electron diffraction (currently unpublished work A. Patwardhan *et al.*). A simplified exploded view of the magnet-electrode assembly is shown

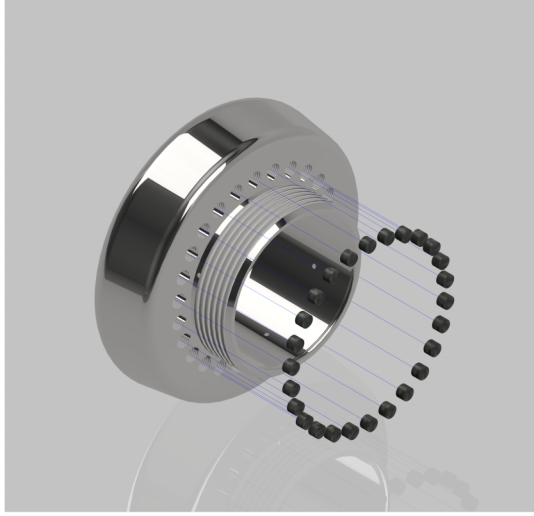


FIG. 6. Magnet and High-Voltage Electrode Assembly: Exploded view of the permanent magnet and high-voltage electrode assembly.

in Fig. 6. In the real implementation of the principle, appropriate precautions were taken to ensure venting of all trapped pockets (to prevent virtual leaks), recessed features to prevent high electric field gradient at corners and edges, and nonmagnetic materials of construction.

The presented design ensures the zero-field point with the same magnetic field gradient (0.2 T/m) at the source point.

By mechanically referencing the magnet assembly to the high-voltage electrode, the misalignment (as defined by the true position geometric dimensioning tolerance) of the magnet to the high-voltage electrode can be better than 20 μm even with routine manufacturing techniques. This is sufficiently accurate for even the most demanding situations in terms of the apparent emittance growth.

IX. ANALYSIS OF ERRORS AND MITIGATION STRATEGIES: EXTRAORDINARY PERFORMANCE FROM ORDINARY COMPONENTS

We continue our analysis of the alignment accuracy of the magnetic field with an analysis of the magnets themselves, i.e., for the following discussion, we assume that the positioning of the magnets is “exact.” The dimensional accuracy for even off the shelf “fridge magnets” is better than 20 μm [48]. Thus, we confine our attention to errors of the magnetization vector. From earlier work on undulator design, a tolerance of better than 3° is expected in the magnetization vector [49,50]. Regarding scalar errors of the magnetic field, we consider the tolerance grades of Nd magnets, from which we estimate a tolerance of 3%.

At the outset, we note that the beam dynamics is largely invariant for 3% errors of magnitude of the magnetic field if the beam energy is adjusted accordingly. Thus, we explore the angular errors of the magnetization. Any deviation from the desired magnetization vector creates dipole fields on the beam axis. Because the focusing effect of solenoidal fields is quadratic in the magnetic field, and the dipole effects are linear, careful minimization of angular error of magnetization is important for obtaining best performance. The obvious solution to this problem is to enforce strict specification on the magnetization vector. Indeed, this is always an option, although there are always economic and practical limits to the achievable level of precision. Thus, we explore designs through which components with moderate manufacturing tolerance can lead to higher performance, and if (as a last measure), the tolerances are tightened further, the performance can only improve.

A. Enhancing performance through statistics

The standard deviation of the mean is linked to the standard deviation of the distribution and the number of samples as

$$\sigma_{\bar{x}} = \sigma_x / \sqrt{N}, \quad (3)$$

where N is the number of samples. This implies that if instead of using a single magnet, we compose it from tiling several smaller magnets; any variances in the magnetization vector are averaged out on the symmetry axis. The only underlying assumption is that the magnet orientations are sufficiently randomized, which is easily realized in practice. For say 25 magnets, a reduction of a factor of 5 is realized in the effective tolerance for the magnetic field on the axis (3° tolerance of the magnets is reduced to 0.6° tolerance on the axis by averaging). Consequently, the true position of the zero of the magnetic field can be guaranteed to be better than approximately 150 μm of the source position with no additional effort. Measured magnetic field profile of such an assembly and analysis of randomized arrangements can be found in [21].

B. Systematic cancelation of dipole errors

It is worth mentioning that magnets manufactured in the same “batch” (i.e., magnets that have been magnetized together) are expected to have the same errors. This means that the magnetization vector is expected to systematically deviate from the nominal specification. For concreteness, we refer to magnet 2 as defined in Sec. VII and the following discussion is related to error mitigation of the same. Magnetization vector error (of say 3°) can be considered to be two separate components, an axially magnetized “solenoid” with effective strength reduced by a

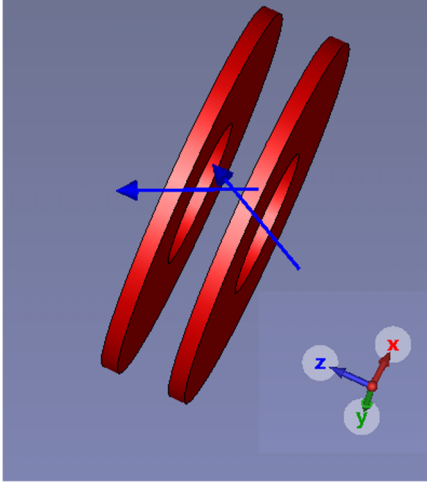


FIG. 7. Exploded view of the split magnet design: Magnetization vector of the two components of the split magnet is depicted by the arrows. For the purpose of illustration, the tilt with respect to the nominal direction has been exaggerated. Generated with the CST Studio Suite [22].

factor of $\cos(3^\circ) \approx 0.999$ and a dipole with strength corresponding to $\sin(3^\circ) \approx 0.05$. The dipole is not uniform along the “ z ” axis of the beam, but has a profile $B_x(z)$ that can be computed by using an electromagnetic solver. Because the dipole and solenoid occupy the same section along the beamline, and because the beam dynamics are linear (in B) for dipoles and nonlinear (in B) for the solenoid, a small vector error of say 3° does not necessarily produce a deviation of 3° , but can be substantially more especially for a tight focussing geometry. When the size of the magnet becomes small (on the order of 1 cm), it is difficult to effectively employ the tiling procedure stated previously. Instead, we exploit the systematic nature of the errors. The magnet is composed of two identical sub-magnets such that the desired magnet configuration is sectioned perpendicular to the symmetry axis. Refer Fig. 7. One of the two sub-magnets is rotated by 180° along the symmetry axis. The resulting dipole moments of the two sub-magnets almost entirely cancel out. The effect of dipole contributions on both beam position and beam properties can be seen in Fig. 8. When the length to diameter ratio of the magnets becomes large (approximately 1 or greater), the beam rotation around the z axis changes the optimum rotation of the second magnet with respect to the first.

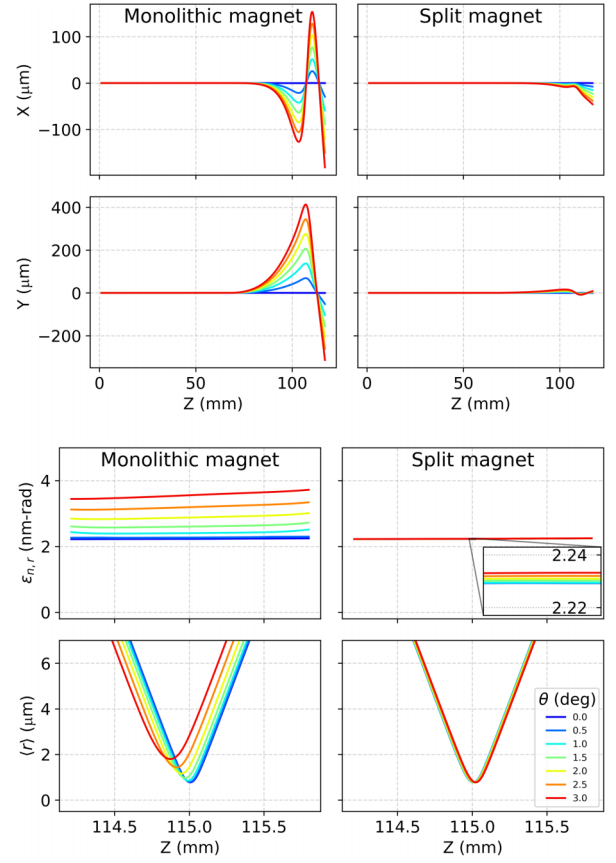


FIG. 8. Beam dynamics with magnetization errors: Top: Beam deflection due to dipole contribution from magnetization vector error. Bottom: Effect on Beam properties due to dipole contribution from magnetization vector error. Only a small fraction of the region near the focus is shown for clarity of illustration. The colors indicate the angular deviation of the magnetization vector with respect to the symmetry axis with red corresponding to 3° of misalignment and blue corresponding to perfect alignment.

X. CONCLUSION AND OUTLOOK

We have presented an innovative design procedure for a high-performance, microfocus beamline for low-energy electrons, employing axially magnetized ring-shaped permanent magnets to ensure magnetic field zeros at both source and focus. We have used this procedure to generate a design for the particular case of a beamline based on the ultracold electron source. In the proposed design, the presence of a magnetic field zero at the source is required,

but the approach is generally relevant to low-emittance applications as these magnetic field zeros avoid apparent emittance growth. Additionally, we propose a number of solutions (self-alignment, statistical averaging, and dipole cancelation) that enable a high-performance magnetic focusing system using simple, off-the-shelf components.

For ultrafast electron diffraction, particularly for samples with large lattice constants, the apparent emittance can be a significant constraint. The presented design methodologies can significantly enhance the applicability of low-energy electron diffraction for complex samples. (See [21] for additional discussion on apparent emittance.) In the context of electron microscopy, of particular interest are designs for low-voltage electron microscopes (LVEM) (electron energies of order 5 keV). These offer many advantages over conventional SEMs/TEMs related to enhanced sensitivity to topography and chemical nature of the sample surface [51]. Indeed, many miniaturized designs for LVEMs have been reported earlier, based on electrostatic lenses [52–54] and magnetic lenses [55–57]. Within the category of magnetic lenses, permanent magnets are used for the obvious advantages of easy mechanical integration and lack of cooling. However, the designs largely resemble conventional yoked-solenoid designs [56,57] with the solenoid being replaced with the permanent magnet. In contrast, the designs presented in this paper go beyond the conventional yoke-based design and instead exploit the field shape of axially magnetized ring magnets, thus providing a new approach to magnetic lens design for LVEM with the added benefits as described above. The same holds for the closely related potential application of electron-beam lithography, including multi-beam electron lithography [58–60]. Currently, endeavors are ongoing to apply such axially magnetized magnets to two beamlines: a “final-focussing” magnet for a 8 keV beamline and the embedded and tiled design to the source of a 100 keV dc-rf beamline design.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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