

Dynamic evolution of ultracold electron beams from nanostructured cathodes

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Dynamic Coulomb expansion of dense, nonuniform particle beamlets can lead to causticlike formations in the charge density profile of the resultant beam. Such features only occur for cold beams where thermal diffusion of the particles is minimized. These phenomena have been observed experimentally for select cases with ion beams but not yet for electron beams as the electron dynamics occur over far faster timescales. We simulate the interaction of electron bunches while varying the initial transverse temperature and density profiles to determine the thresholds that characterize this pattern formation. Furthermore, we extend the simulations to geometric arrangements that are more pertinent to the nanotip cathode application and consider the effects of asymmetries and the impact of a low-density halo on the overall process. The simulations are conducted using a previously documented high-fidelity algorithm for collisional particle dynamics. These studies identify the temperature and density thresholds required for caustic formation and provide guidance for optimizing the design of nanoscale structured cathodes. We also demonstrate that the correlation between the final density patterns and the underlying beam parameters enables a virtual diagnostic tool for measuring ultralow emittance beams solely based on a transverse charge density profile.

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I. INTRODUCTION

The high-intensity, high-brightness frontiers of charged particle beams rely on ultracold electron beams, which are essential for a wide range of applications ranging from ultrafast electron diffraction and microscopy to injectors for free electron lasers (FELs) [1–4]. In such applications, any nonuniformities in the beam distribution and energy profile would degrade the resolution of the imaging techniques and the coherence of the radiation in a FEL. Therefore, it is critical to preserve the quality of the initial beam. A main figure of merit of the electron beam quality is the transverse brightness, defined as the peak current divided by the product of the transverse normalized emittances. At the source, the emittance produced by the cathode is known as the intrinsic emittance [5], which is the ultimate limiting factor of the initial beam quality. The importance of the electron sources as the enabling technology for all advanced applications of electron accelerators was highlighted by the Department of Energy's 2016 report on the Future of Electron Sources [6]. Consequently, many novel electron-source designs have been proposed to meet the various needs of these advanced applications [7–12].

Field emission sources are known to produce the coldest beams in addition to quickly accelerating the particles with the strong applied field. This is important because the self-force between the particles, and thus the degradation of the beam, decreases with increased beam energy. More advanced field emission source designs have the potential to approach the quantum degeneracy limit and the advantage of forming patterned beams [6]. The current state-of-the-art in cold electron sources involves nanostructured cathodes. These can include single nanotips [2,3], thin blades [13], or arrays of nanotips [14–17]. While these are often field emission sources (as those electrons are in the lowest energy state at extraction), they can also be photo-stimulated. In either case, they rely on a sharp tip (or blade) and the geometric field enhancement effect, which can increase the surface field by up to 2 orders of magnitude as compared to the applied gradient.

Cathodes based on arrays of nanoscale structures show promise as high quantum degeneracy sources with modest current output and have the capacity to form beams with specific transverse profiles [8,16,18–21]. The nanoscale pattern of the nanostructure controls the beam's initial distribution, generating density-modulated beams. The sharp nanotip arrays can produce dense ultracold electron beams with highly localized emission sites (i.e., beamlets). There has also been a substantial amount of work in generating cold electron beams using photoemission from cold gases [22] and photocathodes [23,24]. Structured electron beams can be produced by modulating the excitation or ionization laser to create arbitrary density profiles

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using a phase-shifting spatial light modulator [25,26]. It is also possible to achieve structured beams by leveraging plasmonic surfaces [27] or by using cathodes with nanoscale variations in the work function [28].

In the absence of large accelerating gradients, the non-linear Coulomb interactions in dense beams with Gaussian-like distributions will result in substantive expansion that can lead to lasting perturbations in the beam distribution [29–31]. Even with relatively large accelerating gradients, there can be a nontrivial period of time where the Coulombic forces are significant.

The effects of Coulomb expansion have been observed experimentally in the evolution of cold dense ion beams. In the experiment, arrays of rubidium ion beams in close proximity were generated by multilevel laser excitation of rubidium atoms cooled in a magneto-optical trap (MOT) [25]. As they were accelerated, a complex “wheel-and-spokes” pattern was formed in the transverse profile, signaling Coulomb expansion that caused hollowing of the beam cores and density pileup at the boundaries as the individual beams merged [25]. In addition, the low-density halo of secondary emission particles was compressed by the expanding beamlets into the high-density boundaries [25]. These cold halo particles not only increase the density but can also play a role in buffering the Coulombically energized core particles as the expanding beamlets intersect. Similar core hollowing has also been observed in the longitudinal component of a high intensity H^- beam [29] and has even been simulated in the transverse case for a pancake electron bunch [32].

The capability to dynamically shape the ultracold electron source and generate a distinct transverse beamlet profile allows for the investigation of Coulomb expansion and its impact on beam brightness for the high-brightness and high-coherence applications. Since each nanotip will initially produce its own beamlet, it is important to understand the process of merging these individual components to form a single beam. Although the wheel-and-spokes caustics are well-established in ultracold ion beams, their realization in electron beams is not *a priori* implied by the dynamics of ion beams. Indeed, the electron beam dynamics is qualitatively different, owing to their much shorter plasma periods, much smaller mass, and higher intrinsic emission temperatures. In the electron regime, the dynamics is governed by the ratio of the thermal expansion time to the plasma period, describing the competition between collective plasma response and thermal expansion. Such complex electron dynamics requires numerical tools designed to resolve all the electron-electron Coulomb interactions effectively and efficiently (i.e., N -body numerical simulations).

In this paper, we use our high-fidelity collisional N -body algorithm particles’ high-order adaptive dynamics (PHAD) [33] to simulate and study the Coulomb-driven merger of ultracold electron beamlets emitted from nanostructured cathodes. We examine the impact of various initial conditions such as temperature, charge density, and geometric

configuration on the quality of the beam, as shown in our preliminary work described in [34,35]. In addition, we identify the electron-regime control parameter that governs the persistence or washout of wheel-and-spokes caustics and demonstrate how array geometry and charge density control interior-edge thermal asymmetries and pattern coherence. The qualitatively distinguishable results indicate electron-regime dynamics, not a species relabeling of ion results. Our findings demonstrate that array geometry strongly controls the balance between collective heating and field cancellation, leading to pronounced interior-edge asymmetries that directly inform optimal design of the nanoscale structured cathode. Moreover, we show how physically interpretable observables extracted from transverse density profiles can serve as inputs to future machine-learning-based, noninvasive emittance characterization.

The paper is organized as follows: an overview of the beamlet system and the numerical method are described in Sec. II; a detailed investigation of the interaction and merger of multiple electron beamlets for different temperatures, charge densities, and geometric arrangements is given in Sec. III; recommendations for electron-source design are provided in Sec. IV and implications for virtual diagnostics are discussed in Sec. V; and a brief summary is presented in Sec. VI.

II. SYSTEM AND NUMERICAL FRAMEWORK

A. Initial distribution of electron beamlets

The pattern of the cathode’s sharp nanotip arrays generates ultracold electron beamlets that include highly localized dense emission sites. While electrons will primarily be emitted near the apex to form these dense beamlets, some will originate from secondary/alternate emission processes further down the side, forming a low-density halo of electrons around the beamlets. Typically, these halo electrons will have a higher temperature than the dense beamlets as they experience a larger transverse kick during the emission process due to the external electric field profile. The Appendix describes the characterization of the beam quality from the source.

The transverse electron distribution within each beamlet is well approximated by a Gaussian distribution. We use a few figures of merit to describe Gaussian beams, as shown in Fig. 1. The first are the *average* beam figures, which are calculated based on the particles within three standard deviations of the center. Then, the *core* consists of the particles within one standard deviation, and the *peak* includes those within 0.1σ of the center. Figure 1 illustrates these domains along with the vastly different charge densities corresponding to each. Since Coulomb expansion for dense Gaussian beams is largely driven by the dense core, the core and the peak regions are more useful to consider.

Our initial electron beamlet setup is based on the heavy ion configuration from Murphy *et al.* [25]. We adopt the ion geometry as a controlled baseline so differences can be

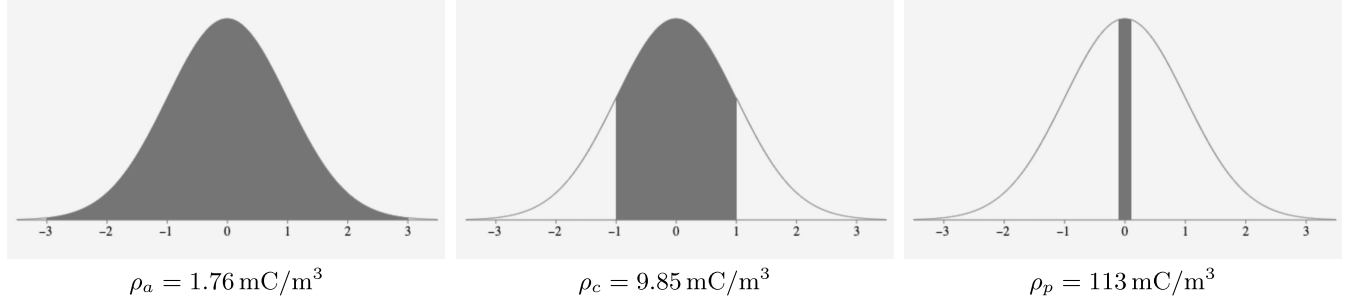


FIG. 1. Charge density and 1D Gaussian plots illustrating the average, core, and peak characteristics of the beamlets given 10^4 electrons per beamlet with 2×10^4 electrons in the halo.

attributed to electron-regime dynamics and timescales, not geometry. This distribution includes nine Gaussian beamlets with a lower density halo of electrons around them (see lower left plot of Fig. 2). The beamlets each have a diameter (full width at half maximum) of $200 \mu\text{m}$ and are uniformly distributed longitudinally with a length of $5 \mu\text{m}$. The halo contains 2×10^4 electrons (3.2 fC) with a uniform density of $n_{\text{halo}} = 1.27 \times 10^{15} \text{ m}^{-3}$, while the beamlets contain 10^4 electrons each (1.6 fC) resulting in a core density (one sigma) of $n_{1\sigma} = 6.15 \times 10^{16} \text{ m}^{-3}$ when factoring in the halo particles.

The initial temperature is inferred from the temperature of the laser-cooled atoms in the MOT [25]. Unlike the ion beams, electron beams from such sources have been observed to be characterized by much higher temperatures [6], due to the rapid heating as a result of the Coulomb repulsion and the much lower mass of the electrons (as compared to rubidium ions). Since the rms momentum spread at the source is described by an effective electron temperature T (see the Appendix), the generated electron beamlet velocity distribution follows Maxwell-Boltzmann statistics, which is given by

$$f(\mathbf{v}) = \left(\frac{m}{2\pi k_B T} \right)^{3/2} \exp\left(-\frac{m \|\mathbf{v}\|^2}{2k_B T} \right),$$

where k_B is Boltzmann's constant, m is the particle's mass, and $\|\mathbf{v}\|^2$ is the square of the velocity's Euclidean norm. The particle velocities can be generated using a 3D form of the Box-Muller transform, which projects randomly sampled uniform data onto a Gaussian distribution. A set of test values $\mathbf{u} = (u_1, u_2, u_3, u_4)$ is sampled from uniform distributions taking values within $(0, 1)$. The velocity vector $\mathbf{v}$ is then given by the following relations:

$$v_1 = \sqrt{-\frac{2k_B T \ln u_1}{m}} \cos(2\pi u_2),$$

$$v_2 = \sqrt{-\frac{2k_B T \ln u_1}{m}} \sin(2\pi u_2),$$

$$v_3 = \sqrt{-\frac{2k_B T \ln u_3}{m}} \cos(2\pi u_4).$$

The accelerating gradient of [25] was in the range of 20–80 kV/m (due to the size of the apparatus), but for this

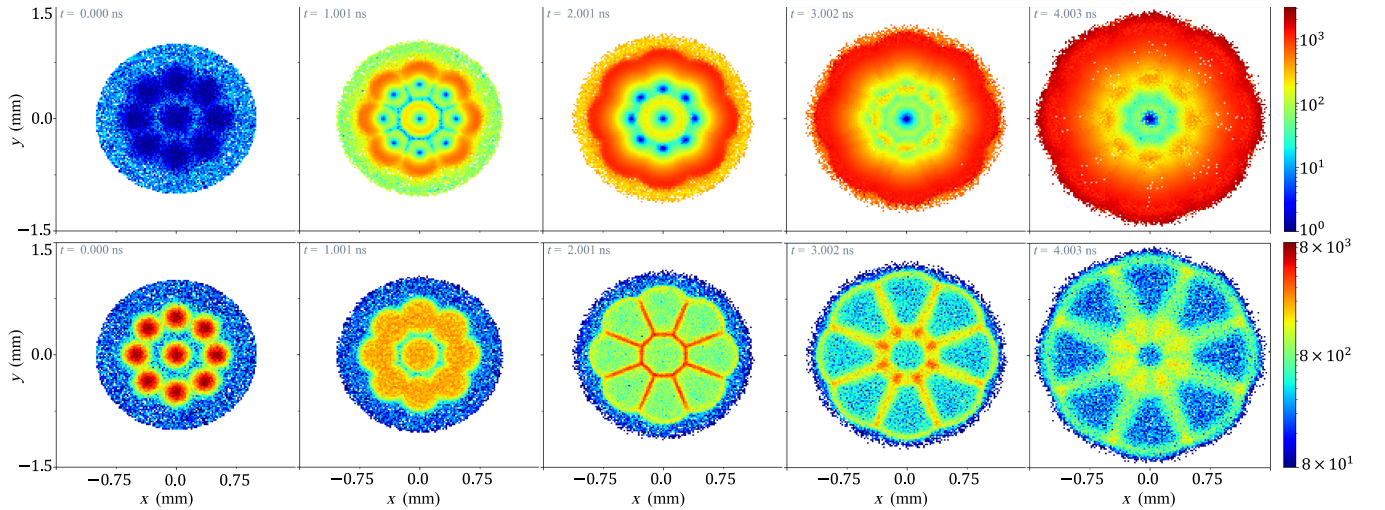


FIG. 2. Time evolution of nine beamlets initially at 1 K with an electron halo initially at 10 K. Plotted is the electron temperature (K) above and charge density ($\mu\text{C}/\text{m}^3$) below at time $t = 0, 1, 2, 3$, and 4 ns from left to right.

application, it makes more sense to use a gradient comparable to that of the nanotip array cathode, $\mathcal{O}(10)$ MV/m. Due to the short timescale of the interactions for electron beamlets, the longitudinal beam acceleration has only a minimal effect on the transverse dynamics (the beam remains nonrelativistic).

B. Pattern observables

The results of the beamlet dynamics in the transverse plane contain multiple features that can be utilized to infer initial parameters of the system. In Sec. V, we discuss the usefulness of these results as a downstream virtual diagnostic tool. As a proof-of-concept, we introduce here a set of physical observables that can be extracted from simulated or measured transverse profiles at a fixed screen plane z . These observables include, but are not limited to, the density contrast, spoke count, anisotropy, dominant power, and interior-rim split.

The density contrast C is computed over the annulus containing the primary ring (excluding obvious hot pixels) and is defined as

$$C \equiv \frac{\rho_{\max} - \rho_{\min}}{\bar{\rho}},$$

where $\rho(r, \phi)$ denotes the 2D transverse charge density (or intensity). This metric tracks the coherence of the pattern with a small- C cutoff serving as an operational threshold for “pattern lost.”

The spoke count (or the dominant azimuthal mode) N_ϕ is obtained from the angular line-out along the primary ring,

$$h(\phi) = \int_{r_1}^{r_2} \rho(r, \phi) w(r) dr,$$

where $[r_1, r_2]$ is a narrow radial window around the ring peak and $w(r)$ is a smooth weight. The discrete Fourier spectrum $|\hat{h}_k|$ yields

$$N_\phi = \arg \max_{k \geq 1} |\hat{h}_k|,$$

with a coherence length estimated from the spectral peak width. This metric is robust to moderate blur and shot noise.

The anisotropy along the ring A quantifies angular intensity variations. Let

$$I(\phi) = \int_{r_1}^{r_2} \rho(r, \phi) dr$$

be the angular profile in a narrow radial window around the ring peak, the anisotropy is then defined as the normalized standard deviation of $I(\phi)$ as follows

$$A \equiv \frac{\sqrt{\frac{1}{2\pi} \int_0^{2\pi} (I(\phi) - \langle I \rangle)^2 d\phi}}{\langle I \rangle}, \quad \langle I \rangle = \frac{1}{2\pi} \int_0^{2\pi} I(\phi) d\phi.$$

Larger A indicates stronger angular modulation (i.e., more pronounced spokes). Using the same $I(\phi)$, the discrete Fourier transform $\hat{I}_k$ allows the computation of the normalized power spectrum for $k \geq 1$. The dominant power P_{dom} is then

$$P_{\text{dom}} = \frac{|\hat{I}_{k_{\text{dom}}}|}{\sum_{k \geq 1} |\hat{I}_k|}, \quad k_{\text{dom}} = \arg \max_{k \geq 1} |\hat{I}_k|,$$

which quantifies the fraction of the total angular power in the dominant spoke mode. High P_{dom} indicates a coherent spoke pattern; low P_{dom} indicates blurred or weak spokes.

The interior-rim split $\Delta\mathcal{T}$ (an MTE proxy) is defined by separating the core and rim regions, Ω_{core} and Ω_{rim} , and evaluating

$$\Delta\mathcal{T} \equiv \mathcal{M}(\Omega_{\text{rim}}) - \mathcal{M}(\Omega_{\text{core}}),$$

where $\mathcal{M}$ is either a direct transverse-temperature/MTE estimate in simulations or an intensity-based proxy in experiments after flat-fielding. Monotonic trends of $\Delta\mathcal{T}$ with the transverse temperature $T_\perp$ and per-beamlet charge make it a useful metric for virtual diagnostics. There are other observables that can be extracted and machine learning methods may reveal latent features that serve as stronger indicators of the system’s initial parameters.

C. Collisional simulations of particle dynamics

The dynamical evolution of the electron beamlets was simulated using the collisional N -body code particles’ high-order adaptive dynamics (PHAD) [33]. PHAD is characterized by the use of Strang splitting to divide the fundamental forces on each particle due to other particles and external fields into near- and far-range components. Far-field forces are efficiently computed using an adaptive fast multipole method [36], while near-field interactions and particle dynamics are accurately captured by the collisional high-order adaptive Simò integrator [37]. This combination yields an algorithm capable of resolving close encounters with high fidelity while preserving the symplectic structure of long-term simulations to machine precision [33]. Reference [33] not only describes PHAD but also illustrates the established accuracy properties of the integrator and field solver.

Because PHAD can describe electron beam dynamics on timescales comparable to the plasma period [down to $\mathcal{O}(\text{fs})$] with exceptional detail, it is well suited for modeling the evolution of electron beamlets and exploring the relevant initial parameter space. Our PHAD simulations of beamlets generated from the nanostructured cathodes

will describe Coulomb expansion and merging of these beamlets, give insight for system-parameter optimization, and motivate a new virtual diagnostic tool.

At the fs-ns timescales that PHAD resolves, the integral observables we report—contrast C , spoke count N_ϕ , anisotropy A , dominant power P_{dom} , interior-rim split $\Delta\mathcal{T}$ —are dominated by collective, order-one features (pileup vs washout) and are insensitive to small perturbations in discretization. We scanned the beamlet parameter space in our simulations using the same numerical settings, including the macroparticle count per beamlet, time step control, near- and far-field split, and multipole/order tolerances. The simulations were performed with real electrons and used a Strang splitting time step of ~ 16.7 ps. We set the FMM order to 6 while the Simò integrator's order was set to 10. Additionally, the Simò integrator used 10 equal-width time bins and maintained an estimated maximum error of 1.75×10^{-15} ; that is, while diagnostic frames are written at a nominal 16.7 ps Strang step, near-field integrations within each step resolve faster encounters as needed, ensuring fidelity to the rapid electron interactions. We note that the same tolerances were applied uniformly and no case-by-case tuning was performed.

III. CHARACTERIZATION OF BEAM CAUSTICS AS A FUNCTION OF BEAMLET PARAMETERS

We next characterize the electron beamlets in the time between generation and the first possible experimental observations. Preliminary results of the simulations presented in this section were described in [34,35]. It is worth noting the electron-regime control parameters necessary to characterize our results. These are the electron plasma period τ_p (see the Appendix), and the thermal expansion/merger times τ_{exp} of the beamlet expressed as

$$\tau_{\text{exp}} \sim \frac{a}{v_\perp}, \quad v_\perp \simeq \sqrt{\frac{k_B T_\perp}{m_e}},$$

where a is the relevant transverse scale (e.g., beamlet radius or half the pitch), and m_e is the electron mass. Because $\tau_p \propto m_e^{1/2}$ and $v_\perp \propto m_e^{-1/2}$, electrons inhabit different stability windows than ions even at comparable geometry and charge per beamlet. The dimensionless ratio between the relevant timescales can be expressed as

$$\Theta \equiv \frac{\tau_{\text{exp}}}{\tau_p},$$

which controls pattern persistence for different electron cases.

When the transverse scale a is the radius of a single beamlet, the time τ_{exp} describes the beamlet's own thermal expansion. This is the time it takes for particles to travel

from the core to the edge, driven by their initial temperature. If this time is short compared to the plasma period [$\Theta \lesssim \mathcal{O}(1)$], the beamlet itself will diffuse before it has a chance to interact with its neighbors to form spokes and caustics. When a equals half the distance between beamlet centers, the time τ_{exp} explains the merger of two adjacent beamlets. It is the time it takes for particles from two neighboring beamlets to meet at the midpoint. This time is the most relevant for the wheel-and-spokes pattern since the spokes are basically the high-density pileups formed at these midpoints. If this time is long compared to the plasma period [$\Theta \gtrsim \mathcal{O}(1)$], then the plasma's collective response has time to act, and coherent pileup/spoke formation is favored and the pattern is sustained. Θ predicts persistence except when nonlinear shock or overlap dynamics dominate at very high density.

Our first simulation consisted of the beamlet profile described earlier in Sec. II A with the typical case where the beamlet temperature is lower than the halo temperature. The beamlet temperature was set to 1 K and the halo temperature was set to 10 K. The bottom row of Fig. 2 illustrates the dynamics of the beamlets as they merge to form a single beam over a period of 4 ns. Once allowed to evolve, the initial Gaussian profile leads to a Coulomb explosion as the dense core electrons gain transverse energy and expand. As beamlets impinge on their neighbors, high-density regions are formed at the boundaries after about 2 ns from the combined effect of the expanding beamlet cores and the halo electrons pushed together. A less distinct dense outer ring begins to form at this time, and the full wheel-and-spokes pattern, similar to [25], is observed around 4 ns. The ratio for the density pileup and the wheel-and-spokes pattern formation in this case is $\Theta \sim \mathcal{O}(10^2)$, and hence the pattern is sustained.

The upper row of Fig. 2 illustrates the evolution of the transverse temperature given by

$$T = \frac{|\mathbf{p}_\perp|^2}{2m_e k_B}, \quad (1)$$

throughout the process, where $\mathbf{p}_\perp$ is the transverse momentum. Notably, the proximity of adjacent beamlets produces a damping effect on the temperature gain, with the hottest electrons located in the outer ring. While the halo particles may play a small role in moderating the temperature increase, the predominant damping factor comes from the neighboring beamlets. This effect persists as the beam continues to evolve, with a core temperature that is orders of magnitude cooler than the extremity electrons after 4 ns.

For the remainder of this paper, it is assumed, unless referring to the initial simulation parameters, that the temperature is defined as in Eq. (1). We will also use the temperature images to estimate the interior-rim split $\Delta\mathcal{T}$ for our simulation results.

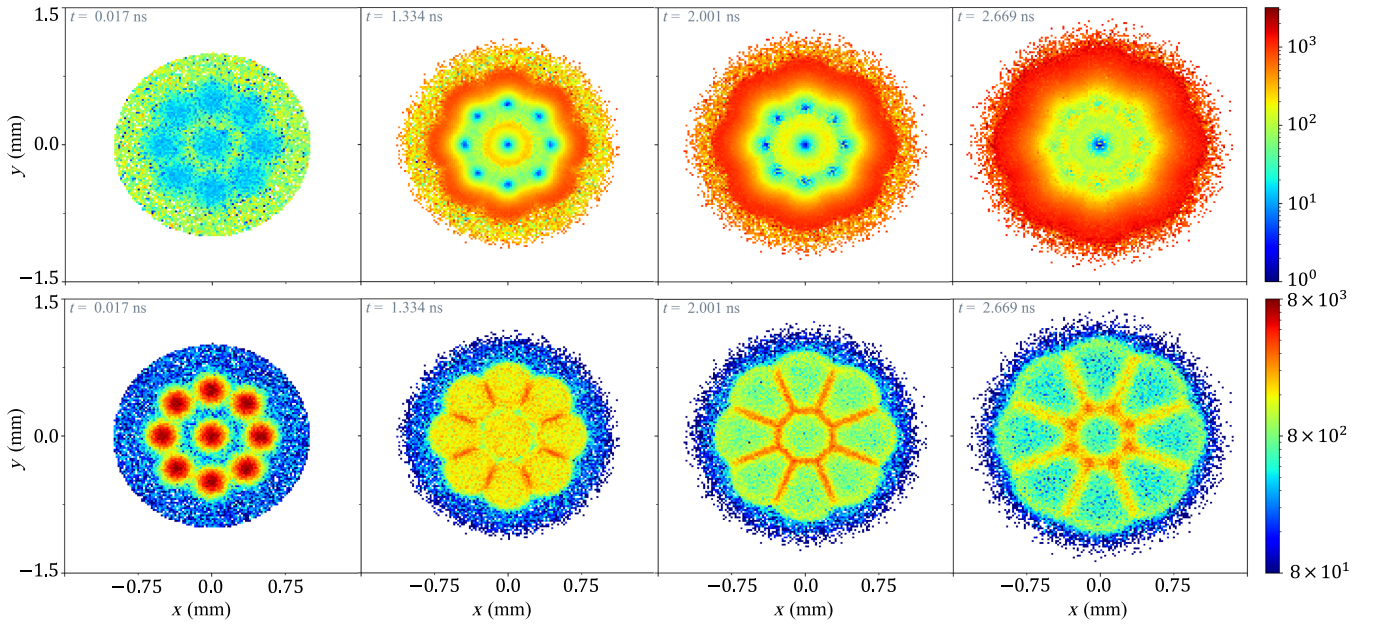


FIG. 3. Time evolution of nine beamlets initially at 10 K with an electron halo initially at 100 K. Plotted is the electron temperature (K) above and charge density ($\mu\text{C}/\text{m}^3$) below at time $t = 0, 1.3, 2,$ and 2.7 ns from left to right.

A. Role of the initial electron temperature

The density pileup at the boundaries of the merging beamlets is fundamentally a phenomenon of ultracold particle dynamics driven by Coulombic interactions. The nature and coherence of the pattern formation depend on the initial temperature of the beamlets and, to a much lesser degree, the temperature of the halo particles. Increasing the initial temperature of the beamlets and the halo by an order

of magnitude (10 and 100 K, respectively), as shown in Fig. 3, leads to a degradation in the pattern. The general behavior is still apparent, but the high-density structures are broadened, and the temperature, especially at the periphery, is higher. The inverse temperature configuration (100 and 10 K for the beamlets and the halo, respectively) is shown in Fig. 4. Here, there is a much higher degradation of the interaction regions, and the core temperature, while still

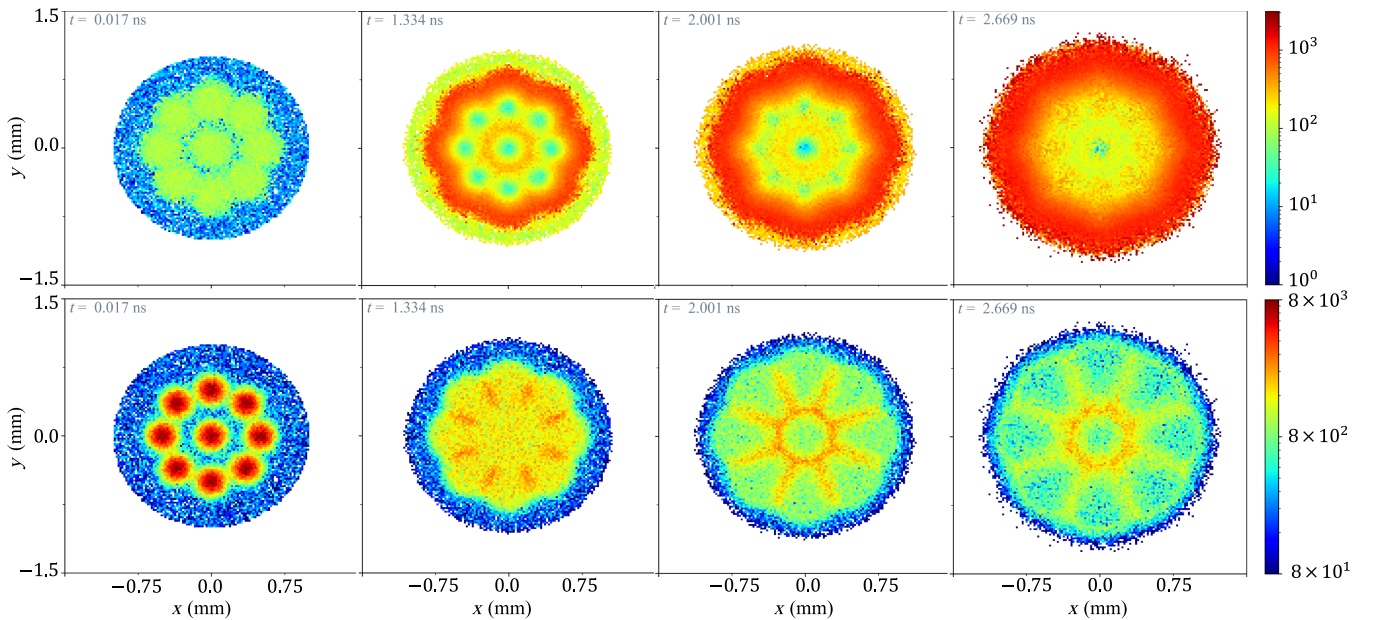


FIG. 4. Time evolution of nine beamlets initially at 100 K with an electron halo initially at 10 K. Plotted is the electron temperature (K) above and charge density ($\mu\text{C}/\text{m}^3$) below at time $t = 0, 1.3, 2,$ and 2.7 ns from left to right.

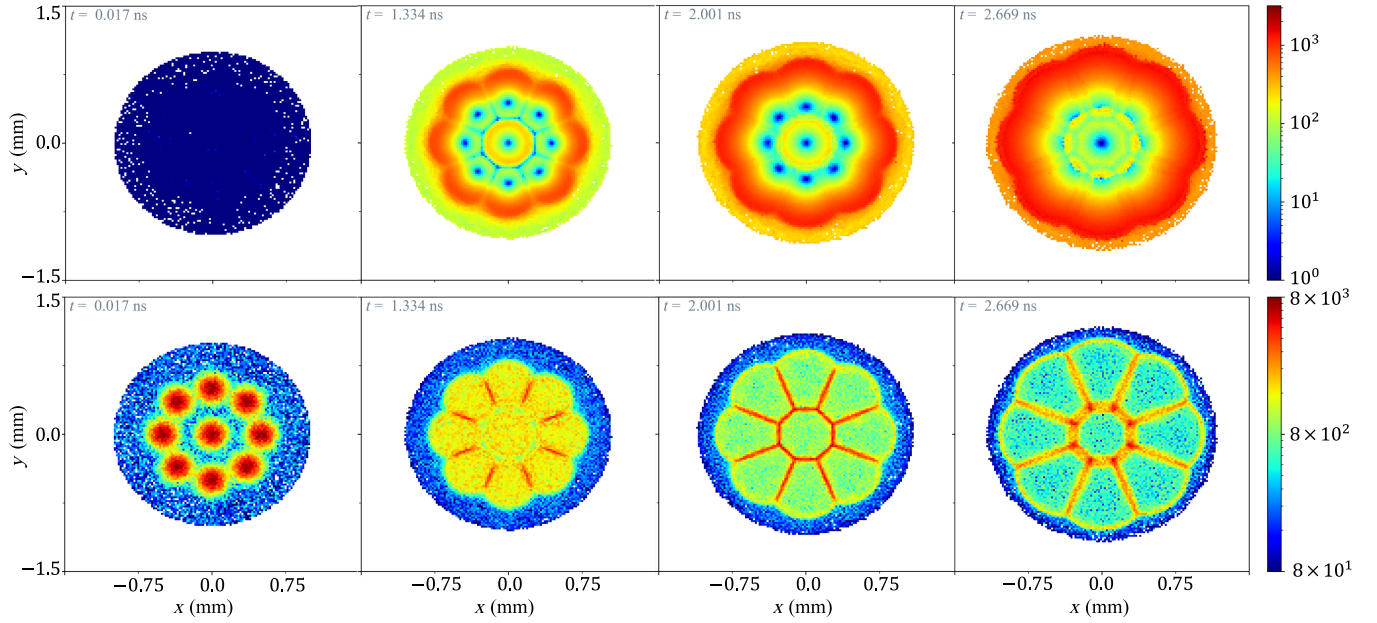


FIG. 5. Time evolution of nine beamlets with an electron halo both initially at 0.2 K. Plotted is the electron temperature (K) above and charge density ($\mu\text{C}/\text{m}^3$) below at time $t = 0, 1.3, 2$, and 2.7 ns from left to right.

cooler than the exterior, is clearly worse than previous two cases. As the initial beamlet temperature is increased, the pattern blurs until it is barely visible. For beamlet temperatures above 100 K (≈ 8.6 meV), the pattern formation is completely swamped by the internal thermal noise. For intermediate cases with $\Theta \sim \mathcal{O}(10)$, caustics still form, but features broaden, consistent with the observed partial degradation relative to the colder cases.

By contrast, decreasing the initial temperatures of both beamlets and halo to 0.2 K (effectively zero) shows only subtle differences from the base case. Here, the pattern formation ratio $\Theta \sim \mathcal{O}(10^2)$ suggests a persistent pattern. Figure 5 shows that the patterns are subtly crisper, especially in the temperature maps, but the overall profiles are very similar. This implies that once the beamlet temperature drops below 10 K, there is very little impact in the overall profiles due to the initial temperature.

For a fixed electron density, varying the initial temperature showed that the low temperatures 0.2 and 1 K resulted in high Θ values and displayed a sustained pattern while the high temperatures 10 and 100 K resulted in low Θ values and a slightly degraded, washed-out pattern. As $T_\perp$ increased, the spoke structures were the first to broaden and wash out, which perfectly aligns with the $\Theta \lesssim \mathcal{O}(1)$ washout condition.

The halo only plays a minor role in the interior region (by providing additional electrons to increase the density pileup) and a modest role in the periphery density ring, as increasing the temperature (even to 100 K or more) does not have a significant impact. Indeed, the resulting density structures will still form similarly if no halo is present. Since the temperature increase is dominated by Coulomb

expansion, there will be a high degree of correlation between transverse position and momentum for exterior particles.

We list in Table I the physical observables for the temperature scan estimated from the images at ~ 2 ns. As expected, all results have a spoke count of 8. All the other observables decrease and the pattern starts to degrade as the temperature increases. The density contrast values indicate that images with $C \lesssim 0.6$ exhibit pattern washout. Very small values of anisotropy ($A < 0.05$) indicate the ring is nearly isotropic where spokes are weak or absent. The dominant power of ~ 0.1 describes the clearly visible spokes while we see weak or washed-out spokes for $P_{\text{dom}} < 0.06$. Since the rim temperature does not significantly change as $T_\perp$ increases while the core gets warmer, the interior-rim split ΔT should decrease. Indeed, ΔT decreases with $T_\perp$ and this monotonic change is a good indicator for virtual diagnostics. In general, we see a slight difference in the observable values between the 1 and 0.2 K, and thus these temperatures may represent the lower bound for detectability. These lower bound temperatures could be

TABLE I. Physical observables for different initial transverse temperatures $T_\perp$ of the beamlet system estimated from the image at ~ 2 ns.

Temperature (K)	C	N_ϕ	Anisotropy	P_{dom}	ΔT (K)
0.2	0.94	8	0.083	0.095	797
1	0.89	8	0.078	0.095	785
10	0.61	8	0.058	0.061	751
100	0.53	8	0.043	0.0063	705

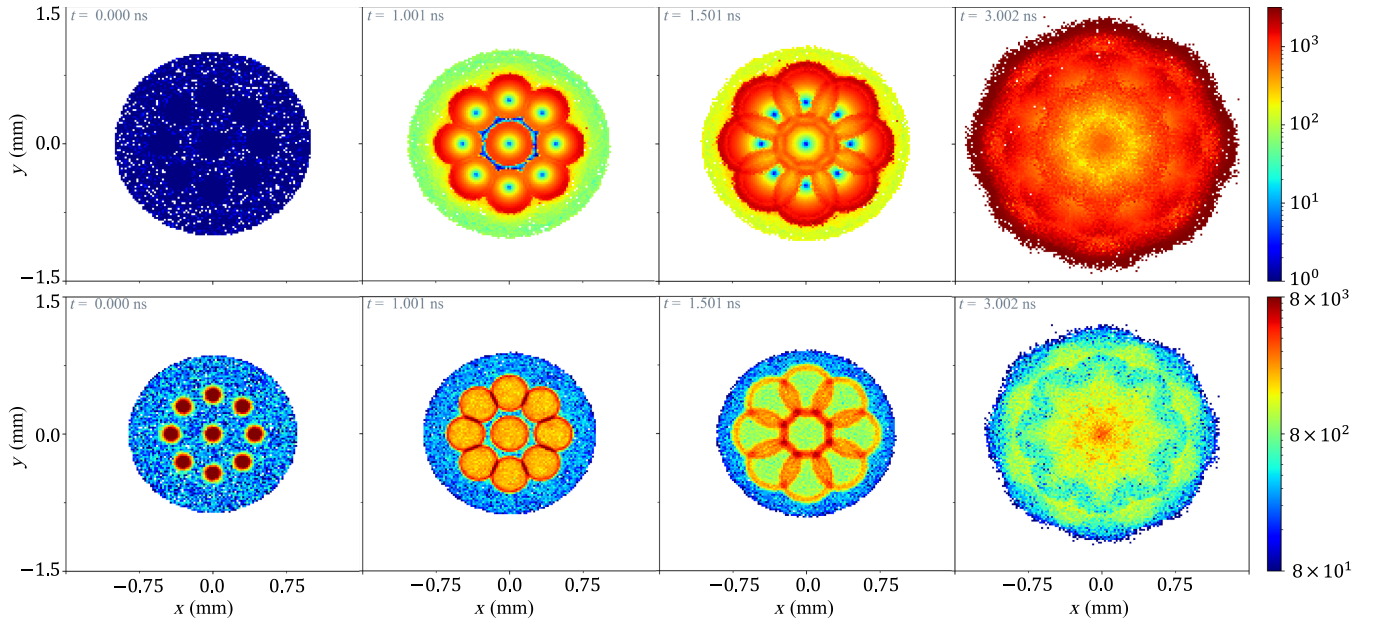


FIG. 6. Time evolution of nine beamlets with the beamlet radius halved (quadrupled charge density); all electrons initially at 0.2 K. Plotted is the electron temperature (K) above and charge density ($\mu\text{C}/\text{m}^3$) below at time $t = 0, 1, 1.5$, and 3 ns from left to right.

close to the quantum degeneracy limit depending on the charge density. Moreover, any source should have a much lower temperature than 100 K (i.e., ≤ 10 K) to see meaningful density pileups form.

B. Variations in the initial beamlet charge density

Varying the initial density of the beamlets also significantly impacts the nature of the beamlet merger and the form of patterns that arise. Decreasing the initial beamlet radius by a factor of 2 (quadrupling the density) leads to a more rapid Coulomb expansion causing the beamlets to gain transverse momentum more rapidly than previous cases. Due to the beamlet center locations remaining fixed, it takes the beamlets about the same time to merge (around 1.5 ns). Unlike previous cases, Fig. 6 shows that the dense exteriors of the beamlets cross each other due to higher energy from the Coulomb explosion rather than forming the high-density pileup. While some structure remains, it does not resemble the familiar wheel-and-spokes pattern. The pattern formation ratio is $\Theta \sim \mathcal{O}(10^2)$, which suggests that the pattern is not degraded or washed out by thermal expansion. However, this case is no longer a simple race between the pileup and thermal expansion. While indeed the pattern is not blurred or degraded, the plasma dynamics are extremely fast. The rapid Coulomb expansion from the quadrupled density is so strong that the beamlets cross each other instead of forming a stable pileup at the boundary. This is a density-driven regime where the outer shocks overtake and soften the classic wheel-and-spokes pattern rather than sharpen it. Additionally, the beam thermal profile is much hotter throughout than the corresponding cases with lower initial density.

Another scenario involves decreasing the distance of the outer beamlets from the center beamlet commensurately with the decrease in beamlet radius. These results, shown in Fig. 7, reveal that the wheel-and-spokes pattern again forms (this time in less than 1 ns). However, the rapid Coulomb expansion injects enough energy into the system that the pattern breaks down over time. In both of the increased charge density cases, there remains a higher density core in the beam. This core is actually more uniform than in the lower density cases because the density shock waves pass through each other and overlap more broadly than in the clear wheel-and-spokes patterns. Unfortunately, while the density profile is perhaps more ideal, the thermal profile in both cases is notably worse, indicating that the final state will continue to evolve rapidly. The pattern formation ratio $\Theta \sim \mathcal{O}(10^2)$ is similar to the case with the smaller density in Fig. 5. This predicts that the thermal diffusion does not dominate and that the pattern forms. Yet, the subsequent breakdown of the pattern over time is caused by the strong Coulomb expansion in this scaled-down, high-density system.

At ~ 1.5 ns, the first case of the density increase exhibited a slightly higher contrast $C \sim 1$, anisotropy $A = 0.093$, and dominant-mode power $P_{\text{dom}} = 0.68$ than the second case, which had $C \sim 0.9$, $A = 0.091$, and $P_{\text{dom}} = 0.51$. This indicates that the beamlet system in the first case maintained a relatively strong spokelike modulation with a defined angular pattern compared to a more washed-out angular structure in the second case. The spoke count for both cases is 8 as expected from the geometry. A large difference in the interior-rim split ΔT is observed between the two cases with $\Delta T \approx 810$ K and

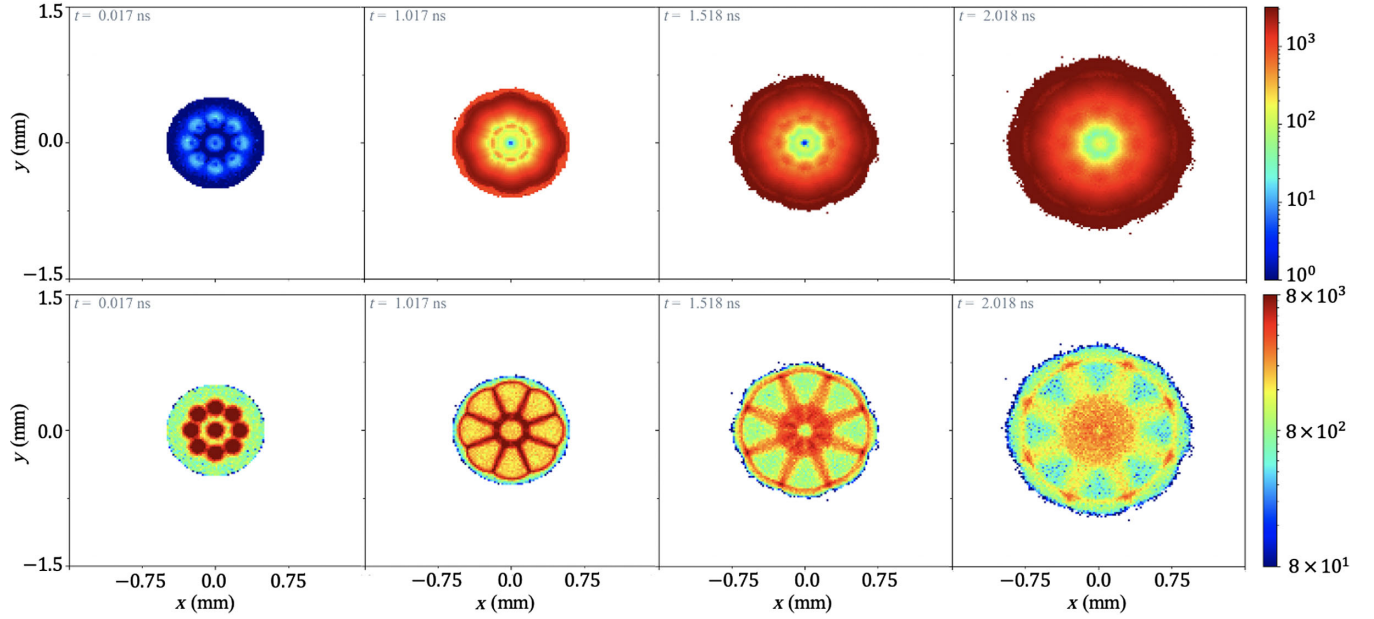


FIG. 7. Time evolution of nine beamlets spatially scaled down by 50% (quadrupled charge density); all electrons initially at 0.2 K. Plotted is the electron temperature (K) above and charge density ($\mu\text{C}/\text{m}^3$) below at time $t = 0, 1, 1.5$, and 2 ns from left to right.

≈ 1600 K for the first and second cases, respectively. This is because the rim heating is much higher for the second case than the first case. The overall behavior of both cases is directly correlated with the electron scaling parameter Θ for the first case being approximately twice that of the second case [both values remaining on the order of $\mathcal{O}(10^2)$]. The larger Θ value implies persisting coherent angular structure, whereas the relatively smaller Θ value amplifies the washout of the spokes. Although Θ remains large in this regime, the classic Θ -based coherence criterion breaks down because the nonlinear Coulomb shock velocity itself becomes the dominant ordering scale.

C. Extensions to large arrays of beamlets

Considering the thermal plots in Fig. 2, it is clear that the proximity of neighboring beamlets has a mitigating effect on the electron heating due to Coulomb expansion. For a single charged beam in isolation (regardless of species), this explosion leads to a core hollowing with an isotropic expansion and thermalization of the particles [29], resulting in a higher density, higher temperature ring (or shock) around the exterior of the beam. By placing a ring of eight beamlets around a central beamlet, it is possible to preserve a cooler region in the center of the unified beam which retains a relatively high particle density if the characteristic beamlet parameters are optimized. Since high energy electrons are concentrated at the extremity of the beamlet array, they can be selectively removed (i.e., via collimation), efficiently cooling the beam with a minimal loss of particles. This would be especially effective for sources that generate a large array (i.e., thousands) of beamlets (such as

the silicon nanoarray cathode in [8,20]) if this phenomenon persists on that scale.

To evaluate this prospective interaction, four different beamlet array configurations are evaluated; each case includes at least 25 beamlets. The first is a rectangular 5×5 grid, the second is a slant grid with six columns alternating with 5 and 4 rows, the third is a radial hexagonal array with a core and four rings of 6, and the fourth is a radial octagonal array with a core and three rings of 8. For consistency, the nearest neighbor distance in the grids and the radial distance in the rings are the same distance as in the original beamlet configuration, r_b . For the rings, the radial distances for the n th ring are given by $r_b(n+1)/2$. To isolate the geometric factors, the initial temperature of the beamlets and the halo is set to a minimal 0.2 K, and the halo shape and density are defined for each case to have a consistent density and to extend beyond the beamlets by approximately the beamlet nearest neighbor distance. Due to software architecture limitations, the total number of particles in the simulation must be reduced from the original number per beamlet. The use of macroparticles can cause distortions, especially in the high charge density regime, so instead, the beamlet radius (and all other corresponding distances) are decreased by a factor of 2, and the number of particles is decreased by a factor of 4. This ensures that the relative magnitude of the forces stays the same, since it scales by charge divided by distance squared.

Figure 8 includes snapshots of the evolution of the rectangular array of 5×5 beamlet grid to the point where the beamlets have fully merged to form a single beam. The third and fourth plots in the sequence show a cooler and

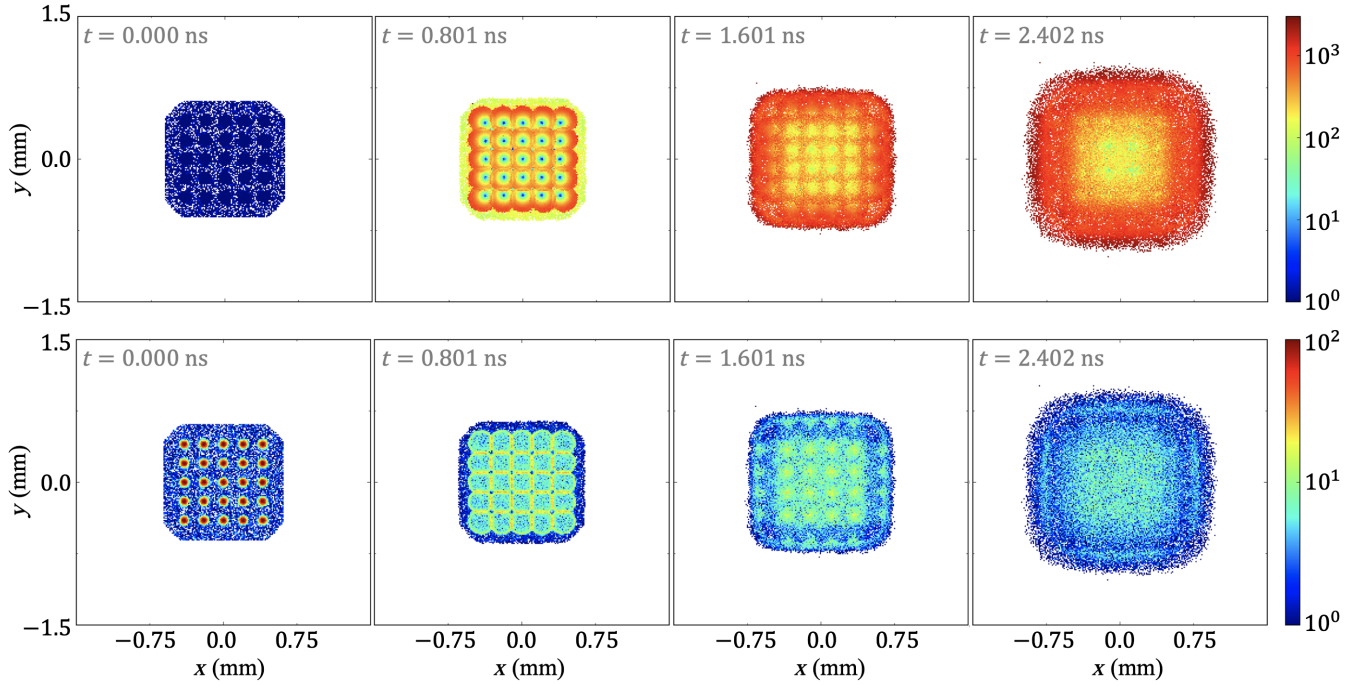


FIG. 8. Dynamic evolution of a rectangular beamlet array with beamlet charge reduced by 75% and spatially scaled down by 50% with respect to the original beamlet parameters. Plotted is the electron temperature (K) above and charge density ($\mu\text{C}/\text{m}^3$) below at time $t = 0, 0.8, 1.6$, and 2.4 ns from left to right.

denser region in the interior of the outer beamlets, with peak densities and lowest temperatures occurring at the intersection points between sets of four adjacent beamlets. By the final plots, the interior density has lost most of its structure (though the average remains an order of magnitude higher than the exterior). However, the temperature retains some of the geometric structure, with the coolest values at the central four intersection points, as shown in the upper row of Fig. 8.

The slant array, shown in Fig. 9, takes the same basic arrangement as the rectangular array, but adds in an asymmetry in the overall shape. In this case, there are 14 interior beamlets compared to 16 in the rectangular case. Here, the evolution shows that the core structure at 1.6 ns is less clear than the rectangular case, though the exterior higher density ring is less pronounced. Comparing the interior region to that in the rectangular array, the latter has a larger interior region that appears to have a slightly higher density in general. The overall temperature profile at 2.4 ns for the slant array is slightly lower than that for the rectangular case, but the cooler interior space is larger for the former.

The ring arrays are based upon extending the original arrangement with additional concentric rings, while maintaining the number of beamlets per ring. Figure 10 includes snapshots of the dynamics for the hexagonal array, while Fig. 11 includes the corresponding plots for the octagonal array. In both cases, the overall behavior is similar to that of the grid arrays. The beamlet intersections occur more

quickly for the octagonal array and somewhat faster for the hexagonal due to some of the interbeamlet distances being shorter (this is especially clear for the octagonal case). There is a cool dense core, a hot exterior, and a thin high-density ring to the outside (these trends are also true for the standard nine beamlet configuration). With the larger arrays, the “spokes” do not persist as long as in the standard configuration, but the dense core remains. Similar to the grid case, fewer exterior beamlets have a greater mitigating effect on the core temperature growth and thus better preserves a higher core density.

In these multibeamlet arrays, the electron scaling parameter for pattern formation $\Theta \sim 160$ remains large enough for the pattern to form and for the systems to be above the thermal washout regime. This allows coherent plasma-driven structures to develop, with complex high-density regions forming preferentially in the array interior as seen in our simulations. The emergence of cooler and denser interior regions is a direct consequence of $\Theta \gg \mathcal{O}(1)$ for which plasma pileup effects are pronounced and density modulations persist. In addition, Θ also governs the observed interior-edge asymmetry. Interior beamlets experience partial cancellation of neighbor fields, leading to reduced effective transverse heating and enhanced coherence, whereas rim beamlets are subject to unbalanced fields that increase heating and accelerate spoke attenuation. As a result, the geometry-dependent preservation or degradation of angular coherence is directly tied to the fraction of interior beamlets supported by a given array layout.

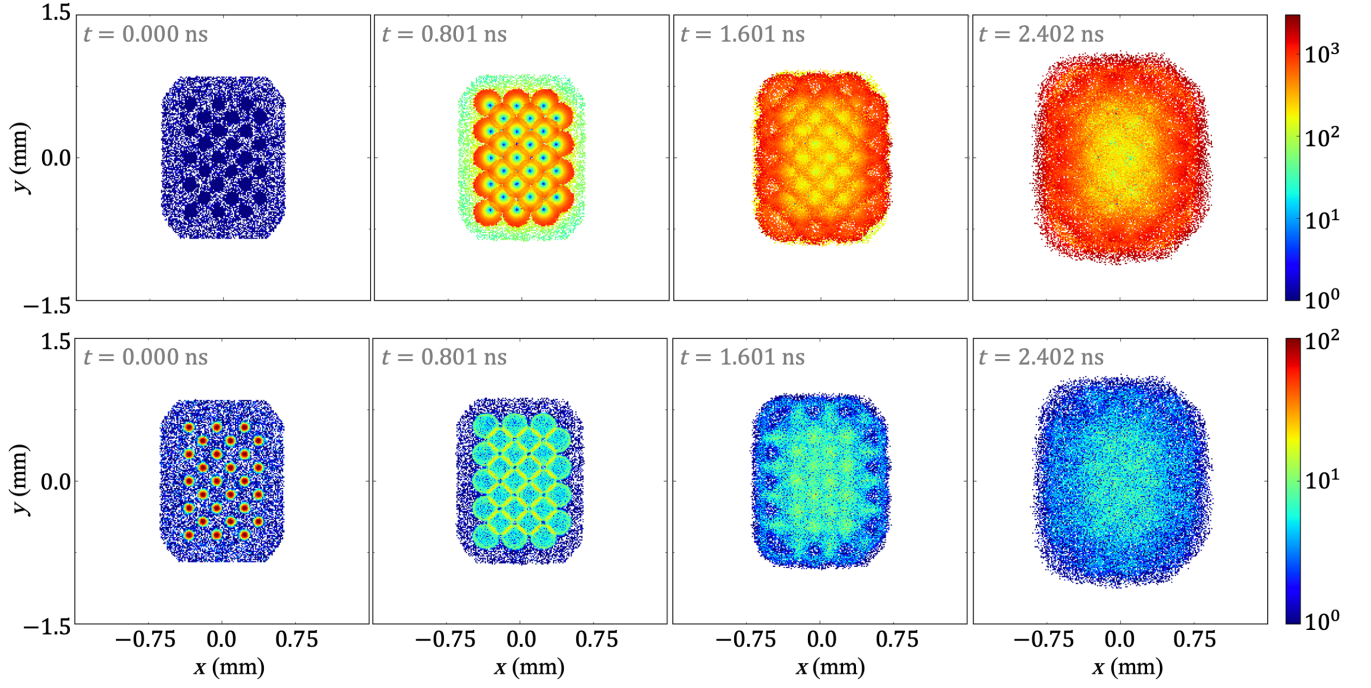


FIG. 9. Dynamic evolution of a slant beamlet array with beamlet charge reduced by 75% and spatially scaled down by 50% with respect to the original beamlet parameters. Plotted is the electron temperature (K) above and charge density ($\mu\text{C}/\text{m}^3$) below at time $t = 0, 0.8, 1.6$, and 2.4 ns from left to right.

The geometric array scan further highlights the role of symmetry and neighbor-field cancellation in setting the observable angular structure. As seen in Table II, the rectangular geometry produces the strongest geometry-imprinted

angular mode, while the slanted and hexagonal arrays exhibit significantly weaker dominant power. The octagonal case remains intermediate, consistent with partial but incomplete interior-field cancellation. The corresponding density

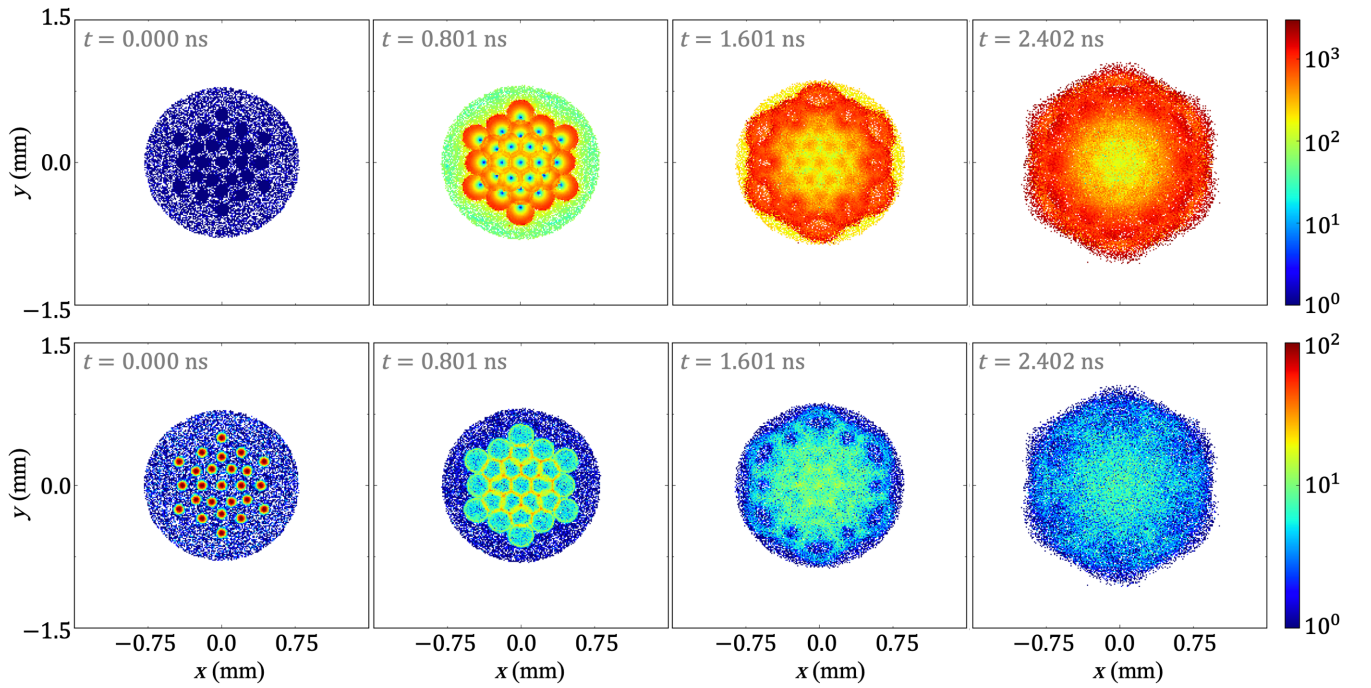


FIG. 10. Dynamic evolution of a hexagonal beamlet array with beamlet charge reduced by 75% and spatially scaled down by 50% with respect to the original beamlet parameters. Plotted is the electron temperature (K) above and charge density ($\mu\text{C}/\text{m}^3$) below at time $t = 0, 0.8, 1.6$, and 2.4 ns from left to right.

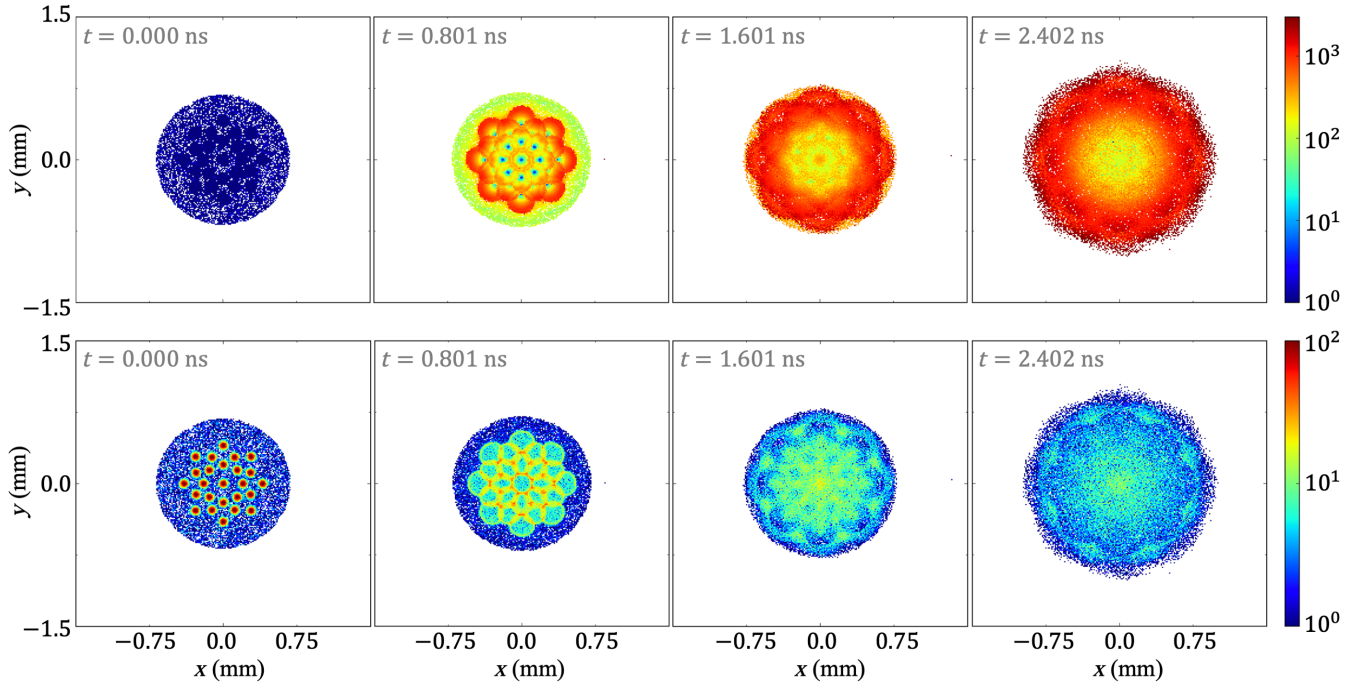


FIG. 11. Dynamic evolution of an octagonal beamlet array with beamlet charge reduced by 75% and spatially scaled down by 50% with respect to the original beamlet parameters. Plotted is the electron temperature (K) above and charge density ($\mu\text{C}/\text{m}^3$) below at time $t = 0, 0.8, 1.6$, and 2.4 ns from left to right.

contrast values follow a similar trend, indicating that although angular coherence is weak in most cases, residual geometric modulation remains measurable across all layouts. For the anisotropy, the values for all four geometries remain small ($A \lesssim 0.06$) confirming that none of these cases exhibit strong global symmetry breaking or fully developed spoke instabilities. Instead, the observed angular structure is dominated by weak geometry-seeded modes rather than collective space-charge-driven effects.

The rectangular and octagonal lattices produce the strongest rim heating, while the slanted pattern yields the weakest, indicating that the rim-core temperature split depends on how each injection geometry couples to the natural azimuthal modes of the expanding beam. Although the hexagonal lattice has higher mathematical symmetry, the ring formation dynamics preferentially amplify low-order modes $N_\phi = 4, 8, \dots$, which arise from radial expansion and density-field coupling; the rectangular (fourfold)

and the octagonal (eightfold) patterns align with these modes and therefore reinforce azimuthal compression, spoke formation, and energy concentration at the rim, resulting in larger ΔT . By contrast, the hexagonal arrangement (sixfold) and especially the slanted configuration, which mainly imprints a weak $N_\phi = 2$ signature, do not match the natural dominant modes and thus produce weaker overlap and reduced rim heating. Overall, the ΔT trends support—but do not yet definitively prove—that geometry-driven symmetry matching influences rim energy deposition.

In all of the simulations, the trends establish Θ as the primary control parameter governing whether electron beamlets reside in a coherent pileup regime, a thermally washed-out regime, or a density-driven shock regime. We now translate these regime boundaries into concrete design rules for nanostructured cathodes. We also emphasize that the Θ -regime favorable for virtual diagnostics ($\Theta \gtrsim 1$), where coherent density pileup and azimuthal modes are maximized, is not necessarily the same regime desired for routine accelerator operation, where collective instabilities and excessive Coulomb heating must be suppressed ($\Theta \lesssim 1$). In practice, diagnostic operation may therefore rely on short dedicated tuning pulses, while steady-state high-brightness operation remains in the stability-optimized regime.

Collectively, the interior cooling with hot rim, the density-driven shock-overtake breakdown of spokes, and the geometry-dependent coherence windows constitute

TABLE II. Physical observables for different geometric configurations of multibeamlet array systems estimated from the image at ~ 2.4 ns.

Array	C	N_ϕ	Anisotropy	P_{dom}	ΔT (K)
Rectangular	0.83	4	0.052	0.21	600
Slant	0.94	2	0.060	0.064	480
Hexagonal	0.84	6	0.050	0.077	525
Octagonal	0.81	8	0.044	0.13	610

electron-specific outcomes not implied by ion-beam studies.

IV. ELECTRON-SOURCE DESIGN RULES

One advantage of understanding the process of merging the individual beamlet components to form a single beam is that it will directly inform the design specifications of the nanotip array. As each nanotip produces its own beamlet initially, the array design can be optimized based on the optimum beam parameters produced. For the ultracold beams, we have generally observed that the arrangements that maximize the number of fully interior beamlets, such as the rectangular grid and the hexagonal ring arrays here, appear to result in the largest favorable interior regions. Therefore, they are the best candidates for large beamlet arrays. Yet, it should be noted that the ring arrays are not as easily scalable due to the average beamlet separation increasing for each subsequent ring. It may be possible to further reduce the beamlet heating by developing a source that produces electrons in a preordered state [30]. For the ultrabright beams, our results are not enough to quantify the optimal design, though our temperature and density variations seem to indicate an ultrabright core.

From our simulation results, we conclude several rules for optimal ultracold electron-source design. As mentioned above, the number of interior beamlets should be maximized by selecting array geometries and chip designs that increase the interior-to-edge ratio, such as rectangular or hexagonal layouts. Interior beamlets benefit from partial neighbor-field cancellation, which reduces effective transverse heating and helps preserve coherence. The strongly edge-biased emission should be avoided, as concentrating large charge at the rim promotes shock overtaking and softens the emergent pattern, and so current should be distributed to favor the interior. It is also advantageous to include a guard ring when fabrication allows because surrounding the active emitters with a ring of lower-current “dummy” tips helps balance edge fields and suppresses rim heating and spoke attenuation.

For a given transverse temperature and density, keep $\Theta \lesssim 1$ near launch to suppress nonlinear density pileup and azimuthal mode growth. The beamlet radius or the pitch a and the charge density n should be chosen such that τ_{exp} does not greatly exceed τ_p . This condition provides an upper bound on the acceptable MTE for a given density or a density bound for a given MTE. When $\Theta \gg 1$, strong spoke formation and density pileup are observed. However, the multibeamlet sources tolerate larger Θ values than single-beamlet emitters due to interbeamlet field cancellation. This explains the geometry-dependent persistence or washout of structure observed in our simulations.

In addition, optimal source operation should be in an intermediate temperature window. Our simulations at very low temperatures ($T_\perp \lesssim 1$ K) illustrated that the system is highly susceptible to collective instabilities and spoke

formation. At higher temperatures ($T_\perp \gtrsim 10$ K), thermal velocity spread induces rapid geometric washout. Therefore, $T_\perp$ should be low enough to maintain the ultracold coherence across beamlets, but high enough to suppress strong collective instabilities. Moreover, early optics should remain gentle and symmetric, as strong or asymmetric focusing prior to the first ring disrupts the beneficial neighbor-field balance experienced by interior beamlets. Weak, symmetric launch optics better preserve the coherence advantages of the electron regime.

Finally, the diagnostic plane should be placed near the first caustic since a short drift enhances the spokes contrast C and improves the fidelity of observables such as N_ϕ , A , P_{dom} , and ΔT for inferring MTE and beamlet density.

V. IMPLICATIONS FOR VIRTUAL DIAGNOSTICS

In the ultracold regime, beam fluctuations at the source strongly affect the beam quality and the accelerator performance. An accurate and effective diagnostic tool is crucial to measure the quality of the electron beam. However, the complex nature of the electron beam dynamics is very challenging to measure, especially due to the very short relevant timescales (on the order of the plasma period). Conventional beam measurement/diagnostic tools directly interact with the beam to characterize its quality. Typical diagnostics include beam profile monitors such as scintillation screens, optical transition radiation monitors, wire scanners, and others [9]. Since these devices directly interact with the beam, they may alter or even destroy the beam, and typically the beam cannot be used for the actual application at the same time. In addition, the accuracy of these diagnostics occasionally faces technical limitations as in the case of very intense or very low charge beams. Other well-known techniques to measure the transverse emittance are quadrupole scans and multiple screens, yet they do not provide detailed information about the beam distribution.

Consequently, virtual diagnostics, which indirectly measure the beam quality, are being pursued and developed. Because the downstream beam parameters are correlated with a set of upstream machine parameters in this case, the beam parameters are indirectly measured using nondestructive measurements of the upstream accelerator parameters. There are different virtual diagnostic methods including mathematical techniques such as linear and nonlinear interpolations. A more attractive tool involves machine learning where various models were created with high precision, efficiency, adaptability, and reliability [38–43].

The image-domain observables $\{C, N_\phi, A, P_{\text{dom}}, \Delta T\}$ provide a compact, physically interpretable basis for a virtual diagnostic mapping downstream patterns to source MTE and beamlet density. In each of the cases presented in Sec. III, variations in the initial temperature, charge density, and geometric arrangement lead to distinct downstream transverse profiles. Thus, the final profiles have the potential to encode the aforementioned characteristics

and other important metrics like intrinsic emittance. However, there are ultimately so many variables to optimize—even just factoring in geometric arrangements and charge density—that the problem appears intractable. We have only evaluated initial temperatures in the range 0.2 to 100 K, as we found that the pattern formation will be completely swamped by the internal thermal noise for temperatures higher than 100 K. The charge density was varied twice here where we pushed to the maximum desirable density (due to the rapid Coulomb heating), but lower densities could also be relevant. While we have considered different geometric arrangements, there are other possible array configurations, including a more random approach. Most of our tests kept the beamlet spacings relatively consistent, but it is also possible to vary the initial separation in both directions. Other parameters to inspect include distributions different from Gaussian within each beamlet and various configurations of asymmetries in the beam.

Because much of the outcome is based on two-dimensional profiles which cannot easily be reduced to simple parameters, this problem is well suited for an image-based machine learning approach. This holds promise for use as a downstream virtual diagnostic/measurement tool. The machine learning approach can treat this problem as an inverse problem between the downstream images and the underlying cathode parameters. The data for the machine learning would include information of the initial parameters at the cathode and downstream images of the bunch for different upstream parameters. Ideally, the downstream images would be experimental, but such measured images cannot be obtained by the current measurement tools. Hence, the image data could be acquired from simulations similar to what is included in this paper. Our detailed PHAD simulation typically requires about two days of computation to generate a downstream image, motivating the use of surrogate models such as physics-informed neural networks (PINNs) to accelerate data generation. Convolutional neural networks (CNNs) are natural candidates for this task due to their strong performance in image-based learning, and transfer learning strategies may be employed when training data are limited.

Because the scaling parameter Θ determines which collective regime the beam occupies, the transverse image itself implicitly encodes the source thermal and density state. In experimental operation, Θ is not known *a priori*; instead, the observed transverse pattern implicitly encodes whether the beam resides in a thermally dominated washout regime [$\Theta \lesssim O(1)$] or a collective pileup regime [$\Theta \gtrsim O(1)$]. Therefore, virtual diagnostics act as an inverse map, using image-domain observables to infer the effective Θ -regime and, by extension, the underlying source temperature and density. In this sense, Θ functions as a latent physical classifier that regularizes the interpretation of image-based diagnostics.

As discussed in Sec. II B, a set of physically interpretable image observables should be sufficient to characterize the collective beam regime. These may include the density contrast C , the dominant azimuthal power P_{dom} , the anisotropy A , and the interior-rim temperature split $\Delta\mathcal{T}$. Each of these quantities exhibits monotonic or near-monotonic dependence on key source parameters such as transverse temperature and beamlet charge density. The thermally dominated washout regime [$\Theta \lesssim O(1)$] is characterized by low C , suppressed P_{dom} , and weak anisotropy A . In contrast, the coherent pileup regime [$\Theta \gtrsim O(1)$] is marked by elevated C , A , and P_{dom} . A third, density-driven shock regime at very high beamlet density is identified by the outer shocks overtaking and softening the classical wheel-and-spokes pattern while broadening the ring even for $\Theta \gtrsim O(1)$. This regime structure enables classification-based virtual diagnostics prior to regression-based inference of detailed source parameters.

The interior-rim temperature split $\Delta\mathcal{T}$ is particularly useful as it decreases monotonically with the transverse temperature. In simulations, $\Delta\mathcal{T}$ can be computed directly from the transverse kinetic energy, while in experiments it can be inferred from appropriately flat-fielded intensity profiles. This makes $\Delta\mathcal{T}$ a robust proxy for the source mean transverse energy (MTE), enabling single-shot, noninvasive inference of intrinsic emittance from downstream images without requiring dedicated emittance measurement hardware.

Within this framework, CNNs may be trained either directly on transverse images or on hybrid inputs consisting of images augmented with physics-informed features such as C , A , P_{dom} , and inferred Θ -regime labels. This hybrid strategy is especially advantageous for limited training data, as it embeds known physical scaling behavior directly into the learning architecture. PINNs further offer a complementary route for enforcing constraints derived from collective plasma expansion during training. At the same time, fully image-based machine learning models remain capable of discovering additional, potentially non-linear correlations between the measured transverse profiles and the initial source conditions beyond those explicitly captured by the chosen observables.

All diagnostic observables introduced here should be extracted from standard transverse screens placed near the first density caustic, requiring no additional invasive instrumentation. As a result, the proposed virtual diagnostic framework is directly compatible with existing beamline hardware and can be deployed with minimal experimental overhead, enabling real-time inference of beam quality during operation.

VI. SUMMARY AND CONCLUSIONS

There are numerous applications for the state-of-the-art electron sources that push the ultracold, high-brightness

frontier including free electron lasers and various imaging techniques such as electron microscopy and diffraction. The quality of the electron source is the ultimate limiting factor in the overall quality of the beam, and many novel designs have been proposed to meet the varied needs of these advanced applications. Cathodes based on arrays of nanoscale structures show promise as a high quantum degeneracy source with modest current output that also have the capacity to form beams with specific transverse profiles. Dynamic Coulomb expansion of dense nonuniform particle beamlets can lead to causticlike formations in the charge density profile of the resultant beam. These phenomena have been observed experimentally for select cases with ion beams, but not yet for electron beams as the electron dynamics occur over far faster timescales.

To investigate these processes, we simulated the dynamical evolution of beamlets across a variety of source configurations. These simulations reveal how beamlet density pileup and pattern formation occur, as well as how multiple beamlets can merge into a single, larger beam. In our scans, we have observed that the maximum temperature where meaningful density pileup occurs is ~ 100 K, and thus any source should have a much lower temperature (i.e., < 10 K) for this pattern to form. Since there is a slight difference visually between 1 and 0.2 K, this temperature may represent the lower bound for detectability. These lower bound temperatures could be close to the quantum degeneracy limit depending on the charge density. In both of the high charge density cases, there was a more uniform high-density core in the beam, but the thermal profile was much hotter, indicating that the beam may continue to evolve rapidly. For large arrays with optimal characteristics, internal beamlets remained cooler due to the stabilizing interactions with their neighbors, illustrating the potential for beams from large emitting arrays to retain a cool core over longer periods than for a single beam in isolation.

Our results also inform practical design rules for ultracold, high-brightness nanostructured cathodes. The arrays should maximize the fraction of interior beamlets and use close-packed or symmetric geometries to reduce global anisotropy. Moderate extraction times ($\Theta \lesssim 1$) should be maintained to suppress collective instabilities and wheel-and-spokes formation. It is also essential to operate at intermediate transverse temperatures to balance the suppression of disorder heating with the prevention of geometric washout. Gentle and symmetric early optics further preserve multibeamlet coherence.

Furthermore, the patterns observed in the simulations provide a basis for virtual diagnostics as early-time extracted physical observables such as density modulations and anisotropy measures can be used to infer initial beamlet temperature, emittance, and density distribution. Given the high-dimensional parameter space, these design and diagnostic challenges may benefit from image-based machine

learning approaches, enabling optimized source operation and characterization.

In summary, our studies establish a framework linking ultracold beam physics, array geometry, and operational parameters, providing clear suggested guidelines for designing nanoscale cathode arrays and leveraging virtual diagnostics to monitor and optimize ultracold electron sources.

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DATA AVAILABILITY

The source code used to generate the results in this study is available at [44]. A detailed description of the initial setup and model parameters is provided within the article.

APPENDIX: EVALUATION OF BEAM QUALITY

The overall quality of an electron beam can be expressed by the brightness B . In 6D, the brightness is defined by the charge Q per the normalized 6D emittance as $B_{6D} = Q/\epsilon_6$, where $\epsilon_6 = \epsilon_x \epsilon_y \epsilon_z$ for uncoupled motion. For applications where the transverse to longitudinal coupling is very small, the 4D brightness (or the transverse brightness) is usually used to describe the beam quality. The 4D brightness is characterized by the peak current $I_p = Q/\sigma_t$ and the normalized transverse emittances as

$$B_{4D} = \frac{I_p}{\epsilon_x \epsilon_y}.$$

At the source, there is no correlation between the position and the momentum coordinates and the normalized transverse emittance is given by $\epsilon_{\perp} = \sigma_{\perp} \sigma_{p_{\perp}}/mc$ (c is the speed of light). In addition, the momentum spread $\sigma_{p_{\perp}}$ at the source can be described by the temperature T as $\sigma_{p_{\perp}} = \sqrt{mk_B T}$. This gives the normalized transverse emittance $\epsilon_{\perp,c} = \sigma_{\perp} \sqrt{k_B T/(mc^2)}$ known as the source intrinsic emittance. Assuming a cylindrically symmetric beam, the achievable brightness of an electron source is given by

$$B_{4D} \leq \frac{I_p}{\epsilon_{\perp,c}^2} = \frac{mc^2 I_p}{\sigma_{\perp}^2 k_B T}.$$

It can be seen that the brightness is inversely proportional to the initial temperature in addition to being proportional to the peak current. Hence, the brightness of the electron sources can be improved by either increasing the peak current at the source or decreasing the temperature of the

source. Much of the effort focused on improving the brightness by decreasing the source emittance to increase the current density at the source. Yet, the smaller the emittance, the larger the nonlinear space-charge forces that can spoil the brightness, and the increase of the current is limited. As a result, the source temperature is considered the figure of merit for the quality of the initial electron beam. This signifies how the ultracold electron sources may outperform conventional electron sources in terms of the electron beam quality or brightness.

In practice, the source mean transverse energy $MTE = k_B T$ determines the source emittance and brightness, and it is the limiting factor of beam brightness. Due to collective effects and beamline imperfections, the emittance of the generated beam is always larger than the intrinsic emittance set by the MTE; and thus the achieved B_{4D} is never equal to the source brightness.

After the beam is generated, the electron beam dynamics occur at a rate given by the plasma frequency

$$\omega_p = \sqrt{\frac{nq^2}{\epsilon_0 m}},$$

where n is the particle density, q gives the charge species, and ϵ_0 represents the permittivity of free space. This is related to the fundamental timescale given by the plasma period $\tau_p = 2\pi/\omega_p$. The particle density for a cylindrically symmetric beam is generally defined by

$$n = \frac{N}{\pi \sigma_r^2 \sigma_z},$$

where N gives the total number of particles, and $\sigma_{r/z}$ gives the transverse and longitudinal beam size. For an electron density of $\mathcal{O}(10^{16}) \text{ m}^{-3}$, the corresponding characteristic timescale is $\mathcal{O}(1)$ ns.

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