

Analytical modeling and experimental verification of oscillatory ponderomotive instabilities in closed-loop superconducting rf cavities

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Superconducting radio-frequency (SRF) cavities with high-loaded-Q are susceptible to ponderomotive instabilities arising from nonlinear electromechanical coupling. Under low-level rf feedback control, stability analysis is challenging and has, until now, lacked comprehensive experimental verification. In this work, we linearize the coupled cavity electromagnetic and mechanical equations about the operating point, deriving explicit small-signal stability criteria for both open- and closed-loop operation. A characteristic equation approach is used to map the oscillatory instability threshold in the detuning–gradient plane. Time-domain simulations confirm the predicted onset of self-excited oscillations at these thresholds. Furthermore, experiments on a 162.5 MHz SRF linac (CAFE2) show that the observed oscillation onset agrees with our analytical predictions within a small margin. By combining analytical modeling, numerical simulation, and direct experimentation, this study provides comprehensive validation of oscillatory ponderomotive instability theory under feedback control, offering practical guidance for the stable operation of high-gradient SRF accelerators.

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I. INTRODUCTION

The stable operation of continuous-wave (cw) superconducting radio-frequency (SRF) linacs at high beam power is contingent upon the rf stability of their SRF cavities. With loaded half-bandwidths of only a few tens of hertz [1–8], such cavities can be driven off-resonance by even modest external disturbances (microphonics) or internal Lorentz-force detuning (LFD) [9–12]. When the resulting electromechanical coupling becomes regenerative—implying positive feedback between the electromagnetic field and mechanical motion—two types of ponderomotive instabilities may develop: a monotonic jump (or drop) and an oscillatory growth in cavity field amplitude and detuning.

Among these, oscillatory instabilities often pose greater operational challenges for routine SRF cavity operation.

The theoretical foundations of ponderomotive instabilities were established decades ago. Schulze first identified stability thresholds for generator-driven resonators [13], and Delayen later extended these analyses to cavities operating in self-excited loops [14]. More recent studies at ANL, TRIUMF, and DESY have further generalized these stability criteria to control schemes of greater practical relevance [15–20]. Despite the rigor of these theoretical treatments, comprehensive experimental validation under real SRF cavities remains limited. Building upon these theoretical foundations, our work introduces a more experimentally relevant formulation. In contrast to previous studies, we derive analytic expressions for the instability thresholds and present the first direct experimental comparison on real SRF cavities.

In this work, we develop a small-signal model of the cavity system capturing the coupled electromagnetic and mechanical dynamics under amplitude-phase feedback control. Using an algebraic elimination method, we derive characteristic equations for both open- and closed-loop configurations, yielding explicit instability boundaries in the detuning–gradient plane. Notably, this approach

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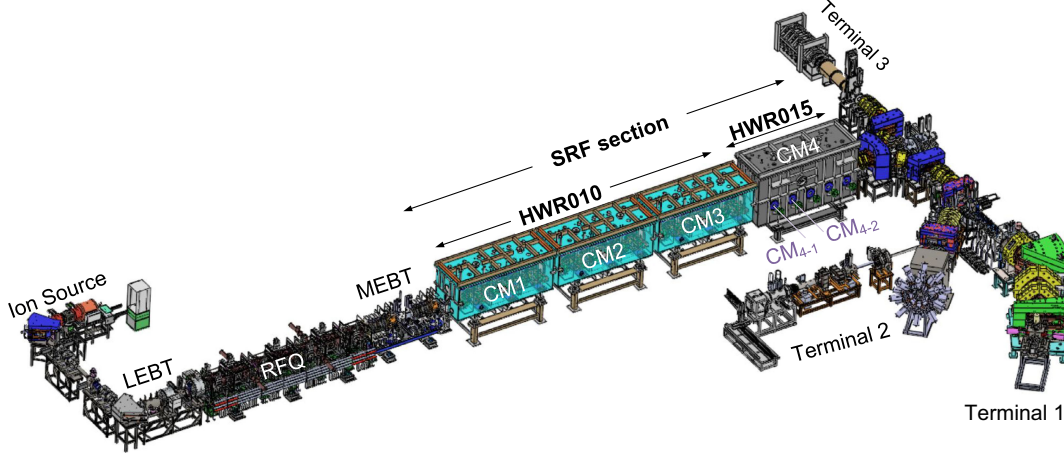


FIG. 1. Schematic layout of the CAFE2 facility. The SRF section consists of four cryomodules (CM1–CM4): CM1–CM3 each house six HWR010 cavities, while CM4 contains five HWR015 cavities. The two HWR015 cavities under study (CM₄₋₁ and CM₄₋₂) are highlighted. In the notation CM_{*m-n*}, *m* denotes the cryomodule index and *n* the cavity position within the module. Downstream experimental terminals are also indicated.

provides analytic criteria for ponderomotive instabilities in a cavity operated under closed-loop control, and these criteria are directly supported by experimental measurements through comparison with simulation results.

Dynamic simulations confirm the theoretical predictions by reproducing the onset of oscillatory instabilities under various feedback conditions. Further corroboration is provided through dedicated experiments conducted on two SRF cavities at the China Accelerator Facility for superheavy Elements (CAFE2, see Fig. 1), a 162.5 MHz high-power SRF linac [21–24]. Our systematic experimental measurements show excellent agreement with analytical results, providing comprehensive and direct validation of the theory under realistic, high- Q_L operating condition.

The remainder of this paper is organized as follows: Sec. II introduces the CAFE2 facility and its low-level rf (LLRF) control system. Section III develops the coupled electromechanical small-signal model and defines the instability criteria. Section IV presents dynamic simulations validating the analysis. Section V reports on experimental measurements, and Sec. VI concludes with implications for accelerator operations.

II. CAFE2 FACILITY AND LLRF CONTROL SYSTEM

To provide a concrete experimental platform for instability modeling and validation, this work makes use of the CAFE2 facility. This section introduces the facility layout and technical specifications relevant to the observed ponderomotive instabilities.

The CAFE2 is an upgrade of the former Chinese ADS Front-end (CAFe) linac [25–28]. It is a high-power, 162.5 MHz heavy-ion superconducting linac dedicated to experiments in superheavy element research and isotope production [21–24]. As illustrated in Fig. 1, CAFE2 comprises an ion source, a low-energy beam transport line, a normal-conducting radio-frequency quadrupole accelerator, a medium-energy beam transport line, and 23 superconducting half-wave resonator (HWR) cavities housed in four cryomodules (CM1–CM4). The cavities operate with high-loaded quality factors (Q_L) in the range of 3×10^5 to 1×10^6 , corresponding to narrow half-bandwidths in the range of a few tens to several hundred hertz. Table I summarizes the key rf and mechanical parameters [24]. For clarity, $f_{0.5}$ in the table refers to the half-bandwidth in

TABLE I. Operating parameters of the CAFE2 superconducting cavity.

Cavity parameters	Units	HWR010 (CM1–CM3)	HWR015 (CM4)
f_{rf} (operational rf frequency)	MHz	162.5	162.5
Q_L (cavity loaded Q)	arb. units	$3 \times 10^5 - 1 \times 10^6$	$6 \times 10^5 - 8 \times 10^5$
$f_{0.5}$ (cavity half-bandwidth)	Hz	80–270	100–135
Normalized shunt impedance	Ω	225	382
E_{peak} (peak gradient)	MV/m	25–35	~30
V_c (cavity voltage)/ E_{peak}	m	0.038	0.066
Optimized β (v/c)	arb. units	0.10	0.15
K_{LFD} (LFD coefficient)	Hz/(MV/m) ²	–0.4–0.2	~ –0.2

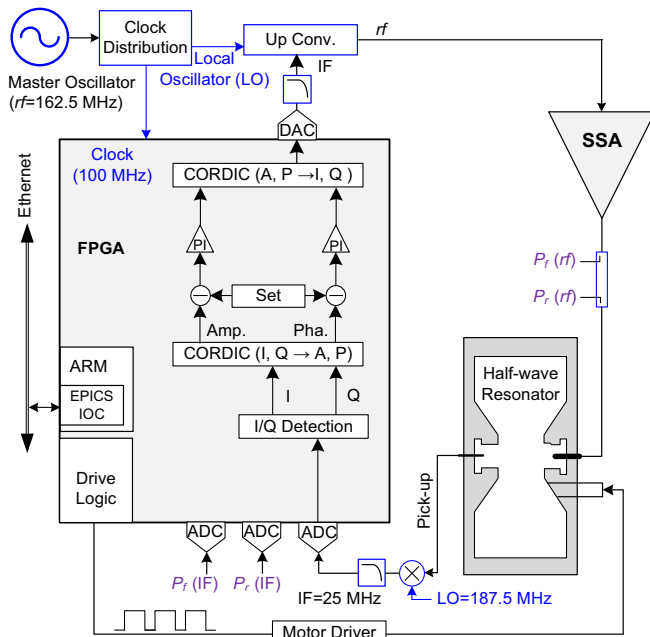


FIG. 2. Block diagram of the LLRF system at the CAFE2 facility.

hertz, while $\omega_{0.5}$ used in later equations denotes its angular frequency equivalent in rad/s.

Figure 2 shows a schematic of the low-level rf (LLRF) control system deployed at CAFE2 [24,29,30]. The cavity pickup signal at 162.5 MHz is first down-converted to an intermediate frequency (IF) of 25 MHz, and subsequently digitized at a sampling rate of 100 MHz. Digital signal processing within the field-programmable gate arrays (FPGA) extracts the amplitude and phase (A/P) of the cavity field in real time. A proportional-integral (PI) feedback controller is digitally implemented to stabilize the A/P of the cavity field by dynamically adjusting the cavity forward signal. The corrected signal is then converted back to analog at 162.5 MHz via a digital-to-analog converter (DAC), amplified by a solid-state amplifier (SSA) [31], and delivered to the superconducting cavity.

Each cavity is equipped with a motorized mechanical tuner for resonant frequency adjustment. The tuner can achieve a mechanical tuning range of up to 5 MHz by elastically deforming the cavity structure. During typical operation, the tuner shaft rotates at up to 50 rpm ($\approx 15 \text{ Hz s}^{-1}$ detuning slew rate), enabling smooth, continuous compensation of slow detuning drifts and environmental perturbations.

III. SMALL-SIGNAL PERTURBATION MODEL

In this section, we derive a linear small-signal perturbation model of the coupled electromagnetic–mechanical dynamics of the cavity to analyze ponderomotive instabilities. This formulation yields the characteristic equation that governs the system stability under feedback control.

Throughout this paper, complex-valued phasor quantities (e.g., cavity voltage $\mathbf{V}_c$, forward signal $\mathbf{V}_f$) are indicated in boldface to improve readability and facilitate distinction from real-valued quantities.

A. Model formulation and linearization

For a strongly overcoupled ($\beta \gg 1$) SRF cavity without beam loading, the complex cavity-voltage dynamics $\mathbf{V}_c(t)$ in generator-driven operation can be written as (see Refs. [32–34]):

$$\frac{d\mathbf{V}_c}{dt} + (\omega_{0.5} - j\Delta\omega)\mathbf{V}_c(t) = 2\omega_{0.5}\mathbf{V}_f(t), \quad (1)$$

where $\omega_{0.5}$ is the cavity half-bandwidth, $\Delta\omega$ is the cavity detuning, and $V_f(t)$ is the cavity forward voltage.

We consider a steady-state operating point where the cavity voltage $\mathbf{V}_{c0}$ is normalized to be on the real axis, i.e., $\mathbf{V}_{c0} = 1 + j0$. This convention simplifies the analysis without loss of generality. At this operating point with a constant detuning $\Delta\omega_0$, the required steady-state forward voltage is

$$\mathbf{V}_{f0} = 0.5 - j \cdot \frac{\Delta\omega_0}{2\omega_{0.5}}. \quad (2)$$

The phase of this steady-state forward voltage is given by $\theta_0 = \tan^{-1}(-\Delta\omega_0/\omega_{0,5})$.

To analyze the system's response to small deviations from this steady state, we express the perturbed cavity voltage in polar coordinates as $\mathbf{V}_c(t) = r(t)e^{j\phi(t)}$. The amplitude $r(t)$, phase $\phi(t)$, and detuning $\Delta\omega(t)$ are represented as small perturbations around their steady-state values:

$$r(t) = 1 + \delta_r(t)$$

$$\phi(t) = \delta_\phi(t)$$

$$\Delta\omega(t) = \Delta\omega_0 + \delta_\omega(t) \quad (3)$$

Here, $\delta_r(t)$, $\delta_\phi(t)$, and $\delta_\omega(t)$ are the small-signal deviations. This analysis assumes that the perturbations are small ($|\delta_r| \ll 1$, $|\delta_\phi| \ll 1$, and $|\delta_\omega| \ll \omega_{0.5}$), justifying a first-order linearization.

Using the first-order Taylor approximation $e^{j\delta_\phi} \approx 1 + j\delta_\phi$, the perturbed cavity voltage is linearized as

$$\mathbf{V}_c(t) \approx (1 + \delta_r)(1 + j\delta_\phi) \approx 1 + \delta_r + j\delta_\phi, \quad (4)$$

where the second-order term $\delta_r \delta_\phi$ is neglected. The time derivative of Eq. (4) is therefore

$$\frac{d\mathbf{V}_c}{dt} \approx \dot{\delta}_r + j\dot{\delta}_\phi. \quad (5)$$

B. Open-loop characteristic equation

Substituting the perturbed variables defined in Eq. (3) and the approximations from Eqs. (4) and (5) into the approximated cavity dynamics [Eq. (1)]:

$$\begin{aligned} \dot{\delta}_r + j\dot{\delta}_\phi + [\omega_{0.5} - j(\Delta\omega_0 + \delta_\omega)](1 + \delta_r + j\delta_\phi) \\ = 2\omega_{0.5}\mathbf{V}_f(t). \end{aligned} \quad (6)$$

Equating real and imaginary parts gives:

$$\begin{aligned} \Re\{2\omega_{0.5}\mathbf{V}_f(t)\} &= \dot{\delta}_r + \omega_{0.5}(1 + \delta_r) + (\Delta\omega_0 + \delta_\omega)\delta_\phi \\ \Im\{2\omega_{0.5}\mathbf{V}_f(t)\} &= \dot{\delta}_\phi + \omega_{0.5}\delta_\phi - (\Delta\omega_0 + \delta_\omega)(1 + \delta_r). \end{aligned} \quad (7)$$

Under open-loop operation, the $\mathbf{V}_f$ remains at its steady-state value [see Fig. 3(a)]:

$$2\omega_{0.5}\mathbf{V}_f(t) = 2\omega_{0.5}\mathbf{V}_{f0} = \omega_{0.5} - j\Delta\omega_0. \quad (8)$$

Substituting Eq. (8) into Eq. (7):

$$\begin{aligned} \dot{\delta}_r + \omega_{0.5}(1 + \delta_r) + (\Delta\omega_0 + \delta_\omega)\delta_\phi &= \omega_{0.5} \\ \dot{\delta}_\phi + \omega_{0.5}\delta_\phi - (\Delta\omega_0 + \delta_\omega)(1 + \delta_r) &= -\Delta\omega_0. \end{aligned} \quad (9)$$

Neglecting second-order terms and subtracting steady-state components (such as $\delta_\omega\delta_\phi$, $\delta_\omega\delta_r$) gives:

$$\begin{aligned} \dot{\delta}_r &= -\omega_{0.5}\delta_r - \Delta\omega_0\delta_\phi \\ \dot{\delta}_\phi &= -\omega_{0.5}\delta_\phi + \Delta\omega_0\delta_r + \delta_\omega. \end{aligned} \quad (10)$$

Applying the Laplace transform (zero initial conditions):

$$\begin{aligned} s\delta_r(s) &= -\omega_{0.5}\delta_r(s) - \Delta\omega_0\delta_\phi(s) \\ s\delta_\phi(s) &= -\omega_{0.5}\delta_\phi(s) + \Delta\omega_0\delta_r(s) + \delta_\omega(s). \end{aligned} \quad (11)$$

LFD arises from cavity wall deformation, approximately proportional to the field amplitude squared. Linearizing near $r = 1$:

$$\Delta\omega(t) \propto r^2(t) = (1 + \delta_r)^2 \approx 1 + 2\delta_r. \quad (12)$$

Hence,

$$\delta_\omega(t) \approx h(t) * 2\delta_r(t), \quad (13)$$

where $h(t)$ denotes the mechanical impulse response of the cavity structure, and the symbol $*$ represents the convolution operation. The Laplace transform of this relationship is given by

$$\delta_\omega(s) = 2H(s)\delta_r(s), \quad (14)$$

where $H(s) = \mathcal{L}\{h(t)\}$ is the transfer function corresponding to the mechanical response.

From Eq. (11),

$$\delta_\phi(s) = -\frac{(s + \omega_{0.5})}{\Delta\omega_0}\delta_r(s). \quad (15)$$

Substituting Eqs. (14) and (15) into the second line of Eq. (11), we obtain:

$$\begin{aligned} -\frac{s(s + \omega_{0.5})}{\Delta\omega_0}\delta_r(s) &= \frac{\omega_{0.5}(s + \omega_{0.5})}{\Delta\omega_0}\delta_r(s) \\ &+ \Delta\omega_0\delta_r(s) + 2H(s)\delta_r(s). \end{aligned} \quad (16)$$

Eliminating $\delta_r(s)$ yields the open-loop characteristic equation:

$$D_{ol}(s) = (s + \omega_{0.5})^2 + \Delta\omega_0^2 + 2\Delta\omega_0H(s) = 0. \quad (17)$$

For the system to be stable, it is necessary and sufficient—by the Routh-Hurwitz stability criterion—that all roots of the characteristic equation $D_{ol}(s) = 0$ lie in the open left half of the s -plane, i.e., $\text{Re}\{s\} < 0$.

C. Closed-loop dynamics and stability analysis

To model the forward signal $\mathbf{V}_f(t)$ under small perturbations in cavity amplitude (δ_r) and phase (δ_ϕ), linear negative feedback is applied with gains K_r and K_ϕ :

$$2\mathbf{V}_f(t) = (2|\mathbf{V}_{f0}| - K_r\delta_r)e^{j(\theta_0 - K_\phi\delta_\phi)}. \quad (18)$$

As shown in Fig. 3(b), this expression captures amplitude and phase modulation of the forward signal. Applying the first-order Taylor approximation $e^{j(\theta_0 - K_\phi\delta_\phi)} \approx e^{j\theta_0}(1 - jK_\phi\delta_\phi)$, the forward signal becomes

$$\begin{aligned} 2\omega_{0.5}\mathbf{V}_f(t) &\approx 2\omega_{0.5}|\mathbf{V}_{f0}|e^{j\theta_0} - \omega_{0.5}K_r\delta_r e^{j\theta_0} \\ &- 2j\omega_{0.5}K_\phi\delta_\phi|\mathbf{V}_{f0}|e^{j\theta_0}. \end{aligned} \quad (19)$$

Second-order terms (e.g., $\delta_r\delta_\phi$) are neglected. By separating Eq. (19) into real and imaginary components, and substituting the expression for $2\omega_{0.5}\mathbf{V}_{f0}$ from Eq. (8), we obtain:

$$\begin{aligned} \Re\{2\omega_{0.5}\mathbf{V}_f(t)\} &\approx \omega_{0.5} - \omega_{0.5}K_r\delta_r \cos \theta_0 - \Delta\omega_0K_\phi\delta_\phi \\ \Im\{2\omega_{0.5}\mathbf{V}_f(t)\} &\approx -\Delta\omega_0 - \omega_{0.5}K_r\delta_r \sin \theta_0 - \omega_{0.5}K_\phi\delta_\phi. \end{aligned} \quad (20)$$

By combining Eqs. (7) and (20), subtracting the steady-state components (e.g., $\omega_{0.5}$ and $\Delta\omega_0$), and neglecting second-order terms (as in the open-loop derivation), the closed-loop perturbation dynamics are obtained as

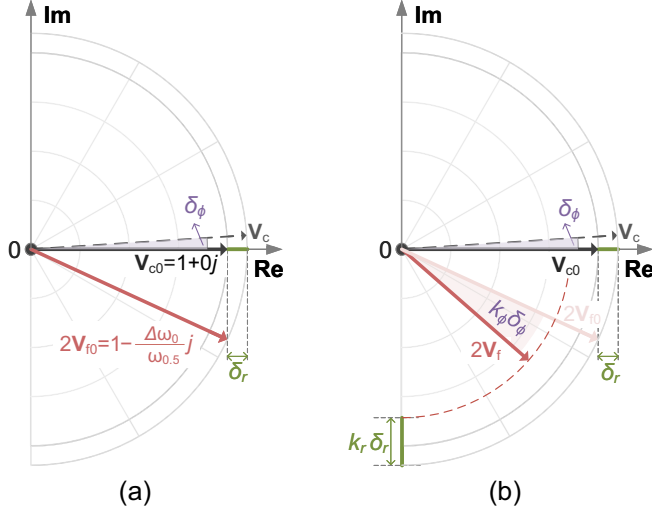


FIG. 3. Small-signal vector perturbation diagrams illustrating the cavity voltage $\mathbf{V}_c$ and the scaled cavity forward voltage $2\mathbf{V}_f$ in the complex plane. (a) Open-loop configuration: depicts the steady-state cavity voltage $\mathbf{V}_{c0} = 1 + j0$ and the corresponding nominal scaled forward voltage $2\mathbf{V}_{f0} = 1 - j\frac{\Delta\omega_0}{\omega_{0.5}}$. (b) Closed-loop configuration under amplitude-phase feedback: illustrates the modulated forward voltage vector $2\mathbf{V}_f$, influenced by feedback control actions characterized by gains K_r and K_ϕ .

$$\begin{aligned}\dot{\delta}_r &= -\omega_{0.5}(1 + K_r \cos \theta_0)\delta_r - \Delta\omega_0(1 + K_\phi)\delta_\phi \\ \dot{\delta}_\phi &= -\omega_{0.5}(1 + K_\phi)\delta_\phi + (\Delta\omega_0 - \omega_{0.5}K_r \sin \theta_0)\delta_r + \delta_\omega\end{aligned}\quad (21)$$

Applying the Laplace transform to Eq. (21) and incorporating the LFD model from Eq. (14) yields:

$$\begin{aligned}s\delta_r(s) &= -\omega_{0.5}(1 + K_r \cos \theta_0)\delta_r(s) \\ &\quad - \Delta\omega_0(1 + K_\phi)\delta_\phi(s) \\ s\delta_\phi(s) &= -\omega_{0.5}(1 + K_\phi)\delta_\phi(s) \\ &\quad + (\Delta\omega_0 - \omega_{0.5}K_r \sin \theta_0)\delta_r(s) + 2H(s)\delta_r(s).\end{aligned}\quad (22)$$

Following a procedure analogous to the open-loop analysis, $\delta_\phi(s)$ is solved from the first equation of Eq. (22) and substituted into the second. After simplifying the resulting expression and employing the identity $\theta_0 = -\Delta\theta_0$, where $\Delta\theta_0 = \tan^{-1}(\Delta\omega_0/\omega_{0.5})$ is the commonly defined steady-state cavity detuning angle, the final closed-loop characteristic equation $D_{cl}(s)$ is obtained:

$$\begin{aligned}D_{cl}(s) &= [s + \omega_{0.5}(1 + K_r \cos \Delta\theta_0)][s + \omega_{0.5}(1 + K_\phi)] \\ &\quad + \Delta\omega_0(1 + K_\phi)[\Delta\omega_0 + \omega_{0.5}K_r \sin \Delta\theta_0 + 2H(s)] \\ &= 0.\end{aligned}\quad (23)$$

The preceding derivations assume constant feedback gains, K_r and K_ϕ , corresponding to a proportional (P) controller. This framework, however, naturally extends to more general linear controllers, such as the commonly used PI controller. Since time-domain convolution becomes multiplication in the Laplace domain, the characteristic equation in Eq. (23) remains valid if K_r and K_ϕ are replaced with transfer functions $K_r(s)$ and $K_\phi(s)$, respectively.

It is worth noting that Refs. [15–17,19,20] analyze ponderomotive stability from a state-space perspective. Following this approach, Appendix A derives the open-loop characteristic equation $D_{ol}(s)$ and confirms its equivalence to the elimination-based method used in the main text. Moreover, since the CAFE2 employs A/P control, the main text focuses on the corresponding modeling framework. For completeness, a parallel derivation based on the widely used I/Q control scheme is provided in Appendix B.

D. Mechanical system identification

To determine the boundaries of oscillatory instability via the derived characteristic equation, the mechanical transfer function $H(s)$ must be specified. In this work, $H(s)$ is represented as the sum of a low-frequency base term and N higher-order mechanical resonance modes [10,12]:

$$H(s) = \underbrace{\frac{G_0 \cdot \omega_{\text{base}}}{s + \omega_{\text{base}}}}_{H_{\text{base}}(s)} + \sum_{k=1}^N \underbrace{\frac{G_k \cdot \frac{\omega_k^2}{Q_k}}{s^2 + \frac{\omega_k}{Q_k}s + \omega_k^2}}_{H_k(s)}. \quad (24)$$

Notably, to account for the residual low-frequency response that arises when the modal sum is truncated to the first N mechanical modes, we introduce the term H_{base} . Formally,

$$H_{\text{base}}(s) \approx \sum_{k=N+1}^{\infty} H_k(s), \quad H_{\text{base}}(0) = \sum_{k=N+1}^{\infty} H_k(0).$$

Hence, if the sum were extended to infinity, H_{base} would vanish. In practice, we represent this aggregate contribution by a phenomenological low-frequency term with an effective time constant (i.e., $\tau_{\text{base}} = 1/\omega_{\text{base}}$), which captures the net dynamic response of the neglected higher-order modes rather than the measurement system's bandwidth.

In this study, the unknown parameters ($G_0, \omega_{\text{base}}, G_k, \omega_k, Q_k$) in Table II were estimated from cavity operational and fault data, particularly from an oscillatory instability event shown in Fig. 5(c), using the greyest function in MATLAB [12]. First, a state-space model was constructed based on the equations derived in Eq. (24). Then, the greyest method was applied to compare this state-space model with the experimental data and minimize the discrepancy between the model output and the observed behavior. For the CAFE2 cavity CM₄₋₁, a model with two

TABLE II. Parameters for mechanical models.

Parameter	Unit	Mode 1	Mode2
$f_m(\omega_m/2\pi)$	Hz	118	280
G_m	arb. units	-3.4	-1.55
Q_m	arb. units	15	78

mechanical modes ($N = 2$), designated as K_1 , was identified. This model exhibits dominant resonances at approximately 118 and 280 Hz, as shown in the Bode plot [red curve in Fig. 4(a)]. The greyest method uses maximum likelihood estimation to iteratively adjust the model parameters, ensuring that the model closely matches the experimental data, even under noisy conditions. A detailed description of the measurement process can be found in [12].

Model accuracy was validated by comparing the predicted dynamic LFD response against measured data across several representative scenarios, as shown in Fig. 5. These include Fig. 5(a): cavity free decay following rf turn-off; Fig. 5(b): monotonic instability growth; and Figs. 5(c) and 5(d): oscillatory instabilities involving either a single mode (118 Hz) or dual-mode excitation (118 and 280 Hz). At CAFE2, thermal quenches or cryogenic-system faults can induce large helium-pressure swings that drive cavity detuning; when the detuning is positive and its magnitude exceeds the critical threshold, a monotonic instability is

triggered; see Fig. 5(b). Conversely, a sufficiently large negative detuning leads to oscillatory instabilities dominated by one or two mechanical modes; see Figs. 5(c) and 5(d). To facilitate identification of the dynamic characteristics governed by the transfer function $H(s)$, all detuning traces have been baseline-corrected by removing their steady-state offsets. The model accurately reproduces both the transient and oscillatory detuning dynamics across all cases. Figure 5(e) further compares the detuning spectra derived from the measured and simulated signals for the two oscillatory cases, showing excellent agreement in all dominant spectral components. This frequency-domain consistency provides additional evidence that the model reliably captures the cavity's dynamic behavior in both time and frequency domains.

E. Instability boundary analysis

With the model validated, we proceeded to analyze the system's stability. The instability boundaries in the $(\Delta f, E_{\text{peak}})$ plane were computed by substituting $H(s) = K_1$ into the open-loop characteristic equation given in Eq. (17) and identifying the loci where the real part of any characteristic root becomes non-negative.

Oscillatory instabilities are observed in the upper-left region of the plane, corresponding to negative detuning. To isolate the contributions of individual mechanical modes, we define one full model and two reduced submodels as follows:

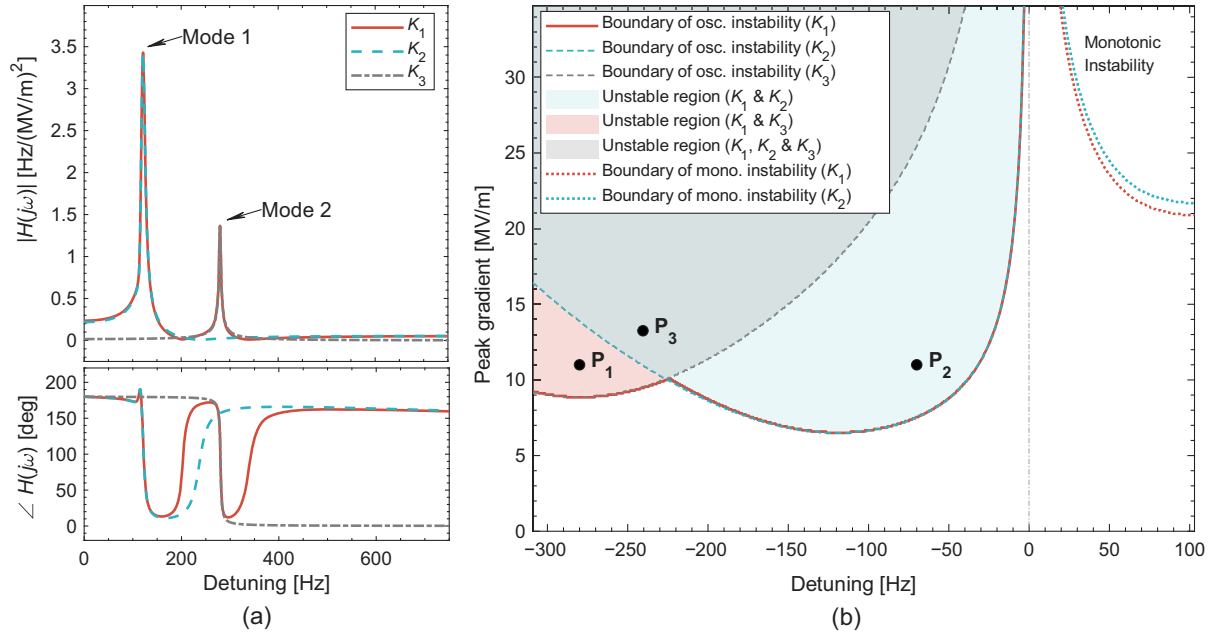


FIG. 4. (a) Estimated frequency responses for K_1, K_2, K_3 , as defined in Eq. (25). (b) The corresponding instability boundaries in the parameter space of detuning and peak gradient, as determined by the open-loop characteristic equation $D_{\text{ol}}(s)$. Solid/dashed curves denote the oscillatory instability boundaries for the models (K_1, K_2, K_3) , while the dotted curves show the corresponding monotonic instability boundaries added for completeness. The shaded areas delineate the theoretically predicted ponderomotive instability regions for different combinations of excited modes. The markers P_1, P_2 , and P_3 denote three typical operating points selected for subsequent dynamic simulation to validate these boundaries.

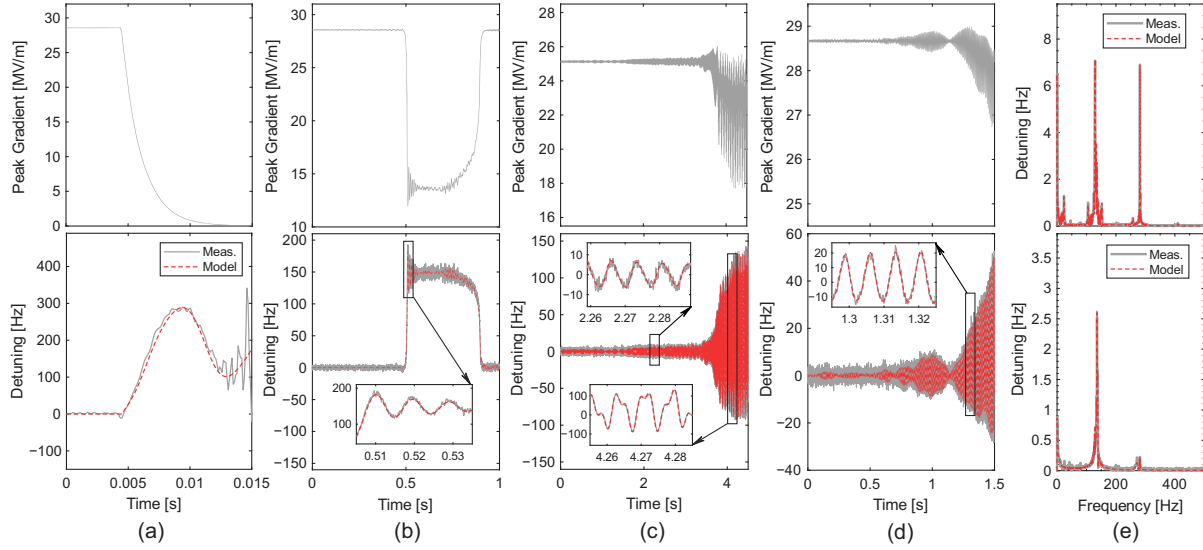


FIG. 5. Validation of the LFD model presented in Fig. 4(a) by comparing its predictions against measured data across four distinct scenarios: (a) an rf turn-off, (b) a monotonic instability, and ponderomotive oscillatory instabilities involving (c) two modes and (d) a single mode. Subplot (e) shows the detuning spectra derived from the measured data and model outputs for cases (c) and (d), with the upper plot corresponding to (c) and the lower plot corresponding to (d). For clarity and to facilitate identification of the dynamic behavior of $H(s)$, the steady-state detuning offsets have been removed in all cases.

$$\begin{aligned} K_1 &= H_{\text{base}} + H_1 + H_2, \\ K_2 &= H_{\text{base}} + H_1, \\ K_3 &= H_2, \end{aligned} \quad (25)$$

where K_1 includes both identified mechanical resonances, while K_2 and K_3 isolate the contributions from the 118 and 280 Hz modes, respectively.

Although the monotonic instability lies beyond the main scope of this study, its boundaries [also obtained from $D_{\text{ol}}(s)$] are included for completeness. This type of instability typically occurs in the region of positive

detuning, where the feedback between the cavity field and the mechanical mode leads to a nonoscillatory but divergent response. As a qualitative note, the boundary of the monotonic instability usually depends on the steady-state component of $H(s)$. For models K_1 and K_2 , whose steady-state components are relatively large, the corresponding monotonic-instability boundaries can be clearly identified on the right-hand side of Fig. 4(b). In contrast, since the steady-state component of model K_3 is negligibly small, no monotonic instability associated with K_3 is expected within the detuning-gradient range shown in Fig. 4(b).

In Fig. 4(b), the red solid, cyan dashed, and gray dashed lines represent the oscillatory instability boundaries for

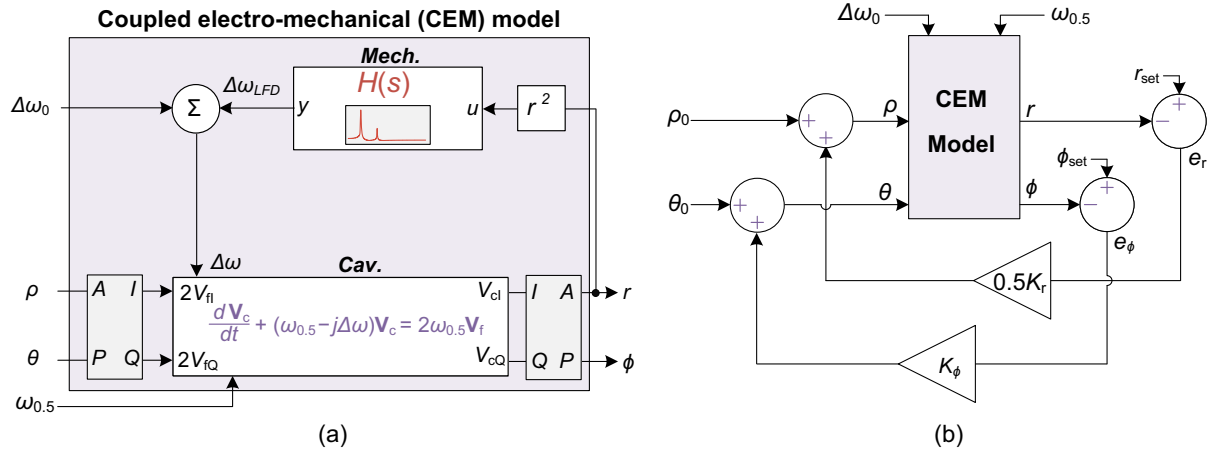


FIG. 6. Block diagrams of the simulation models. (a) Open-loop configuration (model 1), capturing the coupled cavity and mechanical dynamics. (b) Closed-loop configuration (model 2), incorporating external PI amplitude and phase control. The CEM model (coupled electromechanical) describes the interaction between electromagnetic fields and Lorentz-force-induced mechanical detuning.

models K_1 , K_2 , and K_3 , respectively. Each boundary defines the parameter space where instability for that mode begins. As model K_1 is a superposition of modes K_2 and K_3 , its instability region—the area enclosed by its boundary—is a superset that fully contains the individual instability regions of both constituent modes.

IV. DYNAMIC SIMULATION AND MODEL VALIDATION

This section presents a numerical validation of the analytical framework developed in the preceding sections. The validation proceeds in two stages. First, the theoretically derived instability boundaries are benchmarked using simplified dynamic models to confirm the characteristic equation approach. Next, a more realistic simulation model incorporating hardware nonidealities is developed and compared it with experimental measurements (see Sec. V).

A. Validation via idealized simulation

Two simplified dynamic models are constructed for verification: model 1 (open loop), which captures the intrinsic electromechanical feedback loop (ponderomotive loop), and model 2 (closed loop), which adds the LLRF A/P control. Their block diagrams are shown in Figs. 6(a) and 6(b), respectively.

In model 1, cavity amplitude perturbations δr excite detuning fluctuations $\delta\omega_{\text{LFD}}$ via the Lorentz force, forming a natural feedback loop. In model 2, proportional feedback is applied to both amplitude and phase, modifying the forward signal as described in Eq. (18). To facilitate the construction and interpretation of the dynamic simulation

TABLE III. Definitions of the key parameters used in the simulation model.

Symbol	Description
ρ, θ	Amplitude and phase of the complex signal $2\mathbf{V}_f$
ρ_0, θ_0	Amplitude and phase of the complex signal $2\mathbf{V}_{f0}$
$V_{\text{II}}, V_{\text{IQ}}$	I/Q components of the complex signal $\mathbf{V}_f$
$V_{\text{cI}}, V_{\text{cQ}}$	I/Q components of the complex signal $\mathbf{V}_c$
r, ϕ	Amplitude and phase of the complex signal $\mathbf{V}_c$
$\Delta\omega_0$	Cavity static detuning
$\Delta\omega_{\text{LFD}}$	Cavity Lorentz force detuning
$\Delta\omega$	Total cavity detuning ($\Delta\omega_0 + \Delta\omega_{\text{LFD}}$)
$r_{\text{set}}, \phi_{\text{set}}$	Setpoints for the $\mathbf{V}_c$ for the A/P feedback loops
e_r, e_ϕ	Error signals for the A/P feedback loops
K_r, K_ϕ	Feedback controller of A/P feedback loops

models (model 1 and model 2), key variable definitions are summarized in Table III.

We simulate the cavity response at three representative operating points (P_1 – P_3) on the detuning-gradient map [Fig. 4(b)], selected to explore different instability scenarios. Prior to presenting the results, we clarify the expected behavior: (i) P_1 lies inside the mode 2 (280 Hz) instability region but outside the mode 1 (118 Hz) region. We expect oscillations dominated by the 280 Hz mode. (ii) P_2 lies inside the mode 1 region only. Oscillations should occur at 118 Hz. (iii) P_3 lies within the overlap of both regions. A dual-mode beating is anticipated.

The simulation results are shown in Fig. 7. Each subplot presents the time evolution of cavity detuning under two mechanical models: the full two-mode model (K_1) and a partial model (K_2) including only the 118 Hz mode.

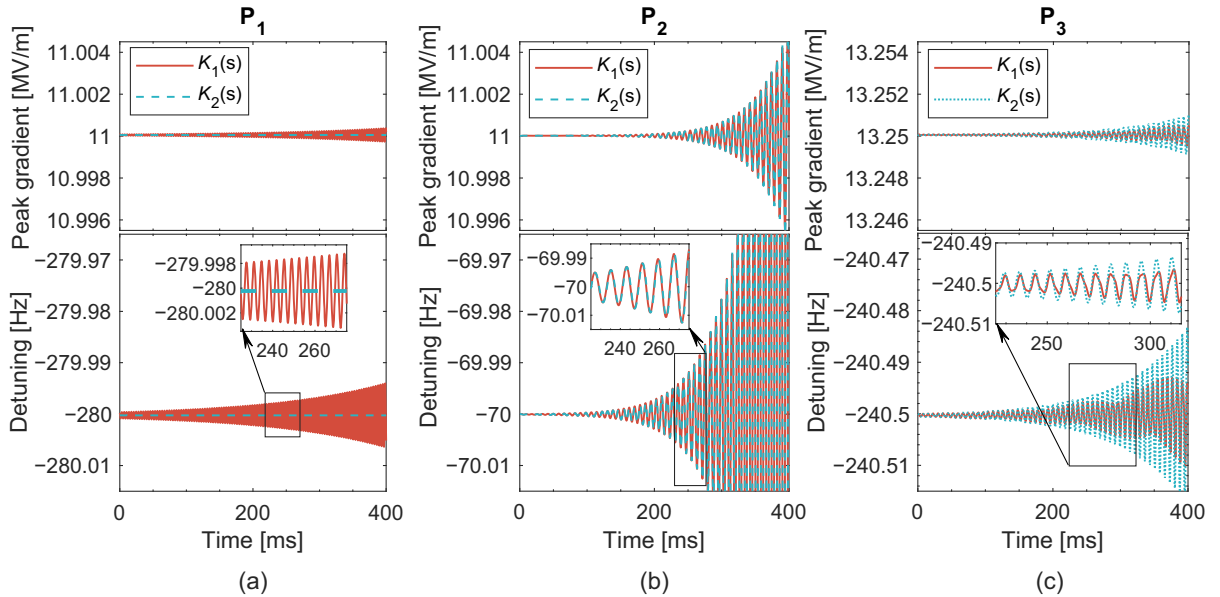


FIG. 7. The oscillatory states at the points P_1 , P_2 , and P_3 for K_1 and K_2 are simulated in (a)–(c), respectively. The oscillation states K_1 and K_2 in these three figures are in agreement with the state in Fig. 4(b).

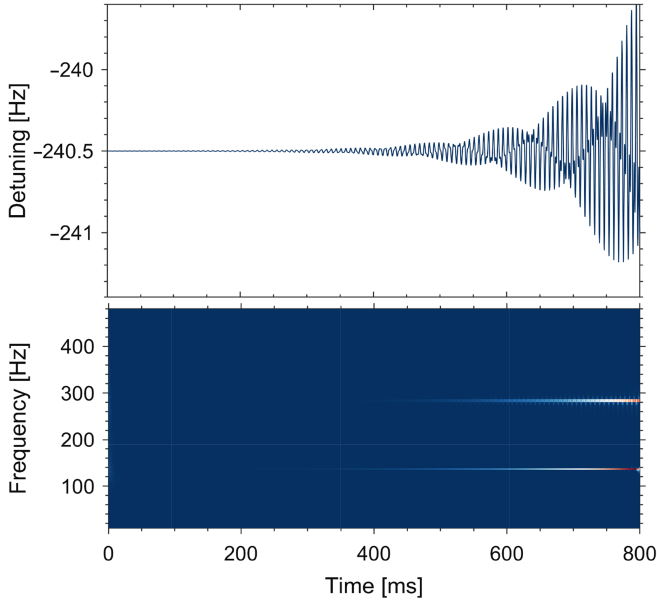


FIG. 8. The spectral response of P_3 at K_1 mode of Fig. 7(c). The lower plot shows at the P_3 point where the oscillatory instability has two mechanical modes.

As expected, at P_1 , the partial model is stable, while K_1 shows 280 Hz oscillation; at P_2 , both models show identical 118 Hz oscillation; and at P_3 , K_1 exhibits a clear beating pattern due to simultaneous excitation of both modes.

To further highlight the dual-mode nature at P_3 , Fig. 8 presents the wavelet spectrogram of the detuning signal. Two distinct frequency components at 118 and 280 Hz are clearly observed, validating the predicted overlap region.

A quantitative comparison between analytical predictions and dynamic simulation results is shown in Fig. 9.

Subfigures (a) and (b) show the open-loop (model 1) and closed-loop (model 2) configurations, respectively, as defined in Fig. 6. In each plot, the red curves indicate instability boundaries obtained from the characteristic equation [$D_{ol}(s) = 0$ for open loop, $D_{cl}(s) = 0$ for closed loop], while the black dots mark the onset of oscillation observed in simulation. The close agreement between analytic boundaries and simulated results across both scenarios confirms the validity of the theoretical framework. For the closed-loop simulation, the specific PI feedback controller parameters used are listed in Table IV of Sec. VI (Gainset 1).

B. Realistic model with hardware imperfections

To bridge the gap between the idealized linear model and the actual experimental setup, we developed a more realistic simulation framework, referred to as Model 3 [see Fig. 10(a)]. This model incorporates two critical hardware imperfections observed in the CAFE2 facility: the nonlinear characteristics of the solid-state amplifier (SSA) [35,36] and impedance mismatch due to the absence of a high-power circulator [37,38].

The SSA introduces amplitude-dependent nonlinearities, generally categorized as amplitude modulation to amplitude modulation (AM/AM) and amplitude modulation to phase modulation (AM/PM). For the SSA used at CAFE2, the AM/PM distortion is negligible within the normal operational amplitude range, thus only AM/AM nonlinearity is considered here [35,36]. As illustrated in Fig. 11, the SSA output exhibits a quasilinear amplitude response (highlighted in red) followed by a nonlinear saturation region. Its behavior can therefore be modeled as

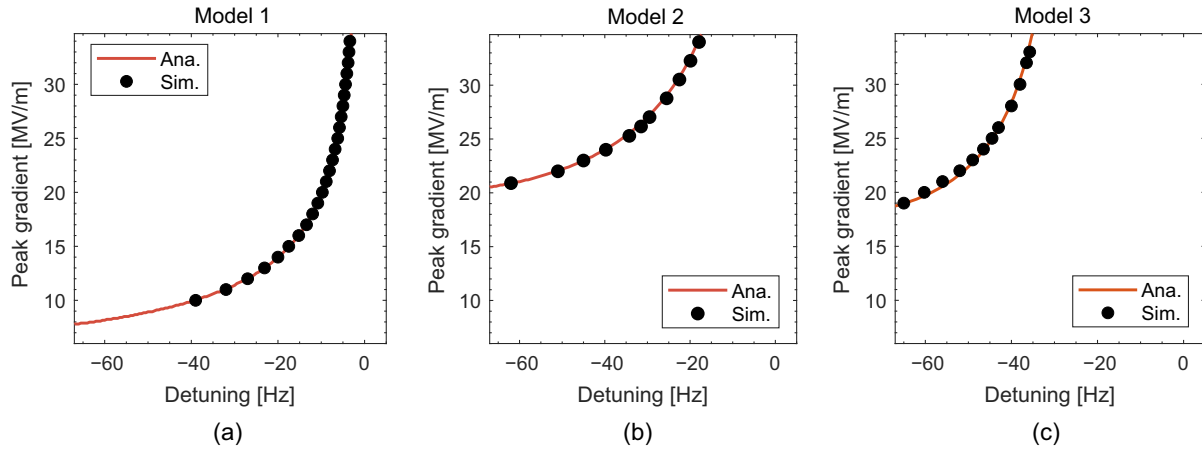


FIG. 9. Comparison of analytical instability boundaries with dynamic simulation results for three models: (a) model 1, (b) model 2, and (c) model 3. The analytical boundaries (Ana., red lines) are derived from the corresponding closed-loop characteristic equations, while the simulation results (Sim., black dots) are obtained from time-domain simulations. All three models exhibit good agreement with the theoretical predictions. The slight deviations observed in model 3 are likely due to the approximation of the power amplifier by a constant gain g , which may not fully capture its behavior even within the nominally linear operating region.

TABLE IV. Control gain sets for the PI controllers.

Gain set	(k_{rp}, k_{ri})	$(k_{\phi p}, k_{\phi i})$
Gain set 1	(1, 150)	(20, 2000)
Gain set 2	(1.4, 200)	(10, 1000)
Gain set 3	(0.7, 150)	(20, 2000)
Gain set 4	(1, 250)	(20, 2000)

$$\mathbf{V}_{\text{SSA}} = f_A(|V_{\text{LLRF}}|)e^{j\theta_{\text{LLRF}}}, \quad (26)$$

where $f_A(a)$ is the measured AM/AM response function, and θ_{LLRF} is the phase of the LLRF output.

Additionally, since CAFE2 does not employ a high-power circulator [37,38], a portion of the reflected wave from the cavity is reflected back into the forward path via

the power source. This is modeled by a complex reflection coefficient Γ , giving rise to a modified forward signal:

$$\begin{aligned}\mathbf{V}_f &= (1 - \Gamma)\mathbf{V}_{\text{SSA}} + \Gamma\mathbf{V}_r, \\ \mathbf{V}_c &= \mathbf{V}_f + \mathbf{V}_r,\end{aligned}\tag{27}$$

where $\mathbf{V}_r$ is the cavity-reflected signal. The coefficient Γ is experimentally measured using the method described in Ref. [37]. Expanding on this further, the forward signal $\mathbf{V}_f$ can be written as

$$\mathbf{V}_f = c\mathbf{V}_{\text{SSA}} + 0.5\alpha\mathbf{V}_c, \quad (28)$$

where $c = \frac{1-\Gamma}{1+\Gamma}$ and $\alpha = \frac{2\Gamma}{1+\Gamma}$.

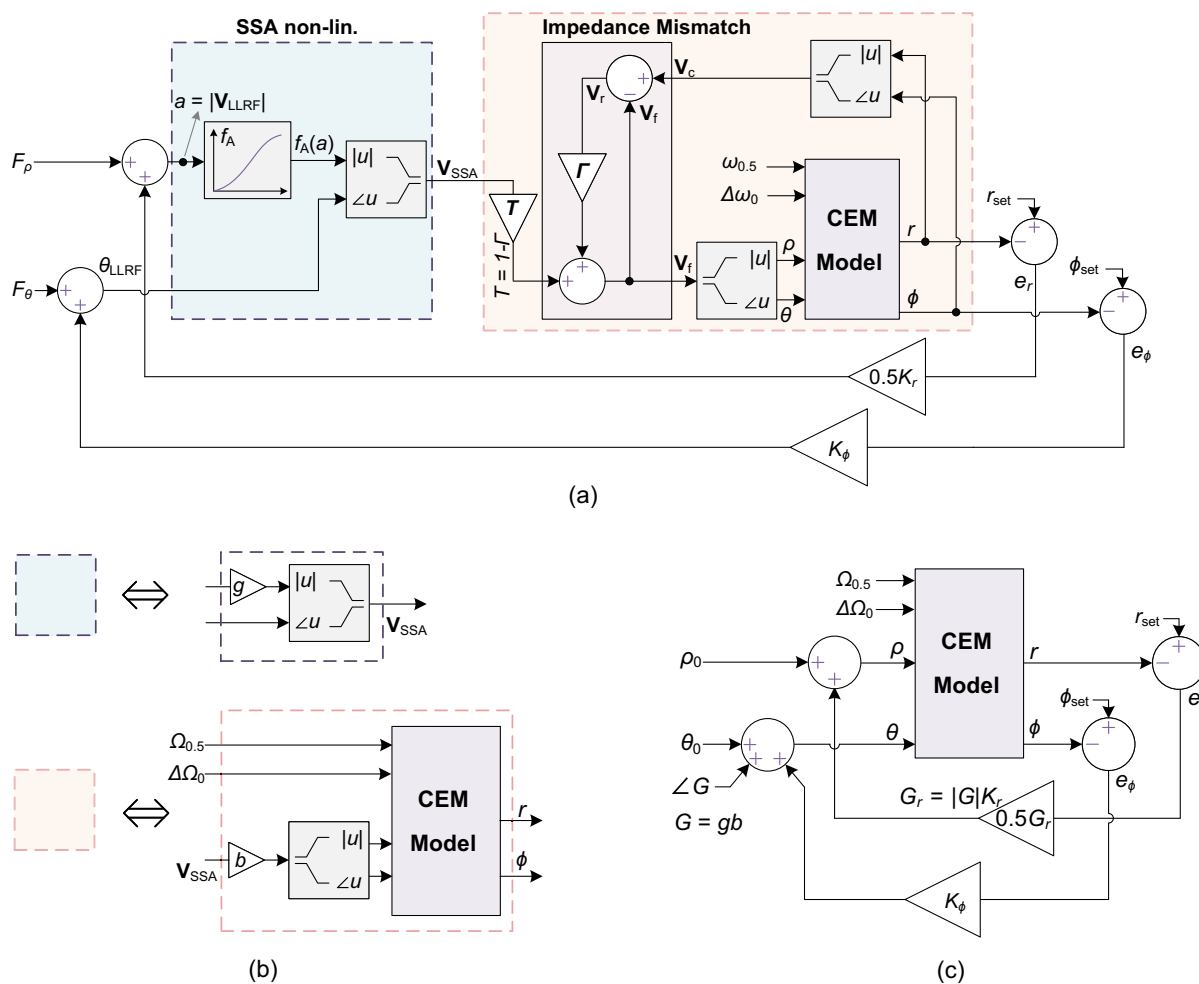


FIG. 10. Block diagram representations related to the realistic simulation model. (a) Full structure of model 3, which incorporates two key hardware imperfections observed in the CAFE2 system: the nonlinear behavior of the solid-state amplifier (SSA) and impedance mismatch at the power source. (b) Intermediate simplification: the SSA nonlinearity is approximated by a constant small-signal gain g , and the cavity with impedance mismatch is equivalently represented as a generalized cavity with modified parameters $\Omega_{0,5}$ and $\Delta\Omega_0$. (c) Final abstraction of the model, whose topology becomes identical to the idealized closed-loop structure shown in Fig. 6(b). This equivalence enables the direct application of the same characteristic equation for stability analysis.

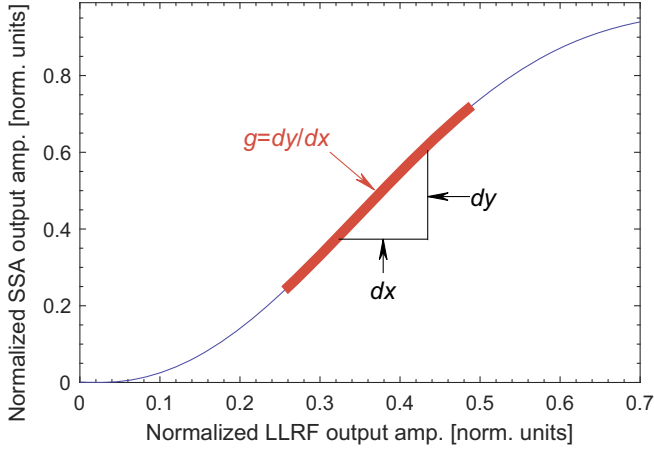


FIG. 11. Amplitude characteristics of the SSA. The LLRF output amplitude (horizontal axis) is normalized to the saturation point of the amplifier. The highlighted red segment of the curve indicates the linear region, which is the typical operating range for the SSA at CAFE2.

Clearly, the presence of SSA nonlinearity and impedance mismatch prevents the direct application of the original closed-loop characteristic equation [Eq. (23)] for stability boundary analysis. However, by introducing suitable approximations, we can transform the system into a form compatible with the existing theoretical framework.

First, as shown in Fig. 10(b), the SSA operates predominantly within its quasilinear region. Hence, its non-linear response can be reasonably approximated by a constant small-signal gain g , corresponding to the slope at the operating point in Fig. 11. This linear approximation simplifies the subsequent stability analysis.

Second, according to Eq. (28), impedance mismatch introduces an additional feedback path from the cavity output back to its input, effectively creating a “generalized cavity” system. As illustrated in Fig. 10(b), this generalized cavity has modified equivalent parameters, including the half-bandwidth $\Omega_{0.5}$, static detuning $\Delta\Omega_0$, and a new gain factor b . These parameters relate to the original cavity parameters via:

$$\begin{aligned}\Omega_{0.5} &= \omega_{0.5}(1 - \alpha_i), \\ \Delta\Omega_0 &= \Delta\omega_0 + \alpha_q\omega_{0.5}, \\ b &= \frac{1}{(1 + \Gamma)(1 - \alpha_i)},\end{aligned}\quad (29)$$

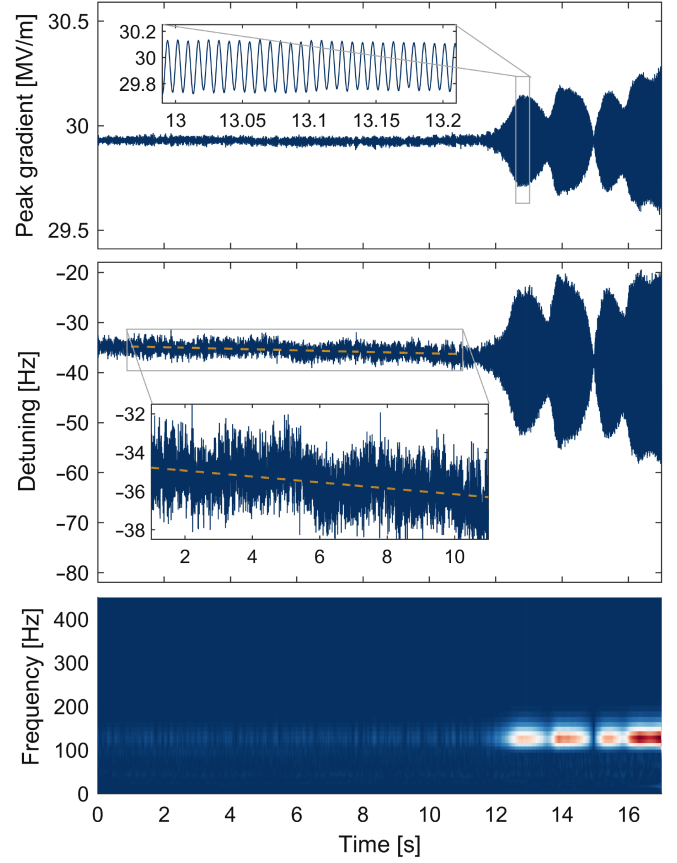


FIG. 12. Measurement procedure for determining an instability boundary. The top panel shows the gradual negative detuning of the cavity at a fixed gradient until oscillations occur, while the middle panel provides a detailed view of the oscillation onset. The bottom panel, showing a wavelet transform of the detuning signal, reveals that a 120 Hz mode is excited. A slow tuning rate is used near the threshold to ensure precision.

where α_i and α_q denote the real and imaginary parts of the complex coefficient α . A detailed derivation of these expressions is provided in Appendix C.

Finally, after incorporating these transformations, the block diagram shown in Fig. 10(a) simplifies structurally into the equivalent form shown in Fig. 10(c). In this simplified representation, the combined gain factor $G = gb$ introduces an additional gain correction to the amplitude control loop. Thus, the simplified Model 3 aligns structurally with the idealized closed-loop configuration depicted in Fig. 6(b). The corresponding closed-loop characteristic equation now becomes

$$\begin{aligned}D_{\text{cl,real}}(s) &= [s + \Omega_{0.5}(1 + G_r \cos \Delta\Theta_0)][s + \Omega_{0.5}(1 + K_\phi)] \\ &\quad + \Delta\Omega_0(1 + K_\phi)[\Delta\Omega_0 + \Omega_{0.5}G_r \sin \Delta\Theta_0 + 2H(s)] = 0,\end{aligned}\quad (30)$$

where $G_r = |G|K_r$, and $\Delta\Theta_0 = \tan^{-1}(\Delta\Omega_0/\Omega_{0.5})$.

Figure 9(c) demonstrates excellent agreement between the analytical instability boundaries computed from Eq. (30) and the dynamic simulation results of model 3. The minor discrepancies observed may primarily result from approximating the SSA nonlinearity with a constant linear gain. Nevertheless, this simplified approach of incorporating SSA linearization and generalized cavity parameters effectively captures key hardware imperfections, providing a robust theoretical foundation for the experimental validations described in Sec. V.

V. EXPERIMENTAL VERIFICATION

To verify the theoretical predictions and validate the characteristic equation [Eq. (30)], we conducted closed-loop tests on two HWR010 SRF cavities, CM_{4,1} and CM_{4,2}, at the CAFE2 facility (see Fig. 1). The oscillatory instability threshold was experimentally determined by slowly increasing the magnitude of cavity detuning (in the negative direction) under fixed gradient until self-sustained oscillations emerged. During the procedure, both amplitude and phase loops, as well as the frequency tuning loop, were closed to emulate realistic operating conditions.

We employed a two-speed tuning strategy, with the initial threshold detuning (or detuning angle) setpoint guided by analytical result from Eq. (30). The tuner motor operated at a high speed when the detuning was far from this predicted threshold, then decelerated to a rate of 0.1–0.15 Hz/s for fine-grained adjustments near the instability point. The onset detuning was precisely identified by applying a linear fit to the data points immediately preceding the instability, a method that effectively mitigates the influence of external disturbances such as environmental microphonics and random noise.

A typical measurement process for cavity CM_{4,1} at an operating gradient of 29.8 MV/m is illustrated in Fig. 12. A divergent oscillation is observed to begin around the 12-second mark, corresponding to a detuning of approximately −37 Hz. This experimentally determined threshold is plotted as the larger black filled circle shown in Fig. 13(a). Following this method, we sequentially set the superconducting cavity gradient from high to low and scan the detuning, which allows us to obtain a set of onset points ($\Delta f, E_{\text{peak}}$), as shown by the red dots in Fig. 13(a). A wavelet transform of the detuning signal, shown in the lower panel of Fig. 12, confirms that a 118 Hz mechanical mode was excited during this event.

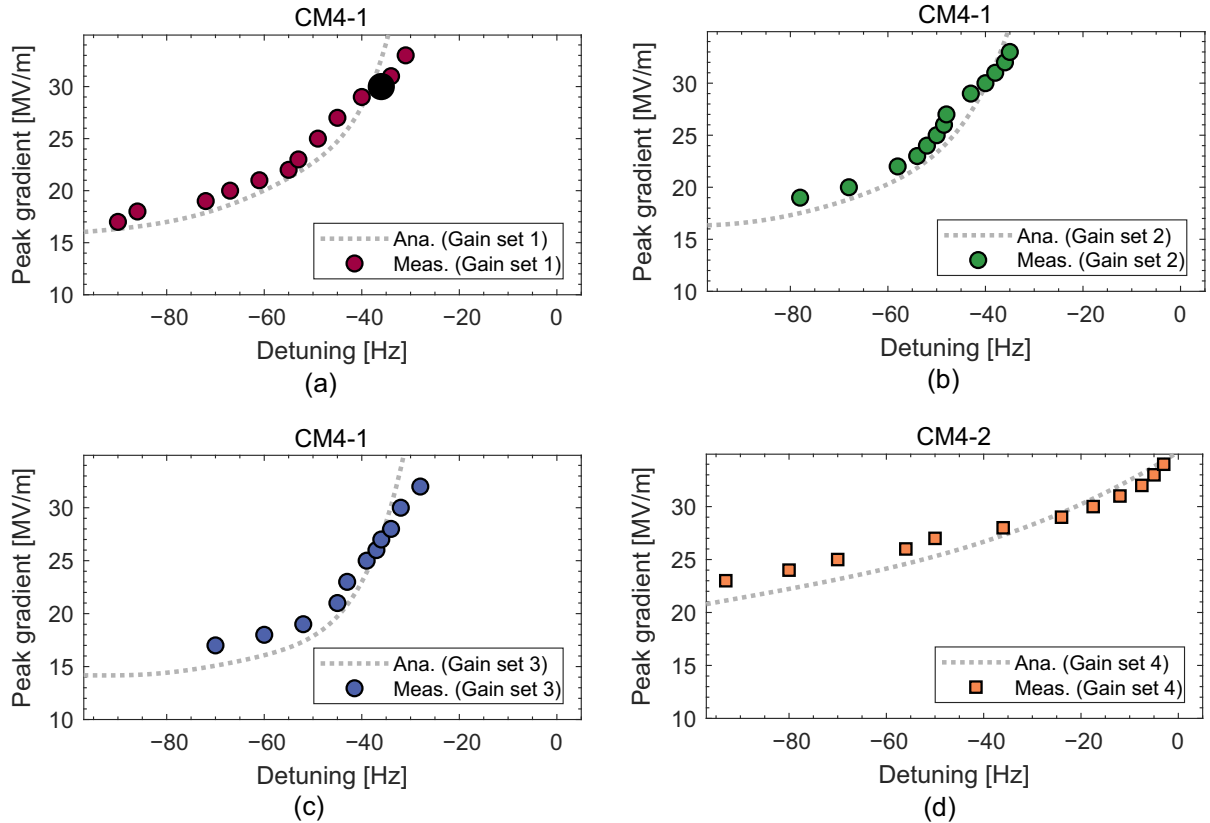


FIG. 13. Comparison of measured (markers) and analytical (dashed curves) oscillation boundaries. The first three subplots, (a)–(c), show results for cavity CM_{4,1} under three different PI gain settings (gain set 1, 2, and 3, respectively). In subplot (a), the larger black filled circle specifically represents the measurement result from the procedure detailed in Fig. 12. Subplot (d) shows the results for a different cavity, CM_{4,2}, using gain set 4. In all cases, the experimental data show good agreement with the analytical results.

To further demonstrate the model's predictive power and adaptability, we performed additional comparisons using different feedback gains. The feedback control system utilizes two independent PI controllers to stabilize the system's amplitude and phase, with transfer functions defined as

$$K_r(s) = k_{rp} + \frac{k_{ri}}{s} K_\phi(s) = k_{\phi p} + \frac{k_{\phi i}}{s} \quad (31)$$

The first controller, $K_r(s)$, governs the amplitude loop, where k_{rp} and k_{ri} are its proportional and integral gains. Similarly, $K_\phi(s)$ governs the phase loop, with $k_{\phi p}$ and $k_{\phi i}$ as its respective gains. The specific gain configurations used in our tests, designated gain sets 1–4, are detailed in Table IV. Note that, the open-loop gain $|G|$ in Fig. 10 has been normalized to unity (0 dB).

The complete set of comparisons between experimental measurements and analytical predictions is shown in Fig. 13. For cavity CM_{4,1}, with a $f_{0.5}$ of approximately 103 Hz, the measured instability thresholds (markers) exhibit excellent agreement with the analytical results (dashed curves) across all three tested gain sets [Figs. 13(a)–13(c)]. This consistency is further confirmed for cavity CM_{4,2} ($f_{0.5} \sim 113$ Hz) under gain set 4, as shown in Fig. 13(d).

Minor discrepancies between the experimental data and analytical boundaries may stem from uncontrollable variations in the cryogenic environment, such as small helium pressure fluctuations, which complicate precise identification of the instability onset. Another source of deviation lies in the treatment of the source reflection coefficient, Γ . In practice, Γ varies with the operating state of the power source, whereas a fixed value was assumed in the analytical model (e.g., $\Gamma \approx 0.15e^{j70^\circ}$ for CM_{4,1}). Additionally, the mechanical transfer function $H(s)$ used in the analytical calculation is based on a simplified identification method, which captures only a few dominant modes and neglects minor contributions.

Between January and February 2024, the proposed analytical model was directly applied to diagnose operational instabilities in cavities CM_{4,1} and CM_{4,2}. Both cavities exhibited frequent rf faults induced by oscillatory electromechanical instabilities, which stemmed from misconfigured PI feedback gains (see gain sets 1 and 4). The root cause was a change in the attenuator within the feedback chain, performed without recalibration of the loop parameters. When helium-pressure fluctuations drove the detuning beyond -37 Hz in CM_{4,1} and -24 Hz in CM_{4,2}, the system frequently entered unstable regimes. We analyzed the rf control loops in detail and reconfigured the PI gains using the closed-loop stability criterion given by Eq. (30). After the gain corrections on January 23 and February 1, the daily fault rates of CM_{4,1} and CM_{4,2} dropped from approximately 17 and 5, respectively, to fewer than 0.4. These results, summarized in Fig. 14,

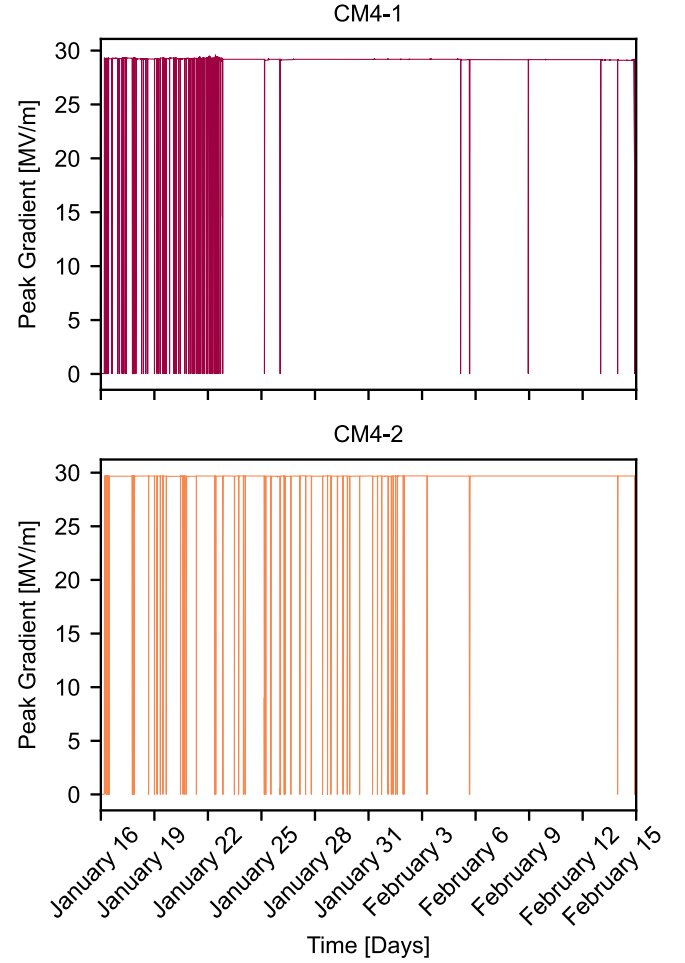


FIG. 14. Operating logs of two superconducting cavities, CM_{4,1} (top) and CM_{4,2} (bottom), during January–February 2024. Prior to January 23 and February 1, respectively, both cavities experienced frequent rf faults triggered by oscillatory ponderomotive instabilities due to misconfigured PI feedback gains. After restoring the controller parameters, the cavity fault rates dropped significantly—from approximately 17/day (CM_{4,1}) and 5/day (CM_{4,2}) to less than 0.4/day.

highlight the model's practical utility in diagnosing and suppressing closed-loop instabilities in high- Q_L superconducting rf systems.

Overall, these experimental validations confirm the effectiveness and accuracy of the developed analytical framework. The demonstrated capability of the model to diagnose and resolve practical operational issues underscores its potential value for future high-performance accelerator designs and routine operational stability assessments.

VI. SUMMARY

In summary, this work establishes a comprehensive theoretical and experimental framework for understanding oscillatory ponderomotive instabilities in the SRF cavities

under feedback control. Based on small-signal perturbation analysis, we derived the characteristic equations for the coupled electromechanical system in both open-loop and closed-loop conditions, leading to analytical stability criteria. These criteria enable accurate mapping of instability thresholds in the detuning–gradient plane, which were found to be in strong agreement with results from dynamic simulations.

Furthermore, we conducted systematic experiments on the CAFE2 linac to measure the instability thresholds under various feedback gain settings. The experimental results show excellent agreement with the predictions from the analytical results in Fig. 13, which incorporate key hardware imperfections. This direct and comprehensive validation of ponderomotive instability theory under realistic closed-loop conditions fills a long-standing gap in the field. Our findings not only reinforce the theoretical stability criteria but also offer practical guidance for the design and stable operation of future continuous-wave SRF accelerators operating at high gradients.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX A: DERIVATION BASED ON STATE-SPACE EQUATIONS AND EIGENVALUES

This appendix demonstrates that the “elimination method” used in the main text to derive the system’s characteristic equation is mathematically equivalent to the state-space approach found in other literature (e.g., TRIUMF [15–18] and DESY [19,20]). The following derivation shows that both methods yield identical stability criteria.

1. State-space representation of $H_m(s)$

The dynamics of a single, arbitrary m th mechanical mode are described by the second-order transfer function $H_m(s)$:

$$H_m(s) = \frac{N}{M(s)}. \quad (\text{A1})$$

The numerator N and the denominator $M(s)$ are defined by the mode’s parameters:

$$N = \frac{G_m \omega_m^2}{Q_m}, \quad (\text{A2})$$

$$M(s) = s^2 + \frac{\omega_m}{Q_m} s + \omega_m^2. \quad (\text{A3})$$

This transfer function corresponds to the second-order ordinary differential equation for the detuning $\Delta\omega(t)$:

$$\frac{d^2 \Delta\omega}{dt^2} + \frac{\omega_m}{Q_m} \frac{d\Delta\omega}{dt} + \omega_m^2 \Delta\omega = \frac{G_m \omega_m^2}{Q_m} r(t)^2. \quad (\text{A4})$$

At the steady-state operating point, where the normalized field amplitude $r(t) \equiv 1$, the static detuning $\Delta\omega_0$ is

$$\omega_m^2 \Delta\omega_0 = \frac{G_m \omega_m^2}{Q_m} (1)^2 \Rightarrow \Delta\omega_0 = \frac{G_m}{Q_m}. \quad (\text{A5})$$

Linearizing Eq. (A4) around this steady state using the small-signal perturbations $\Delta\omega(t) = \Delta\omega_0 + \delta_\omega(t)$ and $r^2(t) \approx 1 + 2\delta_r(t)$ yields the final equation for the detuning perturbation:

$$\ddot{\delta}_\omega + \frac{\omega_m}{Q_m} \dot{\delta}_\omega + \omega_m^2 \delta_\omega = 2 \frac{G_m \omega_m^2}{Q_m} \delta_r. \quad (\text{A6})$$

2. Deriving the 4×4 state-space equations of the system

For clarity, we present the derivation for a single mechanical mode, $H_m(s)$; the extension to multiple modes is straightforward—replace $H_m(s)$ with the aggregate transfer function $H(s) = \sum_m H_m(s)$. To construct the complete state-space model, we define four state variables: the field amplitude perturbation $\delta_r(t)$, the field phase perturbation $\delta_\phi(t)$, the mechanical detuning perturbation $\delta_\omega(t)$, and its time derivative $\delta_v(t) = \dot{\delta}_\omega(t)$. The linearized cavity field equations and the mechanical Eq. (A6) lead to the following system of first-order differential equations:

$$\begin{aligned} \dot{\delta}_r &= -\omega_{0.5} \delta_r - \Delta\omega_0 \delta_\phi \\ \dot{\delta}_\phi &= \Delta\omega_0 \delta_r - \omega_{0.5} \delta_\phi + \delta_\omega \\ \dot{\delta}_\omega &= \delta_v \\ \dot{\delta}_v &= -\omega_m^2 \delta_\omega - \frac{\omega_m}{Q_m} \delta_v + 2 \frac{G_m \omega_m^2}{Q_m} \delta_r. \end{aligned} \quad (\text{A7})$$

The state vector $\mathbf{x}(t)$ is defined as

$$\mathbf{x}(t) = [\delta_r(t) \ \delta_\phi(t) \ \delta_\omega(t) \ \delta_v(t)]^T. \quad (\text{A8})$$

The system can then be written as $\dot{\mathbf{x}}(t) = A\mathbf{x}(t)$. The state matrix A is

$$A = \begin{bmatrix} -\omega_{0.5} & -\Delta\omega_0 & 0 & 0 \\ \Delta\omega_0 & -\omega_{0.5} & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 2\frac{G_m\omega_m^2}{Q_m} & 0 & -\omega_m^2 & -\frac{\omega_m}{Q_m} \end{bmatrix}. \quad (\text{A9})$$

For system stability, all eigenvalues λ of the matrix A must have negative real parts. The eigenvalues are found by solving the characteristic equation $\det(A - \lambda I) = 0$, where I is the identity matrix.

$$\det(A - \lambda I) = \begin{vmatrix} -\omega_{0.5} - \lambda & -\Delta\omega_0 & 0 & 0 \\ \Delta\omega_0 & -\omega_{0.5} - \lambda & 1 & 0 \\ 0 & 0 & -\lambda & 1 \\ 2\frac{G_m\omega_m^2}{Q_m} & 0 & -\omega_m^2 & -\frac{\omega_m}{Q_m} - \lambda \end{vmatrix}. \quad (\text{A10})$$

3. Solving the characteristic equation

We expand the determinant $\det(A - \lambda I)$ along the first row:

$$\det(A - \lambda I) = (-\omega_{0.5} - \lambda)A_1(\lambda) + \Delta\omega_0 A_2(\lambda), \quad (\text{A11})$$

where $A_1(\lambda)$ is the cofactor corresponding to the element A_{11} of $\det(A - \lambda I)$:

$$A_1(\lambda) = \begin{vmatrix} -\omega_{0.5} - \lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 0 & -\omega_m^2 & -\frac{\omega_m}{Q_m} - \lambda \end{vmatrix}. \quad (\text{A12})$$

Calculating this 3×3 determinant yields:

$$\begin{aligned} A_1(\lambda) &= (-\omega_{0.5} - \lambda) \left[(-\lambda) \left(-\frac{\omega_m}{Q_m} - \lambda \right) - (1)(-\omega_m^2) \right] \\ &= (-\omega_{0.5} - \lambda)M(\lambda), \end{aligned} \quad (\text{A13})$$

where $M(\lambda)$ is the characteristic polynomial of the mechanical mode, as defined in Eq. (A4) with s replaced by λ .

The cofactor $A_2(\lambda)$ corresponding to the element A_{12} of $\det(A - \lambda I)$:

$$A_2(\lambda) = \begin{vmatrix} \Delta\omega_0 & 1 & 0 \\ 0 & -\lambda & 1 \\ 2\frac{G_m\omega_m^2}{Q_m} & -\omega_m^2 & -\frac{\omega_m}{Q_m} - \lambda \end{vmatrix}. \quad (\text{A14})$$

Expanding this determinant:

$$\begin{aligned} A_2(\lambda) &= \Delta\omega_0 \begin{vmatrix} -\lambda & 1 \\ -\omega_m^2 & -\frac{\omega_m}{Q_m} - \lambda \end{vmatrix} \\ &\quad - 1 \cdot \begin{vmatrix} 0 & 1 \\ 2\frac{G_m\omega_m^2}{Q_m} & -\frac{\omega_m}{Q_m} - \lambda \end{vmatrix} \\ &= \Delta\omega_0 M(\lambda) + 2N, \end{aligned} \quad (\text{A15})$$

where N is given by Eq. (A3).

Substituting the expressions for $A_1(\lambda)$ from Eq. (A14) and $A_2(\lambda)$ from Eq. (A17) into Eq. (A12), the characteristic equation $\det(A - \lambda I) = 0$ becomes

$$(\lambda + \omega_{0.5})^2 M(\lambda) + \Delta\omega_0^2 M(\lambda) + 2\Delta\omega_0 N = 0. \quad (\text{A16})$$

Recalling from Eq. (A2) that $H_m(\lambda) = N/M(\lambda)$, the characteristic equation for the open-loop system is

$$D_{ol}(\lambda) = (\lambda + \omega_{0.5})^2 + \Delta\omega_0^2 + 2\Delta\omega_0 H_m(\lambda) = 0. \quad (\text{A17})$$

This result is identical to the characteristic equation derived in the main text, confirming the equivalence of the state-space and elimination methods. While only the open-loop case is detailed here, the closed-loop derivation follows an analogous procedure.

APPENDIX B: SMALL-SIGNAL MODELING OF SUPERCONDUCTING CAVITIES UNDER I/Q FEEDBACK

This appendix details an alternative derivation of the small-signal model in the I/Q domain, in contrast to the amplitude-phase ($r - \phi$) framework used in the main text. The I/Q representation is widely adopted in the LLRF field because its inherent linear characteristics simplify the design and analysis of feedback controllers.

1. Cavity differential equation and perturbation setup

To facilitate small-signal modeling, the $\mathbf{V}_c$ and $\mathbf{V}_f$ signals can be decomposed into their I/Q components as follows:

$$\mathbf{V}_c(t) = V_{cl}(t) + jV_{cq}(t), \quad (\text{B1})$$

$$\mathbf{V}_f(t) = V_{fl}(t) + jV_{fq}(t). \quad (\text{B2})$$

Assuming the nominal operating point lies on the real axis, the steady-state values are chosen as $V_{c0l} = 1$ and $V_{c0q} = 0$, as illustrated in Fig. 15(a). Small perturbations around this operating point are defined as

$$V_{cl} = 1 + \delta_i, \quad V_{cq} = \delta_q, \quad (\text{B3})$$

where $\delta_i, \delta_q \ll 1$ denote the small-signal deviations in the I and Q directions, respectively.

The total detuning frequency is subsequently linearized as

$$\Delta\omega(t) = \Delta\omega_0 + \delta_\omega, \quad \delta_\omega = 2\delta_i(t) \cdot H(s), \quad (\text{B4})$$

where $H(s)$ is the mechanical transfer function of the cavity structure. The derivation of Eq. (B4) is detailed in the main body of the paper and is therefore not repeated here.

2. Open-loop perturbation dynamics

Substituting Eqs. (B1) and (B2) into the dynamic Eq. (1) yields a state-space representation form [32,33]:

$$\frac{d}{dt} \begin{pmatrix} V_{cI} \\ V_{cQ} \end{pmatrix} = \begin{pmatrix} -\omega_{0.5} & -\Delta\omega \\ \Delta\omega & -\omega_{0.5} \end{pmatrix} \begin{pmatrix} V_{cI} \\ V_{cQ} \end{pmatrix} + \omega_{0.5} \begin{pmatrix} 2V_{fI} \\ 2V_{fQ} \end{pmatrix}. \quad (\text{B5})$$

Under open-loop conditions, the forward voltage remains constant at its nominal values:

$$V_{f0I} = 0.5, \quad V_{f0Q} = -\frac{\Delta\omega_0}{2\omega_{0.5}}. \quad (\text{B6})$$

As shown in Fig. 15(a), the vector $2V_f$ is fixed and does not vary with perturbations. Substituting Eqs. (B3) and (B6) into Eq. (B5), and retaining only linear terms, we obtain:

$$\begin{pmatrix} \dot{\delta}_i \\ \dot{\delta}_q \end{pmatrix} = \begin{pmatrix} -\omega_{0.5} & -\Delta\omega_0 - \delta_\omega \\ \Delta\omega_0 + \delta_\omega & -\omega_{0.5} \end{pmatrix} \begin{pmatrix} 1 + \delta_i \\ \delta_q \end{pmatrix} + \begin{pmatrix} \omega_{0.5} \\ -\Delta\omega_0 \end{pmatrix}. \quad (\text{B7})$$

Neglecting second-order terms and simplifying yields the open-loop small-signal dynamic equations:

$$\begin{aligned} \dot{\delta}_i &= -\omega_{0.5}\delta_i - \Delta\omega_0\delta_q, \\ \dot{\delta}_q &= \Delta\omega_0\delta_i - \omega_{0.5}\delta_q + \delta_\omega. \end{aligned} \quad (\text{B8})$$

Applying the Laplace transform to Eq. (B8) gives:

$$\begin{aligned} s\delta_i(s) &= -\omega_{0.5}\delta_i(s) - \Delta\omega_0\delta_q(s), \\ s\delta_q(s) &= \Delta\omega_0\delta_i(s) - \omega_{0.5}\delta_q(s) + \delta_\omega(s). \end{aligned} \quad (\text{B9})$$

Solving the first equation for $\delta_q(s)$ yields:

$$\delta_q(s) = -\frac{s + \omega_{0.5}}{\Delta\omega_0}\delta_i(s). \quad (\text{B10})$$

As previously discussed, by applying $\delta_\omega(s) = 2\delta_i(s) \cdot H(s)$ in Eq. (B4) and substituting it together with Eq. (B10) into the second equation of (B9), we derive:

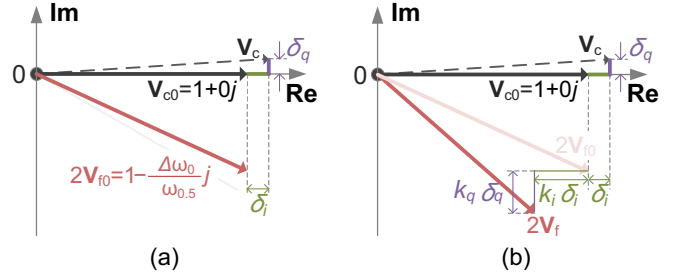


FIG. 15. Small-signal vector perturbation diagrams illustrating the cavity voltage V_c and the scaled forward voltage $2V_f$ in the complex plane. (a) Open-loop configuration: steady-state voltage $V_{c0} = 1 + j0$ and nominal forward voltage $2V_{f0} = 1 - j\frac{\Delta\omega_0}{\omega_{0.5}}$. (b) Closed-loop configuration: the modulated vector $2V_f$ under feedback with gains K_i and K_q .

$$\begin{aligned} -\frac{s(s + \omega_{0.5})}{\Delta\omega_0}\delta_i(s) &= \frac{\omega_{0.5}(s + \omega_{0.5})}{\Delta\omega_0}\delta_i(s) + \Delta\omega_0\delta_i(s) \\ &\quad + 2H(s)\delta_i(s), \end{aligned} \quad (\text{B11})$$

which leads to the characteristic equation of the open-loop system:

$$D_{ol}(s) = (s + \omega_{0.5})^2 + \Delta\omega_0^2 + 2\Delta\omega_0H(s) = 0. \quad (\text{B12})$$

3. Closed-loop dynamics under I/Q feedback

In the closed-loop case, negative feedback is applied to both the in-phase and quadrature components of the cavity voltage. The resulting forward voltage is expressed as

$$V_{fI} = 0.5 + \delta_{V_{fI}}, \quad V_{fQ} = -\frac{\Delta\omega_0}{2\omega_{0.5}} + \delta_{V_{fQ}}. \quad (\text{B13})$$

Substituting into Eq. (B5), the system becomes

$$\begin{aligned} \frac{d}{dt} \begin{pmatrix} V_{cI} \\ V_{cQ} \end{pmatrix} &= \begin{pmatrix} -\omega_{0.5} & -\Delta\omega \\ \Delta\omega & -\omega_{0.5} \end{pmatrix} \begin{pmatrix} V_{cI} \\ V_{cQ} \end{pmatrix} \\ &\quad + \omega_{0.5} \begin{pmatrix} 1 + 2\delta_{V_{fI}} \\ -\frac{\Delta\omega_0}{\omega_{0.5}} + 2\delta_{V_{fQ}} \end{pmatrix}. \end{aligned} \quad (\text{B14})$$

Linearizing around the nominal point yields:

$$\begin{aligned} \dot{\delta}_i &= -\omega_{0.5}\delta_i - \Delta\omega_0\delta_q + 2\omega_{0.5}\delta_{V_{fI}}, \\ \dot{\delta}_q &= \Delta\omega_0\delta_i - \omega_{0.5}\delta_q + \delta_\omega + 2\omega_{0.5}\delta_{V_{fQ}}. \end{aligned} \quad (\text{B15})$$

The I/Q feedback control law is defined as

$$2\delta_{V_{\Pi}} = -K_i\delta_i, \quad 2\delta_{V_{IQ}} = -K_q\delta_q.$$

As illustrated in Fig. 15(b), the feedback mechanism dynamically modulates both I and Q components of the cavity forward signal $\mathbf{V}_f$ in response to small perturbations in the δ_{V_c} in-phase (δ_i) and quadrature (δ_q). Substitute it into Eq. (B15):

$$\begin{aligned} \dot{\delta}_i &= -\omega_{0.5}(1 + K_i)\delta_i - \Delta\omega_0\delta_q, \\ \dot{\delta}_q &= \Delta\omega_0\delta_i - \omega_{0.5}(1 + K_q)\delta_q + \delta_\omega. \end{aligned} \quad (\text{B16})$$

Applying the Laplace transform:

$$\begin{aligned} s\delta_i(s) &= -\omega_{0.5}(1 + K_i)\delta_i(s) - \Delta\omega_0\delta_q(s), \\ s\delta_q(s) &= \Delta\omega_0\delta_i(s) - \omega_{0.5}(1 + K_q)\delta_q(s) + \delta_\omega(s). \end{aligned} \quad (\text{B17})$$

Solving for $\delta_q(s)$ from the first equation:

$$\delta_q(s) = \frac{-(1 + K_i)\omega_{0.5}\delta_i(s) - s\delta_i(s)}{\Delta\omega_0}. \quad (\text{B18})$$

By substituting $\delta_q(s)$ and $\delta_\omega(s) = 2\delta_i \cdot H(s)$ into the second equation of (B17), and simplifying the result, we obtain the characteristic equation of the closed-loop system:

$$\begin{aligned} D_{\text{cl},iq}(s) &= [s + \omega_{0.5}(1 + K_i)][s + \omega_{0.5}(1 + K_q)] \\ &\quad + \Delta\omega_0^2 + 2\Delta\omega_0H(s) = 0. \end{aligned} \quad (\text{B19})$$

In typical implementations, the I/Q feedback loops use identical controllers, i.e., $K_i = K_q \triangleq K_{IQ}$. Under this assumption, Eq. (B19) reduces to

$$D_{\text{cl},iq2}(s) = [s + \omega_{0.5}(1 + K_{IQ})]^2 + \Delta\omega_0^2 + 2\Delta\omega_0H(s) = 0, \quad (\text{B20})$$

4. Comparison of A/P and I/Q control for oscillatory instability suppression

When the static detuning is zero ($\Delta\omega_0 = 0$), the closed-loop characteristic equation for the A/P control scheme, as given in Eq. (23), simplifies to

$$D_{\text{cl},ap}(s) = [s + \omega_{0.5}(1 + K_r)][s + \omega_{0.5}(1 + K_\phi)] = 0, \quad (\text{B21})$$

while the I/Q form in Eq. (B19) becomes

$$D_{\text{cl},iq}(s) = [s + \omega_{0.5}(1 + K_i)][s + \omega_{0.5}(1 + K_q)] = 0, \quad (\text{B22})$$

Equations (B21) and (B22) are structurally identical, differing only in gain notation, and all of their roots are real

and negative. Hence, under small-signal perturbations and zero static detuning, the closed-loop dynamics of the A/P and I/Q control schemes are mathematically equivalent and intrinsically stable, which is consistent with the empirical observation that oscillatory instabilities generally occur only when significant detuning is present.

While both control schemes are stable under small-signal perturbations and zero static detuning, a deeper comparison of their performance, particularly in suppressing oscillatory ponderomotive instabilities, is of great interest. Although a detailed and rigorous head-to-head comparison between A/P control [Eq. (23)] and I/Q control [Eq. (B19)] for suppressing oscillatory ponderomotive instabilities would be valuable, it lies beyond the scope of the present work. Here, we provide a preliminary, qualitative account consistent with our simulations. Let $K_r = K_\phi = K_i = K_q \equiv K = 1.5 \times (1 + \omega_{0.5}/s)$. From Eq. (B19), the I/Q architecture leaves the electro-mechanical coupling term $\Delta\omega_0^2 + 2\Delta\omega_0H(s)$ independent of the loop gains K . In contrast, Eq. (23) for A/P reveals three features: (i) the amplitude-channel damping is reduced by $\cos \Delta\theta_0 \leq 1$; (ii) the entire coupling is multiplied by $(1 + K_\phi)$; and (iii) an additional cross term $\propto K_r \sin \Delta\theta_0$ appears. Since $|H(j\Omega_m)|$ is large near mechanical resonances, these effects make the A/P stability boundary more sensitive to detuning setpoint and gain; for the same K , the unstable region is therefore typically larger than for I/Q (see Fig. 16).

To isolate the dominant role of point (ii), we perform a didactic “what-if” modification: we set the $(1 + K_\phi)$ scaling in front of the coupling term in Eq. (23) to unity and obtain the surrogate

$$\begin{aligned} D_{\text{su}}(s) &= [s + \omega_{0.5}(1 + K_r \cos \Delta\theta_0)][s + \omega_{0.5}(1 + K_\phi)] \\ &\quad + \Delta\omega_0[\Delta\omega_0 + \omega_{0.5}K_r \sin \Delta\theta_0 + 2H(s)] = 0, \end{aligned} \quad (\text{B23})$$

referred to as Eq. (B23). Figure 16 compares the instability boundaries from the original A/P [Eq. (23)], the I/Q [Eq. (B19)], and the surrogate Eq. (B23). Once the $(1 + K_\phi)$ amplification of the coupling is removed, Eq. (B23) nearly collapses onto the I/Q result, highlighting that the $(1 + K_\phi)$ factor is the primary driver behind the relatively larger unstable region observed with A/P. This modification is solely for explanatory purposes; the actual closed-loop equation of a real system should, of course, not be altered.

APPENDIX C: EQUIVALENT SYSTEM

The transfer function corresponding to Eq. (1) is given by

$$\frac{\mathbf{V}_c(s)}{\mathbf{V}_f(s)} = \frac{2\omega_{0.5}}{s + \omega_{0.5} - j\Delta\omega}. \quad (\text{C1})$$

According to Eq. (28), the signal path from $\mathbf{V}_{\text{SSA}}$ to $\mathbf{V}_c$ can be represented as the feedback network shown in Fig. 17. Applying the standard closed-loop formula yields:

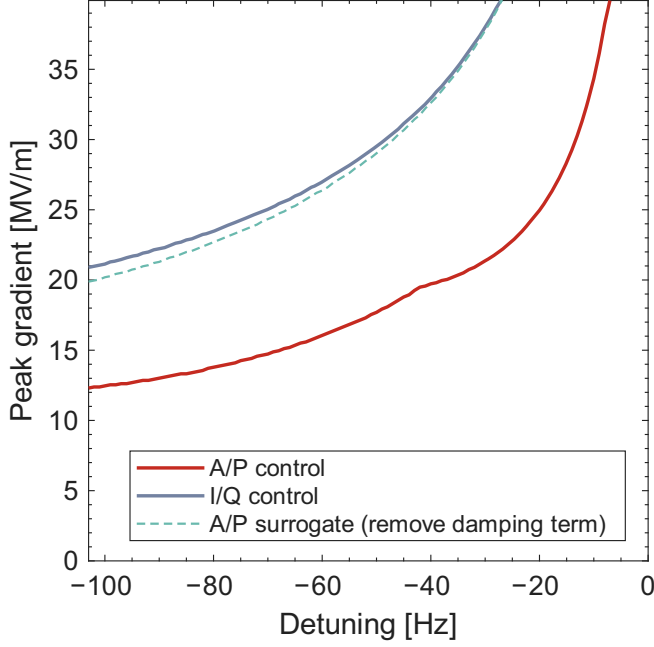


FIG. 16. Instability boundaries under A/P and I/Q control with equal loop gains ($K_r = K_\phi = K_i = K_q \equiv K$). Solid red: A/P [Eq. (23)]. Solid blue: I/Q [Eq. (B19)]. Dashed green: A/P surrogate [Eq. (B23)], obtained by setting the $(1 + K_\phi)$ scaling in front of the coupling term in Eq. (23) to unity for illustration only. Once this amplification is removed, the A/P boundary nearly collapses onto the I/Q result, indicating the dominant role of the $(1 + K_\phi)$ factor in enlarging the unstable region.

$$\begin{aligned} \frac{V_c(s)}{V_{SSA}(s)} &= \frac{c \frac{2\omega_{0.5}}{s + \omega_{0.5} - j\Delta\omega_0}}{1 - \frac{1}{2}\alpha \frac{2\omega_{0.5}}{s + \omega_{0.5} - j\Delta\omega_0}} \\ &= \frac{2c\omega_{0.5}}{s + \omega_{0.5}(1 - \alpha) - j\Delta\omega_0}, \end{aligned} \quad (C2)$$

where $c = \frac{1-\Gamma}{1+\Gamma}$ and $\alpha = \frac{2\Gamma}{1+\Gamma}$.

Equation (C2) can be rearranged as

$$\frac{V_c(s)}{V_{SSA}(s)} = \frac{\frac{1}{1-\alpha_i} \times 2c\omega_{0.5}(1 - \alpha_i)}{s + \omega_{0.5}(1 - \alpha_i) - j(\Delta\omega_0 + \alpha_q\omega_{0.5})}, \quad (C3)$$

where α_i and α_q denote the real and imaginary parts of the complex parameter α .

Finally, Eq. (C3) can be compactly expressed as

$$\frac{V_c(s)}{V_{SSA}(s)} = \frac{2b\Omega_{0.5}}{s + \Omega_{0.5} - j\Delta\Omega_0}, \quad (C4)$$

with the equivalent system parameters defined as

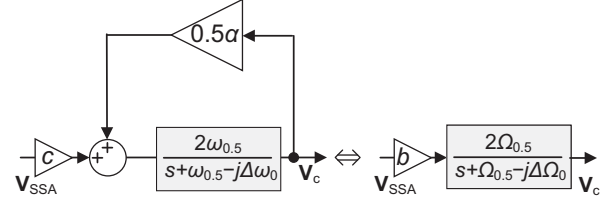


FIG. 17. Equivalent representation of the feedback path from V_{SSA} to V_c .

$$\Omega_{0.5} = \omega_{0.5}(1 - \alpha_i),$$

$$\Delta\Omega_0 = \Delta\omega_0 + \alpha_q\omega_{0.5},$$

$$b = \frac{1}{(1 + \Gamma)(1 - \alpha_i)}. \quad (C5)$$

Equation (C4) is identical to the transfer function of the right-hand block in Fig. 17, thus confirming the equivalence between the feedback network and the simplified generalized-cavity model.

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