

Optimizing injection for the storage ring proton-EDM experiment

Jonathan Lee^{1,2,*}, Haixin Huang², Francois Meot², William Morse³,
Yannis K. Semertzidis⁴, and Nikolaos Tsoupas⁵

¹*Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York 11794, USA*

²*Collider-Accelerator Department, Brookhaven National Laboratory, Upton, New York 11973, USA*

³*Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA*

⁴*Department of Physics, KAIST, Daejeon 34141, Republic of Korea*

⁵*Electron-Ion Collider Project, Brookhaven National Laboratory, Upton, New York 11973, USA*



(Received 17 May 2025; accepted 18 March 2026; published 17 April 2026)

The proposed proton electric dipole moment (pEDM) experiment at Brookhaven National Laboratory (BNL), to be built inside the alternating gradient synchrotron (AGS) tunnel, aims to measure the proton's electric dipole moment with a sensitivity of 10^{-29} e cm. This paper presents the design of the injection line from the AGS booster to the pEDM storage ring, utilizing portions of the existing booster-to-AGS (BtA) transfer line. Building on the symmetric-hybrid lattice design [Z. Omarov *et al.*, Comprehensive symmetric-hybrid ring design for a proton-EDM experiment at below 10^{-29} e cm, *Phys. Rev. D* **105**, 032001 (2022)], our study emphasizes rigorous optics matching, detailed particle and spin tracking, and systematic error mitigation essential for achieving a target sensitivity of 10^{-29} e cm. This design preserves the proton's vertical spin orientation within ± 20 mrad, a critical requirement for the pEDM measurement. Particle and spin tracking simulations using the ray-tracing code Zgoubi [F. Méot, Zgoubi users' guide, Technical Report, Brookhaven National Laboratory (BNL), Relativistic Heavy Ion Collider (RHIC), Upton, NY, 2012] validate the design's performance, demonstrating its feasibility for this precision experiment. The simulation results demonstrate that both clockwise (CW) and counterclockwise (CCW) injection lines meet the stringent beam envelope and polarization requirements.

DOI: [10.1103/physrevacc.29.041601](https://doi.org/10.1103/physrevacc.29.041601)

I. INTRODUCTION

The quest to measure the electric dipole moment (EDM) of fundamental particles is a cornerstone of modern particle physics, probing *CP* violation beyond the Standard Model and potentially explaining the matter-antimatter asymmetry in the universe [1]. The proton-EDM (pEDM) experiment, proposed by the storage ring EDM Collaboration at Brookhaven National Laboratory (BNL) within the alternating gradient synchrotron (AGS) tunnel, targets a sensitivity of 10^{-29} e cm, a significant improvement over the current upper limit of approximately 10^{-25} e cm derived indirectly from nuclear EDM measurements [2,3]. Achieving this precision requires a storage ring with exceptional beam and spin control, housed within the AGS tunnel at BNL.

Earlier works and studies by Weng and Lee [4] and Bracco [5] laid the groundwork for high-quality beam

injection in hadron accelerators, while recent studies [6,7] have stressed the necessity of stringent beam polarization control in pEDM experiments. The symmetric-hybrid ring design proposed by Omarov *et al.* [8] has been widely cited for its ability to support simultaneous clockwise (CW) and counterclockwise (CCW) beam storage, which is key to canceling systematic effects.

The pEDM experiment's sensitivity hinges on minimizing systematic errors, which can mimic the tiny EDM signal (proportional to the spin precession in an electric field) and reduce measurement precision. Key systematic effects include: (i) Magnetic field imperfections: Residual radial magnetic fields can induce vertical spin precession when other lattice imperfections are present, masquerading as an EDM signal [7–10]. (ii) Beam orbit misalignments: Deviations from the ideal orbit can couple with field gradients, producing spurious signals [8,11]. (iii) Injection asymmetries: Differences in beam properties between CW and CCW directions can undermine systematic error cancellation [6,8].

The injection process is pivotal in mitigating these effects. It must deliver a beam with precisely matched optical properties (Twiss parameters and dispersion) and a tightly controlled spin orientation to ensure stability and symmetry in the stored beams [7]. For instance, the two

*Contact author: jlee1@bnl.gov

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](https://creativecommons.org/licenses/by/4.0/). Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

counterrotating beam configuration—where EDM signals add while systematic effects from radial fields subtract—relies on nearly identical beam injection conditions in both directions [7,8]. Any mismatch could amplify systematic errors, reducing the experiment’s sensitivity.

To address these challenges, a single-turn horizontal injection scheme, which minimizes beam perturbations and ensures precise synchronization with the pEDM experiment’s beam bunching specifications [5,7,8,12], was employed. This method reduces injection-induced oscillations that could lead to beam losses or misalignments, both of which exacerbate systematic effects [12]. Protons are extracted from the AGS booster, and the injection line—the so-called booster-to-pEDM (BtP) line—leverages parts of the existing booster-to-AGS (BtA) transfer line. This approach optimizes cost and infrastructure use while meeting the stringent requirements of the experiment.

This paper details the design of the BtP injection line for both CW and CCW directions, emphasizing how it minimizes systematic effects through matched optics, dispersion control, and spin preservation. This paper also validates the design through simulations and provides physics insights into its role in the pEDM experiment.

The paper is organized as follows: Sec. II introduces the physics and ring design of the storage ring proton-EDM experiment. Section III discusses the optimization of injection for the experiment. Section IV concludes with relevant discussions.

II. OVERVIEW OF THE STORAGE RING PROTON-EDM EXPERIMENT

A. Storage ring spin dynamics equations

The spin $\vec{S}$ precession rate for a proton at rest in the presence of the magnetic field $\vec{B}$ and the electric field $\vec{E}$ is given by

$$\frac{d\vec{S}}{dt} = \vec{\mu} \times \vec{B} + \vec{d} \times \vec{E} \quad (1)$$

with the magnetic dipole moment of the proton as $\vec{\mu} = (\frac{gq}{2m})\vec{S}$, and the electric dipole moment of the proton as $\vec{d} = (\eta q/2mc)\vec{S}$. The proton’s g -factor is denoted by g , its charge by q , its mass by m , and its electric dipole moment by η . The speed of light is denoted by c .

Spin motion relative to the momentum for a proton with $\vec{\beta} = \vec{v}/c$ in a cylindrical coordinate system [standard Frenet-Serret (x, y, s) coordinates, with “ x ” as the horizontal coordinate, “ y ” as the vertical coordinate, and “ s ” as the longitudinal coordinate] is given by [13–15]

$$\frac{d\vec{S}}{dt} = (\vec{\omega}_a + \vec{\omega}_\eta) \times \vec{S} \quad (2)$$

with

$$\vec{\omega}_a = -\frac{q}{m} \left\{ G\vec{B} - \frac{G\gamma}{\gamma+1} \vec{\beta}(\vec{\beta} \cdot \vec{B}) - \left(G - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} + \frac{1}{\gamma} \left[\vec{B}_\parallel - \frac{1}{c\beta^2} (\vec{\beta} \times \vec{E})_\parallel \right] \right\}, \quad (3a)$$

$$\vec{\omega}_\eta = -\frac{\eta q}{2m} \left(\frac{\vec{E}}{c} - \frac{\gamma}{\gamma+1} \frac{\vec{\beta}}{c} (\vec{\beta} \cdot \vec{E}) + \vec{\beta} \times \vec{B} \right), \quad (3b)$$

where $\vec{\omega}_a$ and $\vec{\omega}_\eta$ stand for precession due to magnetic and electric dipole moments, respectively, $G = (g - 2)/2$ stands for the proton magnetic anomaly, γ is the Lorentz factor, $\vec{v}$ is the velocity of the proton, and $\parallel$ indicates the horizontal (in-plane) projection of a vector.

By setting $\vec{B} = 0$ and selecting the proton’s momentum as the “magic” momentum, defined by $G = 1/(\gamma^2 - 1)$, with horizontal electric field $\vec{E}$ and $\vec{\beta}$ ($\vec{E} = E_x$, $\vec{\beta} = \beta_x$), all terms in Eq. (3a) vanish, resulting in:

$$\vec{\omega}_a = 0. \quad (4)$$

This condition is known as the frozen spin condition.

In this setting, the spin precesses into the vertical direction only due to the EDM contribution.

$$\frac{dS_x}{dt} = 0, \quad \frac{dS_y}{dt} \approx 0, \quad (5a)$$

$$\frac{dS_y}{dt} = \frac{\eta q}{2mc} E_x S_x. \quad (5b)$$

Measurement of the out-of-plane spin precession rate (dS_y/dt) inside a storage ring probes the intrinsic EDM of the particles. The target sensitivity of $d_p = 10^{-29} e \text{ cm}$ corresponds to a vertical spin precession rate of $dS_y/dt = 1 \text{ nrad/s}$. Any non-EDM originating vertical spin precession rate larger than 1 nrad/s is considered a potential systematic error source.

B. Proton-EDM symmetric-hybrid ring design

The pEDM storage ring features a symmetric-hybrid lattice with 24 FODO cells, yielding a circumference of approximately 800 m [8]. Each FODO cell, illustrated in Fig. 1, comprises focusing (QF) and defocusing (QD) quadrupoles, drift spaces (S), and electric bends (EBEND). The symmetric hybrid configuration and frozen-spin ring design were selected due to their conceptual simplicity and enhanced robustness against systematic errors.

The lattice supports simultaneous CW and CCW beam storage, with the beams experiencing opposite focusing effects from the quadrupoles due to their reversed directions. This two counterrotating beam configuration is essential for averaging out systematic errors [6–8]. The beta (β_x, β_y) and dispersion (D_x, D_y) functions, computed

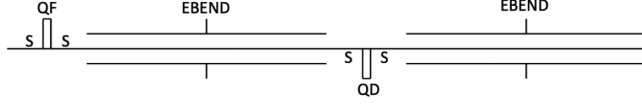


FIG. 1. Schematic of a single pEDM FODO cell (QF: focusing quadrupole, QD: defocusing quadrupole, S: drift space, and EBEND: electric bend) [8].

using the ray-tracing code Zgoubi, are shown in Fig. 2 for both directions. The Zgoubi ray-tracing code for beam and spin dynamics simulation uses a numerical method based on Taylor series to integrate the Lorentz and Thomas-BMT equations, achieving high accuracy and robust preservation of motion invariants while optimizing computational efficiency [16]. Zgoubi has been used as the spin simulation tool and has been experimentally benchmarked for the booster ac dipole and the RHIC spin flipper [17,18]. In this work, a Frenet-Serret coordinates frame ($\hat{x}, \hat{s}, \hat{y}$) is used (radial, longitudinal, vertical); for both CW and CCW rings, the longitudinal axis $\hat{s}$ is oriented in the beam direction, and the radial axis $\hat{x}$ points outward. Referring to Fig. 2(b), the moving frame has its longitudinal axis $\hat{s}$ in the beam direction (CCW), its radial axis $\hat{x}$ points outward (curvature

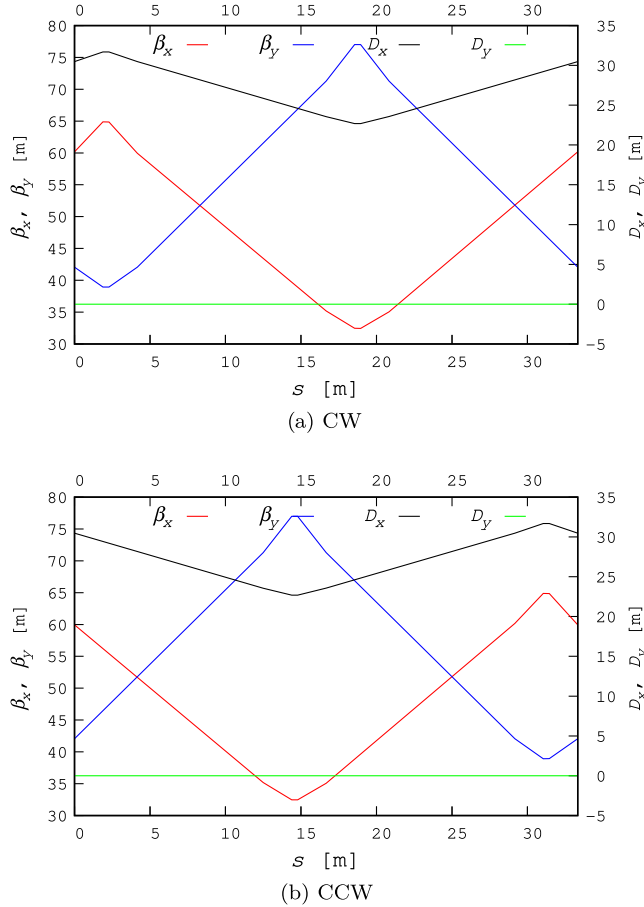


FIG. 2. Beta and dispersion functions for (a) CW and (b) CCW beams in the pEDM ring.

is negative, dispersion is positive), and its vertical axis $\hat{y}$ points down.

Omarov *et al.* [8] employed polarity switching of quadrupoles and simultaneous storage of CR beams (CW and CCW beams) to mitigate and minimize major and potential systematic effects. The measurable EDM signal is then expressed as follows:

$$\left(\frac{dS_y}{dt}\right)_{\text{EDM}} = \frac{1}{2} \left[\frac{1}{2} \left(\frac{dS_y}{dt}\right)_{\text{CW}} - \frac{1}{2} \left(\frac{dS_y}{dt}\right)_{\text{CCW}} \right]_+ + \frac{1}{2} \left[\frac{1}{2} \left(\frac{dS_y}{dt}\right)_{\text{CW}} - \frac{1}{2} \left(\frac{dS_y}{dt}\right)_{\text{CCW}} \right]_- \quad (6)$$

Note that we denote the positive polarity of the currents in the magnetic quadrupoles with “+” sign, the negative polarity of the currents in the magnetic quadrupoles with “−” sign.

The storage ring EDM Collaboration proposes to construct the pEDM storage ring within the existing 800-m circumference tunnel of the alternating gradient synchrotron at Brookhaven National Laboratory. Utilizing the AGS tunnel eliminates the need for new tunnel construction and reduces costs.

III. OPTIMIZING INJECTION FOR THE STORAGE RING PROTON-EDM EXPERIMENT

A. The injection design overview

The single-turn horizontal injection scheme, depicted in Fig. 3, displaces the injected beam horizontally at the septum and uses a kicker to align it with the closed orbit. This method ensures precise timing with the pEDM experiment’s beam bunching specifications and minimizes perturbations to the stored beam [5,8,12].

The design specifications for the pEDM ring, as detailed in Alexander *et al.* [7], stipulate that the spin direction of the injected beam should be as closely aligned with the vertical axis as possible—ideally within ± 20 mrad—to minimize beam depolarization and to ensure that the counterrotating beams have comparable polarization levels and profiles. In this study, we designed the injection line for

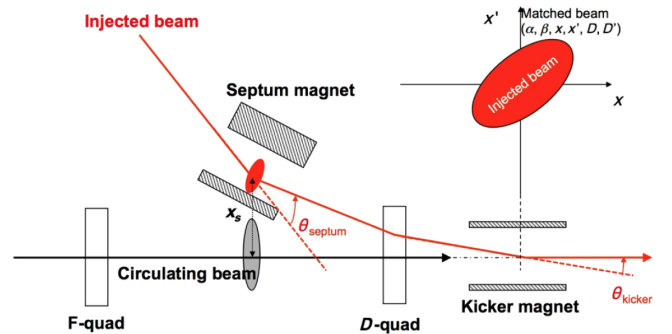


FIG. 3. pEDM single-turn horizontal injection scheme [5].

the BtP system, addressing both CW and CCW horizontal injection schemes. The design process followed these steps: (i) Determination of initial beam parameters: We calculated the beam parameters and dispersion functions at the exit of the AGS booster, specifically at the AGS booster F6 septum. (ii) Analysis of circulating beam properties: We computed the beam parameters and dispersion functions for the circulating beam within the pEDM ring at the designated injection points: the CW injection septum (X) and the CCW injection septa (X2A/X2B). (iii) Beam matching procedure: Using a Zgoubi model of the BtP line, we matched the properties of the injected beam—including Twiss parameters $[\alpha(s), \beta(s)]$, horizontal position (x), horizontal momentum slope (x'), horizontal dispersion $[D_x(s)]$, and horizontal dispersion derivative $[D'_x(s)]$ —to those of the circulating beam at the CW and CCW injection points in the pEDM ring.

This systematic approach ensures that the BtP injection line satisfies the stringent spin alignment and beam dynamics requirements of the pEDM experiment.

B. CW/CCW BtP injection kicker parameters

The kicker angle θ_{kicker} is given by

$$\theta_{\text{kicker}} = \frac{X_s}{\sqrt{\beta_k \beta_s} \sin(\mu)}, \quad (7)$$

where X_s is the beam displacement at the septum, β_k and β_s are the beta functions at the kicker and septum, respectively, and μ is the phase advance [19]. The displacement X_s must exceed the sum of several contributions: (i) Radius of the beamline vacuum pipe: The beamline vacuum pipe is assumed to have a radius of $r_0 = 0.06$ m [7]. (ii) Thickness of the septum blade: The injection septum is assumed to have a thickness of $l_0 = 0.01$ m, with clearances of $l_c = 0.002$ m on both sides [7]. (iii) Injected beam envelope: From the specifications of the pEDM ring lattice [8], the injected beam has an rms horizontal beam size of $\sigma_x = \sqrt{\epsilon_x^{\text{rms}} \beta_x} \approx 3.72$ mm, with rms emittances $\epsilon_x^{\text{rms}} = 0.214$ mm mrad and $\epsilon_y^{\text{rms}} = 0.25$ mm mrad. (iv) Beam size increase due to energy spread: The momentum spread from the AGS booster is $\Delta p/p \sim 1.4 \times 10^{-4}$ [7]. The total horizontal beam size, including the effect of energy spread, is calculated as $\sigma_x = \sqrt{\epsilon_x^{\text{rms}} \beta_x + D_x^2 (\Delta p/p)^2}$. (v) Closed-orbit distortion: According to Ref. [8], for the pEDM experiment, the closed-orbit distortion caused by field errors is expected to be on the order of a few micrometers. (vi) Alignment errors: Refs. [7,8] indicate that for the pEDM ring lattice, tolerable alignment errors are limited to a few hundred micrometers.

Considering all the length quantities mentioned above, the beam displacement (X_s) at the injection septum magnet must be at least approximately 8 cm to ensure successful injection.

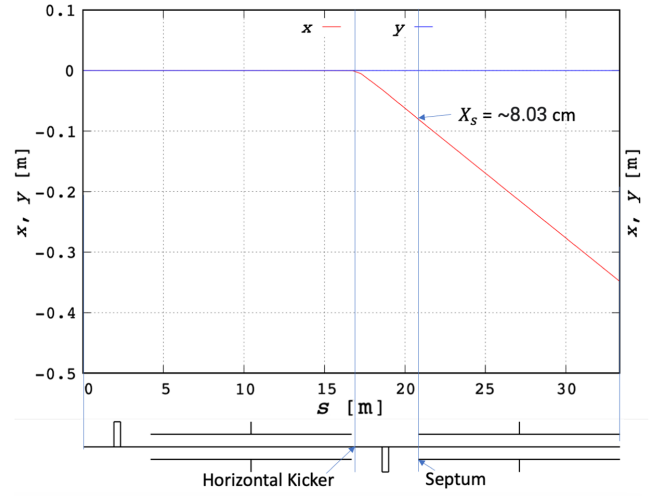


FIG. 4. Beam trajectory for a QD extraction.

The electrode spacing of the electrostatic bending plates is 4 cm. The 4.16 m straight sections, consisting of drift space, focusing/defocusing quadrupoles, and additional drift space between electrostatic bends, house the injection system segment, which includes the injection septum magnet and the injection kicker. Notably, this design leaves the pEDM lattice optical structure unchanged. To minimize kicker strength, the injection septum and kicker are positioned on either side of the defocusing quadrupole (QD).

To simplify the calculation of the kicker angle at the injection point of the pEDM lattice, we simulated beam extraction from the lattice rather than injection. By adjusting the injection kicker strength, a beam displacement (X_s) of approximately 8.03 cm at the injection septum location was achieved. The beam trajectory for this QD extraction is depicted in Fig. 4, with the corresponding kicker parameters listed in Table I. This kicker design is feasible and compatible with existing technologies [20].

C. CW/CCW BtP injection line design and beam matching

The pEDM experiment demands stringent spin alignment and beam dynamics to detect the electric dipole moment signal. Magnetic field imperfections must be

TABLE I. Horizontal QD injection kicker parameters.

Proton P (GeV/c)	0.707
Beam displacement (cm)	8.03
Kicker dimensions (cm)	3.2 (W) $\times$ 2.2 (H) $\times$ 50 (L)
Bunch spacing (ns)	200 [7]
Kicker strength (kG)	0.947
Kicker current (A)	1658.16
Kicker angle (mrad)	22.06
Kicker voltage (kV)	7.58

TABLE II. Comparisons of designed and matched Twiss parameters at the injection point of pEDM BtP CW and CCW injection lines.

	pEDM BtP CW line		pEDM BtP CCW line	
	Designed value	Matched value	Designed value	Matched value
α_x	7.6161622×10^{-1}	7.6161628×10^{-1}	7.6161622×10^{-1}	7.6161625×10^{-1}
β_x	3.5178441×10^1	3.5178440×10^1	3.5178441×10^1	3.5178442×10^1
α_y	-1.4761344	-1.4761353	-1.4761344	-1.4761349
β_y	7.1280776×10^1	7.1280778×10^1	7.1280776×10^1	7.1280777×10^1
D_x	2.3549917×10^1	2.3549917×10^1	2.3549917×10^1	2.3549917×10^1
D'_x	$-4.5545157 \times 10^{-1}$	$-4.5545154 \times 10^{-1}$	$-4.5545157 \times 10^{-1}$	$-4.5545155 \times 10^{-1}$
D_y	0.0000000	$-2.5102151 \times 10^{-5}$	0.0000000	1.9301415×10^{-5}
D'_y	0.0000000	$-1.3527664 \times 10^{-6}$	0.0000000	6.1954814×10^{-7}

minimized through counterrotating beams with identical properties, requiring precisely matched optics and dispersion for both CW and CCW directions. Beam stability is crucial to prevent oscillations that lead to emittance growth and increased sensitivity to field gradients and is achieved through single-turn injection and precisely matched Twiss parameters. Maintaining vertical spin polarization within ± 20 mrad is essential to avoid systematic errors.

Matched optical functions and injection symmetry are crucial for beam stability and systematic error cancellation in the counterrotating configuration of the pEDM experiment, where identical beam properties are essential [7,8]. Anastassopoulos *et al.* [9] underscore the importance of injection symmetry for controlling systematic errors, while Lehrach *et al.* [6] notes that injection asymmetries can compromise error cancellation and beam stability in EDM rings. Emphasis of this design on symmetry mitigates these issues, enhancing precision, as shown by the matched beam profiles (α, β, D, D') in Table II. Additionally, Farley *et al.* [21] highlight spin control techniques in storage rings,

aligning with our approach. Unlike the multiturn injection used in muon $g-2$ experiments [22], the pEDM adopts a single-turn injection method, common in synchrotrons [5], prioritizing simplicity and accuracy.

The BtP line adapts the BtA line from the AGS booster F6 septum, modeled with MAD-X [23] and Zgoubi [16], and adds new sections for CW and CCW injections. Matching the Twiss parameters (α, β) and dispersion (D, D') of the injected beam to the circulating beam minimizes betatron oscillations, ensuring rapid orbit settlement and reducing systematic errors from emittance growth or misalignments [11]. The geometric layout of the CW and CCW BtP injection lines is shown in Fig. 5.

The layout of the CW BtP injection beamline is depicted in Fig. 6(a). It originates at the AGS booster F6 septum, which also serves as the starting point of the BtA line. The existing BtA line comprises [20]: three bending dipole magnets (DH1, DH2, and DH3); seven focusing and defocusing quadrupole magnets (Q1–Q7). From Q7, the



FIG. 5. Geometric layout of CW and CCW BtP injection lines. The injection lines are in orange, the proposed pEDM ring is in red.

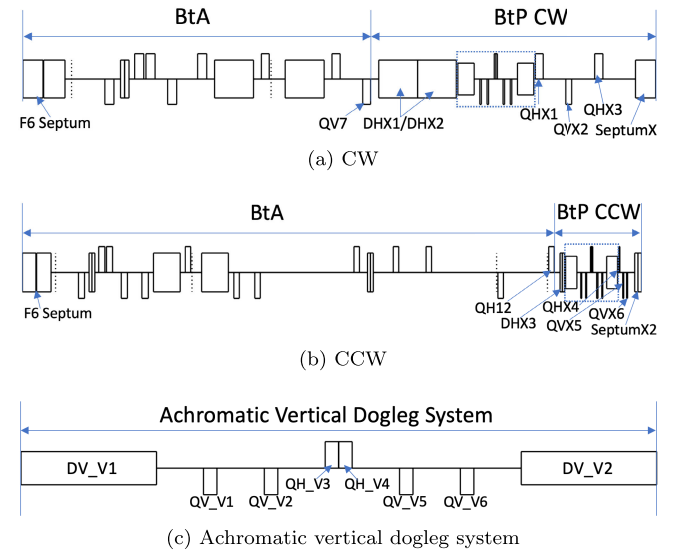


FIG. 6. Beamline layout drawing of (a) CW and (b) CCW BtP injection lines. The achromatic vertical dogleg system is highlighted with a blue box in (a) and (b), and its beamline layout is shown in (c).

CW BtP injection line extends approximately 17.78 m. This section includes: two bending magnets (DHX1 and DHX2), which together provide a bending angle of about 159° ; an achromatic vertical dogleg system (highlighted within a blue box); three quadrupole magnets (QHX1, QVX2, and QHX3). The injection angle is approximately 3° , with the injection septum (SeptumX) contributing about 1.74° and the injection kicker providing approximately 1.26° to direct the beam into the main pEDM CW ring orbit.

The layout of the CCW BtP injection beamline is illustrated in Fig. 6(b). Like the CW beamline, it begins at the AGS booster F6 septum, the starting point of the BtA line. For this configuration, the existing BtA line consists of [20]: four bending dipole magnets (DH1, DH2, DH3, and DH4); 12 focusing and defocusing quadrupole magnets (Q1–Q12). From Q12, the BtP CCW injection line spans approximately 7.86 m. This segment includes: one bending magnet (DHX3), which provides a bending angle of about 8° ; an achromatic vertical dogleg system (highlighted within a blue box); three quadrupole magnets (QHX4, QVX5, and QVX6). The injection angle is approximately 7° . The injection septa (SeptumX2A and SeptumX2B) account for about 5.74° of this angle, while the injection kicker contributes approximately 1.26° to guide the beam into the main pEDM CCW ring orbit.

The pEDM rings are positioned above the AGS ring, with a vertical elevation difference of ~ 40 inches [7]. To enable beam transfer from the AGS booster to the pEDM rings, a vertical beam transport system—an achromatic vertical dogleg—is required to align the beam with the elevation of both the pEDM CW and CCW rings. The beamline layout of this achromatic vertical dogleg is shown in Fig. 6(c). The system comprises two vertical bending magnets (DV_V1 and DV_V2)—one to deflect the beam upward and the other to deflect it downward to the horizontal plane—together with six quadrupole magnets (QV_V1, QV_V2, QH_V3, QH_V4, QV_V5, and QV_V6) interspersed between them to optimize the beam optics. The total length of this achromatic vertical dogleg system is approximately 4.9 m.

The parameters of the injected beam—including Twiss parameters $[\alpha(s), \beta(s)]$ in both horizontal and vertical, horizontal position (x), horizontal momentum slope (x'), vertical position (y), vertical momentum slope (y'), horizontal dispersion $[D_x(s)]$, horizontal dispersion derivative $[D'_x(s)]$, vertical dispersion $[D_y(s)]$, and vertical dispersion derivative $[D'_y(s)]$ —are matched to those of the circulating beam at the pEDM CW and CCW injection points. The corresponding matched optical functions are presented in Fig. 7.

In the pEDM experiment, phase-space mismatch between the injected beam and the ring Twiss parameters can induce closed-orbit deviations, potentially mimicking EDM signals. For CW and CCW ring injections, a 0.1% mismatch causes closed-orbit deviations

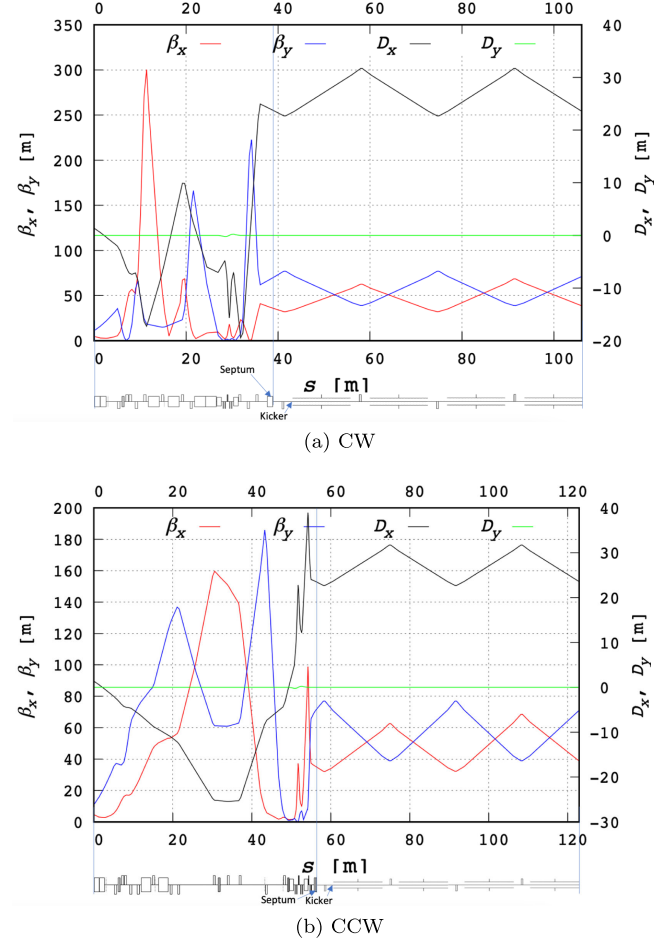
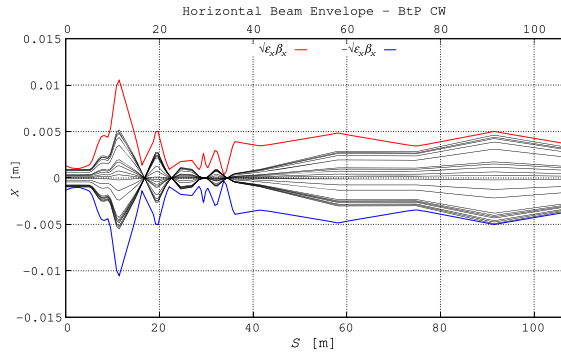


FIG. 7. Matched optical functions for (a) CW and (b) CCW BtP injection lines.

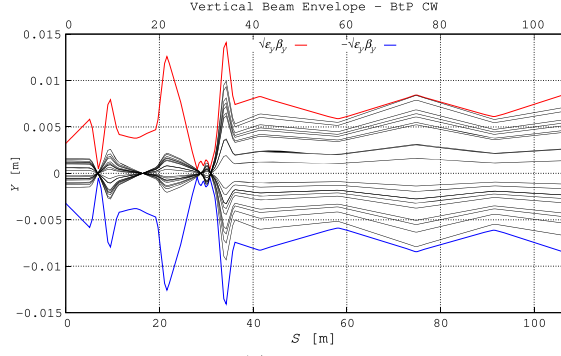
$(\Delta x)_{\max} \approx 1.86 \times 10^{-6}$ m and $(\Delta y)_{\max} \approx 2.20 \times 10^{-6}$ m, comparable to closed-orbit distortions caused by 1 nT radial magnetic fields ($\approx 7.07 \times 10^{-6}$ m for harmonic $N = 2$), which could potentially mimic EDM signals arising from a second-order systematic effect due to the interplay between stray radial magnetic fields and vertical electric quadrupole and sextupole fields [24]. The design of the pEDM BtP CW and CCW injection lines emphasizes rigorous and precise optics matching. The comparisons of designed and matched Twiss parameters at the injection point of pEDM BtP CW and CCW injection line are shown in Table II. Precise optics matching in the pEDM BtP CW and CCW injection lines achieves differences in Twiss parameters of 14 to 270 ppb, reducing closed-orbit deviations to $(\Delta x)_{\max} \approx 4.8 \times 10^{-10}$ m and $(\Delta y)_{\max} \approx 5.7 \times 10^{-10}$ m, rendering phase-space mismatch effects negligible and mitigating systematic EDM backgrounds.

D. Particle and spin tracking results

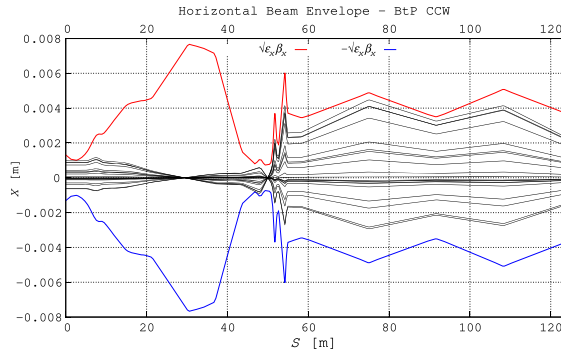
Zgoubi simulations [16] tracked 20 particles with random initial coordinates within the pEDM beam ellipse,



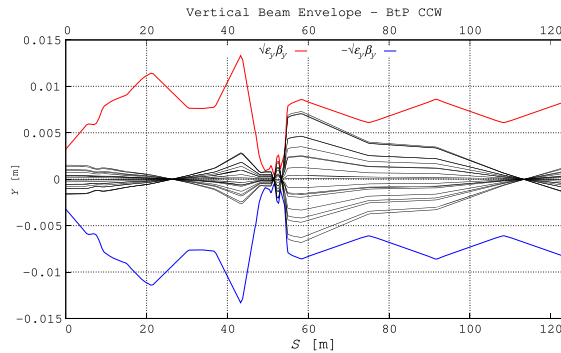
(a) CW Horizontal



(b) CW Vertical

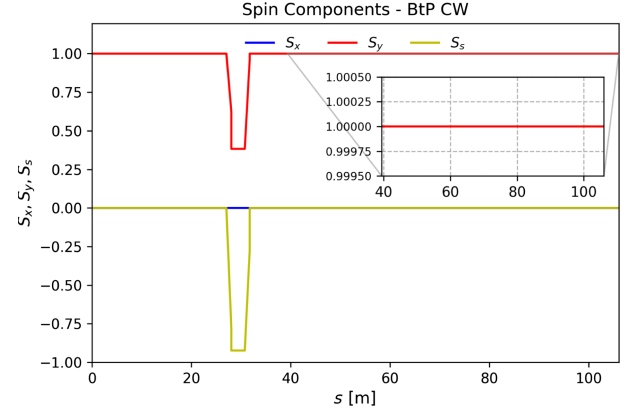


(c) CCW Horizontal

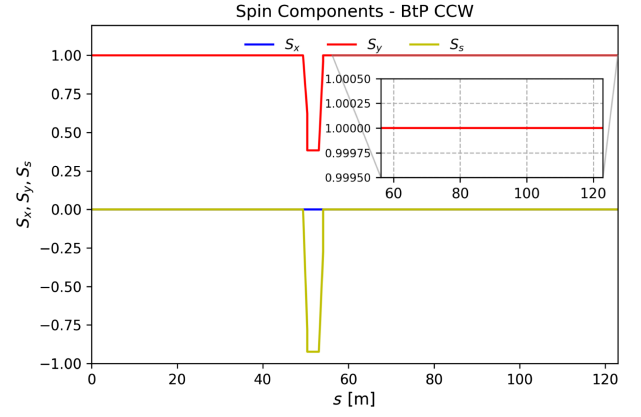


(d) CCW Vertical

FIG. 8. Beam envelopes for (a) CW horizontal, (b) CW vertical, (c) CCW horizontal, and (d) CCW vertical.



(a) CW



(b) CCW

FIG. 9. Spin components for (a) CW and (b) CCW injection lines with kicker on.

which confirms beam containment within the beamline acceptance (Fig. 8). Spin tracking of a single particle (Fig. 9) with the kicker on verifies that the vertical spin (+1 in $\hat{y}$) remains within ± 20 mrad. Spin tracking of vertical spin component of a bunched beam (20 particles with random initial coordinates within the pEDM beam ellipse) with the kicker on (Fig. 10), incorporating the momentum spread from the AGS booster further verifies that the vertical spin (+1 in $\hat{y}$) of this bunched beam remains within ± 20 mrad. The design specifications for the pEDM experiment are achieved, and unwanted spin rotations [7] are avoided. This precision is vital, as spin tilts can introduce systematic errors in the EDM measurement [3,7,8].

IV. DISCUSSION AND CONCLUSION

The BtP injection line is designed to suppress systematic errors in the pEDM experiment, addressing key challenges identified in EDM literature [3,6,8,9,21]: (i) Magnetic field imperfections: Residual radial fields can mimic the EDM

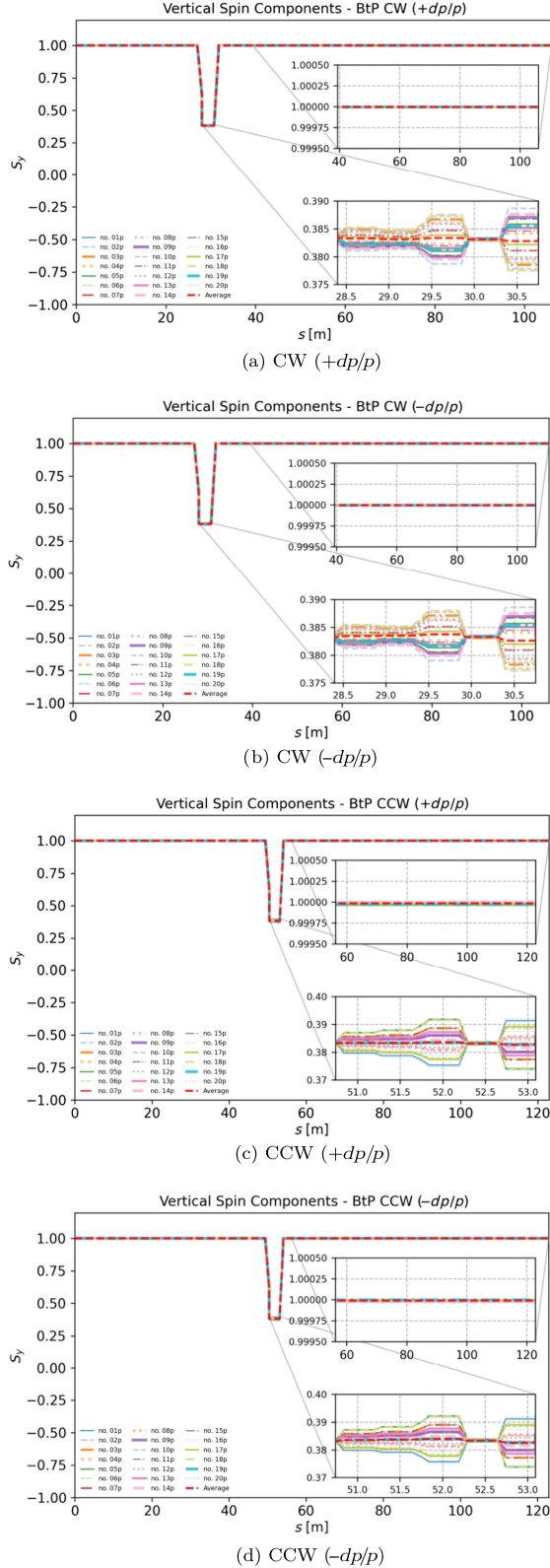


FIG. 10. Vertical spin component of a bunched beam (20 particles) with the kicker on, incorporating the pEDM beam size and momentum spread from the AGS booster, for (a) CW (+ dp/p), (b) CW ($-dp/p$), (c) CCW (+ dp/p), and (d) CCW ($-dp/p$).

signal via vertical spin precession when other lattice imperfections are present [3,8–10]. The counterrotating beams cancel this effect, but only if injection ensures identical beam properties. The precisely matched optics and dispersion for CW and CCW beams minimize asymmetries, enhancing cancellation [6,7]. (ii) Beam stability: Injection-induced oscillations can lead to emittance growth or losses, increasing sensitivity to field gradients [19]. The single-turn scheme and precise matched Twiss parameters reduce these oscillations, stabilizing the beam onto the closed orbit [11]. (iii) Polarization preservation: A spin tilt beyond ± 20 mrad could introduce systematic errors [3,7,8]. Our design preserves vertical spin within this tolerance, drawing on techniques from the muon $g-2$ experiment [22].

This paper describes the design of CW/CCW BtP injection line that meets both optical matching and spin direction preservation requirements, enabling a highly stable beam environment for the counterrotating beam technique, a cornerstone of the pEDM experiment’s ability to distinguish the EDM signal from background effects. By reducing injection-induced errors, stabilizing closed orbits, and ensuring robust systematic error cancellation, the BtP injection line design approach not only addresses longstanding limitations in EDM measurements but also paves the way for exploring physics beyond the Standard Model, potentially revealing new sources of CP violation critical to understanding matter-antimatter asymmetry in the universe.

ACKNOWLEDGMENTS

This work was supported by U.S. Department of Energy under Grant No. DE-SC0012704.

DATA AVAILABILITY

The data that support the findings of this article are openly available [25].

- [1] M. Pospelov and A. Ritz, Electric dipole moments as probes of new physics, *Ann. Phys. (Berlin)* **318**, 119 (2005).
- [2] J. Engel, M. J. Ramsey-Musolf, and U. van Kolck, Electric dipole moments of nucleons, nuclei, and atoms: The Standard Model and beyond, *Prog. Part. Nucl. Phys.* **71**, 21 (2013); A. Faessler, Editorial for the special volume: “Fundamental symmetries in the era of the LHC”, *Prog. Part. Nucl. Phys.* **71**, 1 (2013).
- [3] Y. K. Semertzidis, Electric dipole moments of fundamental particles, *Nucl. Phys. B, Proc. Suppl.* **131**, 244 (2004).
- [4] W. Weng and Y. Y. Lee, Design and construction status of the AGS booster accelerator, Report No. BNL-41030, Brookhaven National Laboratory, Upton, NY, 1988.
- [5] C. Bracco, Injection: Hadron beams, in *Proceedings of the CAS-CERN Accelerator School: Beam Injection*,

- Extraction and Transfer, Erice, Italy, School Proceedings* (2018), Vol. 5/2018; CERN-2018-008-SP, <https://e-publishing.cern.ch/index.php/CYRSP/article/view/700>.
- [6] A. Lehrach, S. Martin, and R. Talman (cpEDM Collaboration), Design of a prototype EDM storage ring, *Proc. Sci. SPIN2018* (**2019**) 144.
 - [7] J. Alexander *et al.* (pEDM Collaboration), The storage ring proton EDM experiment, [arXiv:2205.00830](https://arxiv.org/abs/2205.00830).
 - [8] Z. Omarov, H. Davoudiasl, S. Hacıömeroğlu, V. Lebedev, W. M. Morse, Y. K. Semertzidis, A. J. Silenko, E. J. Stephenson, and R. Suleiman, Comprehensive symmetric-hybrid ring design for a proton-EDM experiment at below 10^{-29} e cm, *Phys. Rev. D* **105**, 032001 (2022).
 - [9] V. Anastassopoulos *et al.*, A storage ring experiment to detect a proton electric dipole moment, *Rev. Sci. Instrum.* **87**, 115116 (2016).
 - [10] S. Hacıömeroğlu and Y. K. Semertzidis, Hybrid ring design in the storage-ring proton electric dipole moment experiment, *Phys. Rev. Accel. Beams* **22**, 034001 (2019).
 - [11] A. Wolski, Beam dynamics in high energy particle accelerators, [10.1142/p899](https://arxiv.org/abs/10.1142/p899) (2014).
 - [12] P. J. Bryant and K. Johnsen, *The Principles of Circular Accelerators and Storage Rings* (Cambridge University Press, Cambridge, England, 1993).
 - [13] V. Bargmann, L. Michel, and V. L. Telegdi, Precession of the polarization of particles moving in a homogeneous electromagnetic field, *Phys. Rev. Lett.* **2**, 435 (1959).
 - [14] T. Fukuyama and A. J. Silenko, Derivation of generalized Thomas–Bargmann–Michel–Telegdi equation for a particle with electric dipole moment, *Int. J. Mod. Phys. A* **28**, 1350147 (2013).
 - [15] A. J. Silenko, Equation of spin motion in storage rings in the cylindrical coordinate system, *Phys. Rev. ST Accel. Beams* **9**, 034003 (2006).
 - [16] F. Méot, Zgoubi users’ guide, Technical Report No. BNL–99611-2013-IR; BNL–98726-2012-IR; CA/AP/#470; KB0202011, Brookhaven National Laboratory (BNL), Relativistic Heavy Ion Collider (RHIC), Upton, NY, 2012, [10.2172/1062013](https://arxiv.org/abs/10.2172/1062013).
 - [17] K. Hock, A. Curcio, H. Huang, F. Méot, J. Jamilkowski, R. Michnoff, P. Oddo, M. Paniccia, V. Schoefer, N. Tsoupas, J. Tuozzolo, and K. Zeno, Commissioning results of the BNL alternating gradient synchrotron booster AC dipole, *Nucl. Instrum. Methods Phys. Res., Sect. A* **1059**, 168999 (2024).
 - [18] F. Méot, P. Adams, H. Huang, J. Kewisch, P. Oddo, and T. Roser, Numerical simulation of RHIC polarized proton run 17 spin flipper experiments, *Phys. Rev. Accel. Beams* **27**, 071002 (2024).
 - [19] K. L. Brown, A first and second order matrix theory for the design of beam transport systems and charged particle spectrometers, *Adv. Part. Phys.* **1**, 71 (1968), <https://inspirehep.net/literature/187522>.
 - [20] AGS booster design manual, Brookhaven National Laboratory, Upton, NY, 1986, <https://www.bnl.gov/cad/accelerator/docs/pdf/boosterdesignmanual.pdf>.
 - [21] F. J. M. Farley, K. Jungmann, J. P. Miller, W. M. Morse, Y. F. Orlov, B. L. Roberts, Y. K. Semertzidis, A. Silenko, and E. J. Stephenson, New method of measuring electric dipole moments in storage rings, *Phys. Rev. Lett.* **93**, 052001 (2004).
 - [22] J. Grange *et al.*, Muon ($g - 2$) technical design report, [arXiv:1501.06858](https://arxiv.org/abs/1501.06858).
 - [23] K. Hock, BtA line MAD-X input files (private communication).
 - [24] J. Lee, W. Morse, F. Meot, Y. Semertzidis, Z. Omarov, and S. Hacıomeroglu, Vertical quadrupole electric field systematics and its mitigation in the proton-EDM ring, in *Proceedings of the IPAC-2024* (JACoW, Geneva, Switzerland, 2024), [10.18429/JACoW-IPAC2024-TUPS36](https://arxiv.org/abs/10.18429/JACoW-IPAC2024-TUPS36).
 - [25] J. Lee, pEDM injection line data, GitHub repository, 2025, https://github.com/jlee-2025/pEDM_injection_line_data.