

Machine learning based closed orbit correction for steady state microbunching storage ring

Liwei Chen,^{1,*} Zhilong Pan¹,^{*} Xiujie Deng,² Zizheng Li,¹ Xiaoyang Zhang¹,^{*} Wenhui Huang,¹ Alexander Chao,^{2,3} and Chuanxiang Tang^{1,†}

¹*Department of Engineering Physics, Tsinghua University, Beijing 100084, China*

²*Institute for Advanced Study, Tsinghua University, Beijing 100084, China*

³*Stanford University, Stanford, California 94309, USA*



(Received 24 March 2025; accepted 3 March 2026; published 17 March 2026)

Closed orbit correction is the initial step in error correction, and its results significantly influence subsequent correction of higher-order errors. This is particularly important for an steady state microbunching (SSMB) storage ring, where both the transverse dimensions approach the diffraction limit and the longitudinal length is on the order of hundreds of nanometers. The precision and speed of closed orbit correction are essential for achieving high average power and high brightness radiation in the SSMB storage ring. Here, we introduce a new model to address this issue. Our model integrates the attention mechanism with convolutional neural network (CNN). This combination enables the model to capture both global dependencies and local patterns. Furthermore, we considered the effects of various error distributions on the lattice optics function. By incorporating diverse error scenarios into the training dataset, we enhance the model's robustness. Through simulations of the SSMB storage ring, we compared the correction results of our model with the singular value decomposition method. It achieves the desired residual closed orbit after only two iterations and shows encouraging correction outcomes across various error distributions. We tested the stability of our model by adjusting the strengths of specific correctors and incorporating beam position monitor (BPM) noise. Additionally, to address the issue of BPM failures, we integrated a Gaussian process model to ensure that our correction process remains effective even when BPMs malfunction.

DOI: [10.1103/h677-zjdg](https://doi.org/10.1103/h677-zjdg)

I. INTRODUCTION

The synchrotron light source is one of the most powerful tools in modern science and technology. It has already evolved from first-generation to fourth-generation, and the transverse emittance of the beam has been progressively reduced to the diffraction limit to provide highly brilliant and stable synchrotron radiation. Notably, in the steady state microbunching (SSMB) [1–4] storage ring scheme, the beam exhibits not only a small transverse emittance but also a very short longitudinal bunch length, which presents the potential to deliver a high-power light source for extreme ultraviolet lithography. To achieve high brightness and high average power, it is essential to ensure high orbit stability to meet the demands of these light sources. In

particle accelerators, the beam orbit inevitably deviates from the ideal orbit due to magnet errors and alignment errors, resulting in closed orbit distortion (COD). These distortions can adversely affect the beam dynamics, degrade the beam quality and lifetime, and limit overall light source performance. Therefore, closed orbit correction is fundamental to error correction in the accelerator, and implementing effective closed orbit correction techniques is a critical task for optimizing accelerator operation. Studies on potential schemes for stabilizing the beam in a storage ring were conducted as early as the 1980s [5,6]. The conventional closed orbit correction method employs a response matrix and the singular value decomposition (SVD) algorithm [7,8]. Additionally, the closed orbit correction module in software such as ELEGANT [9] is also based on this approach.

The response matrix represents a linear mapping between the change in COD and the corrector strength (θ). The equation is given by $\Delta X = R\theta$, where R denotes the response matrix, and each element within this matrix is [10]:

$$R_{ij} = \frac{\sqrt{\beta_i \beta_j}}{2 \sin(\pi \nu)} \cos(|\psi_i - \psi_j| - \pi \nu) - \frac{\eta_i \eta_j}{\alpha_c L_0}, \quad (1)$$

*Contact author: clw20@mails.tsinghua.edu.cn

†Contact author: tang.xuh@tsinghua.edu.cn

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](https://creativecommons.org/licenses/by/4.0/). Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

where i represents the index of the beam position monitor (BPM), j indicates the index of the corrector, β represents the beta function, η signifies the dispersion function, α_c is the momentum compaction factor, L_0 refers to the ring circumference, ψ is the betatron phase advance, and ν represents the working point. Additionally, because of the COD, there exists a discrepancy between the circumference of the actual orbit and that of the ideal orbit, leading to longitudinal asynchrony. To address this issue, the second term that includes the dispersion function is necessary to maintain the synchronization condition, meaning that the actual orbit circumference matches that of the ideal orbit.

In a storage ring, the number of BPMs is typically different from the number of correctors. This discrepancy results in a nonsquare matrix R . To process this nonsquare response matrix, the singular value decomposition (SVD) algorithm is employed. By this method, the matrix R can be expressed as $R = USV^T$, where U and V are unitary matrices of dimensions $N \times N$ and $M \times M$, respectively, and S is a diagonal matrix of size $N \times M$. This formulation allows us to calculate the pseudoinverse matrix as $R^{-1} = VS^{-1}U^T$. Consequently, the strength of the correctors can be expressed as

$$\theta = -VS^{-1}U^T \Delta X. \quad (2)$$

Although the SVD method is relatively straightforward, it has certain limitations in practical applications. The response matrix provides only a linear approximation of the relationship between the correctors and the BPMs in the current state of the storage ring. In reality, accelerator systems often exhibit nonlinear behaviors that are not adequately captured by the linear model, leading to inevitable discrepancies between the ideal linear matrix and the measured response matrix. To ensure accurate correction, it is generally necessary to measure the actual response matrix, and the SVD method requires multiple iterations to converge on the ground truth. This makes closed orbit correction a time-consuming and tedious process. To address these limitations, more advanced orbit correction techniques based on machine learning [11–16] have gained attention in recent years. These approaches are better equipped to capture the nonlinear dynamics of accelerators, potentially offering more accurate and robust solutions for orbit correction.

The SSMB storage rings require more precise and rapid closed orbit correction techniques compared to traditional accelerator facilities. In the SSMB storage ring, the beam not only achieves a diffraction-limited condition in the transverse direction but also possesses a very short longitudinal length of less than 100 nm. Traditional rf modulation has been replaced by a laser, while the modulation wavelength is 1 μm . Furthermore, the momentum compaction factor of the ring is extremely small, on the order of 1×10^{-6} . Therefore, precise closed orbit control is crucial

to ensure longitudinal synchronization. Due to the extreme parameters of the SSMB storage ring, the impact of errors on the optics functions is particularly pronounced.

The measured response matrix consists of three components. The primary component is the linear response of the ideal model itself. The other two components result from the inherent nonlinearities of the magnets and the additional effects caused by the nonzero closed orbit at the locations of the quadrupole and sextupole magnets due to various errors. In the context of our SSMB storage ring, the introduction of errors significantly exacerbates the CODs. The additional effects generated at the quadrupole and sextupole magnet locations cause nonlinear discrepancies between the measured response matrix and the ideal response matrix.

Our objective is to develop an algorithm for rapid closed orbit correction that effectively addresses errors associated with magnets, including magnet strength, alignment, and rotation. To achieve this, we utilized the encoder component of the transformer [17] architecture, supplemented by a convolutional neural network [18] for feature extraction. This approach enabled us to train a model capable of executing rapid and precise corrections for COD resulting from various types of errors.

This paper is organized as follows. Section II introduces the architecture of the model employed in this study, along with the theoretical foundation of the multihead self-attention. Section III details the hyperparameter settings and training results, as well as a comparison of our model with the SVD method regarding closed orbit correction. We conducted tests to evaluate the stability of our model. Additionally, we addressed the impact of faulty BPMs on the correction process by incorporating an additional Gaussian process model. Section IV summarizes our work and outlines future perspectives.

II. METHODS

From Eq. (1), we can see that COD differs from typical sequential data in several key characteristics. First, COD represents a stable periodic solution, indicating that errors will impact both downstream and upstream processes. Additionally, COD at various positions is determined by the beta function and the cosine of the phase advance. Notably, COD does not attenuate with distance, indicating that it exhibits characteristics of long-range dependencies. Furthermore, when different error sources contribute to the COD, their effects can combine, resulting in the emergence of localized features in the distortion pattern.

This requires our model not only to capture local features but also to have a stronger capability for global feature extraction. Common models for handling sequential data include recurrent neural networks (RNNs) [19], gated recurrent neural networks [20], long short-term memory networks (LSTMs) [21], and attention mechanism [22–24]. The encoder-decoder architecture [25] is a widely used structure for processing sequential data. The attention

mechanism, particularly transformer, is better suited than RNN and LSTM to capture the global features of sequences, making it more appropriate for feature extraction in long sequences.

In the transformer model, multihead self-attention serves as the core component, ensuring effective global feature extraction from extended sequences. Therefore, we adopted the encoder part of the transformer architecture as the foundational framework of our model. Additionally, we incorporated convolutional layers to enhance the model's ability to extract local features. The integration of convolutional layers with attention models has been explored in [26–28]. By combining the convolutional layers with the transformer's encoder module, we ensure that the model is well-equipped to capture both local and global features, resulting in more reliable closed orbit correction.

The following part will explore the concepts of self-attention and multihead attention. Subsequently, we will outline the overall framework of our network, detailing the flow of data through the network and the changes in dimensions at each stage.

A. Self-attention

Self-attention is a particular type of attention mechanism [29]. The attention mechanism employs key-value pairs to map a query sequence to an output. The output is calculated as a weighted sum of the values, with the weights determined by the similarity between the query and the corresponding keys.

The compatibility between the queries and the keys represents the concept of attention. This method allows the model to focus on the relevant parts of the input data and adjust its output based on the significance of various elements relative to the query. This enhances the model's ability to identify meaningful patterns and relationships within the data.

There are several methods for calculating attention, including context-based attention, additive attention, dot-product attention, and scaled dot-product attention. In the transformer architecture, scaled dot-product attention is utilized.

To calculate the scaled dot-product attention, the input consists of queries Q and keys K of dimension d_k , and values V of dimension d_v . For instance, in the context of closed orbit correction, Q represents the closed orbit distortion that needs to be corrected, while K and V denote the closed orbit distortions and their corresponding corrector strengths in the dataset, respectively.

First, compute the dot products of the queries with all keys, which are then divided by the scaling factor $\sqrt{d_k}$ to prevent excessively large values. Next, the softmax function is applied to normalize these scores, yielding the attention weights corresponding to each key. Finally, these weights are utilized to perform a weighted sum of the values, resulting in the output of the attention mechanism.

$$\text{Attention}(Q, K, V) = \text{softmax}\left(\frac{QK^T}{\sqrt{d_k}}\right)V. \quad (3)$$

In the attention mechanism, key-value pairs must be provided as additional inputs. In this context, the key-value pairs correspond to the existing dataset of COD and their associated corrector strengths. When a new COD (query) arises, we can compute attention relative to the existing COD and correction magnet setting to derive the corresponding output for the query. However, this typically necessitates storing a sufficiently large number of key-value pairs to ensure accurate predictions for any new query.

To eliminate the need to provide additional key-value pairs, self-attention—sometimes referred to as intra-attention—generates queries, keys, and values by multiplying the input sequence by three weight matrices (W^Q, W^K, W^V).

This process enables the model to compute attention within the input sequence itself. Following the application of attention calculations, the model can effectively learn the internal features and the inherent relationships in the sequence.

To learn more nuanced features, multihead attention introduces multiple sets of weight matrices (W_i^Q, W_i^K, W_i^V), resulting in several attention heads. Each head can focus on different aspects of the relationships within the sequence, enabling the model to capture more complex features. By aggregating the output of these multiple heads, the model can achieve a richer representation of the input data, thereby enhancing its ability to understand and process intricate patterns and dependencies across the sequence.

In our model, multihead self-attention plays a crucial role in accurately capturing the relationships between different BPM readings, particularly in addressing long-range dependencies and global features, ultimately enhancing the model's ability to perform precise closed orbit corrections.

B. Improvement based on transformer

Figure 1 illustrates the structure of our model. In the process of decoding the encoded BPM data to obtain the corrector strength, we do not utilize the inherent decoder structure of the transformer. Instead, we employed a straightforward decoding method. We directly passed the output from the encoder through two layers of a feedforward neural network to achieve the decoding and final output.

Although the attention mechanism effectively captures the global features of COD, it may not adequately emphasize the periodic characteristics and local features. To address this limitation, we employ a convolutional layer to extract its local features. By adding the processed values back to the original input data, we enhance the representation of periodic and local features within the sequence. Subsequent experiments have shown that our model can indeed sensitively detect local features within the sequences.

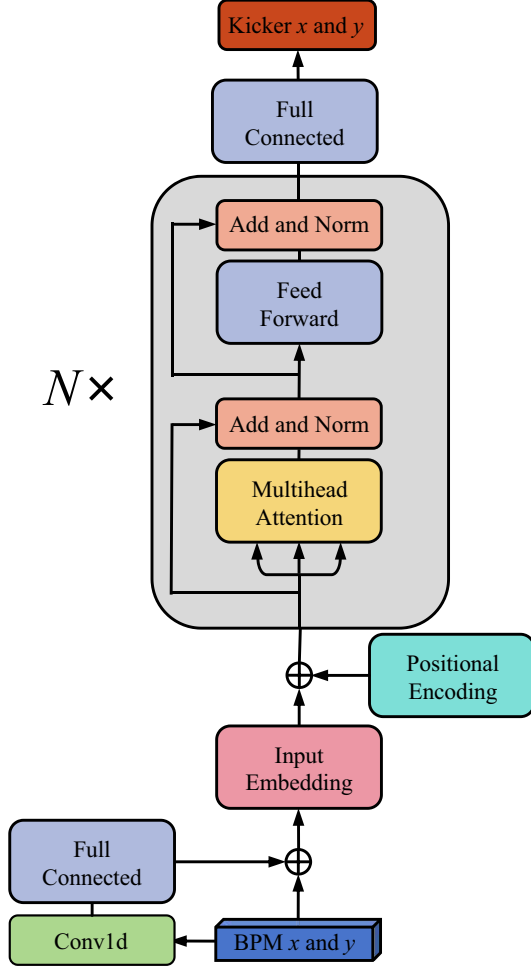


FIG. 1. Structure of our model. The core component of our model consists of a convolutional layer and an encoder block derived from the transformer architecture. The BPM readings first undergo a one-dimensional convolutional layer, where local features are extracted. These extracted features are then summed with the original data before being passed into the embedding layer. Following this, the model employs multihead self-attention layers. Finally, the processed information is decoded through a fully connected layer to produce the output of corrector strengths.

The input consists of BPM data with dimensions of (N_b, L_{in}, d) , where N_b represents the batch size, L_{in} denotes the number of BPMs used for closed orbit correction, and d indicates the number of features. Since the BPM data encompass both the x direction and the y direction, d is set to 2. Before embedding the input data, it first passes through a one-dimensional convolutional layer designed to extract local features and periodic features.

This convolutional layer employs two convolutional kernels of size 1×3 and two of size 1×5 . The padding method used is circular padding, with padding sizes of 1 and 3, respectively, ensuring that the output sequence length remains unchanged. The output of the convolutional layer has dimensions of $(N_b, L_{in}, 2d)$. After this, the array is flattened, resulting in dimensions of $(N_b, L_{in} \times 2d)$.

Subsequently, the flattened array is sent into a fully connected layer, transforming the dimensions to $(N_b, L_{in} \times d)$. Then, this output is reshaped back to (N_b, L_{in}, d) and added to the original input. This facilitates the integration of the extracted features with the initial data, allowing the model to capture both global and local information about the COD.

Next, the input data, which now incorporate both periodic and local features, are passed through the input embedding layer. This layer maps the low-dimensional data into a higher-dimensional space, resulting in dimensions of (N_b, L_{in}, D) . To facilitate the attention mechanism in extracting positional information, we add positional encoding of the same dimension to the embedded input. This positional encoding ensures that the model can effectively recognize the order of the sequence elements, thus enhancing its understanding of the relationships among them.

Similar to the structure of the transformer, our model also incorporates N multihead self-attention blocks. We set the number of heads to h , and reshape the input data dimensions to $(N_b, L_{in}, h, D/h)$. Each head processes the input through three fully connected layers, each with dimensions $(D/h, D/h)$ to obtain the Q_i , K_i , and V_i matrices. The attention is then computed, and the outputs from different heads are concatenated, resulting in an output with the dimension of (N_b, L_{in}, D) . This output is subsequently passed through a fully connected layer with dimensions (D, D) , added to the initial input via a residual connection [30], and subjected to layer normalization [31].

Following this, the data are processed through two layers of feedforward neural network with dimensions (D, nD) and (nD, D) , respectively, to extract features. Finally, a residual connection and another layer normalization are applied, resulting in an output dimension of (N_b, L_{in}, D) .

In the decoder part, the output is processed through two layers of neural networks. The first layer has dimensions $(D, 1)$, transforming the input data dimensions to $(N_b, L_{in}, 1)$. The second layer has dimensions (L_{in}, L_{out}) , resulting in an output dimension of (N_b, L_{out}) , which corresponds to the predicted corrector strengths.

In our model, the input length L_{in} is set to 32 with a dimension $d = 2$. The output length L_{out} is 64 and batch size N_b is 32. The embedding dimension D is set to 64, with $h = 8$ heads in the multihead attention mechanism. The model comprises encoder blocks $N = 6$, the dimension of the fully connected layer is $nD = 256$, and the final fully connected output dimension is also 64.

III. RESULTS AND DISCUSSION

A. Data acquisition and training result

Training data are essential to train the machine learning model, and the quality of these data directly influences the model performance. To enable rapid and accurate corrections for COD caused by magnet errors, we must collect a

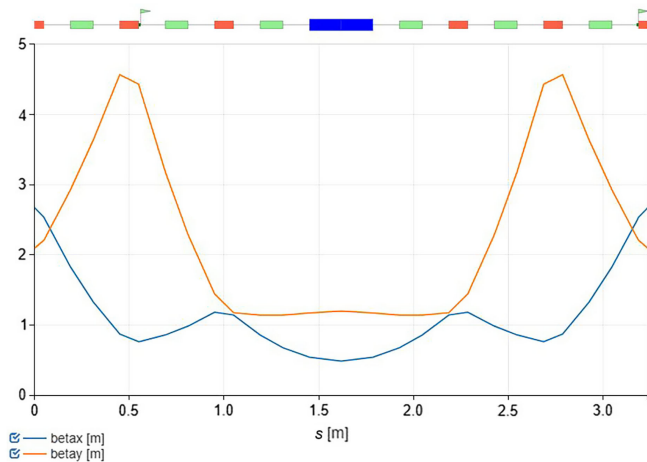


FIG. 2. An example layout of the SSMB superperiod cell and its corresponding Twiss parameters. The blue, orange, and green boxes represent dipoles, quadrupoles, and sextupoles, respectively. The green flags represent BPMs. The deep green boxes adjacent to the green flags represent the correctors.

substantial amount of data on COD generated by various random errors within the range of magnet errors, along with the corresponding corrector strengths required for correction.

Due to limitations in the linear method, the SVD approach may not reliably achieve stable closed orbit correction, particularly when errors are significant. To ensure the reliability of our dataset, we record only data for which the residual closed orbit after correction is less than 1 nm. At this threshold, we can consider the corresponding set of corrector strengths as the optimal solution for the given magnetic error. This approach guarantees that the data we use for training our model are accurate and effective in addressing the complexities of COD.

We utilize the simulation software ELEGANT to generate the dataset. ELEGANT allows us to introduce various errors to each magnet in the storage ring and compute the corresponding COD. Additionally, it can iteratively solve the corrector strengths using the SVD method.

Figure 2 illustrates the layout of the SSMB superperiod cell and its optics function. The SSMB storage ring consists of 14 of these superperiod cells.

We introduce alignment errors and relative strength errors to all 24 dipoles and 48 quadrupoles in the SSMB storage ring using ELEGANT. The magnitudes of these errors are detailed in Table I. When generating the training dataset, BPM noise was not introduced.

When generating various error distributions, we utilize different random seeds for each set of errors. This method ensures that each data group is as distinct as possible from the others, thus minimizing similarities among the datasets. Furthermore, we ensure that the data in the training set and the validation set are different, which is crucial for accurately evaluating the model performance.

TABLE I. Error settings for all dipoles and quadrupoles. All errors conform to a Gaussian distribution, with the values indicating the standard deviation of this distribution.

Element type	Error type	Value
Dipole	Fractional strength error	1×10^{-4}
Dipole	Roll error	0.1 mrad
Quadrupole	Fractional strength error	1×10^{-4}
Quadrupole	Misalignment (x/y)	10 $\mu\text{m}/10 \mu\text{m}$
Quadrupole	Roll error	0.1 mrad
BPM	BPM noise	1 μm

In the SSMB storage ring, 32 BPMs and 32 correctors are used for closed orbit correction. Both the BPMs and the correctors provide two outputs: the x and y signals. Using ELEGANT, we can calculate the COD caused by these errors. A response matrix is then employed to iteratively adjust the corrector strengths. We set a maximum of 10 iterations. The solution for the correction strengths after 10 iterations ensures a very small residual closed orbit (specifically below 1 nm). These corrector strengths represent the optimal solution under the specific errors introduced.

Ultimately, we obtained 40,000 sets of training data, with 30,000 data points designated as the training set and 10,000 data points allocated for the validation set. The distributions of the COD and the corrector strengths are illustrated in Fig. 3.

Under error conditions, the COD can deviate by up to several millimeters. In our injection scheme, the septum is only a few millimeters away from the center. Such a significant orbit distortion can directly lead to considerable beam loss. Therefore, a rapid and effective corrections of these severe CODs are crucial for the stable operation of the SSMB storage ring.

Due to differences in the magnitude of the data collected from ELEGANT, the data cannot be directly used for training. The magnitude of the variations may differ significantly for different BPMs and correctors due to their unique characteristics and states. Therefore, it is essential to perform standardized preprocessing. The formula for the standard scaler is as follows: $X = \frac{x - \mu}{\sigma}$, where μ represents the mean of the samples and σ denotes the standard deviation of the samples. This approach ensures that all training data are reduced to the same range, which minimizes the impact of large variance data, enhances the generalization ability of our model, and optimizes the training result.

We also trained a multilayer perceptron (MLP) model and an attention model without convolutional layers. The MLP consists of six layers, structured similarly to an encoder-decoder architecture. The number of neurons in each layer is as follows: 512, 256, 128, 128, 256, and 512, respectively. We employ the ReLU function as the nonlinear activation function. The hyperparameters for all three models were optimized to achieve optimal performance. The training and testing loss curves are illustrated in Fig. 4.

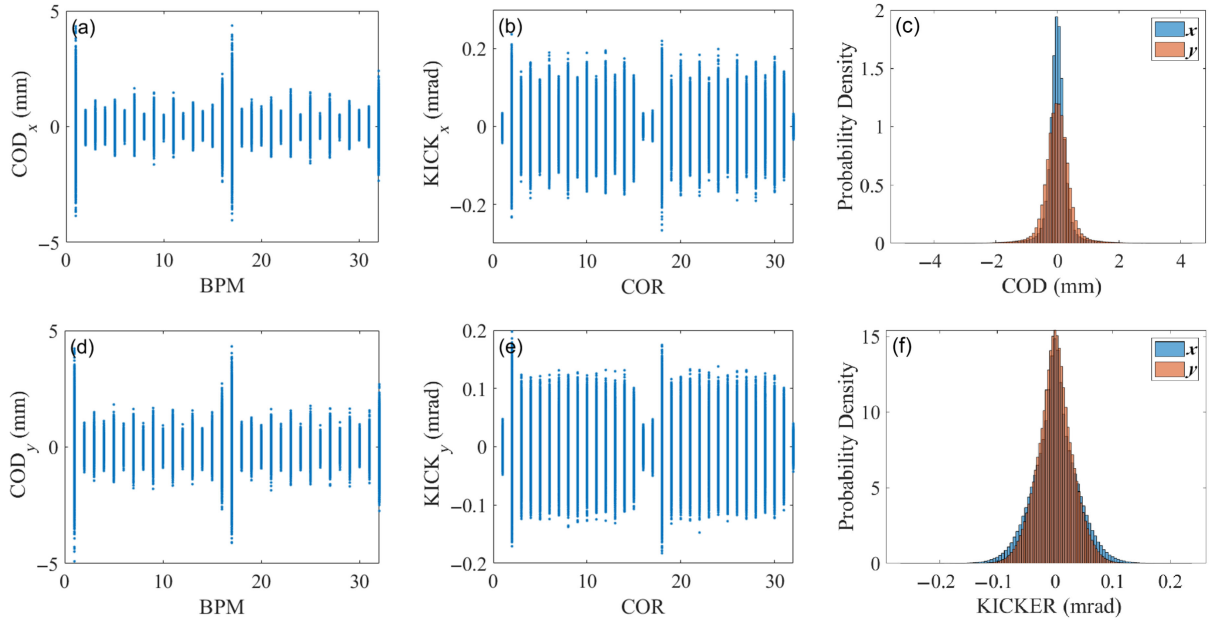


FIG. 3. Distribution of COD and corrector strengths in the dataset. (a) and (d) Depict the COD in the x and y directions. (b) and (e) Present the corresponding optimal corrector strengths. (c) and (f) Display the probability density distributions of the COD and the optimal corrector strengths, respectively.

After 1000 training epochs, all three models have essentially converged, with the model that combines convolutional layers and attention mechanisms demonstrating the best performance. For the simple MLP, the error rapidly converged to approximately 0.3, indicating that its ability to correct the issues encountered by our SSMB storage ring is somewhat limited. This result underscores the need to employ more complex network architectures to effectively address problems involving multiple inputs and outputs with significant nonlinearities. The attention model demonstrates a significant improvement over the simple MLP, indicating its effectiveness in addressing our problem. Comparing the attention model with our combined

convolutional and attention model reveals that incorporating convolutional layers further enhances performance. This improvement confirms the model's ability to extract local features, which is essential for accurately capturing the system's complexities.

For our attention model with convolution layers, the initial learning rate is set at 5×10^{-3} , and it decays by a factor of 0.98 after every 5 iterations. The batch size is 64, the optimizer used is Adam, and the loss function is the mean squared error (MSE). The maximum number of iterations is set to 500. After reaching the maximum number of iterations, we observed that the loss function for both the training and the testing sets continued to fluctuate downward, albeit at a decreasing rate. Consequently, we set the learning rate to 1×10^{-5} , disabled the learning rate decay, and proceeded with an additional 500 training iterations. After 1000 iterations, the MSE loss in both the training and validation sets converged to approximately 2×10^{-4} , with no signs of overfitting.

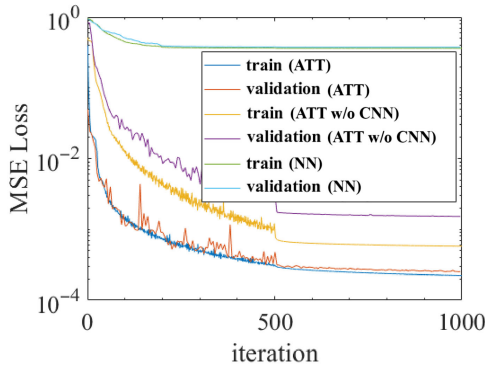


FIG. 4. Evolution of training and validation loss. The top two curves represent the results from the simple MLP, while the middle curves illustrate the performance of the attention model. The bottom curves display the outcomes from the attention model with convolutional layers.

B. Orbit correction simulation on SSMB

After training, we conducted simulations to evaluate the performance of our model. We introduced random errors into the storage ring to obtain the corresponding COD. Using ELEGANT, we performed both single and 10 iteration corrections, while using our model for the single correction. We recorded the corrector strengths and the resulting closed orbit for the three correction settings. The results of the 10 iterations of correction using ELEGANT served as the ground truth for comparison, enabling us to evaluate the deviations

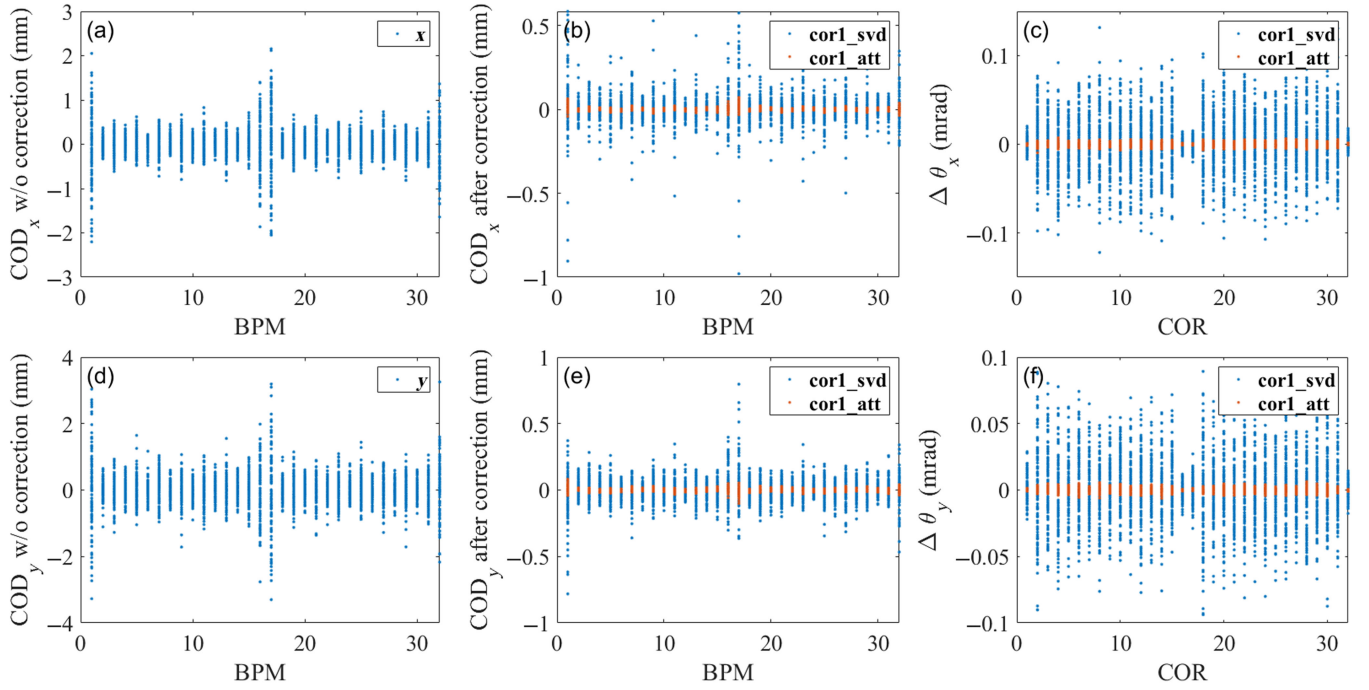


FIG. 5. Results of a single correction over 100 simulations. (a) and (d) Depict the COD before correction. (b) and (e) Show the residual closed orbit after correction, with blue points representing the results from the SVD method and orange points from our model. (c) and (f) Display the deviations between the corrector strengths and the optimal correction strengths.

between the single correction using the SVD method and those of our model.

To ensure reliability, we used a different random seed from those used for the previous dataset, while maintaining error magnitudes consistent with dataset generation. We conducted 100 simulations to test our model, incorporating BPM noise in the simulations. The results are illustrated in Fig. 5.

Compared to the COD before correction, our model significantly reduced the residual closed orbit after a single correction. In particular, the correction results obtained from our model outperform those from the SVD method by approximately an order of magnitude. This improvement is also evident when examining the deviations of the corrector strengths relative to the optimal solution, where the results produced by our model are clearly closer to the ideal values than those from the SVD method. This indicates that our model not only achieves superior performance in reducing COD but also enhances the precision of the correction process.

To ensure that the residual COD has a variation of less than 10% compared to the beam size, we performed an additional correction based on the initial adjustment. To estimate the beam size at different positions, we utilized the parameter $\sqrt{\epsilon\beta}$, where ϵ is the emittance of the beam and β is the beta function at the specified position. We then compared the results of two iterations of correction using the SVD method with the results obtained by using our model.

The results of performing two iterations of correction are presented in Fig. 6 and Table II. After two iterations of correction, the result from our model remains an order of magnitude smaller than those obtained from the SVD method. From Table II, it is evident that the results obtained from two iterations of the SVD method are only comparable to those achieved by a single iteration of our model. This highlights the efficiency of our approach, as it achieves similar or better correction outcomes without the need for extensive iterative corrections. Furthermore, following the two iterations of correction, the maximum closed orbit for the model is less than 10 μm , and the size of the closed orbit at each BPM position varies by less than 10% relative to the beam size at those positions, meeting the required specifications.

To ensure that the convolutional layers effectively contribute to the model's performance, we conducted an experiment in which we disabled the convolutional layers. We then compared the deviations between the predictions and the true values with those obtained when the convolutional layers were active. From Fig. 7, it is evident that disabling the convolutional layers leads to a significant increase in the deviation between the model's output and the true values. This observation further confirms that the convolutional layers effectively enhance the model's ability to extract local features, which is crucial for accurately capturing the underlying patterns in the data.

By integrating the attention mechanism with convolutional layers, our network achieved superior training performance compared to both a simple MLP and a basic attention

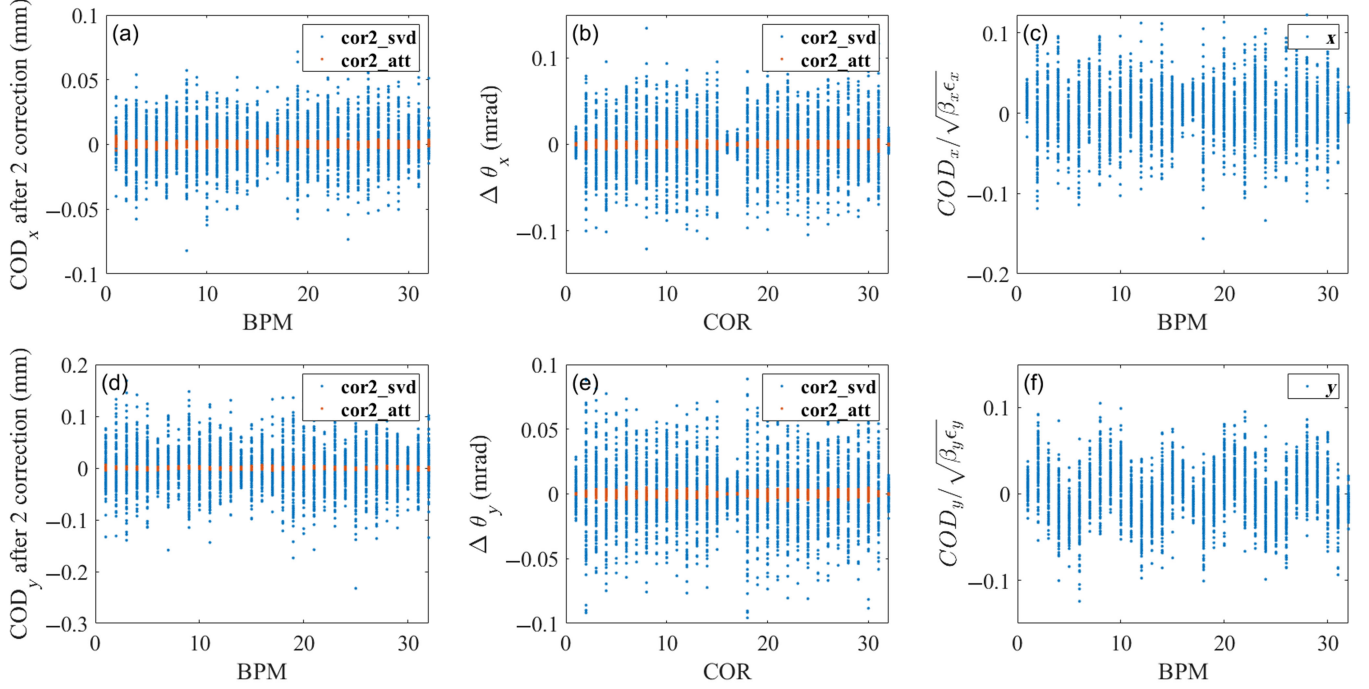


FIG. 6. Results of two iterations of correction over 100 simulations. (a) and (d) Show the residual closed orbit after correction. (b) and (e) Display the deviations between the corrector strengths and the optimal correction strengths. (c) and (f) Illustrate the relative bunch size variation predicted by our model.

model. In cases of COD correction, our model demonstrates better results than traditional methods, achieving calibration objectives more quickly and producing corrector strengths closer to optimal values. This improvement is expected to contribute positively to the long-term stable operation of the storage ring.

C. Test of model stability

For long-term operation of storage rings, the stability of correction algorithms is especially critical. We primarily consider the impact of BPM noise on correction results and the potential for corrector saturation during long-term operations.

In the SVD method, retaining smaller eigenvalues makes the corrections more susceptible to BPM noise. This effect can generally be mitigated by reducing the number of retained eigenvalues. However, selecting too few eigenvalues can reduce the overall correction effectiveness.

Therefore, it is essential to strike a careful balance in choosing the number of eigenvalues to retain.

Furthermore, during prolonged correction processes, the strengths of the correctors can continuously accumulate, potentially leading to saturation in certain correctors and thereby affecting subsequent correction outcomes. Our previous simulation results also indicate that the solutions obtained using the SVD method significantly deviate from the ideal solutions. This suggests that the SVD approach does not fully utilize the most effective correctors. Consequently, each correction iteration adjusts the strengths of all correctors, increasing the likelihood of saturation over time. If the model can focus on utilizing only the most effective correctors at each correction step, it can ensure optimal correction performance while simultaneously preventing strength accumulation and saturation in other correctors.

To assess the impact of BPM noise, we generated 100 sets of closed orbit data. For each set, we repeated the correction process 100 times, introducing a 1 μm error into

TABLE II. Peak-to-peak values of the COD observed at all BPMs, along with the differences between the corrector strengths and the optimal strengths.

Peak-to-peak value	COD _x	COD _y	$\Delta\theta_x$	$\Delta\theta_y$
Before correction	4.36 mm	6.55 mm	...	...
Single iteration of SVD	1.56 mm	1.58 mm	0.254 mrad	0.183 mrad
Two iterations of SVD	0.154 mm	0.405 mm	0.256 mrad	0.184 mrad
Single iteration of our model	0.113 mm	0.183 mm	16.9 μrad	12.6 μrad
Two iterations of our model	11.4 μm	11.3 μm	14.1 μrad	11.4 μrad

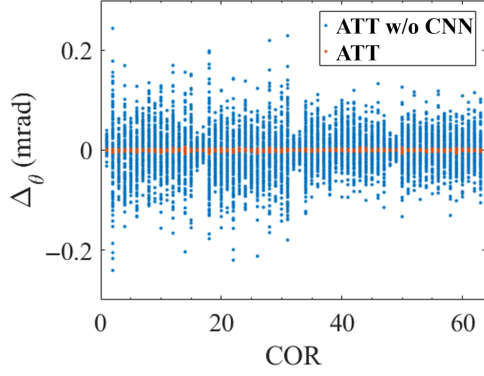


FIG. 7. Deviation between the predicted corrector strengths generated by the model and the true values.

the BPM readings. We then calculated the standard deviation of the corrector strengths resulting from the BPM noise for each closed orbit. The standard deviations of our model's outputs were compared to those obtained using the SVD method. The SVD method utilized half of the available eigenvalues, a selection that typically does not significantly affect correction effectiveness while simultaneously enhancing noise robustness.

The results, presented in Fig. 8, show that our model demonstrates noise robustness at least comparable to the SVD method and, in certain scenarios, outperforms it. Compared to the SVD method, our model not only achieves better correction results but also maintains robustness without any degradation in performance.

To validate our model's ability to prevent corrector saturation, we randomly assigned some correctors a strength of 0.1 mrad based on a Gaussian distribution, while other correctors were set to zero. The resulting closed orbit corresponds to the most effective correctors, which are those with nonzero strengths. We evaluated the model's ability to prevent corrector saturation by calculating the standard deviation of each corrector's output relative to the

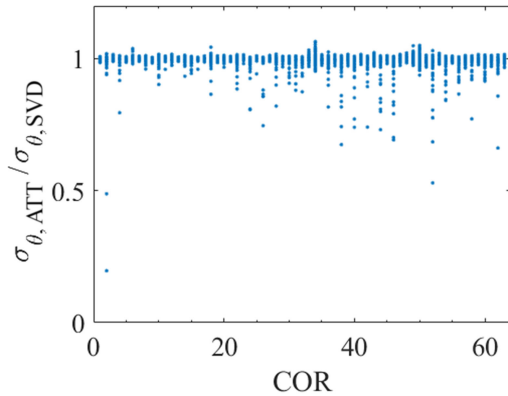


FIG. 8. Comparison between our model and the SVD method by applying random BPM noise to 100 sets of closed orbits. A ratio of less than 1 indicates that our model demonstrates greater noise resistance than the SVD method.

true values for both our model and the SVD method. We conducted tests by randomly selecting 10%, 50%, and 90% of the correctors, with the results shown in Fig. 9.

The findings indicate that our model's predictions for all correctors are closer to the true values compared to the SVD method. Specifically, when using 10% of the correctors, the standard deviation of our model's predictions is approximately 0.03 mrad, significantly lower than the 0.1 mrad standard deviation of the corrector strengths. When using 90% of the correctors, our model still demonstrates superiority over the SVD method. This finding confirms that our model consistently selects the most effective correctors for each correction, thereby avoiding saturation issues associated with other correctors.

D. Test of faulty BPMs

In large facilities, numerous BPMs and correctors are often utilized. Therefore, failures of BPMs or correctors become relatively common occurrences. When a BPM or a corrector fails, it implies that some of our inputs become unknown, or certain outputs must be forced to zero. Since the input and output of our model are fixed, any failure in BPMs or correctors means we may need to retrain the network, which can be time-consuming and inefficient. In contrast, the SVD method offers a more straightforward solution: it only requires the removal of the corresponding rows (for BPM failures) or columns (for corrector failures) from the response matrix. We can then compute the new inverse matrix, allowing for a convenient way to address the fault issue.

To address the issue of BPM failures, we use readings from operational BPMs to estimate the values of the faulty ones. In this context, we employ a Gaussian process (GP) model to accomplish this task. GP models are widely used in accelerators [32–34], due to their advantages, including requiring minimal training parameters and enabling rapid training. The principle of GP is based on the assumption that all data come from an infinite-dimensional Gaussian distribution. Consequently, the inputs and outputs of new data follow the same Gaussian distribution as those of the existing data. This allows us to leverage the properties of Gaussian distributions to make predictions about unknown outputs.

In our approach, we randomly selected 2000 data points from our dataset to serve as known data for the GP model. When a BPM failure occurs, we use this dataset along with readings from the functioning BPMs to predict the readings of the faulty BPMs. We tested scenarios involving a single BPM, as well as cases where 10% and 50% of the BPMs failed. Using the GP model, we predicted the readings for the faulty BPMs. The updated BPM values were then input into our network to determine the corrector strengths.

The simulation results are shown in Fig. 10. It is evident that the GP model provides an excellent fit for the readings of the faulty BPMs, and the corrected BPM readings have minimal impact on the overall correction effectiveness.

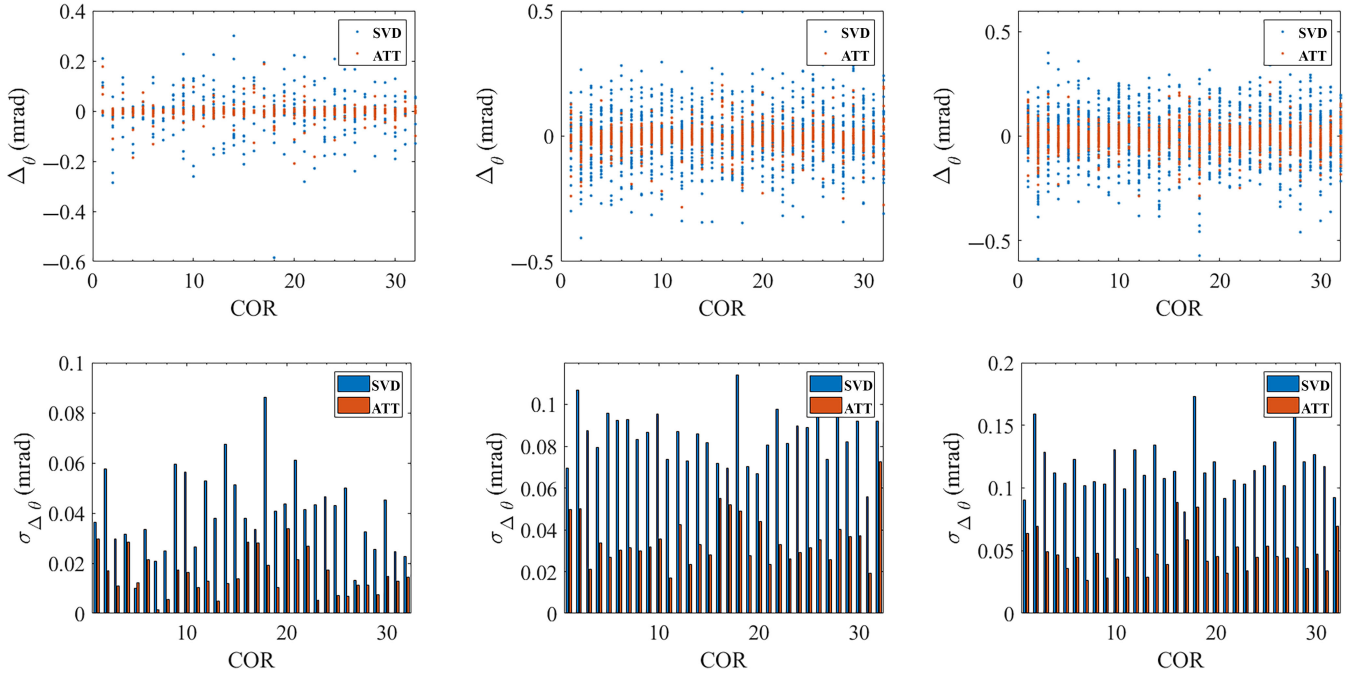


FIG. 9. Results when the closed orbit is generated by a subset of correctors. The left column shows the deviations between the corrector strengths obtained from different methods and the true values, along with the standard deviations of these deviations when CODs are caused by 10%, 50%, and 90% of the correctors.

Training of the GP model is particularly fast, as it requires optimizing only one distance parameter, completing the training in just a few seconds. By integrating the GP model with our correction approach, we can effectively

handle BPM failures. However, the situation becomes more complex when dealing with corrector failures, for which there is currently no quick or suitable method to resolve the issue.

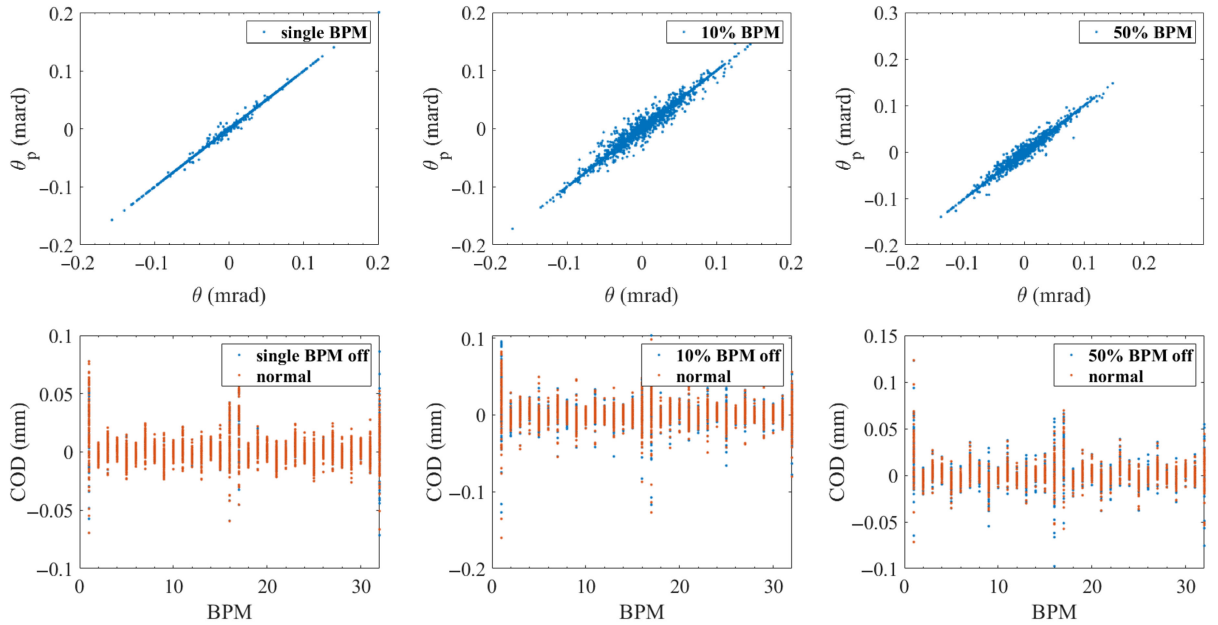


FIG. 10. Results of using the GP model to fit faulty BPMs. The top row shows the predicted values from the GP model and the actual BPM readings. The bottom row shows the correction results using the predictions from the GP model and actual BPM readings. The three columns correspond to single, 10%, and 50% faulty BPMs.

IV. CONCLUSIONS

Both the accuracy and speed of the closed orbit correction significantly impact the performance of the storage ring. When confronted with various error distributions, the SVD method requires a remeasurement of the response matrix, which greatly limits the speed of correction. This paper proposes a closed orbit correction model based on an attention mechanism and a convolutional layer that can effectively address a wide range of errors. By integrating a convolutional layer into the encoder module of the transformer architecture, our model not only captures long-range dependencies within the sequence but also enhances its performance on local features.

In the simulation of closed orbit correction, our model demonstrates an order of magnitude improvement over the SVD method after a single correction, without the need for actual measurements of the response matrix. After two iterations of correction, our model successfully reduces closed orbit distortions to the level of 10 μm , with disturbances in the bunch size below 10%, which is sufficient to meet our requirements.

The BPM noise experiment demonstrated that our model not only maintains excellent correction performance but also exhibits slightly better noise resilience compared to the SVD method. This finding highlights our model's ability to effectively handle measurement uncertainties while still delivering accurate corrections.

The corrector saturation experiment demonstrated that our model can effectively utilize the most effective correctors in each correction iteration, thereby preventing the accumulation of strengths in less effective correctors. This capability highlights our model's proficiency in extracting local features, enabling it to optimally leverage the most effective correctors without risking saturation.

By integrating the Gaussian process model, we have overcome the limitation of fixed input parameters in the neural network model, enabling us to effectively address the issue of BPM failures. This enhancement lays the foundation for broader and more stable applications of our model. However, the issue of corrector failures remains a challenge, and continued research is essential.

ACKNOWLEDGMENTS

This work is supported by the National Key R&D Program of China (Grant No. 2022YFA1603400), the National Natural Science Foundation of China (NSFC Grant No. 12035010), and the Tsinghua University Initiative Scientific Research Program.

DATA AVAILABILITY

The data that support the findings of this article are openly available [35].

- [1] D. F. Ratner and A. W. Chao, Steady-state microbunching in a storage ring for generating coherent radiation, *Phys. Rev. Lett.* **105**, 154801 (2010).
- [2] Y. Jiao, D. F. Ratner, and A. W. Chao, Terahertz coherent radiation from steady-state microbunching in storage rings with x-band radio-frequency system, *Phys. Rev. ST Accel. Beams* **14**, 110702 (2011).
- [3] C. Tang, A. W. Chao, X. Deng, J. Feikes, C. Feng, E. Granados, A. Hoehl, W. Huang, B. Jiang, J. Li, D. Ratner, M. Ries, T. Rui, and X. Wang, An overview of the progress on SSMB, in *Proceeding of 60th ICFA Advanced Beam Dynamics Workshop (FLS'18)*, Shanghai, China (JACoW, Geneva, Switzerland, 2018), pp. 166–170, [10.18429/JACoW-FLS2018-THP2WB02](https://doi.org/10.18429/JACoW-FLS2018-THP2WB02).
- [4] X. Deng, A. W. Chao, J. Feikes, A. Hoehl, W. Huang, R. M. Klein, A. Kruschinski, J. Li, A. Matveenko, Y. Petenev, M. Ries, C. Tang, and L. Yan, Experimental demonstration of the mechanism of steady-state microbunching, *Nature (London)* **590**, 576 (2021).
- [5] R. Nagaoka, Machine stability and orbit correction in ELETTRA, in *Proceedings of the 5th European Particle Accelerator Conference, Barcelona, Spain* (IOP, Bristol, 1996).
- [6] R. J. Nawrocky, J. W. Bittner, L. Ma, H. M. Rarback, D. P. Siddons, and L. H. Yu, Automatic beam steering in the NSLS storage rings using closed orbit feedback, *Nucl. Instrum. Methods Phys. Res., Sect. A* **266**, 164 (1988).
- [7] Y. Chung, G. Decker, and K. Evans, Closed orbit correction using singular value decomposition of the response matrix, in *Proceedings of the 15th Particle Accelerator Conference, PAC-1993, Washington, DC, 1993* (IEEE, New York, 1993), Vol. 3, pp. 2263–2265.
- [8] Y. Chung, G. Decker, K. Evans, J. Safranek, I. So, Y. Tang, W. Corbett, and R. Hettel, Global dc closed orbit correction experiments on the NSLS x-ray ring and spear, in *Proceedings of the 15th Particle Accelerator Conference, PAC-1993, Washington, DC, 1993* (IEEE, New York, 1993), Vol. 3, pp. 2275–2277.
- [9] M. Borland, ELEGANT: A flexible SDDS-compliant code for accelerator simulation, [10.2172/761286](https://doi.org/10.2172/761286) (2000).
- [10] A. W. Chao, *Lectures on Accelerator Physics* (World Scientific, Singapore, 2020).
- [11] E. S. Bozoki and A. Friedman, Neural networks and orbit control in accelerators, in *Proceedings of the 4th European Particle Accelerator Conference, EPAC 94, London, England* (World Scientific, River Edge, NJ, 1994).
- [12] E. Meier, G. LeBlanc, and Y. E. Tan, Orbit correction studies using neural networks, in *Proceedings of 3rd International Particle Accelerator Conference (IPAC'12)* (JACoW, Geneva, Switzerland, 2012), pp. 2837–2839, <https://proceedings.jacow.org/IPAC2012/papers/weppp057.pdf>.
- [13] Y. Bai, Y. Wei, W. Liu, G. Xu, and J. Wang, Research on the slow orbit feedback of BEPCII using machine learning, *Radiat. Detect. Technol. Methods* **6**, 179 (2022).
- [14] R. Li, Q. Zhang, B. Jiang, Z. Zhao, C. Li, K. Wang, and D. Huang, A machine-learning based closed orbit feedback for the SSRF storage ring, [arXiv:2212.01010](https://arxiv.org/abs/2212.01010).

- [15] D. Schirmer, Orbit correction with machine learning techniques at the synchrotron light source delta, in *Proceedings of the 17th International Conference on Accelerator and Large Experimental Physics Control, ICALEPCS2019* (JACoW, Geneva, Switzerland, 2020).
- [16] S. C. Leemann, P. Amstutz, W. Byrne, M. Ehrlichman, T. Hellert, A. Hexemer, S. Liu, M. A. Marcus, C. N. Melton, H. Nishimura, G. C. Penn, F. Sannibale, D. A. Shapiro, C. Sun, D. M. Ushizima, and M. Venturini, First attempts at applying machine learning to ALS storage ring stabilization, in *Proceedings of the 10th International Particle Accelerator Conference, IPAC2019, Melbourne, Australia* (JACoW, Geneva, Switzerland, 2019).
- [17] A. Vaswani, N. M. Shazeer, N. Parmar, J. Uszkoreit, L. Jones, A. N. Gomez, L. Kaiser, and I. Polosukhin, Attention is all you need, in *Proceedings of the 31st Conference on Neural Information Processing Systems, NIPS 2017, Long Beach, CA, USA* (Curran Associates, Inc., New York, 2017), <https://papers.neurips.cc/paper/7181-attention-is-all-you-need.pdf>.
- [18] Y. LeCun, Y. Bengio, and G. Hinton, Deep learning, *Nature (London)* **521**, 436 (2015).
- [19] J. L. Elman, Finding structure in time, *Cognit. Sci.* **14**, 179 (1990).
- [20] J. Chung, K. Cho, and Y. Bengio, Empirical evaluation of gated recurrent neural networks on sequence modeling, [arXiv:1412.3555](https://arxiv.org/abs/1412.3555).
- [21] S. Hochreiter and J. Schmidhuber, Long short-term memory, *Neural Comput.* **9**, 1735 (1997).
- [22] F. Informatik, Y. Bengio, P. Frasconi, and J. Schmidhuber, Gradient flow in recurrent nets the difficulty of learning long-term dependencies, in *A Field Guide to Dynamical Recurrent Neural Networks* (Wiley-IEEE Press, Hoboken, NJ, 2003).
- [23] D. Bahdanau, K. Cho, and Y. Bengio, Neural machine translation by jointly learning to align and translate, [arXiv:1409.0473](https://arxiv.org/abs/1409.0473).
- [24] L. Kaiser and S. Bengio, Can active memory replace attention?, in *Neural Information Processing Systems* (Curran Associates, Inc., New York, 2016).
- [25] K. Cho, B. van Merriënboer, C. Gulcehre, D. Bahdanau, F. Bougares, H. Schwenk, and Y. Bengio, Learning phrase representations using RNN encoder–decoder for statistical machine translation, in *Proceedings of the 2014 Conference on Empirical Methods in Natural Language Processing (EMNLP)*, edited by A. Moschitti, B. Pang, and W. Daelemans (Association for Computational Linguistics, Doha, Qatar, 2014), pp. 1724–1734.
- [26] Z. Peng, Z. Guo, W. Huang, Y. Wang, L. Xie, J. Jiao, Q. Tian, and Q. Ye, Conformer: Local features coupling global representations for recognition and detection, *IEEE Trans. Pattern Anal. Mach. Intell.* **45**, 9454 (2023).
- [27] L. Sasal, T. Chakraborty, and A. Hadid, W-transformers: A wavelet-based transformer framework for univariate time series forecasting, in *Proceedings of the 2022 21st IEEE International Conference on Machine Learning and Applications (ICMLA)* (IEEE Computer Society, Los Alamitos, CA, 2022), pp. 671–676, [10.1109/ICMLA55696.2022.00111](https://doi.org/10.1109/ICMLA55696.2022.00111).
- [28] H. Wang, J. Xu, R. Yan, C. Sun, and X. Chen, Intelligent bearing fault diagnosis using multi-head attention-based CNN, *Procedia Manuf.* **49**, 112 (2020).
- [29] V. Mnih, N. M. O. Heess, A. Graves, and K. Kavukcuoglu, Recurrent models of visual attention, in *Neural Information Processing Systems* (MIT Press, Cambridge, MA, 2014).
- [30] K. He, X. Zhang, S. Ren, and J. Sun, Deep residual learning for image recognition, in *Proceedings of the 2016 IEEE Conference on Computer Vision and Pattern Recognition (CVPR)* (IEEE Computer Society, Los Alamitos, CA, 2016), pp. 770–778, [10.1109/CVPR.2016.90](https://doi.org/10.1109/CVPR.2016.90).
- [31] J. L. Ba, J. R. Kiros, and G. E. Hinton, Layer normalization, [arXiv:1607.06450](https://arxiv.org/abs/1607.06450).
- [32] X. Huang, M. Song, and Z. Zhang, Multi-objective multi-generation Gaussian process optimizer for design optimization, [arXiv:1907.00250](https://arxiv.org/abs/1907.00250).
- [33] A. Hanuka, X. Huang, J. Shtalenkova, D. Kennedy, A. Edelen, Z. Zhang, V. R. Lalchand, D. Ratner, and J. Duris, Physics model-informed Gaussian process for online optimization of particle accelerators, *Phys. Rev. Accel. Beams* **24**, 072802 (2021).
- [34] R. Roussel *et al.*, Bayesian optimization algorithms for accelerator physics, *Phys. Rev. Accel. Beams* **27**, 084801 (2024).
- [35] L. Chen, the raw data for training and testing, Tsinghua Cloud (2025), <https://cloud.tsinghua.edu.cn/d/82df2cce6c7b4c8eac6e>.