

Simulations of single-cycle waveform control of undulator radiation

Jenny Morgan¹,* Nicholas Sudar, Claudio Emma², and Agostino Marinelli
SLAC National Accelerator Laboratory, Menlo Park, USA



(Received 16 March 2026; accepted 1 September 2026; published 25 September 2026)

We show that plasma-accelerated electron beams enable single-cycle control of undulator radiation, including control of its polarization and time-frequency structure as described by the Wigner distribution. Using the ultrashort, high-current electron bunch produced by a plasma-wakefield accelerator reported in Emma *et al.* Terawatt attosecond x-ray source driven by a plasma accelerator, APL Photonics **6**, 076107 (2021), we consider the regime in which the resonant radiation wavelength exceeds the effective bunch length, so that emission is coherently generated within each undulator period and follows the undulator magnetic waveform. This allows subcycle control of the electric field through undulator design, effectively turning the undulator into a tunable source of waveform-shaped radiation. Using numerical simulations, we model two compact undulator configurations: a linearly tapered undulator that produces a chirped optical waveform, and an undulator that generates radiation with controlled, cycle-to-cycle polarization variation. Together, these results establish a route to subcycle waveform and polarization control of ultrashort radiation.

DOI: [10.1103/g653-qmwq](https://doi.org/10.1103/g653-qmwq)

I. INTRODUCTION

X-ray free-electron lasers (FELs) can generate intense, coherent radiation down to EUV [1,2] and x-ray wavelengths [3–6] with a peak brightness that surpasses tabletop high-harmonic generation and synchrotron radiation sources by several orders of magnitude. Steady advances in accelerator and undulator design have enabled increasingly sophisticated control over the properties of this radiation [7,8]. Efforts to directly shape the x-ray pulses include schemes to produce two-color radiation [9,10], attosecond pulses and pulse pairs [11–14], and radiation that is spectrotemporally shaped [15,16]. These approaches reflect a continued progression toward more precise tailoring of the radiation field, expanding the reach and versatility of FEL-based light sources.

There is significant interest in using plasma accelerators to provide electron beams to undulator-based light sources, owing to their compact size and ability to produce beams with properties that differ substantially from those achievable with conventional accelerator technology [17,18]. In particular, plasma-wakefield injectors [19,20] (PWFA) can generate beams with strong energy chirps that can be compressed into ultrashort, high-current spikes [21]. The

natural bunching factor of these short spikes has significant contributions at short wavelengths, enabling them to seed FEL interactions and produce high-power (terawatt-level) x-ray radiation in the attosecond regime. Coherent harmonic generation techniques can further reduce the wavelength by driving nonlinear bunching at a higher harmonic [22].

In this paper, we explore how a compressed beam from PWFA can be used to precisely tailor the electric field of the emitted radiation. Through simulations, we show that when the radiation wavelength exceeds the length of the current spike, the electron beam emits coherently within each undulator period. In this regime, the waveform of the emitted field closely replicates the shape of the undulator's magnetic field, allowing subcycle control of the electric field through undulator design, effectively turning the undulator into a tunable source of waveform-shaped radiation. We predict that such fine control over the electric field could enable new classes of experiments requiring attosecond-scale polarization shaping, structured excitation fields, or waveform-resolved light-matter interactions in the extreme ultraviolet and soft x-ray regimes, particularly for probing and controlling ultrafast dynamics in molecular and atomic systems such as light-induced conical intersections [23] and light-driven magnetization dynamics [24].

II. COHERENT RADIATION REGIME OF ULTRASHORT BEAMS

An overview of the setup used throughout this work is shown in Fig. 1, with example results for the tapered-undulator configuration shown here and discussed in detail

*Contact author: jmorgan@slac.stanford.edu

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](https://creativecommons.org/licenses/by/4.0/). Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

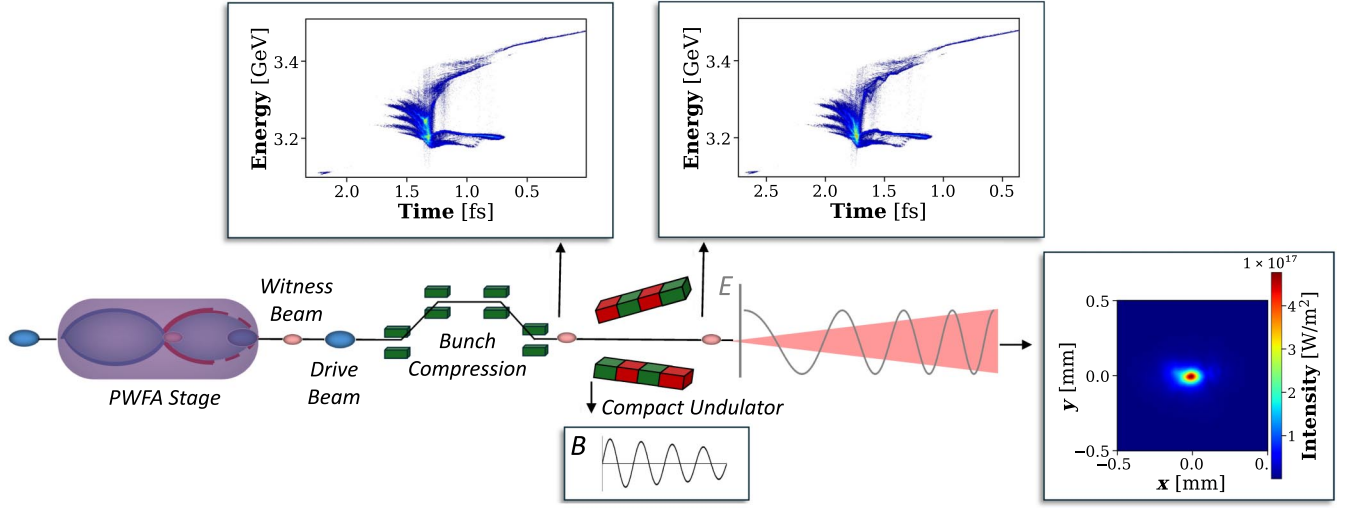


FIG. 1. Schematic of the simulation setup for the tapered-undulator configuration. The PWFA-compressed electron beam from Ref. [21] is injected into the compact undulator. The simulated longitudinal phase space of the electron beam is shown before and after the undulator, with the head of the beam on the right ($E = 3.2$ GeV, slice energy spread $\sigma_E/E = 0.3\%$ at the undulator entrance). The transverse radiation intensity profile is also shown.

in Sec. III A. The simulations use the compressed electron beam produced by the plasma-wakefield accelerator described by Emma *et al.* as the initial condition at the entrance to the compact undulator. The upstream transport line, including the bunch-compression stage, is therefore not modeled explicitly. Instead, its influence is incorporated through the electron-beam phase-space distribution adopted from Ref. [21].

The current profile of the electron beam is shown in Fig. 2(b). The beam features an ultrashort current spike with a full width at half maximum (FWHM) of 22.2 nm and a peak current of 0.6 MA. The asymmetric shape of the spike means that the FWHM does not fully capture the relevant length scale of the current profile, since one side of the spike rises more steeply than the other. The portion of the beam that most strongly influences the coherent

emission is the leading edge of the current spike. Measured from the peak to the half-maximum point on the front side, this leading edge has a half-maximum width of 6.55 nm, corresponding to an effective front-edge FWHM of approximately 13.1 nm.

The emitted radiation is driven by the longitudinal bunching of the electron beam weighted by the local beam current. The bunching characteristics of this beam are illustrated in Fig. 2. The bunching factor is defined as:

$$b(\lambda) = \frac{1}{N} \sum_{j=1}^N e^{-i2\pi z_j/\lambda}, \quad (1)$$

where z_j is the longitudinal position of the j th electron and N is the total number of electrons. The local bunching

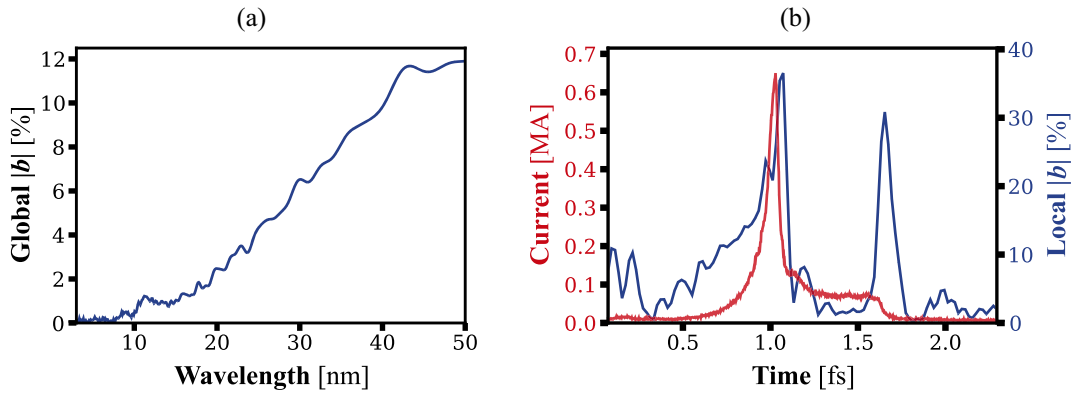


FIG. 2. The bunching properties of the electron beam. (a) Global bunching factor $|b(\lambda)|$ of the full beam as a function of wavelength, showing strong bunching at short wavelengths. (b) Local bunching factor calculated over a sliding window one radiation wavelength (30 nm) long, plotted on top of the current profile. The bunching peaks near 40% at the position of the current spike.

factor is obtained by evaluating the same expression within a sliding longitudinal window one radiation wavelength long. The global bunching factor, calculated over the full beam, increases with wavelength, showing significant modulation down to short wavelengths. At a wavelength of 30 nm, the global bunching reaches about 7%, while the local bunching, evaluated over a sliding window one radiation wavelength long ($\lambda_r = 30$ nm), is much stronger and approaches 40%. This strong local bunching coincides with the region of highest current along the beam.

A. Analytical description of single-spike emission

If we propagate this electron beam through an undulator with normalized strength K and period λ_u , it emits radiation at a wavelength given by the resonance condition:

$$\lambda_r = \frac{\lambda_u}{2h\gamma^2} (1 + K^2), \quad (2)$$

where γ is the relativistic Lorentz factor of the beam and h is the harmonic number. Within each undulator period, the electron beam slips back relative to the light by one resonant ($h = 1$) wavelength. When the resonant wavelength exceeds the length of the current spike, the beam emits coherently over each undulator period, effectively acting as a single radiating bunch [7]. This contrasts with the typical case in which the beam spans many radiation wavelengths, resulting in interference between emission from different electrons and undulator periods. In the single-spike regime, each undulator period produces a distinct radiation cycle, enabling direct temporal shaping of the emitted field.

This regime was explored both numerically and analytically in previous studies. Tibai *et al.* [25] numerically demonstrated that nanobunched electron trains can produce single-cycle, carrier-envelope-phase-stable pulses, with each bunch radiating a field that closely follows the undulator's magnetic waveform. A complementary analytical treatment by Shamuilov *et al.* [26] confirmed this relationship in the limit $K \ll 1$, showing that the radiated field is directly proportional to the derivative of the undulator magnetic field. These studies established the physical picture in which each nanobunch radiates coherently within one optical cycle, generating a waveform that mirrors the undulator field.

In our case, however, the situation differs. The beam produced by the plasma-wakefield accelerator has an ultrashort current spike with a front-edge FWHM of 13.1 nm. To maintain coherence across the entire bunch, the resonant wavelength must be longer than the effective bunch length, which places a strong constraint on the undulator parameters through the resonance condition above. For the energy of the PWFA-generated electron beam ($\gamma \approx 6250$), achieving a resonant wavelength

comparable to the bunch length requires a large undulator parameter K . There is no practical choice of undulator period that would yield such wavelengths in the small- K limit, so operation in the large- K regime arises naturally from the beam and undulator configuration rather than as a free design parameter.

In the large- K regime, the radiation from a single electron becomes distorted compared to the sinusoidal undulator field. The on-axis electric field consists of a sequence of narrow spikes with alternating polarity, and its spectrum contains strong harmonic components [27]. As K increases, these spikes become progressively sharper and more intense, with increasing harmonic content, reflecting the transition from undulatorlike to wigglerlike emission.

At large K , this distortion breaks the direct correspondence between the emitted field and the magnetic waveform of the undulator. The field no longer reproduces the magnetic structure but instead develops a strongly nonsinusoidal profile dominated by higher harmonics. However, this apparent loss of correspondence can be mitigated when the finite length of the electron bunch is taken into account. When the bunch length L_b (FWHM) becomes comparable to half the resonant wavelength ($L_b \sim \lambda_r/2$), the radiation from different electrons overlaps in phase, and the electron-beam current profile acts as a spectral filter on the emitted field. In this regime, there are no bunching components capable of radiating efficiently at the higher harmonics, while the fundamental remains strongly coherent. As a result, the emitted field is dominated by the fundamental wavelength and recovers the temporal structure of the undulator magnetic field, restoring the correspondence between $E(t)$ and $B_u(z)$ that is lost in the single-particle limit.

To illustrate this behavior, we start from the single-electron solution of Ref. [26], in which the undulator field is described via a transverse vector potential $A_x = A_0 f_u(z)$. In contrast to the specific short-radiator field profile used in that work, we consider a sinusoidal undulator magnetic field with $f_u(z) \propto \cos(k_u z)$, representative of a generic single-period undulator. This choice enables a closer correspondence with our numerical simulations and highlights how the bunch length filters the harmonic content in the large- K regime. The on-axis far-field electric field of a single electron takes the form:

$$E(t) \propto \frac{f'_u(Z_{\text{ret}})}{[1 + K^2 f_u^2(Z_{\text{ret}})]^2}, \quad (3)$$

where Z_{ret} denotes the retarded longitudinal coordinate determined by the electron trajectory and the retardation condition. In the limit $K \ll 1$, longitudinal velocity modulation is negligible, and Eq. (3) reduces to $E(t) \propto B_u$. At larger K , the full retardation solution redistributes emission into higher harmonics.

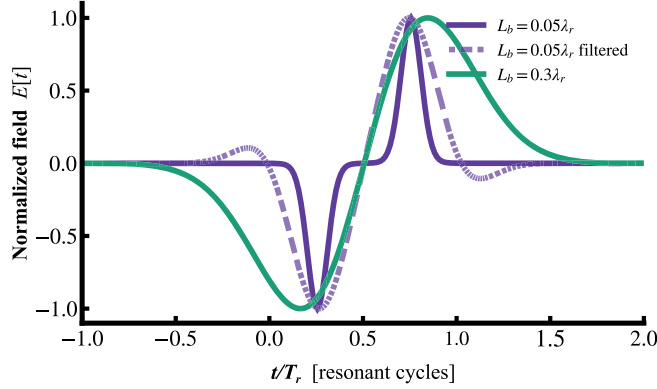


FIG. 3. Field distortion and recovery in the large- K regime. The plot shows the normalized far-field electric field for $K = 6$, with two different electron-bunch lengths. For a short bunch ($\sigma_{L_b} = 0.05\lambda_r$), the field is strongly distorted and dominated by higher harmonics. As the bunch length increases to $\sigma_{L_b} = 0.3\lambda_r$, the higher-harmonic components are filtered by the current profile, and the field recovers the smooth temporal structure of the undulator magnetic field. The short-bunch waveform after applying an explicit spectral filter that removes higher-harmonic content without shifting the central wavelength is also shown. Each trace is normalized to its own peak field amplitude.

While Eq. (3) describes the single-electron emission, the waveform is further shaped by the longitudinal current profile of the bunch. The emitted field is obtained by convolution with the temporal current distribution, evaluated following Eq. (20) of Ref. [26], allowing bunches of different lengths to be considered.

Figure 3 shows the resulting far-field electric field for the case $K = 6$, comparing two representative limits using an ideal Gaussian current profile with rms length σ_{L_b} . For a very short bunch ($\sigma_{L_b} \ll \lambda_r$), the field is highly asymmetric and dominated by higher harmonics, consistent with the single-particle wigglerlike waveform. For a longer bunch ($\sigma_{L_b} \gtrsim \lambda_r/5$), the field becomes nearly symmetric and closely follows the undulator magnetic field, demonstrating that the bunch length effectively filters out the harmonic content and restores the field-magnet correspondence.

In addition to the temporal smoothing, the results also reveal a gradual spreading of the waveform as the bunch length increases. This broadening corresponds to a slight red-shift in the dominant frequency component of the emitted field, arising from the convolution between the single-electron response and the finite electron-bunch profile. Physically, the longer bunch emits over a wider range of field phases, leading to partial destructive interference at higher frequencies and a net shift of the spectral maximum toward lower frequencies. This behavior is intrinsic to the single-cycle undulator regime, where the finite interaction length defines a limited temporal window for coherent emission, and it similarly arises at the edges of

multiperiod undulator radiation. For comparison, Fig. 3 also includes the result of applying an explicit spectral filter to the short-bunch waveform, with higher-harmonic content removed and no shift in the central wavelength.

B. Tapered undulators

The second configuration considered in this work is the tapered undulator, in which the undulator parameter varies along the device, $K \rightarrow K(z)$. The resulting behavior can be understood intuitively from the resonance condition, Eq. (2), with the resonant wavelength determined locally by $K(z)$. As K varies along the undulator, the resonant frequency evolves accordingly, producing a gradual chirp in the emitted field. A decrease in K leads to a progressive red-shift of the waveform, while an increase produces a corresponding blue-shift.

As discussed above, the longitudinal current profile plays an important role in shaping the emitted waveform by filtering harmonic content. In the tapered case, where the spectral structure evolves along the device, this effect becomes particularly relevant. We therefore model this configuration numerically using the full undulator field $B_u(z)$ together with the realistic longitudinal current profile of the PWFA-generated beam, rather than an idealized Gaussian distribution.

III. SIMULATION RESULTS

The simulation code Puffin [28] is used for modeling. Puffin is an unaveraged code and does not use the slowly varying envelope approximation (SVEA) [29]. In contrast to SVEA-based models [30,31], which average the radiation field and electron dynamics over the optical period and therefore neglect fast oscillatory terms at the radiation frequency, Puffin evolves the full electromagnetic field without this averaging. Puffin directly tracks the absolute value of the transverse electric field components, E_x and E_y , on a Cartesian mesh, sampling the field at multiple nodes per wavelength along the time axis. This makes the code particularly well suited to model ultrashort electron beams and capture subwavelength features in the radiation. As Puffin resolves the FEL interaction on the radiation wavelength scale, the electron beam must be represented with sufficient longitudinal particle density to accurately model shot noise and microbunching. The beam was therefore sampled using the Joint Distribution Function algorithm of Traczykowski *et al.* [32], producing approximately 2.3×10^6 macroparticles with Poissonian shot-noise statistics. The simulations take the PWFA-compressed beam from Ref. [21] as the initial condition at the undulator entrance and therefore do not include the upstream transport line. Consequently, coherent synchrotron radiation generated in the compression chicane is not propagated in these simulations, although its effect on the

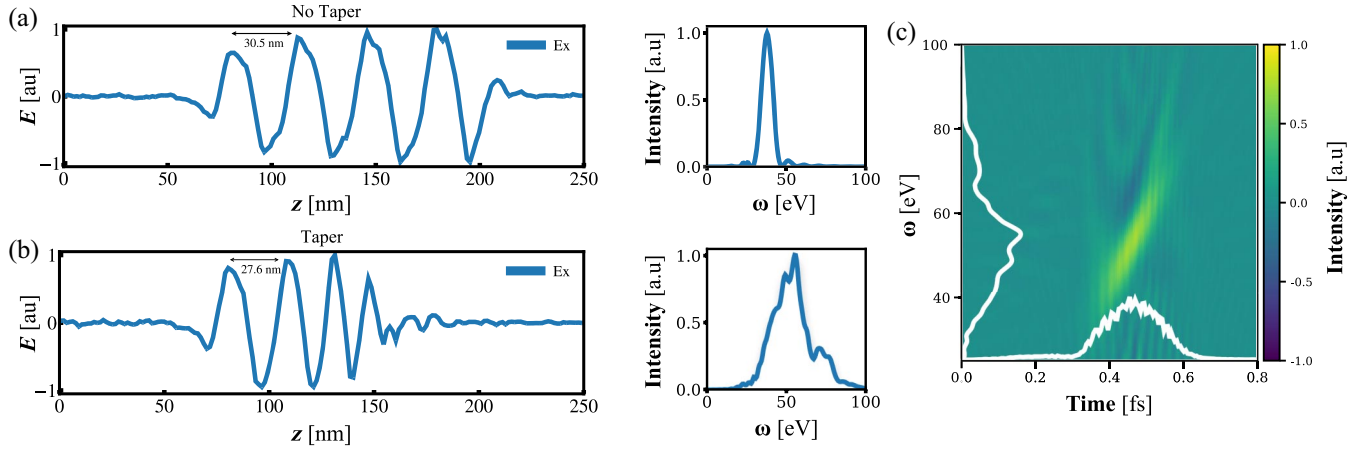


FIG. 4. Comparison of radiation from untapered and linearly tapered undulators. (a) On-axis electric field and corresponding spectrum for an untapered undulator. (b) On-axis electric field and corresponding spectrum for a strongly linearly tapered undulator. (c) Wigner-Ville transform (normalized to its maximum) of the on-axis radiation emitted from the strongly tapered undulator, showing its temporal-spectral structure.

electron beam is incorporated through the initial phase-space distribution.

This work considers compact undulators, with the longest containing only four periods and the remainder even shorter. Because we only consider a short interaction region, longitudinal space-charge effects are expected to have minimal impact on the radiation. The same plasma-accelerated beam was previously studied in Ref. [22], where full space-charge modeling showed a $\approx 20\%$ increase in bunch length over five periods ($K_{\text{rms}} = 3.7$). Scaling that result to the present configuration ($K_{\text{rms}} = 6$, four periods) suggests a worst-case broadening of order 30%. Applied to the effective front-edge width of the current spike, this corresponds to an increase from approximately 13 nm to about 17 nm. Although this represents a measurable change in bunch length, the beam remains within the regime where a well-defined current edge produces a single-cycle, phase-coherent radiation waveform. We therefore leave the implementation of space-charge effects and the associated computational effort required to a future study.

A. Tapered undulator

The first ultrashort undulator simulations apply a continuous linear taper, which modifies the resonance condition along the undulator length and imprints a strong frequency chirp on the emitted radiation. We model this with a planar undulator with a magnetic field defined as:

$$\begin{aligned} B_x &= 0, \\ B_y &= \alpha(z)B_0 \sin(k_u z), \end{aligned} \quad (4)$$

where B_0 is the peak magnetic field, $k_u = 2\pi/\lambda_u$ is the undulator period, and $\alpha(z) = \frac{dz}{dz}z + 1$ defines the linear chirp. The undulator has an initial magnetic field

$B_0 = 0.92$ T and spans four undulator periods with $\lambda_u = 70$ mm. This idealized field does not include dedicated entrance and exit field tapers. Realistic entrance and exit fields are instead treated explicitly for the single-period undulators discussed in Sec. III B, where the detailed magnetic structure is essential. The taper is applied with $\frac{d\alpha}{dz} = -1.578$, so that the undulator parameter K decreases from 6 to 3.5 over the four periods. The longitudinal field mesh used 31 grid points per initial resonant wavelength. Since Puffin requires approximately 11 grid points per wavelength for accurate field propagation, this provides sufficient resolution as the resonant wavelength decreases along the taper.

Consistent with earlier studies of this PWFA-compressed beam, the high peak current enables significant power to be emitted even over this short undulator. At the end of the undulator, the peak power reaches ~ 100 GW. Figures 4(a) and 4(b) compare the emitted electric field and radiation spectrum for the tapered undulator against an otherwise identical untapered case. In the tapered configuration, the changing field amplitude leads to a continuous shift in the radiation frequency, visible in the electric field as a gradual change in the spacing between peaks. The corresponding spectrum is broader, reflecting both the chirp and the reduced temporal duration of the pulse.

To better illustrate the local frequency shift, Fig. 4(c) shows the Wigner-Ville transform of the on-axis electric field for the tapered case. This time-frequency representation clearly reveals the continuous chirp, as the frequency decreases along the pulse duration.

B. Alternating polarization

We now return to the single-cycle configuration, leveraging this setup to control the polarization of successive

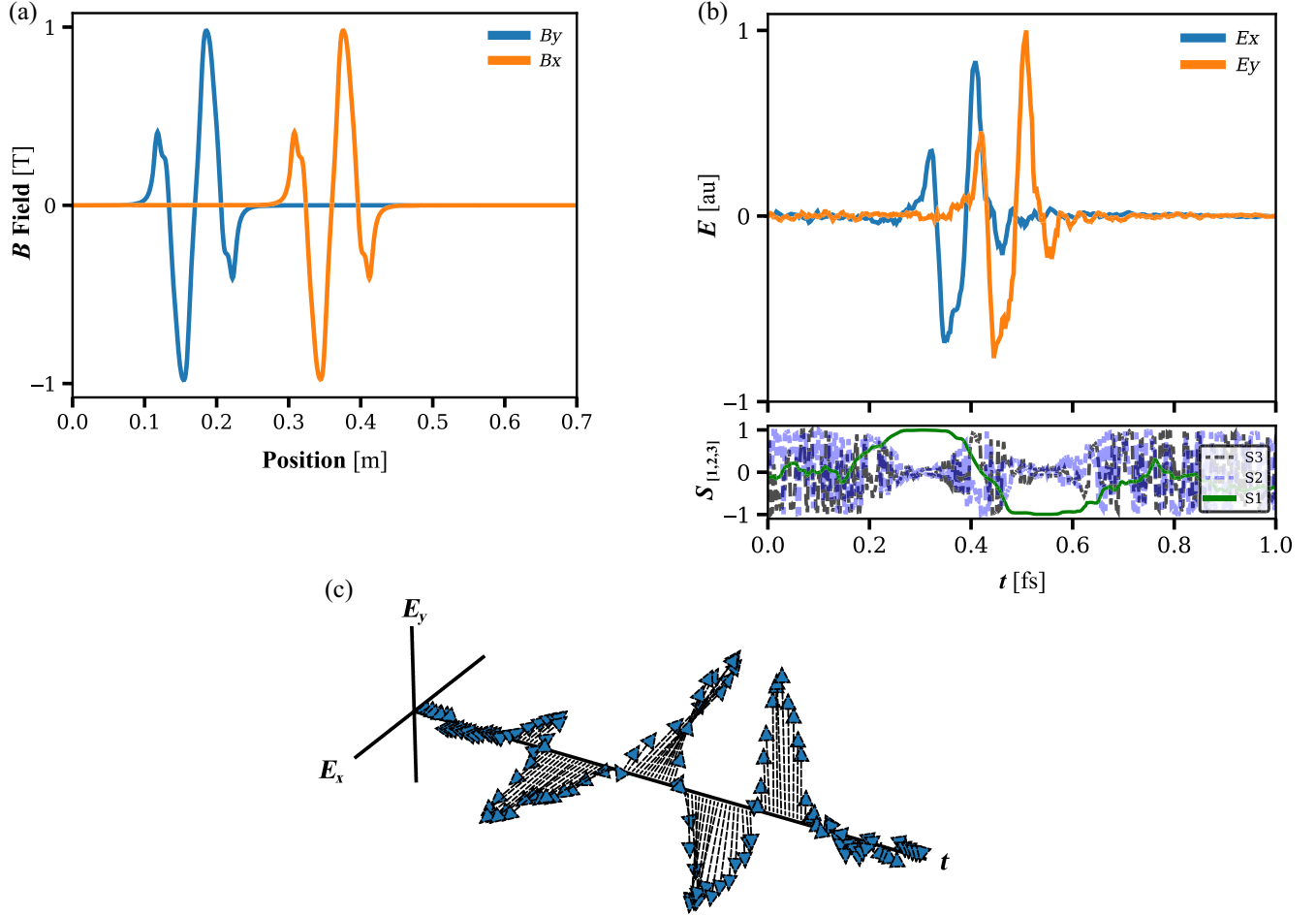


FIG. 5. Radiation from two consecutive single-period undulators with orthogonal linear polarizations. (a) On-axis magnetic field of the undulators including realistic entry and exit tapers. (b) On-axis electric field (top) and corresponding normalized Stokes parameters S_1 , S_2 , and S_3 (bottom), showing alternating linear polarization with intermediate elliptical states during transitions. (c) Three-dimensional evolution of the on-axis electric field vector illustrating the polarization transition from E_x to E_y .

radiation cycles. To model polarization switching, we simulate a sequence of orthogonally polarized single-period undulators that produce alternating linear or circular polarization states.

The idealized field, $\sin(k_u z)$, used in the theoretical model is convenient for analysis but unphysical for a single-period device, since it lacks the entrance and exit tapers required to maintain a zero net field integral. Without these regions, the electron would receive a net transverse kick and leave the undulator on a divergent trajectory, disrupting phase alignment between successive modules. While such taper regions can be ignored for long undulators, they constitute a significant fraction of the total field in a one-period device and strongly influence the electron trajectory.

Accurately modeling the magnetic fields of such short undulators, particularly the entry and exit regions, is not supported by the native field definitions in Puffin. To overcome this, the code was adapted to accept externally defined magnetic field maps. These maps were generated

using the RADIA magnetostatics package developed at European Synchrotron Radiation Facility and Laboratoire de l'Accélérateur Linéaire, which is widely used for realistic three-dimensional (3D) modeling of insertion device fields [33,34]. The resulting fields are provided as 3D meshes and linearly interpolated to the electrons' positions at each time step within Puffin.

1. Linear polarization

Having established a realistic description of single-period undulators, including their entrance and exit tapers, we now apply this model to explore the generation of radiation with varying polarization from cycle to cycle. To do so, we simulate a pair of orthogonally polarized single-period undulators driven by the same PWFA-compressed electron beam. The undulator fields are obtained from the RADIA package, which provides full 3D magnetic field maps with realistic Halbach-style entry and exit tapers.

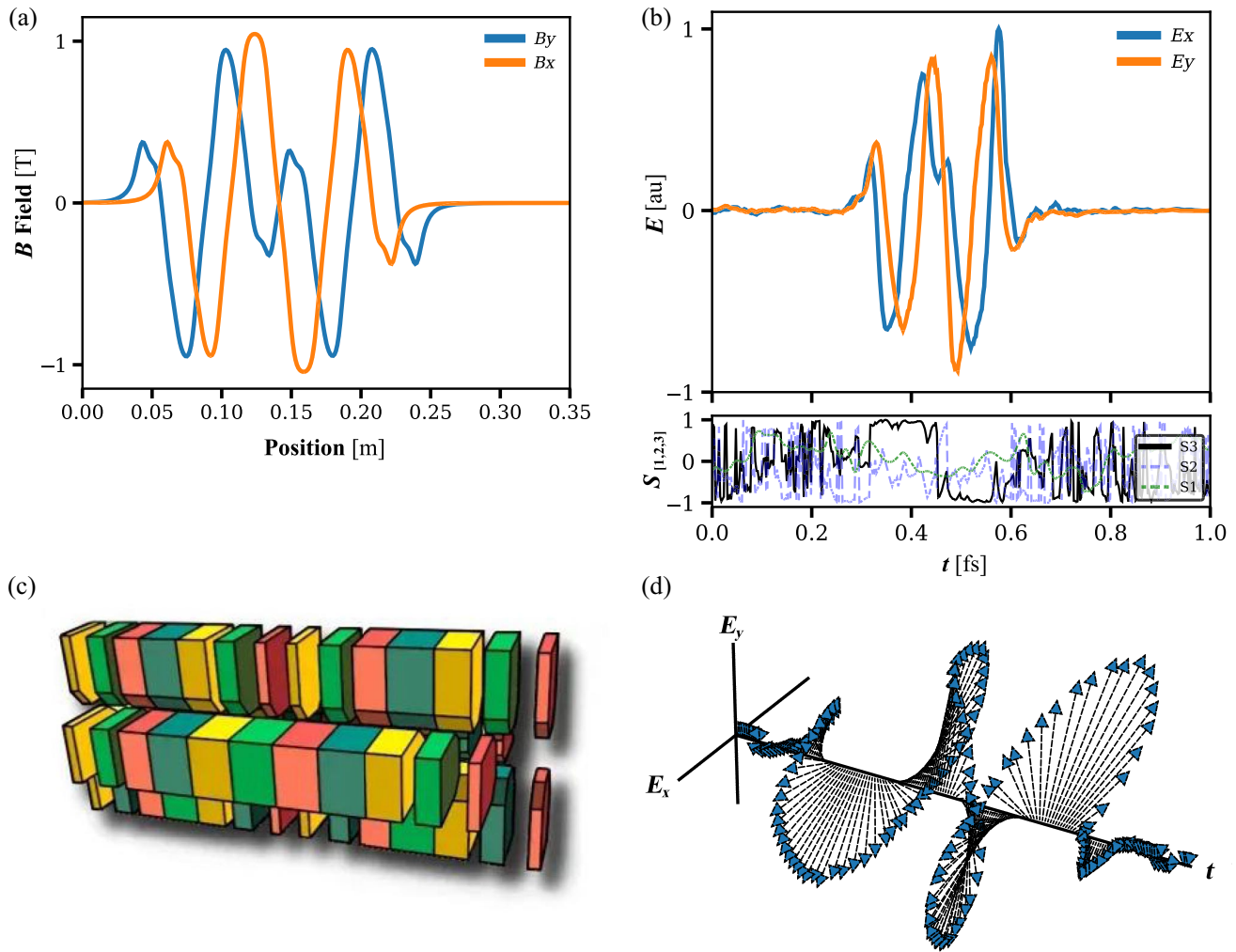


FIG. 6. Radiation from a single-period alternating circularly polarized undulator. (a) On-axis magnetic field. (b) On-axis electric field (top) and corresponding normalized Stokes parameters S_1 , S_2 , and S_3 (bottom), illustrating the transition between right- and left-handed circular polarization. The sharp transition in S_3 reflects the rapid reversal of the electron's transverse motion as the field passes through zero between the two polarization states. (c) Schematic layout of the undulator magnet structure. (d) Time evolution of the on-axis electric field vector showing a reversal of circular polarization handedness between cycles.

Each undulator has a period of $\lambda_u = 80$ mm, and the beam radiates at a resonant wavelength of $\lambda_r = 30$ nm. The two undulators are placed in immediate succession, with the first polarized along the x direction and the second along y . The on-axis magnetic field components of the combined arrangement are shown in Fig. 5(a).

The resulting on-axis electric field of the radiation is shown in Fig. 5(b). As expected from the undulator geometry, the radiation emitted in each section is orthogonally polarized and follows the profile of the corresponding magnetic field. Because the radiation waveform extends over the full duration of the electron beam, and the entry and exit tapers do not provide sufficient slippage between undulators to fully separate the pulses, the emission from each section partially overlaps in time. This overlap is a direct consequence of the waveform

spread described above, where fields radiated from the head and tail of the current spike contribute simultaneously. It is visible in the figure as the two polarization components begin to merge near the pulse center. While introducing a chicane between the undulators could separate the different polarized fields, doing so would potentially distort the temporal structure of the field by distorting the current spike.

To characterize the evolving polarization structure, we compute the normalized Stokes parameters S_1 , S_2 , and S_3 , corresponding to the degrees of linear (horizontal vs vertical), diagonal ($\pm 45^\circ$), and circular polarization, respectively. The bottom panel of Fig. 5(b) shows S_1 switching between values near ± 1 , indicating that the pulse transitions from x - to y -polarized light across successive radiation cycles. During these transitions,

nonzero values of S_2 and S_3 appear, signifying intermediate elliptical polarization states as the two orthogonal components overlap in time. This behavior is visualized directly in Fig. 5(c), where the on-axis electric field vector is seen to transition from alignment along E_x to alignment along E_y .

2. Circular polarization

Building on the previous case of alternating linear polarization, we now explore a configuration that produces alternating circular polarization. Rather than using two separate undulators, we designed a permanent-magnet undulator with RADIA in which the polarization flips within one undulator period and reverses in the subsequent period [35,36]. This is achieved by introducing a half-period phase shift in one of the planes of the magnetic field, which closes the electron orbit and reverses the handedness of the transverse motion. The undulator has a period of $\lambda_u = 70$ mm, and the PWFA-driven beam radiates at a resonant wavelength of $\lambda_r = 28$ nm. The on-axis magnetic field components and 3D rendering of the undulator magnetic layout are shown in Figs. 6(a) and 6(c).

In contrast to the linear case, where radiation from successive orthogonal undulators partially overlaps due to the finite bunch length and limited slippage in the entry and exit tapers, the circularly polarized configuration produces a much sharper and more localized transition between polarization states. Because the handedness reversal occurs within a continuous magnetic structure, the field evolution remains phase-synchronous with the electron motion, avoiding the temporal spreading that caused overlap between orthogonal components in the linear case. As a result, the Stokes parameter S_3 shows a much steeper transition.

Figure 6(b) shows the on-axis electric field and the corresponding normalized Stokes parameters. The S_3 parameter flips rapidly between right- and left-handed circular polarization, while S_1 and S_2 briefly rise during the transition, indicating transient linear polarization as the field crosses through zero. In this configuration, the emitted electric field closely follows the local undulator field, and the sharper reversal arises as one transverse magnetic field component advances in phase by π , reversing the sense of the electron's transverse rotation and therefore the helicity of the emitted light.

The time-dependent electric field vector, shown in Fig. 6(d), illustrates the resulting waveform: the radiation initially rotates in one direction, then reverses to the opposite handedness after half a period. This configuration demonstrates that continuous-field undulators with built-in phase shifts can produce controlled, cycle-resolved helicity modulation.

IV. CONCLUSION

We have presented simulations demonstrating how plasma-accelerated electron beams with ultrashort current spikes can generate tailored radiation waveforms when propagated through compact undulators. By exploiting the single-spike regime, where the beam length is shorter than the resonant radiation wavelength, we demonstrate that the emitted electric field can closely follow the structure of the undulator field. This enables direct temporal shaping of the radiation. We show that even for large undulator parameters ($K \gg 1$), the emitted field can be shaped through controlled variations of the electron trajectory. In this regime, harmonic content may be suppressed either intrinsically by a finite ultrashort bunch length, as demonstrated here, or, in the limit of extremely short beams, through downstream spectral filtering of the emitted radiation.

Two specific examples were explored. A strongly tapered undulator was used to generate radiation with a frequency chirp, while a pair of orthogonally polarized single-period undulators produced waveforms with alternating linear polarization. Extending this concept, we demonstrated an undulator design that yields alternating circular polarization, where the handedness of the emitted field reverses between cycles. The three configurations presented here are fixed undulator designs, chosen to illustrate different approaches to waveform control. The number of radiation cycles is determined by the number of undulator periods, while the polarization state is determined by the magnetic field configuration.

These results show that structured, ultrashort light pulses can be generated directly from compact undulators without external seeding or postundulator optics. The ability to shape both the temporal and polarization structure of the emitted field on a cycle-by-cycle basis may open new opportunities for probing ultrafast dynamics and developing light sources for polarization-sensitive applications.

ACKNOWLEDGMENTS

This work was supported by the U.S. Department of Energy, Office of Science, and Office of Basic Energy Sciences under Contract No. DE-AC02-76SF00515. A. M. and J. M. acknowledge funding from the DOE-BES Accelerator and Detector Research Program. C.E. acknowledges support from the Department of Energy Early Career Research Program.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

- [1] E. Allaria, R. Appio, L. Badano, W. Barletta, S. Bassanese, S. Biedron, A. Borga, E. Busetto, D. Castronovo, P. Cinquegrana *et al.*, Highly coherent and stable pulses from the fermi seeded free-electron laser in the extreme ultraviolet, *Nat. Photonics* **6**, 699 (2012).
- [2] W. a. Ackermann, G. Asova, V. Ayvazyan, A. Azima, N. Baboi, J. Bähr, V. Balandin, B. Beutner, A. Brandt, A. Bolzmann *et al.*, Operation of a free-electron laser from the extreme ultraviolet to the water window, *Nat. Photonics* **1**, 336 (2007).
- [3] P. Emma, R. Akre, J. Arthur, R. Bionta, C. Bostedt, J. Bozek, A. Brachmann, P. Bucksbaum, R. Coffee, F.-J. Decker *et al.*, First lasing and operation of an ångström-wavelength free-electron laser, *Nat. Photonics* **4**, 641 (2010).
- [4] W. Decking, S. Abeghyan, P. Abramian, A. Abramsky, A. Aguirre, C. Albrecht, P. Alou, M. Altarelli, P. Altmann, K. Amyan *et al.*, A MHz-repetition-rate hard x-ray free-electron laser driven by a superconducting linear accelerator, *Nat. Photonics* **14**, 391 (2020).
- [5] H.-S. Kang, C.-K. Min, H. Heo, C. Kim, H. Yang, G. Kim, I. Nam, S. Y. Baek, H.-J. Choi, G. Mun *et al.*, Hard x-ray free-electron laser with femtosecond-scale timing jitter, *Nat. Photonics* **11**, 708 (2017).
- [6] E. Prat, R. Abela, M. Aiba, A. Alarcon, J. Alex, Y. Arbelo, C. Arrell, V. Arsov, C. Bacellar, C. Beard *et al.*, A compact and cost-effective hard x-ray free-electron laser driven by a high-brightness and low-energy electron beam, *Nat. Photonics* **14**, 748 (2020).
- [7] C. Pellegrini, A. Marinelli, and S. Reiche, The physics of x-ray free-electron lasers, *Rev. Mod. Phys.* **88**, 015006 (2016).
- [8] E. Hemsing, G. Stupakov, D. Xiang, and A. Zholents, Beam by design: Laser manipulation of electrons in modern accelerators, *Rev. Mod. Phys.* **86**, 897 (2014).
- [9] A. Marinelli, D. Ratner, A. Lutman, J. Turner, J. Welch, F.-J. Decker, H. Loos, C. Behrens, S. Gilevich, A. Miahnahri *et al.*, High-intensity double-pulse x-ray free-electron laser, *Nat. Commun.* **6**, 6369 (2015).
- [10] A. A. Lutman, T. J. Maxwell, J. P. MacArthur, M. W. Guetg, N. Berrah, R. N. Coffee, Y. Ding, Z. Huang, A. Marinelli, S. Moeller *et al.*, Fresh-slice multicolour x-ray free-electron lasers, *Nat. Photonics* **10**, 745 (2016).
- [11] J. Duris, S. Li, T. Driver, E. G. Champenois, J. P. MacArthur, A. A. Lutman, Z. Zhang, P. Rosenberger, J. W. Aldrich, R. Coffee *et al.*, Tunable isolated attosecond x-ray pulses with gigawatt peak power from a free-electron laser, *Nat. Photonics* **14**, 30 (2020).
- [12] Z. Guo, T. Driver, S. Beauvarlet, D. Cesar, J. Duris, P. L. Franz, O. Alexander, D. Bohler, C. Bostedt, V. Averbukh *et al.*, Experimental demonstration of attosecond pump-probe spectroscopy with an x-ray free-electron laser, *Nat. Photonics* **18**, 691 (2024).
- [13] E. Prat, A. Al Haddad, C. Arrell, S. Augustin, M. Boll, C. Bostedt, M. Calvi, A. L. Cavalieri, P. Craievich, A. Dax *et al.*, An x-ray free-electron laser with a highly configurable undulator and integrated chicanes for tailored pulse properties, *Nat. Commun.* **14**, 5069 (2023).
- [14] J. Yan, W. Qin, Y. Chen, W. Decking, P. Dijkstal, M. Guetg, I. Inoue, N. Kujala, S. Liu, T. Long *et al.*, Terawatt-attosecond hard x-ray free-electron laser at high repetition rate, *Nat. Photonics* **18**, 1293 (2024).
- [15] R. R. Robles, K. A. Larsen, D. Cesar, T. Driver, J. Duris, P. Franz, D. Garratt, V. Guo, G. Just, R. Lemons *et al.*, Spectrotemporal shaping of attosecond x-ray pulses with a fresh-slice free-electron laser, *Phys. Rev. Lett.* **134**, 115001 (2025).
- [16] W. Hu, G. Aeppli, C. Arrell, M. Calvi, S. Carbajo, A. Dax, Y. Deng, P. Dijkstal, D. Dunning, S. Gerber *et al.*, Demonstration of mode-locked frequency comb for an x-ray free-electron laser, *Phys. Rev. Lett.* **135**, 265001 (2025).
- [17] E. Esarey, C. B. Schroeder, and W. P. Leemans, Physics of laser-driven plasma-based electron accelerators, *Rev. Mod. Phys.* **81**, 1229 (2009).
- [18] C. Lindström, S. Corde, R. d’Arcy, S. Gessner, M. Gilljohann, M. Hogan, and J. Osterhoff, Beam-driven plasma-wakefield acceleration, *arXiv:2504.05558*.
- [19] B. Hidding, G. Pretzler, J. B. Rosenzweig, T. Königstein, D. Schiller, and D. L. Bruhwiler, Ultracold electron bunch generation via plasma photocathode emission and acceleration in a beam-driven plasma blowout, *Phys. Rev. Lett.* **108**, 035001 (2012).
- [20] X. L. Xu, F. Li, W. An, T. N. Dalichaouch, P. Yu, W. Lu, C. Joshi, and W. B. Mori, High quality electron bunch generation using a longitudinal density-tailored plasma-based accelerator in the three-dimensional blowout regime, *Phys. Rev. Accel. Beams* **20**, 111303 (2017).
- [21] C. Emma, X. Xu, A. Fisher, R. Robles, J. P. MacArthur, J. Cryan, M. J. Hogan, P. Musumeci, G. White, and A. Marinelli, Terawatt attosecond x-ray source driven by a plasma accelerator, *APL Photonics* **6**, 076107 (2021).
- [22] R. Hessami, J. Morgan, R. Robles, K. A. Larsen, A. Marinelli, and C. Emma, Wavelength scaling and multi-color operation of a plasma-driven attosecond x-ray source via harmonic generation, *Phys. Rev. Accel. Beams* **27**, 070701 (2024).
- [23] S. M. Cavaletto, D. Keefer, J. R. Rouxel, F. Aleotti, F. Segatta, M. Garavelli, and S. Mukamel, Unveiling the spatial distribution of molecular coherences at conical intersections by covariance x-ray diffraction signals, *Proc. Natl. Acad. Sci. U.S.A.* **118**, e2105046118 (2021).
- [24] O. Neufeld, N. Tancogne-Dejean, U. De Giovannini, H. Hübener, and A. Rubio, Attosecond magnetization dynamics in non-magnetic materials driven by intense femtosecond lasers, *npj Comput. Mater.* **9**, 39 (2023).
- [25] Z. Tibai, G. Tóth, M. Mechler, J. Fülöp, G. Almási, and J. Hebling, Proposal for carrier-envelope-phase stable single-cycle attosecond pulse generation in the extreme-ultraviolet range, *Phys. Rev. Lett.* **113**, 104801 (2014).
- [26] G. Shamuilov, A. Mak, P. Salén, and V. Goryashko, Analytical model of waveform-controlled single-cycle light pulses from an undulator, *Opt. Lett.* **43**, 819 (2018).
- [27] A. Hofmann, *The Physics of Synchrotron Radiation* (Cambridge University Press, Cambridge, UK, 2004), Vol. 20.
- [28] L. T. Campbell and B. W. J. McNeil, Puffin: A three dimensional, unaveraged free electron laser simulation code, *Phys. Plasmas* **19**, 093119 (2012).

- [29] F. Arecchi and R. Bonifacio, Theory of optical maser amplifiers, *IEEE J. Quantum Electron.* **1**, 169 (1965).
- [30] S. Reiche, Genesis 1.3: A fully 3D time-dependent FEL simulation code, *Nucl. Instrum. Methods Phys. Res., Sect. A* **429**, 243 (1999).
- [31] W. M. Fawley, in *Proceedings of FEL, BESSY, Berlin, Germany* (JACoW, Geneva, Switzerland, 2006), pp. 218–221.
- [32] P. Traczykowski, L. Campbell, and B. McNeil, Up-sampling of electron beam simulation particles with addition of shot-noise, *Comput. Phys. Commun.* **286**, 108661 (2023).
- [33] P. Elleaume, O. Chubar, and J. Chavanne, Computing 3d magnetic fields from insertion devices, in *Proceedings of the 1997 particle accelerator conference (Cat. No. 97CH36167)* (IEEE, New York, NY, USA, 1997), Vol. 3, pp. 3509–3511.
- [34] O. Chubar, P. Elleaume, and J. Chavanne, A 3D magneto-statics computer code for insertion devices, *J. Synchrotron. Rad.* **5**, 481 (1998).
- [35] K. Halbach, Application of permanent magnets in accelerators and electron storage rings (invited), *J. Appl. Phys.* **57**, 3605 (1984).
- [36] J. A. Clarke, *The Science and Technology of Undulators and Wigglers* (Oxford University, Oxford, 2004), Vol. 4.