

Precise evaluation of time-varying quadrupole field errors through closed-orbit measurements in the J-PARC main ring

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In the main ring (MR) of the Japan Proton Accelerator Research Complex, one of the primary sources of beam loss is the betatron resonance. A factor determining the strength of betatron resonances is the symmetry of optics, which relies on the quadrupole field errors in the ring. In the beam tuning of the MR, correcting quadrupole field errors plays a significant role in reducing beam loss. This paper introduces and examines a precise method to evaluate time-varying quadrupole field errors at individual quadrupole magnets around the ring through closed-orbit measurements in the MR. By applying this method, we experimentally determined substantial quadrupole field errors due to eddy currents on the vacuum ducts during acceleration. The methods developed in this paper are employed in the beam tuning for the user operations of the MR.

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I. INTRODUCTION

The main ring (MR) of the Japan Proton Accelerator Research Complex (J-PARC) is a high-intensity synchrotron that accelerates protons from 3 to 30 GeV, providing them to neutrino and hadron experimental facilities. Table I outlines the fundamental parameters. The MR is meticulously designed to exhibit threefold symmetry and comprises three arc and straight sections. During operation, eight bunches of protons are injected during the “flat bottom period” with a time duration of 130 ms, where beam energy and main magnet currents are controlled to be constant. Two bunches are injected together 4 times at 40 ms intervals during the flat bottom period. Acceleration starts after all eight bunches are injected.

In the MR, stable operation providing a 515 kW intensity beam to the long baseline neutrino experiment (T2 K experiment) succeeded by 2021. Now, a plan to upgrade the beam power of the MR to 1.3 MW by 2028 [1] is underway. The plan entails reducing the repetition period and increasing the number of protons. After the completion of the plan, these values will become 1.16 s and 3.3×10^{14} ppp (protons per pulse), respectively. A large-scale hardware

upgrade of accelerator components was carried out by July 2022, and a 1.36 s repetition period operation has become possible [2,3]. The MR’s beam power reached 800 kW with a 2.3×10^{14} ppp intensity by June 2024.

A critical challenge in increasing the beam intensity of high-intensity proton synchrotrons is the diminished maintainability resulting from the radioactivation of accelerator components caused by beam loss. In the MR, precise adjustment of quadrupole fields to recover the threefold symmetry of optics has been one of the most important factors in mitigating beam loss. Conventionally, quadrupole field errors in the MR are assessed by measuring betatron functions, betatron phase advances, and tunes. These parameters have been evaluated using two main approaches: frequency analysis techniques considering turn-by-turn beam position [4,5] and observation of the orbit response matrix (ORM) [6]. The quadrupole field errors are iteratively

TABLE I. Basic parameters of MR. Steering magnets are small bending magnets used to correct closed-orbit distortion.

Parameter	Value
Circumference	1567.5 m
Superperiodicity	3
Injected particles	Proton p
Injection energy	3 GeV
Extraction energy	30 GeV
Harmonic number	9
Number of bunches	8
Number of quadrupole magnets	216
Number of horizontal steering magnets	93
Number of vertical steering magnets	93

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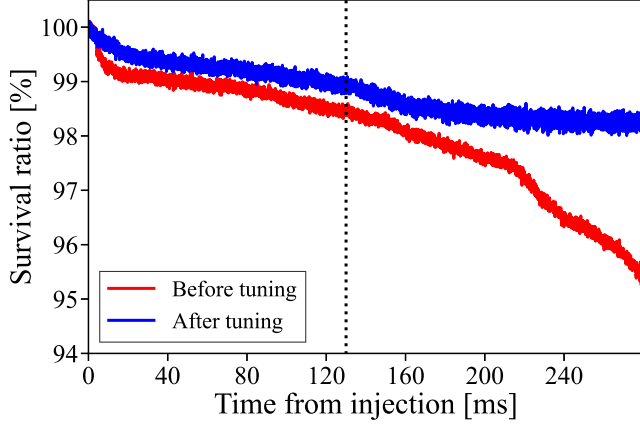


FIG. 1. Remarkable achievement in beam loss mitigation through conventional optics tuning after the large-scale hardware upgrade [7]. Intensity was 4.4×10^{13} ppp with two bunches for both conditions. The red and blue lines represent the beam survival ratio before and after the optics tuning, respectively. The vertical black dotted line marks the start of acceleration.

corrected until $\sigma_\beta \equiv \text{rms}[(\beta_{\text{meas}} - \beta_{\text{design}})/\beta_{\text{design}}]$ reaches 1.5%–5%, where β_{meas} and β_{design} denote the measured and designed values of the betatron function, respectively. The correction knob is the current of 16 power supplies for the main quadrupole magnets, which drive 216 main quadrupole magnets in total. Noteworthy results have been achieved through such conventional beam optics tuning in the MR. For instance, Fig. 1 illustrates the beam survival ratio before and after iterative optics tuning for the low-energy period in an operation cycle of 1.36 s. The tuning led to approximately an 80% reduction of beam loss at 270 ms from beam injection.

The term σ_β is the square root of the sum of two parts: $\sigma_\beta = \sqrt{\sigma_{\beta,\delta K}^2 + \sigma_{\beta,\Delta K}^2}$, where $\sigma_{\beta,\delta K}$ is contributed by lattice errors such as field errors and alignment errors in magnets. These lattice errors have been investigated and corrected, and they are now suppressed within the tolerance range of the optics design of the MR [8,9]. The other part, $\sigma_{\beta,\Delta K}$, is contributed by other quadrupole field errors that are not included in the original optics design. Our following discussion focuses on $\sigma_{\beta,\Delta K}$ errors, hereinafter simply called the “quadrupole field errors.” Any lattice errors in our consideration are explicitly declared.

In the MR, the amount of beam loss must be maintained below a certain threshold for stable operation regardless of the beam intensity. Therefore, it becomes increasingly crucial to correct the quadrupole field errors precisely. After the upgrade, two factors that could break the threefold symmetry of the optics were enhanced. The first factor is that some of the quadrupole magnet power supply families were split without maintaining threefold symmetry. In fact, their compensation has been significant in beam tuning after the upgrade [10]. The second factor is the quadrupole field response of eddy currents on vacuum duct walls due to

the steeper ramping pattern. Since their arrangement along the ring does not necessarily maintain threefold symmetry, it can break the symmetry of optics during the acceleration period. Its effect becomes more significant as the repetition rate increases in the future.

We can estimate the magnitude of the quadrupole field errors that have persisted in conventional optics tuning in the MR. The quadrupole field errors in a main quadrupole magnets can be expressed as $\Delta K/K$, where K and ΔK are the designed quadrupole field strength and its field deviation, respectively. Although the quadrupole fields in the MR have been adjusted by observing $\sigma_\beta \equiv \text{rms}[(\beta_{\text{meas}} - \beta_{\text{design}})/\beta_{\text{design}}]$, using $\Delta K/K$ is more direct and appropriate. In subsequent discussion, priority is given to $\Delta K/K$ over σ_β . Figure 2 plots the relationship between the quadrupole field error $\Delta K/K$ of a single quadrupole magnet and its contribution to the beta beat ($\sigma_{\beta,\Delta K}$) simulated using the accelerator design code SAD [11]. The magnetic field and alignment errors based on the measurement results were considered and their contribution $\sigma_{\beta,\delta K}$ was deducted from the total beta beat σ_β to obtain $\sigma_{\beta,\Delta K}$. As shown in the figure, 1% errors in the horizontal and vertical $\sigma_{\beta,\Delta K}$ correspond to approximately 0.9% and 0.4% errors in $\Delta K/K$ on average, respectively. The result

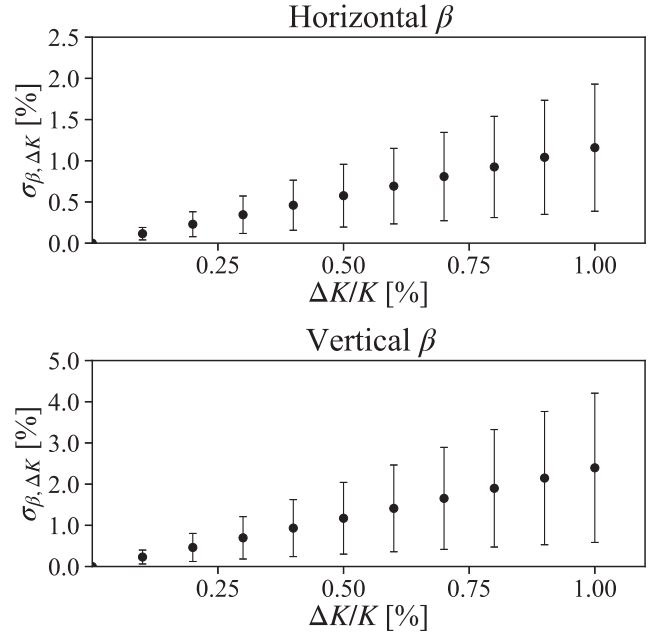


FIG. 2. Relation between $\Delta K/K$ and $\sigma_{\beta,\Delta K}$ simulated using the accelerator design code SAD. An additional $\Delta K/K$ was assigned to each of the 216 quadrupole magnets, and the corresponding σ_β was calculated. The vertical axis represents $\sigma_{\beta,\Delta K}$ averaged among the 216 quadrupole magnets. Further, the error bars indicate the standard deviation of $\sigma_{\beta,\Delta K}$ among the magnets. The model considers both magnetic-field errors and alignment errors and the closed-orbit distortion was corrected before calculating the betatron functions.

indicates that conventional beam optics tuning of quadrupole fields with an accuracy of $\sigma_\beta = 1.5\% - 5\%$ unintentionally retains substantial errors ($\Delta K/K$) of the same order; these errors are 1 order of magnitude larger than the well-investigated field imperfections of quadrupole magnets, whose standard error among magnets is less than $\Delta K/K = 0.05\%$ ($\sigma_{\beta, \delta K} < 1\%$ in beta beat) [12]. These uncorrected substantial errors are referred to as the “remaining quadrupole field errors” in the following discussion.

Typically, to evaluate remaining quadrupole field errors of $\Delta K/K \sim 1\%$ with sufficient significance, a method with an error bar 1 order smaller, or $\Delta K/K \sim 0.1\%$ accuracy, is required. In addition, to discern and characterize the causes of quadrupole field errors, evaluating the errors at each quadrupole magnet in the ring is imperative. As commonly acknowledged, measurement errors profoundly influence fitting results when numerous free parameters are involved. Consequently, a solid method that yields a physically reliable solution regarding individual quadrupole magnets around the ring as free parameters has yet to be devised in the MR.

Based on the discussions thus far, this paper aims to establish and practice a precise method for detecting quadrupole field errors in individual main quadrupole magnets within 0.1% order accuracy.

II. QUADRUPOLE FIELD ANALYSIS BY CLOSED ORBITS USING PERTURBATION-BASED LINEARIZED MODEL

This section introduces a precise method for detecting time-varying quadrupole field errors at individual quadrupole magnets around the MR. The method is hereafter called “quadrupole field analysis by closed orbits using the perturbation-based linearized model” (QCPLM) in this paper. QCPLM involves simultaneously ramping single or multiple steering magnets and discerning quadrupole field errors around the ring from their closed-orbit response. Other methods utilizing the closed-orbit response are widely employed in accelerators worldwide [13–32]. These methods typically involve solving nonlinear least squares problems using iterative techniques. Similar nonlinear iterative methods tailored for the MR are frequently employed in optics tuning [33]. In contrast, QCPLM is distinguished by its approach of minimizing the impact of measurement errors through the utilization of a perturbation-based linearized fit model. This section delves into the principles and intricacies of QCPLM. The precision and validity of QCPLM are experimentally evaluated in subsequent sections.

A. Linearization of fit model

The closed orbit under the effects of error fields from dipole, skew dipole, and quadrupole components can be obtained by solving

$$x'' + k_x^{\text{design}} x = -\Delta k_0 - \Delta k_1 x,$$

$$y'' + k_y^{\text{design}} y = \Delta l_0 + \Delta k_1 y,$$

$$k_x(s) = k_x(s + C), \quad k_y(s) = k_y(s + C), \quad (1)$$

with periodic boundary conditions $\chi(s) = \chi(s + C)$ ($\chi = x, y$), where Δk_0 , Δl_0 , and Δk_1 are the error fields of the normal dipole, skew dipole, and quadrupole field components, respectively. s and C denote the longitudinal position and circumference, respectively. We use a coordinate system such that (x, y, s) are the right-handed coordinates, and $x > 0$ if the particle is further from the center of the ring. k_x^{design} is determined by the designed values of the dipole and quadrupole fields. All other field components are considered systematic errors.

Now, let us assume that normal dipole field $k_0^{\text{actual}}(s)$ and skew dipole field $l_0^{\text{actual}}(s)$ are given to the ring using steering magnets, as shown in Eq. (2). During operation, steering magnets are the small dipole or skew dipole magnets used to correct closed orbits. Equation (1) is then modified as

$$x'' + k_x^{\text{design}} x = -k_0^{\text{actual}} - \Delta k_0 - \Delta k_1 x,$$

$$y'' + k_y^{\text{design}} y = l_0^{\text{actual}} + \Delta l_0 + \Delta k_1 y. \quad (2)$$

Terms $k_0^{\text{actual}}(s)$ and $l_0^{\text{actual}}(s)$ can be decomposed into

$$k_0^{\text{actual}} = (1 + \Delta a_x) k_0^{\text{set}} + \Delta k_0^{\text{offset}},$$

$$l_0^{\text{actual}} = (1 + \Delta a_y) l_0^{\text{set}} + \Delta l_0^{\text{offset}}, \quad (3)$$

where k_0^{set} and l_0^{set} are the set values for the actual fields k_0^{actual} and l_0^{actual} , respectively. Δa_χ ($\chi = x, y$) denotes the calibration errors for kick angles of steering magnets. The terms with offset correspond to the offset field of the steering magnets. Now, considering the solutions of Eq. (2) as $x = x(s, k_0^{\text{set}})$ and $y = y(s, l_0^{\text{set}})$, we define $\Delta x(s)$ and $\Delta y(s)$ as

$$\begin{aligned} \Delta x(s) &\equiv \frac{x(s, k_0^{\text{set}}) - x(s, -k_0^{\text{set}})}{2}, \\ \Delta y(s) &\equiv \frac{y(s, l_0^{\text{set}}) - y(s, -l_0^{\text{set}})}{2}. \end{aligned} \quad (4)$$

By assuming that terms with Δ in the right-hand side of Eqs. (2) and (3) are small and by applying first-order perturbation theory, $\Delta x(s)$ and $\Delta y(s)$ can be expressed as

$$\begin{aligned} \Delta x(s) &\approx \oint -\Delta k_0^{\text{calib}}(s') G_x(s, s') ds' \\ &\quad + \oint \oint \Delta k_1(s') k_0^{\text{set}}(s'') G_x(s, s') G_x(s', s'') ds' ds'', \\ \Delta y(s) &\approx \oint \Delta l_0^{\text{calib}}(s') G_y(s, s') ds' \\ &\quad + \oint \oint \Delta k_1(s') l_0^{\text{set}}(s'') G_y(s, s') G_y(s', s'') ds' ds'', \end{aligned} \quad (5)$$

where $\Delta k_0^{\text{calib}}(s') = \Delta a_x k_0^{\text{set}}$ and $\Delta l_0^{\text{calib}}(s') = \Delta a_y l_0^{\text{set}}$. The validity of this approximation will be experimentally examined in Sec. III. In Eq. (5), $G_\chi(s, s') = \frac{\sqrt{\beta_\chi(s)\beta_\chi(s')}}{2 \sin \pi \nu_\chi} \times \cos(\pi \nu_\chi - |\phi_\chi(s) - \phi_\chi(s')|)$ ($\chi = x, y$) is the Green's function for $\chi'' + K_\chi^{\text{design}} \chi = 0$ with periodic boundary conditions, which are determined only by the designed field strength. The only unknown parameters in Eq. (5) are $\Delta k_0^{\text{calib}}$, $\Delta l_0^{\text{calib}}$, and $\Delta k_1(s')$. $\Delta x(s)$ and $\Delta y(s)$ can be observed using beam position monitors, and all other terms are determined by the unperturbed closed-orbit response and set kick angle values.

Each magnet is thin in the longitudinal direction compared to the circumference of the MR. The core length of the steering magnets is 0.2 m, and that of the quadrupole magnets is less than 2 m, while the circumference of the MR is $C = 1567.5$ m. The strength of each field component can be expressed as

$$\begin{aligned} k_0^{\text{set}}(s) &\equiv \sum_j \theta_x(s_j) \delta(s - s_j), \\ l_0^{\text{set}}(s) &\equiv \sum_j \theta_y(s_j) \delta(s - s_j), \\ \Delta k_0^{\text{calib}}(s) &\equiv \sum_j \alpha_x(s_j) \theta_x(s_j) \delta(s - s_j), \\ \Delta l_0^{\text{calib}}(s) &\equiv \sum_j \alpha_y(s_j) \theta_y(s_j) \delta(s - s_j), \\ \Delta k_1(s) &\equiv \sum_j \Delta K_1(s_j) \delta(s - s_j), \end{aligned} \quad (6)$$

where index j specifies each magnet, θ_χ ($\chi = x, y$) represent the dipole and skew dipole kick angles, and α_χ ($\chi = x, y$) are the factors such that $\alpha_\chi \theta_\chi$ gives the calibration errors of the kick angles. In addition, $\Delta K_1(s_j)$ is the integrated quadrupole field strength in units of $[\text{m}^{-1}]$, which we call the “ K value” hereafter. K value can be explicitly written as $K_1 = qGL_{\text{eff}}/p$, where p is the beam momentum, q is the charge, $G \equiv \frac{\partial B_y}{\partial x}$ is the gradient field, and L_{eff} is the effective length of the magnet. By applying these approximations to Eq. (5), the equation can be rewritten as

$$\begin{aligned} \vec{z} &= A \vec{\Delta K}, \\ \vec{z} &= (\vec{\Delta x}^T \vec{\Delta y}^T)^T, \\ \vec{\Delta K} &= (\vec{\alpha}_x^T \vec{\alpha}_y^T \vec{\Delta K}_1^T)^T, \end{aligned} \quad (7)$$

where vectors represent $\vec{v} = [v(s_1) v(s_2) \dots v(s_n)]^T$. $\vec{z}$ is a vector representing the closed-orbit response. s_j specifies the location of each BPM or magnet. The same notation will be used in the following discussions. Matrix A can be

interpreted as a direct response matrix from the error fields to the closed orbits. The matrix elements of A can be written as

$$A = \begin{pmatrix} A^{(\text{steer}),x} & O & A^{(Q),x} \\ O & A^{(\text{steer}),y} & A^{(Q),y} \end{pmatrix}, \quad (8)$$

$$A_{ij}^{(\text{steer}),x} = -\theta_x(s_j) G_x(s_i, s_j), \quad (9)$$

$$A_{ij}^{(\text{steer}),y} = \theta_y(s_j) G_y(s_i, s_j), \quad (10)$$

$$A_{ij}^{(Q),x} = \sum_k \theta_x(s_k) G_x(s_i, s_j) G_x(s_j, s_k), \quad (11)$$

$$A_{ij}^{(Q),y} = \sum_k \theta_y(s_k) G_y(s_i, s_j) G_y(s_j, s_k), \quad (12)$$

where the rows of the block matrix in Eq. (8) correspond to the horizontal and vertical directions, and the columns correspond to each of the responses from $\vec{\alpha}_x$, $\vec{\alpha}_y$, and $\vec{\Delta K}_1$.

Additionally, indices i and j in Eqs. (9)–(12) represent BPMs and magnets, respectively. The important point is that A is determined only from $G_\chi(s, s')$ ($\chi = x, y$), besides set values $\vec{\theta}_x$ and $\vec{\theta}_y$, which is computationally very efficient.

B. QCPLM

QCPLM is a method that determines the quadrupole field errors around the ring through the following procedure: (i) Use one or multiple steering magnets, to create a nonzero closed orbit $\chi(s, \vec{\theta}_\chi)$ around the ring. (ii) Set the kick angles of each steering magnet to the opposite sign and measure $\chi(s, -\vec{\theta}_\chi)$. (iii) Solve the linear equation $\vec{z} = A \vec{\Delta K}$ in Eq. (7) to acquire $\vec{\Delta K}$.

When applying QCPLM, ensuring that each column of A is not nearly zero vector is vital. If a column is a zero vector, the predicted strength of the field errors will be indeterminate or infinitely large. This situation will arise if an intentionally created closed orbit satisfies $\Delta \chi(s_Q, \vec{\theta}_\chi) \approx 0$ at a specific quadrupole magnet location $s = s_Q$. This issue can be avoided by selecting several steering magnets at different betatron phase locations, ramping them one by one during each measurement, and jointly analyzing the closed-orbit responses using QCPLM. We temporarily refer to this procedure as “QCPLM by a single kick.” In principle, the minimum number of steering magnets necessary for QCPLM is two, with substantially different betatron phases. Herein, QCPLM is always performed in a single kick. Care must be taken not to choose two steering magnets whose difference in betatron phase is near $n\pi$, where n is an arbitrary integer.

Suppose M steering magnets are used for each horizontal and vertical direction for a single kick. Additionally, assume that $\vec{z}_i = A_i \vec{\Delta K}$ ($i = 1, 2, \dots, M$) represents each measurement. These can be expressed as a linear equation as follows:

$$\begin{aligned}
\vec{z}^{\text{sk}} &= A^{\text{sk}} \vec{\Delta K}, \\
\vec{z}^{\text{sk}} &\equiv (\vec{z}_1^{\text{T}} \vec{z}_2^{\text{T}} \cdots \vec{z}_M^{\text{T}})^{\text{T}}, \\
\vec{z}_i &\equiv (\vec{\Delta x}_i^{\text{T}} \vec{\Delta y}_i^{\text{T}})^{\text{T}}, \quad i = 1, 2, \dots, M, \\
A^{\text{sk}} &\equiv (A_1^{\text{T}} A_2^{\text{T}} \cdots A_M^{\text{T}})^{\text{T}}.
\end{aligned} \tag{13}$$

Importantly, the number of elements of $\vec{z}$ increases proportionally with M . The notation “sk” is used to specify that QCPLM by single kick is applied.

If the set values of kick angles are kept constant from injection to extraction, the linear equation can be expressed as

$$\vec{z}^{\text{sk}}(t_{\text{pat}}) = A^{\text{sk}} \vec{\Delta K}(t_{\text{pat}}), \tag{14}$$

where t_{pat} is the time stamp of the pattern measured from the injection timing; hereafter, Eq. (14) is the most basic form of QCPLM, and all the discussions related to QCPLM in this paper assume a single kick. Hereinafter, the notation sk will be omitted, and “QCPLM by a single kick” will be simply written as “QCPLM.”

QCPLM operates through a perturbation-based approach; thus, its application is contingent upon the validity of perturbation. A simple measure is to execute optics correction through a conventional scheme within the MR before employing QCPLM, ensuring that σ_β falls within the range of 1.5%–5%. The array of experiments outlined subsequently indicates that this measure is effective.

C. Experimental setup for QCPLM

This subsection delves into two pivotal topics concerning instruments and experimental conditions necessitating consideration before embarking on QCPLM. The topics are the choice of steering magnets and suppressing random errors.

First, we discuss the choice of steering magnets. As discussed in Sec. II B, selecting steering magnets with various betatron phases is crucial. Typically, four to six steering magnets per direction are used in the conventional ORM method in MR. Conversely, a large kick angle may lead to beam loss, even under low-intensity conditions. Therefore, the kick angle should be chosen to prevent beam loss. For instance, the beam survival ratio and closed orbits measured under the low-intensity condition of 1×10^{12} ppp during the flat bottom period of FX operation mode with several kick angles for the same steering magnet are depicted in Figs. 3 and 4, respectively. In this experiment, two steering magnets—one for the horizontal direction and one for the vertical direction—were ramped simultaneously with constant kick angles from injection to extraction for both magnets. The beam intensity in the MR was measured using direct-current current transformers (DCCT) [34]. Figure 3 illustrates that beam loss occurs when the kick angle reaches 0.3 mrad. In the MR, the kick angle of 0.2 mrad is commonly employed.

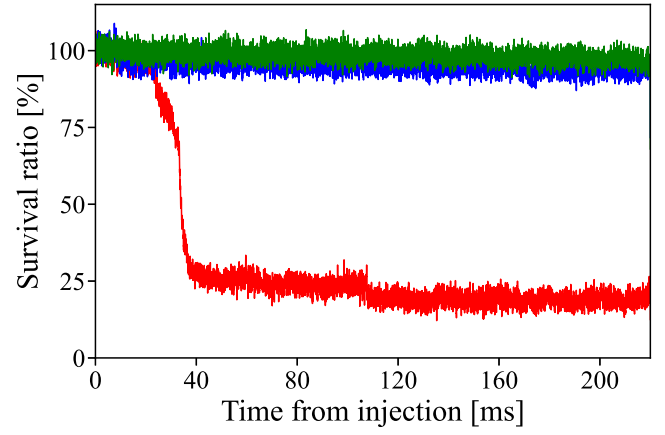


FIG. 3. Beam survival ratio measured by DCCT for various kick angles during the flat bottom period of FX mode operation. The flat bottom period was extended to 220 ms, and beam extraction occurred immediately after the flat bottom period. The time average for the flat bottom period is considered for closed orbits. Green, blue, and red lines correspond to kick angles 0.15, 0.2, and 0.3 mrad, respectively.

Next, we delve into the method to suppress the random errors. Undesirable random fluctuations in closed orbits arise from momentum oscillations and ripples in the output currents of bending magnet power supplies. These fluctuations are directly assessed and subtracted using the method outlined in Appendix C. Additionally, there are random errors from the precision of BPM readings. The precision of closed-orbit measurement using BPM is $30 \mu\text{m}$ [35].

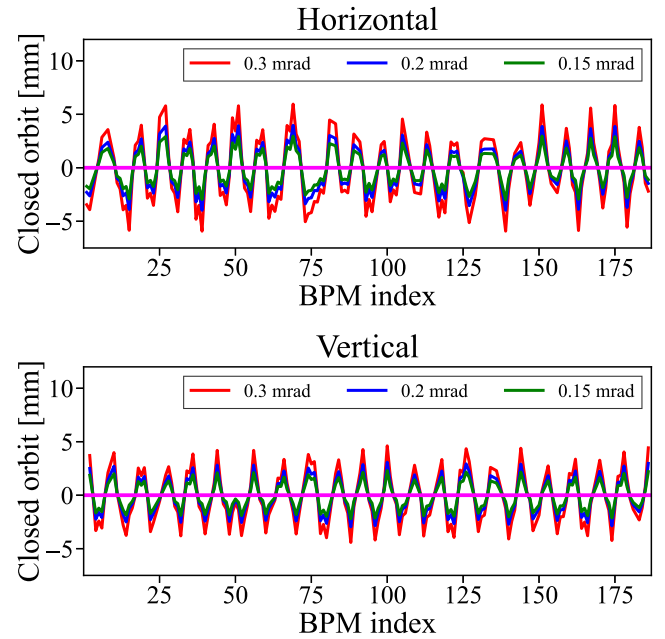


FIG. 4. Closed orbits observed by BPMs at each kick angle. The background closed-orbit distortion, which exists regardless of the single kick, was removed from the results. The magenta line represents the zero closed-orbit response.

This random error can be suppressed as follows. Recall that, in principle, $\Delta\vec{K}(t_{\text{pat}})$ can be obtained from closed orbits for one pulse per steering magnet setting. If multiple pulses are collected for each setting, averaging the pulses can mitigate the impact of remaining random errors. In the experiment, N_{pulse} pulses of $\Delta\vec{K}(t_{\text{pat}})$ were acquired and averaged. The specific values of N_{pulse} are provided in the corresponding discussion. We additionally note that QCPLM is fairly robust against the random errors because it employs a linearized model $\vec{z} = A\Delta\vec{K}$, as elaborated in Appendix B.

III. EXPERIMENTAL BENCHMARK OF QCPLM

The previous section introduced QCPLM, offering a precise means of assessing quadrupole field errors. We executed experimental benchmarks to validate the principle and evaluate its accuracy. The following subsections delve into the validation experiment and its results.

A. Method

This experiment aims to demonstrate the feasibility of detecting the strength, location, and temporal variation of quadrupole field errors for individual main quadrupole magnets using QCPLM. To achieve this, comparing the detected quadrupole field errors with their reference values obtained through another method is imperative. Consequently, the experimental procedure was structured as follows: (i) Five steering magnets with different betatron phases were excited within ± 0.2 mrad to obtain their respective closed-orbit responses. (ii) The current values of all 16 main quadrupole magnet power supplies in the MR were changed, and the closed-orbit response for each steering magnet was reacquired. (iii) QCPLM was

TABLE II. Main quadrupole magnet power supply families in MR. Magnets in the QF* families focus the beam in the horizontal direction, while those in the QD* family defocus the beam horizontally.

Family	Number of magnets	Arc/Straight
QFN	48	Arc
QDN	48	Arc
QFX1	24	Arc
QFX2	24	Arc
QDX1	13	Arc
QDX2	14	Arc
QFP	6	Straight
QFR	9	Straight
QFS1	3	Straight
QFS2	3	Straight
QFT1	3	Straight
QFT2	3	Straight
QDR	6	Straight
QDS1	3	Straight
QDS2	3	Straight
QDT	6	Straight

applied to the closed-orbit data obtained in steps (i) and (ii) using 216 magnets as free parameters. (iv) The difference in field strength between the two conditions detected by QCPLM and that of the set value for power supplies was compared. We write these as ΔK_{meas} and ΔK_{ref} , respectively.

The two conditions outlined in steps (i) and (ii) will be called “condition 1” and “condition 2,” respectively. The power supply families corresponding to each of the 16 power supplies for main quadrupole magnets are listed in Table II. During beam tuning of the MR, the power supply currents are controlled by setting K values to the interface. The set K values are converted to input currents using calibration curves between the currents and output K values. As the calibration curves were evaluated from the magnetic field measurements, the set K values were verified via field measurements. Note that the calibration errors and offsets of the currents are typically 0.01% order or less [36].

B. Setup

The experiment utilized beam optics tailored for the FX operation, with a specific focus on the low-energy region near the onset of acceleration, where beam loss tends to be pronounced in the MR (Fig. 1). Closed-orbit data spanning from the commencement of the flat bottom period (K.E. = 3 GeV) up to 140 ms (K.E. = 7 GeV) after the

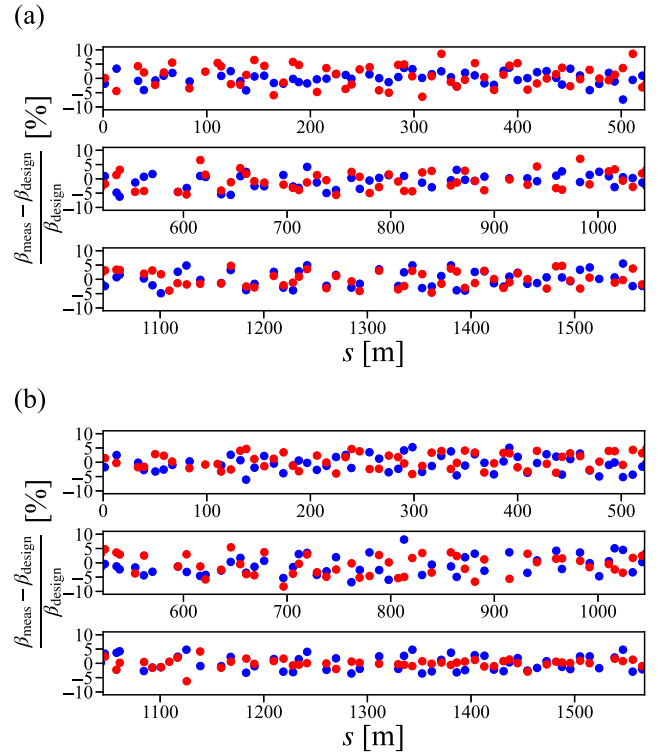


FIG. 5. Measured betatron function relative to the design value, shown as a percentage error, at (a) 10 ms after injection and (b) 100 ms after acceleration starts. Blue and red dots represent horizontal and vertical measurement results, respectively.

TABLE III. Steering magnet combinations used in the experiment. The betatron phases are set such that $\phi_\chi(s=0) = 0$ ($\chi = x, y$), where $s = 0$ is the reference point in the lattice design.

Index	Horizontal	Vertical
	$[\beta_x(\text{m}), \phi_x(2\pi \text{ rad})]$	$[\beta_y(\text{m}), \phi_y(2\pi \text{ rad})]$
1	39.6, 0.91	30.6, 5.70
2	28.6, 6.45	28.1, 8.53
3	27.7, 8.86	32.3, 11.40
4	32.1, 9.64	30.6, 12.84
5	28.9, 16.19	30.6, 19.99

initiation of acceleration were analyzed. Here, “K.E.” denotes kinetic energy. The beam intensity for the experiment was set at 1×10^{12} ppb, with two bunches injected at the onset of the flat bottom period, representing a low-intensity scenario for the MR. The model tune used for the experiment was $(\nu_x, \nu_y) = (21.35, 21.43)$. Closed orbits were measured at intervals of 10 ms from the injection using BPMs.

Before the validation experiment, betatron functions were fine-tuned to an accuracy of approximately 3% using the conventional ORM method. The betatron function around the ring for condition 1 is visualized at two time stamps in Fig. 5.

A list of the steering magnets used in the experiments is provided in Table III, alongside with their respective betatron functions and betatron phases. Five steering magnets were selected to ensure that the difference in

betatron phases between any two magnets out of the five was not approximately $n\pi$, where n represents an arbitrary integer. To facilitate the description of the experimental process, combinations of horizontal and vertical steering magnets are indexed as 1, 2, 3, 4, and 5. Note that two horizontal and vertical steering magnets within each combination are distinct magnets at different locations in the ring. Despite their dissimilarity, both magnets for the respective directions were ramped in unison during the same pulse, ignoring X-Y coupling, and maintaining constant kick angles from injection to extraction. $N_{\text{pulse}} = 100$ samples of $\vec{\Delta K}(t_{\text{pat}})$ were acquired during this experiment.

In this validation, a total of 316 BPMs were utilized, comprising 158 for each of the horizontal and vertical directions. Specifically, 28 BPMs for each horizontal and vertical direction were excluded from the analysis due to their identification as unreliable. As an illustration, Fig. 6 depicts the closed orbit observed at 10 ms after injection using combination 1. This serves to demonstrate the accuracy of the model and correctness of the kick angle calibration.

C. Results and discussion

Figure 7 shows the time variation of the quadrupole field errors detected by QCPLM with $N_{\text{pulse}} = 100$. For this experiment, σ_K was 0.32%, where σ_K is defined as

$$\sigma_K = \text{rms} \left[\frac{\Delta K_{\text{meas}}}{K} - \frac{\Delta K_{\text{ref}}}{K} \right]. \quad (15)$$

K is the designed value of the quadrupole strength. The rms was calculated among all data points in the time direction as well, as shown in Fig. 7.

The quantity σ_K serves as the error bar of QCPLM in evaluating $\Delta K/K$. The σ_K in this validation experiment was $\Delta K/K = 0.32\%$, which is already acceptable in detecting $\Delta K/K = 1\%$ field errors. However, observing Fig. 7, QFX, QFT, and QFP magnets exhibit substantial variance in their predicted values among magnets compared to the other families. The variance seen in these families is larger than 0.002 m^{-1} while it is smaller than 0.001 m^{-1} in other families.

To understand this variance seen in QFP, QFT, and QFX magnets, we examine the arrangement of the quadrupole magnets, as depicted in Fig. 8. The longitudinal thickness and location reflect the actual arrangement of the quadrupole magnets. Notably, a pair of two QFX magnets and a pair consisting of one QFT and one QFP magnet are adjacent in close proximity despite their focusing directions being the same. This pattern persists for all QFX, QFT, and QFP magnets around the ring. Consequently, their responses to the beam are similar, leading to uncertainty in determining the strength of the error fields in these magnet pairs. The substantial variance can also be understood from

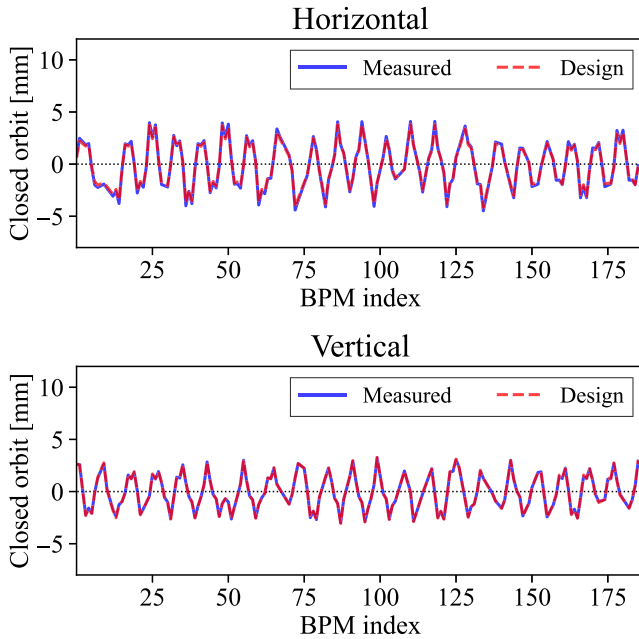


FIG. 6. Measured and designed closed-orbit responses from steering magnets 1 at 10 ms after injection. Closed orbits that are independent of the intentional kicks applied to the steering magnets listed in Table III have been subtracted beforehand.

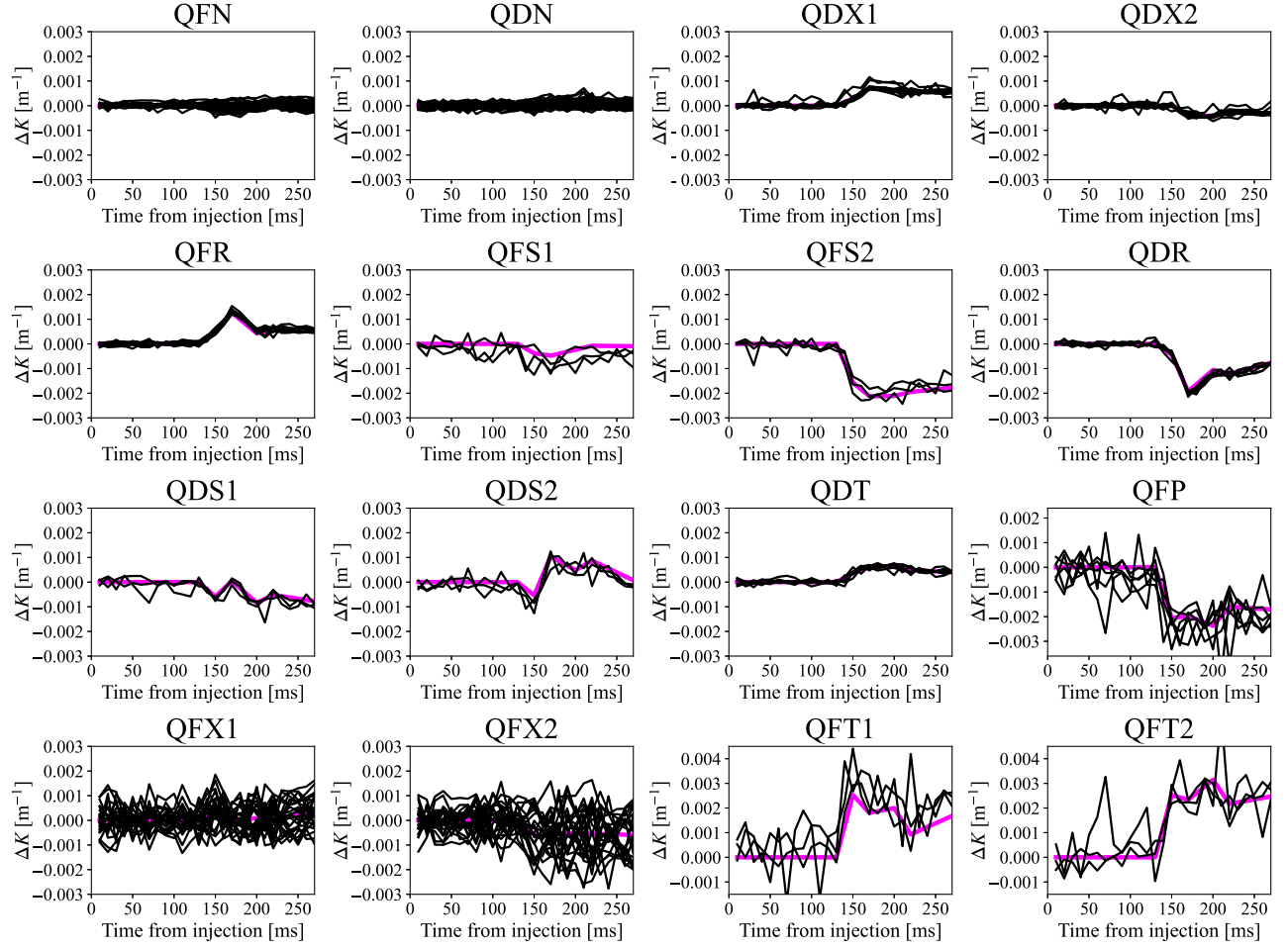


FIG. 7. ΔK_{meas} evaluated by QCPLM for all quadrupole magnets. Each panel represents a different power supply family. The black lines indicate the measured ΔK_{meas} for each magnet, while the magenta lines show the reference values (ΔK_{ref}). The vertical range for each panel is 0.006 m^{-1} .

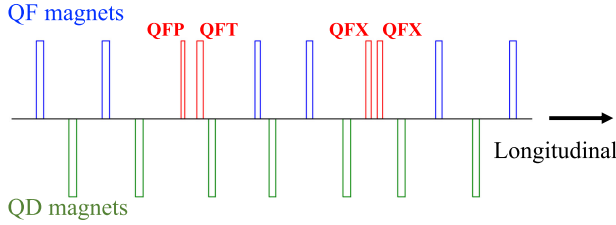


FIG. 8. Longitudinal arrangement of some quadrupole magnets in the ring. Each pair of two QFX magnets and each pair consisting of one QFP magnet and one QFT magnet are adjacent to each other, even though their focusing directions are the same. This arrangement is consistent throughout the entire ring.

the experimental results presented in Fig. 9. This figure illustrates the time variation of the two magnets in a specific pair of QFP and QFT, plotted alongside their averaged time variation. The time variation was smoothed into less than 0.0005 m^{-1} amplitude simply by averaging, indicating that the responses to the beam are indeed similar. In summary, substantial variations in the predicted error fields for the QFP, QFT, and QFX magnets arise because two magnets

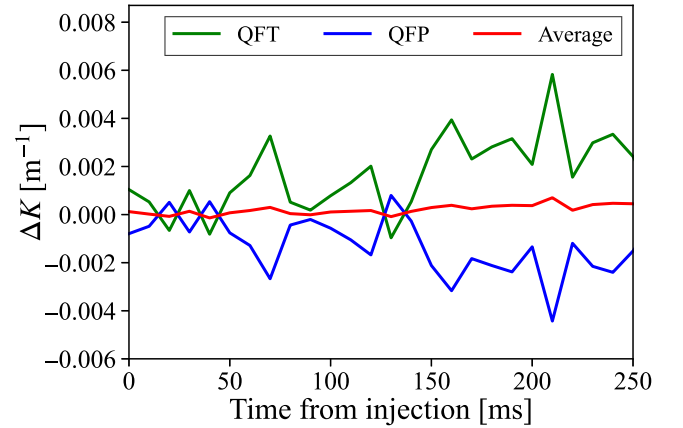


FIG. 9. Time variation of one QFP–QFT pair, including their mean value, is shown as an example. The green and blue lines represent the measured ΔK_{meas} for one QFT magnet and one QFP magnet, respectively. The red line denotes their average value.

with similar responses are treated as distinct free parameters. As a countermeasure, we considered modifying the model.

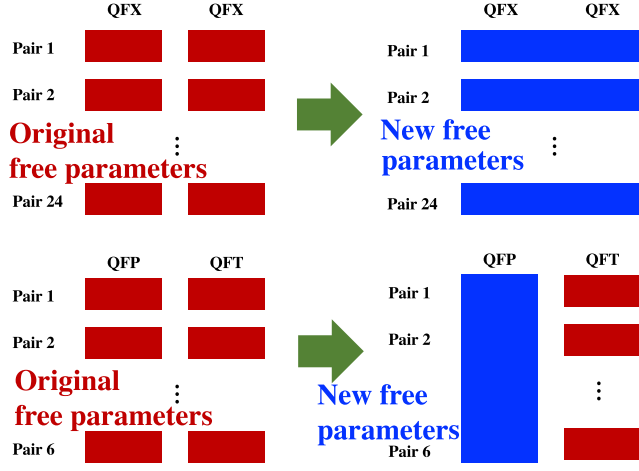


FIG. 10. New model to take measures against large variance seen in QFP, QFT, and QFX magnets.

The fit model should be adjusted to predict error fields with lower variance. One approach to address this phenomenon is to use the averaged value of each pair as the

new free parameters. In this case, if one magnet has an error, it will be predicted as an error in either of the two magnets. However, simple averaging over each QFP–QFT pair loses information on the substantial differences between the QFP and QFT families.

Therefore, a new model—modified QCPLM—was designed so that no information on the output currents of power supply families is lost (Fig. 10). In the new model, QFX is summed within a pair. Meanwhile, for QFP–QFT pairs, the summed value within the magnets in the QFP family is considered a new free parameter. The models of QFT magnets are not modified. In this case, any differences in the QFP magnets are detected as individual field errors in the QFT magnets.

The same data as used in Sec. III A are used but analyzed with the modified QCPLM. Figure 11 shows the fitted results for $N_{\text{pulse}} = 100$. The large variance of the QFX, QFP, and QFT magnets seen in Fig. 7 was successfully reduced to the same level within other families (typically smaller than 0.001 m^{-1}) by applying the modified QCPLM. The σ_K value was reduced from 0.32%

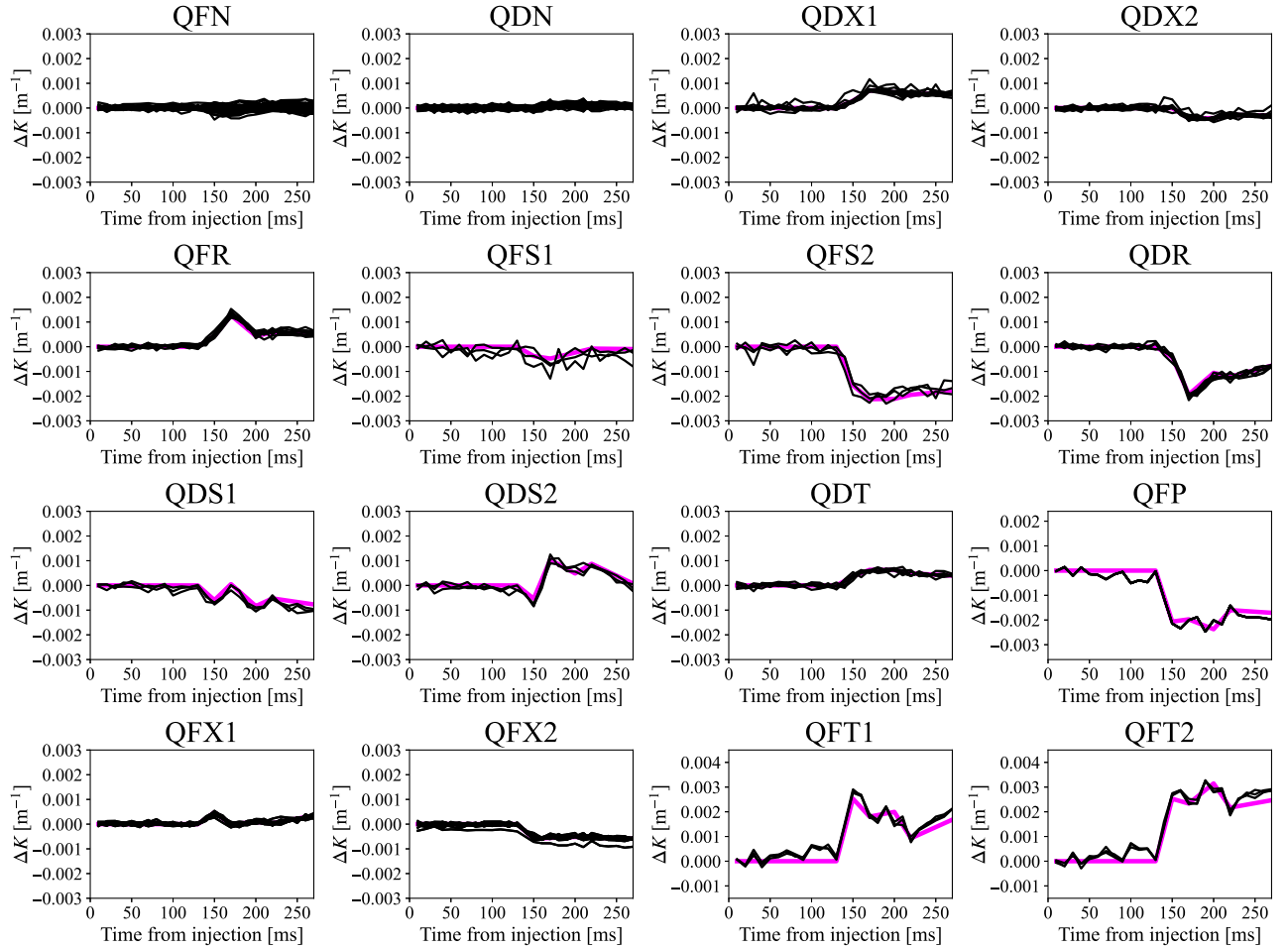


FIG. 11. ΔK_{meas} evaluated by modified QCPLM for all quadrupole magnets. Each panel represents a different power supply family. The black lines indicate the measured ΔK_{meas} for each magnet, while the magenta lines show the reference values (ΔK_{ref}). The vertical range for each panel is 0.006 m^{-1} .

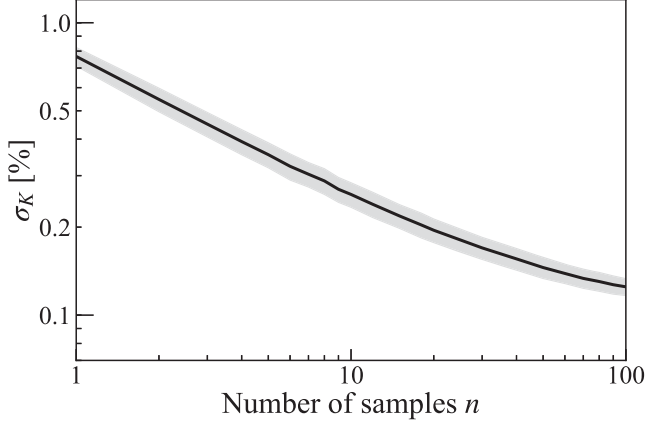


FIG. 12. Dependence of σ_K on the sampling number n . The shaded area represents the error bar, accounting only for random errors.

(QCPLM) to 0.10% (modified QCPLM) in this analysis, fulfilling the objective discussed in Sec. I. Notably, due to the averaging of free parameters when performing modified QCPLM, individual errors among the two magnets in each QFX-QFX and QFP-QFT pair exhibit positional indeterminacy. However, it rarely becomes problematic in evaluating the eddy current effects and family splitting. In the MR, the same type of vacuum ducts are used for magnets in each pair, and this will not be problematic in evaluating the eddy current effects. Meanwhile, there are no model-based systematic errors in evaluating each current of the magnet power supply families; hence, we can precisely evaluate family splitting as well.

D. Further benchmark of QCPLM by changing the number of pulses

The beam tunings must be completed within the severely limited study time. Therefore, how few pulses can be used to shorten the tuning time should be considered. In this respect, we evaluated the sampling number n dependency of σ_K using $N_{\text{pulse}} = 100$ samples of $\vec{\Delta K}(t_{\text{pat}})$.

The evaluated dependency of σ_K on n is illustrated in Fig. 12. To evaluate σ_K for n samples, the bootstrapping method [37] was employed to generate the error bar from random errors. For this purpose, n samples were randomly selected 1000 times from $N_{\text{pulse}} = 100$, permitting multiple selections of identical samples. The average of these 1000 sets represents the estimated expectation value of σ_K , depicted as solid lines in Fig. 12. The standard deviation of these 1000 sets can be interpreted as the random error for σ_K (the shaded area in Fig. 12). Using $N_{\text{pulse}} = 100$ samples yields an accuracy of 0.12%. Furthermore, if the samples used in the analysis are reduced to $N_{\text{pulse}} = 10$, the accuracy becomes 0.26%, which is still acceptable in evaluating $\Delta K/K = 1\%$ order quadrupole field errors.

IV. SURVEY ON TIME-VARYING QUADRUPOLE FIELD ERRORS IN THE MR

Among the remaining quadrupole field errors, one of the most significant concerns in the MR is related to the magnetic field errors induced by eddy currents on the vacuum ducts during acceleration. This phenomenon arises because MR ramps magnets by synchronizing their currents with the beam momentum. Changing magnetic fields generally induce eddy currents in the vacuum ducts in synchrotrons. These eddy currents generate an additional magnetic-field error that affects the beam dynamics [38–42]. Hereafter, we call this phenomenon the “eddy current effects” by vacuum ducts. The MR encompasses over 30 combinations of magnet shapes and vacuum duct types to ensure proper trajectory and physical aperture at each location around the ring. Due to variations in their shapes, materials, and thicknesses, these vacuum ducts’ eddy current effects have distinct strength and time variations. Furthermore, their arrangement along the ring does not consistently maintain threefold symmetry. Regarding these facts, we conducted surveys to identify substantial errors in quadrupole field responses during acceleration using the modified QCPLM, with eddy current effects as the primary target. In this section, the modified QCPLM is simply called QCPLM.

A. Setup

The primary objective of this survey is to identify the eddy current effects by vacuum ducts in the MR. Suppose we write the designed K value of a main quadrupole magnet and its field error due to the eddy current as $K(t)$ and $\Delta K^{\text{eddy}}(t)$, respectively. Assuming that the magnetic field tracks momentum precisely or $K(t) = \text{const}$, a simple approximation of field error by eddy currents [43] becomes

$$\frac{\Delta K^{\text{eddy}}(t)}{K} \sim F_{c_f}(p, \dot{p}) \equiv -c_f \frac{1}{p(t)} \frac{dp(t)}{dt}, \quad (16)$$

where $\dot{p}$ is the time derivative of momentum and c_f is a positive constant. c_f will be referred to as the “form factor” of the eddy current effects hereafter. The form factor c_f does not depend on momentum, time, or beam optics; it only depends on the dimensions and materials of the magnet components. Conducting the survey under various $\frac{1}{p(t)} \frac{dp(t)}{dt}$ values is preferable for identifying eddy current effects, as indicated by Eq. (16).

TABLE IV. Three operation modes of MR in December 2023; K.E. represents kinetic energy.

Operation mode	Extraction K.E. (GeV)	Ramping time (s)
FX	30	0.65
SX	30	1.4
COMET	8	0.9

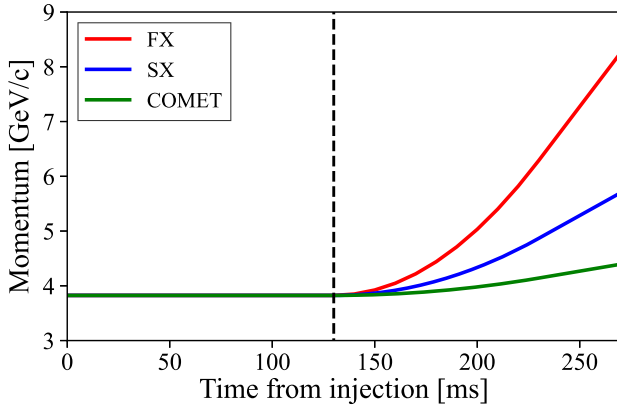


FIG. 13. Time variation of beam momentum $p(t)$ in design for each operation mode. The dashed line represents the start of acceleration.

The MR operates in three modes with distinct time variations of momentum and kinetic energy at extraction, as shown in Table IV. FX and SX represent the fast extraction mode operation for neutrino experiments and the slow extraction mode operation for hadron experiments, respectively. “COMET” mode in Table IV is a particular slow extraction operation mode for the COMET experiment to search for a coherent neutrino-less conversion of a muon to an electron using a pulsed muon beam [44]. The time variation of beam momentum of the reference particle in the design (written as $p(t)$ hereafter) for each operation mode is shown in Fig. 13. In the MR, a smoothing scheme is employed to keep the dp/dt or first derivative of magnet currents continuous in time. In this respect, $p(t)$ is a quadric function of time in the 100 ms interval after the start of the acceleration and before it reaches the maximum. For other periods, $p(t)$ is a linear function. Figure 14 illustrates the time variation of $\frac{1}{p(t)} \frac{dp(t)}{dt}$ for each operation mode. Note that the maximum point is at the end of the smoothing for all modes, where $p(t)$ changes from quadric to linear function. Figure 14 and Eq. (16) show that substantial differences in the strength of eddy current effects are expected among operation modes. Specifically, the largest eddy current effects are anticipated in FX, while the smallest are in COMET. To elucidate the quadrupole field errors attributable to eddy current effects around the ring, this survey was conducted under each of the three MR operation modes.

B. Conditions

The survey conditions are outlined in Table V. All surveys involved injecting two bunches at the onset of the flat bottom period with 2×10^{12} ppp intensity. In SX, a particular kick angle of $\theta_x = 0.3$ mrad is chosen based on iterative optics tuning experiences in the past. To rectify the condition consistent with the experimental benchmark of QCPLM discussed in Sec. III, closed-orbit data from the commencement of the flat bottom period (K.E. = 3 GeV

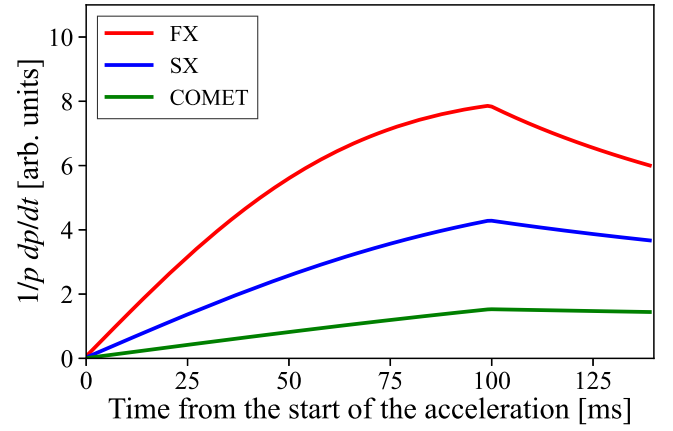


FIG. 14. Time variation of $\frac{1}{p(t)} \frac{dp(t)}{dt}$ during each operation mode.

for FX) to 140 ms (K.E. = 7 GeV for FX) beyond the time stamp after the start of acceleration were analyzed. To account for the time variation of quadrupole field errors, we focused on $\Delta K^{\text{var}}(t) = \Delta K(t) - \Delta K^{\text{bottom}}$, where $\Delta K(t)$ is the K value difference from its designed value at each time stamp, and ΔK^{bottom} is the time average of $\Delta K(t)$ among the data points in the flat bottom period.

C. Results and their interpretation

The survey results for each of the three operation modes are shown in Fig. 15. In these figures, each small panel corresponds to one of the power supply families in the MR, and the lines represent each magnet’s evaluated values of ΔK^{var} . All lines in each figure are drawn in black except for the two magnets, which we highlighted blue and red for convenience in the following discussion. For each magnet, lines are drawn in the same color among the three figures. Note that in SX and COMET operation, the septum magnets used for slow extraction could enhance leakage fields. As discussed in Appendix A, it serves as a non-negligible systematic error in SX mode operation, and therefore, its effect was subtracted before analysis.

We observe the survey results for FX operation in Fig. 15. In this figure, the ΔK^{var} of magnets shown by black lines exhibit consistent time variation among their power supply family, smaller than 0.0005 m^{-1} . However, this indicates that one QFR magnet (colored blue) and one QDT magnet (colored red) exhibit distinct time variations

TABLE V. Survey setup for each operation mode.

	FX	SX	COMET
Number of steering magnets	5	5	5
N_{pulse}	100	10	100
θ_x (mrad)	0.2	0.3	0.2
θ_y (mrad)	0.2	0.2	0.2
ν_x	21.35	22.31	22.30
ν_y	21.43	20.80	20.81

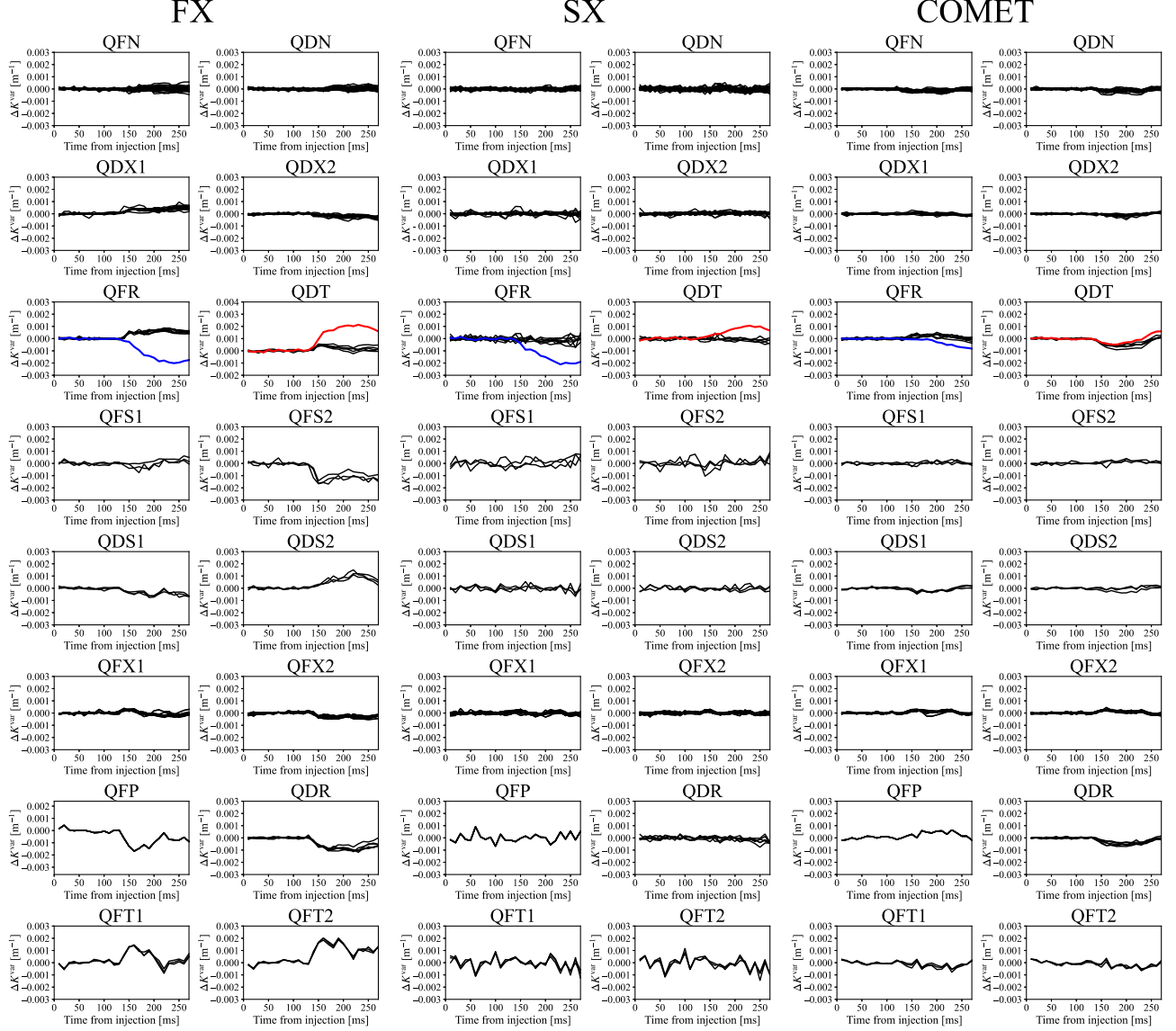


FIG. 15. Time variations of quadrupole field errors ΔK^{var} evaluated by QCPLM for each operation mode. The vertical range for each panel is 0.006 m^{-1} . Each line, including red and blue, corresponds to the fit results for individual quadrupole magnets.

from the other QFR and QDT magnets, with more than 0.002 m^{-1} ($> 1\%$ in $\Delta K/K$) discrepancy at maximum. The same two magnets are shown by blue and red lines for SX and COMET, respectively, in Fig. 15 as well. These magnets are indexed as 154 (blue) and 155 (red) in the MR, where all main quadrupole magnets are indexed from 1 to 216. Therefore, we call these magnets “QFR154” and “QDT155,” respectively. Observing these lines in Fig. 15, the discrepancies are potentially contingent on the operation mode for each magnet, being most pronounced in FX operation and least in COMET, which strongly suggests that they are from the eddy current effects.

In fact, vacuum ducts with unique shapes and thick walls are solely equipped at QFR154 and QDT155. These are called “UFO” ducts, and they are employed to ensure the extraction trajectory of the beam (see Fig. 17 in Sec. IV E).

The shapes of the two UFO ducts vary to accommodate the extraction trajectory. They are designated as “UFO140” and “UFO150.” The numbers “140” and “150” indicate the bore diameters of each quadrupole magnet. QFR154 and QDT155 correspond to UFO140 and UFO150, respectively. These specific time variations observed at QFR154 and QDT155 likely originate from the substantial eddy current effects at UFO ducts. To examine this, we conducted an analysis to compare the survey results with those inferred from the analytical formula $F_{cf}(p, \dot{p})$.

D. Analysis and discussion

Suppose the source of the distinctive behavior of the survey results seen at QFR154 and QDT155 in Fig. 15 are the eddy current effects. Then, for each of the QFR154 and QDT155 magnets, there exists a single constant form factor c_f

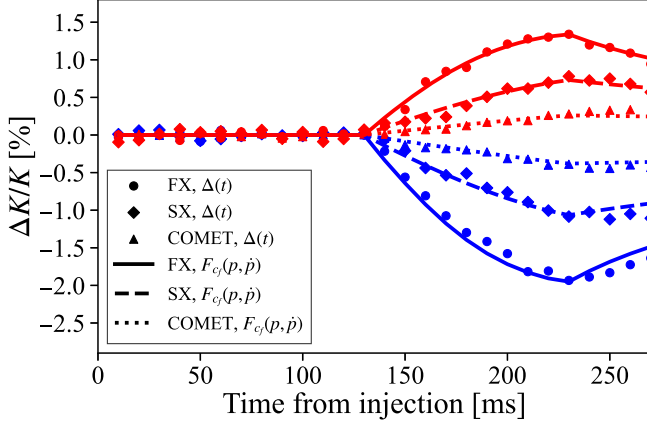


FIG. 16. The experimental results $\Delta(t)$ and evaluated $F_{c_f}(p, \dot{p})$ for each operation mode. Color schemes of blue and red represent QFR154 and QDT155, respectively. c_f was acquired by minimization of $L(c_f)$ for each of the QFR154 and QDT155 magnets. Sign convention was chosen so that the increase/decrease of $\Delta(t)$ matches to that of ΔK .

independent of operation mode whose $F_{c_f}(p, \dot{p})$ explains the behavior within the error bar of QCPLM. To examine this, we evaluated c_f that minimizes a quantity $L(c_f)$ defined as

$$L(c_f) = \sqrt{\frac{1}{MN} \sum_i^M \sum_j^N ([\Delta_i(t_j) - F_{c_f}(p_{ij}, \dot{p}_{ij})]^2)},$$

where i denotes the three operation modes, j represents time stamps in the survey, $p_{ij} = p_i(t_j)$ is the beam momentum for the i th operation mode, and $\Delta_i(t_j)$ is the discrepancy in time variation of quadrupole field errors between the QFR154 or QDT155 magnet and other magnets in their respective families. We evaluated $\Delta_i(t)$ as

$$\Delta(t) = \frac{\Delta K_\chi}{K} - \frac{1}{N-1} \sum_{n \neq \chi} \frac{\Delta K_n}{K}, \quad (17)$$

where we neglect the index i . χ represents either QFR154 or QDR155, N is the number of magnets in the corresponding power supply family (QFR or QDT), n denotes individual magnets in the family, and each $\Delta K/K$ term represents the survey results in Fig. 15. The c_f and resulting $L(c_f)$ determined from the above analysis are shown in Table VI. From this table, we can see that the resulting $L(c_f)$ is below 0.1% or the error bar of QCPLM. Meanwhile, $\Delta(t)$ and $F_{c_f}(p, \dot{p})$ of QFR154 and QDT155 for each operation mode

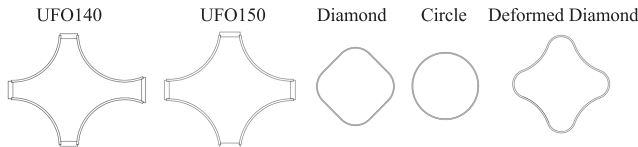


FIG. 17. Classification of vacuum duct shapes at quadrupole magnets into five groups.

TABLE VI. Optimal c_f and the residual $L(c_f)$.

Magnet	c_f (ms)	$L(c_f)$ (%)
QFR154	2.5	0.08
QDT155	1.7	0.05

are displayed together in Fig. 16. This figure exhibits substantial agreement between the survey results $\Delta(t)$ and the analytical formula $F_{c_f}(p, \dot{p})$, which describes the eddy current effects. Table VI and Fig. 16 suggest that the time variation and dependency on operation modes of QFR154 and QDT155 results come from the eddy current effects. To examine this, we conducted a further analysis by simulation approach.

E. Further analysis by simulation

A 3D electromagnetic simulation was performed to validate that the special field errors observed in QFR154 and QDT155 are from the eddy current effects at UFO ducts. The simulation utilized the “LF time domain solver” within the CST Studio Suite software [45] and focused on the FX operation with the steepest acceleration pattern among the three operation modes.

The duct shapes fall into five categories, as illustrated in Fig. 17. For this simulation, we considered five duct shapes with varying bore radii and materials. Table VII provides the dimensions, materials, and corresponding power supply family for each model utilized in the simulation. Hereafter, the longitudinal coordinate is denoted as z instead of s , assuming the longitudinal center of the magnet as $z = 0$. Moreover, a “reference model” was established for each model, where the duct material was replaced by air while retaining the mesh, and simulations were conducted accordingly. Henceforth, we refer to the models with appropriate duct materials as the “regular model.”

We simulated time variation of the magnetic fields in 10 ms intervals up to the time stamp of 140 ms after the initiation of acceleration. The current patterns were designed so that the simulated magnetic fields precisely track the momentum variation. The input currents used in this simulation are shown in Fig. 18. The validity of the input currents is discussed in Appendix D. Figure 19 illustrates the simulated magnetic field and eddy current distribution at 100 ms after the onset of acceleration for UFO140. The simulation focused on the distribution of

TABLE VII. Simulation models.

Shape	Material	Thickness (mm)	Family
UFO140	Ti	4	QFR
UFO150	Ti	4	QDT
Diamond	SUS316L	2	QFR
Circle	Ti	2	QDT
Deformed diamond	Ti	3	QFR

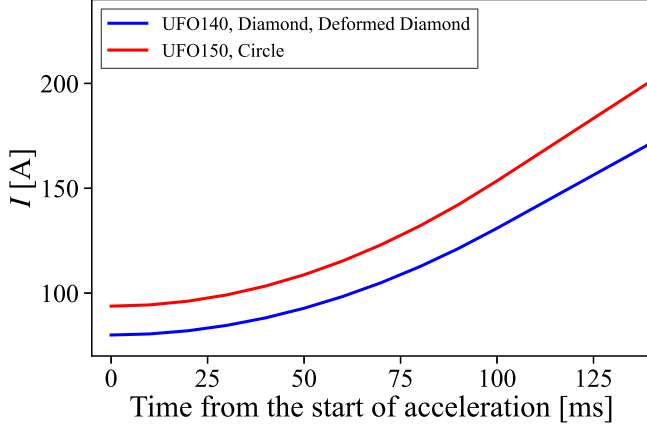
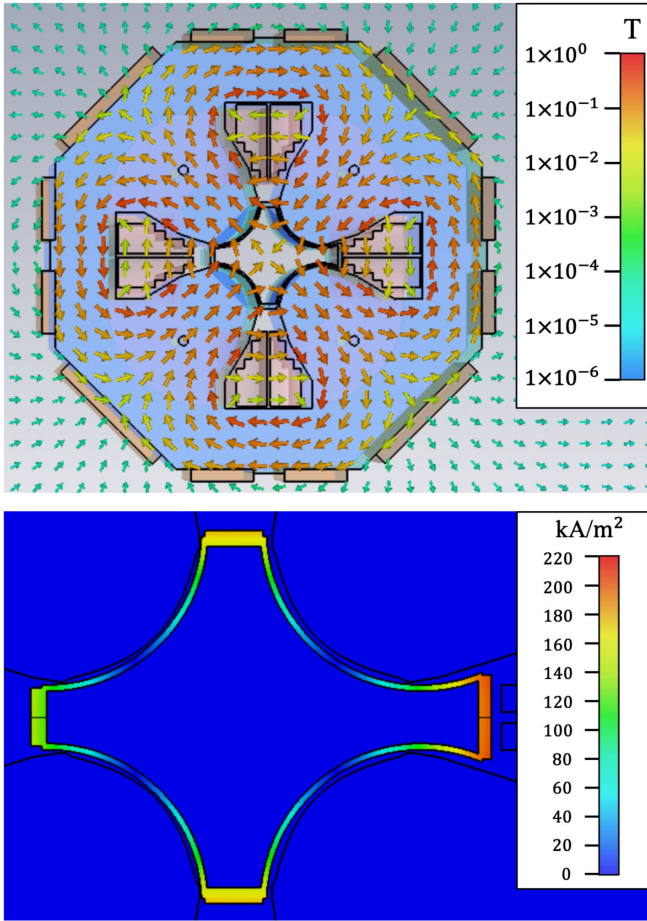


FIG. 18. Input coil current pattern for each models.

FIG. 19. Distributions of magnetic field and eddy current at $z = 0$ for the UFO140 model, 100 ms after the start of acceleration. The upper figure shows arrow plots of the magnetic field, and the lower figure shows color plots of the eddy current distribution.

the magnetic field within the good field region (GFR) in the context of MR. GFR represents the line segment with the equation $|x| < 66$ mm, $y = 0$, and $z = 0$. The main quadrupole magnets were designed to produce a uniform

quadrupole field within GFR, with a relative error of 0.05% or less during the flat bottom period at a kinetic energy of 3 GeV [46]. The simulation must maintain the same level of accuracy as that of the magnet design. Henceforth, the permissible error of 0.05% within GFR will be denoted as the “allowed error.” The uniformity of the quadrupole field can be evaluated by

$$\sigma_{\text{sim}} \equiv \left| \frac{\sum_{m=0}^M \frac{1}{m!} k_m x_{\text{edge}}^m - k_1 x_{\text{edge}}}{k_1 x_{\text{edge}}} \right|, \quad (18)$$

where x_{edge} represents points with $|x| < 66$ mm, $y = 0$ mm, and $z = 0$ mm. M is sufficiently large integer and k_m is the $2(m+1)$ -th pole strength of the magnetic field. To evaluate σ_{sim} , k_m was acquired by polynomial fitting B_y by x in GFR by considering up to the 36th pole or $M = 17$. The allowed multipoles of the quadrupole magnet are the 12, 20, 28, 36, ..., $4(2n+1)$, ...th poles, and the 36th pole corresponds to the fourth largest allowed multipoles. The evaluated σ_{sim} for each reference model is presented in Table VIII, demonstrating compliance with the allowed error criteria.

In Fig. 20, the quadrupole field errors originating from eddy current effects were determined by using the K value differences between the regular model and the reference model. The field strength is shown in $|\Delta K/K| = |(K_{\text{reg}} - K_{\text{ref}})/K_{\text{ref}}|$ where K_{reg} and K_{ref} are the simulated K value for the regular and reference models, respectively. Although the actual simulation was conducted for the acceleration period (starts at 130 ms after injection), the results for the flat bottom period (until 130 ms after injection) are drawn as well by interpolating them to 0%. Evidently, only the two UFO ducts display a quadrupole field response larger than 1%. This result is considered the collective effect of three phenomena: the thicker walls of the UFO ducts than those of other vacuum duct types, wide coverage of the magnet pole by the wall (see Fig. 19), and the larger electric conductivity of Ti than that of SUS316L ($\tilde{1.8}$ MS/m vs $\tilde{1.3}$ MS/m). The simulation result is consistent with the fact that ΔK^{var} in the QCPLM survey exceeded 1% only observed for QFR154 and QDT155. In addition, as discussed in Sec. IV D, all data points $\Delta_i(t_j)$ are well explained by analytical expression of eddy current effects $[F_{c_f}(p, \dot{p})]$ within $\Delta K/K = 0.1\%$ residual. From these two facts, the strength, time variation, and dependency

TABLE VIII. σ_{sim} for each model. The average value is calculated from the two edges for each model.

Model	Mean of σ_{sim} (%)
UFO140	0.04
UFO150	0.02
Diamond	0.04
Circle	0.01
Deformed diamond	0.03

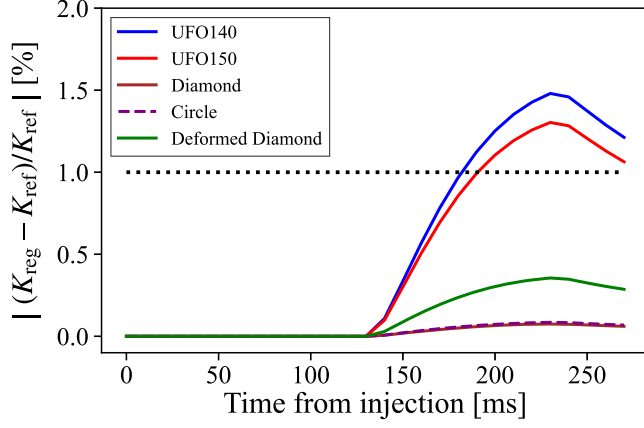


FIG. 20. Simulated $|(K_{\text{ref}} - K_{\text{reg}})/K_{\text{ref}}|$ for five reference models. The black dotted line corresponds to a 1% deviation. Results for the flat bottom period (up to 130 ms after injection) are also shown, interpolated to 0%.

on momentum of $\Delta_i(t)$ evaluated in the experimental survey aligns with that predicted for eddy current effects by UFO ducts. Thus, the primary cause of the quadrupole field errors $\Delta_i(t)$ for QFR154 and QDT155 is the eddy current effects from the vacuum ducts.

We note that there are some discrepancies among the simulation and experimental results. For example, the absolute value of error fields by eddy currents in UFO140 is approximately 0.5% apart between the experiment and the simulation (Figs. 16 and 20, respectively). In addition, the simulation suggests that there are approximately $|(K_{\text{ref}} - K_{\text{reg}})/K_{\text{ref}}| = 0.3\%$ quadrupole field errors in the deformed diamond ducts, where no significant corresponding error fields were observed in the experimental survey. The discrepancies between the simulation and the experiment could come from factors unconsidered in the simulation, such as the tracking errors [47] in the currents of magnet power supplies, delay in magnetic field response during ramping, variance in the imperfection of individual magnets, and eddy current effects from the nearby high-conductivity components. Aligning the simulation results through precise modeling of each factor is not the focus of this paper. The purpose of the simulation was to confirm that the eddy current effects in UFO ducts indeed contribute to 1% order quadrupole field errors, which was successfully attained.

V. CORRECTION OF EDDY CURRENT EFFECTS

Now that substantial eddy current effects have been identified within the ring, we explore potential corrective measures. As part of our experimental efforts, we endeavored to utilize trim coils as a means to address these effects. Trim coils serve as secondary coils alongside the main coil to correct error fields at individual magnets in the MR. Although trim coils are present at each of the main quadrupole magnets, only three are connected to power supplies. Among these three, two of them are equipped at

TABLE IX. Survey setup details for each operation mode.

Number of steering magnets	5
N_{pulse}	20
θ_x (mrad)	0.2
θ_y (mrad)	0.2
Beam intensity	2×10^{12} ppp

QFR154 and QDT155, where UFO ducts are installed. Therefore, the error fields identified by QCPLM can be corrected using trim coils, as demonstrated experimentally during beam tuning for the FX operation.

The setup for the experiment is detailed in Table IX. The currents for the trim coils were computed based on the QCPLM results. The maximum currents required to correct the eddy current effects were 5.6 and 4.8 A for QFR154 and QDT155, respectively. Utilizing QCPLM, the time variation of the quadrupole field errors around the ring was investigated with and without the application of correction for eddy current effects using trim coils. The survey results on the quadrupole field errors for QFR and QDT families with and without utilizing trimcoils are presented in Fig. 21. As observed, the correction of the eddy current effects using trim coils has proven successful with a residual of $\sim 0.0002 \text{ m}^{-1}$ ($\sim 0.1\%$ in $\Delta K/K$) for both QFR154 and QDT155.

Although the correction was conducted in a 1.36 s repetition cycle with 0.65 s acceleration, the same measure is applicable in future 1.16 s repetition cycle operation. The quadrupole field errors by eddy currents at UFO ducts in such operation were predicted from c_f determined in Sec. IV D, as shown in Fig. 22. Because the operation cycle becomes 0.2 s shorter, we considered the steepest case of 0.45 s acceleration. The calculated maximum currents required to correct these field errors in the future are 8.8 and 7.5 A for QFR154 and QDT155, respectively. Meanwhile, the rated

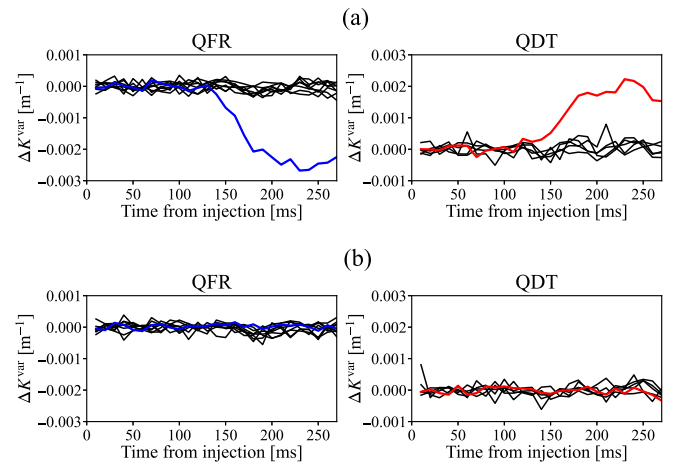


FIG. 21. Time variation of quadrupole fields in the QFR and QDT families (a) before and (b) after applying trim coils to correct eddy current effects. Blue and red lines represent magnets with UFO140 and UFO150 ducts, respectively.

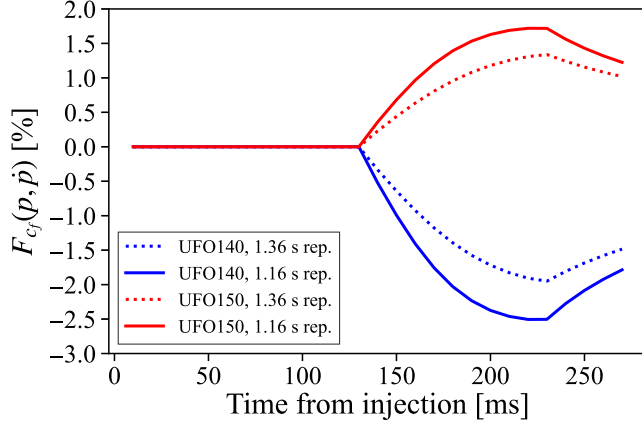


FIG. 22. Eddy current effects by UFO ducts for the current 1.36 s and future 1.16 s repetition operations, predicted from $F_{c_f}(p, \dot{p})$ evaluated in Sec. IV D. The abbreviation “rep.” denotes the repetition cycle.

value of the trim coils utilized in the MR is 160 A. Therefore, we could expect to employ an identical method as in current operation in the future as well.

VI. CONCLUSION AND PERSPECTIVE

We developed a precise method to detect time-varying quadrupole field errors around the ring with 0.1% accuracy. We successfully demonstrated its precision through beam experiments to benchmark the method. Utilizing this method, we identified substantial quadrupole field errors caused by eddy currents on vacuum ducts during acceleration, which were not evaluated or corrected in the past MR beam tuning. To rectify these, we established and demonstrated an approach applicable under higher-repetition rate conditions in the future.

The method of QCPLM developed in this paper is already one of the primary optics tuning methods in the MR and has played an important role in achieving the MR’s 800 kW operation. In addition, the measures for the eddy current effects are already utilized in the beam tuning for the user operations in the MR as well. Further study will be conducted to correct the quadrupole field errors around the whole ring with 0.1% accuracy, anticipating the 1.3 MW operation.

ACKNOWLEDGMENTS

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APPENDIX A: LEAKAGE FIELDS FROM THE SEPTUM MAGNETS FOR SLOW EXTRACTION

In the MR, there are ten septum magnets for slow extraction, which can be categorized into three types: low-field (“SMS1”), middle-field (“SMS2”), and high-field septum (“SMS3”) from the upstream. Their locations in the

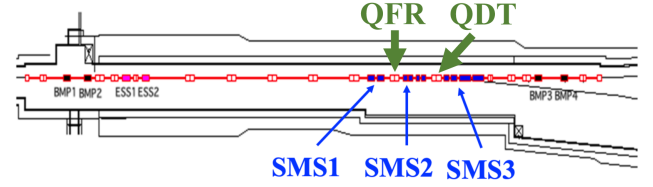


FIG. 23. Septum magnets are located in Ins B [48].

MR are depicted in Fig. 23. There are two, four, and four magnets for SMS1, SMS2, and SMS3, respectively. The strength of quadrupole fields at the flat bottom period (K.E. = 3 GeV) evaluated by field measurement for each of the ten magnets is presented in Table X. Unlike main and steering magnets, septum magnets for slow extraction are ramped with the same currents from the start to end of acceleration. Therefore, the time variation of the K value due to the leakage field from the septum magnets $\Delta K^{\text{sept}}(t)$ can be expressed as

$$\Delta K^{\text{sept}}(t_{\text{acc}}) = K^{\text{F.B.}} \left(\frac{p(t_{\text{acc}})}{p(t_{\text{acc}}=0)} - 1 \right), \quad (\text{A1})$$

where $K^{\text{F.B.}}$ is the K value at the flat bottom determined from Table X, and t_{acc} is the time from the start of the acceleration. From Eq. (14), the closed-orbit response $\vec{z}$ corresponding to $\Delta \vec{K}^{\text{sept}}$ can be expressed as

$$\vec{z}^{\text{sept}}(t_{\text{acc}}) = A \Delta \vec{K}^{\text{sept}}(t_{\text{acc}}), \quad (\text{A2})$$

where A is the linearized model for QCPLM and $\Delta \vec{K}^{\text{sept}}$ is the vector whose elements are $\Delta K^{\text{sept}}(t_{\text{acc}})$ at each of the ten septum magnets. As $\Delta \vec{K}^{\text{sept}}$ serves as the systematic error in the survey using QCPLM, they were subtracted before analysis in the following method. Let us write $\vec{z}^{\text{meas}}$ the raw closed-orbit response vector measured in the survey. Then, the vector $\vec{z}^{\text{meas}} - \vec{z}^{\text{sept}}$ can be considered the closed-orbit response with the error fields caused by the leakage field of septum magnets subtracted. $\Delta \vec{K}^{\text{sept}}$ can be determined from the field measurement results shown in Table X.

Figure 24 shows a comparison of the excerpt of survey results of SX operation using $\vec{z}^{\text{meas}}$ and $\vec{z}^{\text{meas}} - \vec{z}^{\text{sept}}$. In the left panels without subtracting the effect of the leakage field,

TABLE X Quadrupole fields due to leakage from slow-extraction septum magnets, evaluated by field measurements [49]. The quadrupole field strength is expressed as pK_1/q with units in (T), where K_1 represents the K value, p is the beam momentum, and q is the proton charge.

Name of septum magnets	pK_1/q (T)
Sum of two SMS1 magnets	0.03685
Sum of four SMS2 magnets	-0.03596
Sum of two SMS3 in upper stream	-0.02143
Sum of two SMS3 in lower stream	-0.00325

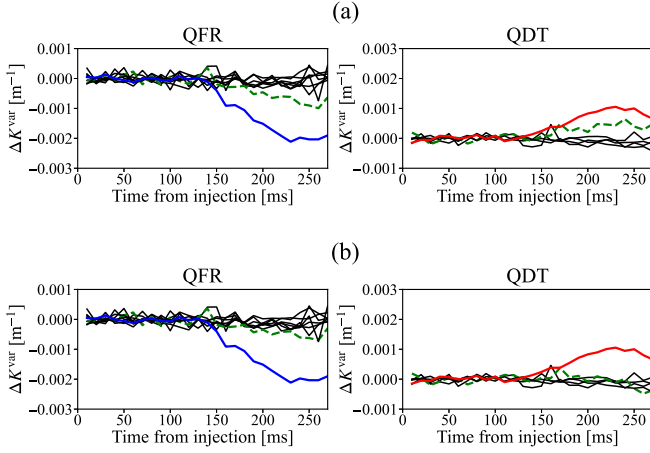


FIG. 24. Time variation of ΔK^{var} in QFR and QDT families deduced from (a) $\vec{z}^{\text{meas}}$ and (b) $\vec{z}^{\text{meas}} - \vec{z}^{\text{sept}}$. Definition of ΔK^{var} is given in Sec. IV B. Blue and red lines represent magnets with UFO140 and UFO150 ducts, respectively. Dashed green lines show fit results that vary between the two analyses.

systematic field errors of approximately 0.001 m^{-1} (typically $\Delta K/K > 0.5\%$) deviation at maximum are observed (shown as green dashed lines). Meanwhile, observing the right panels with subtraction, they diminished to $< 0.0005 \text{ m}^{-1}$ (typically $\Delta K/K < 0.3\%$). Therefore, in Fig. 15, their effects were subtracted from the survey results beforehand using this scheme. Although COMET operation employs these septum magnets as well, the effects of their leakage field are not subtracted in Fig. 15, as no systematic errors that significantly exceed 0.1% order are observed. We can explain this behavior from the fact that both the magnet currents and the $(\frac{p(t_{\text{acc}})}{p(t_{\text{acc}}=0)} - 1)$ term in Eq. (A1) are much smaller than those of SX operation at the initial period of acceleration. This is because the extraction energy of COMET experiments is 8 GeV, while it is 30 GeV for SX operation.

APPENDIX B: ROBUSTNESS OF QCPLM ON RANDOM ERROR

QCPLM is robust against the random errors because it uses a linearized model $\vec{z} = A\Delta\vec{K}$. A detailed explanation is given below.

Consider the singular value decomposition of the response matrix A as $A = \sum_{i=1}^r \sigma_i \vec{u}_i \vec{v}_i^T$, where $\vec{u}_i \cdot \vec{u}_j = \delta_{ij}$ and $\vec{v}_i \cdot \vec{v}_j = \delta_{ij}$. r is the rank of A , and σ_i are singular values. The solution of the linear equation $\vec{z} = A\Delta\vec{K}$ can be expressed as

$$\Delta\vec{K} = \sum_i \vec{v}_i \left(\frac{\vec{z} \cdot \vec{u}_i}{\sigma_i} \right). \quad (\text{B1})$$

Observing Eq. (B1), removing the effects of measurement errors from the term $\vec{z} \cdot \vec{u}_i$ is important. Taking random errors $\vec{\epsilon}$ into consideration, $\vec{z}$ can be expanded as

$$\vec{z} = \alpha_1 \vec{u}_1 + \alpha_2 \vec{u}_2 + \cdots + \alpha_r \vec{u}_r + \vec{\epsilon}. \quad (\text{B2})$$

Given that $\vec{u}_i \cdot \vec{u}_j = \delta_{ij}$, the impact of random errors affects the solution as $\vec{z} \cdot \vec{u}_i = \alpha_i + \vec{u}_i \cdot \vec{\epsilon}$. However, if the dimension of $\vec{z}$ is sufficiently large, the likelihood of the vectors $\vec{u}_i$ and $\vec{\epsilon}$ coincidentally aligning is small, and $\vec{u}_i \cdot \vec{\epsilon} \approx 0$ can be a reasonable approximation. Leveraging this principle, QCPLM mitigates the effects of the random errors.

APPENDIX C: EFFECTS OF CURRENT RIPPLES AND MOMENTUM DEVIATION ON CLOSED ORBITS

QCPLM utilizes closed orbits as an input. The current ripples in bending magnets and momentum deviation serve as a systematic or random error in the closed-orbit measurement. In this paper, these effects are subtracted from the measured closed orbits before applying QCPLM. This section delves into the details of the method. In the first subsection of this appendix, we discuss a method to evaluate momentum deviation used in the MR. In the second subsection, we apply it to suppress the effects of the current ripples and momentum deviation.

1. Measurement of momentum deviation in the MR

In the MR, momentum deviation $\delta = \Delta p/p$ has been monitored during beam operation by the following method. Consider horizontal motion and suppose an arbitrary dipole field error $k_0(s)$ around the ring. The closed orbit observed at BPM location $s = s_1$ can be measured as

$$x(s_1) = \oint G_x(s_1, s) k_0(s) ds + D(s_1) \delta, \quad (\text{C1})$$

where $D(s)$ is the dispersion function and $G_x(s', s)$ denotes the Green's function for the closed-orbit response. As indicated by Eq. (C1), computing δ by simply dividing $x(s_1)$ by the dispersion function is inappropriate due to the dipole errors. However, this issue can be suppressed by utilizing two BPMs with a relative betatron phase difference of $(2n+1)\pi$ along with the same dispersion. Consider two BPMs at $s = s_1, s_2$ ($s_2 > s_1$) with much smaller longitudinal distance $|s_2 - s_1|$ than the circumference. As $G_x(s_1, s) + G_x(s_2, s) = 0$ if $s < s_1$ or $s > s_2$ and $D(s_1) = D(s_2)$, we have

$$\begin{aligned} x(s_1) + x(s_2) &= [D(s_1) + D(s_2)] \delta + \int_{s_1}^{s_2} [G_x(s_1, s) + G_x(s_2, s)] k_0(s) ds \\ &\approx [D(s_1) + D(s_2)] \delta. \end{aligned} \quad (\text{C2})$$

Note that we omitted the integration in the middle term because $|s_2 - s_1|$ is assumed to be small. This approximation is justified by the experimental results presented in Sec. C 2 or Fig. 26.

In the MR, 12 pairs of 2 BPMs, whose betatron phase difference is 3π with the same dispersion, are used to perform Eq. (C2). There are two pairs at each of the six half-arcs. Altogether, 24 BPMs are used to evaluate δ as

$$\delta = \frac{\sum_{i=1}^{24} x(s_i)}{\sum_{i=1}^{24} D(s_i)}. \quad (\text{C3})$$

2. Suppressing the effects of bending magnet power supply current ripples and momentum deviations

Random fluctuation in bending magnet power supply output currents causes undesired closed-orbit distortion (COD). We will call these “BM ripples” in this subsection. A method to suppress these effects on closed orbit is considered below.

The dispersion function at each location is typically determined from all dipole fields around the ring. However, each half-arc of the MR is designed to be achromatic. Additionally, all bending magnets in a half-arc are driven by one power supply. Therefore, if the current deviates in one of the power supplies, COD is seen only along a certain half-arc; along the other half-arcs, it is ideally zero. The model calculations to confirm this using SAD are illustrated in Fig. 25. In this model calculation, the current deviation $\frac{\Delta B_y(s)}{B} = 0.0005$ is given at one of the half-arcs, and COD is only seen at the corresponding half-arc. Therefore, the COD due to the BM ripples observed at the i th BPM can be written as

$$\Delta x_i^{\text{CO}} = \sum_{k=\text{half-arc}} d(i, k) D_i \left[\frac{I_{\text{out}}^{(k)} - I_{\text{set}}^{(k)}}{I_{\text{set}}^{(k)}} \right], \quad (\text{C4})$$

where Δx_i^{CO} is the measured closed orbit. Note that $d(i, k) = 1$ if the i th BPM is within the k th half-arc;

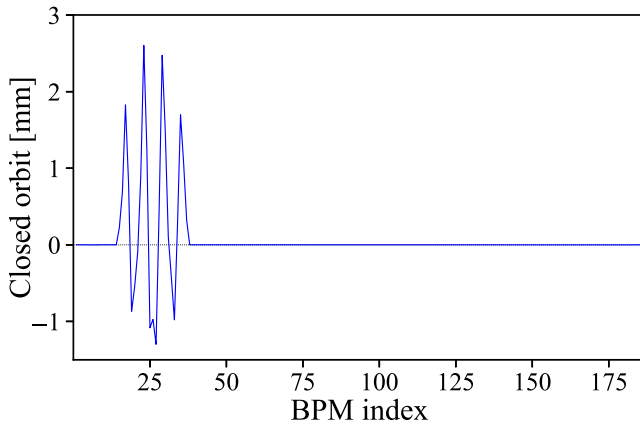


FIG. 25. COD caused by a current deviation $\frac{\Delta B_y(s)}{B} = 0.0005$ applied to one of the bending magnet power supplies.

otherwise, $d(i, k) = 0$. $I_{\text{out}}^{(k)}$ and $I_{\text{set}}^{(k)}$ are the actual output and set value of currents at each half-arc, respectively.

The concept behind the method involves the modification of Eq. (C3). This modified approach capitalizes on achromatic optics. Let us assume four BPMs at the k th half-arc used for δ evaluation in Appendix C 1 are positioned at $s_1^{(k)}$, $s_2^{(k)}$, $s_3^{(k)}$, and $s_4^{(k)}$. The current deviations and δ can be expressed as

$$\xi_k \equiv \left[\delta + \frac{I_{\text{out}}^{(k)} - I_{\text{set}}^{(k)}}{I_{\text{set}}^{(k)}} \right] = \frac{\sum_{i=1}^4 x(s_i^{(k)})}{\sum_{i=1}^4 D(s_i^{(k)})}, \quad (\text{C5})$$

even in the presence of arbitrary dipole errors around the ring. ξ_k can be evaluated from the measured $x(s_i^{(k)})$, and the closed-orbit response by ξ_k observed at the i th BPM can be written as $\xi_k d(i, k) D(s_i)$.

The correctness of Eq. (C5) was experimentally evaluated during beam tuning of the SX operation. In this experiment, we observed the horizontal COD around the ring at 10 ms after injection. The beam intensity was 2×10^{12} ppp, and the operation point was $(\nu_x, \nu_y) = (22.31, 20.80)$. The same experiment was conducted 75 times (75 pulses) and the standard deviation of the horizontal CODs of the 75 samples was computed at each BPM. The results are plotted in Fig. 26. The BPMs at the high-dispersion location showed large random fluctuations (0.65 mm. at most), which were removed after subtracting

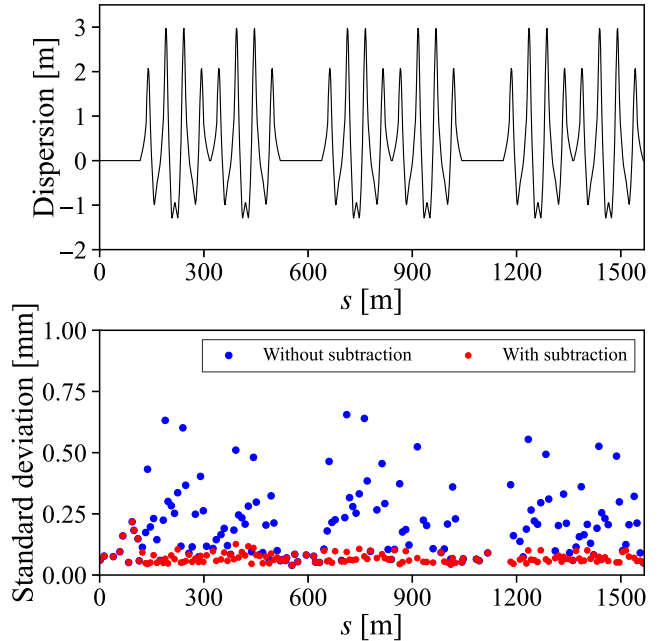


FIG. 26. Ideal dispersion function (top) and standard deviations of the horizontal beam positions at 10 ms after injection of 75 pulses (bottom). In the bottom panel, the blue and red dots are the standard deviations of the horizontal COD before and after subtracting the closed-orbit response of ξ_k [Eq. (C5)], respectively.

the closed-orbit response of ξ_k using Eq. (C5). This result confirms that Eq. (C5) can remove the effects of the error term ξ_k . For all experiments in Secs. III–V, $\xi_k d(i, k) D(s_i)$ is evaluated and subtracted before performing QCPLM.

APPENDIX D: VALIDITY OF INPUT COIL CURRENTS IN THE 3D ELECTROMAGNETIC SIMULATION

In the electromagnetic simulation in Sec. IV E, the input coil currents were designed so that the simulated magnetic fields precisely track the momentum variation [i.e., $K(t) = K_{\text{design}} = \text{constant}$] for the reference models. In general, this condition is not achievable simply by determining coil current $I(t)$ as $I(t) \propto p(t)$, for various reasons, such as the B-H characteristics of the magnet core and eddy currents in the nearby components other than the vacuum ducts. Therefore, we iteratively conducted simulation, adjusting input currents after each iteration. Note that in the real machine, current deviations can be introduced by transfer functions in the system such as back-emf responses. During operation, these effects are corrected by providing an additional input voltage until the relative error between the reference current I_{ref} and output current I_{out} reduces to the order of 10^{-4} or lower [50,51]. As these errors are negligible when evaluating 1% order field errors, they were ignored in our simulation.

The input currents shown in Fig. 18 are the results after several iterations. The maximum discrepancies between the designed and simulated K values are shown in Table XI in terms of $(K_{\text{sim}} - K_{\text{design}})/K_{\text{design}}$. K_{sim} is the simulated K value for the reference models. The K values were assessed by integrating B_y into the z direction along straight lines defined by $x = -64, -60, -56, -52, \dots, 64$ mm with $y = 0$ and polynomial fitting of the integrated B_y for each x coordinate incorporating up to the 36th pole component. We can confirm that $(K_{\text{sim}} - K_{\text{design}})/K_{\text{design}}$ is at maximum of $\sim 0.06\%$ among all models and time stamps, which is sufficiently small for evaluating 1% order field errors. This shows that the magnetic field correctly tracks momentum, and it validates the input currents as well.

TABLE XI Maximum $(K_{\text{sim}} - K_{\text{design}})/K_{\text{design}}$ among time for each model.

Model	MAX $[(K_{\text{sim}} - K_{\text{design}})/K_{\text{design}}]$ (%)
UFO140	0.008
UFO150	0.02
Diamond	0.05
Circle	0.06
Deformed diamond	0.02

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