

Multibend achromat lattice with four dispersion bumps for sextupole cancellation

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Hybrid multibend achromat (MBA) lattice with a pair of dispersion bumps separated by $-I$ transformation is used in some diffraction-limited storage ring (DLSR) designs, which can obtain relatively large dynamic apertures. However, the $-I$ transformation also largely restricts the horizontal tune of the lattice cell. In this paper, an MBA lattice with four dispersion bumps is proposed, which has relatively good flexibility in adjusting the number of bends in the lattice cell and the lattice cell tunes, while also achieving the effective sextupole cancellation within the lattice arc section. The new lattice can have a larger horizontal cell tune than the hybrid MBA lattice with the same number of bends, which is beneficial for reducing the emittance. In the new lattice, $-I$ transformation is not necessary for the sextupole cancellation between paired bumps, and a general relation of sextupole cancellation is used. Following this lattice concept, a 7BA lattice is preliminarily designed for a 1.5 GeV DLSR. This lattice concept can also be extended to the case of MBA lattice with more bumps.

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I. INTRODUCTION

Multibend achromat (MBA) lattices [1] are used in the design of diffraction-limited storage rings (DLSRs) to reduce the emittance. The pursuit of ultralow emittances in the design of DLSRs generally results in serious nonlinear dynamics effects, severely limiting the dynamic aperture (DA) and momentum aperture (MA). The hybrid MBA (HMBA) lattice [2], proposed by ESRF-EBS, is adopted by some DLSRs [3–8] due to its relatively large on-momentum DA and relatively weak sextupole strengths. This is attributed to the two dispersion bumps created at both ends of the lattice arc section, which are separated by $-I$ transformation, enabling very effective sextupole cancellation within a lattice cell. However, the $-I$ transformation also largely restricts the horizontal tune of the lattice cell, and a larger value of the tune is beneficial for reducing emittance. Compared to the 7BA lattice of SLS 2.0 [9], the H7BA lattice of ESRF-EBS [2] has an obviously smaller horizontal cell tune. Recently, an HMBA lattice with $-I$

across a straight section was proposed [10], where the number of bends in the lattice cell and the cell tunes can be flexibly adjusted due to the lattice arc section being free from the constraint of $-I$. However, the phase space tori are distorted in the straight section due to the sextupole cancellation across the straight section. If both the flexible adjustment and the sextupole cancellation can be achieved in the lattice arc section, substantial modifications would need to be made to the typical HMBA lattice.

In Ref. [11], a 5BA lattice with four dispersion bumps was proposed for Sirius. In Ref. [12], we proposed an MBA lattice with four dispersion bumps, where sextupole cancellation for paired bumps is made using $-I$ transformation in an interleaved way. Specifically, in this lattice, $-I$ for one pair of bumps is applied in the arc section, while $-I$ for the other pair is implemented across the straight section. In Ref. [13], a general relation for the nonlinear cancellation of two sextupoles is presented, and $-I$ transformation is not a necessary condition for this cancellation. Inspired by these two works, in this paper, we propose a new MBA lattice with four dispersion bumps for sextupole cancellation, where the cancellation is achieved within the lattice arc section, utilizing the general cancellation relation. This new lattice can have a larger horizontal cell tune than the HMBA lattice with the same number of bends, and the number of bends in the lattice cell and the cell tunes can be relatively flexibly adjusted while satisfying the phase advances required for the general cancellation relation.

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The paper is organized as follows. In Sec. II, the characteristics of the new MBA lattice concept are described, including the relation of sextupole cancellation used. In Sec. III, a 7BA lattice following this concept is designed for a 1.5 GeV DLSR, and the nonlinear dynamics of the lattice is also presented. Finally, conclusion and outlook are given in Sec. IV.

II. LATTICE CONCEPT

The upper part of Fig. 1 shows the conceptual schematic of the new MBA lattice with four dispersion bumps, and the typical HMBA lattice is also shown in the lower as a comparison. Both the HMBA lattice and the new MBA lattice, placing sextupoles in the bumps, can implement the sextupole cancellation within a lattice cell, which is more effective than the cancellation over some cells in enlarging DA. In both lattices, the horizontal and vertical phase advances between paired bumps satisfy the condition:

$$\Delta\mu_x = (2n_x + 1)\pi, \quad \Delta\mu_y = (2n_y + 1)\pi, \quad (1)$$

where n_x and n_y are integers. In the HMBA lattice, the beta functions of paired bumps are symmetric, forming a $-I$ transformation for sextupole cancellation. In the new MBA lattice with four bumps, where the sextupole cancellation is made in an interleaved way, the beta functions of the first and third bumps can be asymmetric. We will show the

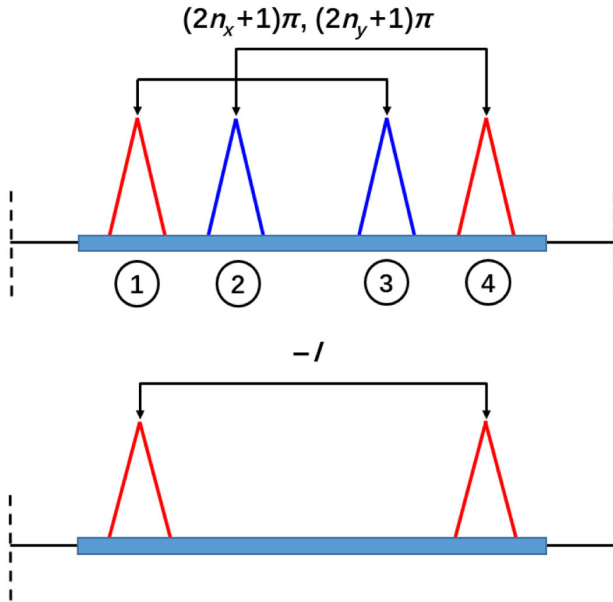


FIG. 1. Schematic of the new MBA lattice with four dispersion bumps (upper) and the typical HMBA lattice (lower). In the new MBA lattice, $-I$ transformation is not a necessary condition for the sextupole cancellation between paired bumps; the beta functions of paired bumps are generally asymmetric, represented by different colors.

details of this sextupole cancellation with asymmetric beta functions later.

An advantage of the new MBA lattice is that, compared to the HMBA lattice with the same number of bends, it can have a larger horizontal tune of lattice cell, which is beneficial for reducing emittance. In the typical H7BA lattice, the phase advances between two bumps with five bend cells in between are $(1.5, 0.5) \times 2\pi$; thus, the average value of the horizontal phase advance of a bend cell μ_x is $0.3 \times 2\pi$. While in the 7BA lattice with four bumps, which will be presented in the next section, there are four bend cells between paired bumps, giving an average μ_x of $0.375 \times 2\pi$. The study in Ref. [14] shows that the emittance of a bend cell is proportional to μ_x^{-3} when μ_x is less than $\sim 0.4 \times 2\pi$. So for these two lattices with the same energy and number of lattice cells, the emittance of the 7BA lattice with four bumps is about 1/2, i.e., $(0.3/0.375)^3$, of that of the H7BA lattice.

The new MBA lattice also has good structural flexibility. If the phase advances between paired bumps are $(1.5, 0.5) \times 2\pi$, the lattice can be a 7BA lattice or other MBA lattices. As shown in Fig. 2, keeping the number of bends between paired bumps at 4 to satisfy the phase advances, it is 7BA when the number of bends between the first and second bumps and the number of bends between the second and third bumps are 1 and 3, respectively. It becomes 8BA when the numbers are 2 and 2, and 9BA when the numbers are 3 and 1. Due to more bends used, 8BA and 9BA can have lower emittances for the same number of lattice cells. In addition, smaller bend cell tunes can be good for improving nonlinear dynamics [15]. Thus, to achieve better nonlinear dynamics performance, the number of bends between paired bumps can be increased

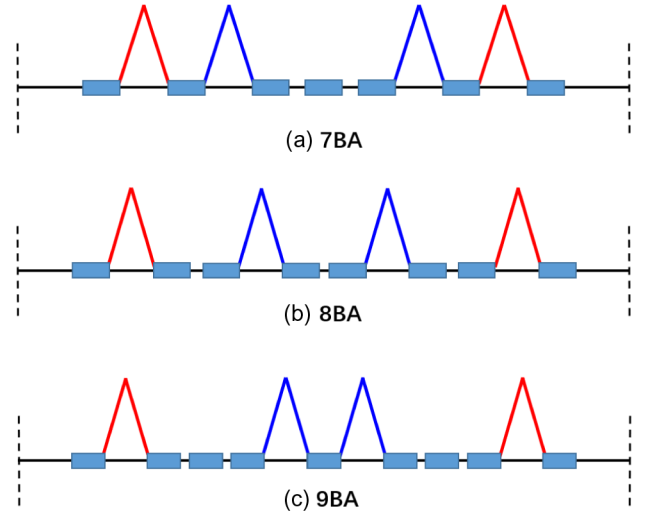


FIG. 2. Schematic of (a) 7BA, (b) 8BA, and (c) 9BA lattices with four dispersion bumps, where the number of bends between paired bumps is 4 to satisfy the horizontal and vertical phase advances of $(1.5, 0.5) \times 2\pi$.

to 5 to satisfy the phase advances of $(1.5, 0.5) \times 2\pi$ as in the H7BA lattice. In this case, by adding a bend between the second and third bumps, the 7BA, 8BA, and 9BA lattices in Fig. 2 will become 8BA, 9BA, and 10BA, respectively.

Next, we describe the sextupole cancellation scheme in the new MBA lattice. The natural idea for this scheme is to create $-I$ transformation between paired bumps as in the HMBA lattice. Thus, in addition to satisfying the phase advance condition of Eq. (1), the beta functions of paired bumps need to be the same. However, this imposes a strong constraint on the lattice design, which can, for example, limit further reduction in emittance. Actually, making the beta functions of paired bumps the same is not necessary for achieving sextupole cancellation.

For two noninterleaved thin-lens sextupoles S_i and S_j with the phase advances satisfying Eq. (1), according to Ref. [13], their nonlinear effects can be canceled when

$$\beta_{xi}/\beta_{yi} = \beta_{xj}/\beta_{yj} \quad (2)$$

and

$$\lambda_i \beta_{xi}^{3/2} = \lambda_j \beta_{xj}^{3/2}, \quad (3)$$

where β_x , β_y , and λ are the horizontal and vertical beta functions at the positions of sextupoles and integrated strengths of sextupoles. When $\beta_{xi} = \beta_{xj}$, the cancellation condition degenerates to $-I$ transformation. Equations (2) and (3) mean that the sextupole cancellation can also be achieved by controlling the relation between β_x , β_y , and λ . It is convenient to implement this relation in the lattice design considering linear optics and nonlinear dynamics simultaneously.

The sextupole cancellation in the new MBA lattice adopts the relations of Eqs. (2) and (3), but it is realized with interleaved bumps, as shown in Fig. 1. For the most important first-order geometric resonance driving terms (RDTs), they are canceled within a lattice cell. The first-order geometric RDTs are expressed as [16]

$$h_{abcd0} \propto \sum_{i=1}^N \lambda_i \beta_{xi}^{\frac{a+b}{2}} \beta_{yi}^{\frac{c+d}{2}} e^{i(a-b)\phi_{xi} + i(c-d)\phi_{yi}}, \quad (4)$$

where $a + b + c + d = 3$, and ϕ_x and ϕ_y are the horizontal and vertical phases at the i th sextupole, respectively. With the cancellation conditions above, the contributions of sextupoles in paired bumps to each first-order geometric RDT are canceled. However, for higher-order geometric RDTs arising from the crossing terms of lower-order geometric RDTs, they are usually not canceled out and are closely related to the nonlinear effects of interleaved sextupoles. As illustrated in Refs. [17,18], smaller variation or fluctuation of the first-order RDTs along the longitudinal

position generally results in smaller higher-order RDTs; and thus the magnitude of the first-order RDT fluctuations can reflect the magnitude of the nonlinear effects of interleaved sextupoles. Therefore, the fluctuation of the first-order RDTs of sextupoles can be used to study the nonlinear effects of interleaved bumps for the new MBA lattice.

In the new MBA lattice, in addition to the fixed phase advances between paired bumps as shown in the upper part of Fig. 1, the phase advances between the first and second bumps are variable. The variable phase advances were scanned to calculate the fluctuation of the first-order geometric RDTs. In the calculation, for simplicity, we considered only one thin sextupole in each bump, located in the middle of the bump; and the horizontal and vertical beta functions at the positions of sextupoles are set to 1 m and the integrated strengths of sextupoles are set to 1 m^{-2} . According to Ref. [17], the fluctuation of the first-order geometric RDTs, denoted as $f_{3,\text{avg}}$, was calculated for normal sextupoles. The calculation result is shown in Fig. 3. It can be seen that the minimum values of the RDT fluctuations occur at the fractional part of $\Delta\nu_x$ equal to 0.5 and the fractional part of $\Delta\nu_y$ equal to 0.5 and 0 (or 1), where $\Delta\nu_x$ and $\Delta\nu_y$ are the horizontal and vertical phase advances between the first and second bumps divided by 2π . That is, these values correspond to the horizontal and vertical phase advances of $(2m_x + 1)\pi$ and $m_y\pi$, with m_x and m_y being integers, which is the condition for the nonlinear cancellation of normal sextupoles [19]. In this case, there is a noninterleaved nonlinear cancellation between the first and second bumps and between the third and fourth bumps. Thus, the new MBA lattice with four bumps can be viewed as having two pairs of noninterleaved

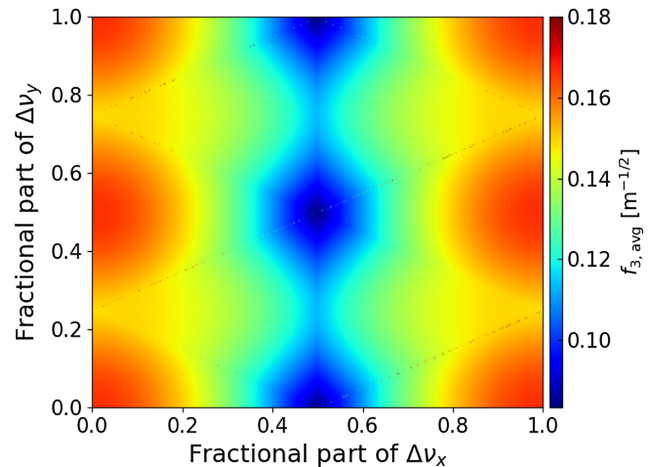


FIG. 3. The fluctuation of the first-order geometric RDTs versus different fractional values of $\Delta\nu_x$ and $\Delta\nu_y$ for the new MBA lattice, where $\Delta\nu_x$ and $\Delta\nu_y$ are equal to the horizontal and vertical phase advances between the first and second bumps divided by 2π . The blue color denotes smaller RDT fluctuations and red denotes larger ones.

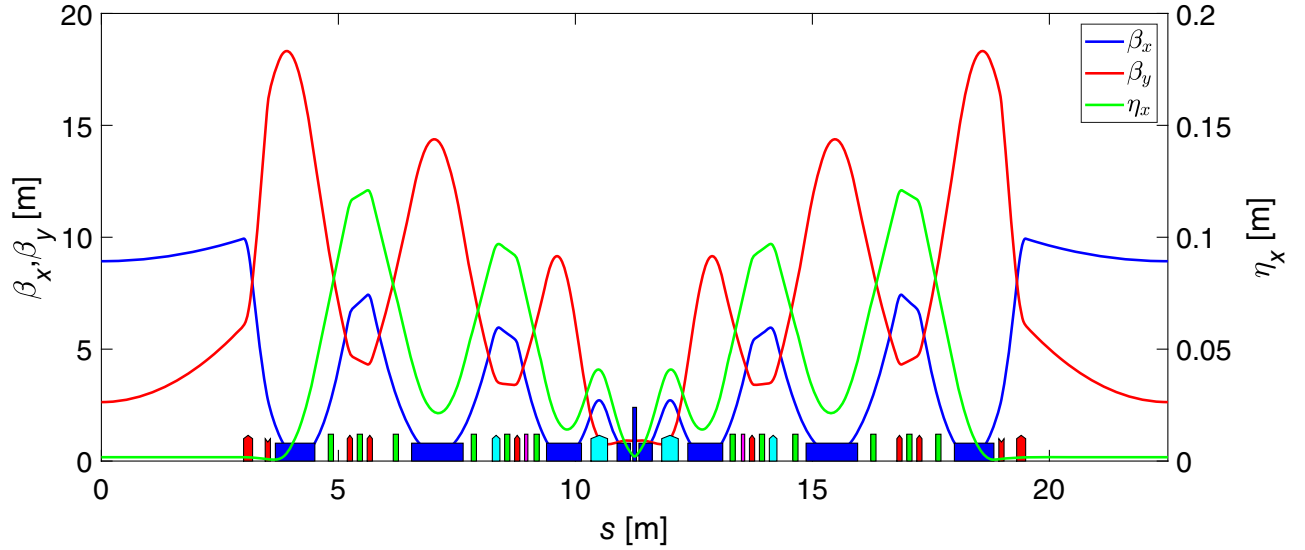


FIG. 4. Magnet layout and optical functions of the designed 7BA lattice with four bumps. In the magnet layout, main bends are in blue, RBs in cyan, quadrupoles in red, sextupoles in green, and octupoles in magenta. The central part of the middle main bend has a high dipole field of 3 T.

bumps, which is like a combination of two HMBA lattices [20,21]. This means that in this case, the interleaved cancellation scheme becomes a noninterleaved cancellation scheme. In addition, from Fig. 3, we can also know how to choose the phase advances between the first and second bumps so as to reduce the first-order RDT fluctuations and hence higher-order RDTs.

III. A 7BA LATTICE

Following the concept above, we preliminarily designed a 7BA lattice with four bumps for a 1.5 GeV storage ring light source with an emittance goal of lower than 250 pm rad. The storage ring consists of eight identical lattice cells with a cell length of 22.5 m and a straight section length of 6 m. Figure 4 shows the designed 7BA lattice cell and its linear optics. As described in Sec. II, the phase advances between the first and third dispersion bumps are limited to close to 3π , π . Except for the middle main bend, all main bends are combined-function bends (CBs) with transverse gradients in order to save space. The maximum dipole field of CBs is 0.82 T, and their quadrupole strengths are lower than 15 T/m. The middle main bend has two CB parts on both sides and a 3 T high-field bend part in the center, similar to that of Sirius [22], which can be used as a hard x-ray source. Two families of reverse bends (RBs) [23] are used for reducing the emittance. The dipole field and quadrupole strength of the RB family located in dispersion bumps are 0.23 T and 29.0 T/m, respectively; and they are 0.31 T and 42.9 T/m for the RB family close to the middle main bend. If the longitudinal gradient bends (LGBs) [24,25] are used in this lattice, the emittance can be further reduced. Additionally, the maximum strength of quadrupoles is about 40 T/m.

The main parameters of the storage ring are listed in Table I. The natural emittance is 248 pm rad. A comparison of emittance is made between this 7BA lattice and two H7BA lattices, as shown in Table II. The first H7BA lattice (H7BA-1) designed for the study of Hefei Advanced Light Facility (HALF) [26] uses CBs and RBs. The second H7BA lattice (H7BA-2) is the APS-U lattice [3], where LGBs are also used. According to the emittance scaling law of E^2/N^3 , where E is the beam energy and N is the number of lattice cells, the natural emittances of H7BA-1 and H7BA-2, when scaled to 1.5 GeV and 8 lattice cells, are 487 and 328 pm rad, respectively. For the designed 7BA lattice and the H7BA-1 lattice, which have relatively close arc lengths and also employ CBs and RBs, the natural

TABLE I. Main storage ring parameters of the designed 7BA lattice.

Parameter	Value
Energy (GeV)	1.5
Circumference (m)	180
Number of cells	8
Length of straight section (m)	6
Natural emittance (pm rad)	248
Betatron tunes (H/V)	23.11/7.40
Natural chromaticities (H/V)	-45.3/-32.5
Momentum compaction factor	5.78×10^{-4}
Damping partition numbers (H/V/L)	1.61/1.00/1.39
Natural damping times (H/V/L) (ms)	13.1/21.1/15.2
Energy lost per turn (keV)	85.2
Natural energy spread	0.81×10^{-3}
Total absolute bending angle (°)	414
β_x/β_y at the center of straight section (m)	8.93/2.64

TABLE II. Comparison of emittance-related parameters of the designed 7BA lattice and two H7BA lattices.

Parameter	7BA	H7BA-1 ^a	H7BA-2 ^b
Energy (GeV)		1.5	
Number of cells		8	
Horizontal tune of cell	2.89	2.41	2.38
Designed/scaled natural emittance ϵ_x (pm rad)	248	487	328
Horizontal damping partition J_x	1.61	2.06	2.25
Emittance normalized to J_x ($\epsilon_x \cdot J_x$) (pm rad)	399	1003	738

^aH7BA-1 is scaled from the design in Ref. [26], where the natural emittance is 67 pm rad with 2.2 GeV and 20 cells.

^bH7BA-2 is scaled from the design in Ref. [3], where the natural emittance is 42 pm rad with 6.0 GeV and 40 cells.

emittance of the former is approximately 1/2 of the scaled natural emittance of the latter. In addition, the horizontal damping partition number J_x of the designed 7BA lattice is obviously smaller than those of the two H7BA lattices. Therefore, the natural emittance of the designed 7BA lattice can be further reduced by using more RBs to increase J_x .

TABLE III. Comparison of integrated strengths of sextupoles and nonlinear terms for the optimized and ideal nonlinear solutions of the designed 7BA lattice.

	Optimized solution ^a	Ideal solution ^b
Integrated strengths		
SD1 (m ⁻²)	-17.60	-17.80
SF2 (m ⁻²)	19.65	18.28
SD3 (m ⁻²)	-8.69	-8.23
SD4 (m ⁻²)	-6.92	-13.22
SF5 (m ⁻²)	23.37	25.90
SD6 (m ⁻²)	-33.81	-25.90
Nonlinear terms		
h_{21000} (m ^{-1/2})	14.45	0.85
h_{30000} (m ^{-1/2})	10.17	5.96
h_{10110} (m ^{-1/2})	17.65	0.94
h_{10020} (m ^{-1/2})	0.51	4.18
h_{10200} (m ^{-1/2})	2.75	0.40
$\partial\nu_x/\partial J_x$ (m ⁻¹)	11,500	-16,599
$\partial\nu_x/\partial J_y$ (m ⁻¹)	-9534	-10,587
$\partial\nu_y/\partial J_y$ (m ⁻¹)	6498	-11,351
$\partial^2\nu_x/\partial\delta^2$	123.3	72.9
$\partial^2\nu_y/\partial\delta^2$	-5.7	-4.5
$\partial^3\nu_x/\partial\delta^3$	-81.2	-3650.7
$\partial^3\nu_y/\partial\delta^3$	-57.3	-368.8

^aThe optimized nonlinear solution was obtained by multi-objective optimization and an octupole family was also employed.

^bThe ideal nonlinear solution has sextupole strengths close to the ideal cancellation condition of Eq. (3).

In designing this lattice, the nonlinear cancellation of six families of sextupoles was considered by approximately satisfying Eqs. (2) and (3), and we obtained an ideal nonlinear solution, where the first-order geometric RDTs are basically canceled out. Actually, in the nonlinear optimization, apart from the first-order geometric RDTs, the amplitude dependent tune shifts (ADTSs) and higher-order chromaticities also need to be primarily controlled to enlarge DA and MA. Therefore, controlling all these nonlinear terms in the nonlinear optimization needs more degrees of freedom, which requires breaking the ideal cancellation of the first-order geometric RDTs to some extent. Then, the nonlinear dynamics taking into account these nonlinear terms was optimized using a multiobjective optimization algorithm. In the optimization, a family of octupoles was also employed to better control the ADTS terms. Table III compares the integrated strengths of sextupoles and nonlinear terms for the optimized and ideal nonlinear solutions, where the sextupole families, denoted as SD1, SF2, SD3, SD4, SF5, and SD6, are located from left to right in the left half of the magnet layout of Fig. 4. It is seen that compared to the ideal nonlinear solution, the ADTS terms and higher-order chromaticities of the optimized solution are significantly changed or well controlled, with the first-order geometric RDTs increasing to some extent. The variation of the first-order geometric RDTs along the ring for the optimized solution is shown in Fig. 5. We see that, although the ideal cancellation of RDTs is compromised, there is still a rough cancellation within a lattice cell to a large extent.

The frequency map analysis of the on-momentum DA, tracked with ELEGANT [27], is shown in Fig. 6, and Fig. 7 shows the on-momentum DAs with rf cavity and error effects. For simplicity, similar to Ref. [28], sufficiently small random errors, including alignment and magnetic field errors, are added to magnets to obtain the level of

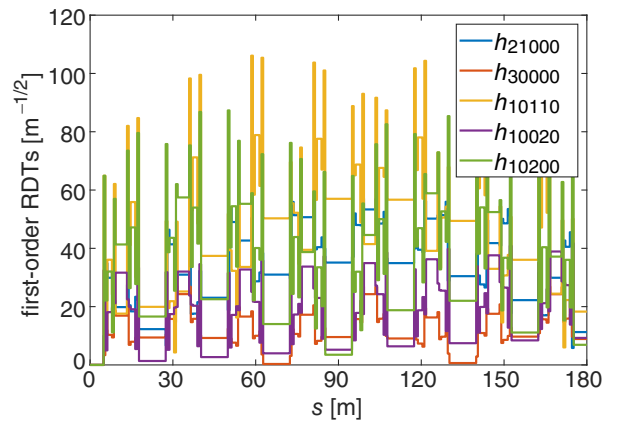


FIG. 5. Variation of the amplitudes of the first-order geometric RDTs along the ring. The amplitude of each RDT at a position s is calculated by summing the RDT from the starting position to the position s according to Ref. [16].

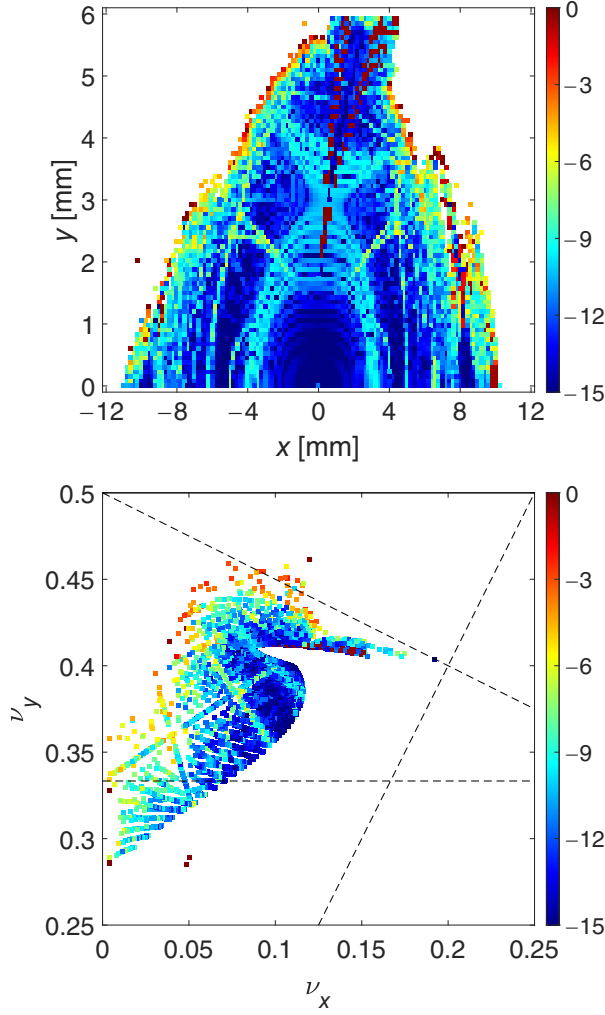


FIG. 6. Frequency map analysis of the on-momentum DA, tracked at the center of straight section. The color bar represents the tune diffusion. A small part of particles with tunes crossing the integer resonance line are not shown in the tune space.

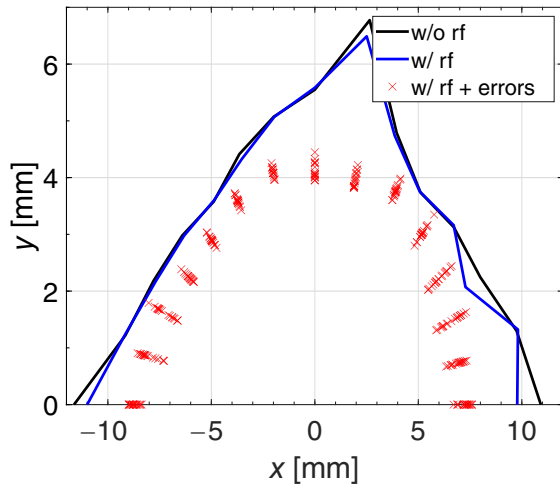


FIG. 7. On-momentum DAs with rf cavity and errors, tracked at the center of straight section.

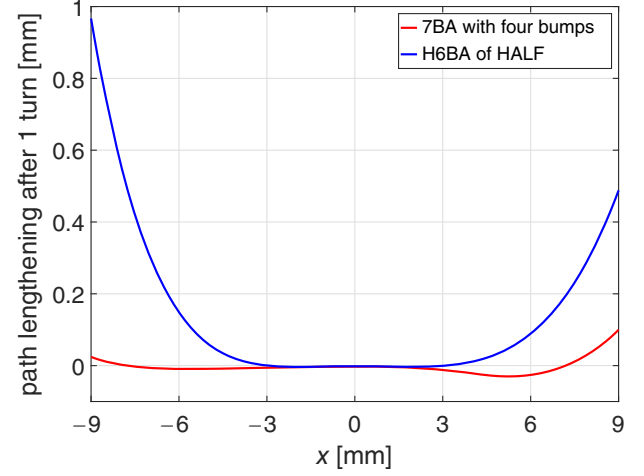


FIG. 8. Comparison of path lengthening effects of the designed 7BA lattice and the HALF H6BA lattice.

residual error effects, and there is no error correction. The residual error effects include closed orbit distortion of tens of μm , beta beat of $\sim 1\%$, and very small transverse coupling. It is seen that the horizontal DA in the negative direction is about 11 mm, and with errors included, it decreases to approximately 8.5 mm, which can allow off-axis injection. The DA with rf cavity and longitudinal motion included is a little smaller than the 4D DA due to small path lengthening effect. Compared to the conventional MBA lattice with distributed chromatic correction, the HMBA lattice with two dispersion bumps has a larger path lengthening effect [29]. The MBA lattice with four dispersion bumps in this paper can be seen as an MBA lattice between the conventional MBA and HMBA

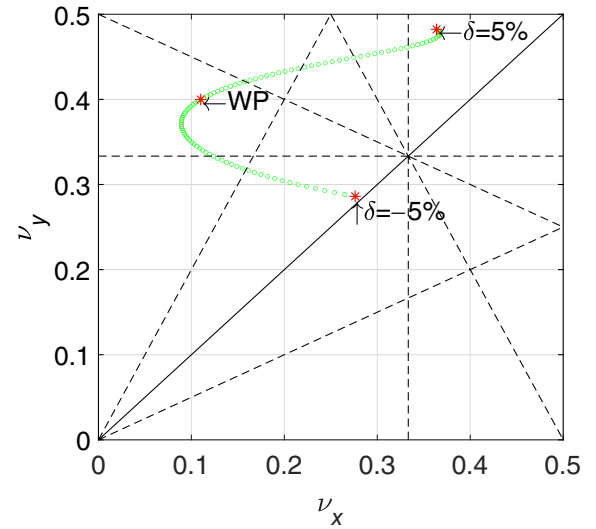


FIG. 9. Chromatic tune footprints up to $\pm 5\%$ momentum deviations, shown by the green circles with a momentum deviation step of 0.1%. The working point and tunes of $\pm 5\%$ momentum deviations are marked by the red asterisks.

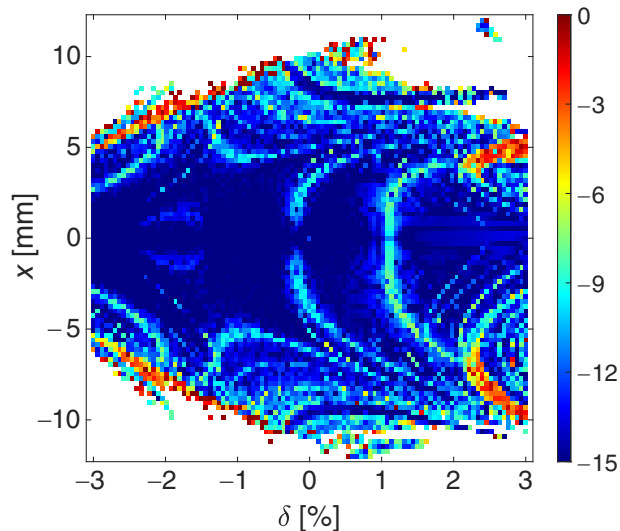


FIG. 10. Horizontal DAs of momentum deviations of -3% – 3% , tracked at the center of straight section. The color bar denotes the tune diffusion.

lattices. Compared to the HMBA lattice, the MBA lattice with four bumps has a smaller path lengthening effect. We calculated the path lengthening of the designed 7BA lattice, which is obviously smaller than that of the H6BA lattice of HALF [30], as shown in Fig. 8. Figure 9 shows the tune footprints of momentum deviations of -5% – 5% , where the tunes are controlled to avoid crossing the half-integer resonance line and linear difference resonance line. Figure 10 shows the horizontal DAs of momentum deviations of -3% – 3% . The local MAs without and with errors are shown in Fig. 11. With errors included, the average Touschek lifetime is about 6 h for a 10%-coupling beam with bunch charge of 1 nC and bunch length stretched by a factor of 3.

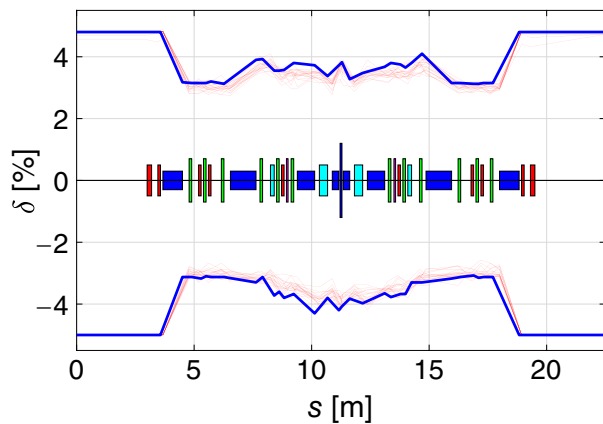


FIG. 11. LMAs without (blue color) and with errors (red color) along a lattice cell. The rf momentum acceptance is set to about 5%.

IV. CONCLUSION AND OUTLOOK

In this paper, we proposed a new MBA lattice with four dispersion bumps, which has the following characteristics. (i) A general relation of sextupole cancellation is used between paired bumps to make the effective cancellation within a lattice cell. The $-I$ transformation is a special case of this general relation. (ii) The lattice cell can have a larger horizontal tune than that of the HMBA lattice with two bumps, which is good for reducing the emittance. (iii) The lattice also has good structure flexibility. The number of bends in the lattice cell and thus the tunes of the lattice cell can be relatively freely changed, while keeping the phase advances between paired bumps unchanged.

As an example, a 7BA lattice with four bumps was preliminarily designed. The natural emittance of the designed 7BA lattice is about 1/2 of the scaled natural emittance of the H7BA lattice with similar arc length and bend types in Ref. [26]. In the nonlinear optimization, although the ideal cancellation of first-order geometric RDTs needs to be compromised to consider the control of ADTSs and higher-order chromaticities, the variation of RDTs along the ring is still controlled in the optimized solution. Taking into account errors, the optimized DA in the horizontal plane is approximately -8.5 – 7.5 mm, and the local MA gives a reasonable Touschek lifetime.

This lattice concept can also be extended to the case of MBA lattice with more bumps. If we denote the sextupole cancellation between the m th and n th bumps of the lattice cell as (m, n) , the sextupole cancellation in the MBA lattice with six bumps can be $(1, 4)$, $(2, 5)$, and $(3, 6)$. Here $(2, 5)$ uses $-I$ transformation, since the beta functions of the second and fifth bumps are symmetric. Thus, in the lattice design, only $(1, 4)$ or $(3, 6)$ needs to satisfy the more general relation of cancellation, similar to the case of the MBA lattice with four bumps. In addition, the cancellation in the MBA with six bumps can also be $(1, 3)$, $(2, 5)$, and $(4, 6)$.

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