

Development and application of a linear optics model for dielectric wall accelerators

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Proton therapy presents an interesting and important challenge to accelerator physics. Practical restrictions such as room size, gantry weight, and cost have led to a demand for more compact and affordable devices. Success would benefit a significant number of patients every year. Dielectric wall accelerators could provide this breakthrough, but there is limited work available regarding their feasibility or possible integration into medical devices, and no solutions exist for longitudinal beam control. In this work, we discuss the development of a linear beam dynamics model for dielectric wall accelerators. The model is implemented as part of the open-source code TRANSOPTR. It is tested against the rf LINAC subroutine in TRANSOPTR and is used to study bunch injection and transport. Building on recent work, the effect of time-varying fields on beam dynamics is also investigated. Conditions for limiting transverse beam size and maintaining longitudinal stability are discussed, and considerations are given to accelerator and beamline design.

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I. INTRODUCTION

Proton beams provide a number of dosimetric advantages over photon beams for the treatment of cancer. As a result, it is possible to achieve similar tumor doses while significantly reducing the dose to normal tissues, thereby reducing the number and severity of treatment-induced side effects without compromising tumor control [1–4]. Proton therapy (PT) is of particular interest today, as global rises in both cancer incidence and 5-year survival rates [5] are leading to an ever-increasing number of radiotherapy patients living with the side effects of their treatment. However, although the number of global PT centers is

growing, access remains insufficient to meet current demands [4]. A key limitation is the size and cost of current equipment [4,6,7], which includes the accelerator (typically a cyclotron), gantry, and beam transport system. This contributes to the significant disparities observed in the number of PT centers based on economic status of a country [4], and in access to existing centers based on the socioeconomic status [8,9], geographic location [9,10], and race [11] of patients. In Canada, a high-income country without a PT center, patients are sent out of country (typically to the United States) at significant cost to the public healthcare system [12]. Breakthrough accelerator technologies that prioritize compactness, affordability, and ease of installation (e.g., designed to directly replace photon machines in existing treatment rooms) could therefore effect significant change in the global proton therapy landscape. In this paper, we focus on one such technology: the dielectric wall accelerator (DWA).

DWAs are a class of nonresonant linear accelerators. Their operation is based on the coordination of a large number of independent acceleration cells, each of which is made up of one or more modules. These modules, which

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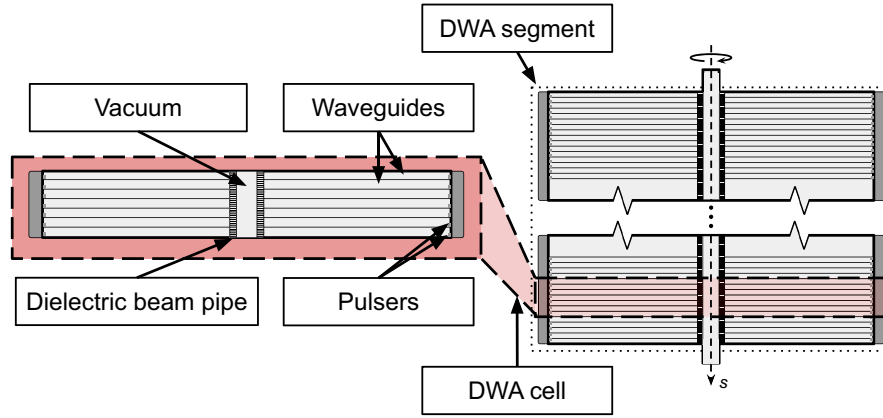


FIG. 1. A cross section of a DWA segment made up of a number of independent acceleration cells. Each cell in this schematic contains seven modules, each comprising pulsers (high-voltage circuits with semiconductor-based switches) connected at multiple injection points around a radial parallel plate waveguide. The pulsers inject a high-voltage impulse into the waveguide when the switches are triggered, which then propagates it to the beam pipe where fringe fields extend across and accelerate passing particle bunches. The structure has axisymmetry, and the proton trajectory is from top to bottom.

are composed of a high-voltage power source, one or more triggering switches, and some configuration of parallel plate waveguides, are stacked along the beam axis. When triggered, they produce short-lived (nanosecond-scale) electric fields that extend across the dielectric beam pipe. A simplified schematic is shown in Fig. 1. The overall effect is that when the modules are timed correctly, particle bunches experience a virtual traveling wave, even as each section of the beam pipe is only subjected to the high electrical stress for (approximately) the duration of the pulse. This allows for high-gradient acceleration while limiting the risk of dielectric breakdown.

Also referred to as pulsed synchronous linear accelerators, DWAs stem from core-less linear induction accelerator research conducted in the mid-late 20th century [13,14]. The idea was revived by a group at the Lawrence Livermore National Laboratory (LLNL) [15] following the development of high-gradient insulators [16] capable of withstanding field strengths of around 100 MV/m for nanosecond-scale pulse durations [17]. As 100 MV/m fields would allow for 250 MeV protons (the maximum needed for PT) within only a few meters, this group proposed its application to PT [18,19]. They made substantial progress on a prototype [20,21] but were limited by issues with their optical switches and parasitic coupling between modules [22]. A group at the Chinese Institute of Fluid Physics (IFP) has achieved a low-energy prototype [23], featuring a unique approach to overcoming the problem of parasitic coupling [24]. An alternative semiconductor-based switching approach pioneered at the SLAC National Accelerator Laboratory [25,26] promises high-voltage, nanosecond-scale pulses [27]. Additionally, these pulses would be formed independently of the waveguides [28], which should allow for some degree of pulse shaping prior to injection. The ability to generate fields with arbitrary time profiles could have important

consequences for longitudinal (and transverse) beam dynamics in a DWA.

From a beam physics perspective, there are some important difficulties inherent to DWAs. Regarding transverse dynamics, magnetic focusing fields cannot be applied to particle bunches in the beam pipe, although the use of interspersed DWA cells (short stacks of modules) and “pancake” magnets has been suggested for electron transport [29]. Furthermore, early simulations suggest that the large field gradient experienced by particle bunches entering 100 MV/m DWA fields can lead to a significant increase in bunch width during acceleration [30,31]. Longitudinally, the flattop pulses generated by the Blumlein-based pulse-forming networks used at LLNL and IFP do not generate a focusing force along the direction of motion. This can possibly be mitigated with sufficiently compressed, low-emittance (and low space charge) bunches at injection [30,31], although even over short distances there is a marked difference compared to accelerators with longitudinal focusing [23]. In addition, any longitudinal focusing that could be applied will necessarily result in transverse defocusing. In short, no clear solution exists for maintaining both transverse and longitudinal stability during acceleration. Chen *et al.* [32] proposed an interesting approach based on a sinusoidal deformation of the high-gradient insulator surface, although this has not been realized.

Previous studies of particle transport in a DWA have used the results of 3D electric field simulations as the input to particle-in-cell codes [30,31,33] or have used combined electric field and particle-in-cell codes [32]. Although multiparticle simulation programs are powerful numerical techniques that can provide valuable information on the dynamics of complex systems, they are not necessarily well suited for initial studies of novel machines. They require far more detailed input to be run than linear optics codes and

are more vulnerable to numerical noise, integration error, and instabilities, making it difficult to identify systematic issues. In contrast, a linear optics code with a flexible, analytical field model would provide fast, reliable results, allowing for a broader study of injection dynamics and the conditions for maintaining bunch stability. It would allow for beamline design studies, where the configuration and design parameters (e.g., field strength) of external components could be optimized as part of a complete device, and the impact of imperfections (e.g., pulse timing errors) could be more easily taken into account. Of course, this comes at the cost of reduced model complexity and the error associated with excluding higher order effects. See Saltelli [34] for a relevant discussion on mathematical modeling. A multiparticle model could then build on this, providing information on beam loss, higher order effects, and sources of aberrations. This approach could provide important feedback for hardware development.

In this work, we describe the development and application of an analytical model of beam dynamics in the DWA. Starting from a set of symmetry conditions and a known on-wall excitation, we derive a Hamiltonian for charged particles in a DWA and use it to formulate the envelope equations representing a segment of DWA modules. An algorithm for pulse timing is also presented. We describe the implementation of our linear optics model in the open-source envelope code TRANSOPTR [35]. This model is then applied to questions of injection criteria, transverse defocusing, and longitudinal stability. Finally, we consider the effect of adding a linear time variation to the accelerating fields.

II. THEORY

A. TRANSOPTR: Principle of operation

TRANSOPTR is an envelope code that models first-order beam optics. As originally introduced in TRANSOPTR [36], a particle bunch is represented by a six-by-six matrix called the beam matrix (σ), which is the covariance matrix of the particle distribution in 6D phase space. The beam matrix defines the width, length, divergence, energy spread, and parameter correlations of a bunch at a given point. Its dynamics are defined with respect to the reference trajectory, which is a curvilinear path parameterized by s . The reference trajectory is defined by the optics of a given system. Transversely, this generally corresponds to the center of the beam pipe. In the longitudinal direction, however, the reference time and energy at s are not always known analytically and are therefore integrated alongside the beam matrix. This allows for accurate tracking through elements with time-varying fields, such as radio-frequency quadrupoles (RFQs) and complicated multigap cavities [37,38]. The beam matrix obeys the equation of motion:

$$\frac{d\sigma}{ds} = \mathbf{F}\sigma + \sigma\mathbf{F}^T, \quad (1)$$

where $\mathbf{F}$ is a matrix describing the linear system of equations of motion, which directly follow from Hamilton's equations of motion (see Appendix). This is a set of 36 (21 independent) ordinary differential equations that TRANSOPTR integrates through using a Runge-Kutta algorithm with adaptive step size. In addition, TRANSOPTR automatically includes a self-consistent linear space charge calculation [39], allowing for the handling of bunches with arbitrary charge.

B. Derivation of the accelerating field in a DWA

A DWA, like a linear induction accelerator, consists of a number of independent acceleration modules. Each module produces a short-lived, localized field, which we can generally assume to be identical between modules. The total field at any point in the beam pipe is then given by a superposition of these individual fields. To represent the fields in TRANSOPTR, they must be expressed in terms of the electromagnetic potentials, as required by the Hamiltonian formulation. A benefit of this approach is that the exact analytic expression for these potentials in free space is not actually required; instead, using symmetry arguments and the precise on-axis field, we only require that the expanded potential formulation satisfies Maxwell's equations to first order.

Let us begin by making the simplifying assumption that the fields are axially symmetric. This is true in the case of radial parallel-plate waveguides, which are of interest in DWA research [28]. For stripline or Blumlein geometries, some modifications would be necessary, but the process would be the same. Now, considering the axial symmetry, the only nonzero electromagnetic field component on axis will be the longitudinal electric field, $\mathcal{E}_z$. In the electrostatic limit, we can write the on-axis field $\mathcal{E}_{z,i}(x, y, s, t)|_{x=y=0} = \mathcal{E}_{z,i}(s, t)$ for a single module i centered at $(s = 0, t = 0)$, as

$$\mathcal{E}_{z,i} = \mathcal{E}(s)\mathcal{T}(t), \quad (2)$$

where $\mathcal{E}(s)$ is the spatial field profile in units of the electric field and $\mathcal{T}(t)$ is a unit-less time-scale profile. To calculate the total on-axis field, we need to combine the contributions of all modules, which are stacked along s with pulses that are offset in t . We define the midpoint of each module i to be (s_i, t_i) and write

$$\mathcal{E}_z(s, t) = \sum_{i=1}^N \mathcal{E}(s - s_i)\mathcal{T}(t - t_i). \quad (3)$$

We seek an analytical description of $\mathcal{E}_z(s, t)$. The electrostatic assumption is only used to determine the driving electric field, and as will be shown later, the induced magnetic field will be represented by the potential formulation. Finally, we assume that the on-wall excitation $\mathcal{E}_w(s)$ and the time profile $\mathcal{T}(t)$ are both known from the characteristics of the modules. These could come from

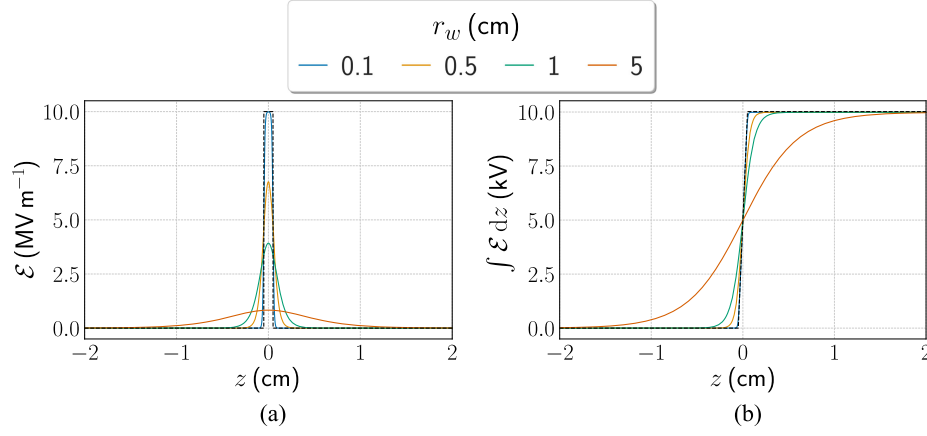


FIG. 2. Effect of aperture radius r_w on on-axis field profiles for a 1 mm DWA module with on-wall excitation $\mathcal{E}_w$ (dotted black lines). The (a) on-axis field strength and (b) field integral over z were calculated for different choices of aperture radii. The on-axis field strength varies with radius, but the cumulative effect of the field does not.

measurements, simulations, analytical models, or could simply be test cases, as we will see below. For clarity, the following derivations will be presented for the single module case, with the total multimodule expressions for the Hamiltonian and $\mathbf{F}$ matrix appearing at the end of their respective subsections.

In the beam pipe, the electric potential follows from Laplace's equation with rotational symmetry, which in (r, z) coordinates is

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \phi}{\partial r} \right) + \frac{\partial^2 \phi}{\partial z^2} = 0. \quad (4)$$

The boundary conditions for the potential are

$$\left. \frac{\partial \phi}{\partial z} \right|_{r=r_w} = -\mathcal{E}_w(z), \quad (5)$$

and

$$\lim_{z \rightarrow \pm\infty} \phi = 0. \quad (6)$$

Equations (4)–(6) can be solved via separation of variables and has the solution:

$$\phi(r, z) = \int_{-\infty}^{\infty} ik^{-1} \tilde{\mathcal{E}}_w(k) \frac{I_0(kr)}{I_0(kr_w)} e^{ikz} dk, \quad (7)$$

where $\tilde{\mathcal{E}}_w(k)$ is the Fourier transform of the on-wall excitation, r_w is the radial distance to the wall, I_0 is the zeroth order modified Bessel function of the first kind, and the integral over k is the inverse Fourier transform. To find the on-axis spatial profile, we take the longitudinal derivative of Eq. (7) and evaluate the results on axis:

$$\mathcal{E}(z) = -\left. \frac{\partial \phi(r, z)}{\partial z} \right|_{r=0} = \int_{-\infty}^{\infty} \frac{\tilde{\mathcal{E}}_w(k)}{I_0(kr_w)} e^{ikz} dk. \quad (8)$$

In practice, evaluating the electric field is done by computing a discrete Fourier transform of an arbitrary on-wall excitation and then computing the inverse discrete Fourier transform of the above integrand to calculate the field. To ensure the field is accurately represented, care must be taken to set sufficiently large integration bounds and sufficiently small grid spacing. Figure 2 shows a set of example calculations and demonstrates the relationship between the on-wall excitation, the on-axis field profile, and their integrals over z for different values of r_w .

C. A formulation of DWA potentials

To implement the fields in TRANSOPTR, we must identify a set of electromagnetic potentials that satisfy Maxwell's equations to first order about the axis. These will be constrained by the symmetry conditions and must reproduce Eq. (3). As above, we start by describing the dynamics of a single module. One such formulation, by ansatz, is

$$\phi(r, z, t) = \frac{r^2}{4} \mathcal{E}'(z) \mathcal{T}(t), \quad (9)$$

$$A_z(r, z, t) = -\mathcal{E}(z) \int_{-\infty}^t \mathcal{T}(\tau) d\tau - \frac{r^2}{4c^2} \mathcal{E}(z) \mathcal{T}'(t), \quad (10)$$

where the transverse components of the vector potential are zero. Equations (9) and (10) produce the following nonzero electromagnetic field components:

$$\mathcal{E}_r = -\frac{r}{2} \mathcal{E}'(z) \mathcal{T}(t), \quad (11)$$

$$\mathcal{E}_z = \mathcal{E}(z) \mathcal{T}(t) + \frac{r^2}{4} \left(\frac{1}{c^2} \mathcal{E}(z) \mathcal{T}''(t) - \mathcal{E}''(z) \mathcal{T}(t) \right), \quad (12)$$

$$B_\theta = \frac{r}{2c^2} \mathcal{E}(z) \mathcal{T}'(t) \quad (13)$$

In the limit as $r \rightarrow 0$, we are left with only

$$\mathcal{E}_z = \mathcal{E}(z)\mathcal{T}(t), \quad (14)$$

which is Eq. (3), as desired.

What is notable about this approach is that by finding potentials consistent with our linear optics framework, we have included the transverse electric and magnetic field effects arising from the changing field consequentially. Furthermore, the potential formulation allows for implementing a field model that requires fewer input parameters and guarantees consistency with Maxwell's laws and is therefore less prone to model input errors.

D. The Hamiltonian for a charged particle in a DWA

The derivation of the expanded Hamiltonian follows the typical procedure for TRANSPORT elements (see [38,40–42]). Namely, the potentials [Eqs. (9) and (10)] are first substituted into the Hamiltonian of a relativistic charged particle in an electromagnetic field. Then, the canonical coordinates of the Hamiltonian are transformed using generating functions to be displacements about the reference trajectory. Finally, the Hamiltonian is Taylor expanded in the new canonical coordinates and truncated at second order (corresponding to linear fields).

The s -dependent Hamiltonian of a relativistic charged particle in an electromagnetic field, where s is the distance along the reference trajectory, is given by

$$H(x, P_x, y, P_y, t, -E; s) = -qA_s - \sqrt{\left(\frac{E - q\phi}{c}\right)^2 - m^2c^2 - (P_x - qA_x)^2 - (P_y - qA_y)^2}. \quad (15)$$

The transverse reference trajectory is on axis, i.e., $x_0(s) = y_0(s) = 0$. However, the longitudinal reference trajectory must be computed from the equations of motion:

$$\frac{dt}{ds} = \frac{1}{c\beta}, \quad (16)$$

$$\frac{dE}{ds} = \mathcal{E}_z(s, t). \quad (17)$$

Let $t_0(s)$ and $E_0(s)$ be the longitudinal reference trajectory that obeys the above equations of motion. For the rest of this derivation, γ and β are the relativistic factors corresponding to the reference particle's longitudinal motion. To expand the motion of the reference particle, the type 2 generating function

$$G_2(t, -\Delta E) = -\left(t - \int \frac{ds}{c\beta}\right)(\Delta E + E_0) \quad (18)$$

is used to transform the longitudinal coordinates by

$$t \rightarrow t_0 + \Delta t, \quad (19)$$

$$E \rightarrow E_0 + \Delta E. \quad (20)$$

It is useful for symmetry between transverse and longitudinal motion to introduce new coordinates z and P_z that are simply time and energy deviations with respect to the reference particle but scaled by the reference speed βc :

$$\Delta t \rightarrow -\frac{z}{c\beta}, \quad (21)$$

$$-\Delta E \rightarrow c\beta P_z. \quad (22)$$

This is accomplished by the generating function:

$$G_2(\Delta t, P_z) = -c\beta\Delta t P_z. \quad (23)$$

Now, taking a Taylor expansion in the expanded coordinates about $s = 0$ and truncating to second order overall, we have the linearized Hamiltonian:

$$H = \frac{P_x^2}{2P} + \frac{P_y^2}{2P} + \frac{q(x^2 + y^2)}{4c^2} \left(\mathcal{E}(s)\mathcal{T}'(t_0) + \frac{c^2}{\beta} \mathcal{E}'(s)\mathcal{T}(t_0) \right) + \frac{P_z^2}{2\gamma^2 P} + \frac{zP_z\beta'}{\beta} + \frac{qz^2}{2c^2\beta^2} \mathcal{E}(s)\mathcal{T}'(t_0). \quad (24)$$

In the multimodule case, Eq. (24) becomes

$$H = \frac{P_x^2}{2P} + \frac{P_y^2}{2P} + \frac{q(x^2 + y^2)}{4c^2} \left[\sum_{i=1}^N \mathcal{E}(s - s_i)\mathcal{T}'(t_0 - t_i) + \frac{c^2}{\beta} \sum_{j=1}^N \mathcal{E}'(s - s_j)\mathcal{T}(t_0 - t_j) \right] + \frac{P_z^2}{2\gamma^2 P} + \frac{zP_z\beta'}{\beta} + \frac{qz^2}{2c^2\beta^2} \sum_{k=1}^N \mathcal{E}(s - s_k)\mathcal{T}'(t_0 - t_k). \quad (25)$$

E. Derivation of the F matrix for a DWA

To construct the **F** matrix, we require the symplectic matrix **J** and the Hessian matrix **D_H** of the (linearized) Hamiltonian in Eq. (25). The symplectic matrix is defined

by the structure of the vector of canonical coordinates, i.e., (x, P_x, y, P_y, z, P_z) , and is given by

$$\mathbf{J} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}. \quad (26)$$

The components of the Hessian matrix are the second-order partial derivatives of the Hamiltonian, with the same coordinate ordering as before:

$$\mathbf{D}_H = \begin{bmatrix} \frac{\partial^2 H}{\partial x^2} & \frac{\partial^2 H}{\partial x \partial P_x} & \frac{\partial^2 H}{\partial x \partial y} & \frac{\partial^2 H}{\partial x \partial P_y} & \frac{\partial^2 H}{\partial x \partial z} & \frac{\partial^2 H}{\partial x \partial P_z} \\ \frac{\partial^2 H}{\partial P_x \partial x} & \frac{\partial^2 H}{\partial P_x \partial P_x} & \frac{\partial^2 H}{\partial P_x \partial y} & \frac{\partial^2 H}{\partial P_x \partial P_y} & \frac{\partial^2 H}{\partial P_x \partial z} & \frac{\partial^2 H}{\partial P_x \partial P_z} \\ \frac{\partial^2 H}{\partial y \partial x} & \frac{\partial^2 H}{\partial y \partial P_x} & \frac{\partial^2 H}{\partial y^2} & \frac{\partial^2 H}{\partial y \partial P_y} & \frac{\partial^2 H}{\partial y \partial z} & \frac{\partial^2 H}{\partial y \partial P_z} \\ \frac{\partial^2 H}{\partial P_y \partial x} & \frac{\partial^2 H}{\partial P_y \partial P_x} & \frac{\partial^2 H}{\partial P_y \partial y} & \frac{\partial^2 H}{\partial P_y^2} & \frac{\partial^2 H}{\partial P_y \partial z} & \frac{\partial^2 H}{\partial P_y \partial P_z} \\ \frac{\partial^2 H}{\partial z \partial x} & \frac{\partial^2 H}{\partial z \partial P_x} & \frac{\partial^2 H}{\partial z \partial y} & \frac{\partial^2 H}{\partial z \partial P_y} & \frac{\partial^2 H}{\partial z^2} & \frac{\partial^2 H}{\partial z \partial P_z} \\ \frac{\partial^2 H}{\partial P_z \partial x} & \frac{\partial^2 H}{\partial P_z \partial P_x} & \frac{\partial^2 H}{\partial P_z \partial y} & \frac{\partial^2 H}{\partial P_z \partial P_y} & \frac{\partial^2 H}{\partial P_z \partial z} & \frac{\partial^2 H}{\partial P_z^2} \end{bmatrix}. \quad (27)$$

The $\mathbf{F}$ matrix is the product of these two matrices:

$$\mathbf{F} = \mathbf{J} \mathbf{D}_H = \begin{bmatrix} 0 & \frac{1}{P} & 0 & 0 & 0 & 0 \\ \mathcal{A}(s) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{P} & 0 & 0 \\ 0 & 0 & \mathcal{A}(s) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\beta'}{\beta} & \frac{1}{r^2 P} \\ 0 & 0 & 0 & 0 & \mathcal{B}(s) & -\frac{\beta'}{\beta} \end{bmatrix}, \quad (28)$$

where $\mathcal{A}(s)$ and $\mathcal{B}(s)$ are the transverse and longitudinal focusing terms, respectively:

$$\mathcal{A}(s) = -\frac{q}{2c^2} \sum_{i=1}^N \left(\mathcal{E}(s - s_i) \mathcal{T}'(t - t_i) + \frac{c}{\beta} \mathcal{E}'(s - s_i) \mathcal{T}(t - t_i) \right), \quad (29)$$

$$\mathcal{B}(s) = -\frac{q}{\beta^2 c^2} \sum_{i=1}^N \mathcal{E}(s - s_i) \mathcal{T}'(t - t_i). \quad (30)$$

F. Pulse timing

Recall that in order to derive the $\mathbf{F}$ matrix, we assumed that the longitudinal reference particle had a known solution: $t_0(s)$ and $E_0(s)$. This solution is necessary for coordinating the pulse times (when to “turn on” each module’s field) with the passing particle bunch. To borrow from radio-frequency (rf) terminology, we define the synchronous phase such that the reference particle arrives at the center of a given module halfway through that module’s “on” time (i.e., the center of the pulse). Importantly, the arrival time and the pulse time are not independent. Not only does the arrival time depend on when the pulse is fired, but if there is significant time variation in the field strength, the energy gain can be quite sensitive to the pulse time. Furthermore, the fringe field overlap between the modules means that neighboring modules will have an effect on the arrival time as well. Solving for the pulse times is therefore a nontrivial multidimensional root finding problem, and we have developed a numerical approach.

To initialize our root finding algorithm, the pulse times of all of the modules are set such they all fire simultaneously as the particle enters the first module. The longitudinal reference trajectory [Eq. (16)] is then integrated through the DWA with these timings. Once integration is complete, the pulse times are updated to the arrival times of the reference particle at the center of each module. This process is repeated until the pulse time matches the arrival time for each module within a prescribed tolerance. The computational cost of this approach increases with module number and with decreasing β , but for all simulations discussed in this work, the algorithm converged to within a tolerance of 10^{-4} (relative to the pulse length) within ten or fewer iterations.

G. Code release

The implementation of our model has been made open source and is part of the public release of TRANSOPTR [43]. For interested users, the auxiliary code PYOPTR [44] provides a Python layer for preparing and running simulations.

III. METHODS

A. Comparison with TRANSOPTR’s LINAC routine

Although our model cannot be validated against experimental evidence for the case of a DWA, it can be compared to established models of other linear accelerating structures by substituting appropriate expressions for the field profiles $\mathcal{E}$ and $\mathcal{T}$. Of particular interest is the rf linear accelerator (rf LINAC), as these devices have fields that vary in time and space, contain electric and magnetic components, and also rely on specific pulse timing (alignment with the synchronous phase).

Consider Eqs. (29) and (30). Setting $N = 1$ and $\mathcal{T} = \cos(\omega t_0(s) + \theta) = \mathcal{C}$ for some angular frequency ω , reference time t_0 , and phase offset θ leads to the focusing terms:

$$\mathcal{A}(s) = -\frac{q}{2\beta c} \left(\mathcal{E}'(s - s_0)\mathcal{C} - \frac{\omega\beta}{c} \mathcal{E}(s - s_0)\mathcal{S} \right), \quad (31)$$

$$\mathcal{B}(s) = \frac{q\omega}{\beta^2 c^2} \mathcal{E}(s - s_0)\mathcal{S}, \quad (32)$$

where $\mathcal{S} = \sin(\omega t_0(s) + \theta)$. These are the equations for an rf LINAC as derived by Baartman [42], which serve as the basis for the LINAC subroutine in TRANSOPTR. This subroutine has been validated against multiparticle codes as well as experimental measurements [38]. To coordinate the acceleration field, users specify the phase θ seen by the reference particle as it enters the rf cavity. To replicate this behavior in the DWA subroutine, which instead coordinates the acceleration field with the reference particle's arrival time at the center of the module, the pulse time must be manually specified. Setting the pulse time to 0 and injecting the beam directly into the module (no drift space before the field region) allows for coordination of the field with the reference particle at the entrance. A cosine with phase offset θ can be input as the time-scale profile $\mathcal{T}$ to fully replicate the behavior of the LINAC subroutine.

To verify the derivation and implementation of our model, an rf cavity was modeled with both the DWA and LINAC subroutines, and the results were compared. The rf cavity was 4 cm long and had a radius of 2 cm. The field amplitude was 10 MV/m and the rf frequency was 5 GHz. The field profile had a full width at half maximum (FWHM) of 1 cm to ensure that it went to 0 before the cavity boundaries. Note that this field was calculated from the method described in Sec. II with a 1 cm long on-wall excitation of 100 kV/m rather than being a measured dataset of a real rf LINAC. This choice was made out of convenience but does not impact the conclusions of this test: whether or not the two subroutines generate the same output for a given input. Similarly, the choices for initial bunch parameters were somewhat arbitrary but were based on realistic source parameters. The initial kinetic energy was set to 60 keV with an rms relative momentum spread $P_z/P = 0.05$. The rms normalized emittance (ϵ_n) was set to 0.25 mm mrad, and the bunch was assumed to be fully uncoupled. To set the transverse Twiss parameters, the bunch was further assumed to be uncorrelated ($\alpha_x = \alpha_y = 0$) with a radius of 1 cm ($0.5r_w$). Longitudinal emittance is more complex, but in the uncoupled, uncorrelated case and in the absence of electric fields (as at the entrance), we can write $\epsilon_{nz} = \beta\gamma z(P_z/P) = zP_z/mc$ and use this to set our initial longitudinal spread z (see [37,45] for a more detailed breakdown of these conditions). The bunch charge was set to $1e$ to minimize space charge

effects, as the primary focus here is on strictly external field-induced dynamics.

B. Bunch transport in a DWA

In this work, we assume a DWA configuration with external pulsers and posit a nonzero linear time variation of the form shown in Fig. 3. Note that by defining $\mathcal{T}$ such that $\mathcal{T} = 1$ halfway through the pulse, the nominal field strength is unaffected by the slope $\mathcal{T}'$.

Large transverse fields present a risk of overfocusing and subsequent rebounding, which, without any means to provide magnetic focusing within the DWA, could ultimately lead to collisions with the beam pipe. To avoid this beam loss and source of machine degradation, any DWA design must seek a set of parameters for which the change in the transverse envelope is smooth and the bunch size does not approach the beam pipe radius. For these tests, we assume a fixed physical configuration of the DWA—namely, 1 mm single-module cells with a 1 cm radius beam pipe—and vary only the field and bunch parameters. We consider the injection of proton bunches with initial energies $E_k \in [0.025, 5]$ MeV directly into DWA segments with field strengths $\mathcal{E}_0 \in [1, 100]$ MV/m. An energy of 25 keV was chosen to represent a generic proton source, while an upper limit of 5 MeV was chosen to represent the 3 GHz CERN RFQ proposed for use in a proton therapy LINAC [46,47]. The rms normalized emittance (ϵ_n) and rms relative momentum spread (P_z/P) were kept constant between tests—specifically, $\epsilon_n = 0.25$ mm mrad and $P_z/P = 0.005$. These are in line with the TRIUMF-licensed D-PACE H- source [48] and the CERN RFQ (see Table 1 in Vretenar *et al.* [46]), if not a bit conservative. For some field strengths, the 25 keV bunches grew longer in time than the pulses, so the tests were repeated with $\epsilon_n = 0.125$ mm mrad and $P_z/P = 0.0025$. The bunch charge was again set to $1e$ to minimize space charge effects, and the remaining bunch parameters were

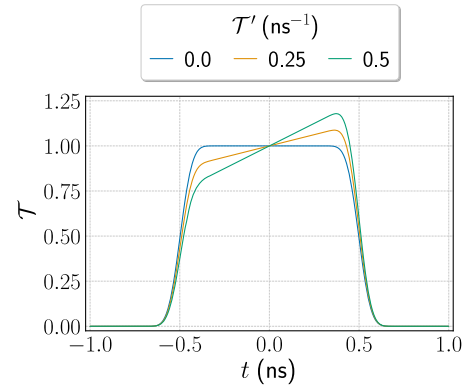


FIG. 3. Time-scale profiles $\mathcal{T}$ used in simulations of DWA segments. The FWHM was 1 ns and the linear time variations $\mathcal{T}'$ were 0.0, 0.25, and 0.5 ns⁻¹.

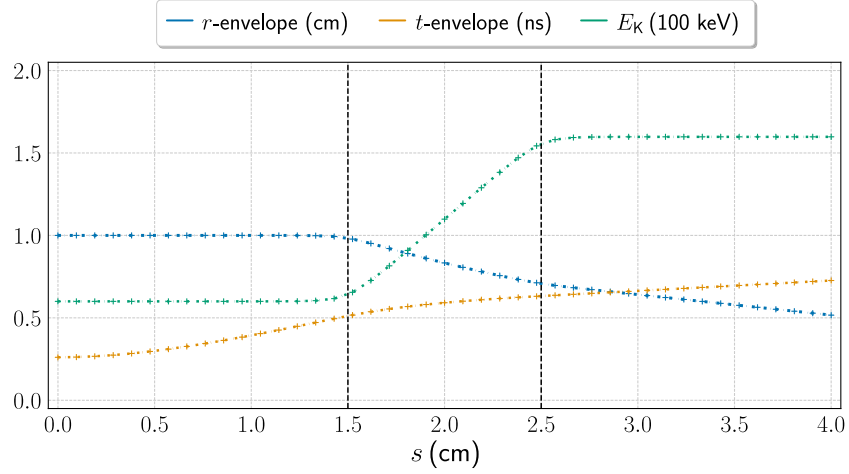


FIG. 4. Comparison of the DWA subroutine with $N = 1$ and $T = \cos(\omega t)$ (+’s) to the built-in LINAC subroutine (squares). To aid visibility, the symbols are regularly spaced and do not show every data point explicitly. The evolution of the r -envelope (2-rms width), t -envelope (2-rms length), and reference kinetic energy (E_K) along the reference trajectory is shown. The field strength was 10 MV/m, the rf frequency was 5 GHz, and the synchronous phase was set to -10° , which roughly corresponded to maximum energy gain. The vertical dashed black lines show the physical edges of the DWA segment/rf cavity along s . The subroutines agreed to within machine precision.

determined in the same way as in Sec. III A. All tests were repeated for linear time slopes $T' = 0.0, 0.25, 0.5 \text{ ns}^{-1}$.

IV. RESULTS

A. Comparison with TRANSPTR’s LINAC routine

A comparison between the DWA and LINAC subroutines for a synchronous phase of -10° (roughly corresponding to maximum energy gain) is shown in Fig. 4. The reference particle energy increased approximately linearly with distance s through the field region, with an overall energy gain of 99.8 keV. The transverse envelope showed an initial focusing effect upon entering the field, and a less apparent

defocusing effect upon exit. Longitudinally, there was a focusing effect within the field region. The residuals for the acceleration curve, transverse envelopes, and length envelope were uniformly 0 (agreed to machine precision), and this was independent of the chosen synchronous phase.

B. Bunch transport in a DWA

The effect of field strength on the transverse and longitudinal dynamics of low-energy proton bunches following injection into DWA segments is shown in Fig. 5. For a field strength $\mathcal{E}_0 = 100 \text{ MV/m}$, a discontinuity characteristic of overfocusing was observed shortly after

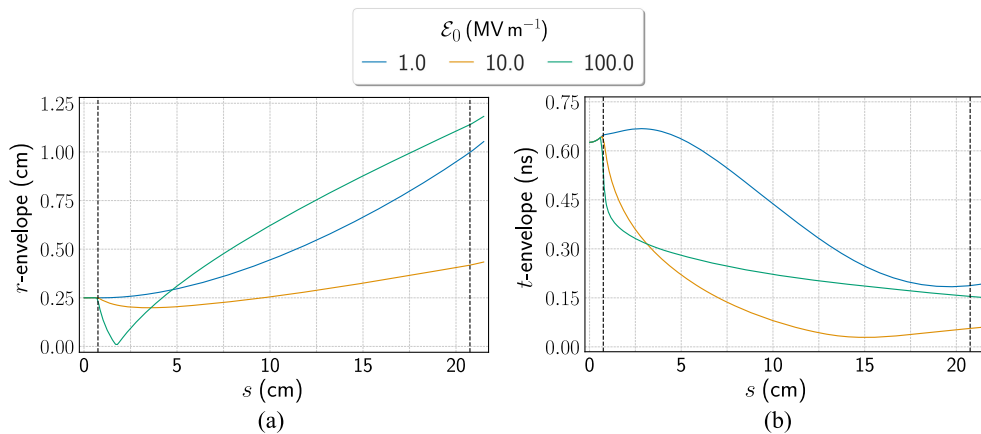


FIG. 5. Effect of field strength on low-energy injection. Beam dynamics of 25 keV, 0.125 mm mrad proton bunches with $P_z/P = 0.0025$ following injection into DWA segments with 1 cm radius apertures. All fields had linearly increasing time-scale profiles with slope $T' = 0.25 \text{ ns}^{-1}$. The evolution of the (a) r -envelope (2-rms width) and (b) t -envelope (2-rms length) along the reference trajectory is shown. Simulations were performed for field strengths $\mathcal{E}_0 \in [1, 100] \text{ MV/m}$. The vertical dashed black lines show the physical edges of the DWA segment along s .

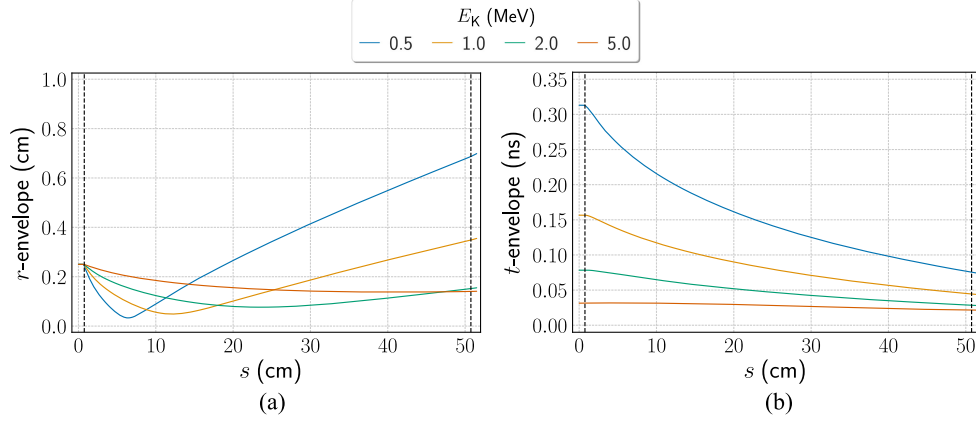


FIG. 6. Effect of initial kinetic energy on bunch injection. Beam dynamics of 0.25 mm mrad proton bunches with $P_z/P = 0.005$ following injection into a 100 MV/m DWA segment with $T' = 0.25 \text{ ns}^{-1}$ and a 1 cm radius aperture. The evolution of the (a) r -envelope (2-rms width) and (b) t -envelope (2-rms length) along the reference trajectory is shown. Simulations were performed for initial energies $E_K \in [0.5, 5] \text{ MeV}$. The vertical dashed black lines show the physical edges of the DWA segment along s .

injection. This resulted in a rapid expansion of the bunch, but the 2-rms width remained smaller than the beam pipe radius over the length of the 15 cm DWA segment. The lower the field strength, the weaker the focusing effect in the transverse plane [Fig. 5(a)] at injection, with no overfocusing observed for $\mathcal{E}_0 \leq 20 \text{ MV/m}$. However, with the exception of the overfocused 100 MV/m case, lower field strengths were associated with faster reexpansion of the beam. For 1 MV/m fields, the r -envelope grew nearly to the size of the beam pipe before the exit window, implying significant wall collisions. Longitudinally, the initial focusing effect increased with $\mathcal{E}_0$, but this was offset (at least in part) by increased adiabatic damping.

The effect of initial proton energy on transverse and longitudinal focusing following injection into 100 MV/m DWA segments is shown in Fig. 6. As the energy of the

proton bunches was increased, the strength of the initial transverse focusing decreased. The overfocusing discontinuity observed in Fig. 5 was not present for any energy. In the longitudinal plane, a smooth decrease in the 2-rms length was observed for all energies. Figure 7 shows the effect of T' on the transverse and longitudinal dynamics over 250 MeV of acceleration. The transverse envelope evolution was smooth, with an initial focusing effect leading to a gradual expansion for all values of T' . However, the value of T' affected the shape of the expansion curve and the final bunch width. For $T' = 0 \text{ ns}^{-1}$, a convex, square-root-like curve led to a 2-rms width equal to the 1 cm beam pipe radius at the exit window, while for $T' = 0.25 \text{ ns}^{-1}$, a more linear trend led to a final width of 8.4 mm. The convex curve observed for $T' = 0.5 \text{ ns}^{-1}$ led to a more rapid expansion despite the

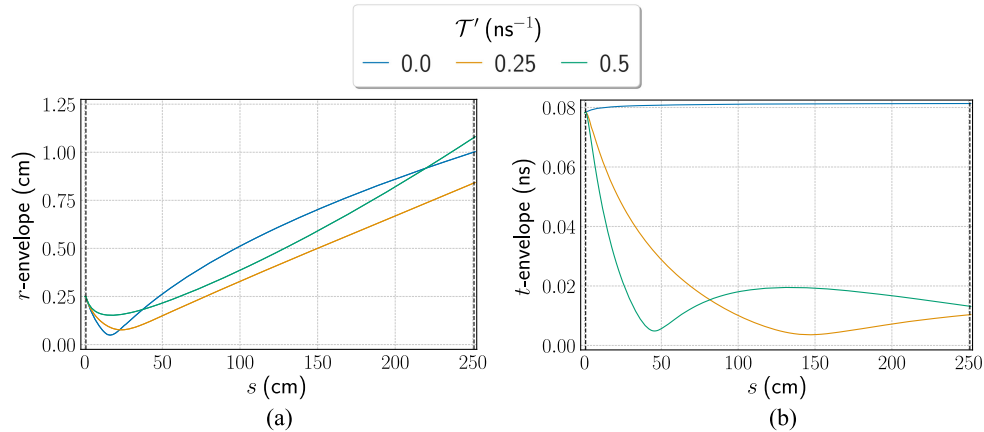


FIG. 7. Effect of T' on bunch injection and transport. Beam dynamics of 2 MeV, 0.25 mm mrad proton bunches with $P_z/P = 0.005$ following injection into a 100 MV/m DWA segment with a 1 cm radius aperture. The evolution of the (a) r -envelope (2-rms width) and (b) t -envelope (2-rms length) along the reference trajectory is shown. Simulations were performed for linearly increasing time-scale profiles with varying slopes T' . The vertical dashed black lines show the physical edges of the DWA segment along s .

reduced focusing (less rebounding) at injection, with the 2-rms exceeding 1 cm by $s = 236.1$ cm. The value of $\mathcal{T}'$ also had a significant impact on the longitudinal dynamics. There was a slow expansion with s for $\mathcal{T}' = 0$ ns⁻¹. The initial focusing was stronger for $\mathcal{T}' = 0.5$ ns⁻¹ than for $\mathcal{T}' = 0.25$ ns⁻¹, but in both cases there was a trend toward oscillatory behavior rather than uncontrolled expansion.

V. DISCUSSION

The analytical expressions derived in Sec. II provide some important insights into DWA operation. First, as shown in Fig. 2, the ratio of the module length to the aperture radius impacts the peak strength of the on-axis field, but it does not impact the spatial field integral. In other words, as long as the pulse length exceeds the particle bunch's transit time, the acceleration gradient (usually expressed in MV/m) is determined by the on-wall excitation rather than the peak on-axis field strength. Next, we note that there are two terms governing transverse focusing [Eq. (29)]. In the $\mathcal{T}' = 0$ case, the $\mathcal{E}'\mathcal{T}$ term is the only contributor and the link between $\mathcal{A}$ and $\mathcal{E}_r$ [Eq. (11)] is clear. The situation becomes more interesting in the case where $\mathcal{T}' \neq 0$ and requires insight from numerical simulations. Consider Fig. 7. As $\mathcal{T}'$ was increased, the initial transverse focusing effect decreased, and the effect of adiabatic damping (seen clearly in the $\mathcal{T}' = 0$ curve) was overcome by an apparent defocusing force. This is the result of two competing effects. First, there is a direct dependence of the $\mathcal{E}'\mathcal{T}$ focusing term on $\mathcal{T}'$. More indirectly, increasing $\mathcal{T}'$ decreases the strength of the $\mathcal{E}'\mathcal{T}$ term (specifically, the summation of all $\mathcal{E}'\mathcal{T}$ terms), as $\mathcal{T} > 1$ for modules upstream of the bunch and $\mathcal{T} < 1$ for modules downstream when $\mathcal{T}' > 0$. Finally, it is worth highlighting the longitudinal focusing effect of Eq. (30). The correlation between a monotonically increasing time-scale function and longitudinal focusing follows from conventional beam physics knowledge. However, it was not obvious that simply applying a linear time gradient to each module independently would translate to longitudinal stability. Demonstrating that this general principle holds is an important argument in favor of external pulse injection-based DWA designs.

The speed, flexibility, and optimization capabilities of the DWA model in TRANSOPTR make it an ideal tool for exploring the design space of not just the DWA acceleration device, but also of complete source-to-patient machines. This is important for prototyping, testing component designs, and gantry configurations, providing feedback for pulser and waveguide development, and building beam models for treatment planning studies. For example, a practical takeaway from Sec. IV B is that a realistic source-to-patient beamline should include a preacceleration stage. This follows from Figs. 5 and 6, which show that there is a minimum acceptable input energy to a 100 MV/m accelerating field. The flexibility of our model also allows for the incorporation of any experimental field data, which can be

used in place of the analytical functions described in this work. This creates a useful feedback loop. However, there are some important limitations of this work that should be mentioned. By definition, a linear optics model cannot account for terms beyond second order in the Hamiltonian. This includes aberrations and chromaticity. Furthermore, linear optics deals with the second moments of the particle bunch, such as width, length, and energy spread. These envelopes are an accurate statistical description of the particle bunch, but they do not describe individual particle dynamics, making it inadequate for a direct, quantitative study of beam loss. To study these effects, particle-by-particle (e.g., particle-in-cell) simulations would be necessary. Used in conjunction with a linear optics framework, such simulations could provide further insight into DWA beam dynamics.

VI. CONCLUSIONS

In this work, an analytical treatment of the fields in a DWA was presented, and the results were used to develop a linear optics model. An implementation of this model has been made publicly available as part of the open-source code TRANSOPTR. It was tested against TRANSOPTR's rf LINAC subroutine, which itself has been validated against numerical and experimental evidence. It was then used to study particle injection and transport in a DWA for a range of bunch and field parameters. These data indicate that longitudinal control is possible in DWAs with linearly increasing time profiles, and that appropriate choices of injection energy, acceleration gradient, aperture radius, and segment length can provide adequate transverse confinement. The time-varying fields cause a transverse defocusing effect, creating an optimization problem where increasing longitudinal focusing and limiting transverse beam growth are in opposition. This tool is ready to be used in the design and study of DWA beamlines and could prove useful for prototyping.

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APPENDIX: HAMILTONIAN LINEAR OPTICS

1. Origin of the F matrix

The equation of motion for a single particle, without loss of generality, is

$$\frac{d\vec{X}}{ds} = \mathbf{J}\nabla_{\vec{X}}H, \quad (\text{A1})$$

where $\mathbf{J}$ is the symplectic matrix and H is the system's Hamiltonian. In a linear system, H can be expressed precisely as

$$H = \frac{1}{2} \vec{X}^T \mathbf{D}_H \vec{X}, \quad (\text{A2})$$

where $\mathbf{D}_H$ is the matrix of second-order partial derivatives of the Hamiltonian. This allows Eq. (A1) to be reexpressed as

$$\frac{d\vec{X}}{ds} = \mathbf{J} \nabla_{\vec{X}} H = \mathbf{J} \mathbf{D}_H \vec{X} = \mathbf{F} \vec{X}, \quad (\text{A3})$$

where we have defined $\mathbf{F} = \mathbf{J} \mathbf{D}_H$. These equations of motion are a system of ordinary linear differential equations, sometimes referred to as a matrix differential equation. The solution to this system is

$$\vec{X}(s) = \mathbf{M}(s) \vec{X}(s_0), \quad (\text{A4})$$

where the transfer matrix

$$\mathbf{M}(s) = \exp \left(\int_{s_0}^s \mathbf{F}(\zeta) d\zeta \right) \quad (\text{A5})$$

allows us to transport $\vec{X}$ from s_0 to s .

2. Origin of the envelope equations

To describe the dynamics of an ensemble of particles, we introduce the beam matrix σ , which is the covariance matrix of an arbitrary distribution of particles in 6D phase space. The beam matrix can be transported by applying the same transfer matrix $\mathbf{M}$ as a matrix transformation:

$$\sigma(s) = \mathbf{M}(s) \sigma(s_0) \mathbf{M}^T(s). \quad (\text{A6})$$

In practice, however, $\mathbf{M}$ is often too complex to express analytically. Instead, it is more useful to integrate the equation of motion of the beam matrix directly. To obtain this equation, we first consider the transfer matrix $\mathbf{M}$ over a small positive length δs along the reference trajectory. By Taylor expansion, we get

$$\mathbf{M}(s_0 + \delta s) = \mathbf{I} + \mathbf{F}(s_0) \delta s + \mathcal{O}(\delta s^2), \quad (\text{A7})$$

where $\mathbf{I}$ is the identity matrix. Transporting the beam matrix over δs then leads to

$$\sigma(s_0 + \delta s) = \mathbf{M} \sigma(s_0) \mathbf{M}^T, \quad (\text{A8})$$

$$= \sigma(s_0) + \mathbf{F} \sigma(s_0) \delta s + \sigma(s_0) \mathbf{F}^T \delta s + \mathcal{O}(\delta s^2), \quad (\text{A9})$$

where we have dropped the arguments from $\mathbf{M}$ and $\mathbf{F}$ for readability. Finally, we ignore the higher order terms and use the definition of the derivative to obtain

$$\frac{d\sigma}{ds}(s_0) = \lim_{\delta s \rightarrow 0} \frac{\sigma(s_0 + \delta s) - \sigma(s_0)}{\delta s} = \mathbf{F} \sigma(s_0) + \sigma(s_0) \mathbf{F}^T, \quad (\text{A10})$$

which is Eq. (1).

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