

Acceleration and spin direction control of tensor-polarized deuteron beams in a synchrotron

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To investigate the internal spin structure of nucleons and explore fundamental phenomena such as parity and time-reversal symmetry violations, polarized deuteron beams are pivotal because they serve as a unique alternative to polarized neutron beams and allow for the exploration of tensor polarization—a critical feature of spin-1 systems that offers distinct insights beyond vector polarization. This study advances the understanding of tensor-polarized deuteron beam dynamics in synchrotrons, with direct relevance to the Electron-Ion Collider in China (EicC). By deriving spin tensor transfer matrices and validating their accuracy through numerical simulations, we demonstrate the tensor polarization evolution in bending magnets, solenoids, and quadrupoles. The analysis of depolarizing resonances during acceleration reveals negligible polarization losses under the EicC's design parameters, ensuring the feasibility of high-precision spin experiments without dedicated polarization maintenance. Furthermore, the planar-constrained translational dynamics of spin tensors, fundamentally distinct from the rotational mechanisms governing spin vectors, offer novel strategies for spin manipulation in future accelerators. These results provide theoretical and computational foundations for spin tensor control in the EicC and broader applications in spin-correlation experiments.

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I. INTRODUCTION

To investigate the internal spin structure of nucleons, an electron-ion collider in the energy region of sea quark dominance has been proposed in China, i.e., the Electron-Ion Collider in China (EicC) [1]. Polarized deuteron beam, serving as a substitute for polarized neutron beam, is one of the key particle species to be collided in the EicC. It plays a crucial role in studying the spin properties of neutrons and their differences relative to proton. Another ideal substitute for a neutron beam is ^3He . Due to their different isospins and the different environments of neutrons within these two particles, both are required for spin physics experiments at the EicC. Tensor-polarized deuteron beams, uniquely enabling searches for both parity-even and parity-odd

time-reversal violating effects beyond the Standard Model [2], represent a critical experimental tool for advancing spin physics research.

The deuteron, composed of a proton and a neutron, is a spin-1 particle and possesses three spin states: +1, 0, and −1. While spin-1/2 particles, such as proton and ^3He , can be fully described by vector polarization alone, spin-1 systems require an additional tensor polarization to characterize their spin state. For instance, the vector polarization may vanish if the number of particles with +1 spin state equals those with −1 spin state, or if all particles are in the pure 0 spin state. The alignment with respect to an axis defined as $3(n_+ + n_-) - 2$ is added to characterize the state unambiguously. While numerous studies on the dynamics of vector polarization have been carried out, including the use of Siberian snakes to preserve vector polarization [3–5], research on the dynamics of tensor polarization remains limited. Previously, Huang *et al.* derived the turn-by-turn equation of motions for spin tensor based on the definition of tensor polarization and the Thomas-BMT equation [6]. He investigated the evolution of tensor polarization at constant $G\gamma$, where $G = -0.14301$ for the deuteron, and γ is the Lorentz factor. In Ref. [7], a spin invariant frame is

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identified and the evolution of the spin tensor in the laboratory frame is given through the coordinate transformation. Explicit relationships are given for axially symmetric polarization.

To study the effects of spin resonances crossing and to analyze the effects of various magnetic elements on spin tensor, this work adopts an alternative definition of spin tensor that intuitively reflects its direction. From this definition, element-by-element spin tensor equations of motion, spin tensor transfer matrices, and the spin variation of tensor polarization in different magnetic elements are subsequently computed. The case of acceleration in the EicC is analyzed employing the code Depol [8] as well as a self-developed spin tracking code, which is based on element-by-element spin tensor transfer matrices. The derived spin tensor transfer matrices are benchmarked with the Depol.

This paper is organized as follows: Sec. II presents the derivation of spin tensor equation of motion, transfer matrices, and the variation of spin tensors in common magnetic elements, establishing the theoretical foundation for tensor polarization dynamics. Section III introduces the acceleration scheme for polarized deuteron beams in the EicC and calibrates the spin tensor transfer matrices. Section IV discusses the translational characteristics of spin tensor in various magnetic elements and their distinctions from spin vector. Finally, a summary is provided in Sec. V.

II. SPIN TENSOR EQUATION OF MOTION AND THE TRANSFER MATRIX

For spin-1 particles, the vector polarization P , and the alignment A , with respect to, for instance, a vertical quantization axis, are defined respectively as

$$P = \frac{N_+ - N_-}{N_+ + N_0 + N_-} \quad (1)$$

and

$$A = 3\langle S_3^2 \rangle - 2 = \frac{3(N_+ + N_-)}{N_+ + N_0 + N_-} - 2, \quad (2)$$

where $\langle S_3 \rangle$ denote the components of the spin vector along the vertical direction. The subscripts 1, 2, and 3 of S represent the radial, longitudinal, and vertical components in the Frenet-Serret curvilinear coordinate system, respectively. The bracket here, just as those in quantum mechanics, denotes the expectation value of the operators. N_+ , N_0 , and N_- represent the number of particles with spin states $+1$, 0 , and -1 , respectively. The relative population of the spin states $m = +1$, 0 , and -1 along the quantization axis can be written in terms of P and A :

$$\begin{aligned} n_+ &= \frac{N_+}{N_+ + N_0 + N_-} = \frac{A + 2 + 3P}{6}, \\ n_- &= \frac{N_-}{N_+ + N_0 + N_-} = \frac{A + 2 - 3P}{6}, \\ n_0 &= \frac{N_0}{N_+ + N_0 + N_-} = \frac{-A + 1}{3}. \end{aligned} \quad (3)$$

We see that neither the vector polarization P nor the alignment A alone can uniquely determine the population distribution of spin states in the beam.

In one commonly used description [6], the following parameters are used:

$$\begin{aligned} \langle T_0 \rangle &= \frac{1}{\sqrt{2}} (3\langle S_3^2 \rangle - 2), \\ \langle T_{\pm 1} \rangle &= \mp \frac{\sqrt{3}}{2} \langle (S_{\pm} S_3 + S_3 S_{\pm}) \rangle, \\ \langle T_{\pm 2} \rangle &= \frac{\sqrt{3}}{2} \langle S_{\pm}^2 \rangle, \end{aligned} \quad (4)$$

where $\langle T_0 \rangle$ is the alignment with respect to the third axis, which is defined by Eq. (2). $\langle S_1 \rangle$, $\langle S_2 \rangle$, and $\langle S_3 \rangle$ denote the components of the spin vector along the radial, longitudinal, and vertical directions, respectively. S_1 , S_2 , and S_3 are the spin operators in the radial, longitudinal, and vertical directions,

$$S_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad (5)$$

$$S_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad (6)$$

$$S_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad (7)$$

and $S_{\pm} = S_1 \pm iS_2$. Given that the spin tensor components are defined as combinations of the spin vector components, the equation of motion for the spin tensor can be derived from the Thomas-BMT equation. This spin tensor equation of motion can be used to analytically compute the tensor polarization for a fixed energy near a single resonance.

Another commonly used convention to describe tensor polarization through the parameters is [9]:

$$\begin{aligned} P_{ij} &= P_{ji} = \frac{3}{2} \langle S_i S_j + S_j S_i \rangle - 2\delta_{ij}, \quad i, j = 1, 2, 3 \\ P_{11} + P_{22} + P_{33} &= 0, \end{aligned} \quad (8)$$

where P_{11} , P_{22} , and P_{33} correspond to the alignment A in the radial, longitudinal, and vertical directions, respectively. In this paper, the spin tensor is ordered as follows: $\mathcal{P} = (P_{12}, P_{13}, P_{23}, P_{11}, P_{22}, P_{33})$. Taking the wave functions $(\pm \frac{1}{\sqrt{2}}, 0, \pm \frac{1}{\sqrt{2}})$ and $(0, 1, 0)$, substituting them into Eq. (8) and combining with Eqs. (5)–(7), the corresponding spin tensors can be obtained as $\mathcal{P} = (0, 0, 0, 1, -2, 1)$ and $\mathcal{P} = (0, 0, 0, 1, 1, -2)$. These are the two initial values of the spin tensors adopted in this paper. Although Eq. (6) introduces imaginary parts, the imaginary parts of the initial spin tensors used in this paper happen to be zero and are therefore omitted.

In Sec. II A, the element-by-element spin tensor equation of motion is found by differentiating Eq. (8) with respect to the longitudinal path length s . In Sec. II B, the spin tensor transfer matrix is derived by solving the spin tensor equation, which is used for spin tensor tracking. In Sec. II C, the spin tensor transfer matrices for some common magnets will be presented. The evolution of the spin tensor in these elements will be studied using these spin transfer matrices and the Runge-Kutta method.

A. Spin tensor equation

In a synchrotron, the Thomas-BMT equation can be rewritten in the Frenet-Serret curvilinear coordinate system $(\hat{x}, \hat{s}, \hat{y})$ (radial, longitudinal, and vertical) [10]:

$$\begin{aligned}\frac{dS_1}{ds} &= S_2\omega_3 - S_3\omega_2, \\ \frac{dS_2}{ds} &= S_3\omega_1 - S_1\omega_3, \\ \frac{dS_3}{ds} &= S_1\omega_2 - S_2\omega_1.\end{aligned}\quad (9)$$

Considering that x' and y' are on the order of 10^{-3} rad, and neglecting the terms in x' and y' , we have

$$\begin{aligned}\omega_1 &= (1 + G\gamma) \frac{B_1}{B\rho}, & \omega_2 &= (1 + G) \frac{B_2}{B\rho}, \\ \omega_3 &= (1 + G\gamma) \frac{B_3}{B\rho},\end{aligned}\quad (10)$$

where $B\rho$ denotes the magnetic rigidity of the particle, and B_1 , B_2 , and B_3 are the magnetic field components along the radial $\hat{x}$, longitudinal $\hat{s}$, and vertical $\hat{y}$ directions.

By differentiating both sides of Eq. (8) with respect to s and substituting Eq. (9) into it, the spin tensor equation of motion can be derived. For simplicity, bracket symbols are omitted.

$$\begin{aligned}\frac{dP_{12}}{ds} &= \frac{3}{2} \left(S_1 \frac{dS_2}{ds} + \frac{dS_1}{ds} S_2 + S_2 \frac{dS_1}{ds} + \frac{dS_2}{ds} S_1 \right) \\ &= \omega_1 P_{13} - \omega_2 P_{23} + \omega_3 P_{22} - \omega_3 P_{11}, \\ \frac{dP_{13}}{ds} &= \frac{3}{2} \left(S_1 \frac{dS_3}{ds} + \frac{dS_1}{ds} S_3 + S_3 \frac{dS_1}{ds} + \frac{dS_3}{ds} S_1 \right) \\ &= \omega_3 P_{23} - \omega_1 P_{12} + \omega_2 P_{11} - \omega_2 P_{33}, \\ \frac{dP_{23}}{ds} &= \frac{3}{2} \left(S_2 \frac{dS_3}{ds} + \frac{dS_2}{ds} S_3 + S_3 \frac{dS_2}{ds} + \frac{dS_3}{ds} S_2 \right) \\ &= \omega_2 P_{12} - \omega_3 P_{13} + \omega_1 P_{33} - \omega_1 P_{22}, \\ \frac{dP_{11}}{ds} &= 3 \left(S_1 \frac{dS_1}{ds} + \frac{dS_1}{ds} S_1 \right) \\ &= 2\omega_3 P_{12} - 2\omega_2 P_{13}, \\ \frac{dP_{22}}{ds} &= 3 \left(S_2 \frac{dS_2}{ds} + \frac{dS_2}{ds} S_2 \right) \\ &= 2\omega_1 P_{23} - 2\omega_3 P_{12}, \\ \frac{dP_{33}}{ds} &= 3 \left(S_3 \frac{dS_3}{ds} + \frac{dS_3}{ds} S_3 \right) \\ &= 2\omega_2 P_{13} - 2\omega_1 P_{23}.\end{aligned}\quad (11)$$

Introducing the vector $\mathcal{P} = (P_{12}, P_{13}, P_{23}, P_{11}, P_{22}, P_{33})$, Eq. (11) can be rewritten in matrix form as

$$\frac{d}{ds} \begin{pmatrix} P_{12} \\ P_{13} \\ P_{23} \\ P_{11} \\ P_{22} \\ P_{33} \end{pmatrix} = M \begin{pmatrix} P_{12} \\ P_{13} \\ P_{23} \\ P_{11} \\ P_{22} \\ P_{33} \end{pmatrix} \triangleq M\mathcal{P}, \quad (12)$$

where M is given by

$$M = \begin{pmatrix} 0 & \omega_1 & -\omega_2 & -\omega_3 & \omega_3 & 0 \\ -\omega_1 & 0 & \omega_3 & \omega_2 & 0 & -\omega_2 \\ \omega_2 & -\omega_3 & 0 & 0 & -\omega_1 & \omega_1 \\ 2\omega_3 & -2\omega_2 & 0 & 0 & 0 & 0 \\ -2\omega_3 & 0 & 2\omega_1 & 0 & 0 & 0 \\ 0 & 2\omega_2 & -2\omega_1 & 0 & 0 & 0 \end{pmatrix}. \quad (13)$$

At this point, the equations of motion for the spin tensor have been derived, and these equations can be directly used to implement spin tensor tracking. One straightforward approach is to numerically solve the equation of motion of the spin tensor, using methods like the Runge-Kutta method, which has been adopted by BMAD [11] for spin vector tracking. Another approach is to track by the spin tensor transfer matrix, a method previously applied to

spin vector tracking [10,12] and particle coordinate tracking [13]. In this paper, the spin tensor matrices are derived by solving the spin tensor equations.

B. Transfer matrix for the spin tensor

To investigate the spin motion in an accelerator, we shall derive the spin tensor transfer matrix in various magnetic components. Assuming that the fields are piecewise constant, the spin tensor equation of motion is a system of first-order homogeneous differential equations with constant coefficients. The general solution of this equation will therefore be yielded by solving for the eigenvalues of M .

The eigenvalues of M are:

$$\lambda = 0, 0, -i\omega, i\omega, -2i\omega, 2i\omega, \quad (14)$$

where $\omega^2 = \omega_1^2 + \omega_2^2 + \omega_3^2$. Therefore, the general solution for the each of the six components of the tensor polarization can be expressed in the following form:

$$\begin{aligned} \mathcal{P}_i(s) = & C_{1i} + C_{2i} \cos \omega s + C_{3i} \sin \omega s \\ & + C_{4i} \cos 2\omega s + C_{5i} \sin 2\omega s, \end{aligned} \quad (15)$$

where C_{1i} , C_{2i} , C_{3i} , C_{4i} , and C_{5i} are arbitrary real constants. The transformation of the spin tensor from s_0 to s_1 is obtained by substituting these constants into the general solution of the spin tensor at $s_1 = s_0 + \delta s$. Consequently, the next step involves solving for these constants.

From Eq. (15), the derivatives with respect to s of the spin tensor components at $s = s_0$ are:

$$\begin{aligned} \mathcal{P}'_i(s_0) = & -C_{2i}\omega \sin \omega s_0 + C_{3i}\omega \cos \omega s_0 \\ & - 2C_{4i}\omega \sin 2\omega s_0 + 2C_{5i}\omega \cos 2\omega s_0, \\ \mathcal{P}''_i(s_0) = & -C_{2i}\omega^2 \cos \omega s_0 - C_{3i}\omega^2 \sin \omega s_0 \\ & - 4C_{4i}\omega^2 \cos 2\omega s_0 - 4C_{5i}\omega^2 \sin 2\omega s_0, \\ \mathcal{P}'''_i(s_0) = & C_{2i}\omega^3 \sin \omega s_0 - C_{3i}\omega^3 \cos \omega s_0 \\ & + 8C_{4i}\omega^3 \sin 2\omega s_0 - 8C_{5i}\omega^3 \cos 2\omega s_0, \\ \mathcal{P}''''_i(s_0) = & C_{2i}\omega^4 \cos \omega s_0 + C_{3i}\omega^4 \sin \omega s_0 \\ & + 16C_{4i}\omega^4 \cos 2\omega s_0 + 16C_{5i}\omega^4 \sin 2\omega s_0. \end{aligned} \quad (16)$$

These derivatives can also be obtained using Eq. (12). Within a single uniform-field magnet under the hard-edge approximation, ω_1 , ω_2 , and ω_3 are constants. So that each differentiation of Eq. (12) effectively results in left-multiplying by M once. Therefore, when both sides of Eq. (12) are differentiated with respect to s , the following expressions are obtained:

$$\begin{aligned} \mathcal{P}'_i(s_0) &= \sum_{j=1}^6 M_{ij}^2 \mathcal{P}_j(s_0), \\ \mathcal{P}''_i(s_0) &= \sum_{j=1}^6 M_{ij}^3 \mathcal{P}_j(s_0), \\ \mathcal{P}'''_i(s_0) &= \sum_{j=1}^6 M_{ij}^4 \mathcal{P}_j(s_0), \end{aligned} \quad (17)$$

where i and j are integers ranging from 1 to 6. M_{ij}^n denotes the element in the i th row and j th column of the matrix M^n . $\mathcal{P}_n$ with $n = 1, 2, 3, 4, 5, 6$ represents the n th component of the spin tensor $\mathcal{P}$, respectively.

Based on Eqs. (16) and (17), the constants in the general solution of the spin tensor can be determined as follows:

$$\begin{aligned} C_{1i} = & \mathcal{P}_i(s_0) + \frac{5}{4\omega^2} \sum_{j=1}^6 M_{ij}^2 \mathcal{P}_j(s_0) + \frac{1}{4\omega^4} \sum_{j=1}^6 M_{ij}^4 \mathcal{P}_j(s_0), \\ C_{2i} \cos \omega s_0 + C_{3i} \sin \omega s_0 = & -\frac{4}{3\omega^2} \sum_{j=1}^6 M_{ij}^2 \mathcal{P}_j(s_0) - \frac{1}{3\omega^4} \sum_{j=1}^6 M_{ij}^4 \mathcal{P}_j(s_0), \\ C_{4i} \cos 2\omega s_0 + C_{5i} \sin 2\omega s_0 = & -\frac{1}{12\omega^2} \sum_{j=1}^6 M_{ij}^2 \mathcal{P}_j(s_0) - \frac{1}{12\omega^4} \sum_{j=1}^6 M_{ij}^4 \mathcal{P}_j(s_0), \\ -C_{2i} \sin \omega s_0 + C_{3i} \cos \omega s_0 = & \frac{4}{3\omega} \sum_{j=1}^6 M_{ij} \mathcal{P}_j(s_0) + \frac{1}{3\omega^3} \sum_{j=1}^6 M_{ij}^3 \mathcal{P}_j(s_0), \\ -C_{4i} \sin 2\omega s_0 + C_{5i} \cos 2\omega s_0 = & -\frac{1}{6\omega} \sum_{j=1}^6 M_{ij} \mathcal{P}_j(s_0) - \frac{1}{6\omega^3} \sum_{j=1}^6 M_{ij}^3 \mathcal{P}_j(s_0). \end{aligned} \quad (18)$$

For simplicity of notation, let:

$$m = \frac{1}{\omega}M, m^2 = \frac{1}{\omega^2}M^2, m^3 = \frac{1}{\omega^3}M^3, m^4 = \frac{1}{\omega^4}M^4,$$

$$S_1 = \sin \omega \delta s, S_2 = \sin 2\omega \delta s,$$

$$C_1 = 1 - \cos \omega \delta s, C_2 = 1 - \cos 2\omega \delta s. \quad (19)$$

By substituting $s_1 = s_0 + \delta s$ from Eq. (15) and using Eq. (18), the components of the tensor polarization can be expressed as

$$\begin{aligned} \mathcal{P}_i(s_1) &= C_{1i} + C_{2i} \cos \omega s_1 + C_{3i} \sin \omega s_1 \\ &\quad + C_{4i} \cos 2\omega s_1 + C_{5i} \sin 2\omega s_1 \\ &= C_{1i} + (C_{2i} \cos \omega s_0 + C_{3i} \sin \omega s_0) \cos \omega \delta s \\ &\quad + (-C_{2i} \sin \omega s_0 + C_{3i} \cos \omega s_0) \sin \omega \delta s \\ &\quad + (C_{4i} \cos 2\omega s_0 + C_{5i} \sin 2\omega s_0) \cos 2\omega \delta s \\ &\quad + (-C_{4i} \sin 2\omega s_0 + C_{5i} \cos 2\omega s_0) \sin 2\omega \delta s \\ &= \mathcal{P}_i(s_0) + \sum_{j=1}^6 \left[\left(\frac{4}{3}S_1 - \frac{1}{6}S_2 \right) m_{ij} \right. \\ &\quad + \left(\frac{4}{3}C_1 - \frac{1}{12}C_2 \right) m_{ij}^2 + \left(\frac{1}{3}S_1 - \frac{1}{6}S_2 \right) m_{ij}^3 \\ &\quad + \left. \left(\frac{1}{3}C_1 - \frac{1}{12}C_2 \right) m_{ij}^4 \right] \mathcal{P}_j(s_0). \end{aligned} \quad (20)$$

Therefore, the transport matrix of the tensor polarization R from s_0 to s_1 , which satisfies:

$$\mathcal{P}(s_1) = R\mathcal{P}(s_0) \quad (21)$$

is given by

$$\begin{aligned} R &= I + \left(\frac{4}{3}S_1 - \frac{1}{6}S_2 \right) m + \left(\frac{4}{3}C_1 - \frac{1}{12}C_2 \right) m^2 \\ &\quad + \left(\frac{1}{3}S_1 - \frac{1}{6}S_2 \right) m^3 + \left(\frac{1}{3}C_1 - \frac{1}{12}C_2 \right) m^4, \end{aligned} \quad (22)$$

where I denotes the identity matrix. The spin tensor transfer matrix R exhibits greater complexity compared to the spin vector transfer matrix (for instance, in Ref. [14]). The explicit expression of the matrix elements of R are listed in the Appendix. Here, the spin tensor transfer matrix for constant field magnetic elements has been derived, exhibiting a highly intricate structure. For specific magnetic elements, the spin tensor transfer matrix can be simplified significantly, yielding more intuitive descriptions of spin tensor variation dynamics. This simplification occurs in elements with uniform magnetic fields such as bending magnets and solenoids. For gradient field magnets, one key point should be emphasized. During the derivation, particle coordinates between s_0 and s_1 are approximated as constant. This simplification is known as the thin lens

approximation. In this paper, the particle coordinates used to calculate the spin transfer matrix are the average values of the particle coordinates at the entrance and exit of the quadrupole slice. However, calculating particle coordinates does not require the thin lens approximation. Instead, they are computed using the transfer matrix generated by TWISS parameters from MADX [15]. When tracking the spin tensor in gradient field magnets, a sufficient number of slices is required to ensure that the calculation error is smaller than the tolerance error. If the relative error between the calculation results with more slices and fewer slices is less than the tolerance error, such a number of slices is considered appropriate.

C. Spin tracking in single elements

The spin tensor transfer matrices for the following typical magnetic elements are given: horizontal bending magnets (vertical magnetic fields), vertical bending magnets (radial magnetic fields), solenoids (longitudinal magnetic fields), and quadrupoles. The spin-tracking results based on these spin tensor transfer matrices will be compared with those obtained by solving the spin tensor equation of motion using the Runge-Kutta method to integrate Eq. (12).

For horizontal bending magnets, it is

$$\begin{aligned} \omega_1 = \omega_2 = 0, \omega_3 &= \frac{1}{\rho}(1 + G\gamma), \\ S_1 &= \sin \omega \delta s, S_2 = \sin 2\omega \delta s, \\ C_1 &= 1 - \cos \omega \delta s, C_2 = 1 - \cos 2\omega \delta s, \end{aligned} \quad (23)$$

$$R = \begin{pmatrix} 1-C_2 & 0 & 0 & -S_2/2 & S_2/2 & 0 \\ 0 & 1-C_1 & S_1 & 0 & 0 & 0 \\ 0 & -S_1 & 1-C_1 & 0 & 0 & 0 \\ S_2 & 0 & 0 & 1-C_2/2 & C_2/2 & 0 \\ -S_2 & 0 & 0 & C_2/2 & 1-C_2/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \quad (24)$$

Applying the spin tensor transfer matrices of horizontal bending magnets at different bending angles θ to the spin tensor produces variations that align with the data points shown in Fig. 1. Here, the horizontal axis represents the bending angle of the horizontal bending magnet, while the vertical axis shows the spin tensor components at the dipole exit. The initial spin tensor of the particles is $\mathcal{P} = (0, 0, 0, 1, -2, 1)$. This initial value can be derived by taking the expectation values of the spin tensor operator with respect to the wave functions $(\pm \frac{1}{\sqrt{2}}, 0, \pm \frac{1}{\sqrt{2}})$. These include the diagonal components P_{11} , P_{22} , and P_{33} (upper panel) and the off-diagonal components P_{12} , P_{13} , and P_{23} (lower panel). Both P_{13} and P_{23} remain 0.

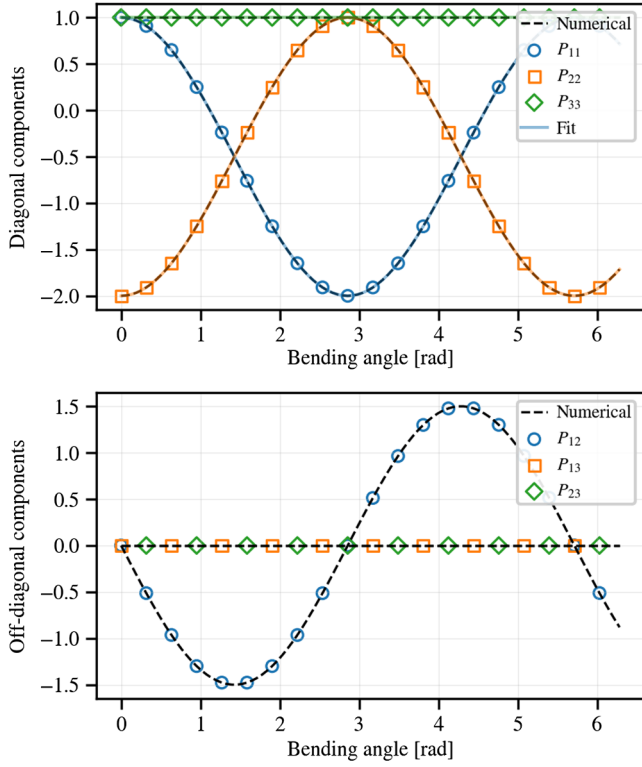


FIG. 1. Variation of spin tensor components vs bending angle for a horizontal bending magnet.

The numerical solutions represented by dashed lines are obtained via the Runge-Kutta method applied to the spin tensor equations. To address numerical divergence caused by large step sizes, an adaptive step size control is implemented in the algorithm. Specifically, the step size is iteratively halved until the difference between consecutive calculations falls below the tolerance error threshold, which is set to 10^{-5} . The spin tensor variations computed using the spin transfer matrices exhibit full agreement with those derived from the adaptive step size Runge-Kutta method, thereby confirming the validity of our expressions for the spin tensor transfer matrices.

The diagonal components of the spin tensor (P_{11} , P_{22} , P_{33}) have been fitted with functions of the form $a \cos(b\theta + c) + d$, yielding the solid curves in Fig. 1. The resulting fitted curves of P_{11} , P_{22} , and P_{33} are:

$$\begin{aligned} P_{11} &= \frac{3}{2} \cos(2(1 + G\gamma)\theta) - \frac{1}{2}, \\ P_{22} &= -\frac{3}{2} \cos(2(1 + G\gamma)\theta) - \frac{1}{2}, \\ P_{33} &= 1. \end{aligned} \quad (25)$$

The sum of these three components strictly satisfies the condition $P_{11} + P_{22} + P_{33} = 0$ [see Eq. (8)].

It is observed that as the bending angle increases, the spin tensor gradually transforms from the initial state with

$P_{22} = -2$ to a state with $P_{11} = -2$. This transformation aligns with the prediction of the Thomas-BMT equation, as expected. This implies a complete conversion of the spin alignment from a 100% population of particles in the longitudinal alignment state to a 100% alignment in the radial direction. In other words, the horizontal bending magnet enables controllable transformations between P_{11} and P_{22} . Subsequently, $P_{11} = -2$ reverts back to $P_{22} = -2$, as further increase, with P_{33} remaining unchanged. A detailed analysis of spin direction control is provided in Sec. IV.

For vertical bending magnets, it is

$$\begin{aligned} \omega_1 &= \frac{1}{\rho} (1 + G\gamma), \omega_2 = \omega_3 = 0, \\ \mathcal{S}_1 &= \sin \omega \delta s, \mathcal{S}_2 = \sin 2\omega \delta s, \\ \mathcal{C}_1 &= 1 - \cos \omega \delta s, \mathcal{C}_2 = 1 - \cos 2\omega \delta s, \end{aligned} \quad (26)$$

$$R = \begin{pmatrix} 1 - \mathcal{C}_1 & \mathcal{S}_1 & 0 & 0 & 0 & 0 \\ -\mathcal{S}_1 & 1 - \mathcal{C}_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 - \mathcal{C}_2 & 0 & -\mathcal{S}_2/2 & \mathcal{S}_2/2 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \mathcal{S}_2 & 0 & 1 - \mathcal{C}_2/2 & \mathcal{C}_2/2 \\ 0 & 0 & -\mathcal{S}_2 & 0 & \mathcal{C}_2/2 & 1 - \mathcal{C}_2/2 \end{pmatrix}. \quad (27)$$

By applying the above spin transfer matrices of vertical bending magnets corresponding to different bending angles θ to the spin tensor, the resulting variations in the spin tensor are shown in Fig. 2. The initial value is set to $\mathcal{P} = (0, 0, 0, 1, 1, -2)$. The results from the spin transfer matrices (represented by data points) closely match those obtained via the Runge-Kutta numerical method (dashed lines). The diagonal components of the spin tensor are fitted by

$$\begin{aligned} P_{11} &= 1, \\ P_{22} &= \frac{3}{2} \cos[2(1 + G\gamma)\theta] - \frac{1}{2}, \\ P_{33} &= -\frac{3}{2} \cos[2(1 + G\gamma)\theta] - \frac{1}{2}. \end{aligned} \quad (28)$$

The sum of these three components satisfies the tensor polarization definition. The spin tensor evolution under vertical bending magnets closely resembles that under horizontal bending magnets. The distinction lies in their control mechanisms as expected from the Thomas-BMT equation: the former enables transformations between P_{22} and P_{33} , while the latter governs the interplay between P_{11} and P_{22} .

For solenoids, it is

$$\begin{aligned} \omega_1 &= \omega_3 = 0, \omega_2 = k_s(1 + G), \\ \mathcal{S}_1 &= \sin \omega \delta s, \mathcal{S}_2 = \sin 2\omega \delta s, \\ \mathcal{C}_1 &= 1 - \cos \omega \delta s, \mathcal{C}_2 = 1 - \cos 2\omega \delta s, \end{aligned} \quad (29)$$

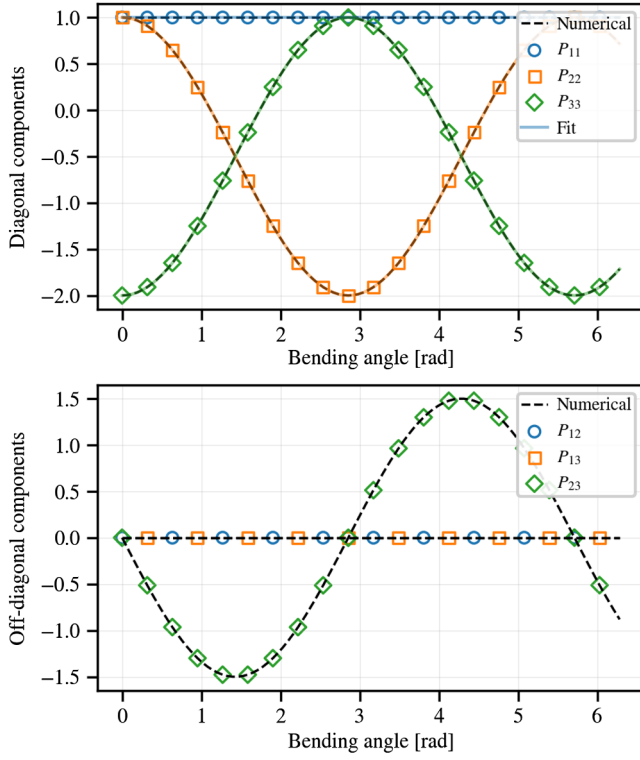


FIG. 2. Variation of spin tensor components vs bending angle for a vertical bending magnet.

$$R = \begin{pmatrix} 1-C_1 & 0 & -S_1 & 0 & 0 & 0 \\ 0 & 1-C_2 & 0 & S_2/2 & 0 & -S_2/2 \\ S_1 & 0 & 1-C_1 & 0 & 0 & 0 \\ 0 & -S_2 & 0 & 1-C_2/2 & 0 & C_2/2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & S_2 & 0 & C_2/2 & 0 & 1-C_2/2 \end{pmatrix}, \quad (30)$$

where k_s is the solenoid strength, which is defined as the ratio of the solenoid magnetic field to the beam momentum rigidity. The evolution of the spin tensor under solenoids of varying lengths L with $k_s = 1 \text{ m}^{-1}$ is illustrated in Fig. 3. Here, the horizontal axis represents the solenoid length, and the initial state is set to $\mathcal{P} = (0, 0, 0, 1, 1, -2)$. The results from the spin transfer matrices closely match those obtained via the numerical method (dashed lines), further validating our expressions for the spin tensor transfer matrices. The fitting results indicate that the diagonal components of the spin tensor follow the variation patterns below:

$$\begin{aligned} P_{11} &= \frac{3}{2} \cos [2(1+G)k_s L] - \frac{1}{2}, \\ P_{22} &= 1, \\ P_{33} &= -\frac{3}{2} \cos [2(1+G)k_s L] - \frac{1}{2}. \end{aligned} \quad (31)$$

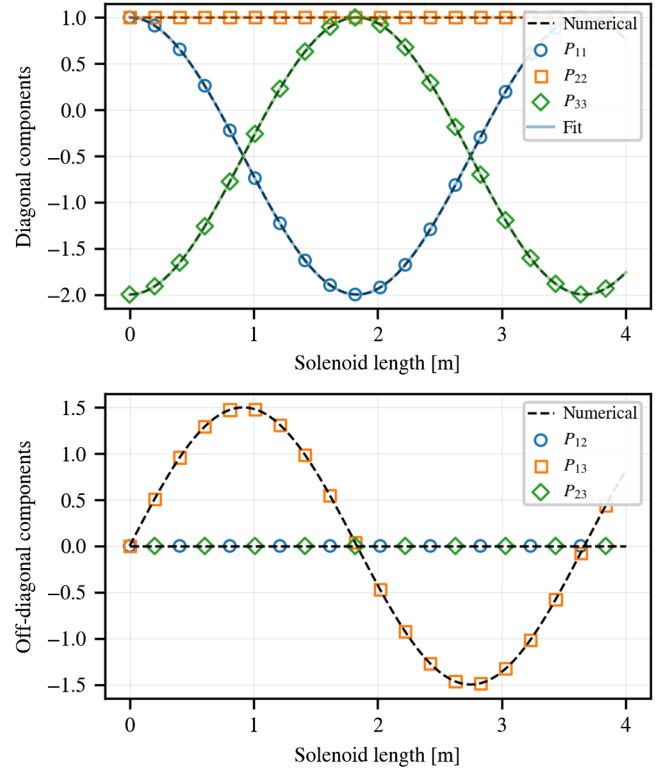


FIG. 3. Variation of spin tensor components vs magnet length for a $k_s = 1 \text{ m}^{-1}$ solenoid.

The sum of these three components satisfies the tensor polarization definition. Notably, the coefficient $2(1+G)$ governing the spin tensor dynamics under solenoids differs from the $2(1+G\gamma)$ term for bending magnets. This discrepancy arises from the different coefficients in the Thomas-BMT equation for magnetic fields parallel versus perpendicular to the particle velocity. The solenoid enables precise control over the interconversion between P_{11} and P_{33} .

For quadrupoles, the spin tensor matrix follows:

$$\begin{aligned} k_1 &= -\frac{dB/dr}{B\rho}, r = \sqrt{x^2 + y^2}, \\ \omega_1 &= k_1(1+G\gamma)y, \omega_2 = 0, \omega_3 = k_1(1+G\gamma)x, \\ S_1 &= \sin \omega \delta s, S_2 = \sin 2\omega \delta s, \\ C_1 &= 1 - \cos \omega \delta s, C_2 = 1 - \cos 2\omega \delta s, \end{aligned} \quad (32)$$

where k_1 is the quadrupole strength, with the unit of m^{-2} . The spin transfer matrices for quadrupoles can be computed by substituting the transfer matrices provided in the Appendix. Figure 4 shows the effect of a quadrupole with $k_1 L = 1 \text{ m}^{-1}$ and $L = 1 \text{ m}$ on the spin tensor of particles with $x = y$ under the thin lenses approximation. Here, the horizontal axis represents the values of the particle coordinates x and y . The results of the spin tensor transfer

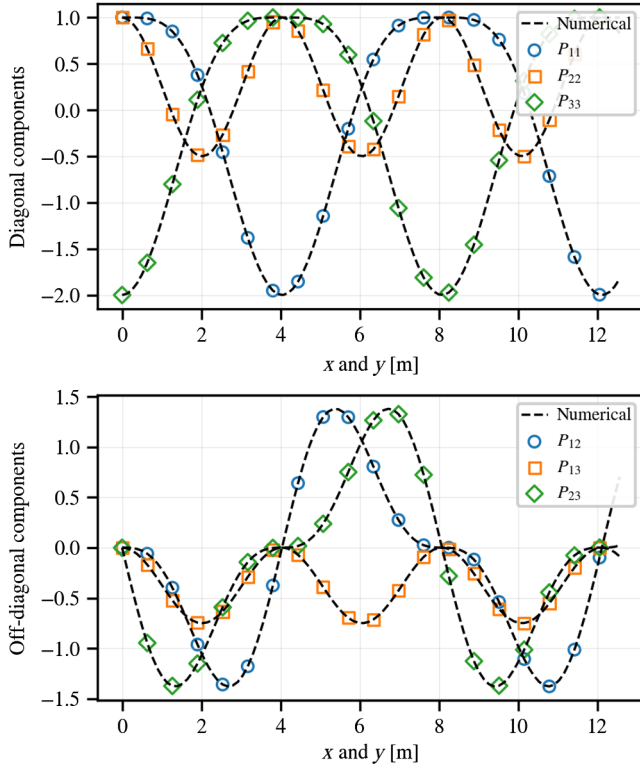


FIG. 4. Spin tensor components at the exit of a quadrupole for different starting values of the particle transverse coordinates with $x = y$.

matrix align with numerical simulations, confirming its validity.

In a quadrupole, the spin precession axis and angle depend upon the particle transverse position and slope along the magnet. It is therefore necessary to sample the particle trajectory at a sufficiently large number of points, for instance, replacing the quadrupole by thin lenses. In this paper, the number of slices of the quadrupoles used for spin tracking is 2. The relative deviation between 4 and 2 slices was of the order of 10^{-2} to be compared to the measurement precision of vector polarization and tensor polarization which are $\pm 2\%$ and $\pm 5\%$, respectively [16]. For stronger quadrupoles and/or if more precise results are required, both the number of orbit and spin slices need to be increased. In [17] accurate methods for orbit and spin tracking are described.

In the next section, we will introduce a self-developed simulation program based on the spin tensor transfer matrices derived above, designed to track and analyze phenomena involving tensor-polarized deuteron beams crossing depolarizing resonances.

III. THE ACCELERATION OF POLARIZED DEUTERON BEAMS

In this section, the behavior of vector- and tensor-polarized deuteron beam crossing depolarizing resonances

during acceleration is analyzed through theoretical formula and spin tracking. The theoretical framework is based on the Depol code [8] and the Froissart-Stora formula, where the depolarizing resonance strength calculated by Depol is substituted into the Froissart-Stora formula to evaluate polarization changes during resonance crossing. For spin tracking, a program is utilized, which relies on the spin tensor transfer matrices derived in Sec. II, the vector polarization is handled using the spin vector transfer matrix algorithm (see [10]), while the particle coordinates are transported through transfer matrices computed from MAD-generated TWISS parameters [15]. The tensor polarization tracking is implemented with the spin tensor transfer matrices derived in this work. Taking the EicC-BRing lattice as an example, all scenarios of tensor-polarized beams crossing depolarizing resonances are examined by theoretical formula and spin tracking: (i) Weak resonance strength: As we will see, in this case there is negligible polarization loss. (ii) Moderate resonance strength: In this case, measurable tensor polarization loss occurs. (iii) Strong resonance strength: In this case, the tensor polarization undergoes adiabatic variation during resonance crossing, resulting in no net polarization loss. The spin tensor direction varies from the $\hat{y}$ direction to the $\hat{x}\hat{z}$ -plane near the resonance tune, subsequently reverting back to the $\hat{y}$ direction.

The Electron-Ion Collider in China (EicC) is based on the underconstruction High Intensity heavy ion Accelerator Facility (HIAF) [18]. Its ion accelerator complex fully utilizes the linear accelerator (iLinac) and the booster ring BRing-N of the HIAF. A new superconducting booster, BRing-S, will be added in the same BRing-N tunnel. BRing-N and BRing-S can increase the ion beam energy to the collision energy of the EicC.

Table I summarizes the parameters used in this work for tracking polarized deuteron beam in the BRing-N and BRing-S synchrotrons, which share a common tunnel and currently employ identical magnetic lattice configurations. Here, α denotes $dG\gamma/d\theta$, where θ represents the cumulated dipole bending angle, k represents the depolarizing resonance tune. According to the current design, the acceleration rates, α through all resonances will be the same. Consequently, the polarization loss at each depolarizing

TABLE I. Parameters of the deuteron beam.

Parameter	Value
Anomalous g-factor G	-0.14301
Acceleration rate for $\nu_{\text{spin}}\alpha$ (rad^{-1})	-1.7938×10^{-6}
$\Delta E/\Delta t$ (GeV/u/s)	36.47184
Transverse tune ν_x/ν_y	$9.47/9.43$
Normalized emittance (π mm mrad)	$20/10$
E_k at BRing-N (GeV/u)	$0.048-4.21$
E_k at BRing-S (GeV/u)	$4.21-11.90$
ν_{spin} at BRing-N	$-0.15-0.79$
ν_{spin} at BRing-S	$-0.79-1.97$

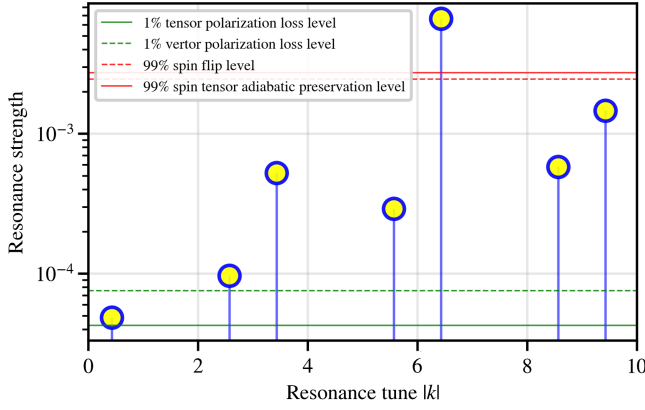


FIG. 5. Strength of the intrinsic depolarizing resonances of the polarized deuteron beam for the emittances in Table I in the BRing calculated by the Depol.

resonance tune depends solely on the depolarizing resonance strength. During the acceleration process of the deuteron beam, two intrinsic depolarizing resonances at -0.43 and -0.57 are crossed in BRing-N, while one imperfection depolarizing resonance at -1 and two intrinsic depolarizing resonances at -1.43 and -1.57 are crossed in BRing-S.

The intrinsic depolarizing resonance strengths at each resonance tune $|k|$ for polarized deuteron beams in the BRing calculated across the range of $G\gamma$ from -0.15 to -10 using the Depol code are shown in Fig. 5. We intentionally extend the energy range of BRing to higher energies only to further benchmark the spin-tracking program. The real and imaginary parts of the $k = -0.57$ resonance strength are below 10^{-6} , falling beneath the default output precision threshold of Depol, and therefore are not included in Fig. 5. Spin tracking simulations across these resonance tunes reveal that the beam polarization remains unchanged during 100 turns of tracking at constant $G\gamma$, indicating that the strength of unmarked depolarizing resonances is sufficiently weak to be neglected.

The theoretical formula describing polarization changes when the spin tune crosses a resonance of strength ϵ at a rate α is the Froissart-Stora formula [19]:

$$\frac{P_{3f}}{P_{3i}} = 2e^{\frac{-\pi\epsilon^2}{2|\alpha|}} - 1, \quad (33)$$

where P_{3i} and P_{3f} are the initial and final vector polarization components, respectively. From this formula, the expressions for the polarization tensor components P_{33i} and P_{33f} obviously follow, namely:

$$\frac{P_{33f}}{P_{33i}} = \frac{3}{2}(2e^{\frac{-\pi\epsilon^2}{2|\alpha|}} - 1)^2 - \frac{1}{2}. \quad (34)$$

Equations (33) and (34) were experimentally verified in Ref. [16]. In Ref. [16], by defining the vector spin flip

efficiency η_v , tensor spin flip efficiency η_t , vector depolarization strength ϵ_v , and tensor depolarization strength ϵ_t , the Froissart-Stora formula was rewritten as

$$\begin{aligned} \frac{P_{3f}}{P_{3i}} \Big|_{\text{emp}} &= (1 + \eta_v)e^{-\pi\epsilon_v^2/(2|\alpha|)} - \eta_v, \\ \frac{P_{33f}}{P_{33i}} \Big|_{\text{emp}} &= \frac{3}{2}[(1 + \eta_t)e^{-\pi\epsilon_t^2/(2|\alpha|)} - \eta_t]^2 - \frac{1}{2}. \end{aligned} \quad (35)$$

Through experimental data, the fitted values of the four defined quantities were obtained as $\eta_v = 91\% \pm 5\%$ and $\epsilon_v = (17.3 \pm 0.6) \times 10^{-6}$; $\eta_t = 99\% \pm 5\%$ and $\epsilon_t = (16.3 \pm 0.3) \times 10^{-6}$. This verifies the correctness of the two formulas, Eqs. (33) and (34), since η is close to 1 within the measurement precision. Additionally, this suggests that the same η and ϵ describe both the vector and tensor polarizations during a resonance crossing.

The four horizontal lines are annotated in Fig. 5 to visualize the impact of the resonance on polarization. From bottom to top, these are sequentially identified as the 1% tensor polarization loss level, 1% vector polarization loss level, 99% spin flip level, and 99% spin tensor adiabatic preservation level. The 99% spin flip level signifies that the vector polarization direction is reversed, with a 1% loss, while the 99% spin tensor adiabatic preservation level indicates that the spin tensor direction reverts to its original direction after crossing the resonance tune, with a 1% loss of tensor polarization. The depolarizing resonances located between the upper two auxiliary lines and the lower two auxiliary lines will result in vector and tensor polarization losses.

Within the energy range of BRing-N, there is only one intrinsic depolarizing resonance at $k = -0.43$. This resonance causes approximately 1.2% tensor polarization loss, while vector polarization loss remains below 1%. No polarization loss induced by the intrinsic depolarizing resonances is expected within the energy range of BRing-S. The deuteron beam in BRing-S is subjected to an imperfection depolarizing resonance at $k = 1$. When the maximum amplitude of the closed orbit is on the order of 10 mm, the corresponding depolarization strength is 4.123×10^{-6} . The corrected closed orbit is smaller than 1 mm. The P_{33f}/P_{33i} and P_{3f}/P_{3i} of crossing this depolarizing resonance are 99.991% and 99.997%, which are significantly below the polarization measurement precision, rendering its polarization loss negligible. In conclusion, polarization losses for the deuteron beam within the EicC energy range are essentially negligible for both vector and tensor polarizations, eliminating the need for dedicated polarization maintenance devices.

The theoretical predictions of polarization evolution through these depolarizing resonances and corresponding spin-tracking results are shown in Fig. 6. The orange dashed line represents theoretical vector polarization

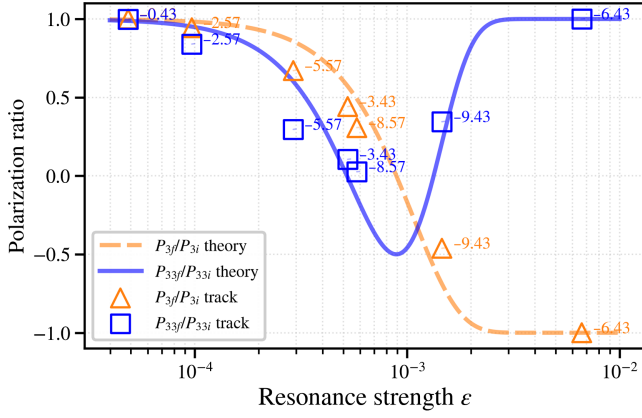


FIG. 6. Ratio of the final to initial polarization as a function of resonance strength ϵ .

variations versus depolarization strength. The orange open triangles denote spin vector tracking results. Each data point is annotated with its corresponding depolarizing resonance tune. Blue solid lines and blue open squares indicate tensor polarization. The spin-tracking results align with theoretical trends, and most simulation outcomes closely match theoretical predictions, demonstrating the reliability of the spin tensor transfer matrices employed in the tracking framework.

The studied resonance tune comprehensively covers all three scenarios where the spin tensor crosses the depolarizing resonances. They are analyzed using the spin-tracking program, with examples at $k = -0.43, -5.57$, and -6.43 , see Fig. 7. The horizontal axis represents the absolute value of $G\gamma$. The vertical axes display the vector polarization

(S_1, S_2, S_3) and the tensor polarization (P_{11}, P_{22}, P_{33}). In the simulation, the deuteron beam is initially either fully vector polarized with $P_3 = 1$ or fully tensor polarized with $\mathcal{P} = (0, 0, 0, 1, 1, -2)$ because the periodic solution of the Thomas-BMT equation on the reference orbit of a planar ring is in the vertical direction. These two different initial states are given by the wave functions $(1, 0, 0)$ and $(0, 1, 0)$, respectively. Dark curves represent the mean polarization components of all particles. Light shaded regions indicate the standard deviation. These results demonstrate that the polarization loss induced by the resonance with strength $\epsilon = 4.85 \times 10^{-5}$ ($k = -0.43$) is negligible. The resonance with $\epsilon = 2.91 \times 10^{-4}$ ($k = -5.57$) introduces significant polarization loss. The polarization decreases near the resonance tune and oscillates around the mean value when moving away from resonance. For the stronger resonance $\epsilon = 6.64 \times 10^{-3}$ ($k = -6.43$), the polarization is adiabatically preserved. S_3 flipping from 1 to -1 (complete spin flip) and P_{33} transitioning from -2 to 1 and back to -2 , indicating the spin tensor varies from the vertical direction to the $\hat{x}\hat{o}\hat{s}$ plane and subsequent recovery.

In summary, the dynamics of polarized deuteron beams crossing depolarizing resonances in the BRing have been systematically investigated. This investigation combined the theoretical formalism of the Depol code with a custom spin-tracking program. The program is based on the spin tensor transfer matrices derived in this work. The consistent agreement between computational results from both methods validates the reliability of the spin tensor transfer matrix. In the actual operational energy range of the BRing, both tensor and vector polarization exhibit negligible

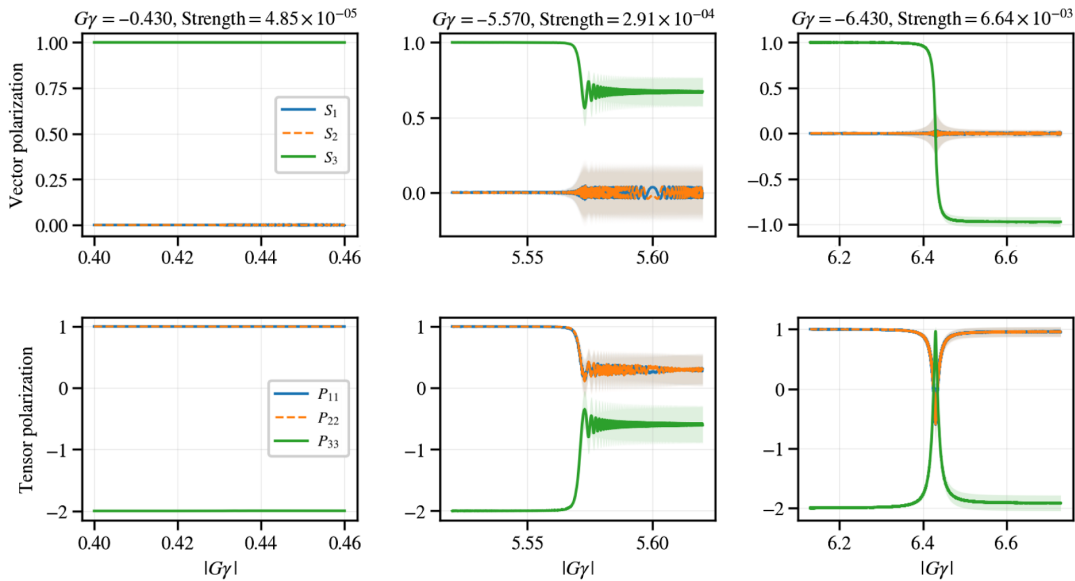


FIG. 7. Evolution of vector polarization and tensor polarization as functions of $G\gamma$ during crossing the resonance at $k = -0.43, -5.57$, and -6.43 .

losses, eliminating the need for additional polarization preservation measures.

IV. SPIN DIRECTION CONTROL OF THE SPIN TENSOR

Unlike the control of spin vector components ($\langle S_1 \rangle$, $\langle S_2 \rangle$, $\langle S_3 \rangle$), which is typically described as rotation, the manipulation of spin tensor components (P_{11} , P_{22} , P_{33}) should be characterized as translation due to their different constraint conditions. The former is governed by the constraint that the sum of their squares remains constant, resulting in circular constraints for perpendicular components under magnetic field effects. This rotational description effectively captures the variation patterns between these components. In contrast, the spin tensor constraint $P_{11} + P_{22} + P_{33} = 0$ defines a planar condition, where the perpendicular components under magnetic fields become linearly constrained. The translational description proves more intuitive for understanding the evolution of spin tensor directions under external magnetic fields.

Regarding the devices required for spin direction control, spin rotators and navigators used for spin vector manipulation are constructed using horizontal bending magnets, vertical bending magnets, and solenoids [5,20–24]. The evolution patterns of spin tensors in these magnetic elements have been established in Sec. II C, see Eqs. (25), (27), and (31). Based on those formulations, the required integral fields for spin tensor control can be derived.

In practical experiments, it is frequently required changing the spin direction from the vertical direction to the radial and longitudinal directions. The integral fields required to achieve such spin tensor translations are specified as follows,

$$\begin{aligned} \int B_{\parallel} dl &= \frac{\pi}{2(1+G)} B\rho, \\ \int B_{\perp} dl &= \frac{\pi}{2(1+G\gamma)} B\rho, \end{aligned} \quad (36)$$

which is in agreement with the integral fields needed for rotating the spin vector direction to these directions [9].

While the integral fields required for translating the spin tensor direction ($\hat{y} \rightarrow \hat{x}/\hat{z}$) match those for rotating the spin vector direction, three fundamental distinctions exist: (i) Control mechanism: Spin vector direction is manipulated through rotation, whereas spin tensor direction is governed by translation. (ii) Field-response relationship: The rotation angle of spin vectors exhibits linear dependence on the integral fields of dipoles and solenoids, while the translation of spin tensor components follows trigonometric rather than linear relationships with bending angles and solenoid fields. (iii) Operational range: Spin vector direction can cyclically evolve from $\hat{y} \rightarrow \hat{x} \rightarrow -\hat{y} \rightarrow -\hat{x} \rightarrow \hat{y}$ with increasing fields, whereas spin tensor direction

reverts to the $\hat{y}$ direction after translating to the $\hat{x}$ direction. This constrained operational range originates from the definition of tensor polarization.

The spin tensor components (P_{11} , P_{22} , P_{33}) correspond to the alignment A along the radial, longitudinal, and vertical directions, respectively, which quantify the population fraction n_0 of particles in the $m = 0$ state [see Eq. (3)]. Since there is no physical quantity corresponding to a negative value of n_0 , the permissible variations of spin tensor directions are strictly confined to the positive domains of radial, longitudinal, and vertical directions.

From the perspective of integral fields required for spin direction manipulation, spin rotators or navigators capable of rotating spin vectors to arbitrary angles can also translate spin tensors to any desired direction with appropriate control parameters. Although the correspondence between spin tensor translation and integral fields involves greater complexity, the hardware requirements for controlling spin tensor directions are less demanding compared to those for spin vector control.

V. CONCLUSION

This work advances the understanding of tensor-polarized deuteron beam dynamics in synchrotrons, with direct relevance to the EicC. By deriving spin tensor transfer matrices and validating their accuracy through numerical simulations, we demonstrate tensor polarization evolution in bending magnets, solenoids, and quadrupoles. The analysis of depolarizing resonances during acceleration reveals negligible polarization losses under the EicC's design parameters, ensuring the feasibility of high-polarization spin experiments without dedicated polarization maintenance. Furthermore, the different translation mechanisms of spin tensors, governed by planar constraints, contrast with the rotational dynamics of spin vectors and provide new insights for spin manipulation. These results can be applied not only to the polarization design of EicC but also to future experiments investigating spin tensor correlations.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data that support the findings of this article are openly available [25].

APPENDIX: SPIN TRANSFER MATRIX

For simplicity of notation, let:

$$w_1 = \frac{\omega_1}{\omega}, \quad w_2 = \frac{\omega_2}{\omega}, \quad w_3 = \frac{\omega_3}{\omega}. \quad (\text{A1})$$

The matrix elements of the spin tensor transfer matrix are as follows, where R_{ij} represents the element in the i th row and j th column of the matrix, with i and j ranging from 1 to 6.

$$\begin{aligned} R_{11} = & 1 + \frac{1}{3}C_1[w_1^4 + w_2^4 + 16w_3^2(w_3^2 - 1) \\ & + w_2^2(17w_3^2 - 4) + w_1^2(14w_2^2 + 17w_3^2 - 4)] \\ & - \frac{1}{12}C_2[w_1^4 + w_2^4 - 4w_3^2 + 16w_3^4 + w_2^2(17w_3^2 - 1) \\ & + w_1^2(14w_2^2 + 17w_3^2 - 1)], \end{aligned}$$

$$\begin{aligned} R_{12} = & \frac{1}{12}[-12C_1w_2w_3(w_1^2 + 5w_2^2 + 5w_3^2 - 4) \\ & + 3C_2w_2w_3(w_1^2 + 5w_2^2 + 5w_3^2 - 1) \\ & - 4S_1w_1(w_1^2 + 7w_2^2 + 7w_3^2 - 4) \\ & + 2S_2w_1(w_1^2 + 7w_2^2 + 7w_3^2 - 1)], \end{aligned}$$

$$\begin{aligned} R_{13} = & \frac{1}{12}[-12C_1w_1w_3(5w_1^2 + w_2^2 + 5w_3^2 - 4) \\ & + 3C_2w_1w_3(5w_1^2 + w_2^2 + 5w_3^2 - 1) \\ & + 4S_1w_2(7w_1^2 + w_2^2 + 7w_3^2 - 4) \\ & - 2S_2w_2(7w_1^2 + w_2^2 + 7w_3^2 - 1)], \end{aligned}$$

$$\begin{aligned} R_{14} = & \frac{1}{12}\{2[S_2 - 8S_1]w_3 \\ & - 4C_1w_1w_2[w_1^2 + 7w_2^2 + 7w_3^2 - 4] \\ & + C_2w_1w_2[w_1^2 + 7w_2^2 + 7w_3^2 - 1] \\ & + 2[2S_1 - S_2]w_3[w_1^2 + 4(w_2^2 + w_3^2)]\}, \end{aligned}$$

$$\begin{aligned} R_{15} = & \frac{1}{12}[2(8S_1 - S_2)w_3 \\ & - 2(2S_1 - S_2)w_3(4w_1^2 + w_2^2 + 4w_3^2) \\ & - 4C_1w_1w_2(7w_1^2 + w_2^2 + 7w_3^2 - 4) \\ & + C_2w_1w_2(7w_1^2 + w_2^2 + 7w_3^2 - 1)], \end{aligned}$$

$$\begin{aligned} R_{16} = & \frac{1}{6}\{w_1w_2[C_2 + 16C_1(w_1^2 + w_2^2 - 1) \\ & - 4C_2(w_1^2 + w_2^2)] \\ & + 3[2S_1 - S_2][w_1 - w_2][w_1 + w_2]w_3 \\ & + 7[4C_1 - C_2]w_1w_2w_3^2\}, \end{aligned}$$

$$\begin{aligned} R_{21} = & \frac{1}{12}[-12C_1w_2w_3(w_1^2 + 5w_2^2 + 5w_3^2 - 4) \\ & + 3C_2w_2w_3(w_1^2 + 5w_2^2 + 5w_3^2 - 1) \\ & + 4S_1w_1(w_1^2 + 7w_2^2 + 7w_3^2 - 4) \\ & - 2S_2w_1(w_1^2 + 7w_2^2 + 7w_3^2 - 1)], \end{aligned}$$

$$\begin{aligned} R_{22} = & 1 + \frac{1}{3}C_1[w_1^4 + 16w_2^2(w_2^2 - 1) + (17w_2^2 - 4)w_3^2 \\ & + w_3^4 + w_1^2(17w_2^2 + 14w_3^2 - 4)] \\ & - \frac{1}{12}C_2[w_1^4 + 16w_2^4 - w_3^2 + w_3^4 \\ & + w_1^2(17w_2^2 + 14w_3^2 - 1) + w_2^2(17w_3^2 - 4)], \end{aligned}$$

$$\begin{aligned} R_{23} = & \frac{1}{12}\{2[8S_1 - S_2]w_3 \\ & - 12C_1w_1w_2[5w_1^2 + 5w_2^2 + w_3^2 - 4] \\ & + 3C_2w_1w_2[5w_1^2 + 5w_2^2 + w_3^2 - 1] \\ & - 2[2S_1 - S_2]w_3[7(w_1^2 + w_2^2) + w_3^2]\}, \end{aligned}$$

$$\begin{aligned} R_{24} = & \frac{1}{12}\{2S_2w_2[w_1^2 + 4w_2^2 + 4w_3^2 - 1] \\ & - 4S_1w_2[w_1^2 + 4(w_2^2 + w_3^2 - 1)] \\ & + w_1w_3[-4C_1(w_1^2 + 7w_2^2 + 7w_3^2 - 4) \\ & + C_2(w_1^2 + 7w_2^2 + 7w_3^2 - 1)]\}, \end{aligned}$$

$$\begin{aligned} R_{25} = & \frac{1}{6}\{3S_2w_2[w_1 - w_3][w_1 + w_3] + 6S_1w_2[w_3^2 - w_1^2] \\ & + w_1w_3[C_2 + 4C_1(4w_1^2 + 7w_2^2 + 4w_3^2 - 4) \\ & - C_2(4w_1^2 + 7w_2^2 + 4w_3^2)]\}, \end{aligned}$$

$$\begin{aligned} R_{26} = & \frac{1}{12}\{-2S_2w_2[4w_1^2 + 4w_2^2 + w_3^2 - 1] \\ & + 4S_1w_2[4(w_1^2 + w_2^2 - 1) + w_3^2] \\ & + w_1w_3[-4C_1(7w_1^2 + 7w_2^2 + w_3^2 - 4) \\ & + C_2(7w_1^2 + 7w_2^2 + w_3^2 - 1)]\}, \end{aligned}$$

$$\begin{aligned} R_{31} = & \frac{1}{12}[-12C_1w_1w_3(5w_1^2 + w_2^2 + 5w_3^2 - 4) \\ & + 3C_2w_1w_3(5w_1^2 + w_2^2 + 5w_3^2 - 1) \\ & - 4S_1w_2(7w_1^2 + w_2^2 + 7w_3^2 - 4) \\ & + 2S_2w_2(7w_1^2 + w_2^2 + 7w_3^2 - 1)], \end{aligned}$$

$$\begin{aligned} R_{32} = & \frac{1}{12}\{2[S_2 - 8S_1]w_3 \\ & - 12C_1w_1w_2[5w_1^2 + 5w_2^2 + w_3^2 - 4] \\ & + 3C_2w_1w_2[5w_1^2 + 5w_2^2 + w_3^2 - 1] \\ & + 2[2S_1 - S_2]w_3[7(w_1^2 + w_2^2) + w_3^2]\}, \end{aligned}$$

$$R_{33} = 1 + \frac{1}{3}C_1[16w_1^4 + w_2^4 - 4w_3^2 + w_3^4 \\ + 2w_2^2(7w_3^2 - 2) + w_1^2(17w_2^2 + 17w_3^2 - 16)] \\ - \frac{1}{12}C_2[16w_1^4 + w_2^4 - w_3^2 + w_3^4 + w_2^2(14w_3^2 - 1) \\ + w_1^2(17w_2^2 + 17w_3^2 - 4)],$$

$$R_{34} = \frac{1}{6}\{6S_1w_1[w_2 - w_3][w_2 + w_3] + 3S_2w_1[w_3^2 - w_2^2] \\ + w_2w_3[C_2 + 28C_1w_1^2 - 7C_2w_1^2 \\ + 16C_1(w_2^2 + w_3^2 - 1) - 4C_2(w_2^2 + w_3^2)]\},$$

$$R_{35} = \frac{1}{12}\{4S_1w_1[4w_1^2 + w_2^2 + 4w_3^2 - 4] \\ - 2S_2w_1[4w_1^2 + w_2^2 + 4w_3^2 - 1] \\ + w_2w_3[-4C_1(7w_1^2 + w_2^2 + 7w_3^2 - 4) \\ + C_2(7w_1^2 + w_2^2 + 7w_3^2 - 1)]\},$$

$$R_{36} = \frac{1}{12}\{2S_2w_1[4w_1^2 + 4w_2^2 + w_3^2 - 1] \\ - 4S_1w_1[4(w_1^2 + w_2^2 - 1) + w_3^2] \\ + w_2w_3[-4C_1(7w_1^2 + 7w_2^2 + w_3^2 - 4) \\ + C_2(7w_1^2 + 7w_2^2 + w_3^2 - 1)]\},$$

$$R_{41} = \frac{1}{6}\{2[8S_1 - S_2]w_3 \\ - 4C_1w_1w_2[w_1^2 + 7w_2^2 + 7w_3^2 - 4] \\ + C_2w_1w_2[w_1^2 + 7w_2^2 + 7w_3^2 - 1] \\ - 2[2S_1 - S_2]w_3[w_1^2 + 4(w_2^2 + w_3^2)]\},$$

$$R_{42} = \frac{1}{6}\{-2S_2w_2[w_1^2 + 4w_2^2 + 4w_3^2 - 1] \\ + 4S_1w_2[w_1^2 + 4(w_2^2 + w_3^2 - 1)] \\ + w_1w_3[-4C_1(w_1^2 + 7w_2^2 + 7w_3^2 - 4) \\ + C_2(w_1^2 + 7w_2^2 + 7w_3^2 - 1)]\},$$

$$R_{43} = S_2w_1(w_2 - w_3)(w_2 + w_3) + 2S_1w_1(w_3^2 - w_2^2) \\ + \frac{1}{3}w_2w_3[C_2 + 28C_1w_1^2 - 7C_2w_1^2 \\ + 16C_1(w_2^2 + w_3^2 - 1) - 4C_2(w_2^2 + w_3^2)],$$

$$R_{44} = 1 + \frac{1}{6}(w_2^2 + w_3^2)\{C_2 + 4C_1[w_1^2 \\ + 4(w_2^2 + w_3^2 - 1)] - C_2[w_1^2 + 4(w_2^2 + w_3^2)]\},$$

$$R_{45} = \frac{1}{6}\{3[4C_1 - C_2]w_1^2w_2^2 + 6[2S_1 - S_2]w_1w_2w_3 \\ - [C_2 + 16C_1(w_1^2 + w_2^2 - 1) \\ - 4C_2(w_1^2 + w_2^2)]w_3^2 + 4[C_2 - 4C_1]w_3^4\},$$

$$R_{46} = \frac{1}{6}\{-w_2^2[C_2 + 16C_1(w_1^2 + w_2^2 - 1) \\ - 4C_2(w_1^2 + w_2^2)] + 6[S_2 - 2S_1]w_1w_2w_3 \\ + [4C_1 - C_2][3w_1^2 - 4w_2^2]w_3^2\},$$

$$R_{51} = \frac{1}{6}[2(S_2 - 8S_1)w_3 \\ + 2(2S_1 - S_2)w_3(4w_1^2 + w_2^2 + 4w_3^2) \\ - 4C_1w_1w_2(7w_1^2 + w_2^2 + 7w_3^2 - 4) \\ + C_2w_1w_2(7w_1^2 + w_2^2 + 7w_3^2 - 1)],$$

$$R_{52} = 2S_1w_2(w_1 - w_3)(w_1 + w_3) + S_2w_2(w_3^2 - w_1^2) \\ + \frac{1}{3}w_1w_3[C_2 + 4C_1(4w_1^2 + 7w_2^2 + 4w_3^2 - 4) \\ - C_2(4w_1^2 + 7w_2^2 + 4w_3^2)],$$

$$R_{53} = \frac{1}{6}\{-4S_1w_1[4w_1^2 + w_2^2 + 4w_3^2 - 4] \\ + 2S_2w_1[4w_1^2 + w_2^2 + 4w_3^2 - 1] \\ + w_2w_3[-4C_1(7w_1^2 + w_2^2 + 7w_3^2 - 4) \\ + C_2(7w_1^2 + w_2^2 + 7w_3^2 - 1)]\},$$

$$R_{54} = \frac{1}{6}\{3[4C_1 - C_2]w_1^2w_2^2 + 6[S_2 - 2S_1]w_1w_2w_3 \\ - [C_2 + 16C_1(w_1^2 + w_2^2 - 1) - 4C_2(w_1^2 + w_2^2)]w_3^2 \\ + 4[C_2 - 4C_1]w_3^4\},$$

$$R_{55} = 1 + \frac{1}{6}(w_1^2 + w_3^2)[C_2 + 4C_1(4w_1^2 + w_2^2 + 4w_3^2 - 4) \\ - C_2(4w_1^2 + w_2^2 + 4w_3^2)],$$

$$R_{56} = -\frac{1}{6}w_1^2[C_2 + 16C_1(w_1^2 + w_2^2 - 1) \\ - 4C_2(w_1^2 + w_2^2)] + (2S_1 - S_2)w_1w_2w_3 \\ - \frac{1}{6}(4C_1 - C_2)(4w_1^2 - 3w_2^2)w_3^2,$$

$$R_{61} = \frac{1}{3}\{w_1w_2[C_2 + 16C_1(w_1^2 + w_2^2 - 1) - 4C_2(w_1^2 \\ + w_2^2)] - 3[2S_1 - S_2][w_1 - w_2][w_1 + w_2]w_3 \\ + 7[4C_1 - C_2]w_1w_2w_3^2\},$$

$$R_{62} = \frac{1}{6} \{ 2\mathcal{S}_2 w_2 [4w_1^2 + 4w_2^2 + w_3^2 - 1] \\ - 4\mathcal{S}_1 w_2 [4(w_1^2 + w_2^2 - 1) + w_3^2] \\ + w_1 w_3 [-4\mathcal{C}_1 (7w_1^2 + 7w_2^2 + w_3^2 - 4) \\ + \mathcal{C}_2 (7w_1^2 + 7w_2^2 + w_3^2 - 1)] \},$$

$$R_{63} = \frac{1}{6} \{ -2\mathcal{S}_2 w_1 [4w_1^2 + 4w_2^2 + w_3^2 - 1] \\ + 4\mathcal{S}_1 w_1 [4(w_1^2 + w_2^2 - 1) + w_3^2] \\ + w_2 w_3 [-4\mathcal{C}_1 (7w_1^2 + 7w_2^2 + w_3^2 - 4) \\ + \mathcal{C}_2 (7w_1^2 + 7w_2^2 + w_3^2 - 1)] \},$$

$$R_{64} = 1/6 \{ -w_2^2 [\mathcal{C}_2 + 16\mathcal{C}_1 (w_1^2 + w_2^2 - 1) \\ - 4\mathcal{C}_2 (w_1^2 + w_2^2)] + 6[2\mathcal{S}_1 - \mathcal{S}_2] w_1 w_2 w_3 \\ + [4\mathcal{C}_1 - \mathcal{C}_2] [3w_1^2 - 4w_2^2] w_3^2 \},$$

$$R_{65} = -\frac{1}{6} w_1^2 [\mathcal{C}_2 + 16\mathcal{C}_1 (w_1^2 + w_2^2 - 1) \\ - 4\mathcal{C}_2 (w_1^2 + w_2^2)] + (\mathcal{S}_2 - 2\mathcal{S}_1) w_1 w_2 w_3 \\ - \frac{1}{6} (4\mathcal{C}_1 - \mathcal{C}_2) (4w_1^2 - 3w_2^2) w_3^2,$$

$$R_{66} = 1 + \frac{1}{6} (w_1^2 + w_2^2) \{ \mathcal{C}_2 + 4\mathcal{C}_1 [4(w_1^2 \\ + w_2^2 - 1) + w_3^2] - \mathcal{C}_2 [4(w_1^2 + w_2^2) + w_3^2] \}.$$

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- [1] D. P. Anderle, V. Bertone, X. Cao, L. Chang, N. Chang, G. Chen, X. Chen, Z. Chen, Z. Cui, L. Dai *et al.*, Electron-Ion Collider in China, *Front. Phys.* **16**, 1 (2021).
- [2] N. Nikolaev, F. Rathmann, A. Silenko, and Y. N. Uzikov, New approach to search for parity-even and parity-odd time-reversal violation beyond the Standard Model in a storage ring, *Phys. Lett. B* **811**, 135983 (2020).
- [3] B. Blinov, C. Chu, E. Courant, D. Crandell, W. Kaufman, A. Krisch, T. Nurushev, R. Phelps, D. Raczkowski, L. Ratner *et al.*, First test of a partial Siberian snake during polarized beam acceleration, *Phys. Rev. Lett.* **73**, 1621 (1994).
- [4] H. Huang, L. Ahrens, M. Bai, K. Brown, E. Courant, C. Gardner, J. Glenn, R. Gupta, A. Luccio, W. MacKay *et al.*, Acceleration of polarized protons in the AGS with two helical partial snakes, in *Proceedings of the 2005 Particle Accelerator Conference* (IEEE, New York, 2005), pp. 1404–1406.
- [5] F. Méot, H. Huang, V. Ptitsyn, V. Ranjbar, G. Robert-Demolaize, V. Schoefer, and V. Morozov, RHIC optics and spin dynamics with snakes and rotators, *Phys. Rev. Accel. Beams* **25**, 121002 (2022).
- [6] H. Huang, S. Lee, and L. Ratner, The evolution of tensor polarization, in *Proceedings of International Conference on Particle Accelerators* (IEEE, New York, 1993), pp. 432–434.
- [7] A. J. Silenko, General dynamics of tensor polarization of particles and nuclei in external fields, *J. Phys. G* **42**, 075109 (2015).
- [8] E. Courant and R. D. Ruth, Acceleration of polarized protons in circular accelerators, Technical Report No. BNL-51270, Brookhaven National Lab (BNL), Upton, NY, 1980.
- [9] S.-Y. Lee, *Spin Dynamics and Snakes in Synchrotrons* (World Scientific Publishing Company, Singapore, 1997).
- [10] H. Wu, T. Katayama, K. Hantanka, Y. Aoki, and N. Saito, The spin tracking study in RHIC for polarized proton, Technical Report No. BNL-103652-2014-IR, Brookhaven National Lab (BNL), Upton, NY, 1996.
- [11] D. Sagan, BMAD: A relativistic charged particle simulation library, *Nucl. Instrum. Methods Phys. Res., Sect. A* **558**, 356 (2006).
- [12] A. U. Luccio, Spin tracking in RHIC (code spink), in *Proceedings of the Adriatic Research Conference on Trends in Collider Spin Physics* (World Scientific, Singapore, 1997), p. 235.
- [13] S.-Y. Lee, *Accelerator Physics* (World Scientific Publishing Company, Singapore, 2018).
- [14] F. Lin, Towards full preservation of polarization of proton beam in the AGS, Ph.D. thesis, Indiana University, 2007.
- [15] H. Grote and F. C. Iselin, The MAD program (methodical accelerator design): Version 8.10; user's reference manual, Technical Report No. CM-P00049316, European Organization for Nuclear Research (CERN), Geneva, Switzerland, 1993.
- [16] V. Morozov, Z. Etienne, M. Kandes, A. Krisch, M. Leonova, D. Sivers, V. Wong, K. Yonehara, V. Anferov, H. Meyer *et al.*, First spin flipping of a stored spin-1 polarized beam, *Phys. Rev. Lett.* **91**, 214801 (2003).
- [17] D. T. Abell, D. Meiser, V. H. Ranjbar, and D. P. Barber, Accurate and efficient spin integration for particle accelerators, *Phys. Rev. ST Accel. Beams* **18**, 024001 (2015).
- [18] J. Yang, J. Xia, G. Xiao, H. Xu, H. Zhao, X. Zhou, X. Ma, Y. He, L. Ma, D. Gao *et al.*, High intensity heavy ion accelerator facility (HIAF) in China, *Nucl. Instrum. Methods Phys. Res., Sect. B* **317**, 263 (2013).
- [19] M. Froissart and R. Stora, Depolarisation d'un faisceau de protons polarisés dans un synchrotron, *Nucl. Instrum. Methods* **7**, 297 (1960).
- [20] J. Buon and K. Steffen, Hera variable-energy “mini” spin rotator and head-on ep collision scheme with choice of electron helicity, *Nucl. Instrum. Methods Phys. Res., Sect. A* **245**, 248 (1986).
- [21] D. Raparia, Development of spin rotator and an absolute polarimeter for polarized He-3 at BNL, Technical Report No. BNL-209705-2019-COPA, Brookhaven National Lab (BNL), Upton, NY, 2019.
- [22] Y. N. Filatov, A. Kondratenko, M. Kondratenko, Y. S. Derbenev, and V. Morozov, Transparent spin method for

- spin control of hadron beams in colliders, *Phys. Rev. Lett.* **124**, 194801 (2020).
- [23] Y. N. Filatov, A. M. Kondratenko, M. A. Kondratenko, E. D. Tsyplakov, A. V. Butenko, S. A. Kostromin, V. P. Ladygin, E. Syresin, I. L. Guryleva, A. A. Melnikov *et al.*, Spin navigator based on correcting dipoles of the JINR nuclotron, *JETP Lett.* **116**, 413 (2022).
- [24] J. Buon, Techniques for longitudinal polarization in electron storage rings, in *Electroweak Effects at High Energies* (Springer, New York, 1983), pp. 413–427.
- [25] M. Li, The acceleration and spin direction control of tensor-polarized deuteron beams in a synchrotron (data), Zenodo (2025), [10.5281/zenodo.15308410](https://doi.org/10.5281/zenodo.15308410).