

Resonant island trapping in a hybrid multibend achromat synchrotron light source

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We report the first direct experimental observations of resonance island formation in a hybrid multibend achromat (HMBA) storage ring lattice. Using a low-emittance electron beam, we achieved high-precision measurements that resolve the onset, evolution, and transient dynamics of particle capture near resonance islands. These measurements are compared with simulation predictions, enabling a direct validation of HMBA lattice models. The experimental results also provide new insight into the capture and trapping mechanisms governing the island population, as well as beam lifetime in resonance islands. Together, these findings establish a detailed experimental basis for understanding nonlinear dynamics in next-generation HMBA-based synchrotron light sources.

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Introduction. Synchrotron light sources are versatile analytical tools, enabling studies in many areas of science and technology. The latest—fourth generation [1]—features highly optimized lattices with a high degree of cancellation of nonlinear aberrations, with the hybrid multibend achromat lattice concept being the technology of choice for high-energy (6 GeV) machines such as the ESRF EBS [2]. The residual nonlinearities may cause a number of adverse effects on electron beam dynamics, such as halo formation, reduction of dynamic aperture, or beam losses at injection, and need to be controlled. Moreover, storage rings are uniquely suitable for general experimental studies of nonlinear oscillations [3]. For these reasons, exploitation and control of nonlinear beam dynamics have been an important subject in accelerator physics (see, e.g., Refs. [4–7]).

Among the most exciting nonlinear beam dynamics phenomena is the formation of resonant islands in the phase space of particle motion [8–15]. Recently, this dynamics has found applications in manipulating photon beam properties in light sources [16–18] and slow multiturn beam extraction from proton synchrotrons [19–21]. In all those cases, betatron frequencies (tunes) of the machine are set close to a resonance condition.

As a result of extensive numerical studies of dynamics of large amplitude particles oscillations in fourth-generation

storage rings, we found that conditions for trapping of particles into third-order resonance islands can be met without the need to set the optics working point close to the third-order resonance. Benefiting from the small natural emittance of EBS, we could carry out experiments to observe and characterize the trapping dynamics in this new regime. In this paper, we first describe the dynamics of resonant island trapping at large amplitudes in a fourth-generation light source, and then present the direct experimental observation of island formation, the dynamics of the capture process, and the island lifetime measurement at ESRF EBS.

Resonant island trapping process. The transverse beam dynamics in synchrotron light sources without taking radiation damping and excitation into account can be qualitatively described by the following Hamiltonian:

$$\mathcal{H} = \bar{h}(J_x, J_y) + h^{(r)}(J_x, J_y, \phi_x, \phi_y), \quad (1)$$

where (J, ϕ) are the action-angle variables and the Hamiltonian is split into an (averaged) integrable part and a resonant part, which gives rise to irregular and chaotic motion. The linear part of the averaged Hamiltonian $\mathcal{H}_0 = -2\pi q_{x,0} J_x$ —with $q_{x,0}$ the fractional tune or working point of the machine—represents the linear betatron motion, while $\bar{h} = \mathcal{H}_0 + h_1(J_x, J_y)$. The action-angle variables are related to the normalized coordinates as (e.g., for the horizontal plane)

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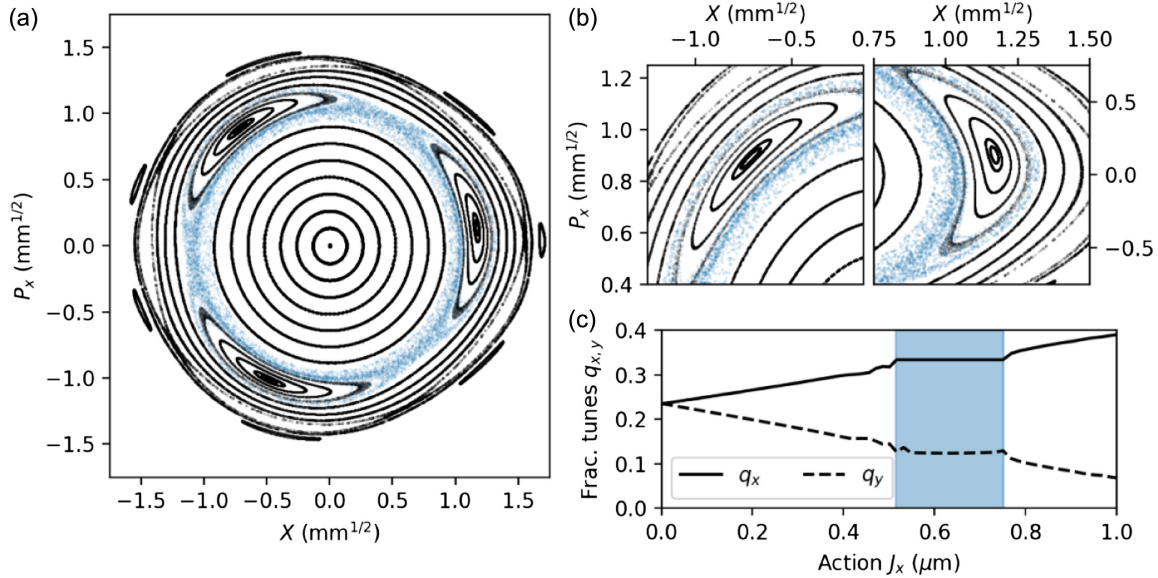


FIG. 1. (a) Phase space portrait of the horizontal betatron oscillations. The working point is $(q_x, q_y) = (0.23, 0.23)$. The blue band represents the separatrix around the third-order resonance islands ($3q_x = 1$). (b) Enlarged view of the islands reveals regular motion inside the island and a quasistochastic layer surrounding them. (c) “Devil’s staircase”: amplitude dependent fractional tune values for particles along $P_x = 0$ remain constant for a wide range of actions.

$$\begin{pmatrix} X \\ P_x \end{pmatrix} = \sqrt{2J_x} \begin{pmatrix} \cos \phi_x \\ \sin \phi_x \end{pmatrix}. \quad (2)$$

In practice the dynamics is studied through direct numerical simulations (particle tracking). Figure 1 shows the transverse phase space of betatron motion of ESRF EBS obtained through tracking. In this plot, synchrotron oscillations are neglected to highlight the horizontal dynamics. The dependency of oscillation frequencies on amplitude and crossing of the third-order resonance at large amplitudes is an important feature in the dynamics.

Tracking simulations are carried out using `pyAT` [22], incorporating lattice imperfections derived from previous experimental measurements [23]. At $J_x \approx 0.6 \mu\text{m}$ the system approaches the third-order resonance [Fig. 1(c)], where the resonance part of $\mathcal{H}$ is dominated primarily by

$$h^{(r)} = \frac{S}{\sqrt{2}} J_x^{3/2} \sin(3\phi_x + 3\mu_S). \quad (3)$$

Then resonance islands emerge in the phase space portrait [Figs. 1(a) and 1(b)]. Although the amplitude of the resonance driving term S is canceled by design for the nominal optics [24], for high action values the nonlinear optics perturb the cancellation condition. The phase μ_S of the resonance driving term determines the orientation of the islands in phase space. Explicit expressions of (S, μ_S) can be found elsewhere, e.g., Ref. [25].

High-amplitude particle behavior cannot be fully described by a simple 1D resonance model and an interplay between various resonance conditions may lead to complex

dynamics with chaotic behavior and loss of stability. Note that the simulation predicts as well a layer overlapping the regular and bounded motion in the resonance islands. Here particle tracking simulations are performed in 4D, therefore the crossing of equipotential lines is not prohibited, but rather triggered by parametric resonances described by $h^{(r)}$. The actions J_{FP} at which Hamiltonian fixed points are found can be influenced by adjusting the strength of both sextupoles and octupoles dedicated to the correction of nonlinear aberrations. This follows directly from the dependence of amplitude detuning coefficients on the octupole settings and resonance driving terms predominantly determined by sextupoles.

To understand the effect of resonance islands on the off-axis beam injection process, numerical simulations were performed, including the effects of synchrotron radiation and quantum excitation. The beam of emittance 85 nm rad [26] is injected at normalized position of $X_{\text{inj}} = -1.16 \text{ mm}^{1/2}$ and is damped toward zero as a result of radiation process. A significant fraction (80%–90%) of the injected beam damps toward the core ($J_x \rightarrow 0$), while the remainder becomes trapped in resonance islands for at least several hundred milliseconds. The island population is then reduced as a result of quantum diffusion. The island lifetime is in the range from hundreds of milliseconds to tens of seconds, exhibiting a strong dependency on exact values of the sextupole configuration, which at EBS is tuned to optimize the Touschek lifetime [27].

The injected beam has larger emittance compared to the stored beam size, and its position in phase space is limited. To characterize the island trapping process better

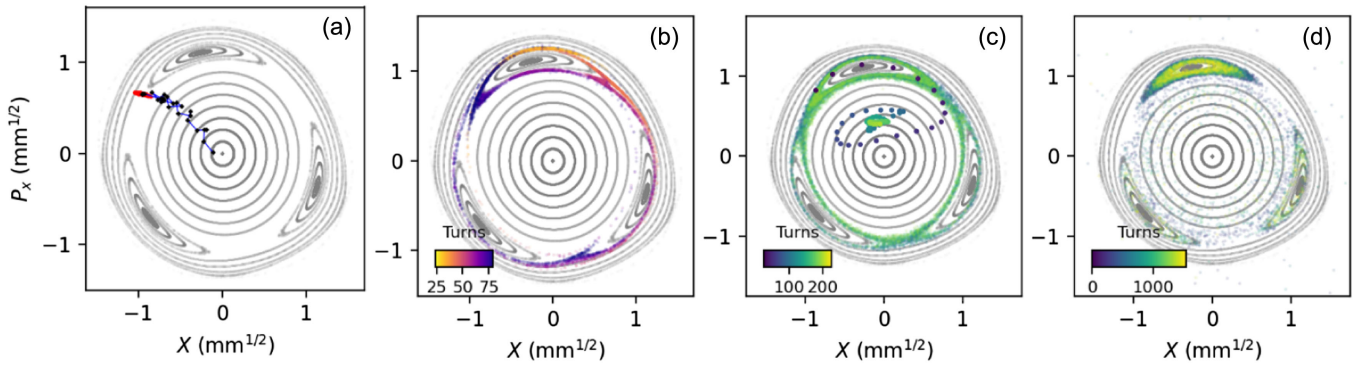


FIG. 2. Island capture dynamics simulation. Equipotential levels are included to guide the eye. Data in (b)–(d) are shown stroboscopically every three turns. The whole dataset acquires the same threefold symmetry as the islands. (a) The centroid of the beam over the kicking process is illustrated as black markers. The red blob depicts the resulting distribution after 25 turns during which the kicking takes place. (b) After the beam is brought to the islands, particles exert free betatron oscillations. The particle distribution thereby exhibits strong filamentation due to nonlinear and chromatic aberration effects. (c) A fraction of the beam is trapped in the islands, whereas the rest damps to the center. The beam centroid is depicted as dots and follows a spiral (see text). (d) After several turns, the radiation damping dominates and brings the trapped electrons closer to the fixed points determined by the underlying dynamics.

experimentally, it is more effective to apply controlled kicks to the stored beam in order to reach arbitrary locations in phase space and observe the resulting dynamics. Simulation studies were performed in preparation for the experiment. The expected behavior is presented in Fig. 2.

An ensemble of 10^4 particles is subjected to kicks over 25 turns, corresponding to the rise time of the kicker pulse [Fig. 2(a)]. The simulation shows an increase in the rms action from the equilibrium value of 133 to 952 pm as a result of the applied kicks. Subsequently, the beam undergoes free betatron oscillations [Figs. 2(b) and 2(c)]. Over the following few hundred turns, strong filamentation develops along the separatrix contour and into the surrounding quasistochastic layer. As a result, some particles are redistributed into neighboring resonance islands or along the separatrix. Consequently, the beam centroid does not remain near the stable fixed points at the centers of the islands, but instead follows a spiraling trajectory through phase space. After this transient capture phase, radiation damping becomes the dominant mechanism, gradually guiding the particles toward the stable fixed points—either within the resonance islands or at the origin. A significant fraction of particles is captured within a single resonance island [Fig. 2(d)]; however, due to diffusion along the separatrix, some particles populate other islands or are damped toward the origin. As a result, the beam centroid converges to a position between the origin and the most populated island [Fig. 2(c)].

Experiment. Dedicated measurements were carried out at EBS in low beam current mode (see Table I). Octupole strengths were reduced by half, and the working point was slightly increased to shift the TRIBs at EBS within the dynamic aperture. Sextupole settings remained unchanged from standard operation, so no additional adjustments were

required to generate TRIBs. The storage ring is filled with a bunch train occupying one-third of the circumference. The electron beam is kicked using four pulsed kicker magnets, and oscillations of its center of mass are recorded with the 320 available beam position monitors (BPMs) in the turn-by-turn fast acquisition mode. The kickers' pulses follow a quarter-sine wave and have a rise time of 70 μ s, corresponding to approximately 25 revolutions of the electron beam. The pulse amplitude can be set independently for each kicker magnet and thus precalculated pulses are set, such that the beam can reach arbitrary points in phase

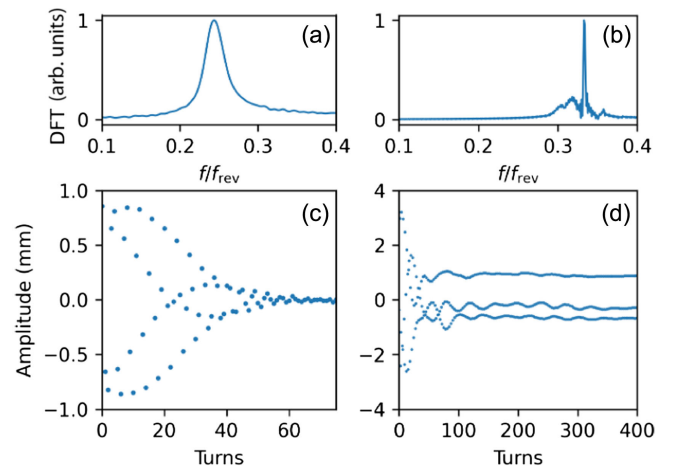


FIG. 3. Beam position recorded at one BPM for two different kicker strengths. Lower panels (c),(d) depict exemplary recorded BPM signals of the beam centroid oscillations after the electrons have been kicked. Top panels (a),(b) illustrate their corresponding spectral content. Left panels (a),(c) correspond to low kick pulse amplitude, whereas right panels (b),(d) show the case where the third-order resonance is excited at higher magnet kicker pulse amplitudes.

TABLE I. Storage ring and beam parameters for the experimental campaign.

Parameter	Symbol	Value
Circumference	C	844 m
Beam energy	E	6 GeV
Beam current	I	5 mA
Fractional tunes	q_x/q_y	0.23/0.23
Chromaticities	ξ_x/ξ_y	9/9
Equilibrium emittance	ϵ_0	133 pm rad
Damping times (ms)	(τ_x, τ_y, τ_z)	(8.6, 13.2, 8.9)
Equilibrium momentum spread	σ_p	0.95×10^{-3}

space. At high pulse amplitudes, the beam reaches limits of the dynamic aperture and beam loss is observed. The phase space is scanned over seven angles in the range $\phi_x \in [0, \pi]$ with steps of $\pi/6$ rad. The amplitude of the magnet kicker pulses is scaled with a single parameter (from 0 to 1) and scanned in ten steps. The full amplitude corresponds to the expected position of the transverse resonance islands. Example BPM data of the centroid oscillations and its corresponding Fourier spectrum are illustrated in Fig. 3. On the left top and bottom panels, a typical signal is illustrated. The beam is kicked, and due to chromatic and nonlinear tune spread, the centroid oscillation signal experiences strong decoherence. On the right top and bottom panels, the case is shown where the third-order resonance is excited, and triple orbits persist.

In addition, the radiation emitted by the beam at dedicated imaging diagnostics was recorded, with the results shown in Fig. 4. Visible-spectrum synchrotron light

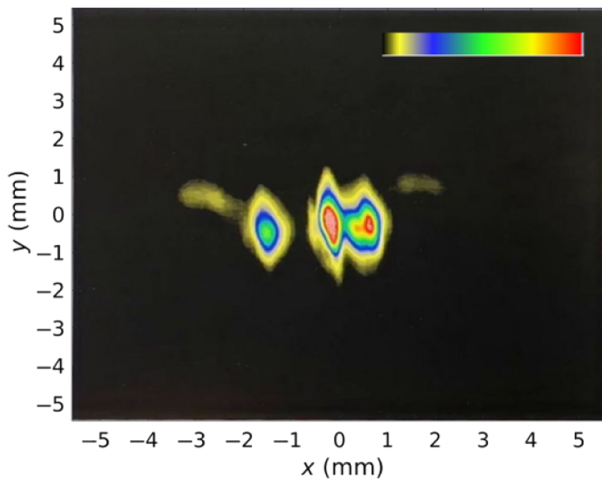


FIG. 4. Visible-light image of light emitted by particles captured in resonance islands. The central spot corresponds to the nominal orbit, while the three horizontally separated spots are the islands (1.3, 0.8, and 0.66 mm). The faint stripes are optical artifacts. Spot shapes are influenced by asymmetric apertures and diffraction, and the color scale is limited by saturation and the use of a screenshot.

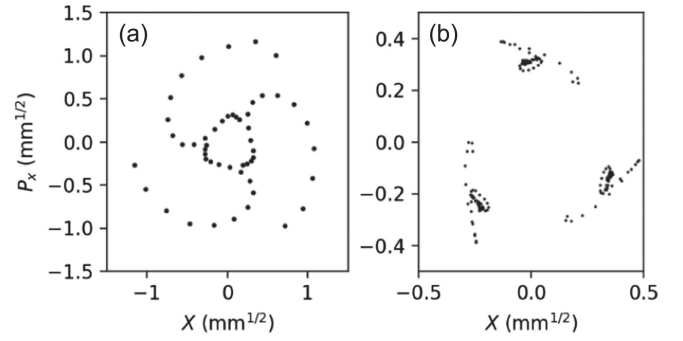


FIG. 5. Reconstructed centroid excursion from BPM data. (a) On the left panel, the first 51 turns after the kick have taken place are illustrated. (b) The right panel depicts the consecutive 400 recorded turns (starting from the 52nd turn).

from the beam was captured using a camera positioned downstream of a bending magnet. The experimental setup is described in detail in [28]. To enhance signal visibility, the acquisition synchronization was delayed by 100 ms after the kick, when the islands are fully developed. The exposure time was approximately 50 ms, and images were acquired at a rate of 15 Hz. Due to low capture efficiency (from 3% to 35%) the image possesses a high intensity contrast.

To determine the island lifetime, the BPM data are retrieved over a period of 20 s in intervals of approximately 2 s with 200 consecutive turns after the island is populated. The amplitude of the BPM signal is proportional to the weighted average of the charge distribution, and the lifetime is determined by fitting the rms of the BPM signal of the three traces with an exponentially decaying function. The result is averaged over the data available from the 320 BPMs. The measured lifetime is $\tau_{\text{island}} = 6.74 \text{ s} \pm (0.23_{\text{stat}} + 0.03_{\text{sys}}) \text{ s}$, 3 orders of magnitude larger than the damping time due to synchrotron radiation (see Table I).

The data from adjacent BPMs can be used to reconstruct the motion of the center of mass of the beam in phase space [29]. An example of beam centroid motion reconstructed from 30 BPM pairs during island trapping is shown in Fig. 5. The top panel shows the first 51 recorded turns after the kicker magnet ramp. The beam centroid spirals inward and then converges to a point between the center and the islands' fixed points, as predicted by our numerical simulations [Fig. 2(c)].

Discussion and outlook. We reported on a new regime of resonant island trapping that occurs in operational optics of low-emittance storage rings. It results from the interplay of strong nonlinear detuning with low-order resonances. The numerical predictions were confirmed experimentally and resonant islands at high oscillation amplitudes with a lifetime of approximately 7 s were observed through beam position observation and through visible synchrotron

radiation. These results have several implications for the operation of fourth-generation synchrotron light sources. They can lead to halo formation and additional beam losses at injection but can also be exploited to control the timing structure of photon pulses and create additional experimental capabilities [17,30]. Further understanding of the dependence of island lifetime on machine nonlinearity can be explored as a tool for diagnostics of nonlinear machine optics. A systematic study on sextupole and octupole settings will enrich the findings for future exploitation of TRIBs. Similar phenomena are expected to occur in the planned PETRA IV synchrotron light source [31], where a lower emittance beam will provide a higher resolution (due to shrunken source point) to study dynamics inside resonance islands, stochastic layers, and separatrices.

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Data availability. The data that support the findings of this article are openly available [32], embargo periods may apply.

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