

Observation and simulation study on bunch fill pattern dependence of absorbed power in cavity HOM dampers

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In the SuperKEKB accelerator, higher-order-mode (HOM) power absorbed by the HOM dampers of the superconducting (SC) cavities limits the attainment of the design electron beam current of the High Energy Ring. As regards cylindrical ferrite dampers installed in the SC cavities, we observed that their effective loss factors considerably depended on the bunch fill pattern. To investigate this behavior in detail, wakefield simulations were conducted. HOM power is generally evaluated from the numerically calculated coupling impedance, which provides only the total beam power loss within the structure. However, when multiple absorbers are present, the absorbed power of each absorber remains unknown. To address this limitation, we introduce calculation methods that utilize the local electromagnetic fields to evaluate the absorbed power. In this study, the theoretical basis of the methods is presented and applied to the SuperKEKB SC cavity.

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I. INTRODUCTION

In high-intensity storage rings, higher-order modes (HOMs) excited in radio-frequency (rf) accelerating cavities cause harmful coupled-bunch instabilities. To suppress these effects, HOM-damped cavities [1–6] are widely used. In such cavities, microwave absorbers, including ferrites [7–9] and ceramics [10,11], are installed at terminations of HOM waveguides or along beam pipe walls, enabling strong damping of monopole and dipole HOMs.

In the SuperKEKB accelerator [12], a luminosity-frontier electron-positron asymmetric-energy collider, two types of HOM-damped cavities are used: the normal-conducting ARES cavity [3] and the superconducting (SC) single-cell cavity [2]. Figure 1 shows the distribution of rf stations in the SuperKEKB Main Ring. The positron beam with a beam energy of 4 GeV circulates in the Low Energy Ring (LER), while the electron beam with a beam energy of 7 GeV circulates in the High Energy Ring (HER). The positron beam is accelerated by 22 ARES cavities, while the electron beam is accelerated by 8 ARES cavities and 8 SC cavities. These cavities have been in operation since the KEKB accelerator era (the predecessor of SuperKEKB).

A major challenge of the SuperKEKB rf system is the extremely high design beam current: 3.6 A for the positron beam and 2.6 A for the electron beam, approximately twice those achieved in KEKB. As the beam current increases, the power absorbed by microwave absorbers also rises proportionally to the beam current square. This results in the thermal expansion of the absorbers, thereby leading to cracking. To prevent such incidents, their absorbed power must be examined. In the SuperKEKB HER, excessive HOM power absorbed by the ferrite dampers in the SC cavities is critical for achieving the design electron beam current. Notably, the increase rate of absorbed power strongly depends on the bunch fill pattern. This unexpected behavior suggests a resonance effect between the beam spectrum and cavity HOM spectrum. To understand this phenomenon, a simulation study of the HOM power was conducted.

HOM power excited in a structure can be evaluated using its coupling impedance [13,14]. However, this approach provides only the total beam power loss within the structure. When multiple dampers are present, as in the SC cavities, the power absorbed by each individual damper cannot be determined.

Bane *et al.* evaluated a distribution of HOM power in the LCLS-II cryomodules using S-matrix and diffusion approaches [15].

On the other hand, there are methods wherein the local electromagnetic fields are directly utilized to evaluate the absorbed power. In the SuperKEKB rf group, a calculation method using CST Wakefield Solver [16] was developed to analyze the power loss of each damper [17]. In this approach, power flow monitors were placed on the

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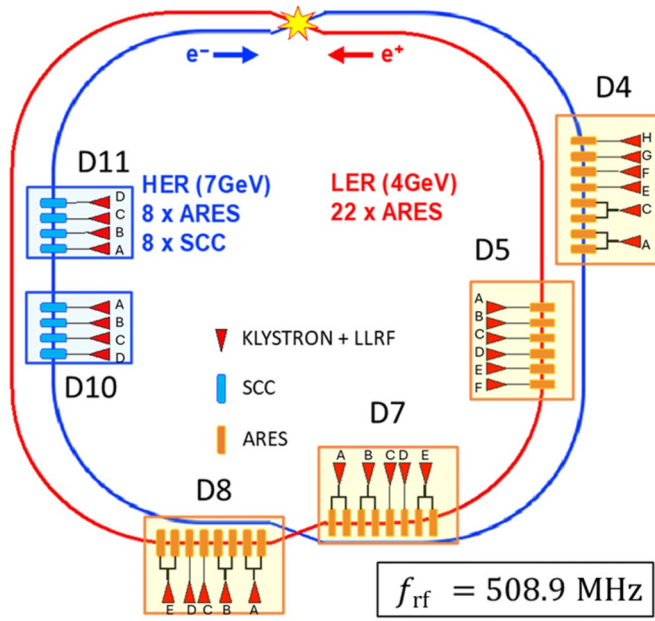


FIG. 1. Distribution of rf stations in the SuperKEKB Main Ring. The 4 GeV positron beam circulates in the LER. The 7 GeV electron beam circulates in the HER.

surfaces of dampers and beam pipe ends. Further, an alternative calculation technique was suggested by a group at BESSY [18]. They calculated the power loss of dielectric absorbers using the electric field inside the material.

In our study, we systematically reformulate the above techniques in more rigorous manner. Using the energy conservation law of electromagnetic field, we discuss absorbed power for general conditions of beam operation (arbitrary bunch fill patterns and bunch length) and absorbers (arbitrary number, locations, and material dispersions). The detailed derivation and systematic analyses are described in this paper. The resulting formulations are then applied to evaluate the absorbed power in HOM dampers of the SC cavity, while the calculated results are compared to the measurement results under high current operation.

This paper is organized as follows: Section II introduces the HOM dampers of the SuperKEKB SC cavities and describes the absorbed power measurements. Section III reviews the evaluation of HOM power using the coupling impedance. Section IV analyzes the electromagnetic energy conservation law to derive formulae for calculating power absorbed by dispersive materials and presents two calculation schemes. Section V applies these techniques to the SuperKEKB SC cavity, compares calculation and measurement results, and verifies the consistency between the presented techniques and the conventional evaluation using the coupling impedance. Section VI discusses our results and possible approaches to reducing the power load. Section VII concludes this paper.

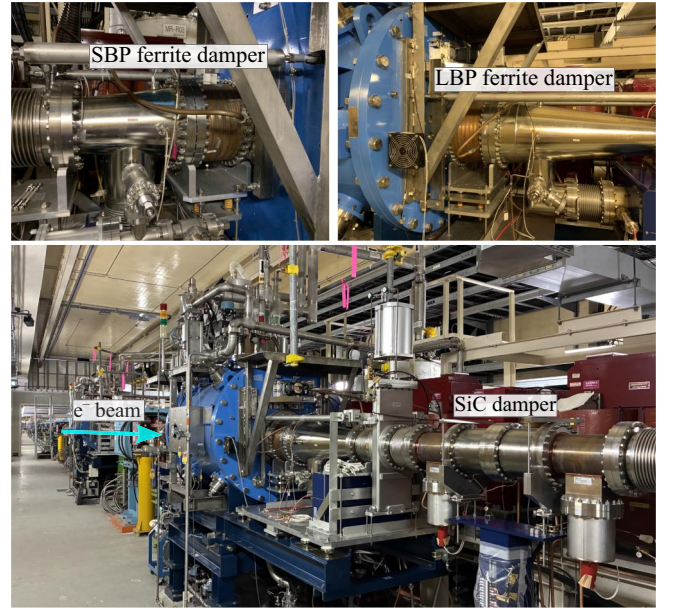


FIG. 2. HOM dampers of the SuperKEKB SC cavity: the SBP ferrite damper (upper left), LBP ferrite damper (upper right), and SiC damper (bottom).

II. BUNCH FILL PATTERN DEPENDENCE OF HOM POWER IN SUPERKEKB SC CAVITIES

A. SuperKEKB SC cavity

Figure 2 shows the SuperKEKB SC cavity and its dampers. The main cavity parameters are presented in Table I. The rf operating frequency of the Main Ring is $f_{\text{rf}} = 508.876$ MHz. The SC cavities are installed in the HER, notably at the D10 and D11 rf stations in the Nikko straight section. The upstream beam pipe with a diameter of 220 mm is referred to as the small beam pipe (SBP),

TABLE I. Main cavity parameters of the SuperKEKB SC cavity.

Parameter	Value
Cavity type	Single-cell
Material	Niobium
Operating temperature	4.4 K (LHe)
rf operating frequency, f_{rf}	508.876 MHz
$R_{\text{sh}}/Q_0 = V_c^2/(\omega_{\text{acc}} W)$	93 Ω
Geometrical factor	251 Ω
$E_{\text{peak}}/E_{\text{acc}}$	1.84
$H_{\text{peak}}/E_{\text{acc}}$	4.03 mT/(MV/m)
Unloaded Q at 1.5 MV, Q_0	$> 1 \times 10^9$
Loaded Q , $Q_L (\approx Q_{\text{ext}})$	$5.3\text{--}7.7 \times 10^4$

V_c : rf voltage, ω_{acc} : resonant frequency of the accelerating mode, W : stored energy, E_{acc} : accelerating gradient, E_{peak} : peak electric field on surface, H_{peak} : peak magnetic field on surface, Q_{ext} : external Q to input coupler.

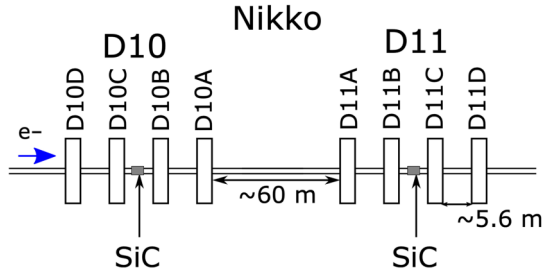


FIG. 3. Layout of the SiC dampers [21] as of the 2025c run.

while the downstream one with a diameter of 300 mm is referred to as the large beam pipe (LBP). Cylindrical ferrite dampers are installed in the SBP and LBP [7]. During the upgrade from KEBK to SuperKEKB, SiC dampers [19] were installed between cavities. These SiC dampers absorb propagating HOM power in beam pipes and reduce the power load of ferrite dampers of downstream cavities [20]. The layout of the SiC dampers as of the 2025c run is shown in Fig. 3. Two SiC dampers have been installed to date, and installation between all cavities is planned in the future.

B. Measurement of bunch fill pattern dependence of absorbed power

During beam operation, the absorbed power of each damper is monitored individually from the temperature rise of the cooling water between its inlet and outlet. These

measurements revealed that the increase rate of absorbed power with beam current strongly depends on the bunch fill pattern. This phenomenon was observed only in LBP ferrite dampers [20–22], which are described in detail below.

At the beginning of SuperKEKB operation, the number of bunches was typically 1565, corresponding with a bunch spacing of three rf periods ($3/f_{\text{rf}} \approx 5.90$ ns). To achieve a higher average beam current, the number of bunches was gradually increased to 2346 while the bunch spacing was shortened to two rf periods ($2/f_{\text{rf}} \approx 3.93$ ns). Figure 4(a) shows the fill pattern for the 2346-bunch configuration. The total number of rf buckets (harmonic number) is 5120. Two abort gaps exist each comprising ~ 160 buckets. The abort gaps provide the standby time required for beam abort system excitation. The fill pattern has a 49-bucket period as shown in Fig. 4(b). This subperiod comes from the common frequency of $f_{\text{rf}}/49 \approx 10.385$ MHz between the Main Ring rf system and the injector linac rf system operated at $275 \times f_{\text{rf}}/49 \approx 2856$ MHz.

The SuperKEKB Main Ring adopts a top-up injection scheme, wherein bunches are continuously injected to each rf bucket. The total beam current can be increased by gradually increasing bunch charge per rf bucket with keeping the bunch fill pattern.

To compare absorbed power under different fill patterns, we use effective loss factors, expressed as follows:

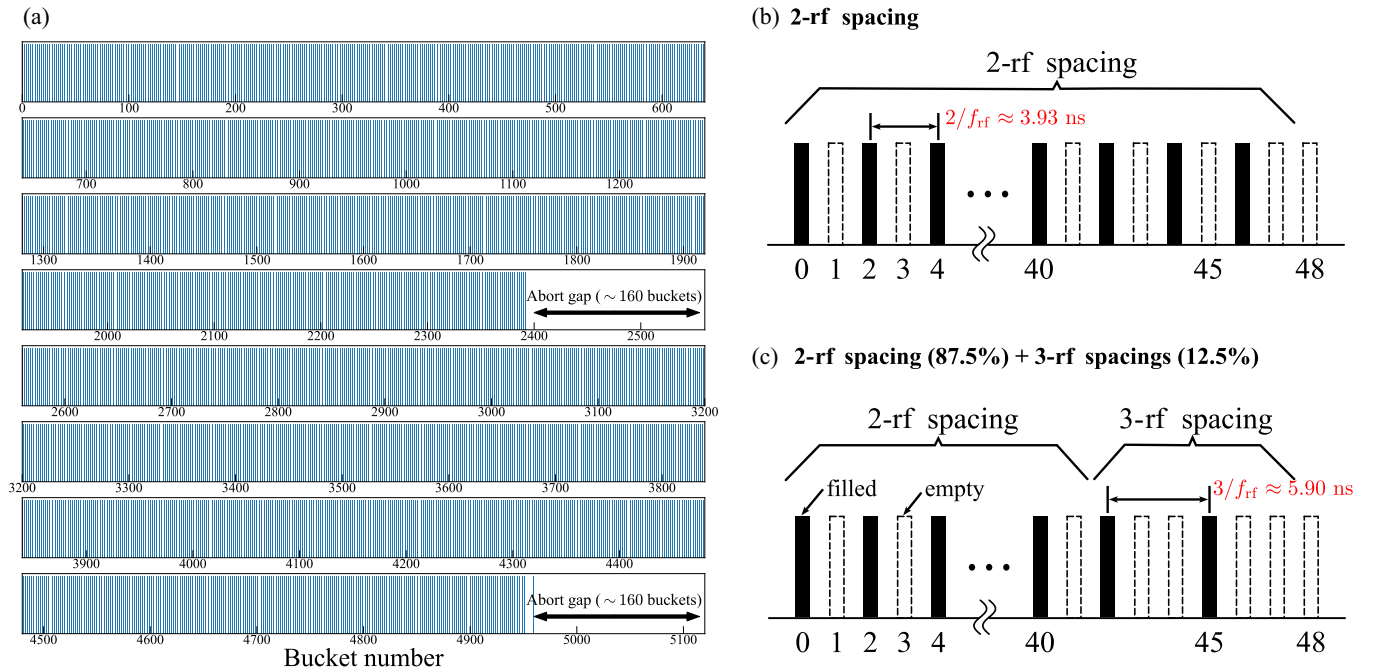


FIG. 4. (a) Typical bunch fill pattern in SuperKEKB with the number of bunches $M = 2346$ and the bunch spacing of 2 rf periods, (b) the 49-bucket subperiod in the case of the 2-rf spacing, and (c) the subperiod where the 2- and 3-rf spacings are mixed. Each bar indicates a relative bunch charge in an rf bucket.

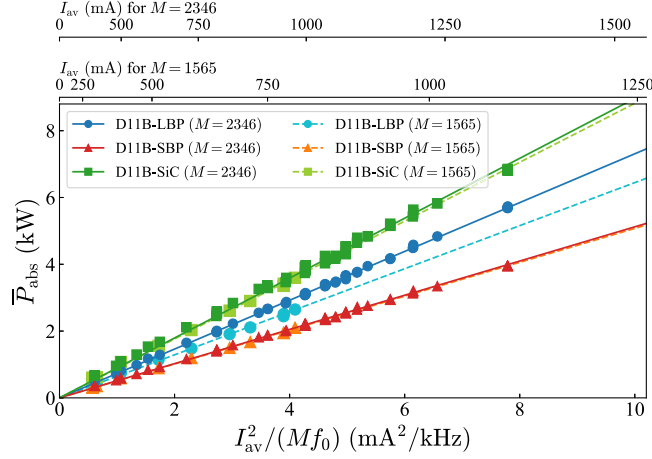


FIG. 5. Measured power absorbed by the SBP and LBP ferrite dampers and the SiC damper in a cavity of the D11B station. The absorbed power at the number of bunches of $M = 1565$ and $M = 2346$ is shown. The solid (dashed) lines show linear fits of the measured data of $M = 2346$ ($M = 1565$). At the top of the figure, the average beam current of $M = 1565$ or 2346 is shown.

$$k_{\text{eff}} = \frac{1}{q_b^2} \overline{\Delta U}_{\text{abs}}, \quad (1)$$

where q_b denotes the bunch charge and $\overline{\Delta U}_{\text{abs}}$ the average energy loss per bunch. If the measured absorbed power for a given fill pattern is $\bar{P}_{\text{abs}}$, then $\overline{\Delta U}_{\text{abs}}$ is expressed as follows:

$$\overline{\Delta U}_{\text{abs}} = \frac{1}{M} \bar{P}_{\text{abs}} T_0, \quad (2)$$

where M denotes the number of bunches and T_0 the revolution time. The effective loss factor is rewritten as [23]

$$k_{\text{eff}} = \frac{\bar{P}_{\text{abs}}}{M q_b^2 f_0} = \frac{\bar{P}_{\text{abs}} M f_0}{I_{\text{av}}^2}, \quad (3)$$

where $f_0 = 1/T_0$ denotes the revolution frequency and $I_{\text{av}} = M q_b f_0$ the average (or total) beam current. The effective loss factor k_{eff} represents the energy loss of a stored beam passing repeatedly through the structure, while the conventional loss factor $k_{\parallel}$ is defined for that of a bunch passing once. In storage rings, only k_{eff} is experimentally observed.

Figure 5 shows an example of the measured absorbed power in the SBP and LBP ferrite dampers and SiC damper. The measurement data of the D11B rf station are shown. The horizontal axis represents $I_{\text{av}}^2 / (M f_0)$. In this measurement, the total beam current I_{av} was increased by gradually increasing the bunch charge q_b with keeping the bunch fill pattern. Data for $M = 1565$ and 2346 are plotted with markers, while the dashed and solid lines show linear fits. The slopes of these fits give the effective loss factors. The effective loss factor of the LBP ferrite damper increases with shortening bunch spacing (increasing M), while those of the SBP ferrite damper and SiC damper remain unchanged.

Figure 6 shows the measured effective loss factors for more various fill patterns. The effective loss factors of the SBP, LBP, and SiC dampers in all the SC cavities are plotted as functions of the number of bunches. These measured values were evaluated by the slopes of linear fits as illustrated in Fig. 5. The bunch spacing is indicated at the top of each plot, where “ n -rf” denotes the spacing of n rf periods. The percentage values indicate cases in which two different bunch spacings are mixed. When two kinds of bunch spacings were mixed, the subperiod of 49 buckets was adjusted as shown in Fig. 4(c). In this example, the

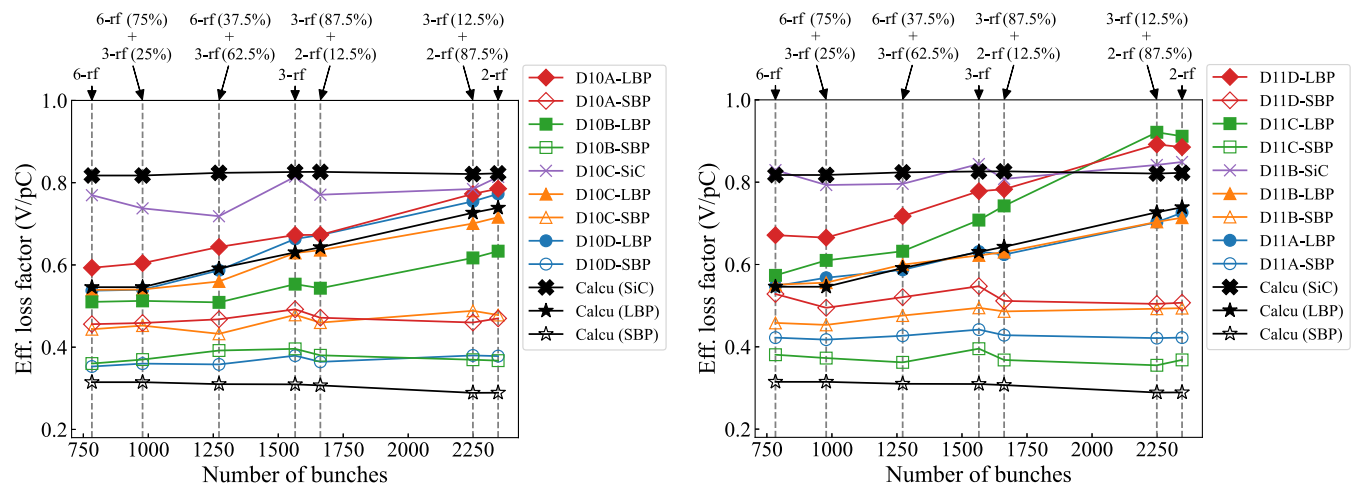


FIG. 6. Measured effective loss factors of HOM dampers located at the D10 (left) and D11 (right) sections, plotted as functions of the number of bunches. The bunch spacing is indicated at the top of each plot, where n -rf denotes a spacing of n rf periods. Percentages are shown when two different bunch spacings are mixed. For comparison, calculated effective loss factors are plotted as black markers (Sec. V).

subperiod is divided to two parts wherein bunches are arranged with 2- or 3-rf spacings. A clear buildup of the effective loss factor is observed only in LBP dampers while those of the SBP and SiC dampers exhibit little dependence on the fill pattern.

This pronounced correlation between k_{eff} and the bunch fill pattern suggests a resonance effect between the beam spectrum and HOM spectrum of the SC cavities. To identify the origin of the buildup effect, a wakefield simulation study of the SC cavity was performed. In Secs. III and IV, calculation methods for evaluating HOM power in arbitrary fill patterns are presented.

III. COUPLING IMPEDANCE AND HOM POWER

This section briefly reviews the evaluation of HOM power using the coupling impedance. First, we introduce the longitudinal wake function. Consider a leading point charge q_s at the speed of light c_0 along the positive z axis through a resonant structure such as an rf cavity. This charge induces electromagnetic fields $\mathbf{E}_\delta(x_1, y_1, x, y, z, t)$ and $\mathbf{H}_\delta(x_1, y_1, x, y, z, t)$ at position (x, y, z) , where (x_1, y_1) denotes the transverse position of the leading charge. The longitudinal wake function is expressed as follows [24]:

$$\begin{aligned} w_{\parallel}(\boldsymbol{\rho}_1, \boldsymbol{\rho}, s) &= -\frac{c_0}{q_s} \int \mathbf{E}_\delta(\boldsymbol{\rho}_1, \boldsymbol{\rho}, c_0 t - s, t) \cdot \mathbf{e}_z dt \\ &= -\frac{1}{q_s} \int_L \mathbf{E}_\delta\left(\boldsymbol{\rho}_1, \boldsymbol{\rho}, z, \frac{s+z}{c_0}\right) \cdot \mathbf{e}_z dz, \end{aligned} \quad (4)$$

where $\mathbf{e}_z$ denotes the unit vector in the longitudinal direction, $\boldsymbol{\rho}_1 = (x_1, y_1)$, and $\boldsymbol{\rho} = (x, y)$. The integration is performed over the extent of the structure. By definition, a positive $w_{\parallel}$ corresponds to particle deceleration. In the following, we assume that particle bunches travel along the beam axis $(x, y) = (0, 0)$, while the transverse dependence of the wake function is omitted.

In a storage ring, particle bunches repeatedly pass through the structure. In a stationary state, the beam-induced voltage is expressed as the convolution of the wake function $w_{\parallel}(s)$ with the instantaneous beam current $I_b(t)$

$$V_b(t) = - \int_{-\infty}^{\infty} w_{\parallel}(c_0(t-t')) I_b(t') dt'. \quad (5)$$

The instantaneous beam current measured at a fixed point in the storage ring is expressed as follows:

$$I_b(t) = \sum_{n=-\infty}^{\infty} \sum_{m=0}^{h-1} c_0 q_m \lambda_{\parallel} \left[c_0 \left(t - nT_0 - m \frac{T_0}{h} \right) \right], \quad (6)$$

where q_m denotes the bunch charge in the m th rf bucket ($m = 0, 1, 2, \dots, h-1$), h the harmonic number, and $\lambda_{\parallel}(s)$ the longitudinal line charge density in a bunch.

The rf frequency is expressed as $f_{\text{rf}} = h/T_0 = hf_0$. Longitudinal collective motions such as dipole or quadrupole oscillations are not considered here. The line density is normalized as

$$\int_{-\infty}^{\infty} \lambda_{\parallel}(s) ds = 1. \quad (7)$$

Using Poisson's sum rule, we have

$$I_b(t) = \frac{1}{T_0} \sum_{p=-\infty}^{\infty} \tilde{q}_p \tilde{\lambda}_{\parallel}(p\omega_0) e^{jp\omega_0 t}, \quad (8)$$

where $\omega_0 = 2\pi f_0$ denotes the revolution angular frequency. The coefficient

$$\tilde{q}_p = \sum_{m=0}^{h-1} q_m e^{-2\pi j p m / h} \quad (9)$$

denotes the discrete Fourier transform of q_m , and

$$\tilde{\lambda}_{\parallel}(\omega) = \int_{-\infty}^{\infty} \lambda_{\parallel}(s) e^{-j\omega s / c_0} ds \quad (10)$$

the Fourier transform of $\lambda_{\parallel}(s)$.

Defining the longitudinal coupling impedance as

$$Z_{\parallel}(\omega) = \int_{-\infty}^{\infty} w_{\parallel}(s) e^{-j\omega s / c_0} \frac{ds}{c_0}, \quad (11)$$

the beam-induced voltage is rewritten as

$$V_b(t) = -\frac{1}{T_0} \sum_{p=-\infty}^{\infty} \tilde{q}_p \tilde{\lambda}_{\parallel}(p\omega_0) Z_{\parallel}(p\omega_0) e^{jp\omega_0 t}. \quad (12)$$

The instantaneous power loss $P_b(t)$ of the stored beam in the structure is expressed as follows:

$$\begin{aligned} P_b(t) &= -V_b(t) I_b(t) \\ &= \frac{1}{T_0^2} \sum_{p=-\infty}^{\infty} \sum_{p'=-\infty}^{\infty} \tilde{q}_p \tilde{q}_{p'}^* \tilde{\lambda}_{\parallel}(p\omega_0) \tilde{\lambda}_{\parallel}^*(p'\omega_0) \\ &\quad \times Z_{\parallel}(p\omega_0) e^{j(p-p')\omega_0 t}. \end{aligned} \quad (13)$$

Using energy conservation, the total power loss could be expressed as follows:

$$P_b(t) = \frac{dW_{\text{em}}(t)}{dt} + P_{\text{wall}}(t) + P_{\text{rad}}(t) + P_{\text{abs}}(t), \quad (14)$$

where $W_{\text{em}}(t)$ denotes the electromagnetic energy stored in the structure, $P_{\text{wall}}(t)$ the wall-loss power, $P_{\text{rad}}(t)$ the power radiated to beam pipes and external couplers, and $P_{\text{abs}}(t)$ the power absorbed in the microwave absorbers.

The external couplers include an rf input coupler and HOM couplers for HOM damping.

In the majority of practical cases, the quantity of interest is the time-averaged power rather than the instantaneous power $P_b(t)$. Hence, we average $P_b(t)$ over one revolution period T_0 :

$$\bar{P}_b = \frac{1}{T_0} \int_{t-T_0}^t P_b(t') dt'. \quad (15)$$

Here, $\bar{P}_b$ is time-independent because $P_b(t)$ is a periodic function associated with T_0 . As the electromagnetic energy $W_{em}(t)$ possesses the same periodicity, the time average of dW_{em}/dt is zero. Hence, the time average of Eq. (14) is expressed as follows:

$$\bar{P}_b = \bar{P}_{wall} + \bar{P}_{rad} + \bar{P}_{abs}, \quad (16)$$

where $\bar{P}_{wall}$, $\bar{P}_{rad}$, and $\bar{P}_{abs}$ denote the time averages of $P_{wall}(t)$, $P_{rad}(t)$, and $P_{abs}(t)$ over T_0 , respectively. Using the following property

$$\frac{1}{T_0} \int_{t-T_0}^t e^{j(p-p')\omega_0 t} dt = \delta_{p,p'}, \quad (17)$$

where $\delta_{p,p'}$ denotes the Kronecker delta, the time average of Eq. (13) is [13,14]

$$\begin{aligned} \bar{P}_b &= \sum_{p=-\infty}^{\infty} \left| \frac{\tilde{q}_p}{T_0} \right|^2 |\tilde{\lambda}_{||}(p\omega_0)|^2 Z_{||}(p\omega_0) \\ &= \sum_{p=-\infty}^{\infty} \left| \frac{\tilde{q}_p}{T_0} \right|^2 |\tilde{\lambda}_{||}(p\omega_0)|^2 \text{Re}[Z_{||}(p\omega_0)]. \end{aligned} \quad (18)$$

Equation (18) is the well-known expression for HOM power. It shows that when spectral lines of the beam coincide with HOM resonant frequencies, the HOM power increases resonantly.

IV. EVALUATION TECHNIQUES OF ABSORBED POWER IN MATERIALS

As discussed in the previous section, the time-averaged HOM power can be evaluated for arbitrary fill patterns using Eq. (18). However, this equation expresses only the total beam power loss in the structure. When multiple HOM dampers are present, as in the SuperKEKB SC cavity, the absorbed power of each individual damper cannot be determined.

In the method previously used in SuperKEKB rf group, time-domain wakefield simulations were conducted and the power flow via the surface of each damper was monitored to evaluate the absorbed energy [17]. This approach yields the energy loss of a single bunch passing through the structure once. Consequently, it does not allow

the analysis of resonance effects between HOM and beam spectra. An alternative method was developed by the BESSY group [18], wherein the power loss density inside dielectric materials is determined by the local electric field and beam spectrum. This formulation includes resonance effects.

In this section, we reconstruct these techniques for arbitrary fill patterns in terms of electromagnetic energy conservation law. Two approaches for calculating absorbed power are considered: (1) evaluating the power loss density inside the absorber and integrating over its volume and (2) calculating the power flow through the absorber surface and integrating over that surface.

A. Energy conservation law in dispersive materials

From the Maxwell equations, the energy conservation law in the electromagnetic field is expressed as follows:

$$-\nabla \cdot \mathbf{S} = \mathbf{J} \cdot \mathbf{E} + \mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} + \mathbf{H} \cdot \frac{\partial \mathbf{B}}{\partial t}, \quad (19)$$

where $\mathbf{E}(\mathbf{r}, t)$ denotes the electric field, $\mathbf{H}(\mathbf{r}, t)$ the magnetic field, $\mathbf{D}(\mathbf{r}, t)$ the electric flux density, $\mathbf{B}(\mathbf{r}, t)$ the magnetic flux density, and $\mathbf{J}(\mathbf{r}, t)$ the current density. The Poynting vector is expressed as $\mathbf{S}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, t) \times \mathbf{H}(\mathbf{r}, t)$.

For a dispersive material, its time response must be considered for the field vectors $\mathbf{D}$ and $\mathbf{B}$ as follows:

$$\mathbf{D}(\mathbf{r}, t) = \epsilon_0 \left[\mathbf{E}(\mathbf{r}, t) + \int_{-\infty}^{\infty} \chi_e(\mathbf{r}, t-t') \mathbf{E}(\mathbf{r}, t') dt' \right], \quad (20)$$

$$\mathbf{B}(\mathbf{r}, t) = \mu_0 \left[\mathbf{H}(\mathbf{r}, t) + \int_{-\infty}^{\infty} \chi_m(\mathbf{r}, t-t') \mathbf{H}(\mathbf{r}, t') dt' \right], \quad (21)$$

where $\chi_e(\mathbf{r}, t)$ denotes an electric response function of the material and $\chi_m(\mathbf{r}, t)$ a magnetic one. Herein, we have assumed a linear and isotropic material. The Fourier transforms of $\chi_e(\mathbf{r}, t)$ and $\chi_m(\mathbf{r}, t)$ are related to the electric permittivity and magnetic permeability, respectively:

$$\begin{aligned} \epsilon_r(\mathbf{r}, \omega) &= \epsilon'_r(\mathbf{r}, \omega) - j\epsilon''_r(\mathbf{r}, \omega) \\ &= 1 + \int_{-\infty}^{\infty} \chi_e(\mathbf{r}, t) e^{-j\omega t} dt, \end{aligned} \quad (22)$$

$$\begin{aligned} \mu_r(\mathbf{r}, \omega) &= \mu'_r(\mathbf{r}, \omega) - j\mu''_r(\mathbf{r}, \omega) \\ &= 1 + \int_{-\infty}^{\infty} \chi_m(\mathbf{r}, t) e^{-j\omega t} dt, \end{aligned} \quad (23)$$

where $\epsilon_r(\mathbf{r}, \omega)$ denotes the relative electric permittivity, $\mu_r(\mathbf{r}, \omega)$ the relative magnetic permeability, ϵ'_r and μ'_r their real parts, and ϵ''_r and μ''_r their imaginary parts.

Generally, $\epsilon_r(\mathbf{r}, \omega)$ and $\mu_r(\mathbf{r}, \omega)$ are complex and their nonzero imaginary parts represent electromagnetic power loss. In vacuum, the second and third terms in the right-hand side of Eq. (19) correspond with the time variations of the electromagnetic energy density $u_{\text{em}}(\mathbf{r}, t)$. In a lossy dispersive material, we must consider its power loss density as

$$\frac{\partial u_{\text{em}}}{\partial t} + p_{\text{em}} = \mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} + \mathbf{H} \cdot \frac{\partial \mathbf{B}}{\partial t}, \quad (24)$$

where $p_{\text{em}}(\mathbf{r}, t)$ denotes the electromagnetic power loss density.

B. Power absorbed by microwave absorbers in storage rings

In a time-domain wakefield simulation, the right-hand side of Eq. (24) can be evaluated directly. However, the power loss density $p_{\text{em}}(\mathbf{r}, t)$ cannot be separated from the time derivative of the energy density without explicitly modeling the material dispersion [25–28]. Instead, we reconsider the situation described in Sec. III, where particle bunches pass through the structure repeatedly and evaluate Eq. (24) using time averaging.

We introduce functions $\mathbf{G}_e$ and $\mathbf{G}_m$ using the electric and magnetic fields $\mathbf{E}_\delta$ and $\mathbf{H}_\delta$ induced by a point charge q_s as

$$\mathbf{G}_e(\mathbf{r}, t) = \frac{1}{q_s} \mathbf{E}_\delta(\mathbf{r}, t), \quad (25)$$

$$\mathbf{G}_m(\mathbf{r}, t) = \frac{1}{q_s} \mathbf{H}_\delta(\mathbf{r}, t), \quad (26)$$

where all the particles are assumed to travel along the beam axis $\boldsymbol{\rho} = (x, y) = (0, 0)$, and the transverse position of the leading particle $\boldsymbol{\rho}_1$ is omitted. In the stationary state, the electric and magnetic fields are expressed similarly to Eq. (5) as follows:

$$\mathbf{E}(\mathbf{r}, t) = \int_{-\infty}^{\infty} \mathbf{G}_e(\mathbf{r}, t - t') I_b(t') dt', \quad (27)$$

$$\mathbf{H}(\mathbf{r}, t) = \int_{-\infty}^{\infty} \mathbf{G}_m(\mathbf{r}, t - t') I_b(t') dt', \quad (28)$$

where $I_b(t)$ has been already defined in Eq. (6).

Time-averaging the energy conservation in Eq. (24) similarly to the analysis in Sec. III, we can derive the time-averaged power loss density $\bar{p}_{\text{em}}(\mathbf{r})$. The detailed discussion is presented in Appendix A. Consequently, $\bar{p}_{\text{em}}(\mathbf{r})$ is derived as

$$\bar{p}_{\text{em}}(\mathbf{r}) = \sum_{p=1}^{\infty} 2p\omega_0 \left| \frac{\tilde{q}_p}{T_0} \right|^2 |\tilde{\lambda}_{\parallel}(p\omega_0)|^2 [\epsilon_0 \epsilon_r''(\mathbf{r}, p\omega_0) |\tilde{\mathbf{G}}_e(\mathbf{r}, p\omega_0)|^2 + \mu_0 \mu_r''(\mathbf{r}, p\omega_0) |\tilde{\mathbf{G}}_m(\mathbf{r}, p\omega_0)|^2], \quad (29)$$

where $\tilde{\mathbf{G}}_e(\mathbf{r}, \omega)$ and $\tilde{\mathbf{G}}_m(\mathbf{r}, \omega)$ are the Fourier transforms of $\mathbf{G}_e$ and $\mathbf{G}_m$, respectively. The time-averaged power loss inside an absorber of volume V_{abs} is expressed as follows:

$$\bar{P}_{\text{abs}} = \iiint_{V_{\text{abs}}} \bar{p}_{\text{em}}(\mathbf{r}) d^3\mathbf{r}, \quad (30)$$

where the integration is performed over the absorber volume V_{abs} . Even in the presence of multiple absorbers, the absorbed power of each absorber can be evaluated by specifying the corresponding integration region.

Alternatively, the absorbed power $\bar{P}_{\text{abs}}$ can be calculated using the Poynting vector. Assuming no conduction current $\mathbf{J} = \mathbf{0}$ inside an absorber, the energy conservation law in the absorber is expressed as follows:

$$\frac{dU}{dt} + P_{\text{abs}} = - \iint_{\Omega_{\text{abs}}} \mathbf{S} \cdot \mathbf{n} d^2\mathbf{r}, \quad (31)$$

where Ω_{abs} denotes the surface of the volume V_{abs} , $\mathbf{n}$ the unit vector normal to the surface Ω_{abs} in the outside direction, and

$$U(t) = \iiint_{V_{\text{abs}}} u_{\text{em}}(\mathbf{r}, t) d^3\mathbf{r}, \quad (32)$$

$$P_{\text{abs}}(t) = \iiint_{V_{\text{abs}}} p_{\text{em}}(\mathbf{r}, t) d^3\mathbf{r}. \quad (33)$$

The Poynting vector $\mathbf{S}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, t) \times \mathbf{H}(\mathbf{r}, t)$ is obtained from the beam-induced electromagnetic fields expressed in Eqs. (A1) and (A2) as follows:

$$\begin{aligned} \mathbf{S}(\mathbf{r}, t) &= \sum_{p=-\infty}^{\infty} \sum_{p'=-\infty}^{\infty} \frac{\tilde{q}_p}{T_0} \frac{\tilde{q}_{p'}^*}{T_0} \tilde{\lambda}_{\parallel}(p\omega_0) \tilde{\lambda}_{\parallel}^*(p'\omega_0) \\ &\quad \times \tilde{\mathbf{G}}_e(\mathbf{r}, p\omega_0) \times \tilde{\mathbf{G}}_m^*(\mathbf{r}, p'\omega_0) e^{j(p-p')\omega_0 t}. \end{aligned} \quad (34)$$

After time-averaging Eqs. (31) and (34), the absorbed power $\bar{P}_{\text{abs}}$ is expressed as follows:

$$\bar{P}_{\text{abs}} = - \iint_{\Omega_{\text{abs}}} \bar{\mathbf{S}}(\mathbf{r}) \cdot \mathbf{n} d^2\mathbf{r}, \quad (35)$$

where the time-averaged Poynting vector is

$$\bar{\mathbf{S}}(\mathbf{r}) = \sum_{p=-\infty}^{\infty} \left| \frac{\tilde{q}_p}{T_0} \right|^2 |\tilde{\lambda}_{\parallel}(p\omega_0)|^2 \times \text{Re}[\tilde{\mathbf{G}}_e(\mathbf{r}, p\omega_0) \times \tilde{\mathbf{G}}_m^*(\mathbf{r}, p\omega_0)]. \quad (36)$$

The time-averaged absorbed power can be evaluated either by the volume integral of the power loss density [Eq. (30)] or by the surface integral of the time-averaged Poynting vector [Eq. (35)].

The conventional loss factor $k_{\text{abs}} = \Delta U_{\text{abs}}/q_s^2$ per absorber can also be calculated in a similar way to that of the effective loss factor, where ΔU_{abs} is the energy loss in an absorber of a single bunch charge q_s . In the calculation of k_{abs} , the bunch charge passes through the structure only once. The details are given in Appendix B.

V. EVALUATION OF ABSORBED POWER USING WAKEFIELD SIMULATION

In the previous section, we derived methods to evaluate absorbed power in each damper. We applied these techniques to the SuperKEKB SC cavity. To evaluate the absorbed power in each damper, we utilized a time-domain wakefield simulation with the following procedure: (i) Prepare the simulation model including HOM dampers. (ii) Set material dispersive properties for the dampers. (iii) Simulate the wakefield in time domain while recording the electromagnetic field inside the dampers and on their surfaces. (iv) Evaluate the absorbed power from the recorded electromagnetic field using Eqs. (30) and (35).

In this section, we describe the simulation setup and obtained results.

A. Simulation setup

Wakefield simulations were conducted using the Wakefield Solver in CST studio suite [16]. An axially symmetric model of the SC cavity (Fig. 7) was utilized. Several structures, including a coupler port, probe port, and vacuum-pumping port, were omitted for simplicity. The leading bunch trajectory was set along the cavity symmetric axis $(x, y) = (0, 0)$. Owing to the axial symmetry of the cavity geometry, the YZ ($x = 0$) and ZX ($y = 0$) planes were defined as magnetic planes, wherein the magnetic field is perpendicular to the plane. The beam pipe end planes were assigned open boundaries so that the outgoing waves were absorbed without reflection.

Table II presents the solver parameters. A Gaussian bunch was used with an rms bunch length of $\sigma_z = 5$ mm, close to the realistic bunch length in the SuperKEKB Main Ring. Wake length is the distance that the solver calculates the wake potential. The wake length of 500 m was enough for the cavity HOMs to be sufficiently damped as mentioned later. The cavity walls were modeled as perfect electric conductors, indicating zero wall-loss power.

For the dispersive properties of the ferrite and SiC absorbers, the first-order Debye model was employed. In this model, the complex permittivity or permeability is expressed as follows:

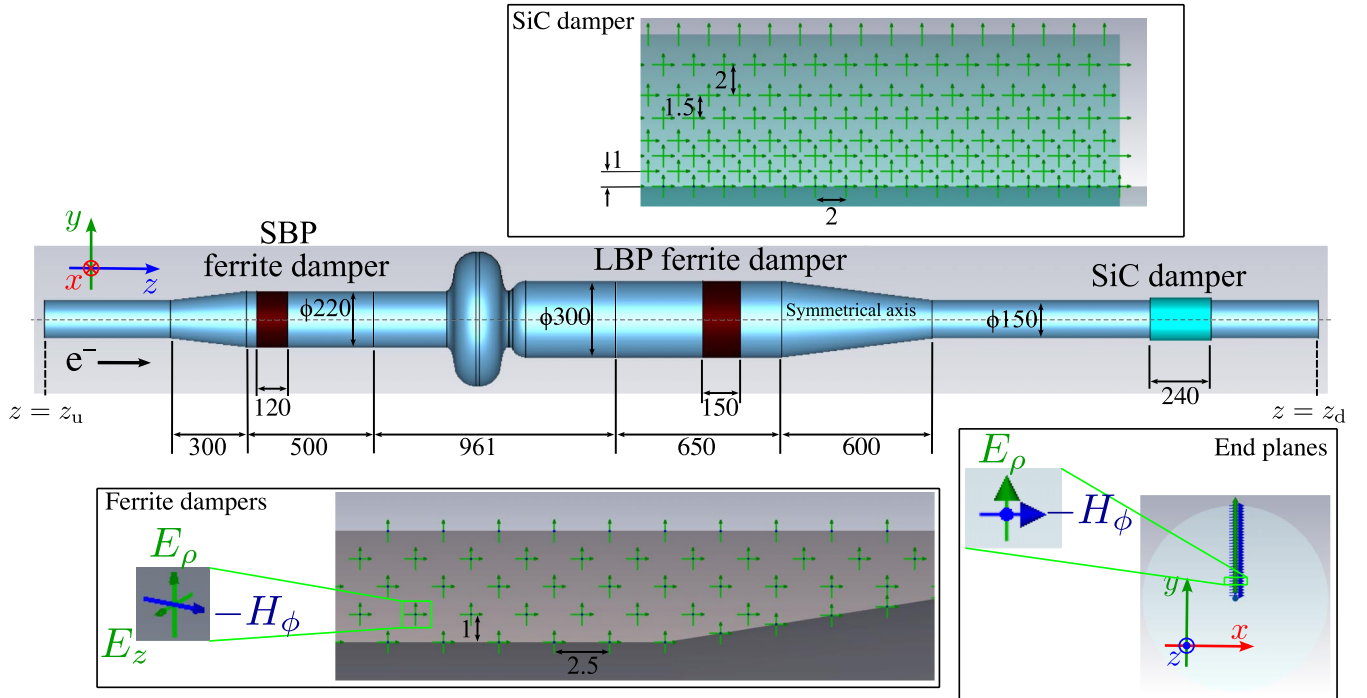


FIG. 7. Simulation model of the SuperKEKB superconducting cavity and field probe distribution (unit: mm).

TABLE II. Simulation setup of the CST Wakefield Solver.

Parameter	Value
rms bunch length, σ_z	5.0 mm
Charge of a leading bunch, q_s	1.0 nC
Wake length	500 m
Velocity (beta)	1.0
Lines per sigma	10.0
Background	Perfect electric conductor
Current injection scheme	Analytic
Wake integration method	Indirect testbeam

$$f(\omega) = f'(\omega) - jf''(\omega) = f_\infty + \frac{f_s - f_\infty}{1 + j\omega\tau_f}, \quad (37)$$

$$f'(\omega) = f_\infty + \frac{f_s - f_\infty}{1 + (\omega\tau_f)^2}, \quad (38)$$

$$f''(\omega) = \frac{(f_s - f_\infty)\omega\tau_f}{1 + (\omega\tau_f)^2}, \quad (39)$$

where $f'(\omega)$ denotes the real part, $f''(\omega)$ the imaginary part, f_s the low-frequency limit, f_∞ the high-frequency limit of $f(\omega)$, and τ_f the relaxation time. For electric permittivity, $f(\omega)$ corresponds with $\epsilon_r(\omega)$; for magnetic permeability, it corresponds with $\mu_r(\omega)$. Table III presents the parameters for the ferrite and SiC materials. These values were obtained by fitting measured electric permittivity and magnetic permeability to the Debye model. The magnetic permeability of SiC was set to $\mu_r = 1$.

The CST time-domain solvers include a function known as Field Probe, which records electric and magnetic field signals at specified spatial points in the time domain. As shown in Fig. 7, numerous field probes were placed inside the ferrite and SiC dampers and on their surfaces. Additional probes were equally positioned at the upstream and downstream beam pipe end planes to evaluate the radiated power from the cavity. As the cavity geometry is axially symmetric and the leading bunch travels along the symmetry axis, the resulting electromagnetic fields are also axially symmetric, with no angular dependence.

TABLE III. Setup of the dispersive properties of the ferrite and SiC.

	Ferrite	SiC
ϵ_s	18	100
ϵ_∞	11	12
$1/\tau_e$	$2\pi \times 8 \times 10^7 \text{ s}^{-1}$	$2\pi \times 3 \times 10^8 \text{ s}^{-1}$
μ_s	38	...
μ_∞	1	...
$1/\tau_\mu$	$2\pi \times 2.3 \times 10^8 \text{ s}^{-1}$	...

Therefore, field probes were placed only on the YZ plane. The time step of the recorded signals was approximately 9.4 ps. In the following evaluations, we analyzed these signals without down-sampling them.

Let $\mathbf{E}_\lambda(\mathbf{r}, t)$ and $\mathbf{H}_\lambda(\mathbf{r}, t)$ denote the electric and magnetic fields induced by a Gaussian leading bunch and recorded by the field probes. These fields can be expressed using the functions $\mathbf{G}_e$ and $\mathbf{G}_m$ as

$$\mathbf{E}_\lambda(\mathbf{r}, t) = q_s \int_{-\infty}^{\infty} \mathbf{G}_e(\mathbf{r}, t - t') \lambda_\parallel(c_0 t') dt', \quad (40)$$

$$\mathbf{H}_\lambda(\mathbf{r}, t) = q_s \int_{-\infty}^{\infty} \mathbf{G}_m(\mathbf{r}, t - t') \lambda_\parallel(c_0 t') dt', \quad (41)$$

where q_s is a charge of the leading bunch assumed in the simulation and

$$\lambda_\parallel(s) = \frac{1}{\sqrt{2\pi}\sigma_z} \exp\left(-\frac{s^2}{2\sigma_z^2}\right). \quad (42)$$

Taking the Fourier transforms of Eqs. (40) and (41), the Fourier transforms of $\mathbf{G}_e$ and $\mathbf{G}_m$ are expressed as follows:

$$\tilde{\mathbf{G}}_e(\mathbf{r}, \omega) = \frac{1}{q_s} \frac{\tilde{\mathbf{E}}_\lambda(\mathbf{r}, \omega)}{\tilde{\lambda}_\parallel(\omega)}, \quad (43)$$

$$\tilde{\mathbf{G}}_m(\mathbf{r}, \omega) = \frac{1}{q_s} \frac{\tilde{\mathbf{H}}_\lambda(\mathbf{r}, \omega)}{\tilde{\lambda}_\parallel(\omega)}, \quad (44)$$

where $\tilde{\lambda}_\parallel(\omega) = \exp[-\omega^2\sigma_z^2/(2c_0^2)]$ for a Gaussian bunch and $\tilde{\mathbf{E}}_\lambda(\mathbf{r}, \omega)$ and $\tilde{\mathbf{H}}_\lambda(\mathbf{r}, \omega)$ represent the Fourier transforms of $\mathbf{E}_\lambda(\mathbf{r}, t)$ and $\mathbf{H}_\lambda(\mathbf{r}, t)$.

In our simulation, we set the rms bunch length of the leading bunch equal to the real bunch length in operation. Then, we could directly use $\tilde{\mathbf{E}}_\lambda$ and $\tilde{\mathbf{H}}_\lambda$ for the power loss evaluation, and did not need deconvolution processes in Eqs. (43) and (44). This simulation setting is beneficial to avoid calculation errors that occur in deconvolving signals. We use deconvolved $\tilde{\mathbf{G}}_e$ and $\tilde{\mathbf{G}}_m$ only for showing their spectra in the next subsection.

The calculated wake potential is plotted in Fig. 8 with blue line. As the accelerating mode does not dissipate energy, its stored energy remains conserved within the cavity cell. Consequently, the wake potential satisfies the asymptotic behavior $W_\parallel(s) \xrightarrow{s \rightarrow +\infty} W_{\text{acc}}(s)$, where $W_{\text{acc}}(s)$ is the wake potential of the accelerating mode. For a Gaussian bunch, $W_{\text{acc}}(s)$ and its asymptotic form are expressed as follows [24]:

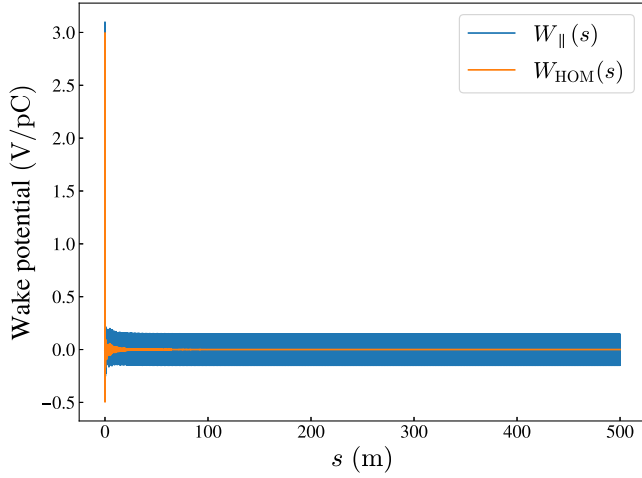


FIG. 8. Simulated wake potential at the rms bunch length of 5 mm. The blue line shows the total wake potential $W_{\parallel}(s)$ and the orange line shows the wake potential $W_{\text{HOM}}(s)$ excluding the accelerating mode.

$$W_{\text{acc}}(s) = 2k_{\text{acc}}(0) \int_{-\infty}^s ds' \cos\left(\omega_{\text{acc}} \frac{s-s'}{c_0}\right) \times \frac{1}{\sqrt{2\pi}\sigma_z} \exp\left(-\frac{s'^2}{2\sigma_z^2}\right) \quad (45)$$

$$\xrightarrow{s \rightarrow +\infty} 2k_{\text{acc}}(0) e^{-\frac{1}{2}\omega_{\text{acc}}^2 \sigma_z^2 / c_0^2} \cos\left(\omega_{\text{acc}} \frac{s}{c_0}\right),$$

where $k_{\text{acc}}(\sigma_z)$ denotes the loss factor of the accelerating mode for a rms bunch length of σ_z , expressed as follows:

$$k_{\text{acc}}(\sigma_z) = \frac{1}{4} \omega_{\text{acc}} \frac{R_{\text{sh}}}{Q_0} e^{-\omega_{\text{acc}}^2 \sigma_z^2 / c_0^2}, \quad (46)$$

where ω_{acc} denotes the resonant angular frequency, R_{sh} the shunt impedance, and Q_0 the unloaded Q of the accelerating mode. We extracted parameters such as ω_{acc} and $k_{\text{acc}}(0)$ from the asymptotic form of $W_{\parallel}(s)$. The wake potential $W_{\text{HOM}}(s) = W_{\parallel}(s) - W_{\text{acc}}(s)$ excluding the accelerating mode is also shown in Fig. 8 with orange line. The wake potential from HOMs was sufficiently damped at a distance of 500 m. The electromagnetic fields recorded by field probes were also damped adequately in this simulated time range.

B. Spectra of beam-induced electromagnetic field inside the absorbers

Figure 9 presents the simulated spectra of the electric and magnetic fields inside the dampers. Herein, we define the spectra inside a damper of volume V_{abs} as

$$S_e(\omega) = \frac{\epsilon_0}{V_{\text{abs}}} \iint \int_{V_{\text{abs}}} |\tilde{\mathbf{G}}_e(\mathbf{r}, \omega)|^2 d^3\mathbf{r}, \quad (47)$$

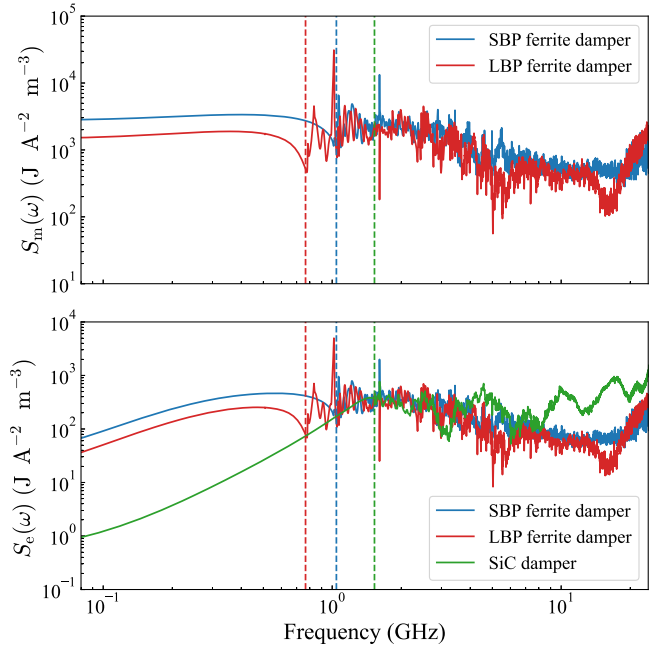


FIG. 9. Frequency spectra of the functions $\mathbf{G}_m(\mathbf{r}, t)$ and $\mathbf{G}_e(\mathbf{r}, t)$ spatially averaged inside the dampers. The upper panel shows the magnetic field spectrum and the lower panel shows that of the electric field spectrum. The vertical dashed lines indicate the cutoff frequencies of the TM_{01} propagating mode in the LBP (0.765 GHz, red line), SBP (1.043 GHz, blue line), and beam pipe (1.523 GHz, green line) outside the taper section.

$$S_m(\omega) = \frac{\mu_0}{V_{\text{abs}}} \iint \int_{V_{\text{abs}}} |\tilde{\mathbf{G}}_m(\mathbf{r}, \omega)|^2 d^3\mathbf{r}, \quad (48)$$

where $\tilde{\mathbf{G}}_e$ and $\tilde{\mathbf{G}}_m$ were evaluated by deconvolutions shown in Eqs. (43) and (44), respectively. The above spectra were evaluated inside the volumes of the SBP ferrite damper V_{sf} , LBP ferrite damper V_{lf} , and SiC damper V_{sic} . In Fig. 9, the vertical dashed lines indicate the cutoff frequencies of the TM_{01} propagating mode in the LBP, SBP, and beam pipe outside the taper section.

Figure 10 shows the enlarged view of the spectra around the second and third harmonics of f_{rf} . Two prominent peaks are observed at approximately 1.0 GHz in the LBP damper and 1.6 GHz in the SBP damper. These peak frequencies correspond with the resonant frequencies of the TM_{011} -like and TM_{020} -like modes, respectively. Notably, the TM_{011} -like mode coincides with the second harmonic of f_{rf} . As the HOM spectrum overlaps with the beam spectrum at this frequency, a resonance enhancement is expected in the LBP ferrite damper.

Equations (47) and (48) are continuous spectra of the electromagnetic field induced by a single bunch passing once. To verify the resonant enhancement between the cavity HOMs [$\tilde{\mathbf{G}}_e(\mathbf{r}, \omega)$ and $\tilde{\mathbf{G}}_m(\mathbf{r}, \omega)$] and the beam spectrum [$\tilde{q}_p \tilde{\lambda}_{\parallel}(p\omega_0)$], we next evaluate discrete spectra of the field induced by a stored beam defined as follows:

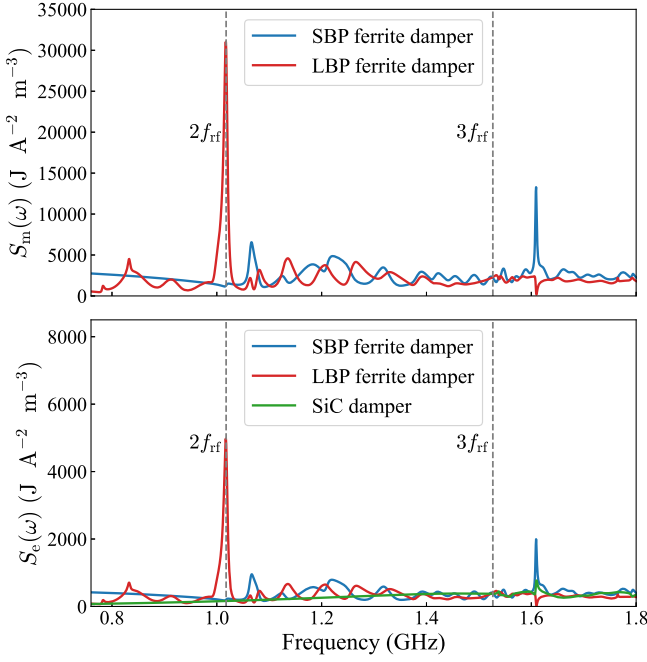


FIG. 10. Enlarged view of Fig. 9 around the second and third harmonics of f_{rf} .

$$S'_e(p) = \frac{\epsilon_0}{V_{\text{abs}}} \frac{|\tilde{q}_p|^2}{M q_b^2} \iiint_{V_{\text{abs}}} \left| \frac{\tilde{\mathbf{E}}_\lambda(\mathbf{r}, p\omega_0)}{q_s} \right|^2 d^3\mathbf{r}, \quad (49)$$

$$S'_m(p) = \frac{\mu_0}{V_{\text{abs}}} \frac{|\tilde{q}_p|^2}{M q_b^2} \iiint_{V_{\text{abs}}} \left| \frac{\tilde{\mathbf{H}}_\lambda(\mathbf{r}, p\omega_0)}{q_s} \right|^2 d^3\mathbf{r}. \quad (50)$$

The amplitudes are normalized by $M q_b^2 = I_{\text{av}}^2 T_0^2 / M$. Using Eqs. (29), (30), (49), and (50), the effective loss factor can be expressed as

$$k_{\text{eff}} = \frac{\omega_0 V_{\text{abs}}}{\pi} \left[\sum_{p=1}^{\infty} p \omega_0 \epsilon''_{\text{r,abs}}(p\omega_0) S'_e(p) + \sum_{p=1}^{\infty} p \omega_0 \mu''_{\text{r,abs}}(p\omega_0) S'_m(p) \right], \quad (51)$$

which is a similar representation to that of the conventional loss factor k_{abs} using $S_e(\omega)$ and $S_m(\omega)$ indicated in Eq. (B4), but the bunch spectrum $\tilde{\lambda}_{\parallel}(p\omega_0)$ is included in $S'_e(p)$ and $S'_m(p)$ in Eq. (51). To examine the dependence on bunch spacing, we first considered uniform fill patterns wherein equal charges are stored with equal bunch spacing. The fill pattern q_m with a uniform n -rf spacing is given by

$$q_m = \begin{cases} q_b & \text{for } m/n = \text{integer,} \\ 0 & \text{for others,} \end{cases} \quad (52)$$

where the abort gaps and 49-bucket subperiod were omitted. Its discrete Fourier transform becomes

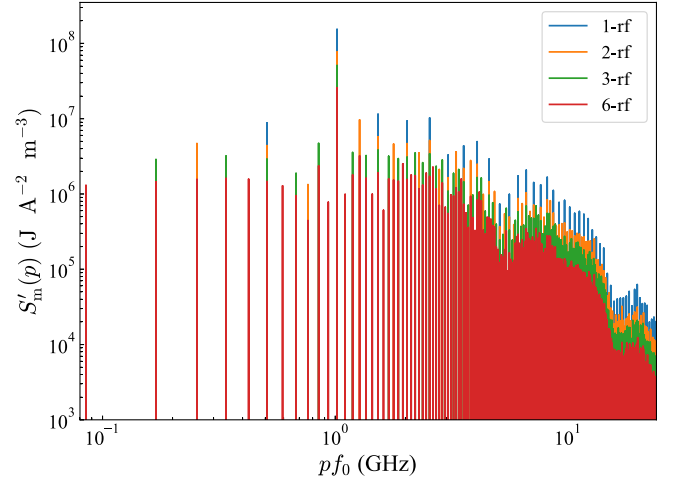


FIG. 11. Magnetic discrete frequency spectra $S'_m(p)$ evaluated for the LBP ferrite damper at several bunch spacings of $n = 1, 2, 3$, and 6 .

$$\tilde{q}_p = \begin{cases} M q_b & \text{for } p/M = \text{integer,} \\ 0 & \text{for others,} \end{cases} \quad (53)$$

where $M = h/n$. A beam with n -rf spacing has frequency components at the harmonics of f_{rf}/n . Figure 11 shows the magnetic discrete spectrum $S'_m(p)$ evaluated for the LBP ferrite damper at bunch spacings of $n = 1, 2, 3$, and 6 . Note that, for the cases of $n = 3$ and 6 , the associated fill patterns are not completely uniform because $h = 5120$ is not divisible by 3 . From Eqs. (49) and (50), each peak height of the spectra builds up proportionally to $1/n$. When a HOM frequency overlaps with a harmonic of f_{rf}/n , the resulting enhancement of k_{eff} becomes larger in shorter bunch spacings as indicated in Eq. (51).

C. Calculation of effective loss factors and absorbed power

Using the field probe signals, we evaluated the absorbed power of each damper. Figure 12 shows the calculated effective loss factors of the SBP, LBP ferrite dampers, and SiC damper as functions of bunch spacing. For comparison, the conventional loss factor of each damper evaluated from Eq. (B5) was indicated by horizontal dashed lines. The results show a clear buildup of the effective loss factor only in the LBP damper. This originates from the resonance between the TM_{011} -like mode and beam spectrum. In the SBP ferrite and SiC dampers, the effective loss factors are almost equal to the conventional ones.

Figure 13 shows absorbed power in each damper normalized by the square of the average beam current. At a fixed average current, the absorbed power decreased in all dampers—including the LBP ferrite damper—as the bunch spacing narrows (as the number of bunches M increases). As expressed in Eq. (3), the normalized absorbed power satisfies $\bar{P}_{\text{abs}}/I_{\text{av}}^2 \propto k_{\text{eff}}/M$. Figure 13

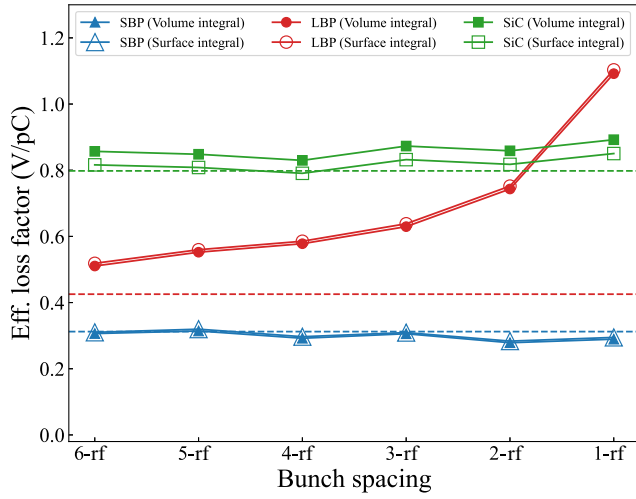


FIG. 12. Effective loss factors of the SBP, LBP, and SiC dampers as functions of bunch spacing calculated by the volume-integration method (filled marker) and the surface-integration method (empty marker). The horizontal dashed lines indicate the conventional loss factors expressed in Eq. (B5) for the SBP ferrite damper (blue line), LBP ferrite damper (red line), and SiC damper (green line).

shows that the increase in M outweighs the buildup of k_{eff} in all dampers.

In Figs. 12 and 13, two calculation methods were compared: (1) volume integration of the power density [Eq. (30)] and (2) surface integration of the power flow [Eq. (35)]. Regarding the SiC damper, the difference between the two methods was $\sim 5\%$. As regards the SBP and LBP ferrite dampers, the difference was 1%–1.8%. As the SiC damper possesses the largest volume, the

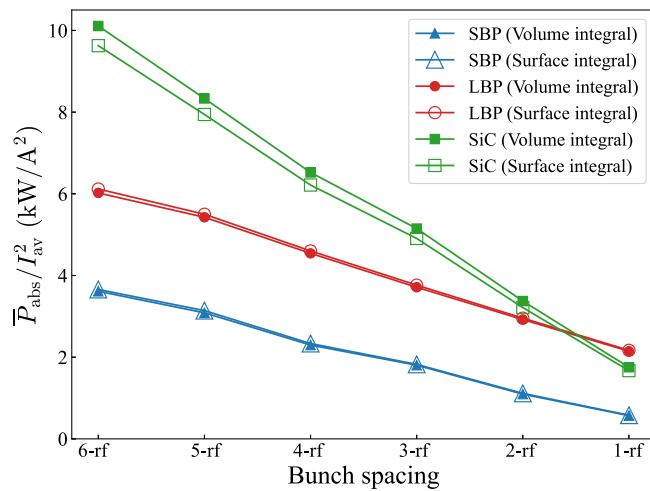


FIG. 13. Absorbed power of the SBP, LBP, and SiC dampers as functions of bunch spacing calculated by the volume-integration method (filled marker) and the surface-integration method (empty marker).

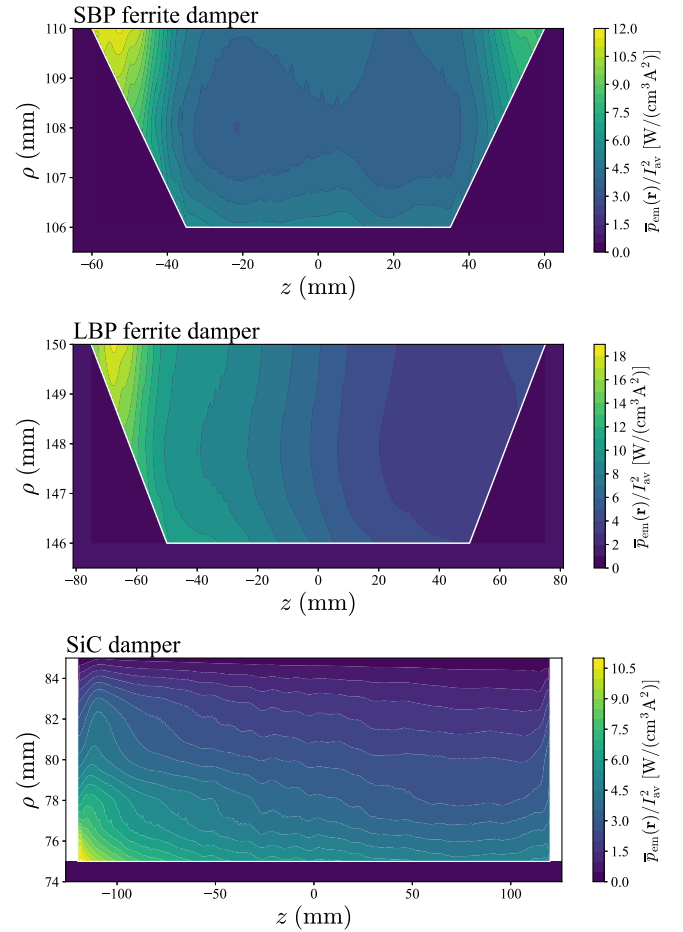


FIG. 14. Contour plot of the calculated absorbed power density normalized by the square of the average beam current for the SBP ferrite damper (top), LBP ferrite damper (center), and SiC damper (bottom) in a fill pattern of 2346 bunches. The horizontal axis represents the longitudinal direction, while the vertical axis represents the radial direction. The beam propagates from negative to positive z .

numerical error associated with the volume integration becomes relatively large.

For comparison with measurements, the calculated effective loss factors of the SBP ferrite, LBP ferrite, and SiC dampers are shown in Fig. 6 (black markers). In this calculation, the surface-integration method was used while realistic operational fill patterns—including abort gaps and 49-bucket subperiod—were assumed. The calculated values correspond with measurements within the experimental uncertainty ($\sim 10\%$), and successfully reproduce the buildup effect observed only in the LBP dampers.

Figure 14 shows contour plots of calculated absorbed power density in dampers for a fill pattern of 2346 bunches. Herein, the absorbed power density was calculated using Eq. (29). The beam propagates from negative to positive z . The absorbed power is concentrated on the upstream side. The maximum power density occurs in the LBP damper and reaches $p_{\text{em}}(\mathbf{r})/I_{\text{av}}^2 \approx 18 \text{ W}/(\text{cm}^3 \text{ A}^2)$.

D. Verification of energy conservation

To validate the absorbed-power evaluation, we verify energy conservation. In the stationary state, the energy conservation law was expressed by Eq. (16). In the wakefield simulation, the loaded Q of the accelerating mode is infinity as the cavity walls are assumed lossless and the input coupler is omitted. To avoid unphysical contributions, we exclude the accelerating mode from the HOM power $\bar{P}_b$ calculation. The total HOM power $\bar{P}_{\text{HOM}}$ excluding the accelerating mode is expressed as follows:

$$\bar{P}_{\text{HOM}} = \sum_{p=-\infty}^{\infty} \left| \frac{\tilde{q}_p}{T_0} \right|^2 |\tilde{\lambda}_{\parallel}(p\omega_0)|^2 \text{Re}[Z_{\text{HOM}}(p\omega_0)], \quad (54)$$

where $Z_{\text{HOM}}(\omega)$ denotes the coupling impedance excluding the accelerating mode

$$Z_{\text{HOM}}(\omega) = \frac{\int_{-\infty}^{\infty} W_{\text{HOM}}(s) e^{-j\omega s/c_0} ds}{c_0 \tilde{\lambda}_{\parallel}(\omega)}. \quad (55)$$

The total HOM power $\bar{P}_{\text{HOM}}$ is separated into radiated and absorbed components:

$$\bar{P}_{\text{HOM}} = \bar{P}_{\text{rad}} + \bar{P}_{\text{abs}}. \quad (56)$$

For the SuperKEKB SC cavity, the total absorbed power is expressed as

$$\bar{P}_{\text{abs}} = \bar{P}_{\text{abs,sf}} + \bar{P}_{\text{abs,lf}} + \bar{P}_{\text{abs,sic}}, \quad (57)$$

where $\bar{P}_{\text{abs,sf}}$, $\bar{P}_{\text{abs,lf}}$, and $\bar{P}_{\text{abs,sic}}$ denote the absorbed power in the SBP ferrite, LBP ferrite, and SiC dampers, respectively.

The radiated power is expressed as follows:

$$\bar{P}_{\text{rad}} = \bar{P}_{\text{rad,u}} + \bar{P}_{\text{rad,d}}, \quad (58)$$

where $\bar{P}_{\text{rad,u}}$ and $\bar{P}_{\text{rad,d}}$ denote the power radiated to the upstream and downstream beam pipes, respectively. These quantities are evaluated using the time-averaged Poynting vector expressed as follows:

$$\bar{P}_{\text{rad,u}} = - \iint_{\Omega_u} \tilde{\mathbf{S}}(\mathbf{r}) \cdot \mathbf{e}_z d^2\mathbf{r}, \quad (59)$$

$$\bar{P}_{\text{rad,d}} = + \iint_{\Omega_d} \tilde{\mathbf{S}}(\mathbf{r}) \cdot \mathbf{e}_z d^2\mathbf{r}, \quad (60)$$

where $\tilde{\mathbf{S}}(\mathbf{r})$ is expressed in Eq. (36), while Ω_u and Ω_d represent the upstream and downstream beam pipe end surfaces ($z = z_u$ and $z = z_d$ in Fig. 7), respectively. In the surface integration, the contribution of the bunch self-field was excluded to avoid divergence at $\rho = 0$. The self-fields of a bunch are expressed as follows [13,24]:

$$\mathbf{E}_{\text{self}}(\mathbf{r}, t) = \frac{q_b}{2\pi\epsilon_0\rho} \lambda_{\parallel}(c_0 t - z) \mathbf{e}_{\rho}, \quad (61)$$

$$\mathbf{H}_{\text{self}}(\mathbf{r}, t) = \frac{c_0 q_b}{2\pi\rho} \lambda_{\parallel}(c_0 t - z) \mathbf{e}_{\varphi}. \quad (62)$$

To exclude the self-fields in Eqs. (59) and (60), we redefine the functions $\tilde{\mathbf{G}}_e(\mathbf{r}, \omega)$ and $\tilde{\mathbf{G}}_m(\mathbf{r}, \omega)$ as

$$\tilde{\mathbf{G}}_e(\mathbf{r}, \omega) = \frac{\tilde{\mathbf{E}}_{\lambda}(\mathbf{r}, \omega) - \tilde{\mathbf{E}}_{\text{self}}(\mathbf{r}, \omega)}{q_b \tilde{\lambda}_{\parallel}(\omega)}, \quad (63)$$

$$\tilde{\mathbf{G}}_m(\mathbf{r}, \omega) = \frac{\tilde{\mathbf{H}}_{\lambda}(\mathbf{r}, \omega) - \tilde{\mathbf{H}}_{\text{self}}(\mathbf{r}, \omega)}{q_b \tilde{\lambda}_{\parallel}(\omega)}, \quad (64)$$

where $\tilde{\mathbf{E}}_{\lambda}$ and $\tilde{\mathbf{H}}_{\lambda}$ denote the Fourier transforms of the electromagnetic fields recorded at the beam pipe end planes and $\tilde{\mathbf{E}}_{\text{self}}$ and $\tilde{\mathbf{H}}_{\text{self}}$ the Fourier transforms of the self-fields expressed in Eqs. (61) and (62), respectively.

TABLE IV. Calculated total HOM power $\bar{P}_{\text{HOM}}$, absorbed power $\bar{P}_{\text{abs}}$, and radiated power to beam pipes $\bar{P}_{\text{rad}}$ for the fill pattern of 2346 bunches. All power values are normalized by the square of average beam current. The rms bunch length is $\sigma_z = 5$ mm.

Parameter	Value (kW/A ²)	
$\bar{P}_{\text{HOM}}/I_{\text{av}}^2$ (wake potential)	9.498	
$\bar{P}_{\text{rad}}/I_{\text{av}}^2 = (\bar{P}_{\text{rad,u}} + \bar{P}_{\text{rad,d}})/I_{\text{av}}^2$	1.600	
$\bar{P}_{\text{rad,u}}/I_{\text{av}}^2$	0.198	
$\bar{P}_{\text{rad,d}}/I_{\text{av}}^2$	1.402	
$(\bar{P}_{\text{HOM}} - \bar{P}_{\text{rad}})/I_{\text{av}}^2$	7.898	
	Volume integral [Eq. (30)]	Surface integral [Eq. (35)]
$\bar{P}_{\text{abs}}/I_{\text{av}}^2 = (\bar{P}_{\text{abs,sf}} + \bar{P}_{\text{abs,lf}} + \bar{P}_{\text{abs,sic}})/I_{\text{av}}^2$	8.057	7.938
$\bar{P}_{\text{abs,sf}}/I_{\text{av}}^2$	1.220	1.240
$\bar{P}_{\text{abs,lf}}/I_{\text{av}}^2$	3.133	3.169
$\bar{P}_{\text{abs,sic}}/I_{\text{av}}^2$	3.704	3.528

The total HOM, radiated, and absorbed powers calculated for the fill pattern of 2346 bunches are presented in Table IV. All power values are normalized by I_{av}^2 . The total HOM power obtained from the coupling impedance $Z_{HOM}(\omega)$ using Eq. (54) was $\bar{P}_{HOM}/I_{av}^2 = 9.50 \text{ kW/A}^2$. The radiated power to the beam pipes was calculated as $\bar{P}_{rad}/I_{av}^2 = 1.60 \text{ kW/A}^2$. From the energy conservation in Eq. (56), the total absorbed power must be $(\bar{P}_{HOM} - \bar{P}_{rad})/I_{av}^2 = 7.90 \text{ kW/A}^2$. On the other hand, the total absorbed power was calculated as $\bar{P}_{abs}/I_{av}^2 = 8.06 \text{ kW/A}^2$ using the volume-integration method [Eq. (30)] and $\bar{P}_{abs}/I_{av}^2 = 7.94 \text{ kW/A}^2$ using the surface-integration method [Eq. (35)]. The total absorbed power $\bar{P}_{abs}$ calculated using the surface-integration method was in better agreement with $(\bar{P}_{HOM} - \bar{P}_{rad})$ than that using the volume-integration method. The relative difference between $\bar{P}_{abs}$ and $(\bar{P}_{HOM} - \bar{P}_{rad})$ is 2.0% for the volume integration and 0.5% for the surface integration.

VI. DISCUSSION

We have demonstrated two methods for calculating the power absorbed by HOM dampers in the SuperKEKB SC cavity. The first includes the volume integration, wherein the power density obtained from the local electromagnetic fields expressed in Eq. (29) is integrated over the damper volume. The primary advantage of this method is that it provides a spatial distribution (heat map) of the absorbed power, which is useful for subsequent thermal and mechanical analyses. The second includes the surface integration, wherein the absorbed power is evaluated by integrating the energy flux over the damper surface. This method is advantageous in that fewer field probes are required, as they need to be placed only on the damper surface. This reduces computational cost while maintaining high accuracy. If only the total absorbed power of a damper is required, the surface-integration method is preferable.

As shown in Fig. 12, by comparing the effective k_{eff} and conventional loss factors k_{abs} of a damper, we can practically estimate how much the resonance effect is dominant in its power loss. If there are no resonance effects, $k_{eff} \approx k_{abs}$ will be valid. The effective loss factor of the LBP damper increases ~ 2.6 times as large as its conventional one at 1-rf spacing, which implies that the resonance nature is dominant in its power loss. In the SBP ferrite damper and SiC damper, since the difference between k_{eff} and k_{abs} is small ($\lesssim 10\%$), then short-range interactions between the damper and each bunch are dominant in their power loss.

Based on the analyses in Sec. V, we consider two approaches to reduce HOM power. The first entails utilizing a fill pattern with 1-rf spacing. Figure 15 shows the measured and calculated absorbed power in the LBP dampers as a function of number of bunches. In the calculation, the absorbed power was evaluated additionally

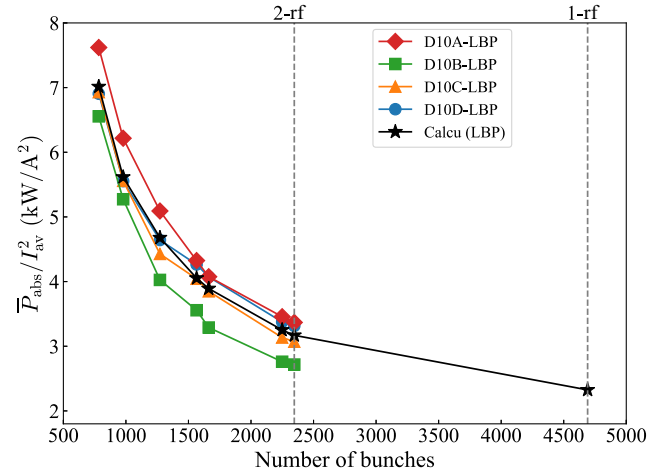


FIG. 15. Absorbed power in the LBP ferrite dampers normalized by the average beam current as a function of number of bunches. Colored markers represent measured values in the D10 section, while black markers the calculated values. The vertical dashed lines indicate the numbers of bunches corresponding with 2-rf and 1-rf spacings, respectively.

for the 1-rf spacing case. Compared to the 2-rf spacing currently used in operation, the absorbed power in the LBP dampers can be reduced by approximately 27% at fixed average beam current. Further, this approach would reduce the absorbed power in the SBP ferrite and SiC dampers. A beam study using the 1-rf spacing is planned to experimentally verify this effect.

The second approach is to shift the resonant frequency of the TM_{011} -like mode. If the resonance can be shifted by approximately $\pm 10 \text{ MHz}$, the effective loss factor in the LBP dampers is expected to decrease by 30%–40%. However, this way is effective only for the LBP dampers.

VII. CONCLUSION

During the operation of the SuperKEKB SC cavities, a strong bunch fill pattern dependence of effective loss factors in the LBP ferrite dampers was observed. To clarify this phenomenon, we conducted a simulation study using time-domain wakefield analysis. To accurately evaluate absorbed power in each damper, two calculation methods were examined: the volume and surface-integration methods. Measured and calculated effective loss factors showed reasonable agreement in their dependence on the bunch fill pattern.

To validate our calculation results, energy conservation in the simulation was verified. The results confirmed that the relationship $\bar{P}_{HOM} = \bar{P}_{abs} + \bar{P}_{rad}$ holds, demonstrating that the present evaluation is consistent with the conventional HOM power evaluation using coupling impedance.

Based on the absorbed power evaluation, approaches to mitigate the power load on the HOM dampers will be further investigated.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: DERIVATION OF THE FORMULA FOR ABSORBED POWER DENSITY

In this appendix, we derive the time-averaged absorbed power density in Eq. (29). Similarly to Eq. (12), the beam-induced electromagnetic fields in Eqs. (27) and (28) are rewritten as

$$\mathbf{E}(\mathbf{r}, t) = \frac{1}{T_0} \sum_{p=-\infty}^{\infty} \tilde{q}_p \tilde{\lambda}_{\parallel}(p\omega_0) \tilde{\mathbf{G}}_e(\mathbf{r}, p\omega_0) e^{jp\omega_0 t}, \quad (\text{A1})$$

$$\mathbf{H}(\mathbf{r}, t) = \frac{1}{T_0} \sum_{p=-\infty}^{\infty} \tilde{q}_p \tilde{\lambda}_{\parallel}(p\omega_0) \tilde{\mathbf{G}}_m(\mathbf{r}, p\omega_0) e^{jp\omega_0 t}, \quad (\text{A2})$$

where

$$\tilde{\mathbf{G}}_e(\mathbf{r}, \omega) = \int_{-\infty}^{\infty} \mathbf{G}_e(\mathbf{r}, t) e^{-j\omega t} dt, \quad (\text{A3})$$

$$\tilde{\mathbf{G}}_m(\mathbf{r}, \omega) = \int_{-\infty}^{\infty} \mathbf{G}_m(\mathbf{r}, t) e^{-j\omega t} dt, \quad (\text{A4})$$

represent the Fourier transforms of $\mathbf{G}_e$ and $\mathbf{G}_m$, respectively. Representing the Fourier transforms of $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{H}(\mathbf{r}, t)$ as $\tilde{\mathbf{E}}(\mathbf{r}, \omega)$ and $\tilde{\mathbf{H}}(\mathbf{r}, \omega)$, respectively, $\mathbf{E}$, $\mathbf{D}$, $\mathbf{H}$, and $\mathbf{B}$ are expressed as

$$\mathbf{E}(\mathbf{r}, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{\mathbf{E}}(\mathbf{r}, \omega) e^{j\omega t} d\omega, \quad (\text{A5})$$

$$\mathbf{D}(\mathbf{r}, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \epsilon_0 \epsilon_r(\mathbf{r}, \omega) \tilde{\mathbf{E}}(\mathbf{r}, \omega) e^{j\omega t} d\omega, \quad (\text{A6})$$

$$\mathbf{H}(\mathbf{r}, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{\mathbf{H}}(\mathbf{r}, \omega) e^{j\omega t} d\omega, \quad (\text{A7})$$

$$\mathbf{B}(\mathbf{r}, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mu_0 \mu_r(\mathbf{r}, \omega) \tilde{\mathbf{H}}(\mathbf{r}, \omega) e^{j\omega t} d\omega. \quad (\text{A8})$$

From the property of a Fourier transform for a real-valued function, we have $\tilde{\mathbf{E}}(\mathbf{r}, -\omega) = \tilde{\mathbf{E}}^*(\mathbf{r}, \omega)$. Using this, $\mathbf{E} \cdot \partial \mathbf{D} / \partial t$ is calculated as

$$\mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} = \frac{1}{(2\pi)^2} \epsilon_0 \iint_{-\infty}^{\infty} \tilde{\mathbf{E}}^*(\mathbf{r}, \omega') \cdot [j\omega \epsilon_r(\mathbf{r}, \omega) \tilde{\mathbf{E}}(\mathbf{r}, \omega)] e^{j(\omega - \omega')t} d\omega d\omega'. \quad (\text{A9})$$

Taking the complex conjugate of the above equation and exchanging the integration variables $\omega \leftrightarrow \omega'$, we also have

$$\mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} = \frac{1}{(2\pi)^2} \epsilon_0 \iint_{-\infty}^{\infty} \tilde{\mathbf{E}}(\mathbf{r}, \omega) \cdot [-j\omega' \epsilon_r^*(\mathbf{r}, \omega') \tilde{\mathbf{E}}^*(\mathbf{r}, \omega')] e^{j(\omega - \omega')t} d\omega d\omega'. \quad (\text{A10})$$

Summing up Eqs. (A9) and (A10), it gives [29]

$$\mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} = \frac{1}{2(2\pi)^2} \epsilon_0 \iint_{-\infty}^{\infty} d\omega d\omega' [j\omega \epsilon_r(\mathbf{r}, \omega) - j\omega' \epsilon_r^*(\mathbf{r}, \omega')] \tilde{\mathbf{E}}(\mathbf{r}, \omega) \cdot \tilde{\mathbf{E}}^*(\mathbf{r}, \omega') e^{j(\omega - \omega')t}. \quad (\text{A11})$$

For the magnetic field, the equivalent equations to Eq. (A11) are valid by changing variables as $\mathbf{E} \rightarrow \mathbf{H}$, $\mathbf{D} \rightarrow \mathbf{B}$, $\epsilon_0 \rightarrow \mu_0$, and $\epsilon_r \rightarrow \mu_r$ as follows:

$$\mathbf{H} \cdot \frac{\partial \mathbf{B}}{\partial t} = \frac{1}{2(2\pi)^2} \mu_0 \iint_{-\infty}^{\infty} d\omega d\omega' [j\omega \mu_r(\mathbf{r}, \omega) - j\omega' \mu_r^*(\mathbf{r}, \omega')] \tilde{\mathbf{H}}(\mathbf{r}, \omega) \cdot \tilde{\mathbf{H}}^*(\mathbf{r}, \omega') e^{j(\omega - \omega')t}. \quad (\text{A12})$$

Substituting the Fourier transforms of Eqs. (A1) and (A2) into the right-hand sides of Eqs. (A11) and (A12), respectively, the energy conservation law in Eq. (24) is expressed as follows:

$$\begin{aligned}
\frac{\partial u_{\text{em}}}{\partial t} + p_{\text{em}} &= \frac{1}{2} \sum_{p=-\infty}^{\infty} \sum_{p'=-\infty}^{\infty} \frac{\tilde{q}_p}{T_0} \frac{\tilde{q}_{p'}^*}{T_0} \tilde{\lambda}_{\parallel}(p\omega_0) \tilde{\lambda}_{\parallel}^*(p'\omega_0) \\
&\times [j\epsilon_0\{p\omega_0\epsilon_r(\mathbf{r}, p\omega_0) - p'\omega_0\epsilon_r^*(\mathbf{r}, p'\omega_0)\} \tilde{\mathbf{G}}_e(\mathbf{r}, p\omega_0) \cdot \tilde{\mathbf{G}}_e^*(\mathbf{r}, p'\omega_0) \\
&+ j\mu_0\{p\omega_0\mu_r(\mathbf{r}, p\omega_0) - p'\omega_0\mu_r^*(\mathbf{r}, p'\omega_0)\} \tilde{\mathbf{G}}_m(\mathbf{r}, p\omega_0) \cdot \tilde{\mathbf{G}}_m^*(\mathbf{r}, p'\omega_0)] e^{j(p-p')\omega_0 t}. \quad (\text{A13})
\end{aligned}$$

In this case, $\mathbf{E}$ and $\mathbf{H}$ contain only the discrete frequency components corresponding with harmonics of the revolution frequency ω_0 . Hence, the continuous frequency integrals in Eqs. (A11) and (A12) are discretized into the summation form shown in Eq. (A13).

We average Eq. (A13) over the revolution time T_0 . As u_{em} and p_{em} are periodic with T_0 , the time average of the left-hand side becomes

$$\frac{1}{T_0} \int_{t-T_0}^t dt' \left[\frac{\partial u_{\text{em}}(\mathbf{r}, t')}{\partial t'} + p_{\text{em}}(\mathbf{r}, t') \right] = \bar{p}_{\text{em}}(\mathbf{r}),$$

where $\bar{p}_{\text{em}}(\mathbf{r})$ denotes the time-averaged power loss density expressed as follows:

$$\bar{p}_{\text{em}}(\mathbf{r}) = \frac{1}{T_0} \int_{t-T_0}^t p_{\text{em}}(\mathbf{r}, t') dt'. \quad (\text{A14})$$

When time-averaging the right-hand side of Eq. (A13) using Eq. (17), the interference terms between different

harmonics $p \neq p'$ vanish. Time-averaging the right-hand side of Eq. (A13), we derive Eq. (29).

APPENDIX B: CONVENTIONAL LOSS FACTOR PER ABSORBER

Here, we consider the “conventional” loss factor per absorber that is defined by the energy loss of a single bunch passing the structure once. The energy conservation of this case is given by Eq. (24) with changing the fields $\mathbf{E}$ and $\mathbf{H}$ to ones $\mathbf{E}_\lambda$ and $\mathbf{H}_\lambda$ induced by a single bunch of charge q_s , respectively. Using the power loss density $p_{\text{em}}(\mathbf{r}, t)$, the energy absorbed by a microwave absorber of volume V_{abs} is

$$\Delta U_{\text{abs}} = k_{\text{abs}} q_s^2 = \iiint_{V_{\text{abs}}} d^3\mathbf{r} \int_{-\infty}^{\infty} dt p_{\text{em}}(\mathbf{r}, t), \quad (\text{B1})$$

where k_{abs} is the conventional loss factor of the absorber. From Eqs. (24), (A11), and (A12), $p_{\text{em}}(\mathbf{r}, t)$ is

$$\begin{aligned}
p_{\text{em}}(\mathbf{r}, t) &= -\frac{\partial u_{\text{em}}}{\partial t} + \frac{1}{2(2\pi)^2} \left[\epsilon_0 \iint_{-\infty}^{\infty} d\omega d\omega' [j\omega\epsilon_r(\mathbf{r}, \omega) - j\omega'\epsilon_r^*(\mathbf{r}, \omega')] \tilde{\mathbf{E}}_\lambda(\mathbf{r}, \omega) \cdot \tilde{\mathbf{E}}_\lambda^*(\mathbf{r}, \omega') e^{j(\omega-\omega')t} \right. \\
&\quad \left. + \mu_0 \iint_{-\infty}^{\infty} d\omega d\omega' [j\omega\mu_r(\mathbf{r}, \omega) - j\omega'\mu_r^*(\mathbf{r}, \omega')] \tilde{\mathbf{H}}_\lambda(\mathbf{r}, \omega) \cdot \tilde{\mathbf{H}}_\lambda^*(\mathbf{r}, \omega') e^{j(\omega-\omega')t} \right]. \quad (\text{B2})
\end{aligned}$$

Before the bunch passage, the electromagnetic field is zero: $u_{\text{em}}(\mathbf{r}, t) \xrightarrow{t \rightarrow -\infty} 0$. After the bunch passage, the electromagnetic field inside dispersive materials decays, and then the electromagnetic energy tends to zero: $u_{\text{em}}(\mathbf{r}, t) \xrightarrow{t \rightarrow +\infty} 0$. As a result, the time integration of $\partial u_{\text{em}}/\partial t$ from $t = -\infty$ to $+\infty$ should be zero. Integrating Eq. (B2) over $t = -\infty$ to $+\infty$ and using the frequency spectra given in Eqs. (47) and (48), the conventional loss factor $k_{\text{abs}} = \Delta U_{\text{abs}}/q_s^2$ is expressed as

$$\begin{aligned}
k_{\text{abs}} &= \frac{1}{\pi} \left[\int_0^\infty d\omega \epsilon_{r,\text{abs}}''(\omega) \iiint_{V_{\text{abs}}} d^3\mathbf{r} \left| \frac{1}{q_s} \tilde{\mathbf{E}}_\lambda(\mathbf{r}, \omega) \right|^2 \right. \\
&\quad \left. + \int_0^\infty d\omega \mu_{r,\text{abs}}''(\omega) \iiint_{V_{\text{abs}}} d^3\mathbf{r} \left| \frac{1}{q_s} \tilde{\mathbf{H}}_\lambda(\mathbf{r}, \omega) \right|^2 \right] \quad (\text{B3})
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{\pi} V_{\text{abs}} \left[\int_0^\infty \omega \epsilon_{r,\text{abs}}''(\omega) |\tilde{\lambda}_{\parallel}(\omega)|^2 S_e(\omega) d\omega \right. \\
&\quad \left. + \int_0^\infty \omega \mu_{r,\text{abs}}''(\omega) |\tilde{\lambda}_{\parallel}(\omega)|^2 S_m(\omega) d\omega \right], \quad (\text{B4})
\end{aligned}$$

where we have used $\tilde{\mathbf{E}}_\lambda(\mathbf{r}, \omega) = q_s \tilde{\lambda}_{\parallel}(\omega) \tilde{\mathbf{G}}_e(\mathbf{r}, \omega)$ and $\tilde{\mathbf{H}}_\lambda(\mathbf{r}, \omega) = q_s \tilde{\lambda}_{\parallel}(\omega) \tilde{\mathbf{G}}_m(\mathbf{r}, \omega)$ and assumed $\epsilon_r(\mathbf{r}, \omega) = \epsilon_{r,\text{abs}}'(\omega) - j\epsilon_{r,\text{abs}}''(\omega)$ and $\mu_r(\mathbf{r}, \omega) = \mu_{r,\text{abs}}'(\omega) - j\mu_{r,\text{abs}}''(\omega)$ for $\mathbf{r} \in V_{\text{abs}}$.

Equation (B4) is calculated by a volume integration over the material volume. The conventional loss factor can also be evaluated in terms of a surface integration as follows:

$$k_{\text{abs}} = -\frac{1}{q_s^2} \int_{-\infty}^{\infty} dt \iint_{\Omega_{\text{abs}}} d^2\mathbf{r} \mathbf{S}_\lambda(\mathbf{r}, t) \cdot \mathbf{n}, \quad (\text{B5})$$

where $\mathbf{S}_\lambda(\mathbf{r}, t) = \mathbf{E}_\lambda(\mathbf{r}, t) \times \mathbf{H}_\lambda(\mathbf{r}, t)$.

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