

Noninvasive ion fraction quantification of dual-species beams in synchrotrons

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The ion composition of dual-species beams in synchrotrons is typically inferred from invasive measurements performed after beam extraction. This paper introduces a complementary noninvasive method to determine the ion composition of such beams directly inside the synchrotron. The approach is applicable to low- and medium-energy ion synchrotrons and to small relative mass-to-charge ratio offsets, typically at the 10^{-4} level. The method exploits dispersive orbit offsets between the two species and corresponding frequency corrections applied by the synchrotron rf radial regulation loop. This capability is of particular interest for ongoing research on online monitoring in carbon ion beam therapy using mixed $^4\text{He}^{2+}$ and $^{12}\text{C}^{6+}$ beams, which feature a relative mass-to-charge ratio offset of 0.065%. The proposed method is analytically derived and tested with particle tracking simulations using XSUITE. Its applicability under realistic experimental conditions is demonstrated at the MedAustron facility using mixed $^4\text{He}^{2+}$ and $^{12}\text{C}^{6+}$ beams. The results show good agreement with independent postextraction measurements.

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I. INTRODUCTION

In recent years, mixed helium and carbon ion irradiation has been proposed as a new concept for online range monitoring in carbon ion therapy [1,2]. The aim of such monitoring is to detect range errors, for example, due to anatomical changes or uncertainties associated with CT-to-stopping-power conversion.

$^4\text{He}^{2+}$ and $^{12}\text{C}^{6+}$ have a similar mass-to-charge ratio m/q , with a relative difference of

$$\frac{\Delta(m/q)}{(m/q)} = \frac{(m/q)_{\text{He}} - (m/q)_{\text{C}}}{(m/q)_{\text{C}}} = 6.5 \times 10^{-4}. \quad (1)$$

This allows both ion species to be simultaneously delivered to the patient at a similar energy-per-mass, at which $^4\text{He}^{2+}$ has approximately 3 times the range in matter of $^{12}\text{C}^{6+}$. As a result, the tumor can be treated with carbon

ions while range errors are inferred from the residual energy of the helium ions traversing the patient (Fig. 1).

Since 2023, both at GSI [3] and MedAustron [4,5], mixed helium and carbon ion beams have been successfully generated, accelerated, and extracted to experimental irradiation rooms for interdisciplinary research in accelerator physics, detector development, and medical physics. Due to differing accelerator capabilities, GSI currently generates the mixed beam directly in a single ion source [3], while at MedAustron the two ion species are generated in separate ion sources and mixed by means of a sequential multiturn injection into the synchrotron [4].

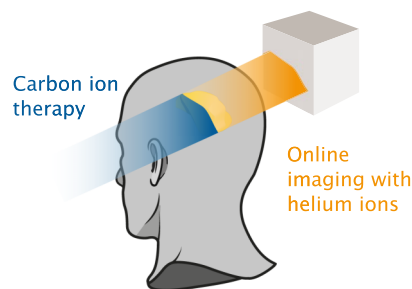


FIG. 1. Schematic of mixed-beam irradiation. The carbon beam is used for tumor treatment while the residual energy of the helium ions is measured downstream of the patient for diagnostic purposes.

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Currently, these beams are provided to research users on an experimental basis, with helium fractions

$$r = \frac{N_{\text{He}}}{N_{\text{He}} + N_{\text{C}}}, \quad (2)$$

typically varying within $0.1 < r < 0.5$, depending on the specific user requirements. N_{He} and N_{C} are the intensities per species. Studies toward future clinical implementation currently consider helium fractions of $r \approx 0.1$ to 0.2 , as this range is expected to provide a helium signal distinguishable from the carbon-ion fragments, while the dose deposited by the helium ions amounts to less than 1% of the total dose [1,6]. However, final clinical target values and tolerances for the helium fraction have not yet been established.

If the beam is generated from a single ion source (e.g., using a helium-methane mixture as source gas), the helium fraction can be controlled by adjusting the helium gas flow to the source plasma. In the case of mixed-beam generation by means of sequential multiturn injection, as implemented at MedAustron [4], it is possible to tailor the helium fraction using the injection settings (Sec. II A). However, particularly in the latter approach, the helium fraction can exhibit noticeable shot-to-shot fluctuations.

At present, the ion composition cannot be determined directly within the accelerator complex and is instead inferred from measurements of the extracted beam in the irradiation room. Such measurements require the installation of a dedicated diagnostic setup, which complicates beam commissioning tasks. Further, in most cases the setup cannot be installed simultaneously with experiments, which means that no cycle-resolved information on the delivered mixing ratio is available.

To address this gap, this work proposes a new noninvasive method for estimating the ion composition prior to extraction in synchrotrons. The method relies on composition-dependent feedback from the radial regulation loops of the synchrotron radio frequency (rf) system. Due to the small but measurable difference in beam rigidity between ${}^4\text{He}^{2+}$ and ${}^{12}\text{C}^{6+}$ (Sec. II B), the two ion species follow different dispersive orbits. In synchrotrons without radial rf regulation, information on the ion composition can be obtained from the measured orbit offset in dispersive regions. In synchrotrons with radial rf regulation it can be retrieved from the applied frequency corrections. While the present work focuses on the latter case, the methodology can in principle also be adapted to synchrotrons without radial regulation loops.

Section II reviews the status of mixed-beam delivery at MedAustron as well as the rf acceleration of such beams. Section III presents the proposed method for determining the ion fraction in the synchrotron. The experimental demonstration is described in Sec. IV, followed by discussion and concluding remarks in Secs. V and VI.

II. BACKGROUND

A. Mixed-ion beam delivery at MedAustron

At the MedAustron center for ion beam therapy and research, which is based on the Proton-Ion Medical Machine Study (PIMMS) [7,8], mixed helium-carbon ion beams are generated and delivered to the irradiation room for nonclinical research studies using a sequential multiturn injection scheme [4]. In nominal single-species operation, helium and carbon ion beams are generated in two separate ion sources as ≈ 8 keV/u ${}^4\text{He}^{2+}$ and ${}^{12}\text{C}^{4+}$ beams, respectively, which are subsequently preaccelerated through a radio frequency quadrupole and linear accelerator to ≈ 7 MeV/u. The beams then pass a stripping foil. The fully stripped ${}^4\text{He}^{2+}$ and ${}^{12}\text{C}^{6+}$ ions are injected into the synchrotron via a conventional multiturn injection.

In mixed-species operation, the beam is generated by successively injecting the ${}^4\text{He}^{2+}$ and ${}^{12}\text{C}^{6+}$ ions from the two distinct ion sources into the synchrotron. First, helium is injected and stored at flat bottom. After this, the injector is reconfigured and the carbon beam is injected using a second multiturn injection. A substantial fraction of the already circulating helium ion beam is lost at the injection septum when ramping the injection bump for the carbon injection. As a result, the two ion species end up occupying different regions in horizontal phase space. One way to tailor the ion fraction is to adjust the amplitude of the second injection bump [4].

Following sequential injection, both ion species are captured together and accelerated to extraction energies between 120 and 402.8 MeV/u. As in single-species operation, the synchrotron rf system [9] employs radial and phase regulation loops. The system operates with a loop cycle time of 10 μs . The two ion species are subsequently extracted together using third-order resonance slow extraction. To date, the clinically employed betatron core-driven slow extraction is not compatible with mixed-beam operation. Therefore the mixed-beam slow extraction has so far been performed [10] with either rf knockout or phase displacement acceleration as driving mechanism, profiting from recent developments on these alternative slow extraction mechanisms at MedAustron [11–13].

B. rf acceleration of mixed-species ion beams

When accelerating mixed-species beams with slightly different m/q in synchrotrons, the rf synchronism condition must be simultaneously fulfilled for all ion species despite the slightly different closed-orbit lengths. As a consequence, the secondary ion species attains a slight offset in momentum-per-mass from the reference species, as reviewed below.

The synchronous particle of the reference species (species I) is described using the rest mass m_I , charge q_I , velocity $v_I = \beta_I c$, with the speed of light c , and the

relativistic factors β_I and γ_I . The synchronous particle of the secondary ion species (species II) is described using m_{II} , q_{II} , β_{II} , and γ_{II} . Note that the synchronous particle of species I may be off-momentum with respect to the on-momentum reference particle, which is denoted by the subscript “0”. The label “sp” is used to indicate offsets between the two ion species.

The relative offset in magnetic rigidity $B\rho = \frac{m\gamma\beta c}{q}$ of species II from species I,

$$\frac{\Delta(B\rho)}{(B\rho)_I} = \frac{\Delta(p/q)}{(p/q)_I} = \frac{m_{II}/q_{II}}{m_I/q_I} \cdot \frac{\beta_{II}\gamma_{II}}{\beta_I\gamma_I} - 1 = \delta_{\text{eff,sp}}, \quad (3)$$

depends on offsets in both m/q and $p/m = \beta\gamma c$. Following the notation of Hermes [14], it is also referred to as *effective relative momentum offset* δ_{eff} .

Similar to the single-species case, a deviation in p/m , i.e., in $\beta\gamma$, will be parametrized by

$$\delta_{\text{sp}} = \frac{\Delta(p/m)}{(p/m)_I} = \frac{\Delta(\beta\gamma)}{(\beta\gamma)_I} = \frac{\beta_{II}\gamma_{II}}{\beta_I\gamma_I} - 1. \quad (4)$$

The offset in m/q is commonly described by the factor

$$\chi_{\text{sp}} = \frac{q_{II}/m_{II}}{q_I/m_I}. \quad (5)$$

Together, Eqs. (4) and (5) are used to rewrite the relative rigidity offset between species I and II, Eq. (3), as

$$\delta_{\text{eff,sp}} = \frac{1 + \delta_{\text{sp}}}{\chi_{\text{sp}}} - 1. \quad (6)$$

The formalism presented in Sec. III is based on linearized models valid for small mass-to-charge ratio offsets $|\chi_{\text{sp}} - 1| \ll 1$. Treating derivations in this work consistently to first order, Eq. (6) can be expressed as

$$\delta_{\text{eff,sp}} \approx \frac{1}{\chi_{\text{sp}}} - 1 + \delta_{\text{sp}}. \quad (7)$$

Due to this rigidity offset, the secondary ion species follows a different dispersive orbit than the reference ion species, with the modified average bending radius

$$R_{II} = R_I (1 + \alpha_c \delta_{\text{eff,sp}}) + \mathcal{O}(\delta_{\text{eff,sp}}^2), \quad (8)$$

where α_c is the linear momentum compaction factor. In the case of off-momentum operation, α_c is evaluated at the momentum offset of the reference species I.

When the rf system is turned on, both synchronous particles must have the same revolution frequency f_{rev} , despite their different closed-orbit lengths. This results in the two ion species acquiring slightly different β , i.e.,

$$\frac{\Delta f_{\text{rev}}}{f_{\text{rev,I}}} = 0 = \frac{\beta_{II}}{\beta_I} \frac{R_I}{R_{II}} - 1 \Rightarrow \frac{\beta_{II}}{\beta_I} = \frac{R_{II}}{R_I}. \quad (9)$$

Inserting Eq. (8), with Eq. (7), and

$$\frac{\beta_{II}}{\beta_I} \approx 1 + \frac{1}{\gamma_I^2} \delta_{\text{sp}} + \mathcal{O}(\delta_{\text{sp}}^2), \quad (10)$$

into Eq. (9) yields an expression for the resulting relative offset in $p/m = \beta\gamma c$ of species II from species I,

$$\delta_{\text{sp}} = \frac{(\beta\gamma)_{II} - (\beta\gamma)_I}{(\beta\gamma)_I} = \frac{\gamma_I^2}{\gamma_{\text{tr}}^2 - \gamma_I^2} \left(\frac{1}{\chi_{\text{sp}}} - 1 \right), \quad (11)$$

with $\gamma_{\text{tr}} = 1/\sqrt{\alpha_c}$ being the transition energy.

Combining Eqs. (11) and (7), we obtain an explicit expression for the effective momentum deviation, valid when the secondary ion species circulates at the same frequency as the reference species,

$$\delta_{\text{eff,sp}} = \frac{1}{1 - \gamma_I^2/\gamma_{\text{tr}}^2} \left(\frac{1}{\chi_{\text{sp}}} - 1 \right). \quad (12)$$

Alternatively, Eq. (11) can also be obtained by logarithmic differentiation, i.e.,

$$\frac{d(p/q)}{(p/q)} = \frac{d(m/q)}{(m/q)} + \frac{d(\beta\gamma)}{(\beta\gamma)}, \quad (13)$$

as presented, for example, in Refs. [15,16].

As an example, we consider a mixed ${}^4\text{He}^{2+}$ and ${}^{12}\text{C}^{6+}$ beam in a PIMMS-like synchrotron with $\alpha_c \approx 0.25$, operated below transition. ${}^{12}\text{C}^{6+}$ is defined as the reference species I. ${}^4\text{He}^{2+}$, species II, features $1 - \chi_{\text{sp}} = 6.5 \times 10^{-4}$. It is therefore slightly more rigid than ${}^{12}\text{C}^{6+}$ and follows an orbit with increased path length. This means that, below transition, ${}^4\text{He}^{2+}$ must be accelerated to slightly higher velocities, $\beta_{\text{He}} > \beta_{\text{C}}$, to ensure synchronism with the rf wave. The introduced relative offset in $\beta\gamma$ for ${}^4\text{He}^{2+}$ in such a synchrotron, Eq. (11), is between $\delta_{\text{sp}} \approx 0.3\%$ and 0.7% for clinical extraction energies of $E_{\text{kin}} = 120$ to 400 MeV/u. This offset is comparable in magnitude to the operational rms momentum spreads δ_{rms} at flattop, prior to an optional phase jump [17]. The related total effective rigidity offset, Eq. (7), ranges from 0.95% to 1.35% for the stated extraction energies.

Above transition, Eq. (11) changes sign, which reduces the effective rigidity offset, Eq. (7). For $\gamma_I \gg \gamma_{\text{tr}}$, $\delta_{\text{sp}} \rightarrow (1 - 1/\chi_{\text{sp}})$, the two contributions in Eq. (7) compensate and $\delta_{\text{eff,sp}}$ vanishes. Both ion species move on a similar orbit. However, when approaching transition energy, the orbit separation increases. Whether both ion species in a mixed-species beam can be maintained during transition crossing would therefore depend on the details

of the transition crossing scheme and the value of χ_{sp} . This effect was made use of in the late 1980s in the CERN PS to clean a S^{16+} beam from an O^{8+} contamination [16,18].

III. ION FRACTION QUANTIFICATION

This section presents a model that exploits the described difference in beam rigidity between two ion species to noninvasively determine the mixing ratio. The model is applicable to low- and medium-energy ion synchrotrons, ion mixtures with small mass-to-charge offsets, $|\chi_{\text{sp}} - 1| = \mathcal{O}(10^{-4})$, and is formulated under the assumption that the horizontal dispersion is approximately constant for the respective rigidity offsets. Furthermore, it requires that the beam position monitors exhibit an approximately linear response across the horizontal range occupied by both ion species, allowing their signals to be superimposed. This condition is met by shoebox-type pickups [19].

A proposal conceptually related to the approach pursued here was put forward by van der Meer in 1978 [20]. He suggested determining the mass difference between p and $\bar{p}$ in the CERN SPS by resolving their orbit separation in a dispersive region when operating close to transition energy. However, this approach was ultimately abandoned, as subsequent analyses indicated that for that specific use case alternative methods could achieve higher accuracy [[15], Chap. 7.7].

A. Dispersive distribution of mixed-species beams

For a dual-species beam with N_{I} ions of species I with charge $Q_{\text{I}} = q_{\text{I}} e$ (charge state q , elementary charge e) and N_{II} ions of species II with charge state q_{II} , it is useful to distinguish between the total number of ions

$$N_{\text{mix}} = N_{\text{I}} + N_{\text{II}}, \quad (14)$$

and the total number of charges

$$N_{\text{q,mix}} = N_{\text{I}} q_{\text{I}} + N_{\text{II}} q_{\text{II}}. \quad (15)$$

Using the previously introduced definition for the ion fraction

$$r = \frac{N_{\text{II}}}{N_{\text{II}} + N_{\text{I}}} = \frac{N_{\text{II}}}{N_{\text{mix}}}, \quad (16)$$

$N_{\text{q,mix}}$ can be expressed by

$$N_{\text{q,mix}} = N_{\text{I}} q_{\text{I}} + N_{\text{II}} q_{\text{II}} = (1 - r) N_{\text{mix}} q_{\text{I}} + r N_{\text{mix}} q_{\text{II}}. \quad (17)$$

At a location with nonzero horizontal dispersion $D_x \neq 0$, the center of gravity of species II features a horizontal offset from the center of gravity of species I of

$$\langle x \rangle_{\text{II}} - \langle x \rangle_{\text{I}} = D_x \delta_{\text{eff,sp}}, \quad (18)$$

with $\delta_{\text{eff,sp}}$ obtained from Eqs. (7) and (11). In the case of off-momentum operation, with the reference ion species featuring a relative momentum offset δ_{I} (as common in a PIMMS-like synchrotron), D_x is the horizontal dispersion evaluated at the momentum offset of the reference ion species, i.e., $D_x = D_x(\delta_{\text{I}})$.

Consequently, the two displaced charge distributions shift the center of gravity position of the entire charge distribution to

$$\langle x \rangle_{\text{q,mix}} = \frac{N_{\text{II}} q_{\text{II}} \langle x \rangle_{\text{II}} + N_{\text{I}} q_{\text{I}} \langle x \rangle_{\text{I}}}{N_{\text{q,mix}}} = A_r D_x \delta_{\text{eff,sp}} + \langle x \rangle_{\text{I}}, \quad (19)$$

with the charge-weighted ion fraction

$$A_r = \frac{q_{\text{II}} r}{q_{\text{II}} r + q_{\text{I}} (1 - r)}. \quad (20)$$

To provide an example, Fig. 2 illustrates the simulated charge distributions for a mixed 262.3 MeV/u ${}^4\text{He}^{2+}$ and ${}^{12}\text{C}^{6+}$ beam with different ion fractions r in a PIMMS [7,8] lattice. ${}^{12}\text{C}^{6+}$ is considered the reference ion species. As in single-species operation, the modeled synchrotron is operated off-momentum, with a radial loop correcting the rf frequency so that the measured center of gravity of the total charge distribution, $\langle x \rangle_{\text{q,mix}}$, features the requested offset x_{RL} at the radial loop pickup, in this example $x_{\text{RL}} = 27.5$ mm. The two distributions are separated by the offset given in Eq. (18). In the convention used throughout this work, the beam moves counterclockwise and hence $x > 0$ and $x < 0$ denote horizontal offsets toward and away from the center of the synchrotron, respectively, which is indicated in Fig. 2 as *inward* and *outward*. ${}^4\text{He}^{2+}$, being more rigid with $\delta_{\text{eff,sp}} > 0$, moves on a closed orbit shifted toward the outside of the synchrotron.

The top row displays the horizontal $1\sigma_{\text{rms}}$ beam envelopes of ${}^4\text{He}^{2+}$ (orange) and ${}^{12}\text{C}^{6+}$ (blue). The black-dashed line marks the radial loop setpoint. The diamond marker indicates the dispersive location ($D_x = -8.3$ m) at which a representative pickup is installed in this model. The envelope is computed as

$\sigma_{\text{rms}} = \sqrt{\beta_x \epsilon_{x,n,\text{rms}} / (\gamma\beta) + D_x^2 \delta_{\text{rms}}^2}$, where β_x denotes the Courant-Snyder β -function. Both ion species are assumed to have Gaussian momentum distributions with similar rms momentum spreads of $\delta_{\text{rms}} \approx 0.5 \times 10^{-3}$ [17]. For this illustration, both ion species are further modeled with Gaussian betatron distributions with a normalized rms emittance of $\epsilon_{x,n,\text{rms}} = 0.9 \mu\text{m}$ [21]. While in practice, the horizontal emittances of the helium and carbon distributions differ significantly when generating the mixed beam

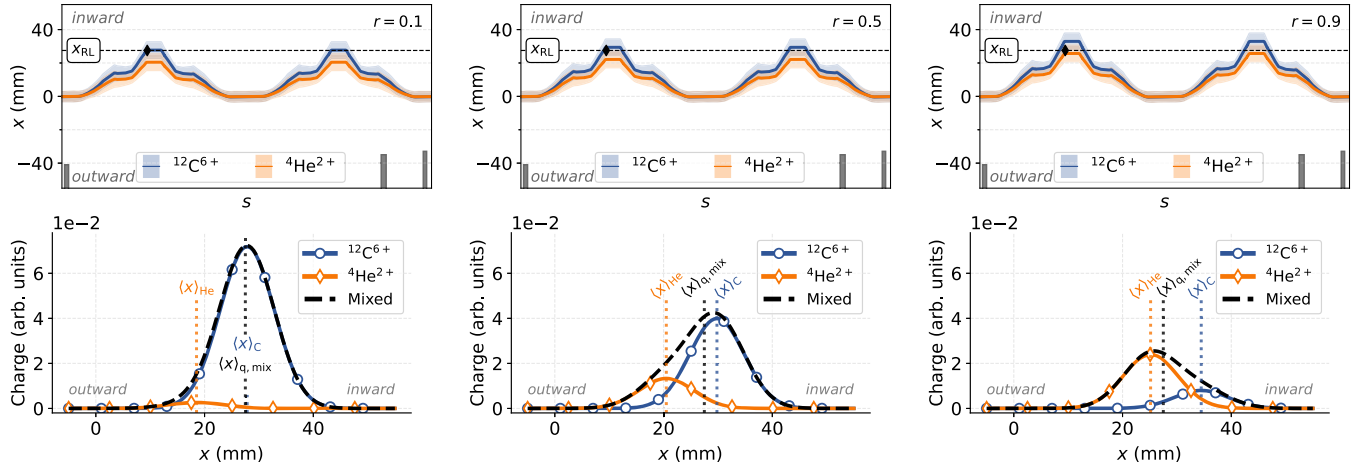


FIG. 2. Mixed-beam charge distributions for different helium fractions r . The radial loop (RL) regulates the horizontal center of gravity of the mixed charge distribution, $\langle x \rangle_{q,\text{mix}}$, to $x_{\text{RL}} = 27.5$ mm. Top: $1\sigma_{\text{rms}}$ beam envelopes along a PIMMS lattice. Bottom: horizontal charge distributions at the radial loop pickup with $D_x = -8.3$ m.

using a double multiturn injection scheme [4], this distinction is not relevant for the present analysis and is therefore omitted here for simplicity. The bottom row shows the modeled horizontal charge densities at the location of the radial loop pickup.

B. Radial loop regulation of mixed-species beams

The radial loop acts on the overall charge distribution (black-dashed line in the bottom row of Fig. 2) and applies a frequency correction $\Delta f(r) = f_{\text{mix}}(r) - f_{\text{ref}}$ to move $\langle x \rangle_{q,\text{mix}}$ to the specified radial loop setpoint. f_{mix} denotes the rf frequency of the mixed beam after regulation to the prescribed offset x_{RL} . f_{ref} is the corresponding reference frequency of a monoisotopic beam of the reference ion species (only species I, i.e., $r = 0$), which is acquired at the same radial loop setpoint. In the remaining section, the subscript “ref” is omitted when describing the reference beam, i.e., $f = f_{\text{ref}}$, $\gamma = \gamma_{\text{ref}}$, $\beta = \beta_{\text{ref}}$, $p = p_{\text{ref}}$, and $R = R_{\text{ref}}$.

After regulation, the displacement of the reference species I with respect to the monospecies reference position follows from Eq. (19) as

$$x_I(r) - x_{\text{RL}} = -A_r D_x \delta_{\text{eff},\text{sp}}. \quad (21)$$

This offset can be related to the induced momentum shift of species I, $\Delta p_I = p_I - p$, by

$$D_x \frac{\Delta p_I}{p} = -A_r D_x \delta_{\text{eff},\text{sp}}. \quad (22)$$

D_x cancels out, assuming it is approximately constant over the relevant range. Finally, the obtained $\Delta p_I/p$ can be related to the induced frequency deviation,

$$\frac{\Delta f}{f} = \eta \frac{\Delta p}{p} = -\eta A_r \delta_{\text{eff},\text{sp}}, \quad (23)$$

with the slippage factor $\eta = \frac{1}{\gamma^2} - \frac{1}{\gamma_{\text{tr}}^2}$. Rearranging with Eqs. (7) and (11) yields the relative frequency offset expected for a specific ion fraction r ,

$$\frac{\Delta f}{f} = -\frac{1}{\gamma^2} A_r \left(\frac{1}{\chi_{\text{sp}}} - 1 \right). \quad (24)$$

Equivalently, the ion fraction r can be obtained from a measured $\Delta f/f$ by rearranging Eq. (24), yielding

$$r = \frac{q_I \gamma^2 \frac{\Delta f}{f}}{\gamma^2 \frac{\Delta f}{f} (q_I - q_{\text{II}}) - q_{\text{II}} \left(\frac{1}{\chi_{\text{sp}}} - 1 \right)}. \quad (25)$$

Notably, Eqs. (24) and (25) remain independent of the machine layout, as the dependence on the momentum compaction factor cancels out in Eq. (23).

Alternatively, the situation can also be conceptualized by means of a virtual particle representing the charge-weighted center of gravity of the mixed-beam distribution. This virtual particle is described by the effective charge-to-mass ratio factor χ_v , which is obtained as a charge-weighted average of $\frac{1}{\chi} - 1$,

$$\frac{1}{\chi_v} - 1 = \frac{\left(\frac{1}{\chi_{\text{sp}}} - 1 \right) q_{\text{II}} r}{q_{\text{II}} r + q_I (1 - r)} \quad (26)$$

consistent with the fact that the change in dispersive offset is linear in $\frac{1}{\chi} - 1$, see Eq. (18). Using $\frac{1}{\chi} - 1 \approx 1 - \chi$, we obtain

$$\chi_v = \frac{\chi_{\text{sp}} q_{\text{II}} r + q_{\text{I}} (1 - r)}{q_{\text{II}} r + q_{\text{I}} (1 - r)}. \quad (27)$$

Also this approach requires that D_x is approximately constant over the relevant range.

The radial loop regulates the dispersive offset of this virtual particle to the same position as the monoisotopic reference beam. This means that the effective rigidity offset of the virtual particle with respect to the reference beam vanishes,

$$\delta_{\text{eff},v} = \frac{1}{\chi_v} - 1 + \delta_v = 0, \quad (28)$$

which requires the virtual particle to feature a relative offset in $\beta\gamma$ of

$$\delta_v = 1 - \frac{1}{\chi_v} \approx \chi_v - 1. \quad (29)$$

Since the radial loop aligns the center of gravity of the mixed beam with the reference orbit, the effective path length of the virtual particle equals that of the reference beam, i.e., $R_v = R$ in Eq. (9). Using Eq. (10), the relative frequency offset becomes

$$\frac{\Delta f}{f} = \frac{\beta_v}{\beta} \frac{R}{R_v} - 1 = \frac{1}{\gamma^2} \delta_v + \mathcal{O}(\delta_v^2). \quad (30)$$

The measured relative frequency shift $\Delta f/f$ can therefore be used to estimate the relative offset in $p/m = \beta\gamma c$ of the virtual particle,

$$\delta_v \approx \gamma^2 \frac{\Delta f}{f}. \quad (31)$$

With Eqs. (26) and (29), one obtains again Eqs. (24) and (25), as derived above. The independence from the machine layout stems in this approach from postulating $R_v = R$, which eliminates any effective path length difference for the virtual particle.

To provide an example, Fig. 3 illustrates the resulting relations for a mixed ${}^4\text{He}^{2+}$ and ${}^{12}\text{C}^{6+}$ beam in a PIMMS synchrotron lattice, with a pure ${}^{12}\text{C}^{6+}$ beam as the reference species. The example explores energies between 120 and 680 MeV/u and ion fractions between $r = 0$ (pure ${}^{12}\text{C}^{6+}$) and $r = 1$ (pure ${}^4\text{He}^{2+}$). While this example assumes on-momentum operation for simplicity, the relations remain valid for off-momentum operation.

Notably, it is evident in Fig. 3 (left and center panels) that the method is not applicable for $\gamma \gg 1$ as the induced frequency offset vanishes in accordance with the $1/\gamma^2$ dependence in Eq. (24). Additionally, we emphasize that the present formalism is not expected to apply in the vicinity of transition, which lies beyond the scope of this work.

To validate Eqs. (24) and (25), simulations are performed with XSUITE [22], which enables tracking of particles with non-nominal mass-to-charge ratios. Details are outlined in the Appendix. In these simulations, Δf is determined numerically by enforcing the charge-weighted average radial displacement $\langle x \rangle_{q,\text{mix}}$ to be x_{RL} . As a substitute for a radial feedback loop, the required frequency offset Δf is determined using the Nelder-Mead optimization algorithm [23], implemented in SCIPY [24]. The resulting frequency offsets and the corresponding radial displacements are shown as gray diamond markers in Fig. 3, in excellent agreement with the expected values.

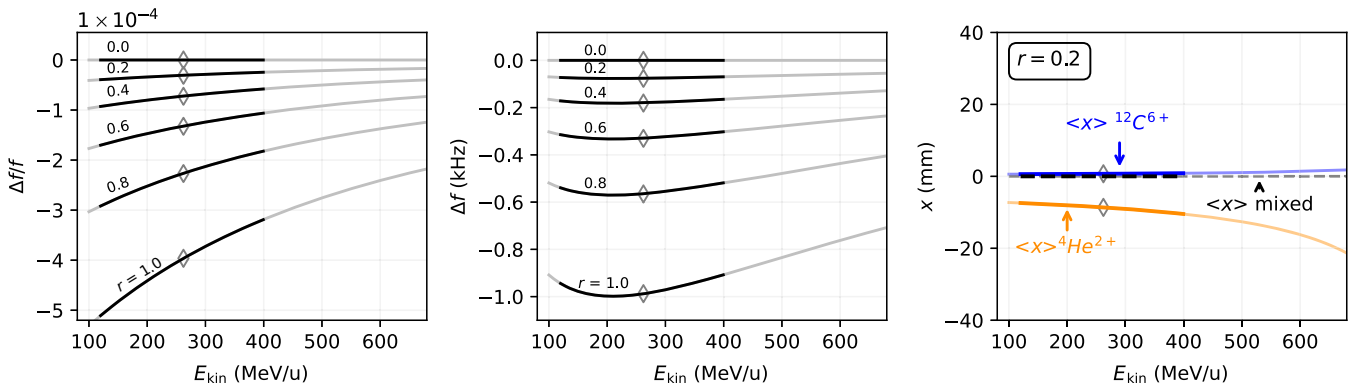


FIG. 3. Frequency and radial position offsets for mixed helium-carbon beams with different ion fractions r , estimated analytically (lines) and in simulations (gray diamond markers). The clinically relevant energy range is indicated by enhanced opacity. It should be noted that the present formalism is not expected to hold in the vicinity of transition energy (here $\gamma_{\text{tr}} \approx 2$). Left: relative frequency offsets with respect to a monoisotopic ${}^{12}\text{C}^{6+}$ beam. Center: absolute frequency offsets in a PIMMS synchrotron. Right: horizontal centroid of the mixed, carbon, and helium charge distributions at the radial loop pickup with $D_x = -8.3$ m. In this example, the radial loop regulates to $x_{\text{RL}} = 0$ mm.

IV. EXPERIMENTAL DEMONSTRATION

The proposed method for estimating the ion fraction of dual-species beams was experimentally tested in 2025 at the MedAustron center for ion beam therapy and research.

A. Setup and measurement campaign

For this purpose, the helium fraction of mixed $^4\text{He}^{2+}$ and $^{12}\text{C}^{6+}$ beams was determined both at flattop in the synchrotron (Sec. IV A 2) and independently after extraction in the research irradiation room (Sec. IV A 3). The setup is schematically illustrated in Fig. 4. Measurements were performed at two extraction energies, 262.3 and 402.8 MeV/u. For 262.3 MeV/u, measurements were carried out with two distinct radial loop setpoints, $x_{\text{RL}} = 27.5$ and 29.9 mm, to further verify the robustness of the measurement procedure. The radial loop offset $x_{\text{RL}} = 27.5$ mm represents the default setting at flattop in the employed extraction setup (Sec. IV A 1). The second setpoint, $x_{\text{RL}} = 29.9$ mm, was selected to be sufficiently close to the initial setpoint to avoid significantly affecting the dynamics during slow extraction. The specific value is not important for the conclusions presented here. It was chosen solely to reproduce the relative frequency offset used in previous tests at a different energy (data not shown). For each energy and radial loop configuration, the ion fraction was varied by means of adapting the sequential injection settings (Sec. IV A 1). For each setting, five measurements were taken.

1. Mixed-beam delivery

The mixed beams were produced using the double multiturn injection scheme described in Sec. II A. Different ion fractions were obtained by varying the amplitude of the second injection bump, which is used for the injection of $^{12}\text{C}^{6+}$ ions. As a full calibration of the bump amplitude was not available at the time of writing, the bump strength is specified in terms of the injection bump current. By adjusting the injection kicker current

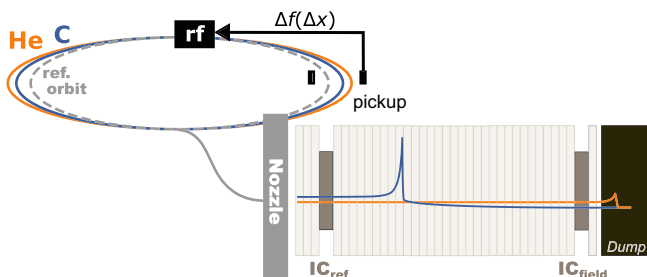


FIG. 4. Schematic of the two complementary methods to determine the ion composition of the dual-species beam: the noninvasive rf-based measurement in the synchrotron and the postextraction measurement with an ionization chamber (IC) telescope.

between $I_{\text{MKI}} = 350$ and 0 A, ion fractions ranging from $r = 0$ (pure $^{12}\text{C}^{6+}$) to $r = 1$ (pure $^4\text{He}^{2+}$) were achieved.

Slow extraction to the experimental room was performed by means of phase displacement acceleration [13,25,26], in which the beam is pushed into the third-order resonance by sweeping an empty rf bucket through a coasting beam stack. This extraction mechanism was used because the clinically employed betatron core is currently not compatible with mixed-species extraction, and the experimentally available rf knockout system was not available for the extraction at the time of the measurements [10].

For all measurements, the extraction parameters were chosen to extract the beam over ≈ 7 s while ensuring complete emptying of the synchrotron and avoiding saturation of the IC in the research irradiation room. For both beam energies, the empty rf bucket was empirically initialized at a frequency offset that did not induce beam losses upon application of rf voltage. A combination of rf voltage, i.e., bucket height, and frequency sweep speed was then empirically selected to empty the entire synchrotron over several seconds.

The extraction efficiency was estimated for pure $^{12}\text{C}^{6+}$ beams by comparing intensity measurements in the synchrotron with readings from the IC in the nozzle of the MedAustron research irradiation room. This yielded extraction efficiencies of $96.1 \pm 0.7\%$ for 262.3 MeV/u with a 27.5 mm radial loop offset, $95.6 \pm 0.6\%$ for 262.3 MeV/u with a 29.9 mm offset, and $96.0 \pm 0.2\%$ for 402.8 MeV/u with a 25 mm offset. However, operation in mixed-beam mode may lead to species-dependent changes in extraction efficiency relative to the reference measurements obtained with a pure carbon beam, which should therefore be taken into account when interpreting the results.

2. Measuring the ion composition in the synchrotron

Figure 5 shows representative low-level rf (LLRF) acquisitions used for the rf-based ion fraction quantification. As expected, the mixed $^4\text{He}^{2+}/^{12}\text{C}^{6+}$ beam (black) exhibits a frequency offset that lies between those of the two pure species (orange and blue). The frequency values required for the mixing-ratio quantification were obtained by averaging the acquired rf frequency signal over a fixed time interval during the flattop period prior to extraction (gray-shaded region in Fig. 5). In the presented measurements, these intervals were 18 ms for 262.3 MeV/u beams and 6 ms for 402.8 MeV/u beams. These time windows were selected as the longest intervals at the end of the LLRF acquisition during which the frequency averages remained unaffected by residual magnetic field settling. The intervals are relatively short for the data presented because the main dipole magnets were operated in current-regulated mode, and the magnetic field had not yet fully stabilized by the end of the LLRF acquisition.

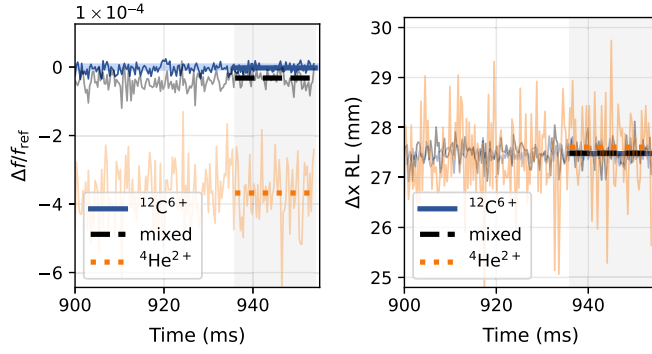


FIG. 5. LLRF acquisitions at flattop for 262.3 MeV/u $^{12}\text{C}^{6+}$, $^4\text{He}^{2+}$, and mixed beams ($r \approx 0.2$), with the radial loop regulating to $x_{\text{RL}} = 27.5$ mm. The gray-shaded time window is used to determine the average frequencies, shown by the solid lines. Left: relative rf frequency offset with respect to a pure $^{12}\text{C}^{6+}$ beam. The blue-shaded band marks the reference frequency $f_{\text{ref}} \pm 3\sigma_{f_{\text{ref}}}$. Right: position measured at the radial loop pickup.

For each configured energy and radial loop setpoint, the reference frequency f_{ref} [f in Eq. (24)] and corresponding standard error $\sigma_{f_{\text{ref}}}$ were determined at the beginning of the measurement campaign by averaging five independent acquisitions with a pure $^{12}\text{C}^{6+}$ beam. The obtained rms frequency spread is $\sigma_{f_{\text{ref}}} < 8$ Hz for all extraction energies and radial loop offsets, which is therefore adopted as common uncertainty for all cases. This spread is mainly attributed to noise in the rf frequency readout and to magnetic field fluctuations of the main dipole magnets. The blue-shaded band in the left subplot of Fig. 5 represents the resulting $^{12}\text{C}^{6+}$ reference frequency interval of $\pm 3\sigma_{f_{\text{ref}}}$.

For each subsequent mixed-beam measurement, the revolution frequency f_{mix} is measured, the corresponding frequency shift $\Delta f = f_{\text{mix}} - f_{\text{ref}}$ calculated, and the ion fraction r obtained via Eq. (25).

Figure 5 further shows that the frequency fluctuations are noticeably larger for the pure $^4\text{He}^{2+}$ beam than for the mixed and pure $^{12}\text{C}^{6+}$ beams. This is attributed to the lower helium beam current and to the gain setting of the pickup amplifier, which is optimized for $^{12}\text{C}^{6+}$ operation. This results in less stable regulation for beams with $r \rightarrow 1$. Nevertheless, the measurement quality is sufficient for the presented proof-of-principle demonstration. Importantly, the pickups installed in the MedAustron synchrotron are of the shoebox type, which provide a linear relationship between beam displacement and induced signal over a wide displacement range [19], as required for the presented model.

Representative parameter sets and the corresponding rms uncertainties used for error propagation are listed in Table I. The uncertainty of the extraction energy is conservatively assumed to be 3 MeV/u to account for non-nominal synchrotron operation in the mixed-beam setup. However, its effect is negligible, as the uncertainty in Δf is the dominant contribution to the uncertainty in r .

TABLE I. Parameters for the helium fraction quantification using the rf-based method in the synchrotron and the IC telescope in the irradiation room.

Parameter	Value	Standard error	Unit
263.2 MeV/u 27.5 mm radial offset			
f_{ref}	$2.405\,142 \times 10^6$	8	Hz
Δf	$\Delta f(r) \in [0, -1000]$	8	Hz
E_{kin}	263.2	3	MeV/u
a_{He}	1.54	4×10^{-3}	
a_{C}	0.01	2×10^{-5}	
ΔE_{He}	5.69×10^{-3}	5%	MeV
ΔE_{C}	5.45×10^{-2}	5%	MeV
402.8 MeV/u 25 mm radial offset			
f_{ref}	$2.757\,010 \times 10^6$	8	Hz
Δf	$\Delta f(r) \in [0, -900]$	8	Hz
E_{kin}	402.8	3	MeV/u
a_{He}	0.77	2×10^{-3}	
a_{C}	0.15	5×10^{-4}	
ΔE_{He}	4.53×10^{-3}	5%	MeV
ΔE_{C}	4.07×10^{-2}	5%	MeV

3. Measuring the ion composition of the extracted beam

Methodology. In the research irradiation room, a two-stage IC telescope was used to quantify the mixing ratio of the extracted beam. As illustrated in Fig. 4, the two ICs are positioned at different penetration depths, with the *reference chamber* IC_{ref} being located in the entrance plateau of the carbon Bragg peak, while the *field chamber* IC_{field} is located in between the $^{12}\text{C}^{6+}$ and the $^4\text{He}^{2+}$ Bragg peaks. The ion composition can be reconstructed from the ratio of the currents measured simultaneously on both ICs, I_{ref} and I_{field} [27].

The method relies on calibration factors that relate the average signal induced by a primary $^4\text{He}^{2+}$ or $^{12}\text{C}^{6+}$ ion, or by their fragments, in IC_{field} to the corresponding signal in IC_{ref}. These factors are determined in dedicated calibration measurements performed at the beginning of the experiment by irradiating the IC telescope with pure helium and pure carbon beams and computing

$$a_j = \frac{I_{\text{field},j}}{I_{\text{ref},j}} \quad j = \text{He and C.} \quad (32)$$

To first order, these calibration factors are independent of the incident beam intensity.

During measurements with the mixed-species beam, the total current in the two ICs is

$$\begin{aligned} I_{\text{ref}}(t) &= I_{\text{ref,He}}(t) + I_{\text{ref,C}}(t) \\ I_{\text{field}}(t) &= I_{\text{field,He}}(t) + I_{\text{field,C}}(t), \end{aligned} \quad (33)$$

where t describes the time during the ≈ 10 s spill. Using the calibration factors defined in Eq. (32), the current in the field IC can be written as

$$I_{\text{field}}(t) = a_{\text{He}} I_{\text{ref,He}}(t) + a_{\text{C}} I_{\text{ref,C}}(t). \quad (34)$$

Solving this system of equations yields

$$\begin{aligned} I_{\text{ref,He}}(t) &= \frac{I_{\text{field}}(t) - a_{\text{C}} I_{\text{ref,C}}(t)}{a_{\text{He}} - a_{\text{C}}} \\ I_{\text{ref,C}}(t) &= \frac{a_{\text{He}} I_{\text{ref,He}}(t) - I_{\text{field}}(t)}{a_{\text{He}} - a_{\text{C}}}. \end{aligned} \quad (35)$$

The integrated reference IC current for each species j over the spill duration T is

$$I_{\text{ref},j} = \sum_{i=0}^{T/\Delta t} I_{\text{ref},j}(t_i) \Delta t, \quad (36)$$

where Δt is the sampling time of the electrometer. The fraction of extracted ions per species is subsequently obtained from the ratio of these integrated currents and the ratio $\Delta E_{\text{He}}/\Delta E_{\text{C}}$ of the energy deposited in the detector volume per primary ${}^4\text{He}^{2+}$ and ${}^{12}\text{C}^{6+}$ ion for the given experimental configuration,

$$\frac{N_{\text{He}}}{N_{\text{C}}} = \frac{I_{\text{ref,He}}}{I_{\text{ref,C}}} \frac{\Delta E_{\text{C}}}{\Delta E_{\text{He}}}. \quad (37)$$

The ion fraction r then follows as

$$r = \frac{\frac{N_{\text{He}}}{N_{\text{C}}}}{\frac{N_{\text{He}}}{N_{\text{C}}} + 1}. \quad (38)$$

$\Delta E_{\text{He}}/\Delta E_{\text{C}}$ is obtained from OPENGATE [28] simulations, with details described below.

Details on the experimental setup. The presented experiment employed PTW 34070 ICs, separated in range by multiple PTW RW3 absorber plates (PTW Freiburg GmbH, Freiburg, Germany). The reference chamber was positioned after 3 plates, with another 33 plates installed in between reference and field IC. Each plate has a physical thickness of 1 cm. Their water equivalent thickness has been characterized with protons and is available internally at the MedAustron facility. Both ICs fully covered the entire beam, which was verified using GafchromicTM EBT3 radiochromic films (Ashland Inc., Wayne, NJ, USA) upstream and downstream of the IC telescope. The chosen configuration assured that the carbon Bragg peak position (around 14 cm for 262.3 MeV/u and 28 cm for 402.8 MeV/u) was in between the two ICs for both energies while the ${}^4\text{He}^{2+}$ Bragg peak (approximately 41 cm for 262.3 MeV/u and 83 cm for 402.8 MeV/u) was located after the field IC. The ICs were read out with a

50 ms integration time using a PTW TANDEM dual channel electrometer.

The setup-specific parameters and the corresponding rms uncertainties used for error propagation are summarized in Table I. The calibration factors a_{He} and a_{C} were obtained by averaging five acquisitions taken at the beginning of the measurement campaign. To obtain ΔE_{He} and ΔE_{C} , a simplified model of the setup was simulated in OPENGATE (version 10.0.3 using GEANT4 [29] version 11.2.3). The PTW RW3 slabs and the ICs were implemented explicitly according to their material composition and internally available data for their water equivalent thickness. The active region of the ICs was modeled as 2 mm air-filled volumes with energy deposition scored over the full sensitive detector thickness and cross section. The simulations employed the GEANT4 SHIELDING_EMZ physics list [30], which provides stopping-power data based on the ICRU Report 90 [31] for protons, alpha particles, and heavy ions in water, graphite, and air. Given the high energies of the helium and carbon ions and the low fragmentation upstream of the reference IC, the ICRU90 stopping-power data are expected to dominate the energy-loss modeling in the detector volumes. This study assumes an rms error of 5% for the estimate on the deposited energy ΔE . On the one hand, this value contains systematic uncertainties related to stopping-power data, which are assumed to be $\pm 2.5\%$ in accordance with recent comparisons of stopping powers obtained with different GEANT4 physics lists [32]. Further, the error budget accounts for statistical simulation uncertainties ($< 1\%$) and systematic errors in the simulation setup. A more precise quantification of these uncertainties is subject to future studies and goes beyond the presented proof of principle.

4. Data analysis

To quantify potential systematic differences between the two independent ion fraction estimates, a simple phenomenological model is introduced. For each method, the inferred helium fraction is written as

$$r_i = \frac{\lambda_i N_{\text{He}}}{\lambda_i N_{\text{He}} + N_{\text{C}}}, \quad i = \text{ic, rf}, \quad (39)$$

where N_{He} and N_{C} denote unknown ground truth, i.e., the intensities prior to both measurements. λ_i accounts for method-specific response differences, such as species-dependent quantification or extraction efficiencies.

Eliminating N_{He} and N_{C} yields a direct relation between the ion fraction

$$\kappa = \frac{\lambda_{\text{rf}}}{\lambda_{\text{ic}}}, \quad (40)$$

the expected relation reads

$$\hat{r}_{\text{rf}}(r_{\text{ic}}) = \frac{\kappa r_{\text{ic}}}{1 - r_{\text{ic}} + \kappa r_{\text{ic}}}. \quad (41)$$

In the presented measurements, the parameter κ was determined by ordinary least-squares regression. Pure-species points ($r = 0$, $r = 1$) are, by design, detected as such by the IC setup, independent of detection and extraction efficiencies. These data points were excluded from the fit. The fitted parameter κ represents an effective relative response between the two methods, incorporating differences in detection and extraction efficiencies for the presented measurement setup. The adequacy of this model was assessed using the residuals $r_{\text{rf}} - \hat{r}_{\text{rf}}(r_{\text{ic}})$.

B. Experimental results

The ion fractions r estimated using the two different methods are compared in Fig. 6. The left panels show a direct comparison between the ion fraction of the extracted beam measured in the research irradiation room

(x axis, r_{ic}) and the noninvasive composition estimate obtained in the synchrotron (y axis, r_{rf}). The colored markers correspond to different injection settings used to tailor the ion fraction.

For a fixed injection setting, i.e., one color in Fig. 6, the measured ion fractions exhibit significant fluctuations in selected cases. The fact that these variations are consistently reflected in both methods indicates that these are real shot-to-shot fluctuations of the beam composition rather than measurement artifacts.

Importantly, the pure-species limit $r = 1$, which is intrinsically identified as such by the IC-based measurement, is also recovered as $r \approx 1$ by the rf-based method. This agreement provides a strong consistency check of the underlying model, as it follows directly from the calculated frequency offset rather than from any interpolation.

At intermediate ion fractions, deviations between the two methods are observed. The fit of the model in Eq. (41) yields a value of $\kappa \approx 0.75$ to 0.83 for the different

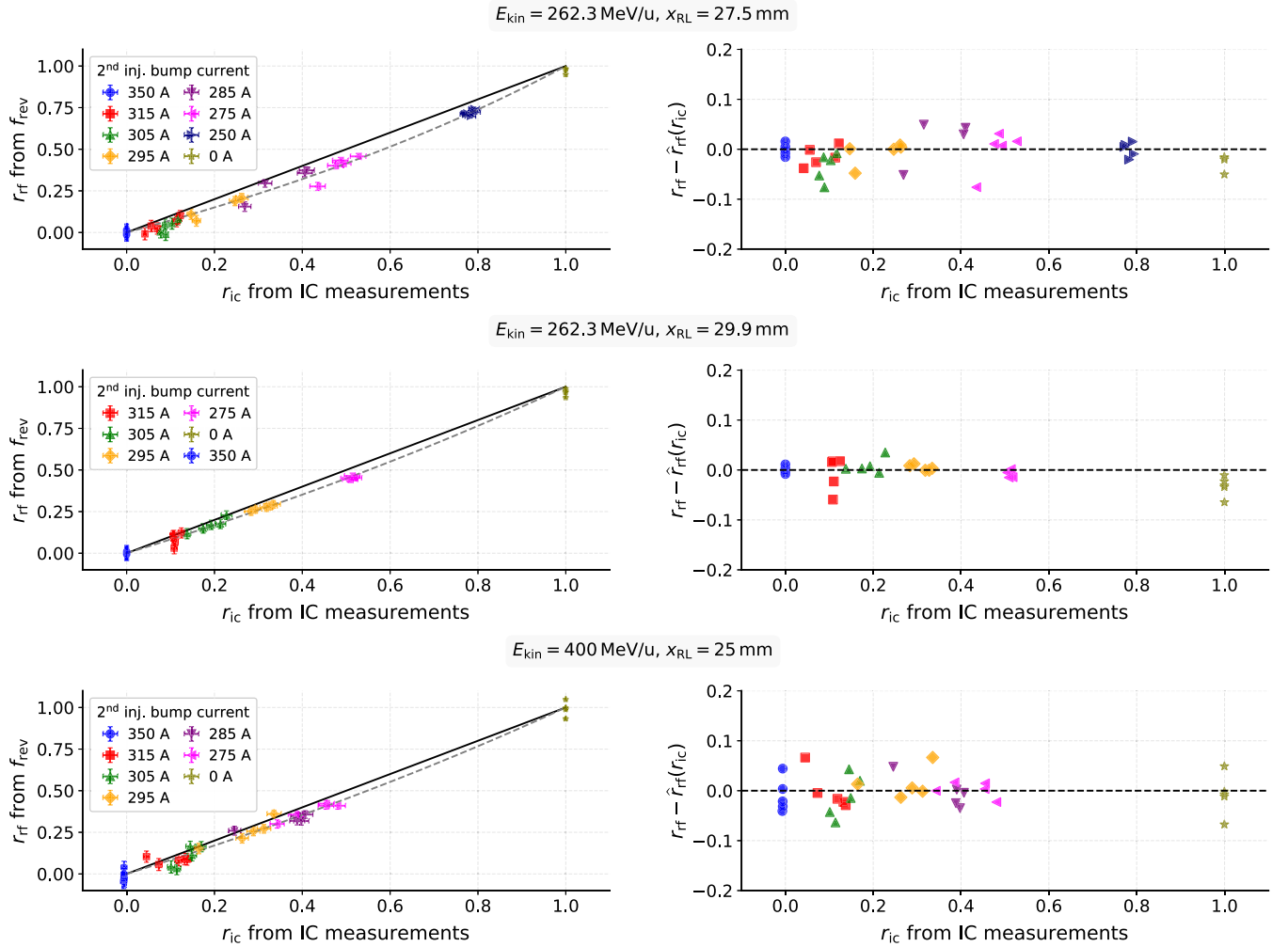


FIG. 6. Left: comparison of the ion fraction obtained with the IC telescope in the irradiation room (x axis) and the noninvasive rf-based quantification in the synchrotron (y axis) for different beam energies and radial loop offsets. The black solid line indicates the identity $r_{\text{ic}} = r_{\text{rf}}$, the gray dashed line shows the linear fit based on the efficiency model. Right: residuals of the fit.

measurement sets. This indicates a difference in the relative response to the initially present $^4\text{He}^{2+}$ and $^{12}\text{C}^{6+}$ between the two methods, which may arise from species-dependent detection efficiencies and/or differences in extraction efficiency. The deviations are largest at intermediate ion fractions ($r \approx 0.5$), where $\kappa \approx 0.8$ corresponds to the in-synchrotron measurement yielding a helium fraction that is approximately up to 6% lower than that inferred from the ICs in the experimental room. However, as the present results serve as a proof of principle, the regression is used only to assess systematic trends. A detailed quantitative and physical discussion of the fit parameters is deferred to future studies.

The residuals of the fitted model are shown in the right panels. For all investigated configurations, the residuals are centered around zero and remain within approximately $\pm 7\%$, with no significant systematic trends observed over the full range. This supports the validity of the model parameterized by κ , Eq. (41).

V. DISCUSSION AND OUTLOOK

A. Method applicability and limitations

The presented results demonstrate the feasibility of noninvasively estimating the ion composition of dual-species beams directly in the synchrotron, with good agreement between the rf-based method and the IC telescope. While the current implementation is not optimized for high-precision measurements, it already provides significant practical value and is regularly used for mixed-beam commissioning. Further, for experiments where the installation of an IC telescope was incompatible with the experimental setup, the presented noninvasive approach enabled cycle-resolved composition monitoring without interfering with beam delivery.

Looking ahead to potential clinical implementation, the results indicate that this measurement principle could possibly be used to monitor the ion composition during treatment and generate interlocks in case of unacceptable mixing ratios, similar to an energy verification system. Further studies are recommended to assess the feasibility of this approach and investigate concepts for directly integrating the ion fraction determination into the LLRF system. For these studies, input from medical physics research groups is needed to define the required accuracy and precision, which are expected to be more stringent than those currently achieved.

The achievable accuracy and precision of the rf-based method, as well as their evaluation, are currently limited by several factors. Regarding accuracy, an assessment remains challenging due to the absence of a ground truth. So far, the helium fraction estimated via the rf-based method could only be compared with the IC measurement in the irradiation room. However, this comparison may be affected by differences in extraction efficiency between the

two ion species, as well as uncertainties in the IC-based reconstruction, both of which require further investigation.

Regarding precision, the key constraints in this study were fluctuations in the acquired frequencies, with $\frac{\Delta f}{f}$ at the $\mathcal{O}(10^{-6})$ level. These fluctuations are attributed mainly to noise in the LLRF acquisition system and cycle-to-cycle magnetic field fluctuations [33].

The LLRF acquisition noise is expected to be reducible by extending the sampling duration. For example, averaging over several tens of milliseconds instead of just a few milliseconds is expected to reduce the rms error due to LLRF noise by a factor of 2–3, following the dependence $\sigma_{\text{rms}} \propto 1/\sqrt{n_s}$ on the sample size n_s . If simple averaging is applied to retrieve the frequency, longer sampling times are only feasible if the magnetic field remains stable during the measurement period. In the presently employed current-regulated magnetic field control, this requires extending the flattop duration by ≈ 2 s to ensure that the field is fully settled prior to the measurement [34]. This is a straightforward mitigation measure for the research setup and is therefore foreseen for future mixed-beam campaigns. However, it is not suitable for potential clinical application, where significantly prolonged cycle times are unacceptable. Alternatively, longer sampling times without an extended flattop could be enabled by fitting a smooth model function to the time evolution of the drifting frequency or by operating the main dipoles in B -field-regulated mode [34].

The second factor that limits measurement precision is cycle-to-cycle B -field fluctuations, which have been assessed to be $\frac{\Delta B}{B} \lesssim 10^{-5}$ for both current- and B -field-regulated modes [33,34]. Using the common differential relations between B -field, frequency, and mean orbit radius [35], this corresponds to a frequency uncertainty $\frac{\Delta f}{f}$ at the $\mathcal{O}(10^{-6})$ level. While these fluctuations therefore clearly limit the precision of the proposed method, they are below the tolerances required for nominal clinical operation, and significant further reduction is not expected in current facilities.

Generally, when aiming to increase precision, the regime with $r \approx 0.1$ to 0.4 should be prioritized, as this is the range of helium fractions most commonly requested by experiments. The reduced signal-to-noise ratio in the LLRF acquisitions for high helium fractions ($r \rightarrow 1$, orange in Fig. 5), which arises because the gain settings are optimized for carbon, is therefore not considered a significant issue within the relevant operational range.

B. Operational implications of radial loop regulation for mixed-species beams

The frequency correction applied by the radial loop for a given ion composition, Eq. (24), induces a composition-dependent shift in $\beta\gamma$ and, consequently, also in the flattop energy of the reference ion species used for tumor

treatment. When the beam is extracted using amplitude-driven slow extraction (rf knock-out), this energy offset is also maintained during extraction and results in a corresponding range offset.

For a PIMMS-like synchrotron and mixed $^4\text{He}^{2+}$ and $^{12}\text{C}^{6+}$ beams with ion fraction $r = 0.2$, preliminary estimates indicate a range shift of <0.1 mm across the entire clinical energy range. At $r = 0.4$, for example, the range shift would reach <0.2 mm. These values confirm that range shifts for expected clinical mixing ratios ($r = 0.1$ to 0.2) fall well within reported clinical range tolerances, e.g., 0.3 mm in Refs. [21,36]. Nevertheless, further quantification of this effect, experimental validation, and assessment of its implications for energy verification systems [37] are recommended for future studies.

VI. CONCLUSION

We have demonstrated a practical, noninvasive method to determine the ion composition of dual-species beams with relative mass-to-charge offsets of order 10^{-4} in low- and medium-energy ion synchrotrons with radial rf regulation loops. An analytical model was derived that relates rf frequency offsets to ion fractions. The method was validated with XSUITE tracking simulations and demonstrated experimentally at MedAustron, where the rf-based mixing ratio estimates were found to be in good agreement with independent ion composition estimates of the extracted beam.

The method is now used at MedAustron for mixed-beam commissioning and diagnostics during user experiments. It offers potential for developing a mixing ratio interlock system, with further improvements expected under optimized measurement conditions.

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E. R.: conceptualization, methodology, investigation, formal analysis, writing—original draft. M. K.: methodology, investigation, and writing—review and editing. D. O.: methodology and writing—review and editing. H. F.: methodology. K. H.: investigation and writing—review and editing. N. O.: investigation, formal analysis, and writing—review and editing.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX: VALIDATION WITH XSUITE SIMULATIONS

Equations (24) and (25) are validated using the beam dynamics simulation framework XSUITE [22], which allows for tracking of particles with non-nominal m/q . The simulations employed `xtrack` v0.89.4 and are used to numerically determine the revolution frequency offset Δf , which is required to regulate the charge-weighted radial displacement $\langle x \rangle_{q,\text{mix}}$ to a specified x_{RL} for mixed $^4\text{He}^{2+}$ and $^{12}\text{C}^{6+}$ beams with ion fractions $r = 0$ to 1 . The simulations are prepared by initializing a generic PIMMS lattice with an on-momentum $^{12}\text{C}^{6+}$ as the reference particle and $E_{\text{kin}} = 262.3$ MeV/u as reference energy. This reference particle moves on a closed orbit with length C , and features the revolution frequency f and the relativistic factors β and γ .

As a substitute for an explicit radial feedback loop, the required rf frequency offset Δf is determined using the Nelder-Mead optimization algorithm [23], as implemented in `SCIPY` [24]. The algorithm varies Δf to minimize the cost function

$$\mathcal{C} = |\langle x \rangle_{q,\text{mix}} - x_{\text{RL}}|. \quad (\text{A1})$$

Each iteration of the optimization proceeds as follows. (i) *Application of an rffrequency shift*: In XSUITE, applying a shifted frequency $f_{\text{RF}} = f + \Delta f$ not only implies that this value is assigned as the new frequency to the `xtrack.Cavity` element but also that a corresponding shift in the longitudinal coordinate ζ ,

$$\Delta \zeta = C \frac{\Delta f}{f}, \quad (\text{A2})$$

is assigned via the `xtrack.ZetaShift` element. The latter is the standard method for programming off-momentum operation in XSUITE [38].

(ii) *Initialization of a $^{12}\text{C}^{6+}$ test particle close to the synchronous $^{12}\text{C}^{6+}$ particle:* The test particle is initialized with the longitudinal phase space coordinates $(\zeta_{\text{C}}, \delta_{\text{C}}) = (0, \frac{\Delta f/f}{\eta})$, with η being the linear slippage factor. Due to numerical and linearization errors, the test particle is initialized close to, but not exactly on, the phase space coordinates of the synchronous particle and therefore performs small synchrotron oscillations around it. Since the test particle will be tracked over multiple synchrotron periods in step (v), these small initial offsets will not affect the averaged solution.

(iii) *Initialization of a $^4\text{He}^{2+}$ test particle close to the synchronous $^4\text{He}^{2+}$ particle:* The initial momentum deviation of $^{12}\text{C}^{6+}$, δ_{C} , is then used to compute γ_{C} , β_{C} and eventually the offset δ_{sp} between $^4\text{He}^{2+}$ and $^{12}\text{C}^{6+}$ with Eq. (11). Subsequently β_{He} , obtained via Eq. (10), is used to determine the momentum deviation δ_{He} of $^4\text{He}^{2+}$ with respect to the lattice reference particle. The $^4\text{He}^{2+}$ test particle is initialized with the longitudinal phase space coordinates $(\zeta_{\text{He}}, \delta_{\text{He}}) = (0, \delta_{\text{He}})$.

(iv) *Tracking and estimation of radial loop offset:* Both test particles are tracked over multiple synchrotron periods. The average radial displacements at the radial loop pickup, $\langle x \rangle_{\text{C}}$ and $\langle x \rangle_{\text{He}}$, are obtained by averaging the respective offsets over multiple synchrotron periods, thereby averaging out radial oscillations arising from small-amplitude synchrotron motion. For a given ion fraction r , $\langle x \rangle_{\text{q,mix}}$ is subsequently computed using Eq. (19).

(v) *Computation of cost function:* The cost function $\mathcal{C}$, Eq. (A1), is evaluated and returned to the optimization algorithm, which iteratively updates Δf until convergence is achieved.

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