

Dynamic phase-driven inverse Smith-Purcell dielectric laser accelerator on a chip

Minghao Liu¹ and Weihao Liu^{1*}

*College of Electronic and Information Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing, Jiangsu, 211106, China
and Key Laboratory of Radar Imaging and Microwave Photonics (Nanjing University of Aeronautics and Astronautics), Ministry of Education, Nanjing, China*

Shengguang Liu

Key Laboratory for Laser Plasmas (Ministry of Education), Shanghai Jiao Tong University, Shanghai 200240, China



(Received 11 March 2025; accepted 4 February 2026; published 4 March 2026)

Pursuing on-chip particle accelerators has long been a focal point for researchers, and the inverse Smith-Purcell dielectric laser accelerator holds great promise as a viable option. However, achieving the cascade acceleration and focusing of subrelativistic electrons on a chip remains one of the most formidable challenges. In this paper, we propose a dynamic phase method to address this issue. This method takes advantage of the phase slippage between the electron and the acceleration wave, enabling transverse alternate focusing, defocusing, and continuous acceleration. Simulation results show that a 29.50 keV electron beam with a 20 eV spread can achieve an acceleration gradient of 39 MV/m with a captured efficiency of 95% within a 120- μm distance in a 200-nm-wide channel. Compared with the alternating phase method, this approach streamlines the fabrication process by removing the need for incremental adjustments of the phase velocity at each accelerating stage to match the electron energy. We anticipate that applying the dynamic phase-driven approach will significantly propel the development of on-chip accelerators.

DOI: [10.1103/4bmd-mrc6](https://doi.org/10.1103/4bmd-mrc6)

I. INTRODUCTION

In the realm of modern physics and advanced technology, on-chip particle accelerators [1–3] have emerged as a subject of intense research and development. Their significance lies in the potential to miniaturize the large-scale, costly particle acceleration systems, which are currently the cornerstone of numerous scientific and industrial applications [4–6]. These on-chip counterparts could bring about a paradigm shift, enabling compact, energy-efficient, and accessible particle acceleration for a wide range of fields, from fundamental high-energy physics research to applications in medical diagnostics and treatment [7–9].

Among the various approaches to realize on-chip particle acceleration, dielectric laser accelerators (DLAs) [10–13] have garnered substantial attention. DLAs based on either the inverse Smith-Purcell effect or the inverse Cherenkov

effect have shown great promise. The inverse Smith-Purcell DLA [14–18], for instance, uses the interaction between an electron beam and the periodic structure of a dielectric grating to generate an accelerating electromagnetic field. Similarly, the inverse Cherenkov DLA [19–22] relies on the phase-matching condition between the electron velocity and the electromagnetic wave along a dielectric surface to achieve acceleration. These DLA concepts offer the advantages of high accelerating gradients, compatibility with semiconductor manufacturing processes, and the potential for high repetition rate operation, making them highly attractive for on-chip implementation.

However, a major challenge in the development of DLAs for on-chip applications is the cascade acceleration and focusing of subrelativistic electrons on a chip. To achieve efficient acceleration and precise control of the electron beam, it is crucial to not only continuously increase the electron energy but also maintain the beam's quality through proper focusing. Over the years, several methods have been proposed and demonstrated to address this challenge. The alternate phase focusing method [23–28], for example, has been widely explored. It alternates the phase of the accelerating field to provide both focusing and defocusing forces periodically, aiming to balance the

*Contact author: liuwhao@nuaa.edu.cn

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](https://creativecommons.org/licenses/by/4.0/). Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

transverse and longitudinal dynamics of the electron beam. Nevertheless, the implementation complexity is considerably increased by the requirement to progressively modify the phase velocity at each accelerating stage and to account for the phase discontinuity between successive stages.

In this paper, we introduce a concept of dynamic phase focusing and acceleration for subrelativistic electrons in the ISP-DLA scheme. Our approach capitalizes on the phase slippage between the subrelativistic electrons and the acceleration waves. By carefully controlling this phase slippage, we can achieve transverse alternate focusing and defocusing, along with continuous acceleration. This dynamic phase-driven mechanism [29], which is similar to phase stability in rf accelerators [30], offers a unique way to manipulate the electron beam's behavior on a chip, potentially overcoming the limitations of previous methods. As will be shown through simulations and theoretical analysis, this new approach has the potential to significantly advance the development of on-chip particle accelerators, opening up new possibilities for high-performance, compact acceleration systems.

II. MODEL DESCRIPTION

The proposed scheme is shown in Fig. 1(a), employing a pair of symmetric p-polarized ultrafast lasers with an angular frequency ω_0 to excite the n th spatial harmonics in the multistaged dual pillar structure [31] through the inverse Smith-Purcell effect. For ISP-DLA, the

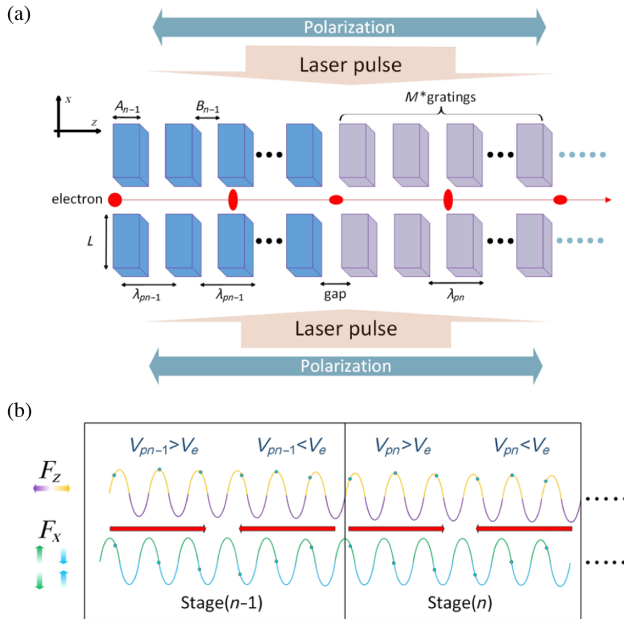


FIG. 1. (a) Schematic diagram of the proposed cascade ISP-DLA. Here $\lambda_{pn} = A_n + B_n$, $\text{gap} = \frac{1}{2}B_{n-1} + \frac{1}{2}B_n$, and M are the periods of each accelerating stage. (b) The relative positions of the particle concerning the spatial harmonics in the accelerating process. The red arrows indicate the direction of phase slippage between the particle and the spatial harmonics.

phase velocity at each stage of the scheme is $V_{pn} = \omega_0 \lambda_{pn} / (2\pi m)$. Here λ_{pn} is the period length of each grating stage, and m represents the order of the spatial harmonic. We select the first order of spatial harmonics ($m = 1$) as the accelerating mode to enhance the acceleration efficiency. Therefore, the synchronization requirement is defined as follows: $\lambda_{pn} = \beta_n \lambda$, where λ represents the wavelength of the incident laser. The gap between the pillars in two consecutive stages is designed as the average value of the gaps of the two stages.

Figure 1(b) shows the acceleration process of dynamic phase-driven synchronization. In this scheme, the phase velocity of each grating stage is adjusted to exceed the initial velocity of the electron while remaining below its final velocity in the acceleration region, with a progressive increase in phase velocity between stages. Upon entering the accelerating stage, the electron will undergo acceleration while trailing the initial phase. When the particle's velocity exceeds that of the spatial harmonics, it will subsequently move in the opposite direction, overshooting the current phase. The particle experiences a cyclical process of departing from and returning to its original phase, remaining in the accelerating phase throughout each acceleration stage due to the disparity in phase velocity between the spatial harmonics and the particle.

When the electron is in the accelerating phase, the Lorentz force exerted on the electrons is as follows [28]:

$$F = qG \begin{pmatrix} \frac{1}{\gamma} \sinh(k_x x) \cos \varphi_a \\ 0 \\ \cosh(k_x x) \sin \varphi_a \end{pmatrix} \quad (1)$$

in which G is the excitation coefficient of the dynamic-synchronous mode, q represents the electric charge of the particle, $k_x = 2\pi/(\beta\gamma\lambda)$, φ_a is the accelerating phase, and $\gamma = \sqrt{1/(1-\beta^2)}$ is the Lorentz factor.

In Eq. (1), the transverse force acting on the electron equals zero when the longitudinal force reaches its maximum at the accelerating phase $-(\pi/2)$. Besides, the direction of the transverse force changes when the phase is on either side of $-(\pi/2)$.

Then, the particle dynamic equation of the accelerated electron [29] is as follows:

$$\begin{aligned} \frac{dw}{dz} &= F_z = qG \cosh(k_x x) \sin \varphi_s \\ \varphi_s &= \delta_\varphi(z) + \varphi_0 \\ \delta_\varphi(z) &= \int_0^z \left(\frac{\omega}{v_{pn}} - \frac{\omega}{v_e(z)} \right) dz \end{aligned} \quad (2)$$

in which φ_0 denotes the initial accelerating phase of the particle and w represents the electron energy.

As shown in Fig. 1(b), the phase experienced by the particle varies periodically throughout the acceleration

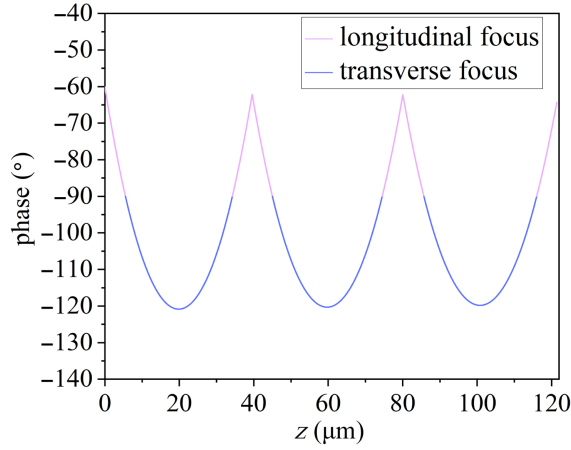


FIG. 2. Theoretically designed phase of dynamic phase-driven DLA.

process, a feature that can be exploited to achieve particle bunching. According to Earnshaw's theorem [32,33], an electron cannot be simultaneously confined in all three spatial dimensions, implying that horizontal confinement inevitably results in longitudinal dispersion. To address this, particles must be periodically subjected to alternating transverse defocusing and focusing phases, ensuring confinement in both transverse and longitudinal dimensions. In the proposed scheme, the initial phase is set within the defocusing zone, allowing particles to alternately experience defocusing and focusing forces, which collectively promote particle clustering. The design assumes the absence of phase discontinuities between successive grating stages. By leveraging the phase slippage of the electron and employing Eq. (2), the periodic length and the number of periods for each grating stage are determined to enable continuous acceleration and effective bunching. The phase design curve is presented in Fig. 2.

III. SIMULATION

In this section, we first derive the frequency-domain electromagnetic field within the acceleration channel by applying a discrete Fourier transform to the time-domain results obtained from the finite-difference time-domain mode in Lumerical. The grating's dielectric is selected as silicon due to its established fabrication process and elevated breakdown threshold. Due to the uniformity of the pillar structure along the y axis, we conduct a two-dimensional simulation on the x - z plane. The symmetrically stimulating lasers possess a center wavelength of $2\text{ }\mu\text{m}$ and a peak electric field strength of 0.3 GV/m . In the simulation, we set periodic boundaries on the z axis and perfectly matched layers on the x axis. The simulated frequency-domain data are subsequently entered into the general particle tracer to model particle dynamics in the time domain. We set the initial electron energy at 29.5 keV , with an energy spread of 0.02 keV , following a Gaussian

TABLE I. Simulation parameters.

Symbol	Physical quantity	Value
L	Length of the grating	$1.89\text{ }\mu\text{m}$
C	Channel width	$0.2\text{ }\mu\text{m}$
M	Periods of each accelerating stage	60
E_k	Initial centered particle energy	29.50 keV
Q	Initial bunch charge	0.004 fC
ε	Initial rms transverse emittance	0.2 pm rad
σ_s	Bunch length	10 nm
λ	Wavelength of the laser	$2\text{ }\mu\text{m}$
E_0	Amplitude of the incident laser	0.3 GV/m
n	Refractive index	1.528
L_z	Longitudinal length of bunch channel	$120\text{ }\mu\text{m}$

distribution in the x direction and a uniform distribution in the y and z directions, and the initial y -spread is set to be $6\text{ }\mu\text{m}$. The phase-space distribution of the bunch is presented in Fig. 5(c). In the particle-tracking simulation, we account for three-dimensional space-charge effects and set the number of macroparticles to 1000 to improve simulation accuracy. The specific simulation parameters are presented in Table I.

The phase-engineered curve of the proposed scheme is illustrated in Fig. 2. In this scheme, the configuration consists of three stages of accelerating gratings, with the phase velocities of the successive stages set to 30.3 , 31.81 , and 33.37 keV , respectively. To streamline the simulation design, the grating periods for all stages (M) are uniformly set to 60. The initial accelerating phase is set to be 60° , and it will slip to 120° and return to 60° due to the mismatch of the phase velocity. The particle undergoes periodic transitions between transverse defocusing and focusing phases while remaining confined within the acceleration zone. This dynamic ensures simultaneous transverse and longitudinal confinement, consistent with the constraints imposed by Earnshaw's theorem.

Figure 3 presents the time-domain electric field distribution within the bunch channel. In each acceleration stage, the longitudinal electric field E_z maintains a constant phase velocity, which increases incrementally between successive stages. Particles in the transverse direction experience nonuniform electric field strengths, with those farther from the channel center deviating more significantly from the intended phase trajectory—an effect particularly pronounced for low-energy particles. The frequency-domain electric field distribution is depicted in Fig. 4. As can be seen from the enlarged view in Fig. 4(a), the phase difference between individual acceleration stages is nearly insignificant across the entire structure.

Figure 5(a) compares the energy gain curve of a single electron obtained from simulation with the theoretical design. The simulation results show excellent agreement with the theoretical predictions. In the dynamic phase-driven synchronization mechanism, the acceleration

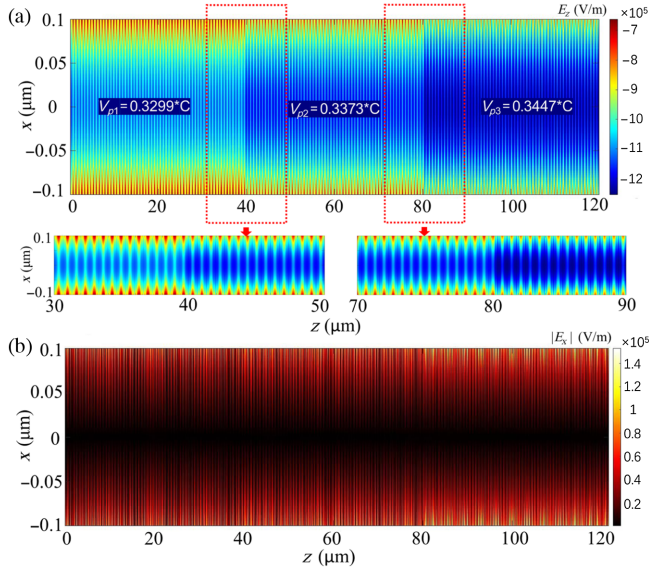


FIG. 3. Simulated contour maps of the time-domain longitudinal E_z (a) and transverse $|E_x|$ (b) electric fields.

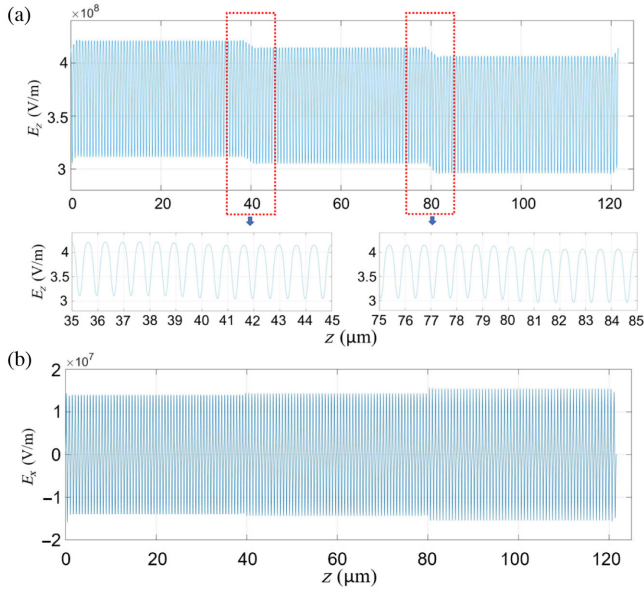


FIG. 4. Simulated real part of the frequency-domain electric field distribution at $x = 20$ nm.

gradient remains constant even when a discrepancy exists between the phase velocity and the electron energy. Within the proposed scheme, a single particle is continuously accelerated from 29.53 to 34.2 keV, achieving an acceleration gradient of 39 MV/m. Figure 5(b) illustrates the variation of β_x during the transmission of a single particle. The particle is observed to periodically traverse regions of transverse defocusing and focusing, enabling confinement of the entire bunch in both transverse and longitudinal dimensions. Figure 5(c) shows that for the single electron, it is captured within a specific phase-energy range across the

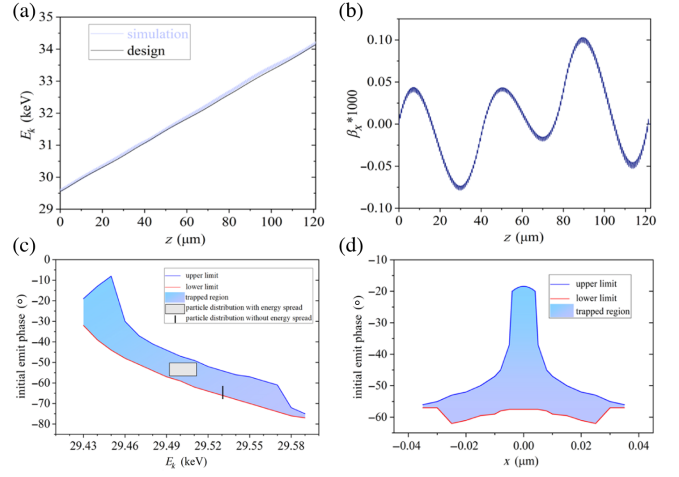


FIG. 5. (a) Simulated and designed particle energy versus the transmission length. (b) Simulated transverse velocity component β_x of a single particle emitted at $x = 10$ nm versus the transmission length. (c) Simulated trapped phase versus electron energy of a single particle emitted at $x = 10$ nm. (d) Simulated trapped phase versus initial emission position (x) of a single particle at 29.5 keV.

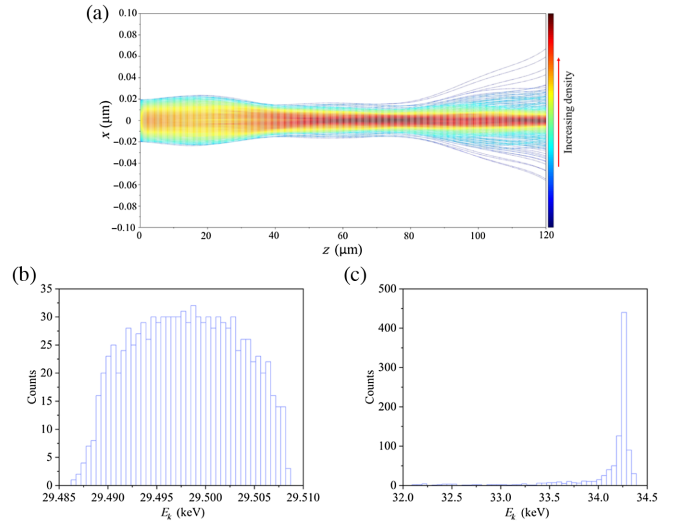


FIG. 6. (a) Simulated density map of trajectories of the bunch with energy spread versus the transmission length. (b) Simulated particle energy distribution at the start of the simulation. (c) Simulated particle energy distribution at the end of the simulation.

three acceleration stages, which demonstrates that there is a tolerance for the energy of the particles and the corresponding emission phase in the dynamic phase-driven method. Figure 5(d) further illustrates that the captured phase varies with the initial emission position, with the central area exhibiting the largest range.

Figure 6(a) shows the trajectories of the entire particle bunch as a function of particle position. All particles are concentrated near the center of the acceleration channel.

As shown in Fig. 6(c), 95% of the particles are accelerated to within the full-width at half-maximum (FWHM) centered at an energy of 34.2 keV, whereas a small fraction of the bunch becomes detrapped due to their initial emission position.

Figures 7 and 8 show the phase space of the electron bunch during the accelerating process. The electron bunch experiences periodic transverse defocusing and focusing forces, which confine it to the center of the bunch channel in Fig. 7, consistent with the theoretical design. Although the initial bunch length is short, it will extend as the transmission length increases due to the nonlinear DLA force in the different transverse direction, which increases the energy spread of the bunch. From Fig. 8(e), we observe

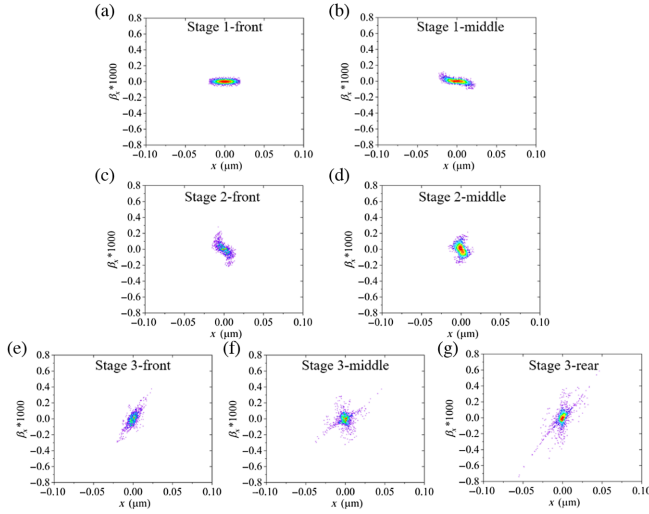


FIG. 7. (a)–(g) Simulated $x - \beta_x$ phase space in the front, middle, and rear of each stage.

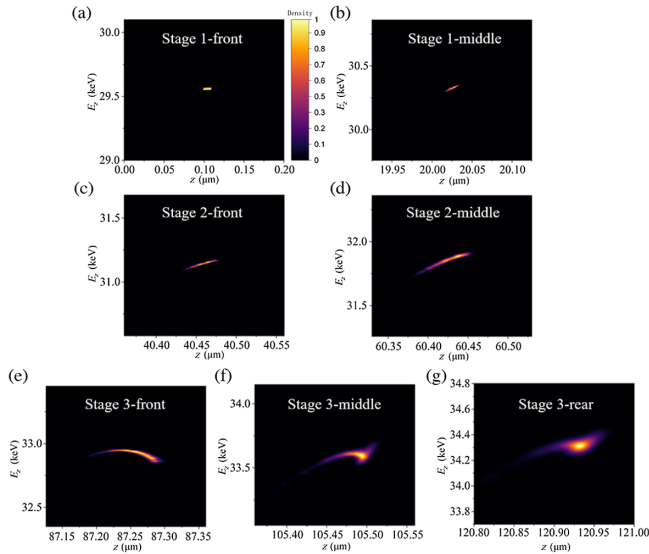


FIG. 8. (a)–(g) Simulated $z - P_z$ phase space in the front, middle, and rear of each stage.

that the energy of the bunch head is lower than the trailing segment, at which point the bunch begins to focus longitudinally. As evident from Fig. 8(g), after propagating 120 μm , most of the bunch achieves longitudinal focusing and is accelerated to high energies.

To study the detrapping mechanism, we simulated a bunch with a spatial distribution identical to that used previously, but with key parameters adjusted: an emittance of 2.1 $\mu\text{m rad}$, a monochromatic energy of 29.53 keV, and an initial emission phase outside the trapping region [see Fig. 5(c)]. Figure 9(b) shows that bunches outside the trapping phase are decelerated following 60 μm of propagation. A large part of the particles that match the phase design are successfully accelerated, while those outside the trapping phase undergo deceleration and will be eliminated in the subsequent acceleration stage. Compared with the situation without laser stimulation, most of the electron beam passed through the acceleration

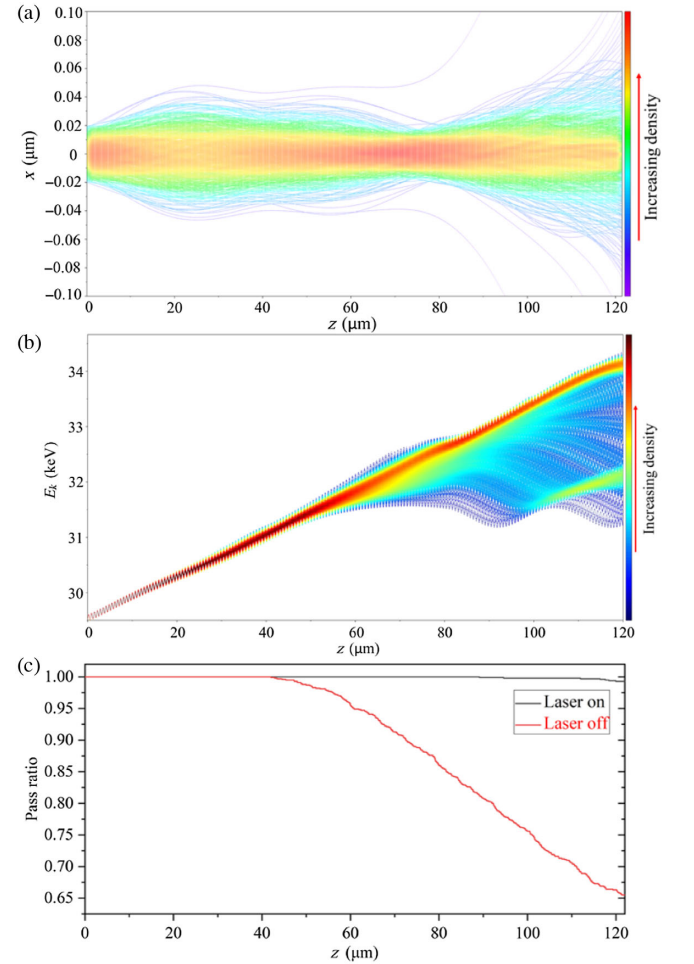


FIG. 9. (a) Simulated density map of trajectories of the bunch without energy spread versus the transmission length. (b) Simulated density map of the particle energy versus the particle position. (c) Simulated pass ratio of the electron bunch as a function of particle position in the condition of laser on and off.

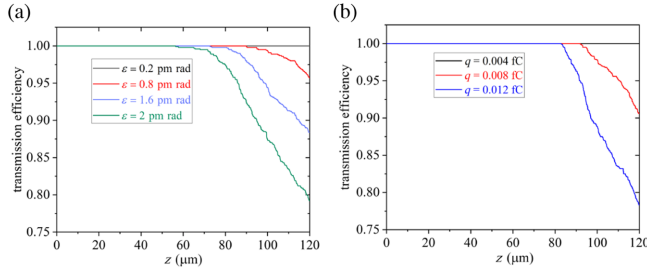


FIG. 10. Simulated transmission efficiency of the whole bunch versus emittance (a) and charge (b) of the electron bunch.

channel. This indicates that the pass ratio of particles with higher emittance can be increased through the control of the initial emission phase, while a trade-off is the detraping of a portion of particles.

The influence of initial emittance and charge on the capture rate of a 29.50 keV electron beam (with a 20 eV spread) is shown in Fig. 10. Under the emittance of 0.2 pmrad and charge of 0.004 fC, most of the electron bunch is captured, but increasing the initial emittance and charge diminishes transmission efficiency. An increase in initial emittance causes the transmission rate to decline as it exceeds the capture range, while the transmission rate drop resulting from increased charge is attributed to the space-charge effect.

It should be noted that the bunch charge used in our simulations is much higher than that employed in previous DLA experiments based on scanning electron microscope electron guns [28,34]. Consequently, space-charge effects must be accounted for in our model. Nevertheless, the proposed method remains fully applicable to single-electron dynamics, where space-charge effects are negligible, as is the case in prior experimental studies. Additionally, we employed 1000 macroparticles in our simulations—far exceeding the number of physical particles in a typical bunch—to ensure the statistical reliability of the simulation results, particularly for the phase-space distributions of the electron bunch.

IV. CONCLUSION

In summary, we introduce the dynamic phase-driven synchronization approach within the ISP-DLA and design a three-stage accelerating grating structure to improve acceleration stability compared to the APF method. The dynamic phase-driven synchronization allows precise regulation of lateral particle dispersion, enabling continuous and simultaneous acceleration. Simulations demonstrate that particles can be accelerated from 29.50 to 34.2 keV with a captured efficiency of 95% within a 200-nm-wide and 120-μm-long bunch channel. This approach offers a compelling alternative for tabletop ultrafast electron accelerators, broadening the accessibility and applicability of particle acceleration technologies across diverse scientific and technological domains.

ACKNOWLEDGMENTS

This work was supported by the Natural Science Foundation of China (Grant No. 12475153), the Key State Laboratory of Advanced Electromagnetic Technology (Grant No. AET 2025KF001), and the Key National Laboratory of Electromagnetic On Environment Effects of Science and Technology (No. 6142205250201).

DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

- [1] N. V. Sapra, K. Y. Yang, D. Vercruysse, K. J. Leedle, D. S. Black, R. J. England, L. Su, R. Trivedi, Y. Miao, O. Solgaard *et al.*, On-chip integrated laser-driven particle accelerator, *Science* **367**, 79 (2020).
- [2] R. J. England, P. Hommelhoff, and R. L. Byer, Microchip accelerators, *Phys. Today* **74**, No. 8, 42 (2021).
- [3] L. Sun, W. Liu, J. Zhou, Y. Zhu, Z. Yu, Y. Liu, Q. Jia, B. Sun, and H. Xu, GV m⁻¹ on-chip particle accelerator driven by few-cycle femtosecond laser pulse, *New J. Phys.* **23**, 063031 (2021).
- [4] S. Kutsaev, Advanced technologies for applied particle accelerators and examples of their use, *Tech. Phys.* **66**, 161 (2021).
- [5] R. W. Hamm and M. E. Hamm, *Industrial Accelerators and Their Applications* (World Scientific, Singapore, 2012).
- [6] M. R. Cleland, Industrial applications of electron accelerators, in *CAS - CERN Accelerator School: Small accelerators* (CERN, 2008), pp. 383–416.
- [7] M. Silari, Applications of particle accelerators in medicine, *Radiat. Protect. Dosim.* **146**, 440 (2011).
- [8] G. Cuttone, Applications of particle accelerators in medical physics, Istituto Nazionale di Fisica Nucleare-Laboratori Nazionali del Sud, VS Sofia 44, 2008.
- [9] S. Kutsaev, Advanced technologies for applied particle accelerators and examples of their use, *Tech. Phys.* **66**, 161 (2021).
- [10] R. J. England, R. J. Noble, K. Bane, D. H. Dowell, C. K. Ng, J. E. Spencer, S. Tantawi, Z. Wu, R. L. Byer, E. Peralta *et al.*, Dielectric laser accelerators, *Rev. Mod. Phys.* **86**, 1337 (2014).
- [11] X. Mei, R. Zha, Y. Pan, S. Wang, B. Sun, C. Lei, C. Ke, Z. Zhao, and D. Wang, Dielectric laser accelerators driven by ultrashort, ultraintense long-wave infrared lasers, *Ultrafast Sci.* **3**, 0050 (2023).
- [12] R. England, U. Niedermayer, L. Schächter, T. Hughes, P. Musumeci, R. Li, and W. Kimura, Considerations for a TeV collider based on dielectric laser accelerators, *J. Instrum.* **17**, P05012.
- [13] T. Hughes, G. Veronis, K. P. Wootton, R. Joel England, and S. Fan, Method for computationally efficient design of dielectric laser accelerator structures, *Opt. Express* **25**, 15414 (2017).

- [14] K. Mizuno, J. Pae, T. Nozokido, and K. Furuya, Experimental evidence of the inverse Smith–Purcell effect, *Nature (London)* **328**, 45 (1987).
- [15] J. Breuer and P. Hommelhoff, Dielectric laser acceleration of 28 keV electrons with the inverse Smith–Purcell effect, *Nucl. Instrum. Methods Phys. Res., Sect. A* **740**, 114 (2014).
- [16] J. Breuer and P. Hommelhoff, Laser-based acceleration of nonrelativistic electrons at a dielectric structure, *Phys. Rev. Lett.* **111**, 134803 (2013).
- [17] K. J. Leedle, R. Fabian Pease, R. L. Byer, and J. S. Harris, Laser acceleration and deflection of 96.3 keV electrons with a silicon dielectric structure, *Optica* **2**, 158 (2015).
- [18] M. Kořak, M. Förster, J. McNeur, N. Schönenberger, K. Leedle, H. Deng, J. Harris, R. Byer, and P. Hommelhoff, Dielectric laser acceleration of sub-relativistic electrons by few-cycle laser pulses, *Nucl. Instrum. Methods Phys. Res., Sect. A* **865**, 84 (2017).
- [19] W. D. Kimura, G. H. Kim, R. D. Romea, L. C. Steinhauer, I. V. Pogorelsky, K. P. Kusche, R. C. Fernow, X. Wang, and Y. Liu, Laser acceleration of relativistic electrons using the inverse Cherenkov effect, *Phys. Rev. Lett.* **74**, 546 (1995).
- [20] W. Liu, Z. Yu, L. Sun, Y. Liu, Q. Jia, H. Xu, and B. Sun, Microscale laser-driven particle accelerator using the inverse Cherenkov effect, *Phys. Rev. Appl.* **14**, 014018 (2020).
- [21] L. Zhang, W. Liu, Y. Liu, Q. Jia, B. Sun, H. Xu, and S. Liu, Inverse Cherenkov dielectric laser accelerator for ultra-relativistic particles, *J. Phys. D* **56**, 045103 (2022).
- [22] B. Sun, Y.-F. He, R.-Y. Luo, T.-Y. Zhang, Q. Zhou, S.-Y. Wang, D. Wang, and Z.-Q. Zhao, Structure and material study of dielectric laser accelerators based on the inverse Cherenkov effect, *Chin. Phys. B* **32**, 094101 (2023).
- [23] I. B. Fainberg, Alternating phase focusing, in *CERN Symposium on High Energy Accelerators and Pion Physics* (CERN, 1956), pp. 91–102.
- [24] P. Broaddus, T. Egenolf, D. S. Black, M. Murillo, C. Woodahl, Y. Miao, U. Niedermayer, R. L. Byer, K. J. Leedle, and O. Solgaard, Subrelativistic alternating phase focusing dielectric laser accelerators, *Phys. Rev. Lett.* **132**, 085001 (2024).
- [25] U. Niedermayer, T. Egenolf, O. Boine-Frankenheim, and P. Hommelhoff, Alternating-phase focusing for dielectric laser acceleration, *Phys. Rev. Lett.* **121**, 214801 (2018).
- [26] U. Niedermayer, T. Egenolf, and O. Boine-Frankenheim, Three dimensional alternating-phase focusing for dielectric-laser electron accelerators, *Phys. Rev. Lett.* **125**, 164801 (2020).
- [27] L. Zhang, H. Xu, and W. Liu, Inverse Cherenkov dielectric laser accelerator with alternating phase focusing for sub relativistic particles, *Phys. Rev. Accel. Beams* **27**, 110401 (2024).
- [28] T. Chlouba, R. Shiloh, S. Kraus, L. Brückner, J. Litzel, and P. Hommelhoff, Coherent nanophotonic electron accelerator, *Nature (London)* **622**, 476 (2023).
- [29] W. Liu, L. Zhang, Y. Liu, Q. Jia, B. Sun, and H. Xu, Toward a laser-driven traveling-wave linac on a chip, *Phys. Rev. Appl.* **19**, 044066 (2023).
- [30] T. P. Wangler, *RF Linear Accelerators* (John Wiley & Sons, New York, 2008).
- [31] A. Aimidula, M. Bake, F. Wan, B. Xie, C. Welsch, G. Xia, O. Mete, M. Uesaka, Y. Matsumura, M. Yoshida *et al.*, Numerically optimized structures for dielectric asymmetric dual-grating laser accelerators, *Phys. Plasmas* **21**, 023110 (2014).
- [32] W. Jones, Earnshaw’s theorem and the stability of matter, *Eur. J. Phys.* **1**, 85 (1980).
- [33] R. Bassani, Earnshaw (1805–1888) and passive magnetic levitation, *Meccanica* **41**, 375 (2006).
- [34] U. Niedermayer, D. S. Black, K. J. Leedle, Y. Miao, R. L. Byer, and O. Solgaard, Low-energy-spread attosecond bunching and coherent electron acceleration in dielectric nanostructures, *Phys. Rev. Appl.* **15**, L021002 (2021).