

Tolerances to driver-witness misalignment in a quasilinear plasma wakefield accelerator

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Plasma-based accelerators offer high accelerating gradients and scalability through staging or long plasma sources, which makes them good candidates for future accelerator and collider concepts. Proton-driven accelerators in particular have the potential to bring particles to high energy in a single stage. In the quasilinear regime, where the plasma wake is only partially evacuated, a witness bunch of electrons drives a cavitating wake that acts to preserve the emittance of the portion of the witness inside this self-blowout. In the case of a misalignment between the driver and witness, this behavior can persist, but its effectiveness is reduced. In this paper, we study transverse witness dynamics in this regime and develop an analytical model to describe the witness motion. We show that the degree of emittance preservation can be predicted by estimating the beam density after phase-mixing. Particle-in-cell simulations using the AWAKE Run 2c baseline parameters show excellent agreement with the predictive model developed. This work allows alignment constraints to be set both for the AWAKE experiment and other wakefield acceleration schemes operating in the quasilinear regime.

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I. INTRODUCTION

Prospective designs of future particle accelerators look to push the energy frontier forward. In order to do this, the high acceleration gradients of plasma-based acceleration are attractive [1,2]. While the accelerating gradient of conventional radiofrequency (rf) cavity machines is limited to the tens of MeV m^{-1} due to rf breakdown, plasmas can support high accelerating gradients at the GeV m^{-1} level and hence have the potential to reduce the real-estate footprint of future accelerator facilities [1,3–5].

In order to use plasma as an accelerator, high electric fields must be set up by separating the electrons and ions. This requires a suitable driver, either an intense short-pulse

laser [6] or an energetic particle beam [7]. This driver displaces plasma electrons, as it propagates, while the much heavier ions move comparatively less. This separation of charges induces electric fields that cause the plasma electrons to oscillate, and a traveling electron-density wave with a phase velocity equal to the driver velocity follows the driver. This is a plasma wake, into which a “witness” bunch of electrons may be injected and accelerated to high energy.

As the driver propagates through the plasma, energy is gradually transferred to the wake, and ultimately to the witness. If the driver propagates sufficiently far, it will lose a significant fraction of its energy, and the wake will collapse, making acceleration no longer possible. Collider-relevant plasma accelerators will need to be hundreds to thousands of meters in length, which exposes them to the possibility of driver depletion.

Two main schemes address this problem. Either periodically replacing the spent driver in order to continue the acceleration of the witness, in a process called “staging.” Staging requires both the extraction of the spent driver and

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the injection of a new one while simultaneously preserving the witness [8].

Alternatively, by choosing a driver with a high enough energy, the depletion length is guaranteed to be longer than the accelerator. Proton beams are the only practical choice for such a scheme [9,10], with their extremely high mass allowing them to be accelerated with low synchrotron losses in existing circular collider infrastructure.

Later, we consider here a proton-driven plasma wakefield accelerator. Proton beams are the only currently available driver that can reach the energy frontier in a single stage.

The AWAKE experiment [10,11] is a proof-of-principle accelerator using the 400 GeV proton beam from the CERN Super Proton Synchrotron (SPS) to accelerate electrons in plasma. The SPS beam is around 5 cm in length, and the plasma is quite sparse, having a plasma wavelength of ~ 1 mm, thus the beam spans many plasma periods. The beam is initially too long to effectively drive a strong wake for acceleration, but nonetheless the periodic focusing and defocusing action causes it to undergo self-modulation. The beam modulates over several meters into a train of microbunches [12,13] with periodicity on the scale of the plasma wavelength. The modulated beam is then suitable for resonant wakefield excitation.

As the proton beam modulates, the phase of the wake shifts. It is therefore undesirable to attempt acceleration until the wake has stabilized [14]. The AWAKE Run 2c experiment will separate the modulation stage from the acceleration stage physically in two sequential plasma sources. This allows the proton beam to completely self-modulate in the first, and then electrons to be injected into a stable wake in the second. Each stage will be 10 m long, with a gap of 1 m to allow for injection [11].

Building upon the proton-driven accelerator concept, the ALiVE scheme [5] proposes to use a single short proton bunch [15,16] to drive the wakefield, thus avoiding the need for modulation entirely.

When accelerating electrons, it is typical to operate in the blowout regime. This is where the driver expels all plasma electrons behind it, leaving only the background ions. This is preferable not only because it produces the strongest possible accelerating fields for a given plasma density but also because the focusing fields on the bunch are linear. By matching the beta function of the witness to the focusing field of the plasma, the witness emittance can be preserved [17]. However, proton-driven wakefields tend to operate in the quasilinear regime, where the wake is not fully evacuated, so we cannot expect to preserve the witness emittance completely. This being said, the witness itself also drives wakefields and, when its charge density is sufficiently high, can drive its own blowout [18]. This affords some degree of emittance control. A schematic view of this is shown in Fig. 1, where we depict a short driver, rather than a train of microbunches.

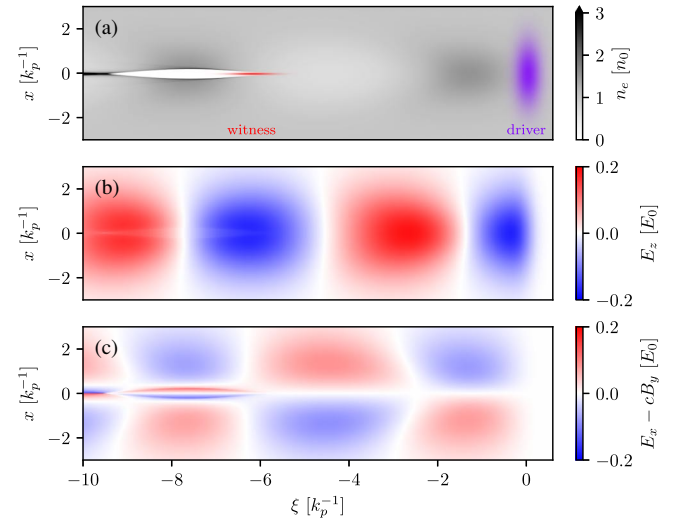


FIG. 1. Schematic view of the simulation setup. For an aligned driver-witness setup, we have (a) the plasma (gray), with driver (purple) and witness (red) moving left-to-right in series. (b) The longitudinal fields, showing the beamloading due to the witness, and (c) the transverse force seen by a relativistic particle. We see that the witness blowout creates a region of focusing fields within the larger wake. Fields are normalized to the wave breaking field $E_0 = m_e e \omega_p / e$. Here ξ is the longitudinal coordinate relative to the driver centroid.

For an externally injected witness, we consider the effect of imperfect transverse alignment between the driver and witness at injection. This will be present to some degree in any real experiment and has been studied for the blowout case [19], where tight tolerances were found. In the context of a quasilinear wake, we look at the effect that a transverse misalignment has on the final beam properties and what experimental tolerances might be extracted from this.

II. SIMULATION SETUP

We performed simulations using the 3D quasistatic Particle-in-cell (PIC) codes QV3D [20] and HiPACE++ [21] and found excellent agreement between the two. The following parameters follow those of Ref. [18], where a similar study was conducted. We present here a more detailed study of emittance preservation in the presence of an initial misalignment. We simulate a plasma of length 10 m and radius 1 mm, with a constant plasma density of $n_0 = 7 \times 10^{14} \text{ cm}^{-3}$, corresponding to a skin depth of $1/k_p = 200 \text{ } \mu\text{m}$. The 400 GeV driver is modeled as a positively charged, nonevolving Gaussian ellipsoid with a radius of $\sigma_{r,p^+} = 200 \text{ } \mu\text{m}$, length $\sigma_{z,p^+} = 40 \text{ } \mu\text{m}$, and total charge $Q = 2.34 \text{ nC}$ (peak driver density $n_d = 0.83n_0$), resulting in a peak accelerating gradient of $\sim 500 \text{ MeV m}^{-1}$. These parameters were chosen to emulate the wakefields driven by the full microbunch train in AWAKE Run 2c [11].

The electron witness bunch is also a Gaussian ellipsoid, with an injection energy of 150 MeV ($\gamma_0 \approx 294$), with 0.1% relative energy spread, a longitudinal length of $\sigma_z = 60 \mu\text{m}$, and a radial size chosen to satisfy the blowout matching condition:

$$\sigma_{\text{matched}} = \left(\frac{2\epsilon_n^2}{k_p^2 \gamma} \right)^{1/4}, \quad (1)$$

where ϵ_n is the normalized emittance and γ is the Lorentz factor of the electron witness. We choose to use the blowout matching condition despite the proton-driven wake being quasilinear, as we expect most of the bunch charge to sit in a self-generated blowout.

As the plasma streams past the witness, the blowout takes some finite time to form, and this time depends on the charge density [18,22]. We vary the witness emittance from $\epsilon_n = 2$ to $16 \mu\text{m}$, corresponding to a range of radial sizes $\sigma_r = 5.76$ – $16.3 \mu\text{m}$. Beam charge is also varied from 100 to 400 pC, leading to a range of peak witness densities n_b from 3 to $115n_0$. The transverse offset between the driver and the witness is varied from 0 to $20 \mu\text{m}$. The longitudinal position of the witness in the wake is chosen so as to minimize energy spread and varies linearly between 6.2 and $5.7/k_p$, equivalent to a delay between the driver and the witness of 4.1 – 3.8 ps, depending on the witness charge (high-low).

III. RESULTS AND DISCUSSION

A schematic of the simulation setup is shown in Fig. 1, where we illustrate the initial configuration for the perfectly aligned witness case. Figure 1(a) shows the relative densities of the plasma, driver, and witness, where we see immediately that the wake is not fully evacuated by the low-density driver, allowing the higher-density witness to drive its own blowout within the larger wake. We also see the effect of beamloading in Fig. 1(b), situated over the witness by, the fact that the accelerating fields are reduced (paler in color). Finally, the transverse fields are shown in Fig. 1(c), where the quasilinear wakefields are disturbed by the small blowout driven by the witness. We notice that the witness-blowout projects a focusing region even when located in the defocusing phase of the encompassing quasilinear wake.

A. Key results

The simulations were allowed to evolve over 10 m, and in this time, projected energy spread increases to 5%–10% depending on charge (low-high). Energy gain varies inversely with bunch charge, falling in the range 2–4 GeV (high-low). Both final energy and projected energy spread are mostly independent of the initial offset.

Figure 2 shows final density maps and a spread of final emittances for each longitudinal slice of the witness beam.

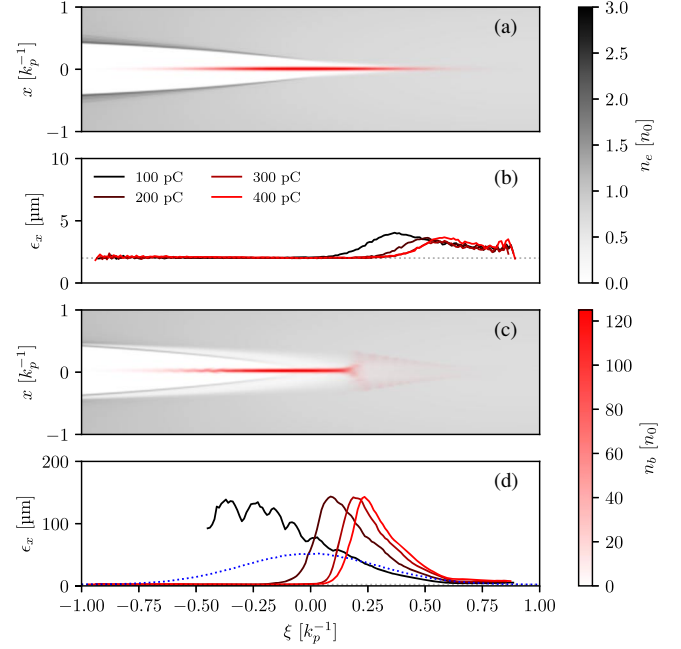


FIG. 2. Snapshots of the 400 pC, $2 \mu\text{m}$ emittance bunch for (a) no offset and (c) $20 \mu\text{m}$ offset after 10 m, and comparative slice emittances for all charge cases 100–400 pC, again for (b) no offset and (d) a $20 \mu\text{m}$ offset. The initial charge density profile of all beams is illustrated by the blue dotted line in panel (d). Here ξ is the longitudinal coordinate relative to the witness centroid.

Figure 2(a) shows a snapshot after 10 m of acceleration for the case of no initial offset. The 400 pC, $2 \mu\text{m}$ case is shown, but this picture is typical; the head of the witness sits in the quasilinear wake generated by the driver, while the tail sits in the self-blowout.

Figure 2(b) shows the longitudinal slice emittance for this case (red line), and all other witness charges with no initial offset and $2 \mu\text{m}$ initial emittance. Using the matched radius ensures that for the portion of the witness that sits in a blowout, there is little change in emittance, and this is reflected in the slice emittances. We see that despite no visible degradation of the beam head in Fig. 2(a), there is still some emittance growth, which saturates early in the acceleration as the witness head reaches a new matching condition with the plasma [18]. The emittance growth drops off, as the local plasma electron density approaches zero and the matching condition approaches Eq. (1). The higher the charge of the bunch, the earlier a blowout forms, and emittance preservation occurs for a larger fraction of the beam. Overall, the projected emittance growth varies from $\epsilon_x = 2$ to 2.1 – $2.5 \mu\text{m}$ depending on charge, with higher charge seeing less growth.

Figure 2(c) shows the equivalent case to Fig. 2(a), but now with a $0.1 k_p^{-1}$ ($20 \mu\text{m}$) initial transverse offset. We see that after 10 m, the witness has taken on a distinctive profile, where the head of the bunch has become smeared out beyond the range of the initial offset. This occurs because the low-density head is unable to drive a blowout

and consequently diverges due to being unmatched [recall that Eq. (1) assumes blowout conditions, but here the focusing fields are smaller]. Thus different longitudinal slices of the beam oscillate at different frequencies and undergo phase-mixing, with the net result being increased emittance [23]. Some length back from the head of the beam, at around $\xi = 0.2k_p^{-1}$, the beam density is high enough that a blowout forms and the tail of the bunch remains tightly focused, oscillating coherently with emittance preservation. The blowout forms further back along the bunch compared to the zero-offset case, and this distance increases with increasing offset.

Figure 2(d) shows the final slice emittances for bunches with a 20 μm offset and 2 μm initial emittance. In the lowest-charge case (black line), the dilution of the bunch head means the transverse wake is no longer strong enough to keep the witness focused, and the entire length of the bunch experiences massive emittance growth, with a final projected emittance of $\epsilon_x = 80 \mu\text{m}$. However, at higher witness charge, emittance can still be partially preserved as the increasing density of the witness still allows a blowout to form, which protects progressively more of the bunch, with the 400 pC case having a final projected emittance of $\epsilon_x = 25 \mu\text{m}$.

B. Witness betatron motion

In Fig. 3(a), we plot the transverse centroid position of the 400 pC bunch with a 20 μm initial offset, as shown in Fig. 2(c), along both the length of the bunch itself (ξ) and along the propagation direction (z). We see there is a visible indicator of the regime change at around $\xi \approx 0.5/k_p$ between the head of the bunch, undergoing phase-mixing,

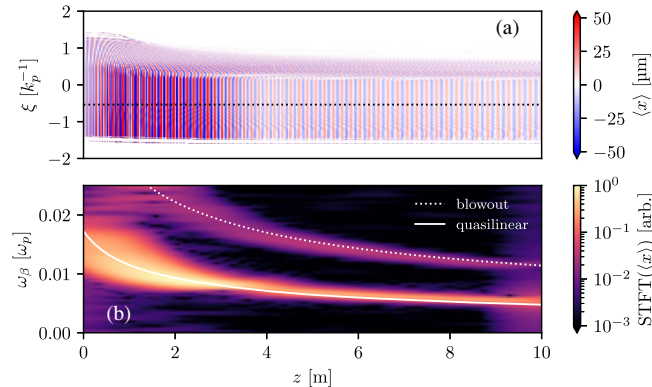


FIG. 3. (a) Waterfall plot of the bunch transverse centroid position over its length ξ and the propagation coordinate z and (b) short-time Fourier transform (STFT), showing the propagation distance-resolved oscillation frequency at the point $\xi = -0.5k_p$ in the witness, indicated by the dotted line in panel (a). The dotted white line shows the blowout betatron frequency $\omega_p/\sqrt{2\gamma}$, and the solid white line shows the numerically calculated quasilinear betatron frequency.

and the rear of the bunch, undergoing coherent betatron oscillations.

Figure 3(b) shows the betatron frequency of the witness as a function of propagation distance. This is obtained by choosing a fixed ξ and taking the witness transverse centroid position along z at this point, indicated by the dotted line in Fig. 3(a). Applying a STFT on this signal yields the betatron wave number k_β as a function of propagation distance z . We can then approximate the temporal frequency as $\omega_\beta = ck_\beta$, representing the effective betatron frequency at the chosen ξ . We see that there are two frequency components to the signal, with the low-frequency component dominant.

This may be explained by analogy. Consider a ball (the witness) placed in a basket (its self-blowout), and the basket is moved back and forth at low frequency (the oscillations of the driver head in the quasilinear wake). The ball's position within the basket is not fixed; however, its own inertia will cause it to undergo high-frequency oscillations depending on the shape of the basket (the transverse potential of the self-blowout). When the frequency of the ball's motion is measured, there will be two components: the low-frequency motion of the basket and the high-frequency motion of the ball within the basket. As the amplitude of the slow motion is much greater than that of the fast motion, the low-frequency component will have the stronger signal.

The high-frequency component in this case is straightforward to determine. In the self-blowout $\rho = en_0$ is constant, the resulting transverse fields are found via Gauss' law to be

$$E_\perp(r) = \frac{n_0 e}{2\epsilon_0} r. \quad (2)$$

The high-frequency component is therefore simply the characteristic betatron frequency in a blowout:

$$\omega_{\beta,b} = \sqrt{\frac{e^2 n_0}{2\gamma m_e \epsilon_0}} = \frac{\omega_p}{\sqrt{2\gamma}}. \quad (3)$$

We may retrieve an estimate of the low-frequency component by modeling the quasilinear wakefield potential via Poisson's equation:

$$\nabla^2 \varphi = -\frac{\rho}{\epsilon_0}, \quad (4)$$

where φ is the wakefield potential, ρ is the plasma charge density, and ϵ_0 is the permittivity of free space. In the quasilinear regime, the longitudinal component of the charge density is sinusoidal in z , allowing us to separate the longitudinal and transverse parts:

$$\rho(r, z) = \rho_\perp(r) \cos(k_p z), \quad (5)$$

where $\perp$ denotes the transverse component. Given that ρ is separable, and the plasma response is linear, the potential can also be expressed in the same form [24]:

$$\varphi(r, z) = \varphi_{\perp}(r) \cos(k_p z). \quad (6)$$

We now insert Eqs. (5) and (6) into Eq. (4) and evaluate derivatives of z to eliminate the longitudinal components. We hence find that the transverse wake potential takes the form of a screened Poisson equation:

$$(\nabla_{\perp}^2 - k_p^2)\varphi_{\perp} = -\frac{\rho_{\perp}}{\epsilon_0}. \quad (7)$$

In general, analytical solutions to Eq. (7) are nontrivial. One may find solutions for pointlike [25] and uniform drivers [24], but a full analytic solution for a Gaussian driver is beyond the scope of this paper.

From PIC simulation, we numerically determine the perturbed plasma density in the wake through as $n_e \approx 0.67n_0$. Taking the transverse density profile to be similar to the driver $\rho_{\perp} \approx -en_e \exp(-r^2/(2\sigma_r^2))$, we solve Eq. (7) numerically for the wake potential and retrieve the fields from $E_{\perp} = -\nabla_{\perp}\varphi$. Assuming the witness displacement from the axis to be small $r \ll k_p^{-1}$, we expand $E_{\perp}$ only to lowest order and find the quasilinear wakefield falls to $E_{\perp,q} \approx 0.175E_{\perp,b}$. We disregard the contributions of the azimuthal magnetic field B_{θ} here, as we explicitly consider only a weakly nonlinear wake, which will not drive significant magnetic fields due to the plasma current. A detailed discussion of magnetic effects is found in Ref. [26]. The low-frequency component is therefore $\omega_{\beta,q} = \omega_{\beta,b} \sqrt{E_{\perp,q}/E_{\perp,b}}$. Comparing this to the simulation, in Fig. 3(b), we overlay our numerically calculated low-frequency (solid line) and high-frequency (dotted line) components and find them both to be in good agreement with the results.

C. Parameter scan

Expanding the scope of our simulations, we can scan several parameters and look for patterns. Taking initial emittance in the range 2–16 μm , charge in 100–400 pC, and offsets in 0–20 μm , we plot the resulting emittance as a function of these three variables in Fig. 4. We have the intuitive result that in general, higher charges results in less emittance growth across the board. This can be seen by observation, and for a visual aid, we mark the isobar for a 30 μm projected emittance on all figures. One can see that for increasing charge, this isobar moves further up, indicating emittance preservation for a wider range of parameters.

With an offset, the emittance in the plane of initial offset will be larger than that of the nonoffset plane. While the emittance in the plane of the offset (here x) grows considerably, as seen in Fig. 2(d), the emittance in the

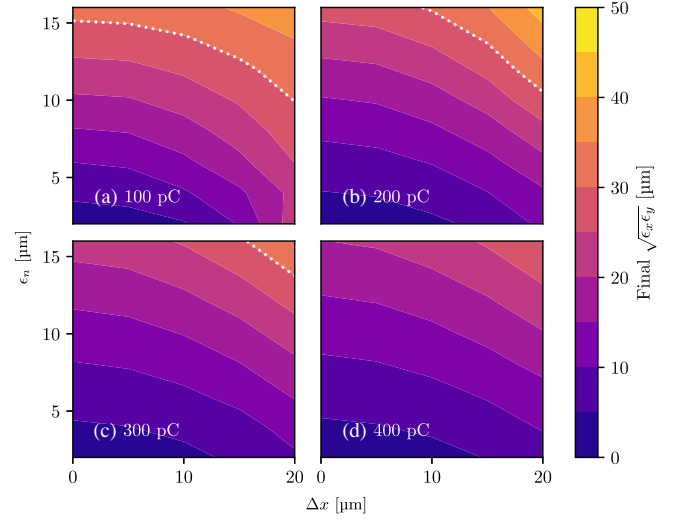


FIG. 4. Projected emittance after 10 m. The geometric average of the x - and y -plane emittance is taken as the representative quantity while varying the initial emittance ϵ_n and the offset Δx . Subplots (a)–(d) show increasing charge from 100 to 400 pC. The isobars for a 30 μm projected emittance are shown as white dotted lines.

orthogonal plane (here y) remains on the order of a nonoffset bunch, e.g., Fig. 2(b). We use the geometric mean of the x - and y -emittances to get a representative quantity for the overall bunch quality, resulting in the more modest increases to the projected emittance recorded in Fig. 4.

In the presence of an initial offset, the head of the bunch will oscillate about the axis and undergo phase-mixing. For a low-charge witness, such that the transverse wakefields are entirely determined by the driver, the entire bunch will spread out in the plane of the offset. The result is a density profile such that:

$$n_{b,\text{eff.}} = \frac{Q}{e(2\pi)^{3/2}\sigma_z\sigma_y\sqrt{\sigma_x^2 + \Delta x^2}}, \quad (8)$$

where $n_{b,\text{eff.}}$ describes the peak density of a bunch that has been uniformly spread out in one direction by a factor $\sqrt{1 + (\Delta x/\sigma_x)^2}$. If the density is high enough, then the witness will drive its own wakefields in addition to those of the driver. The rear of the witness experiences stronger focusing forces than the head, and if the density of the witness is high enough, a blowout will form. The exact point at which a blowout forms depends on the charge density of the bunch head, which will always be proportional to this “effective density,” assuming the longitudinal bunch profile is the same. We may therefore take Eq. (8) as indicative of the ability of a bunch to drive a blowout. Additionally, Eq. (8) encodes multiple beam parameters into one figure: the charge, offset, and emittance [via Eq. (1)], making it a useful point of reference.

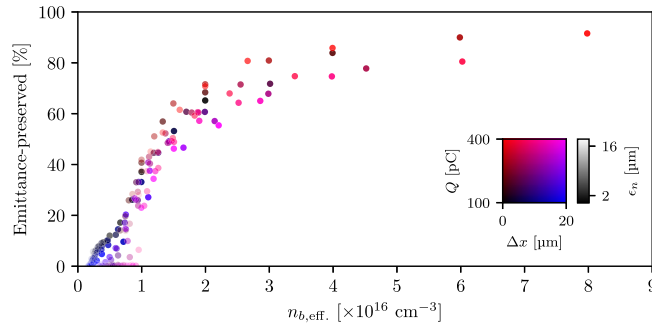


FIG. 5. Fraction of the bunch charge for which emittance is preserved (final emittance < 1.1 times initial emittance). The effective peak density of the bunch is taken as $n_{b,\text{eff.}} = Q / [e(2\pi)^{3/2} \sigma_z \sigma_y \sqrt{\sigma_x^2 + \Delta x^2}]$. The point color reflects the bunch parameters: charge (redness), offset (blueness), and initial emittance (alpha).

We now compute the preserved emittance fraction of each case by examining the longitudinal slice statistics. We look at the slice emittance along the bunch and choose some threshold emittance, under which we consider that slice to be “preserved.” Setting this threshold is arbitrary; we here choose a threshold of 10% over the initial emittance. We then add up the charge of each slice under the threshold and then compare this to the charge of the whole bunch. This gives us a “preserved fraction” for each case.

We now compare the ability of the witness to drive a blowout as per simulation, against the predicted ability given by Eq. (8) in Fig. 5, where we see a strong correlation between the preserved fraction and our effective density. Choosing a different threshold value changes the shape of the curve, lower values flatten it, and higher values steepen it, but the general trend is maintained. There is a collection of points with low effective density that have little-to-no emittance preservation at all. This is expected [23,27,28], as it stands to reason that there will be some critical threshold below which no blowout can be driven at any peak density and thus no emittance preservation.

We may use this correlation to determine tolerances. By nominating a desired final emittance, one can find the maximum offset for which the required performance is achieved, given the initial bunch charge and emittance. Additionally, we can extend this to estimate a pointing error. Assuming the bunch motion is sinusoidal, a transverse offset Δx can be translated into an angle $\Delta\theta$ by considering the trajectory of a test particle as it crosses the axis:

$$\Delta\theta = \arctan(k_{\beta,q} \Delta x) \approx k_{\beta,q} \Delta x, \quad (9)$$

where $k_{\beta,\text{ql.}} = \omega_{\beta,\text{ql.}}/c$ is the betatron wave number of the quasilinear wake, and we assume $\Delta\theta$ remains small. These results are encouraging if translated to angular

misalignment; the largest offset considered here, combined with the betatron wave number at injection $k_{\beta,\text{ql.}} \approx 86 \mu\text{m}^{-1}$ gives an angular offset on the order of 1.7 mrad, which is large compared to currently achievable performance at the 100 μrad level [29]. This arises due to the large focusing fields in plasma; such a small angle offset does not lead to large oscillations.

We find that the results here scale well, with the major requirement for model validity being the quasilinear regime, $n_d/n_0 \lesssim 1$, independent of the charge of the driver species. The picture becomes more complicated if the wake acting on the witness head evolves. This could occur due to dephasing or driver evolution, but neither of these effects is likely to contribute much in a typical acceleration scheme. In the blowout regime, the governing equation is Eq. (4) with $\rho = en_e$, and the physics simplifies as the fields become linear.

IV. CONCLUSION

Transverse or angular misalignments when staging or injecting act to degrade beam quality, and so effective emittance control is a challenge that must be addressed for future plasma-based accelerators and colliders.

We find that plasma acceleration in the quasilinear regime exhibits unique transverse dynamics, which can be split into two modes: the slow betatron motion of particles oscillating in the quasilinear wake and the fast betatron motion of particles oscillating in the self-bubble. The fast motion is described well by existing theory, and we have developed an analytical model for the slow betatron motion, describing witness self-blowout motion in the case of an offset. The witness head oscillates, which leads to oscillation of the self-blowout, within which the witness oscillates at the textbook betatron frequency.

The time for the witness self-blowout to form depends on the charge density of the beam head and greatly influences the degree of emittance preservation. We show that the final emittance correlates strongly with the effective density [Eq. (8)] derived from the beam properties and offset, which estimates the bunch charge density after phase-mixing. The predictive power of this model is tested by simulation and shows strong agreement.

Using the model, tolerances can easily be determined for a desired level of emittance control, and positional offsets can be translated into equivalent angular offsets. A beneficial property of plasma is that typical angular tolerances are relaxed compared to transverse misalignment tolerances due to the strong focusing action of the plasma. The range of applicability for this model covers the linear to quasilinear regime of wakefield acceleration, where the plasma is not fully evacuated.

This work will be useful for the ongoing design of AWAKE Run 2c and beyond and for any other acceleration scheme which operates in the quasilinear regime.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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