

Design and nonlinear optimization of a one-fold lattice for the Super Tau-Charm Facility collider rings

Tao Liu¹, Anton Bogomyagkov², Demin Zhou^{3,4,*}, Penghui Yang⁵, Sangya Li⁵,
Linhao Zhang¹, Ye Zou¹, Jingyu Tang¹, and Qing Luo^{1,5,†}

¹*School of Nuclear Science and Technology, University of Science and Technology of China,
No. 443 Huangshan Road, Hefei 230027, China*

²*Budker Institute of Nuclear Physics SB RAS, Novosibirsk 630090, Russia*

³*KEK, Oho 1-1, Tsukuba, 305-0801, Ibaraki, Japan*

⁴*The Graduate University for Advanced Studies (SOKENDAI), Hayama, Kanagawa 240-0193, Japan*

⁵*National Synchrotron Radiation Laboratory, University of Science and Technology of China,
No. 42, Hezuohua South Road, Hefei 230029, China*



(Received 19 May 2026; accepted 13 August 2026; published 28 August 2026)

The Super Tau-Charm Facility (STCF) is a proposed high-luminosity electron-positron collider operating at beam energies of 1–3.5 GeV, with a nominal peak-luminosity target exceeding $0.5 \times 10^{35} \text{ cm}^{-2} \text{ s}^{-1}$ at 2 GeV. At such low energies, the combined effects of beam-beam interactions in the crab-waist collision scheme, collective effects, and Touschek scattering impose stringent requirements on the dynamic aperture, momentum acceptance, and beam lifetime. This paper presents the design and nonlinear optimization of a one-fold lattice for the STCF collider rings. The design procedure consists of three main steps: lattice-independent optimization of the global machine parameters under luminosity, beam-beam, collective effects, and lifetime constraints; design of a compact interaction region (IR) with local chromatic correction and dedicated crab-waist sextupoles; and full-ring nonlinear optimization. The last step employs a two-stage strategy combining analysis-driven preoptimization with tracking-based refinement. Particular emphasis is placed on controlling momentum- and amplitude-dependent nonlinearities in the IR and preserving the nonlinear transparency of the crab-waist sextupole pair. At 2 GeV, the optimized lattice achieves a theoretical peak luminosity of $1.1 \times 10^{35} \text{ cm}^{-2} \text{ s}^{-1}$ while maintaining sufficient dynamic aperture, momentum acceptance, and a Touschek lifetime of approximately 600 s. These results demonstrate the effectiveness of the proposed staged optimization strategy and provide a systematic framework for the design of high-luminosity, low-energy electron-positron colliders.

DOI: [10.1103/44m8-gk5x](https://doi.org/10.1103/44m8-gk5x)

I. INTRODUCTION

The crab-waist (CW) collision scheme [1] has revolutionized the design of circular e^+e^- colliders by enabling order-of-magnitude luminosity gains without proportional increases in beam current. Its key principle is to combine a large Piwinski angle with dedicated sextupoles that create a “crabbed” waist at the interaction point (IP), thereby suppressing nonlinear beam-beam resonances and enhancing beam stability [2,3]. First demonstrated successfully at DAΦNE [4], the CW scheme has since become a cornerstone concept for new-generation factories. It has been

exploited at SuperKEKB [5] and adopted as a core design strategy in proposed projects such as FCC-ee [6], CEPC [7], and the Super Tau-Charm Facility (STCF) [8,9].

Recent tau-charm collider studies illustrate different implementations of the CW scheme. Both the Tau-Charm Factory (TCF) in Italy [10] and the Super Charm-Tau Factory (SCTF) in Russia [11,12] employ local chromatic correction and dedicated crab-waist sextupoles in the interaction region (IR), following the general approach developed for SuperB [13]. The TCF adopted a short distance of $L^* = 0.2 \text{ m}$ from the IP to the final doublet, which favored nonlinear performance but introduced demanding engineering constraints. In contrast, the SCTF adopted $L^* \simeq 0.9 \text{ m}$, comparable to that of SuperKEKB [14], thereby relaxing the IR layout at the expense of more challenging nonlinear dynamics and a shorter Touschek lifetime. Both designs also considered longitudinal electron-beam polarization at the IP.

Against this background, the STCF is being developed in China as a high-luminosity e^+e^- collider operating at

*Contact author: dmzhou@post.kek.jp

†Contact author: luoqing@ustc.edu.cn

Published by the American Physical Society under the terms of the *Creative Commons Attribution 4.0 International license*. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

beam energies of 1–3.5 GeV. Its IR design follows an approach similar to that of the SCTF, employing an independent local chromaticity correction system and dedicated crab-waist sextupoles. At the nominal beam energy of 2.0 GeV, the facility targets a peak luminosity exceeding $0.5 \times 10^{35} \text{ cm}^{-2} \text{ s}^{-1}$, while the lattice studied here adopts the more ambitious design goal of $1.0 \times 10^{35} \text{ cm}^{-2} \text{ s}^{-1}$ to provide adequate performance margin.

Because the nominal STCF beam energy is lower than the 2.5 GeV design energy of the SCTF, Touschek scattering places particularly stringent requirements on momentum acceptance (MA), nonlinear dynamics, and beam lifetime. These constraints motivated extensive STCF lattice studies [15–19], building upon earlier one-fold collider concepts proposed in the same energy region [10,11].

The configuration analyzed in this paper is the one-fold lattice developed during an earlier phase of the STCF project. It incorporates three Siberian snakes and was designed to preserve the possibility of collisions with a longitudinally polarized electron beam at the IP. All layouts, optical functions, nonlinear analyses, and performance results reported in this paper refer to this configuration.

The strategic priorities of the STCF project subsequently evolved toward a nearly twofold baseline, with greater emphasis on layout flexibility, operational robustness, and the possibility of retaining a second IP. That baseline and its performance are documented in Refs. [9,20] and are not analyzed quantitatively here.

The objective of this work is to provide a complete account of the one-fold lattice and of the optimization methodology developed in the context of this design for tackling the nonlinear challenges inherent in a long- L^* , crab-waist collider ring. Although the same methodology was later adopted for the current nearly twofold baseline, the method itself has not been systematically elaborated in Refs. [9,20]. A full exposition of the method is therefore given in the present paper.

The remainder of this paper is organized as follows. Section II presents the design philosophy and the global machine parameters obtained from lattice-independent parameter optimization. Section III describes the IR optics and the analytical framework for controlling momentum- and amplitude-dependent nonlinearities. Section IV presents the technical sections and their integration into the complete one-fold collider ring. Section V describes the two-stage global nonlinear-optimization strategy and the resulting lattice performance. Finally, Sec. VI summarizes the main results and discusses directions for future work.

II. OPTIMIZATION OF MACHINE PARAMETERS

For a high-luminosity tau-charm factory such as STCF, the choice of global machine parameters plays a decisive

role in determining whether the target luminosity can be achieved in a sustainable and operationally robust manner. Unlike incremental upgrades of existing facilities, the STCF design must simultaneously satisfy stringent luminosity requirements, unavoidable beam-beam effects inherent to the crab-waist collision scheme, and strong limitations arising from collective effects and beam lifetime at relatively low beam energy. As a result, the optimization of global parameters constitutes a critical first step that precedes detailed lattice design.

For flat-beam collisions with a large Piwinski angle and symmetric beam parameters in a CW collider, the luminosity can be expressed as [21,22]

$$L = L_0 F_h, \quad (1)$$

with

$$L_0 = \frac{N_b I_{b+} I_{b-}}{4\pi e^2 f_0 \sigma_x^* \sigma_y^* \sqrt{1 + \phi^2}}, \quad (2)$$

$$F_h \approx 2\sqrt{\pi}\zeta \exp(4\zeta^2) \text{Erfc}(2\zeta), \quad (3)$$

where N_b is the number of bunches, $I_{b\pm}$ are the bunch currents, f_0 is the revolution frequency, $\sigma_{x,y}^*$ are the transverse beam sizes at the IP, and $\text{Erfc}(x)$ is the complementary error function. The Piwinski angle is defined as $\phi = \sigma_z \tan \theta / \sigma_x^*$ with σ_z the bunch length and θ the half horizontal crossing angle. F_h indicates the hourglass factor with $\zeta = \beta_y^* \tan(2\theta) / (2\sigma_x^*)$. Note that Eq. (3) is valid with full crab waist applied to the colliding beams.

Crab-waist colliders are typically designed to operate with a large Piwinski angle ($\phi \gg 1$) and a small crossing angle ($\theta \ll 1$), albeit still significantly larger than those used in earlier colliders. In this regime, the geometric luminosity L_0 scales with the geometry parameters as $L_0 \propto 1/(\sigma_y^* \sigma_z \theta)$. To avoid significant geometric luminosity loss and to suppress strong beam-beam effects associated with the hourglass effect, the condition $\zeta \gtrsim 1$ is typically required, corresponding to $\beta_y^* \gtrsim \sigma_x^* / \theta$. This requirement contrasts with the conventional hourglass condition in earlier colliders, $\beta_y^* \gtrsim \sigma_z$. In the conditions of $\phi \gg 1$ and $\zeta \gtrsim 1$, the beam-beam tune shifts are given by [23]

$$\xi_y = \frac{N_p r_e \beta_y^*}{2\pi\gamma\sigma_x^* \sigma_y^* \sqrt{1 + \phi^2}} \approx \frac{N_p r_e \beta_y^*}{2\pi\gamma\sigma_y^* \sigma_z \theta}, \quad (4)$$

$$\xi_x = \frac{N_p r_e \beta_x^*}{2\pi\gamma\sigma_x^{*2} (1 + \phi^2)} \approx \frac{N_p r_e \beta_x^*}{2\pi\gamma\sigma_z^2 \theta^2}, \quad (5)$$

where symmetric beams are assumed, N_p is the bunch population and γ is the Lorentz factor. Guided by empirical

beam-beam limits observed in existing circular e^+e^- colliders, the STCF design adopts the constraint $\xi_y \lesssim 0.1$.

In the CW scheme, the luminosity is highly sensitive to perturbations that can increase the vertical beam size at the interaction point, σ_y^* , which is designed to be extremely small compared with that in earlier collider configurations. The luminosity is controlled by a tightly coupled set involving the beam currents, Twiss functions at the IP, bunch length, crossing angle, and beam-beam tune shifts. These parameters are further constrained by nonlinear beam dynamics, intrabeam scattering (IBS), Touschek scattering, synchrotron radiation damping and excitation, and impedance-driven collective instabilities. An unfavorable choice of any one of these parameters may become a dominant bottleneck, effectively limiting the achievable luminosity even if all others are optimized. Consequently, a global and self-consistent optimization strategy is required, in which luminosity performance and beam-dynamics constraints are treated on an equal footing.

From the luminosity perspective, the key parameters include the vertical beta function at the IP β_y^* , the vertical emittance ϵ_y , the bunch population N_p , the number of bunches, and bunch length σ_z . Achieving a small β_y^* is essential for high luminosity, yet it inevitably leads to large beta functions in the final-focus quadrupoles, generating strong chromaticity and enhanced nonlinearities. This, in turn, affects the dynamic aperture (DA) and MA, directly impacting the Touschek lifetime. At the same time, a small horizontal beam size at the IP σ_x^* is favored to suppress the hourglass effect and increase the Piwinski angle, but it increases the nonlinearities of the IR optics, thereby affecting the lifetime.

Longitudinal beam dynamics introduces additional couplings among the momentum compaction factor α_c , rf voltage V_{rf} , synchrotron tune ν_s , and energy spread σ_e . These parameters jointly determine the natural bunch length, rf momentum acceptance, and damping times. In particular, maintaining ν_s sufficiently larger than the horizontal beam-beam tune shift ξ_x is necessary to avoid luminosity degradation due to horizontal synchro-betatron resonances and coherent X-Z instability [24], imposing nontrivial constraints on α_c and rf parameters. Meanwhile, the rf acceptance must exceed the lattice MA to avoid rf-induced lifetime limitations.

Single-bunch collective effects further restrict the available parameter space. IBS modifies the transverse emittance, energy spread, and bunch length, while Touschek scattering sets a stringent lower bound on the combined transverse beam size, bunch length, and MA. Impedance-driven effects such as potential-well distortion, microwave instability, and transverse mode-coupling instability impose upper limits on the single-bunch current. These effects are particularly critical for STCF due to its relatively low beam energy, where the Touschek lifetime scales unfavorably with energy. Therefore, the global

TABLE I. Machine parameters for the one-fold STCF obtained from lattice-agnostic global parameter optimization.

Parameter	Unit	Value
Beam energy, E	GeV	2
Circumference, C	m	835.12
Full crossing angle, 2θ	mrاد	60
Horizontal emittance, ϵ_x	nm	5.9
Coupling factor, k	%	0.50
Beta functions at IP, β_x^*/β_y^*	mm	40/0.6
Momentum compaction factor, α_c	$\times 10^{-3}$	1.3
Energy spread, σ_e	$\times 10^{-4}$	9.8
Beam current, I	A	2.02
Bunch population, N_p	$\times 10^{10}$	5.05
Energy loss per turn, U_0	keV	400
Damping times, $\tau_x/\tau_y/\tau_z$	ms	27.8/27.8/13.9
Harmonic number, h	...	1392
rf voltage, V_{rf}	MV	1.93
Bunch length, σ_z	mm	11.2
Piwinski angle, ϕ	rad	21.9
Beam-beam parameters, ξ_x/ξ_y	...	0.002/0.078
Synchrotron tune, ν_s	...	0.016
Hourglass factor, F_h	...	0.9268
Theoretical peak luminosity, L	$\text{cm}^{-2} \text{s}^{-1}$	1.05×10^{35}

parameter optimization must ensure that the operating point remains safely below instability thresholds while still delivering the target luminosity.

These considerations make the STCF parameter choice a multiobjective problem, in which luminosity, beam-beam limits, collective effects, rf acceptance, dynamic aperture, and Touschek lifetime must be optimized consistently. We therefore employ a lattice-agnostic parameter optimization model (POM) to identify a physically viable region of parameter space before committing to a specific optical implementation. In this model, global parameters such as the ring circumference, emittance, momentum compaction factor, IP beta functions, rf voltage, beam current, and bunch length are varied under constraints derived from luminosity formulas, beam-beam limits, thresholds of collective effects, and damping requirements.

The resulting preoptimized parameters, summarized in Table I for the 2-GeV one-fold design, provide the design targets for the lattice developed in the following sections. The parameter correlations and trade-offs identified through this optimization also provide general guidance for the design and nonlinear optimization of high-luminosity, low-energy colliders. For a more detailed discussion of POM, the reader is referred to Ref. [25].

III. OPTICS OF THE IR FOR ONE-FOLD STCF

Two classes of crab-waist IR optics have been developed. In the local chromatic correction (LCC) scheme [13,26], the chromaticity generated by the final doublet is corrected locally by dedicated sextupole pairs, leaving the

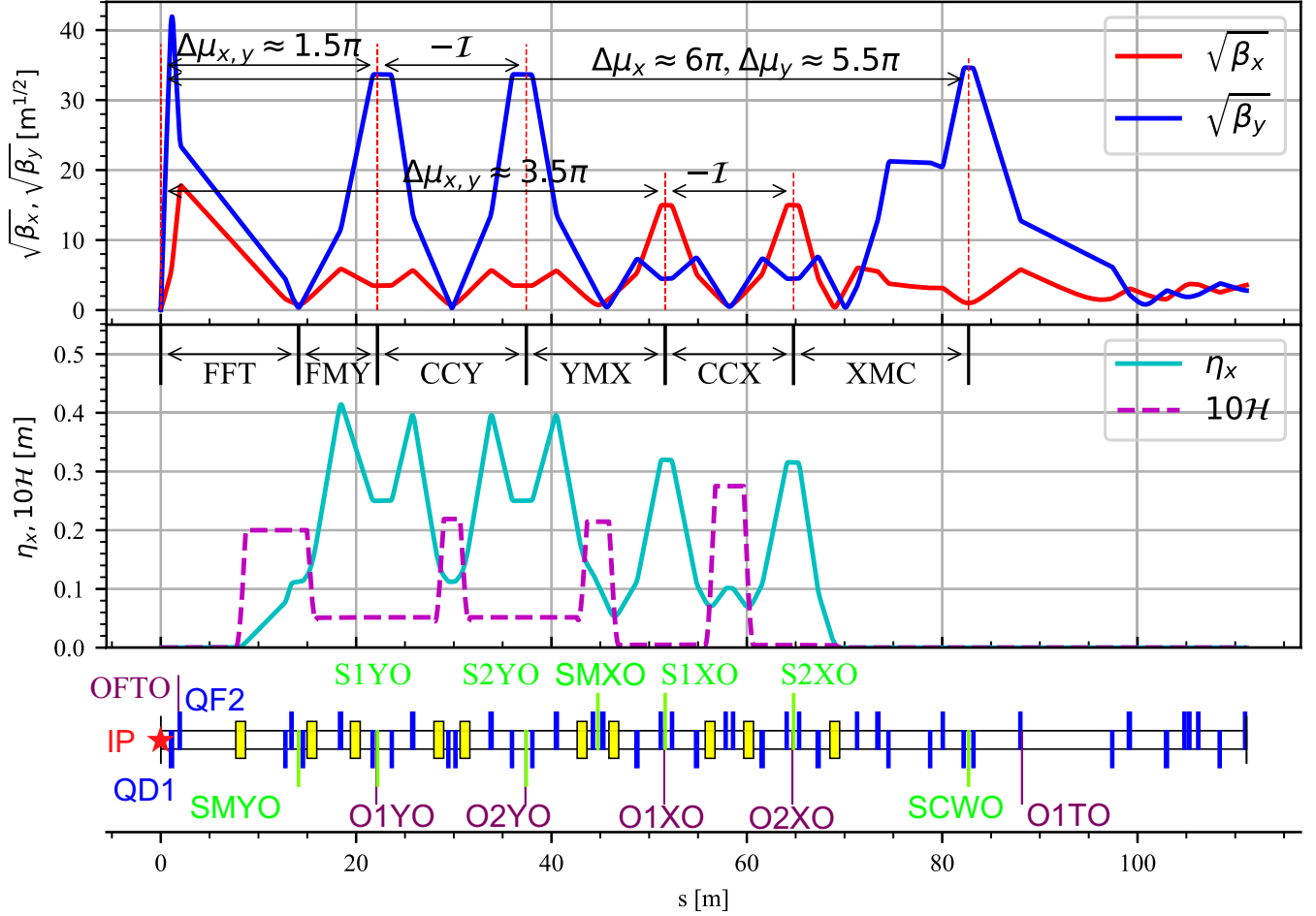


FIG. 1. Linear optics and lattice layout of the right-side IR for the one-fold STCF, where $\mathcal{H}$ is defined in Eq. (31). Yellow, blue, and cyan rectangles denote dipoles, quadrupoles, and sextupoles, respectively, while purple lines denote thin octupoles.

crab-waist sextupoles as independent knobs. In the global hybrid correction scheme [27], the crab-waist transformation is combined with chromatic correction through a controlled strength imbalance of sextupole pairs. The STCF IR adopts the LCC approach because its locality and independent knobs are advantageous for controlling higher-order chromaticity and amplitude-dependent detuning, which are critical for preserving MA and DA at low beam energy.

A. Modular design for the IR

The one-fold STCF IR, based on the LCC scheme, is shown in Fig. 1. The linear optics functions are calculated using our in-house Python package PAMKIT [28], which has been benchmarked against SAD [29]. The modular IR-design concept here has also been adapted to the current nearly twofold configuration [30]. From the IP to the right-side crab sextupole (SCWO), the optics is arranged as: final-focus telescope (FFT) → matching to vertical chromatic correction (FMY) → vertical chromatic correction

(CCY) → matching (YMX) → horizontal chromatic correction (CCX) → matching to the crab sextupoles (XMC). On the left side of the IP, the optics follows a similar structure, with slight modifications in the bending magnets to accommodate the finite crossing angle geometry. In the remainder of this section, we first discuss the physical principles underlying the nonlinear optimization and then describe the configurations of multipole magnets used for local nonlinear correction.

A central limitation of CW colliders is the interplay between the crab-sextupole pair and the nonlinearities accumulated between them [26,31]. Although the leading geometric terms cancel for particles close to the design orbit under appropriate phase conditions [32], this cancellation is degraded by momentum deviation and large betatron amplitudes. Robust performance therefore requires the optics between the crab sextupoles to remain nearly transparent, i.e., weakly dependent on both momentum and amplitude. This motivates the two complementary optimizations discussed below: momentum-dependent chromatic control and amplitude-dependent detuning control.

B. Optimization of momentum-dependent nonlinearities

Momentum-dependent nonlinearities are characterized by the tune expansion

$$\nu_u = \nu_{u0} + \xi_{u1}\delta + \xi_{u2}\delta^2 + \xi_{u3}\delta^3, \quad (6)$$

where $u \in \{x, y\}$, ν_{u0} is the nominal tune, and $\xi_{u1,2,3}$ denote the first- to third-order chromaticities. Including contributions from quadrupoles, sextupoles, and octupoles, the chromaticities through third order can be written as [33,34]

$$\xi_{u1} = \frac{\delta_u}{4\pi} \int_0^C \beta_u(s) G_1(s) ds, \quad (7)$$

$$\begin{aligned} \xi_{u2} = & -\xi_{u1} - \frac{\delta_u}{4\pi} \int_0^C \beta_u(s) G_2(s) ds \\ & + \frac{\delta_u}{8\pi} \int_0^C \beta_u(s) B_{u1}(s) G_1(s) ds, \end{aligned} \quad (8)$$

$$\begin{aligned} \xi_{u3} = & \xi_{u1} - \frac{\delta_u}{4\pi} \int_0^C \beta_u(s) W_{u1}^2(s) G_1(s) ds \\ & + \frac{\delta_u}{4\pi} \int_0^C \beta_u(s) (G_2(s) + G_3(s)) ds \\ & + \frac{\delta_u}{4\pi} \int_0^C \beta_u(s) B_{u2}(s) G_1(s) ds, \end{aligned} \quad (9)$$

with $\delta_u = -1$ for $u = x$ and $\delta_u = 1$ for $u = y$. The gradient-dependent functions are

$$G_1(s) = K_1(s) - K_2(s)\eta_{x0}(s), \quad (10)$$

$$G_2(s) = K_2(s)\eta_{x1}(s) + \frac{1}{2}K_3(s)\eta_{x0}^2(s), \quad (11)$$

$$G_3(s) = -K_2(s)\eta_{x2}(s) - K_3(s)\eta_{x0}(s)\eta_{x1}(s), \quad (12)$$

where $K_{1,2,3}(s)$ are the normalized strengths of quadrupoles, sextupoles, and octupoles, respectively. The horizontal orbit is expanded as

$$x(s, \delta) = \eta_{x0}(s)\delta + \eta_{x1}(s)\delta^2 + \eta_{x2}(s)\delta^3, \quad (13)$$

defining the dispersion functions up to third order. For simplicity, we assume that the on-momentum orbit vanishes (i.e., $x_0(s) = 0$), thereby neglecting feed-down effects arising from nonzero orbit offsets.

Off-momentum optical distortions enter Eqs. (8) and (9) through the Montague functions [35]. The first-order Montague W function is

$$W_{u1}(s) = \sqrt{A_{u1}^2(s) + B_{u1}^2(s)}, \quad (14)$$

with

$$A_{u1} = \frac{\partial \alpha_u}{\partial \delta} - \frac{\alpha_u}{\beta_u} \frac{\partial \beta_u}{\partial \delta} \Big|_{\delta=0}, \quad B_{u1} = \frac{1}{\beta_u} \frac{\partial \beta_u}{\partial \delta} \Big|_{\delta=0}, \quad (15)$$

and explicit forms

$$A_{u1}(s) = -\frac{\delta_u}{2 \sin(2\pi\nu_u)} \int_0^C \beta_u(s') G_1(s') \sin(\phi_{ss'}) ds', \quad (16)$$

$$B_{u1}(s) = -\frac{\delta_u}{2 \sin(2\pi\nu_u)} \int_0^C \beta_u(s') G_1(s') \cos(\phi_{ss'}) ds', \quad (17)$$

where $\phi_{ss'} = 2\pi\nu_u - 2|\mu_u(s) - \mu_u(s')|$ and μ_u is the phase advance. Second-order analogs A_{u2}, B_{u2} are defined as [36,37]

$$\begin{aligned} A_{u2} = & \frac{1}{2} \left[\frac{\partial^2 \alpha_u}{\partial \delta^2} - \frac{\alpha_u}{\beta_u} \frac{\partial^2 \beta_u}{\partial \delta^2} \right] \Big|_{\delta=0}, \\ B_{u2} = & \frac{1}{2\beta_u} \frac{\partial^2 \beta_u}{\partial \delta^2} \Big|_{\delta=0}. \end{aligned} \quad (18)$$

We follow [36] to take Eqs. (15) and (18) as Montague functions. In the present work, the off-momentum beta-beating terms are retained up to second order, which is sufficient to describe the dominant distortions relevant for the present IR optimization; higher-order extensions exist but are not required here [36,37]. For simplicity, the bending-magnet contribution to chromaticity is neglected in the analytical discussion below.

1. Local chromaticity compensation in the IR

The dominant IR chromaticity source is the final doublet of the quadrupoles (FD). To illustrate the local correction principle, we consider the FFT-CCY segment in Fig. 2. On the right of the IP, the FD is formed by QD1 and QF2; the vertical chromaticity is corrected locally by the CCY sextupole pair S1YO/S2YO, separated by a $-\mathcal{I}$ section to cancel the leading geometric terms [32]. Neglecting other elements, the first-order contributions from FD quadrupoles and CCY sextupoles follow from Eq. (7):

$$\xi_{yQ} = \sum_i \langle \beta_y K_1 \rangle_{Q_i}, \quad \xi_{yS} = \sum_j \langle \beta_y K_2 \eta_{x0} \rangle_{S_j}, \quad (19)$$

where $\langle \cdot \rangle$ denotes integration over finite magnet length. It should be noted that ξ_{yQ} and ξ_{yS} are not exactly the first-order chromaticity (without the $1/(4\pi)$ factor and the opposite sign of ξ_{yS}) as defined in Eq. (7). Local cancellation requires $\xi_{yQS} \equiv \xi_{yQ} - \xi_{yS} = 0$. In CW optics, the very large β functions in the FD imply strong sextupoles for local compensation, which makes controlling higher-order chromaticity and detuning essential.

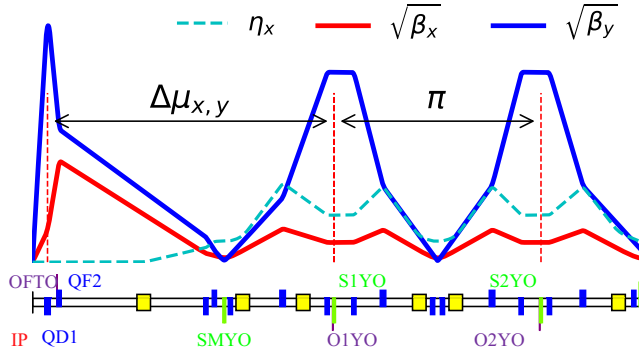


FIG. 2. Schematic layout of the FD quadrupoles (QD1/QF2) and the CCY sextupole pair (S1YO/S2YO) in the one-fold STCF IR. Here, $\Delta\mu_{x,y}$ denotes the horizontal and vertical phase advances from QD1 to S1YO.

2. Phase constraints for higher-order chromaticity

Second- and third-order chromaticities depend on the dispersions and off-momentum beta beating through Montague functions in Eqs. (8) and (9). Because the FD produces both large β and a relatively small phase advance between QD1 and QF2, it is beneficial to choose the phase advance from QD1 to the CCY sextupoles close to an integer multiple of π (see Fig. 2), yielding the approximation

$$B_{y1}(s) \approx -\frac{\cos \Pi_y(s)}{2 \sin(2\pi\nu_y)} \xi_{yQS}, \quad (20)$$

with $\Pi_y(s) = 2(\pi\nu_y - |\mu_{yQ1} - \mu_y(s)|)$ and μ_{yQ1} the phase at QD1. Under the conditions $\xi_{yQS} = 0$ and $\Delta\mu_y \simeq k\pi$, the residual second-order chromaticity is dominated by second-order dispersion:

$$\xi_{y2} \approx -\frac{1}{4\pi} \sum_j \langle \beta_y K_2 \eta_{x1} \rangle_{s_j}, \quad (21)$$

and, analogously, ξ_{y3} is governed mainly by η_{x1} and η_{x2} when the local first-order cancellation and the phase constraint are satisfied.

3. Sensitivity to phase-advance deviations

Deviations from the ideal phase relations (see Fig. 2), between QD1 and QF2 ($\Delta\mu_{yQQ}$), between QD1 and S1YO ($\Delta\mu_{yQS}$), and between S1YO and S2YO ($\Delta\mu_{ySS}$), perturb the chromaticities. In design, these deviations can be used as weak knobs to redistribute higher-order chromaticity; in operation, they must be tightly controlled and/or implemented as beam-based tunable knobs. For small $\Delta\mu_{yQS}$, the induced variations are approximately

$$\Delta\xi_{y2} \approx \frac{\Delta\mu_{yQS}}{4\pi} \xi_{yQ} \xi_{yS}, \quad (22)$$

$$\begin{aligned} \Delta\xi_{y3} \approx & -\frac{\Delta\mu_{yQS}}{2\pi} \xi_{yQ} \xi_{yS} + \frac{\Delta\mu_{yQS}}{4\pi} \xi_{yQ} \sum_j \langle \beta_y K_2 \eta_{x1} \rangle_{s_j} \\ & - \frac{\Delta\mu_{yQS}}{2\pi \tan(2\pi\nu_y)} \xi_{yQ} \xi_{yS} \xi_{yQS}. \end{aligned} \quad (23)$$

Similarly, small deviations inside the FD or inside the CCY pair produce second-order contributions

$$\Delta\xi_{y2} = \frac{\Delta\mu_{yQQ}}{4\pi} (\xi_{yQS} - \langle \beta_y K_1 \rangle_{Q2}) \langle \beta_y K_1 \rangle_{Q2}, \quad (24)$$

$$\Delta\xi_{y2} = \frac{\Delta\mu_{ySS}}{4\pi} (\xi_{yQS} - \langle \beta_y K_2 \eta_{x0} \rangle_{s2}) \langle \beta_y K_2 \eta_{x0} \rangle_{s2}. \quad (25)$$

When the local cancellation condition $\xi_{yQS} = 0$ is satisfied, Eqs. (23)–(25) can be further simplified. Nevertheless, the expressions presented here remain valid for $\xi_{yQS} \neq 0$, as discussed further in the following paragraph. The above expressions are presented for the vertical plane; analogous formulations can be derived for the horizontal plane and are omitted here for brevity. These relations highlight a general feature of the low-beta IRs with local chromaticity correction: the very large β functions at the FD and at the local chromatic sextupoles amplify the chromatic sensitivity to phase errors, making phase control a critical design and operational requirement.

4. Practical considerations

The discussion above isolates the FD-CCY subsystem to expose the basic correction mechanism. In the full IR, however, additional quadrupoles and matching sections, as illustrated in Fig. 1, modify these idealized conditions and contribute further to the chromaticity. In particular, the local cancellation condition $\xi_{yQS} = 0$, derived by considering only the quadrupoles in the FD and the sextupoles in the CCY section, is no longer satisfied exactly. These additional contributions are included in the full IR optimization and global matching of the STCF ring optics. Additional sextupoles at IP image points (locations with a phase advance of an integer multiple of π with respect to the IP), such as SMYO and SMXO in Fig. 1, are introduced to provide further flexibility for higher-order chromatic correction [26].

C. Optimization of amplitude-dependent nonlinearities

Amplitude-dependent nonlinearities primarily appear as nonlinear tune shifts, which degrade the effective cancellation of the crab-sextupole pair and reduce the DA. In the IR, the dominant contribution is typically the first-order amplitude-dependent tune shift (ADT), so the optimization targets the leading detuning coefficients:

$$\begin{aligned} \Delta\nu_x &= \alpha_{xx} J_x + \alpha_{xy} J_y, \\ \Delta\nu_y &= \alpha_{yx} J_x + \alpha_{yy} J_y, \end{aligned} \quad (26)$$

with $\alpha_{xy} \simeq \alpha_{yx}$ in most practical cases. In CW IRs, the main contributions arise from: (i) the kinematic term in the final drift, (ii) Maxwellian fringe fields of the FD quadrupoles, and (iii) finite-length effects of strong $-\mathcal{I}$ sextupole pairs [38]. Below, we employ the analytical expressions of Ref. [38] and apply them directly to the STCF optics.

1. Kinematic term in the final drift

For the drift between the IP and QD1 (length L^*), modeling QD1 as a thin lens at its center adds $L_q/2$ to the effective drift length and yields [31,37]

$$\begin{aligned}\alpha_{xx} &= \frac{3}{16\pi} \oint \gamma_x^2(s) ds \approx \frac{3}{16\pi} \frac{(L^* + \frac{L_q}{2})}{\beta_x^{*2}}, \\ \alpha_{xy} &= \frac{1}{8\pi} \oint \gamma_x(s) \gamma_y(s) ds \approx \frac{1}{8\pi} \frac{(L^* + \frac{L_q}{2})}{\beta_x^* \beta_y^*}, \\ \alpha_{yy} &= \frac{3}{16\pi} \oint \gamma_y^2(s) ds \approx \frac{3}{16\pi} \frac{(L^* + \frac{L_q}{2})}{\beta_y^{*2}},\end{aligned}\quad (27)$$

where $\gamma_{x,y}(s)$ denote the Twiss parameters along the beam line and $\beta_{x,y}^*$ are the beta functions at the IP. Because CW colliders require very small $\beta_{x,y}^*$, this contribution can be significant even when L^* is constrained by the machine-detector interface [30].

2. FD fringe fields

Fringe-field contributions from the FD can be written as [38]

$$\begin{aligned}\alpha_{xx} &= -\frac{1}{32\pi} \oint K_1'' \beta_x^2 ds \approx -\frac{1}{8\pi} \sum_i \delta_i K_{1,i} \alpha_{x,i} \beta_{x,i}, \\ \alpha_{xy} &= \frac{1}{8\pi} \oint K_1' (\alpha_y \beta_x - \alpha_x \beta_y) ds \\ &\approx \sum_i \frac{\delta_i K_{1,i}}{8\pi} (\alpha_{y,i} \beta_{x,i} - \alpha_{x,i} \beta_{y,i}), \\ \alpha_{yy} &= \frac{1}{32\pi} \oint K_1'' \beta_y^2 ds \approx \frac{1}{8\pi} \sum_i \delta_i K_{1,i} \alpha_{y,i} \beta_{y,i},\end{aligned}\quad (28)$$

where $K_{1,i}$ denotes the normalized gradient of the quadrupole adjacent to the i th fringe field, $\alpha_{(x,y),i}$ and $\beta_{(x,y),i}$ are the corresponding Twiss parameters, and $\delta_i = 1$ (-1) denotes the entrance (exit) fringe, respectively. The exceptionally large β functions and strongly asymmetric edge optics of the FD make these terms particularly important, often dominating the cross-plane detuning.

3. $-\mathcal{I}$ sextupole pairs and detuning knobs

A thick $-\mathcal{I}$ sextupole pair with small phase errors $\Delta\varphi_{x,y}$, as illustrated in Fig. 3, contributes to detuning as [33,39]

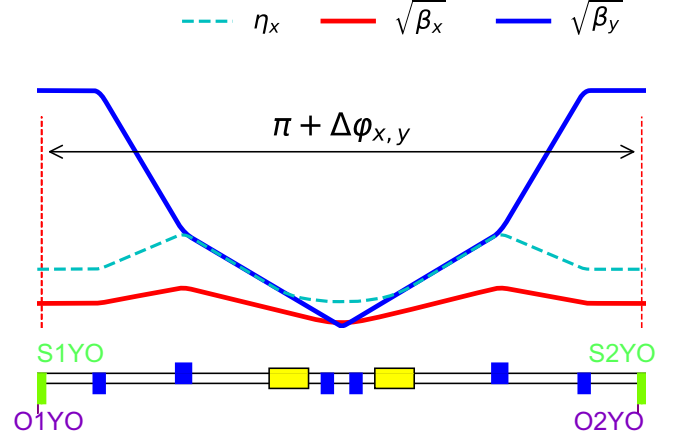


FIG. 3. Schematic of the thick sextupole pair S1YO/S2YO connected by a nonexact $-\mathcal{I}$ transformation. The phase advances from the center of S1YO to the center of S2YO are adjusted to $\pi + \Delta\varphi_{x,y}$ in the horizontal and vertical planes.

$$\begin{aligned}\alpha_{xx} &= -\frac{(K_2 L)^2 \beta_x^2 (L - 3\Delta\varphi_x \beta_x)}{16\pi}, \\ \alpha_{xy} &= -\frac{(K_2 L)^2 \beta_x \beta_y (L - 3(\Delta\varphi_x \beta_x - 2\Delta\varphi_y \beta_y))}{24\pi}, \\ \alpha_{yy} &= -\frac{(K_2 L)^2 \beta_y^2 (L - 3\Delta\varphi_y \beta_y)}{16\pi},\end{aligned}\quad (29)$$

where K_2 , L are the sextupole strength and length. This relation clarifies how small phase adjustments in CCY/CCX can selectively tune ADT components (notably α_{xy} and α_{yy} in regions with $\beta_y \gg \beta_x$), as suggested in Ref. [26].

4. Octupoles

A flexible and largely orthogonal control of detuning is provided by dedicated octupoles:

$$\begin{aligned}\alpha_{xx} &= \frac{1}{16\pi} \oint K_3(s) \beta_x^2(s) ds, \\ \alpha_{xy} &= -\frac{1}{8\pi} \oint K_3(s) \beta_x(s) \beta_y(s) ds, \\ \alpha_{yy} &= \frac{1}{16\pi} \oint K_3(s) \beta_y^2(s) ds,\end{aligned}\quad (30)$$

where $K_3(s)$ is the octupole strength. The tuning efficiency is therefore dictated by the local optics: regions with $\beta_y \gg \beta_x$ are optimal for α_{yy} control, $\beta_x \gg \beta_y$ for α_{xx} , and comparable β_x, β_y for α_{xy} . For an LCC IR, this naturally suggests octupole locations near CCY (see Fig. 3, to tune α_{yy}), near CCX (to tune α_{xx}), and near the FD (to tune α_{xy}).

D. Application to the STCF IR

Following the principles discussed above, the STCF IR implements a compact LCC-based CW layout with dedicated local knobs for chromaticity correction, crab-waist

transformation, and detuning control. Referring to Fig. 1 for the right-side IR, the sextupole pairs S1YO/S2YO and S1XO/S2XO provide vertical and horizontal chromaticity correction, respectively, primarily providing the first- and second-order chromaticity correction. The IP image point sextupoles SMYO and SMXO are used for third-order chromaticity correction (see Eq. (9)). The octupole pairs O1YO/O2YO and O1XO/O2XO, placed near the chromaticity-correction sextupoles, provide local control of the dominant ADTs in the vertical and horizontal planes. To simplify the optimization workflow, the octupoles within the $-\mathcal{I}$ pairs are constrained to have identical strengths. The octupole OFTO near QF2 is used to correct the coupling ADTs α_{xy} and α_{yx} within the crab-sextupole pair, while O1TO outside the crab-sextupole pair provides an additional knob for global nonlinear optimization.

Chromatic optimization follows the analytical considerations outlined previously. The phase advances between FD and CCY/CCX are set close to integer multiples of π to suppress chromatic beta beating at the correction sextupoles, while each CCY/CCX pair is arranged close to form a $-\mathcal{I}$ transformation to cancel leading geometric aberrations. Within these constraints, the dispersions at the LCC sextupoles are chosen by balancing sextupole strength against MA. Larger dispersion reduces the required sextupole strength and hence geometric aberrations (see Eq. (29)) and also reduces sensitivity to orbit errors [26]; however, it increases the dispersion invariant

$$\mathcal{H} = \gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta'_x + \beta_x \eta_x'^2, \quad (31)$$

which enhances off-momentum distortions and can limit the local momentum acceptance (LMA) and Touschek lifetime. The STCF solution therefore adopts a moderate dispersion level that preserves both practical sextupole strengths and sufficient MA.

The CW phase conditions between the IP and the crab sextupoles are imposed in the standard form,

$$\Delta\mu_x = m\pi, \quad \Delta\mu_y = \left(n + \frac{1}{2}\right)\pi, \quad (32)$$

with integers m, n , while the surrounding matching sections are used to minimize the momentum and amplitude dependence of the optical functions at the crab sextupoles. The resulting octupole configuration follows the optics-based placement criteria discussed in Sec. III C. In the present design, $-\mathcal{I}$ phase tuning is not used in routine optimization in order to keep the on-momentum optics simple; the primary detuning control is instead provided by the dedicated octupoles.

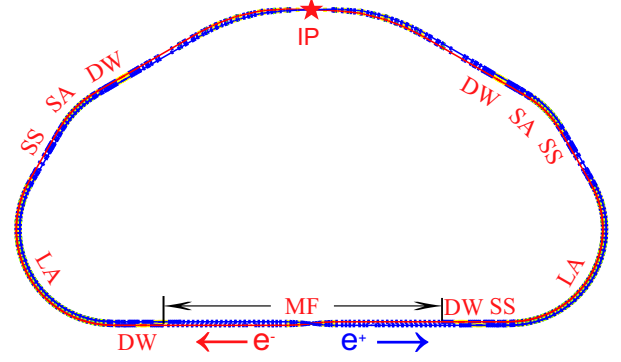


FIG. 4. Layout of the one-fold STCF collider rings.

IV. OPTICS OF TECHNICAL SECTIONS FOR THE ONE-FOLD STCF

A. Layout

Following the IR design described in the previous section, the remaining sections of the one-fold STCF ring are designed to connect the IR, provide the required radiation damping and polarization control, and preserve the optical conditions needed for global nonlinear optimization.

The lattice design is based on the preoptimized global parameters listed in Table I. As shown in Fig. 4, the collision ring consists of one IR, four arc sections including two long-arc (LA) sections and two short-arc (SA)

TABLE II. Optimized machine parameters of the one-fold STCF lattice.

Parameter	Unit	Value
Beam energy, E	GeV	2
Circumference, C	m	885.232
Beam current, I	A	2
Full crossing angle, 2θ	mrad	60
Beta functions at IP, β_x^*/β_y^*	mm	40/0.6
Coupling factor, k	%	0.50
Betatron tunes ν_x/ν_y	...	33.538/33.58
Horizontal emittance, ϵ_x (SR/IBS)	nm	5.77/6.22
Synchrotron tune, ν_s	...	0.0165
Momentum compaction factor, α_c	$\times 10^{-3}$	1.18
Energy spread, σ_e (SR/IBS)	$\times 10^{-4}$	9.70/9.91
Bunch population, N_p	$\times 10^{10}$	5.0
Energy loss per turn, U_0	keV	382.7
Damping times, $\tau_x/\tau_y/\tau_z$	ms	30.6/30.8/15.4
Harmonic number, h	...	1476
rf voltage, V_{rf}	MV	2.0
Bunch length, σ_z (SR/IBS)	mm	9.76/9.99
rf energy acceptance, δ_{rf}	%	1.62
Piwinski angle, ϕ	...	19.0
Beam-beam parameters, ξ_x/ξ_y	...	0.0025/0.084
Hourglass factor, F_h	...	0.9235
Theoretical peak luminosity, L	$\text{cm}^{-2} \text{s}^{-1}$	1.1×10^{35}

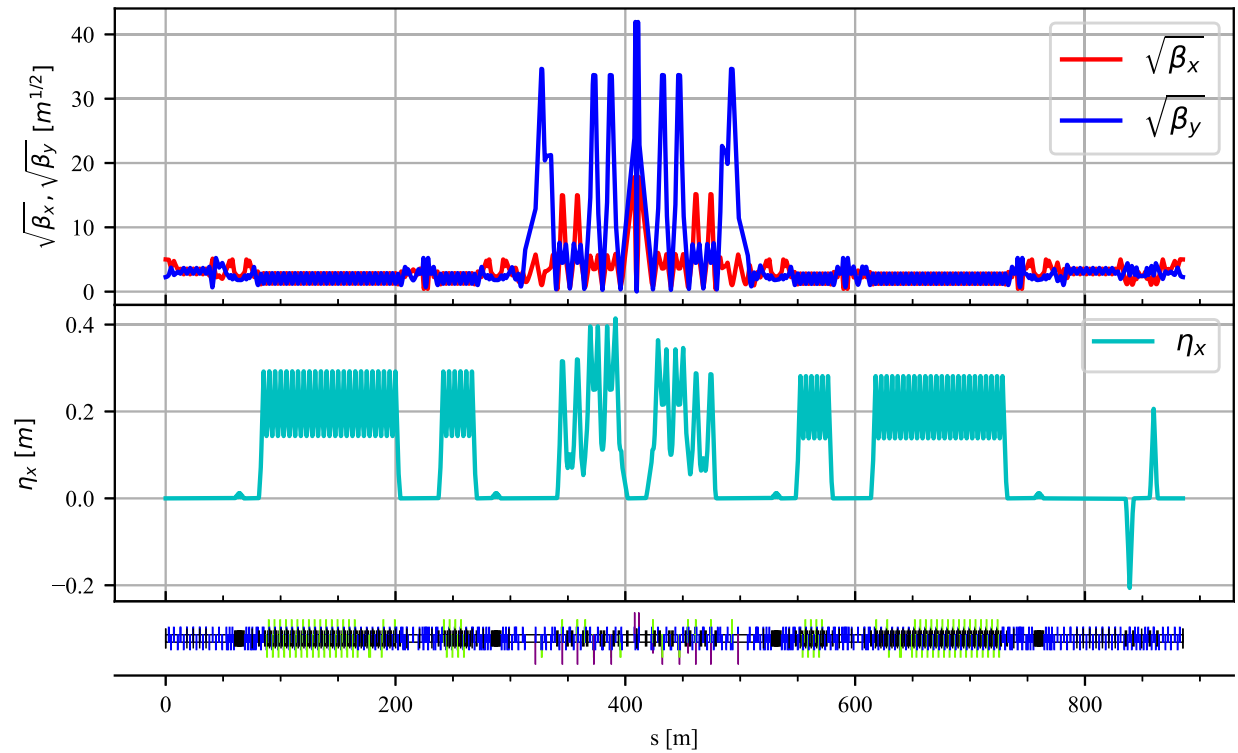


FIG. 5. Full-ring linear optics and lattice layout of the one-fold STCF, with $s = 0$ corresponding to the injection point.

sections, three Siberian snake (SS) sections, four damping-wiggler (DW) sections, and one multifunction (MF) region. The positron ring and electron ring are mirror-symmetric about the IP. The dual-ring configuration is realized by inner and outer half-rings that differ only in dipole curvature radii within the arc sections, while all other complex units share identical optical structures.

The full linear optics functions of the collision ring are presented in Fig. 5. The detailed design of these sections is discussed in the following subsections. After integrating and jointly optimizing all sections under these constraints, the resulting machine parameters for the one-fold STCF are summarized in Table II.

B. Arc sections

The arc sections are designed to realize the dual-ring geometry within a common tunnel. To accommodate the dual-ring configuration, each ring is divided into inner and outer half-rings. Each half-ring consists of one long arc with a total bending angle of 120° and one short arc with a bending angle of 30° , as illustrated in Fig. 4. The choice of these bending angles ensures uniform azimuthal distribution of the three Siberian snakes, which is necessary to achieve longitudinal polarization of the electron beam at the IP.

Both long and short arcs are constructed from FODO cells with horizontal and vertical phase advances of 90° per cell and a dipole bending angle of 5° . The inner and outer

arcs share the same layout, differing only in the dipole curvature radii. The inner half-ring uses dipoles with a curvature radius of 10 m, whereas the outer half-ring employs a curvature radius of 12.14 m. Their centers coincide, so that the transverse separation between the electron and positron beam centers remains constant at 2.14 m throughout the arc sections. Quadrupole, sextupole, and drift lengths are kept identical in both half-rings in order to preserve symmetry and simplify correction strategies. Figures 6 and 7 present the lattice and corresponding linear optics functions of the long and short-arc sections in the outer half-ring.

The short-arc region consists of five periodic FODO cells. Among them, four consecutive FODO cells are configured such that a sextupole is placed downstream of each quadrupole for correcting both horizontal and vertical chromaticity, and these four FODO cells are completely identical (including the strengths of their sextupoles). This configuration represents a balance between mitigating nonlinear resonances and correcting chromaticity. A high-filling scheme is beneficial for locally correcting the chromaticity of the IR matching section and DW sections as much as possible, which helps achieve good off-momentum dynamics performance. These four identical FODO cells form a higher-order achromat (HOA) structure [32], in which first- and second-order geometric driving terms largely cancel. This arrangement significantly reduces nonlinear resonance excitation while maintaining strong chromatic correction capability. The nonlinearity

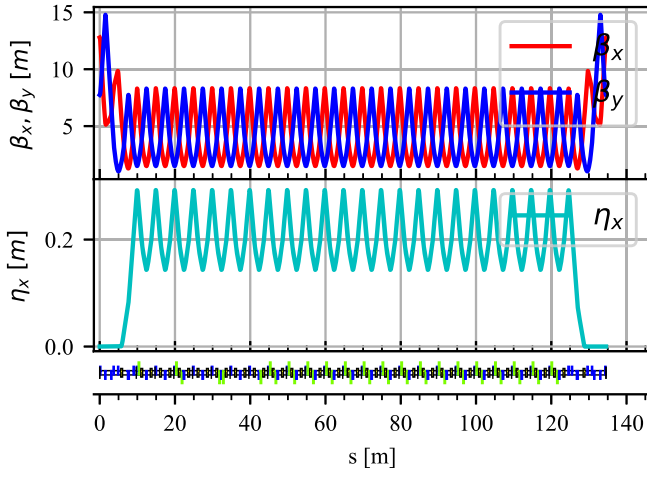


FIG. 6. Linear optics and lattice layout of the long-arc section in the outer half-ring of the one-fold STCF.

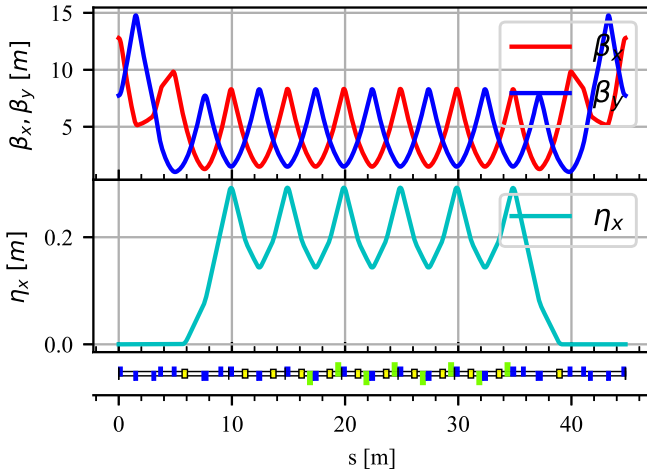


FIG. 7. Linear optics and lattice layout of the short-arc section in the outer half-ring of the one-fold STCF.

configured in this way is better than that in the case where sextupoles are installed downstream of quadrupoles in all five periodic FODO cells.

The long arc contains 23 FODO cells and is organized into two functional regions. The first seven cells employ three noninterleaved $-\mathcal{I}$ sextupole pairs for chromaticity correction and Montague W -function control. Since the dispersion function is smaller near locations of maximum vertical β , two sextupole pairs are dedicated to vertical chromaticity correction and one pair to horizontal correction, thereby improving correction efficiency while limiting nonlinear side effects. In the remaining sixteen cells, a higher sextupole filling scheme is adopted, with one sextupole placed downstream of each quadrupole. These sextupoles form interleaved $-\mathcal{I}$ pairs. Every four identical FODO cells also constitute a HOA structure. In this region, the Montague W function oscillates periodically and is

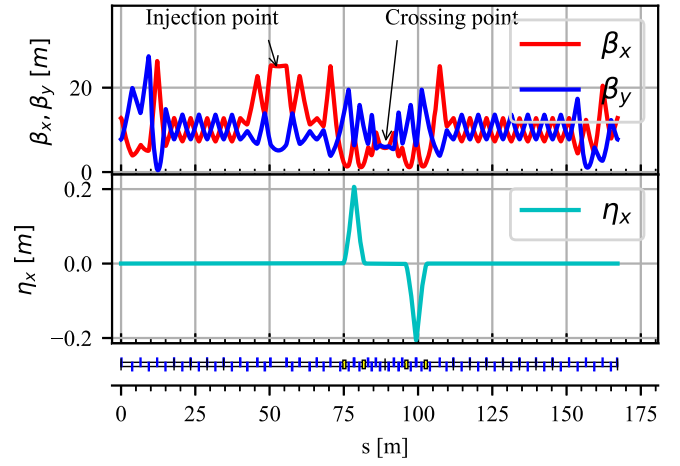


FIG. 8. Linear optics and lattice layout of the multifunction section for the one-fold STCF.

approximately matched at the entrance and exit of the module, preserving optical transparency to adjacent sections.

Both long and short arcs are connected to the straight sections through dispersion-suppressing matching sections.

C. Multifunction section

The multifunction region is the section located opposite to the IR (see Fig. 4), with a total bending angle of zero. It serves as the primary operational section of the ring, providing both geometric connection and optical matching between the inner and outer half-rings, while accommodating essential subsystems such as rf cavities and injection elements. The injection point and crossing point locations are labeled in Fig. 8. With a crossing angle of 0.2 rad, the most compact magnet spacing occurs for the first quadrupole adjacent to the crossing point: this element has a gradient of 3.4 T/m, with a maximum allowable distance of 200 mm from the beam center to the magnet edge. Preliminary checks confirm sufficient clearance, though further optimization of the local magnet layout will be performed in subsequent work.

From the optics perspective, this region is configured to offer flexibility in tuning the working point over a wide range, with minimal impact on amplitude-dependent detuning coefficients. At the same time, the optics is arranged to preserve chromatic properties, particularly first- and higher-order chromaticities, as much as possible, thereby avoiding additional nonlinear burden on the correction system. The resulting linear optics functions are presented in Fig. 8.

D. Straight sections

1. Damping Wiggler section

To provide flexibility in controlling radiation damping time, emittance, and beam parameters across the operating

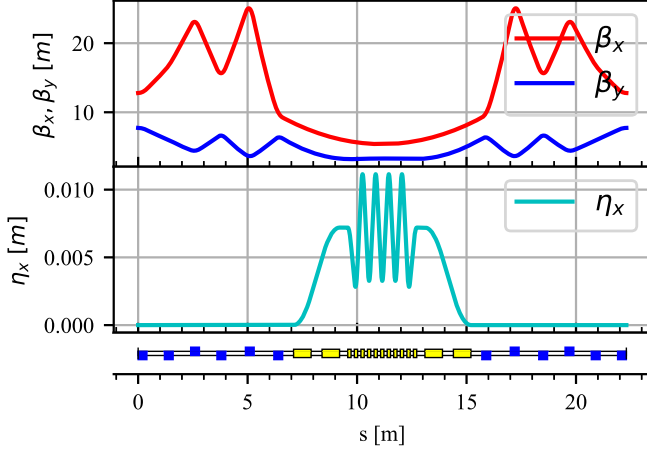


FIG. 9. Linear optics and lattice layout of the damping-wiggler section for the one-fold STCF.

beam energy range of 1.0–3.5 GeV, two damping wigglers are installed in each of the inner and outer half-rings, as shown in Fig. 4. The lattice and linear optics of the damping-wiggler sections are shown in Fig. 9. The wigglers are placed at the center of dedicated straight sections. A controlled nonzero dispersion is generated at the wiggler location by bending magnets positioned on both sides.

By adjusting the magnetic field strength of these bending magnets, the local dispersion at the wiggler can be tuned, allowing controlled modification of the horizontal emittance and damping time. This feature is particularly important when the beam energy changes, as it enables compensation of energy-dependent variations in damping time and equilibrium beam size while preserving the overall lattice structure. Six quadrupole families are used for optics matching, ensuring that the working point and beta functions remain essentially unchanged when the wiggler field is varied. In this way, damping optimization can be achieved without perturbing the global tune or chromatic correction scheme.

2. Siberian snake section

Siberian snake sections are introduced to achieve longitudinal polarization of the electron beam at the IP. The snakes are distributed around the ring, with equal azimuthal spacing of 120° between neighboring sections. Figure 10 shows the lattice configuration together with the corresponding linear optics functions of a representative snake section.

In the second phase of STCF operation, collisions with polarized electron beam will be implemented, and solenoids will be installed at both ends of each snake section. To preserve longitudinal polarization at the IP and to compensate the transverse coupling and betatron perturbations induced by the solenoidal fields, the optics is carefully matched to realize an $\mathcal{I}$ transformation in the

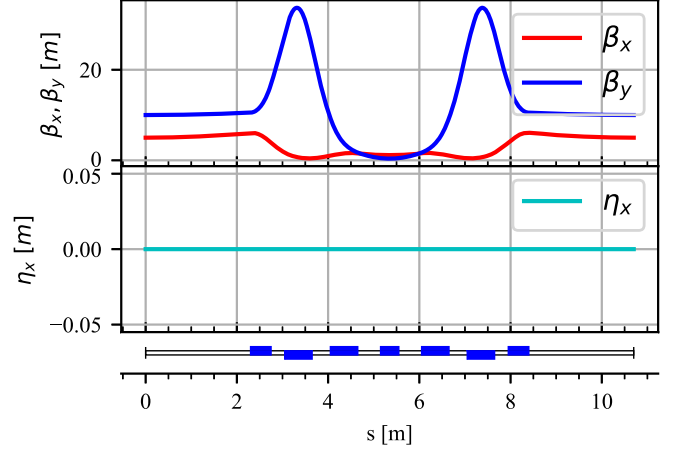


FIG. 10. Linear optics and lattice layout of the Siberian-snake section in the one-fold STCF. Solenoids are not included in this schematic.

horizontal plane and a $-\mathcal{I}$ transformation in the vertical plane. Three Siberian snake sections are installed per ring to maintain symmetry. Notably, the Siberian snakes in the positron ring are not equipped with solenoids in either Phase 1 or Phase 2 of the STCF project.

V. GLOBAL NONLINEAR OPTIMIZATION AND LATTICE PERFORMANCE

After the IR and technical sections of the one-fold lattice have been individually designed and locally optimized, they are integrated into the full ring, as illustrated in Fig. 4. The corresponding linear optics is shown in Fig. 5. The remaining performance limitations arise primarily from global nonlinear effects, which require coordinated optimization across the complete one-fold ring. This section presents the global nonlinear-optimization strategy and the resulting DA, MA, and Touschek lifetime.

The nonlinear multiobjective optimization uses sextupole and octupole strengths as control variables. The sextupole variables consist of 22 individual or paired magnets, including the IR chromatic sextupoles, IP image point sextupoles, arc $-\mathcal{I}$ pairs, and HOA sextupoles. Selected IR octupoles are also varied when ADT control is included. Their locations and local functions are described in Sec. III.

During the nonlinear optimization, the first-order chromaticities are constrained to small positive values, typically within $1 \lesssim \xi_{x1}, \xi_{y1} \lesssim 3$, to maintain beam stability. With this constraint imposed, a two-stage optimization strategy is then applied, as described in the following subsections.

A. Stage 1: Analysis-driven nonlinear optimization

The first stage targets the global nonlinear properties that primarily limit the DA and Touschek lifetime, while

avoiding the computational cost of long-term 6D tracking. Fast surrogate indicators are used, including resonance driving terms evaluated following Ref. [33], the first-order Montague W function, and the chromatic β -beating coefficients B_{x1} and B_{y1} at selected locations such as the IP and the centers of the crab-waist sextupoles. These quantities provide efficient measures of the dominant nonlinear behavior and allow a broad exploration of the high-dimensional parameter space.

A key control knob in this stage is the phase advance between the IR and arc sections, as well as between adjacent arc modules. These phase relations determine how sextupole-driven nonlinear terms and chromatic beta beating accumulate or cancel around the ring. Using the PAMKIT package [28], the phase advances in the straight sections are varied using built-in Tuning elements. These elements apply a pure betatron phase rotation via a linear matrix for the reference (on-energy) particle; while applicable to all particles, the off-energy phase advance is intrinsically determined by the lattice optics rather than being actively matched. This adjustment preserves other lattice functions while allowing precise control of the on-energy optics.

The sextupole strengths are optimized together with these phase advances using a multiobjective genetic algorithm (NSGA-II) [40]. The objectives include minimizing the Montague W function at the IP, reducing chromatic beta beating at the crab-waist sextupoles, maximizing MA, and suppressing dominant nonlinear driving terms. The IR octupoles are adjusted as pairs of identical octupoles. They are initially tuned to compensate the leading ADTs, and during the sextupole optimization, their strengths are adjusted to keep the detuning coefficients within the target ranges $0 \leq \alpha_{xx} < 1000 \text{ m}^{-1}$, $0 \leq \alpha_{xy} < 2000 \text{ m}^{-1}$, and $0 \leq \alpha_{yy} < 10000 \text{ m}^{-1}$. Once an optimal phase configuration is identified, the virtual Tuning elements are replaced by a physically realizable

solution by adjusting quadrupole strengths in the straight sections.

Figures 11–13 show the on- and off-momentum DA and LMA, computed via the SAD code [29], for three representative lattices obtained in stage 1: lattice 1 without octupoles, lattice 2 with octupoles activated inside the CW sextupole pair, and lattice 3 with all IR octupoles enabled. As expected, ADT correction by octupoles enlarges the on-momentum DA. However, the additional octupole fields also enhance higher-order chromatic effects, reducing the off-momentum DA and LMA. The corresponding Touschek lifetimes evaluated with SAD are about 180, 320, and 310 s, respectively.

These results expose the main limitation of the analysis-driven stage: surrogate indicators are useful for identifying favorable nonlinear configurations, but they are insufficient to simultaneously optimize the on-momentum DA,

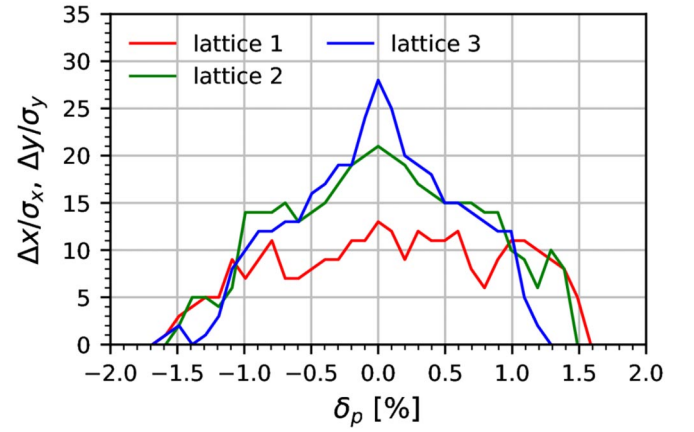


FIG. 12. Off-momentum dynamic aperture at injection point after stage 1 optimization for the three representative lattices. The initial transverse coordinates are chosen along $\Delta y/\sigma_y = \Delta x/\sigma_x$, corresponding to the dashed black line in Fig. 11.

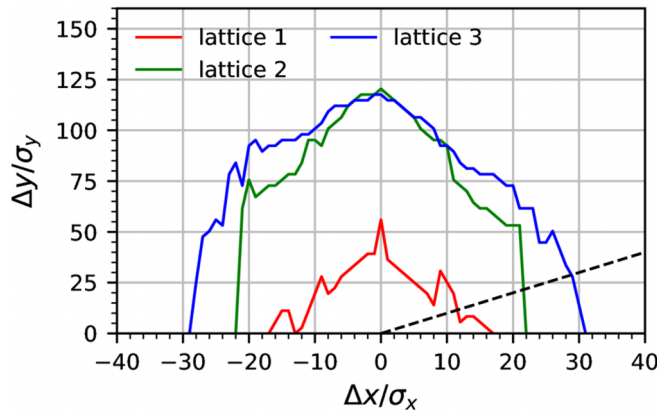


FIG. 11. On-momentum dynamic aperture at injection point after stage 1 optimization for the three representative lattices. The dashed black line is defined by $\Delta y/\sigma_y = \Delta x/\sigma_x$.

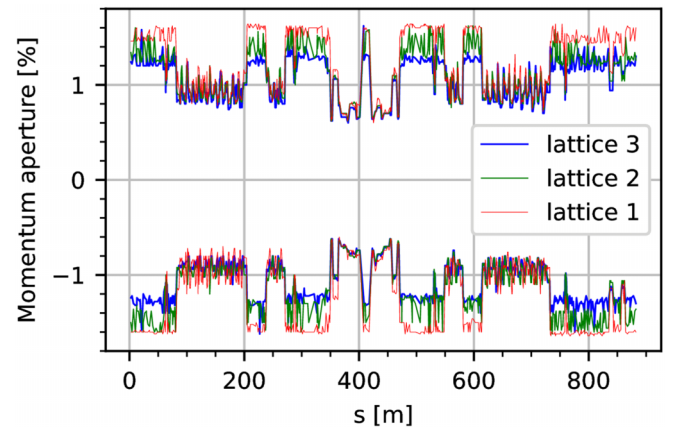


FIG. 13. Local momentum acceptance evaluated with SAD for the three representative lattices after stage 1 optimization.

off-momentum DA, and LMA. This motivates the tracking-based refinement described below.

B. Stage 2: Tracking-based optimization

Starting from the stage 1 solution, stage 2 directly optimizes lattice performance using full 6D particle tracking. In contrast to stage 1, which relies on analytical surrogate indicators, this stage evaluates the on-momentum DA, off-momentum DA, and Touschek lifetime from tracked particle stability. The phase advances obtained in stage 1 are kept fixed, while sextupole strengths and, when activated, selected octupole strengths are varied. Unlike in stage 1, the ADT coefficients are not explicitly constrained; their net impact is assessed through the tracked DA and lifetime. The first-order chromaticities are maintained within the prescribed positive range. Due to the computational cost of tracking, this stage is executed on high-performance computing resources.

To evaluate the Touschek lifetime efficiently, we use the fast Touschek lifetime tracking method [41], in which off-momentum particles are mapped to a reference reduced phase space (x, p_x, δ) and tested against the tracked stable region. In practice, the simplified implementation available in SAD [29] is used. The tracking model includes exact drift kinematics, Maxwellian fringe fields of the final doublet, and finite-length sextupoles.

The optimization is carried out by varying the strengths of 22 sextupoles (14 in the arcs and 8 in the IR, as described previously), along with selected octupoles when activated, using SAD. To reduce unnecessary tracking evaluations, several preselection criteria are imposed: (i) momentum acceptance exceeding $\pm 1.4\%$, (ii) Montague W function at the IP below 20, (iii) differences of chromatic beta-beatings B_{x1} and B_{y1} between the two crab sextupoles below 20, and (iv) Touschek lifetime from short (100-turn) tracking exceeding 300 s. Only lattices satisfying these conditions are further evaluated with more detailed tracking.

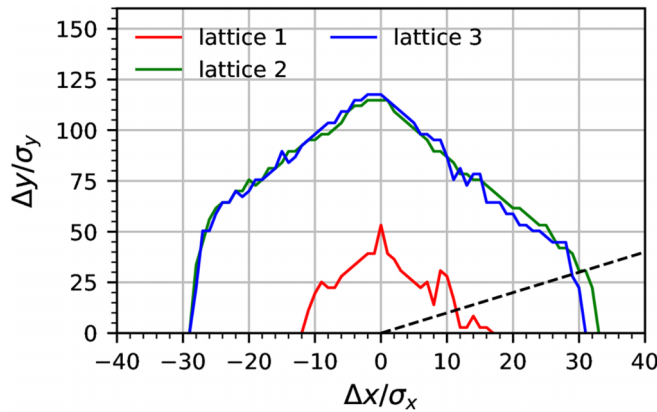


FIG. 14. On-momentum dynamic aperture at injection point after stage 2 optimization for the three representative lattices. The dashed black line is defined by $\Delta y/\sigma_y = \Delta x/\sigma_x$.

The optimization objectives are to maximize both the on-momentum DA and the Touschek lifetime. The DA is evaluated from 1000-turn tracking, and the lifetime is computed consistently based on the resulting off-momentum aperture. Initial particle coordinates are sampled in transverse phase space along the coordinate (x, y) and momentum (p_x, p_y) directions as

$$X_{1,ij} = \{n_i\sigma_x, 0, n_i\sigma_y, 0, 0, n_j\sigma_e\}, \quad (33)$$

$$X_{2,ij} = \{0, n_i\sigma_{p_x}, 0, n_i\sigma_{p_y}, 0, n_j\sigma_e\}, \quad (34)$$

where n_i and n_j denote the transverse and energy scan indices, respectively. The vertical amplitude is scaled with the horizontal amplitude to avoid physically undesirable small vertical apertures during the optimization. The

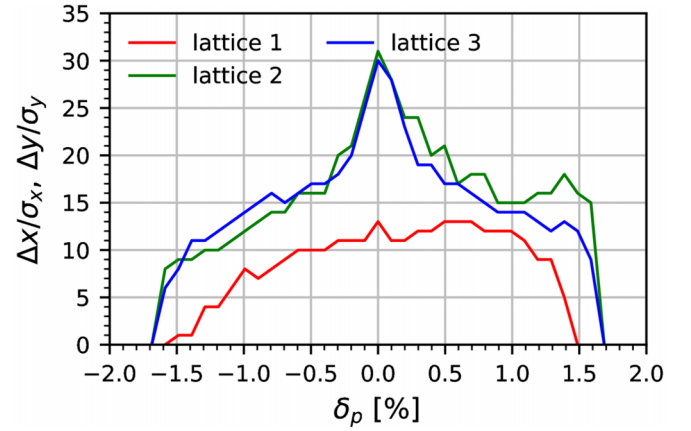


FIG. 15. Off-momentum dynamic aperture at injection point after stage 2 optimization for the three representative lattices. The initial transverse coordinates are chosen along $\Delta y/\sigma_y = \Delta x/\sigma_x$, corresponding to the dashed black line in Fig. 14.

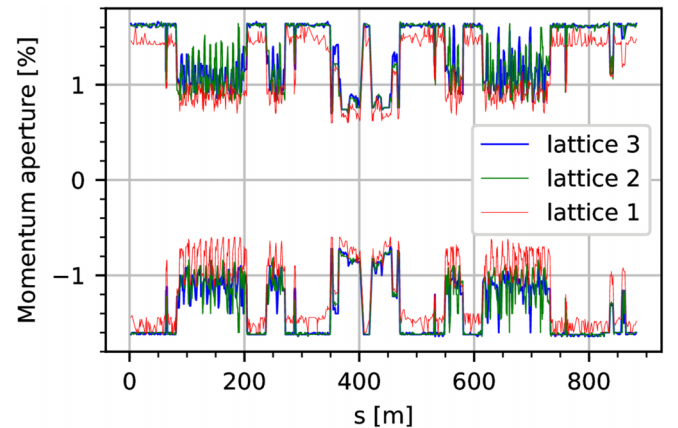


FIG. 16. Local momentum acceptance evaluated with SAD for the three representative lattices after stage 2 optimization.

off-momentum DA is defined as the intersection of the stable regions obtained from the coordinate- and momentum-dominated initial conditions.

To quantify the computational overhead of this stage 2 tracking-based optimization: the multiobjective genetic algorithm is executed for 120 generations with a population size of 1000. For each fitness evaluation, 270 particles are tracked for 1000 turns; a single evaluation requires

approximately 5 s when parallelized across 100 CPU cores on a computing cluster equipped with AMD EPYC 9654 processors.

Figures 14–16 show the resulting on- and off-momentum DA and LMA for the three representative lattices obtained in stage 2. Compared with Figs. 12 and 13, the tracking-based optimization substantially improves both the off-momentum DA and the LMA. The corresponding

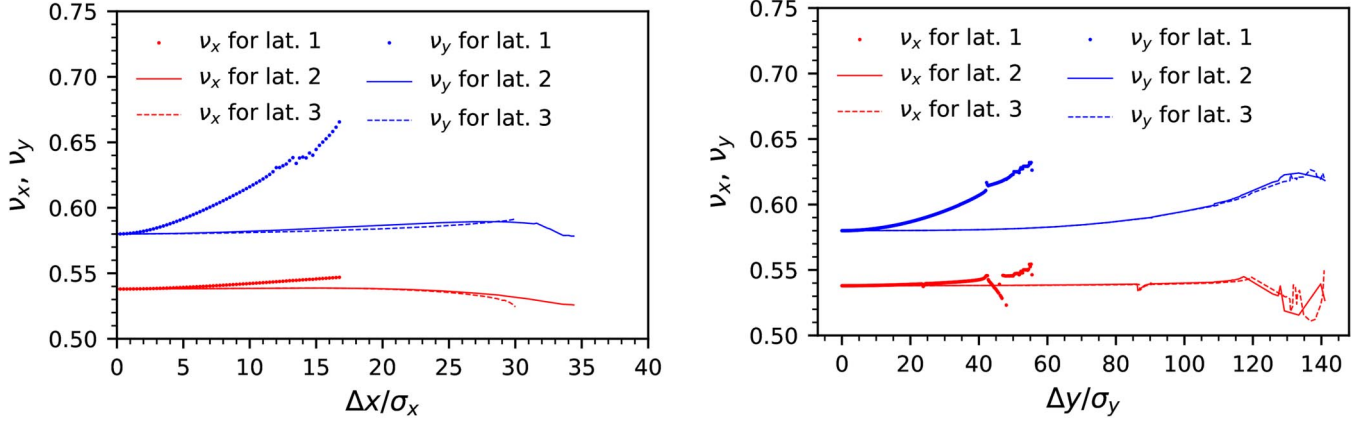


FIG. 17. Amplitude-dependent tune shifts with respect to the horizontal (left) and vertical (right) amplitudes. Lattices 1, 2, and 3 denote the lattices without octupoles, with octupoles inside the CW sextupole pair, and with octupoles both inside and outside the CW sextupole pair, respectively.

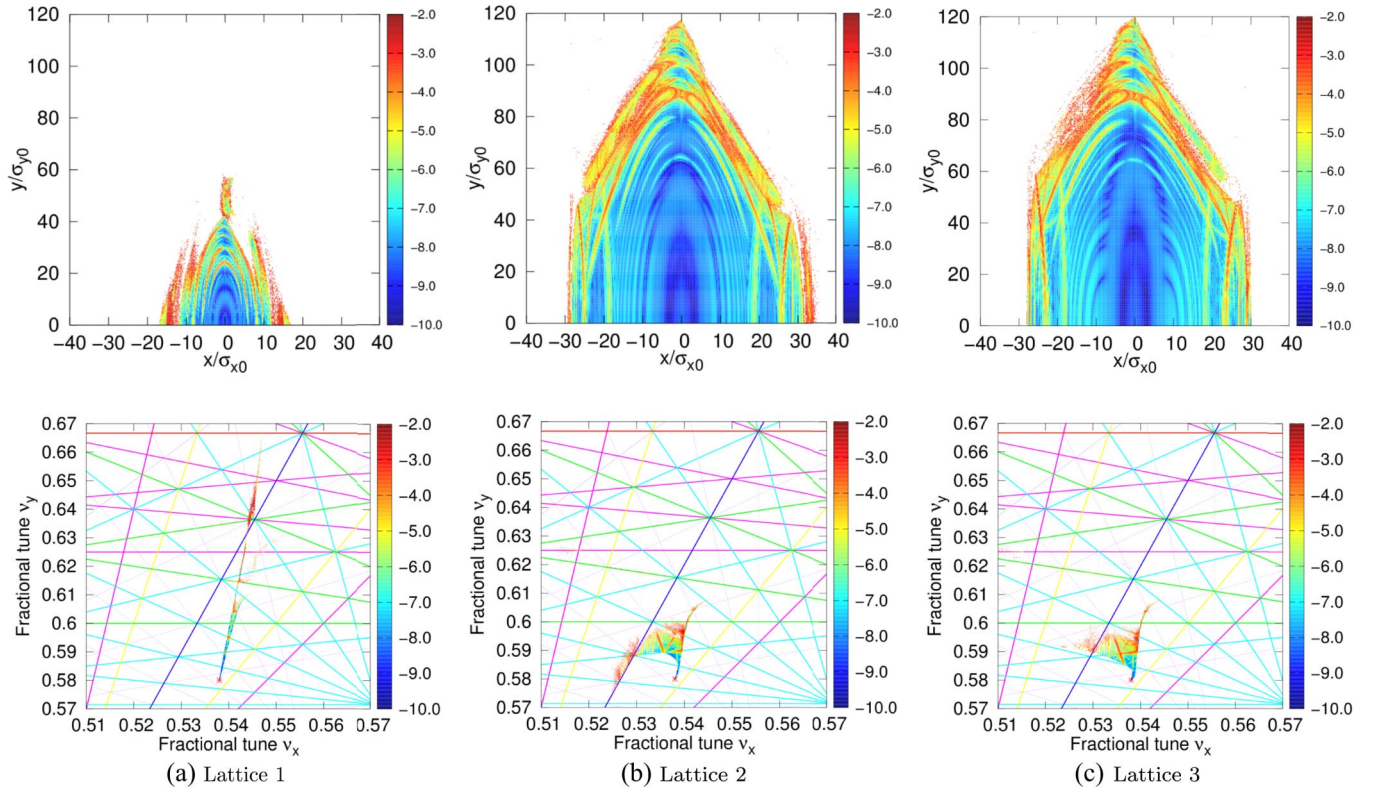


FIG. 18. Frequency map analysis for the three stage 2 lattices. The upper row shows the diffusion rates in transverse coordinate space, and the lower row shows the corresponding tune footprints. The color scale indicates the diffusion rate. From left to right, (a) lattice 1 has no octupoles, (b) lattice 2 has octupoles inside the CW sextupole pair, and (c) lattice 3 has all IR octupoles enabled.

Touschek lifetimes increase to approximately 260, 540, and 600 s, respectively. These results demonstrate that the adverse off-momentum effects introduced by octupoles can be mitigated when the optimization directly includes tracking-based aperture and lifetime objectives.

To diagnose the role of octupoles in the stage 2 solutions, Fig. 17 compares the one-dimensional amplitude-dependent tune shifts, while Fig. 18 shows the corresponding frequency map analysis diffusion maps in the transverse physical space and tune footprints in the betatron-tune space. The octupole-corrected lattices exhibit reduced tune variation with amplitude and a more compact tune footprint, confirming that the improved DA is associated with effective suppression of the leading ADTs.

The comparison also shows that the octupoles outside the CW sextupole pair provide only marginal additional benefit for the present optics. This suggests that, once the dominant detuning inside the CW region is compensated, the need for extra outer octupoles may be reduced. Further studies are nevertheless required to test the robustness of this conclusion.

VI. CONCLUSION AND OUTLOOK

In this work, we have presented the design and nonlinear optimization of the one-fold STCF collider lattice developed during an earlier phase of the project. The study combines lattice-independent global parameter optimization, an LCC-based crab-waist IR design, and full-ring nonlinear optimization. At 2 GeV, the optimized lattice achieves a theoretical peak luminosity of approximately $1.1 \times 10^{35} \text{ cm}^{-2} \text{ s}^{-1}$ while maintaining sufficient dynamic aperture, momentum acceptance, and a Touschek lifetime of about 600 s.

A central result is that the nonlinear performance is governed mainly by the transparency of the crab-waist region. The two-stage optimization strategy, consisting of analysis-driven preoptimization followed by tracking-based refinement, provides an effective way to balance on-momentum dynamic aperture and off-momentum dynamic aperture. The results also indicate that octupoles inside the CW region are the most effective detuning knobs, whereas additional octupoles outside this region provide only limited further benefit for the present optics.

The present study has established the method and demonstrated its effectiveness under ideal lattice conditions. Future work will address a systematic assessment of robustness against machine imperfections and the interplay among lattice nonlinearities, beam-beam effects, and collective effects.

ACKNOWLEDGMENTS

T. Liu thanks P. Raimondi for insightful discussions and detailed guidance on crab-waist optics design. This work

was supported by the National Key R&D Program of China under Contract No. 2022YFA1602201, the National Natural Science Foundation of China No. 12341501 and No. 12405174, and the International Partnership Program of the Chinese Academy of Sciences Grant No. 211134KYSB20200057. We would like to thank the Hefei Comprehensive National Science Center for their strong support on the STCF key technology research project.

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

-
- [1] P. Raimondi, *Status on SuperB Effort, 2nd SuperB Workshop* (INFN-LNF, Frascati, 2006).
 - [2] P. Raimondi, D. N. Shatilov, and M. Zobov, Beam-beam issues for colliding schemes with large Piwinski angle and crabbed waist, [arXiv:physics/0702033](#).
 - [3] P. Raimondi, D. Shatilov, and M. Zobov, Suppression of beam-beam resonances in crab waist collisions, in *Proceedings of the 11th European Particle Accelerator Conference (EPAC 2008)* (2008), pp. 2620–2622.
 - [4] M. Zobov, D. Alesini, M. E. Biagini *et al.*, Test of crab-waist collisions at the DAΦNE Φ factory, *Phys. Rev. Lett.* **104**, 174801 (2010).
 - [5] Y. Ohnishi, T. Abe, K. Akai, Y. Arimoto, K. Egawa, S. Enomoto, H. Fukuma, Y. Funakoshi, K. Furukawa, N. Iida *et al.*, SuperKEKB operation using crab waist collision scheme, *Eur. Phys. J. Plus* **136**, 1023 (2021).
 - [6] A. Abada, M. Abbrescia, S. S. AbdusSalam, I. Abdyukhanov, J. Abelleira Fernandez, A. Abramov, M. Aburaia, A. O. Acar, P. R. Adzic, P. Agrawal *et al.*, FCC-ee: The Lepton collider: Future circular collider conceptual design report volume 2, *Eur. Phys. J. ST* **228**, 261 (2019).
 - [7] CEPC Study Group, CEPC conceptual design report: Volume 1-accelerator, [arXiv:1809.00285](#).
 - [8] H. Peng, Y.-H. Zheng, and X.-R. Zhou, Super Tau-Charm Facility of China, *Physics* **49**, 513 (2020).
 - [9] X.-C. Ai, L.-P. An, S.-Z. An, Y. Bai, Z.-H. Bai, O. Bakina, J.-C. Bao, V. Batzskaya, A. Belias, M. E. Biagini *et al.*, Conceptual design report of the Super Tau-Charm Facility: The accelerator, *Nucl. Sci. Tech.* **36**, 242 (2025).
 - [10] M. Biagini, R. Boni, M. Boscolo, A. Chiarucci, R. Cimino, A. Clozza, A. Drago, S. Guiducci, C. Ligi, G. Mazzitelli *et al.*, Tau/Charm factory accelerator report, [arXiv:1310.6944](#).
 - [11] V. Anashin *et al.*, Super Charm-Tau Factory: Conceptual design report, Tech. Rep. (Budker Institute of Nuclear Physics, Novosibirsk Russia, 2018).
 - [12] A. Y. Barnyakov *et al.*, The project of the Super Charm-Tau Factory in Novosibirsk, *J. Phys. Conf. Ser.* **1561**, 012004 (2020).

- [13] SuperB Collaboration, SuperB progress reports—accelerator, [arXiv:1009.6178](https://arxiv.org/abs/1009.6178).
- [14] SuperKEKB design report.
- [15] Q. Luo, W. Gao, J. Lan, W. Li, D. Xu *et al.*, Progress of conceptual study for the accelerators of a 2–7 GeV Super Tau Charm Facility at China, in *Proceedings of the 10th Int. Particle Accelerator Conf. (IPAC'19), Melbourne, Australia, 19–24 May 2019* (JACOW Publishing, Geneva, Switzerland, 2019), pp. 643–645.
- [16] J. Q. Lan, Q. Luo, C. Zhang, W. W. Gao, Y. Xu, and Y. Bai, Design of beam optics for a Super Tau-Charm Factory, *J. Instrum.* **16**, T07001 (2021).
- [17] T. Liu, C. Zhang, L. Wang, and Q. Luo, Design of a hybrid seven-bend-achromat-based lattice for a Super Tau Charm Facility, in *Proceedings of Ipac2023, Venice, Italy, 07–12 May 2023* (JACoW Publishing, Venice, Italy, 2023), Vol. 14, pp. 768–771.
- [18] T. Liu, L. Zhang, S. Li, L. Wang, and Q. Luo, Recent progress and future plan for STCF collider ring lattice design, *Mod. Phys. Lett. A* **39**, 2440005 (2024).
- [19] T. Liu, L. Wang, and Q. Luo, Study of chromaticity correction methods for a hybrid Multi-bend-achromat lattice, *J. Phys. Conf. Ser.* **2709**, 012001 (2024).
- [20] Y. Zou, L. Zhang, T. Liu, P. Yang, W. Li, T. He, D. Zhou, K. Ohmi, S. Li, Z. Yu *et al.*, Optics design of the Super Tau-Charm Facility collider rings, *Nucl. Instrum. Methods Phys. Res., Sect. A* **1084**, 171191 (2025).
- [21] N. S. Dikansky and D. V. Pestrikov, Effect of the crab waist and of the micro-beta on the beam–beam instability, *Nucl. Instrum. Methods Phys. Res., Sect. A* **600**, 538 (2009).
- [22] D. Zhou, Formulae of luminosity and beam-beam tune shifts for flat-beam asymmetric colliders, [arXiv:2212.12706](https://arxiv.org/abs/2212.12706).
- [23] P. Raimondi and M. Zobov, Tune shift in beam-beam collisions with a crossing angle, Technical note G-58 (DAΦNE, 2003).
- [24] K. Ohmi, N. Kuroo, K. Oide, D. Zhou, and F. Zimmermann, Coherent beam-beam instability in collisions with a large crossing angle, *Phys. Rev. Lett.* **119**, 134801 (2017).
- [25] T. Liu *et al.*, Nonlinear optimization for the Super Tau-Charm Facility, in *The 7th International Workshop on Future Tau Charm Facilities (FTCF2025), Huangshan, China* (2025).
- [26] P. Raimondi, S. M. Liuzzo, L. Farvacque, S. White, and M. Hofer, Local chromatic correction optics for future circular collider e^+e^- , *Phys. Rev. Accel. Beams* **28**, 021002 (2025).
- [27] K. Oide, M. Aiba, S. Aumon, M. Benedikt, A. Blondel, A. Bogomyagkov, M. Boscolo, H. Burkhardt, Y. Cai, A. Doblhammer *et al.*, Design of beam optics for the future circular collider e^+e^- collider rings, *Phys. Rev. Accel. Beams* **19**, 111005 (2016).
- [28] T. Liu, PAMKIT-Particle Accelerator Modeling Kit, <https://pypi.org/project/PAMKIT> (2025).
- [29] SAD Home Page, <https://acc-physics.kek.jp/SAD>.
- [30] L. Zhang, T. Liu, Y. Zou, P. Yang, D. Zhou, J. Bao, Z. Yu, Y. Jin, Y. Mo, S. Li, T. He, Q. Luo, and J. Tang, Crab-waist interaction region design and integration for the Super Tau-Charm Facility, [arXiv:2510.09198](https://arxiv.org/abs/2510.09198).
- [31] A. Bogomyagkov, E. Levichev, and P. Piminov, Final focus designs for crab waist colliders, *Phys. Rev. Accel. Beams* **19**, 121005 (2016).
- [32] K. L. Brown and R. V. Servranckx, First- and second-order charged particle optics, in *AIP Conference Proceedings* (AIP, 1985), Vol. 127, pp. 62–138.
- [33] J. Bengtsson, The sextupole scheme for the Swiss Light Source (SLS): An analytic approach, *SLS-Note* **9**, 97 (1997). [10.55402/psi:86776](https://arxiv.org/abs/10.55402/psi:86776).
- [34] A. Bogomyagkov, Chromaticity correction of the interaction region, in *The First IHEP-BINP CEPC Accelerator Collaboration Workshop, Beijing* (2016).
- [35] B. W. Montague, Linear optics for improved chromaticity correction, Tech. Rep. (SCAN-0009131, 1979).
- [36] Y. Cai, Y. Nosochkov, J. S. Berg, J. Kewisch, Y. Li, D. Marx, C. Montag, S. Tepikian, F. Willeke, G. Hoffstaetter, and J. Unger, Optimization of chromatic optics in the electron storage ring of the electron-ion collider, *Phys. Rev. Accel. Beams* **25**, 71001 (2022).
- [37] A. Bogomyagkov, Single-particle effects limiting the parameters of modern synchrotron radiation sources and electron-positron colliders, Dr.Sc. thesis (Doktor Nauk) [Budker Institute of Nuclear Physics, Siberian Branch, Russian Academy of Sciences, Siberian, 2024 (in Russian)], https://www.inp.nsk.su/images/Bogomyagkov_dissertation.pdf.
- [38] A. Bogomyagkov, E. Levichev, and S. Sinyatkin, Touschek lifetime and luminosity optimization for Russian Super Charm Tau Factory, *J. Instrum.* **19**, P02018 (2024).
- [39] B. Wei, Z. Bai, J. Tan, L. Wang, and G. Feng, Minimizing the fluctuation of resonance driving terms in dynamic aperture optimization, *Phys. Rev. Accel. Beams* **26**, 84001 (2023).
- [40] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, A fast and elitist multiobjective genetic algorithm: NSGA-II, *IEEE Trans. Evol. Comput.* **6**, 182 (2002).
- [41] B. Riemann, M. Aiba, J. Kallestrup, and A. Streun, Efficient algorithms for dynamic aperture and momentum acceptance calculation in synchrotron light sources, *Phys. Rev. Accel. Beams* **27**, 94002 (2024).