

Post long shutdown 2 CERN proton synchrotron transverse impedance model: Description and beam-based validation

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In the framework of the LHC injectors upgrade project, the CERN proton synchrotron underwent hardware changes during the long shutdown 2. These changes required an update of the CERN proton synchrotron impedance model and an assessment of their impact on beam quality. In the following, the updated CERN proton synchrotron impedance model is presented and then validated with beam-based measurements, comparing impedance-induced tune shifts and instability growth rates with simulations performed with a macroparticle tracking code, PyHEADTAIL, which includes the effects of beam-coupling impedance, betatron motion, nonlinear synchrotron motion, and space charge. The interplay between first- and second-order chromaticity, space-charge forces, and their effect on beam observables is thus studied in detail. Their respective impacts have been assessed and benchmarked with measurements to validate the PS transverse impedance model.

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I. INTRODUCTION

The pursuit of high-brightness beams in many facilities across the world has intensified the impact of collective effects, particularly those driven by impedance and space charge. The consequences of these effects range from beam quality degradation (e.g., emittance blowup) to beam losses. For example, the interplay between space-charge-induced tune spread and resonances can strongly limit the choice of the working point in GSI's SIS100 [1]. In addition to this, transverse instabilities arising from the combined effect of impedance and space charge prevent increasing the maximum beam intensity by causing severe beam losses as observed in the Fermilab Recycler [2], ISIS RCS [3,4], and JPARC RCS [5,6]. In addition, space-charge effects were recently predicted to be also relevant in fourth generation light sources [7].

One of the most demanding projects to push beam performance further is the high-luminosity LHC (HL-LHC) project [8], which aims at increasing the integrated luminosity, i.e., the ratio of the total number

of events over a period divided by the event cross section, by one order of magnitude. It would allow us to surpass the initial LHC goal of approximately 300 fb^{-1} to reach 3000 fb^{-1} after the 12 years following the upgrade [8].

In preparation for HL-LHC, its injectors were upgraded with the LHC injectors upgrade project (LIU) [9] to enable a twofold increase of the injected beam intensity with a slightly smaller transverse emittance required for the higher luminosity. The PSB, PS, and SPS underwent significant hardware refurbishment and upgrade during the long shutdown 2 (LS2, 2019–2021). The PS has a history of transverse instabilities [10–12], and the hardware modifications introduced during LS2 raised concerns about its beam stability with the new LIU parameter range. To address these concerns, the PS impedance model has been updated and validated with beam-based measurements. The updated impedance model proved to be critical to find mitigation for recent instabilities [13,14]. It is also expected to provide key insights into PS beam stability in future scenarios, such as those envisioned in the Physics Beyond Collider project [15].

In this paper, we first describe the updated transverse impedance model of the PS. Then, beam-based reference measurement campaigns, carried out to validate the transverse impedance model, are reviewed. Both tune shifts and instability growth rates were acquired to study respectively the imaginary and real parts of the PS transverse impedance. Several mechanisms prone to impacting the impedance-induced beam observables, namely, the first- and second-order chromaticity and direct space-charge force,

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are then studied through simulations with `PyHEADTAIL` [16–18], a macroparticle tracking code, and a detailed assessment of the impact of each mechanism on the impedance-induced beam observables is performed. The beam observables are finally assessed against `PyHEADTAIL` simulations to validate the impedance model.

II. TRANSVERSE IMPEDANCE MODEL OF THE CERN PS

The role of an impedance model [19] is first to predict instability thresholds and ensure that the nominal parameters of the accelerator can be achieved. Second, it allows identifying the dominant impedance sources and quantifying their impact on observed instabilities. Third, it provides the necessary input for macroparticle tracking simulations that include the interplay of beam-coupling impedance with other effects (e.g., betatron coupling, space charge, potential well distortion, beam-beam, and feedback system).

The use of impedance models is well established in the study of collective effects in both particle colliders [20,21] and light sources [22–25]. Once an impedance model is deemed accurate, its Fourier transform—the wake function—can be used as input in macroparticle tracking codes (e.g., `PyHEADTAIL`, `Xsuite` [26], `ELEGANT` [27], and `MBTRACK2` [28]) to simulate the turn-by-turn motion. These simulations can reproduce impedance-induced beam observables such as the tune shift [24,29,30], closed-orbit distortion [31,32], or instability growth rate [29,33,34]. The validity of the impedance model is determined through its agreement with the beam-based measurements.

A. PS transverse impedance model workflow and overview

The PS transverse impedance model is generated through the following workflow. First, the Twiss parameters are computed at each relevant ring element using `MAD-X` [35]. These parameters are used to calculate the beta weighting—defined as the ratio between the local beta function at an element and the average beta function around the ring, $\beta_{x,y}^{\text{element}} / \langle \beta_{x,y} \rangle$ —which is then passed to the impedance model handled by the `xWakes` code (formerly `PyWIT`) [36].

The model includes the impedance contributions of the known elements that have the greatest impact on the overall impedance. These contributions are obtained either from electromagnetic simulations (e.g., using `CST Studio Suite` [37]), analytical formulas (e.g., `1W2D` [21]), or direct impedance measurements performed on a bench. Each contribution is scaled by its corresponding beta weighting and summed to obtain the total transverse impedance of the accelerator.

The resulting impedance model is then used to generate the total PS wake function from the computation of the Fourier transform of the impedance using the `NEFFINT`

[38] algorithm. Finally, the wake function is used as an input in macroparticle tracking simulations to simulate the impedance-induced tune shift and instability growth rate, which will be discussed in a later section.

The current PS transverse impedance model is the result of the work of several people, in particular recently, over the years: Ventura [39], Persichelli [40], Biancacci [41], and Popovic [42]. In recent years, it was upgraded to its post long shutdown 2 version, which is described in detail in [43]. The PS impedance model is composed of a vacuum chamber made of Inconel alloy (30%) and stainless steel (70%) sections, kicker magnets for injection and extraction with their external circuits, accelerating cavities operating at 10, 40, 80, and 200 MHz, a Finemet cavity for mitigating longitudinal instabilities, an electrostatic septum for extraction, vacuum instrumentation such as vacuum ports, bellows, valves, and metallic flanges, a stripline pickup for tune acquisition, and two beam dumps. The individual contributions of these elements to the impedance model are illustrated in Fig. 1.

A complete layout of the machine and the specific elements taken into account in the impedance model can be found in [44]. All the impedance components' stacked contribution plots are shown in Appendix A.

According to Sacherer's formalism [45], the effective impedance Z_t^{eff} characterizes the interaction between a particle bunch and the impedance spectrum. This quantity is defined as the sum of the impedance overlapping with the mode power spectrum [offset by the chromatic frequency, defined in Eq. (6)] along betatron lines:

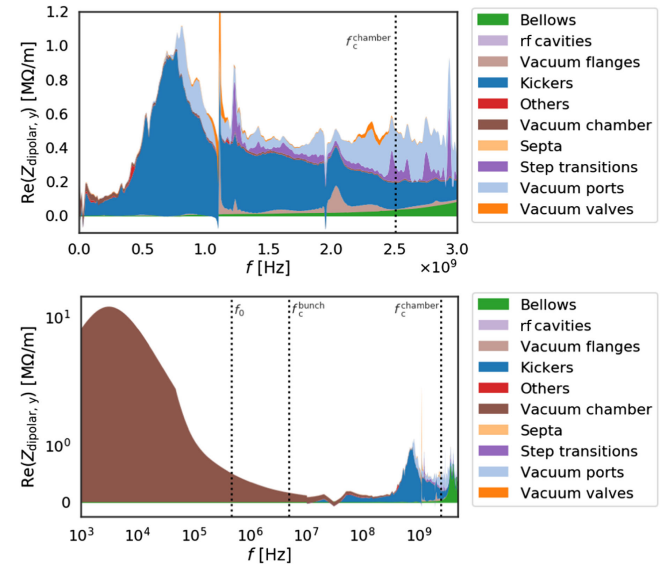


FIG. 1. Representation of the stacked contributions to the real part of the vertical dipolar impedance accounted for in the PS impedance model, at injection energy, in linear and logarithmic scales. The revolution frequency f_0 , the frequency corresponding to the bunch length at injection f_c^{bunch} , and the vacuum chamber cutoff frequency f_c^{chamber} are displayed as dashed black lines.

$$Z_t^{\text{eff}} = \frac{\sum_{p=-\infty}^{\infty} Z_t(\omega') h_l(\omega' - \omega_\xi)}{\sum_{p=-\infty}^{\infty} h_l(\omega' - \omega_\xi)}, \quad (1)$$

where ω is the angular frequency, $Z_t(\omega)$ the transverse impedance spectrum, h_l the mode power spectrum of the azimuthal mode l , and ω_ξ the chromatic frequency. The betatron lines are sampled according to the following relation:

$$\omega' = [(p + Q_0) + lQ_s]\omega_0, \quad (2)$$

where p is the spectral line number, Q_0 the unperturbed tune, Q_s the synchrotron tune, and ω_0 is the angular revolution frequency.

The revolution frequency, $f_0 \equiv \frac{\omega_0}{2\pi}$, is also a measure of the density of the spectral lines in the bunch spectrum, while f_c^{bunch} (frequency corresponding to the bunch length) gives the qualitative extent of the mode power spectrum ($l = 0$). The convention used for the frequency corresponding to the bunch length is $f_c^{\text{bunch}} = \frac{1}{\tau_b} = 5$ MHz assuming a Gaussian distribution of full bunch length at injection energy $\tau_b = 200$ ns (4 times the standard deviation—this is the typical bunch length for LHC beams produced in the PS). Far above this value, the interaction of the bunch spectrum with the impedance spectrum is typically negligible.

Above the chamber cutoff frequency f_c^{chamber} , most of the EM fields generated by the bunch are no longer trapped by accelerator components and instead propagate into the surrounding vacuum chamber. Approximating the chamber as a circular waveguide with a radius equal to the smallest semiaxis of the vacuum chamber $r = 35$ mm, the cutoff frequency for the dominant TE_{11} mode is $f_c^{\text{chamber}} = \frac{j'_{1,1}c}{2\pi r} \approx 2.5$ GHz, where $j'_{1,1} \approx 1.84$ is the first zero of the derivative of the Bessel function of order one.

In the next section, we will study how the hardware modifications implemented during long shutdown 2 affected the PS transverse impedance model, with particular focus on the most significant change: the increased injection energy and its impact on the vacuum chamber impedance.

B. Hardware changes introduced during the long shutdown 2

One of the limitations in the PS before the LS2 and the LIU upgrade was the dominant effect of space charge at injection energy [9]. Its impact translates into an incoherent tune shift depending on the particle's position in the bunch. In the presence of a strong space-charge force, a significant emittance blowup [46] caused by resonance crossing [47] was observed during the injection plateau.

Besides, the space charge tune spread [48] scales with the beam intensity and the inverse of the transverse emittance. Thus, doubling the beam intensity while preserving the transverse emittance could not be considered in

this configuration. Yet, the space charge tune spread also scales inversely with the beam energy. For a round beam, it is defined as [48]:

$$\Delta Q_{\text{sc}} = \frac{RNr_0}{2\varepsilon\beta^2\gamma^3}, \quad (3)$$

where R is the accelerator radius, N the number of particles per unit length, r_0 the classical radius of the proton, and ε the RMS emittance. Consequently, increasing the injection energy can considerably reduce the detrimental impact of space charge [9]. This solution was considered for the PS and the injection kinetic energy has been raised from 1.4 to 2 GeV. The injection region of the accelerator was renovated during LS2 to accommodate this new energy range.

In addition, raising the injection energy also reduces the indirect space charge contribution to the vacuum chamber impedance. The indirect space charge impedance is defined as the contribution that would be there if the chamber boundaries were perfectly conducting [49], it only depends on the vacuum chamber geometry and the beam energy. The indirect space charge contribution scales inversely in $\beta\gamma^2$ and thus vanishes for an ultrarelativistic beam. At injection energy, this contribution is dominant in the hundreds of kHz to the GHz range, and the new injection energy results in a 25% reduction [43,50] of the impedance in this frequency range. The real part of the impedance is, however, left almost untouched with only a slight impedance increase [43,50] between 1 and 100 kHz.

While the energy dependence of the vacuum chamber impedance is well understood, calculating this impedance for the PS elliptic vacuum chamber in the nonultrarelativistic regime ($\beta \approx 0.95$) proves challenging. In this regime, the Yokoya factors [51] do not apply, and a different formalism [43,52] relying on the Mathieu functions can be employed. When calculating the impedance model used previously, the PS vacuum chamber was approximated as a circular chamber whose radius matches the smallest ellipse semiaxis. Now, it is accurately calculated for its elliptic cross section as illustrated in Fig. 2.

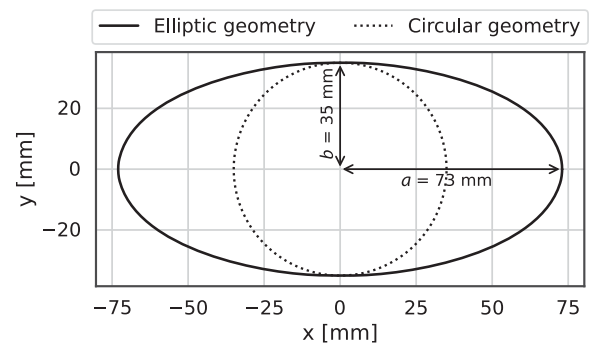


FIG. 2. Comparison of the circular and elliptic vacuum chamber geometries used in impedance calculations.

While the vertical dipolar impedance is not changed too strongly (about 20%) between both geometries, the horizontal dipolar impedance is reduced by approximately a factor of 2 in the elliptic geometry. The calculations for the elliptic geometry also properly account for the quadrupolar impedance, which was negligible in the circular geometry.

In addition to the vacuum chamber impedance, LS2 introduced several other changes to the PS impedance model, including the removal of the continuous transfer equipment [50,53], the installation of new beam dumps, rf equipment, and beam instrumentation. Further details can be found in [43].

Based on the updated PS impedance model, macro-particle tracking simulations can be performed to predict impedance-induced beam observables, in particular, the impedance-induced tune shift and instability growth rate.

III. BEAM-BASED VALIDATION OF THE PS TRANSVERSE IMPEDANCE MODEL

Two single-bunch beam observables of interest, caused by the presence of transverse impedance, are the instability growth rate and the impedance-induced tune shift, both affecting the centroid motion in the following way:

$$x(t) \propto e^{j2\pi f_0(Q_0 + \Delta Q)t} e^{j2\pi f_0[\Re(\Delta Q)]t} e^{-2\pi f_0\Im(\Delta Q)t}, \quad (4)$$

where ΔQ is the complex tune shift, Q_0 is the unperturbed tune, and f_0 is the revolution frequency. The real part of the impedance induces the instability growth rate $[2\pi f_0\Im(\Delta Q)]$, which leads to an exponential increase of the bunch center of mass turn after turn. The imaginary part generates a tune shift, resulting in an intensity-dependent shift of the coherent tune.

To study these effects, macroparticle tracking codes can be used, such as PyHEADTAIL, Xsuite, ELEGANT, and mbtrack2. It can be noted that Vlasov solvers such as DELPHI [54] can also be used to study these effects. However, as they currently do not account for tune shift with amplitude (e.g., from octupolar order nonlinearities) or space-charge effects, tracking codes were preferred. Here, PyHEADTAIL is being used as it enables the study of the interplay between wakefields, chromaticity, and space-charge forces, which will be examined in the following sections.

All the parameters used in the PyHEADTAIL simulations were measured during the reference measurement campaigns and are listed in Appendix B. The injection energy cycle was based on a neutron time of flight (nTOF) cycle, whereas the extraction energy one was based on an East Area experimental facility (EAST) cycle.

While simulations provide nearly ideal signals, beam-based measurements face several real-world limitations. These include limited equipment resolution, statistical and electronic noise, and various sources of nonlinearities that cause tune spread. Consequently, methods to enhance the

signal quality are essential to fully exploit the measured data. The following methods were used with all the beam-based measurements and the simulations presented in this paper.

A. Tune acquisition method

The measurement of impedance-induced tune shift requires a sufficient accuracy to allow the observation of its variation with beam intensity in the presence of tune spread. To achieve this precision, an external excitation is employed to enhance the frequency spectrum of coherent beam oscillations. The optimal approach uses a chirp signal, a sinusoidal excitation whose frequency varies linearly from $f_{\min}$ to $f_{\max}$ throughout the signal acquisition period. The excitation is produced by the PS transverse feedback system.

In addition to exciting the beam, the frequency spectrum is calculated by using the Numerical Analysis of Fundamental Frequencies (NAFF) method [55], as implemented in PyNAFF [56]. While an FFT spectrum remains useful to qualitatively observe the spectral content of a signal, its accuracy can be outperformed by NAFF algorithms, which allow us to determine the tune with the highest accuracy while requiring smaller sample sizes than an FFT.

B. Instability growth rate acquisition method

When an instability develops, the transverse beam amplitude exhibits exponential growth with respect to the number of turns. To quantify this growth, we first extract the oscillation envelope using a Hilbert transform of the signal. The (exponential) growth rate is then determined by applying a modified least-squares fitting technique [57], which optimizes the parameters of an exponential function to minimize the sum of squared residuals between the fitted curve and the measured envelope.

Different approaches to measure and analyze the tune and instability growth rates are described in [43].

IV. EFFECT OF CHROMATICITY (LINEAR AND NONLINEAR)

The chromaticity is a crucial mechanism in the study of the impact of transverse impedance on beam dynamics as it couples the transverse and longitudinal planes. It gives rise to a tune shift experienced by off-momentum particles, which can be written as

$$\Delta Q_\xi = Q'\delta + Q''\delta^2 + \mathcal{O}(\delta^3), \quad (5)$$

where δ is the relative particle momentum deviation with respect to the synchronous particle, Q' is the first-order chromaticity, and Q'' is the second-order chromaticity. The first-order chromaticity is responsible for the head-tail phase shift χ [58,59], which may allow the amplification of the impact of wakefields on a bunch after each synchrotron period and eventually lead to head-tail instabilities. The head-tail phase shift is usually expressed through the chromatic frequency f_ξ , defined as

$$f_{\xi} = \chi / \tau_b = \frac{f_0 Q'}{|\eta|}, \quad (6)$$

where τ_b is the full bunch length and η the slippage factor. The chromatic frequency intervenes in Eq. (1) under the form of $\omega_{\xi} = 2\pi f_{\xi}$ and causes a shift of the mode spectrum over the impedance spectrum. As a result, scanning the chromatic shift frequency by modifying the first-order chromaticity can be leveraged to probe the accelerator's effective impedance over different frequency ranges.

In the PS, the working point of particle beams with kinetic energy below 3.5 GeV is primarily controlled using combined function magnets and additional quadrupoles distributed along the ring [60]. Above this energy, these elements become insufficient due to their limited number and gradient. Instead, Pole Face Windings (PFW) and Figure of Eight Loops (F8L) are used for working point control, though this comes with the drawback of introducing nonlinear chromaticity [61]. Still, the chromaticity can be controlled to some extent using a measured response matrix that relates the currents in the PFW/F8L (5 degrees of freedom) to the tunes $Q_{x,y}$, first-order chromaticities $Q'_{x,y}$, and either horizontal or vertical second-order chromaticity Q''_x or Q''_y . However, since the chromaticity control knobs were primarily designed for correction rather than wide-range scanning, the achievable chromaticity range during measurements is limited.

To understand these operational boundaries, we measured the achievable chromaticity ranges at both injection ($E_{\text{kin}} = 2.0$ GeV, under transition crossing) and extraction ($E_{\text{kin}} = 25.4$ GeV, above transition crossing) energies. At injection, we could vary first-order chromaticity between $Q' = 0$ and -9 , allowing to explore the impedance spectrum from 0 to 80 MHz. While extraction energy measurements had a narrower chromaticity range ($Q' = 0-1.5$), the smaller slippage factor still enabled the exploration of the impedance spectrum up to 60 MHz. In addition, the bunch spectrum at extraction energy spans a wider frequency range than at injection energy due to the shorter bunch length, resulting from adiabatic damping during acceleration.

As it can be noted from the stacked contribution plots shown in Appendix A, the 0 to 100 MHz frequency range of the PS impedance is dominated by the vacuum chamber impedance. Moreover, the imaginary part of the impedance remains almost constant with frequency in this frequency range, except for the peak at low frequencies. Thus, independently of the chromatic shift, the tune shift is anticipated to be mainly affected by the proximity to the low frequency peak introduced by the vacuum chamber impedance. Consequently, Q' is expected to have a minimal impact on the tune shifts once the bunch spectrum is sufficiently far from the peak.

The observation of the main mode tune shift has been proven challenging in simulations, especially for the

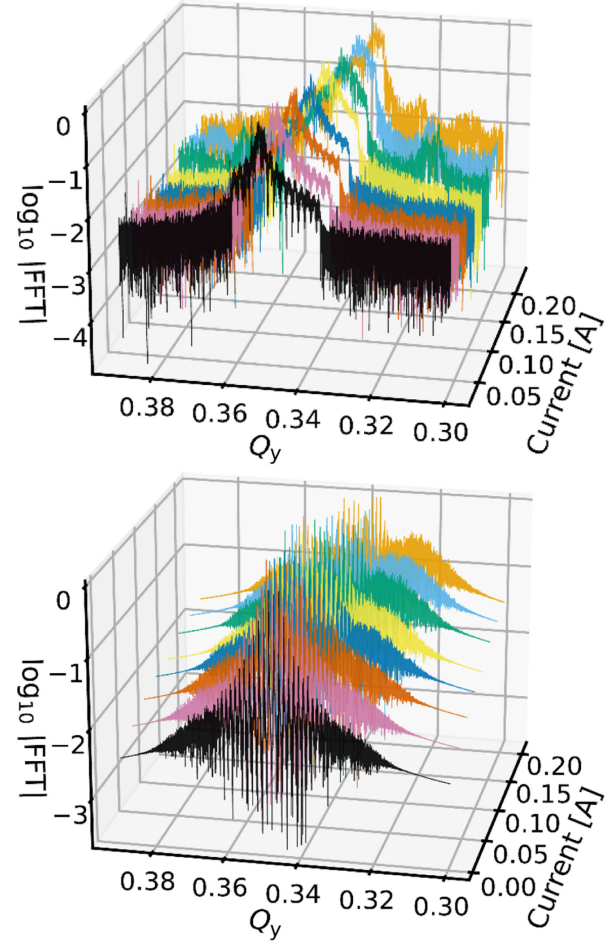


FIG. 3. Vertical tune shift measurements performed in 2022 (top) and simulations (bottom) at $E_{\text{kinetic}} = 2$ GeV for $Q'_y \approx -7$. Both figures correspond to successive tune spectra for various bunch current values.

injection energy case. All the azimuthal and radial modes exhibit a comparable magnitude, hence hiding the main mode, as shown in Fig. 3, for simulations whose parameters can be found in Table V, including a negative first-order chromaticity, without nonlinear chromaticity, octupolar order nonlinearities, or space-charge effects. In comparison, the main mode is well noticeable in measurements and its magnitude surpasses by orders of magnitude the one of the other modes. At this stage, the simulations cannot reproduce the experimental observations; the following sections will therefore introduce the missing effects—nonlinear chromaticity and octupolar order nonlinearities, as well as space-charge effects—step by step until the simulated spectrum reproduces the experimental one.

Despite this effect, the main mode remains present in the simulations and can be observed for low chromaticities (i.e., $Q'_y \geq -1$). As the focus is put on the impedance-induced tune shift, a workaround can be employed to increase the main mode magnitude. To do so, a positive Q' is used, as a positive chromaticity below transition results in

an unstable mode 0, thus further exciting the main mode peak in the tune shift spectrum. Then, it is known that the imaginary part of the transverse impedance exhibits even parity as a function of frequency [62]. As a result, the overlap of the imaginary part of the impedance with the bunch power spectrum frequency (shifted by f_ξ) is independent of the chromaticity sign.

The method's validity was assessed by using $Q'_y = -1$ and 1 at injection energy, where the main mode could be observed in both cases, as depicted in Fig. 4. The main mode was identified in both cases using the knowledge of the vertical tune in the zero-current limit. By comparing both chromaticities, comparable tune shift slopes were found, as well as a significant increase of the main mode magnitude for intensities below 5×10^{11} p with a positive chromaticity, compared to a negative one. Based on this result, the method is deemed valid and has been used in simulations to estimate the tune shift slopes for the various chromaticities probed during the measurement campaigns.

To fully understand the role of chromaticity on the impedance-induced tune shift, we analyze its effects systematically. We first examine how Q' influences the measured and simulated tune shift slopes. Then, we investigate the impact of Q'' , which proved crucial to interpret a discrepancy between measurements and simulations.

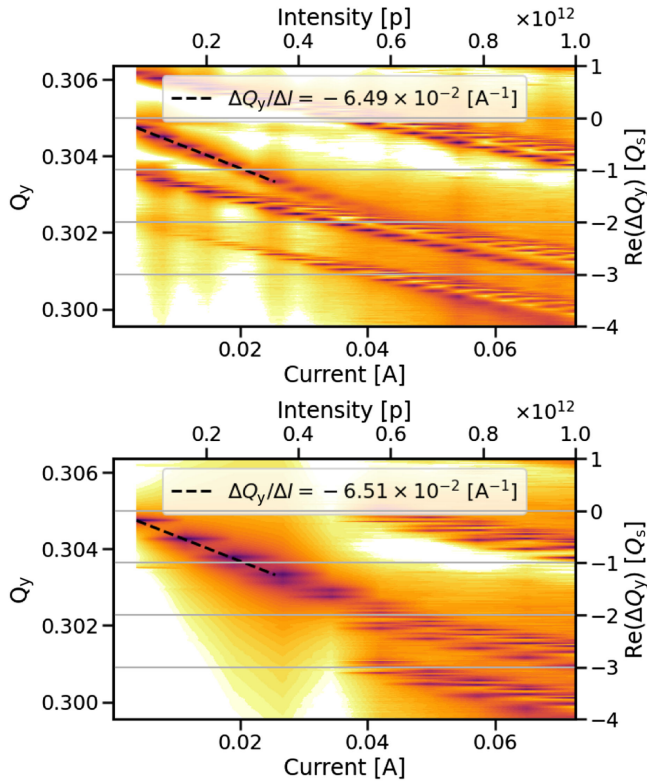


FIG. 4. Vertical tune shift for $Q'_y = -1$ (top) and $Q'_y = 1$ (bottom), from simulations at injection energy. The color scale represents the spectral magnitude, from high (purple) to low (yellow).

A. First-order chromaticity

Making use of the method described above to observe the simulated main mode tune shift for $|Q'_y| \geq 1$, we now compare the measured and simulated tune shift slopes across different Q' values. Figure 5 shows the vertical tune shift slopes obtained at both injection and extraction energies for various chromaticities. The simulation parameters can be found in Table V (with $Q''_{x,y} = 0$), and all tune shift slopes are plotted against the chromatic frequency defined in Eq. (6).

First, the tune shift slopes show a clear beam energy dependence: at injection energy, they are approximately one order of magnitude larger than at extraction energy, regardless of chromaticity. This difference originates from the reduced indirect space charge impedance contribution at higher beam energy.

Then, it can be observed that the 2015 and 2022 measurements exhibit different behaviors with f_ξ at injection energy. The 2015 data show significant variation in tune shift slopes with increasing chromaticity, while the 2022 measurements remain more stable. Additionally, while simulations agree well with 2022 measurements at injection energy, they underestimate the tune shift slopes at extraction energy for both 2015 and 2022 data. A similar discrepancy was observed in [63] for nonzero chromaticity values.

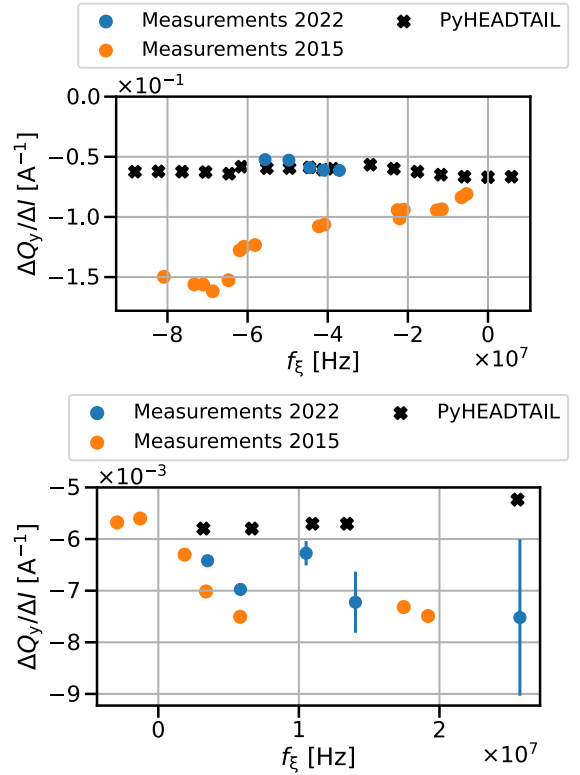


FIG. 5. Comparisons between measured and simulated vertical tune shift slopes against chromatic frequency f_ξ at injection (top) and extraction (bottom) energies.

Since the PS transverse impedance remained largely unchanged between 2015 and 2022, these observations suggest a missing mechanism in our simulations. A plausible candidate to explain these discrepancies is the second-order chromaticity Q'' .

B. Second-order chromaticity

To investigate whether second-order chromaticity could explain the observed discrepancies, we performed additional PyHEADTAIL simulations with the extraction energy scenario, where the largest discrepancy between measurements and simulations was observed.

The same simulation parameters are being used (i.e., from Table V) except a parameter scan is performed with $Q' = 0$ while varying Q'' between ± 1000 , a range achievable in the PS. The resulting simulated tune shifts are shown in Fig. 6.

A significant dependency of the main mode tune shift with Q'' is found in both planes. It can be observed that a negative Q'' increases the tune shift slope and a positive one decreases it. The same observation was made in the LHC, where they studied the generation of transverse Landau damping through the introduction of Q'' [64]. A chromaticity value of $Q''_y = -500$ can be responsible for an increase of the tune shift slope by 30%. Although initially observed in simulations at extraction energy, this correlation between Q'' and tune shift slopes extends to the injection energy case as well. Additionally, measurements at both injection and extraction energies, shown in Fig. 7, also confirm this behavior.

By including the second-order chromaticity measured during measurements in the simulations, we can refine the comparison with measurements at extraction energy, shown in Fig. 5. The updated comparison is presented in Fig. 8, where both the simulated tune shift slopes with and without Q'' are plotted.

During these measurements, a correlation has been observed between Q' and Q'' . In other words, an acquisition exhibiting a low Q' also shows a low Q'' . Therefore, the simulated tune shift slope for low chromatic frequency

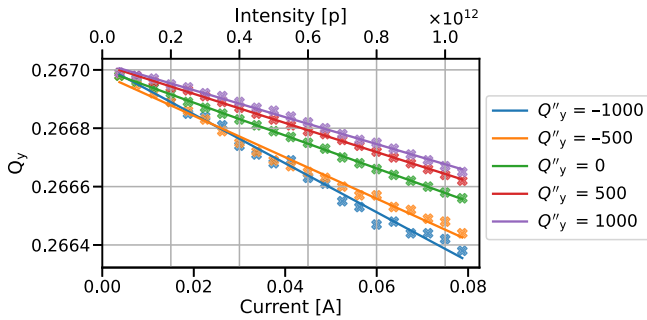


FIG. 6. Impact of second-order chromaticity on simulated vertical tune for various Q'' at extraction energy. Crosses represent the tune values used for the fit by a single tune shift slope (shown in solid lines).

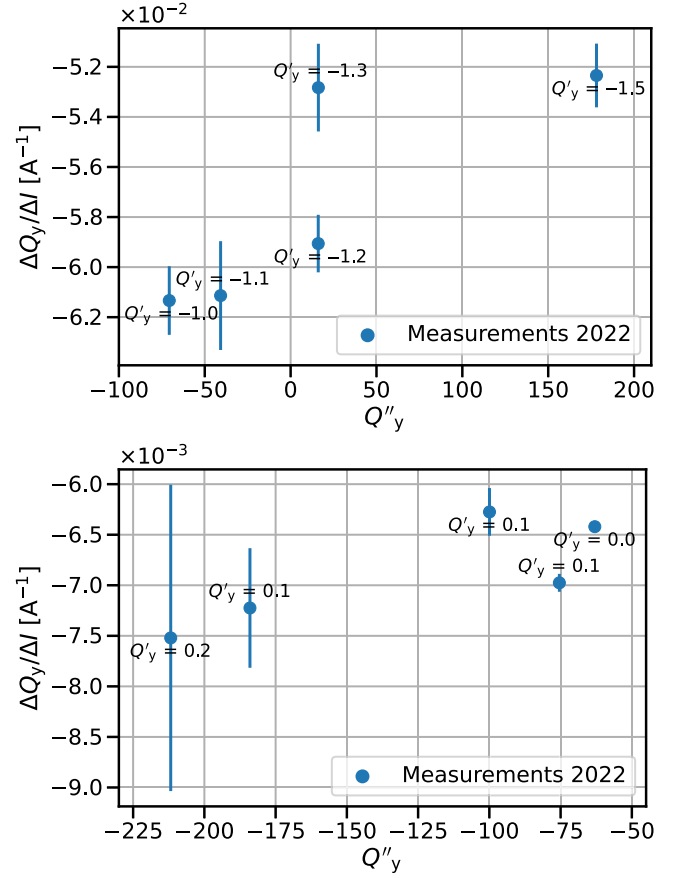


FIG. 7. Measured vertical tune shift slope at injection (top) and extraction energy (bottom) versus second-order chromaticity.

values is similar whether Q'' is accounted for or not. In any case, the discrepancy between measurements and simulations has been reduced for large chromatic frequencies. Nonetheless, Q'' is not sufficient to fully explain it. To address this, we now focus on two mechanisms still missing from the simulations—the octupolar order nonlinearities and the space-charge force.

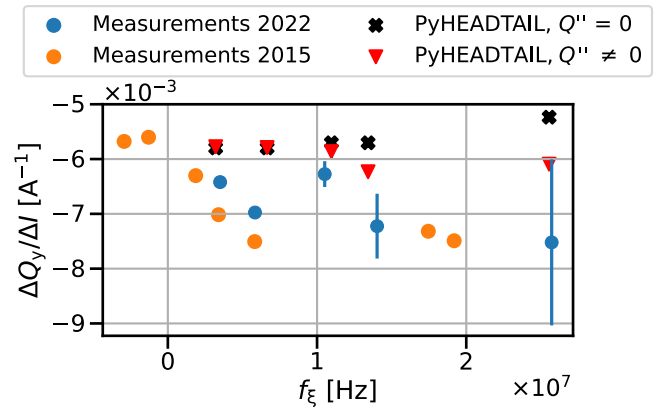


FIG. 8. Comparisons between measured and simulated vertical tune shift slopes against chromatic frequency f_ξ at extraction energy, including the effect of Q'' .

V. EFFECT OF OCTUPOLAR ORDER NONLINEARITIES AND DIRECT SPACE CHARGE

While second-order chromaticity explains some of the observed discrepancies, other mechanisms also have an important interplay with impedance effects. As highlighted in Fig. 3, the simulated main mode cannot be observed similarly to the measured one over a wide range of beam intensities. It remains visible for intensities lower than 5×10^{11} p before being masked by higher-order modes. The simulations differ from the measurements by the absence of Landau damping from octupolar order nonlinearities and direct space charge.

The strength of the octupolar order nonlinearities is quantified by the anharmonicity (or detuning) coefficients [65] a_{xx}, a_{xy}, a_{yy} , which result in an incoherent tune shift $Q_{x,y}^{\text{AD}}$ depending on the particle's transverse actions $J_{x,y}$ as

$$\begin{aligned}\Delta Q_x^{\text{AD}} &= a_{xx}J_x + a_{xy}J_y, \\ \Delta Q_y^{\text{AD}} &= a_{yy}J_y + a_{xy}J_x.\end{aligned}\quad (7)$$

The anharmonicity coefficient values are obtained from MAD-X results. At injection energy and for the custom EAST cycle, the anharmonicity coefficients are $a_{xx} = 6.11 \text{ m}^{-1}$, $a_{xy} = -53.7 \text{ m}^{-1}$, and $a_{yy} = 32.6 \text{ m}^{-1}$. They give rise to amplitude-dependent tune shifts, also called tune spread. For a particle located at $1 \sigma_{x,y}$, it amounts to $\Delta Q_x \sim 5 \times 10^{-5}$ and $\Delta Q_y \sim 1.5 \times 10^{-6}$ for $\varepsilon_x = 4.5 \text{ mm mrad}$, $\varepsilon_y = 1.3 \text{ mm mrad}$ and a beam intensity of 1×10^{12} p. The anharmonicity coefficients for the extraction energy cycle used during simulations can be found in Table VI.

The direct space-charge force is introduced in the simulations by using a 3D particle-in-cell (PIC) solver [66] to compute the interactions between particles. Its principle lies in the overlap of the bunch distribution with a 3D grid in which the Poisson equation is solved cell by cell, and the resulting change of momentum is propagated to all particles of a cell.

Both effects are closely tied to the beta functions and dispersion variations along the ring. Thus, the simulations require the accelerator to be divided into a sufficient number of segments to realistically model these interactions. It is handled by PyHEADTAIL by using the Floquet matrix defined in Eq. (8) for the i th macroparticle in the n th segment.

$$\begin{pmatrix} u_{i,n+1} \\ u'_{i,n+1} \end{pmatrix} = \begin{pmatrix} \cos(2\pi\mu_{i,n}) & \beta \sin(2\pi\mu_{i,n}) \\ -\frac{1}{\beta} \sin(2\pi\mu_{i,n}) & \cos(2\pi\mu_{i,n}) \end{pmatrix} \begin{pmatrix} u_{i,n} \\ u'_{i,n} \end{pmatrix} + \begin{pmatrix} \eta[1 - \cos(2\pi\mu_{i,n})] \\ \frac{\eta}{\beta} \sin(2\pi\mu_{i,n}) \end{pmatrix}. \quad (8)$$

Here u and u' represent the horizontal or vertical position and angle, β is the beta function, $\mu_{i,n}$ is the betatron phase

TABLE I. Parameters for the PyHEADTAIL tune shift simulations with space charge.

$n_{\text{macroparticles}}$	$n_{\perp}^{\text{cell}}$	Transverse grid span
3×10^6	128	$\pm 5\sigma_{x,y}$
n_{segments}	$n_{\parallel}^{\text{cell}}$	Longitudinal grid span
240	32	$\pm 4\sigma_z$

advance, and η is the dispersion. The impact of the chromaticity and anharmonicity coefficients is accounted for through $\mu_{i,n}$. The convergence tests on the required number of segments and PIC solver parameters can be found in [43]. The PyHEADTAIL PIC solver parameters are summarized in Table I.

Compared to previous simulations, the Landau damping will change for each intensity due to the intensity-dependent transverse emittance in the PS and the scaling of the space-charge force with intensity. The intensity dependency of the transverse emittance follows an affine function and is referenced in Appendix B. As intensity increases, so does the emittance, thus the Landau damping from the octupolar order nonlinearities increases ($\propto \varepsilon$) while the one from the direct space charge slightly increases ($\propto N/\varepsilon$), as the brightness asymptotically increases toward its high-intensity value. The overall Landau damping thus increases with the beam intensity, especially for head-tail modes. The main mode remains unaffected by direct space charge as its effect is invariant from the bunch position in the vacuum chamber [67], but still gets more stable with higher octupolar order nonlinearities at high intensities.

To study this, PyHEADTAIL simulations were performed with all the relevant mechanisms: space charge, octupolar order nonlinearities, first- and second-order chromaticity, and nonlinear synchrotron motion. The simulation parameters, present in Table V, originate from the measurements to which they are compared. Two different acquisitions with different PFW settings were used for each plane. The measurements exhibit a horizontal chromaticity of $Q'_x = 0$, $Q''_x = -236$ and a vertical chromaticity of $Q'_y = -9.4$, $Q''_y = 178$. Both sets of measurements are compared with simulations in Figs. 9 and 10. The synchrotron sidebands are displayed on an additional axis, where the zeroth sideband is placed at the intercept of the tune shift slope with the vertical axis.

First, it can be noted in Figs. 9 and 10 that the measured unperturbed tune differs from the simulated one. The simulations use the working point $(Q_x, Q_y = 6.064, 6.317)$ obtained from the main mode tune measured at the lowest intensity. However, due to the nonlinear intensity-dependency of the measured main mode tunes at low intensities, the fitted tune shift intercepts differ from the simulation working point. Second, the inclusion of space charge in simulations is essential to reproduce the measurements, particularly the visibility of the main mode over other

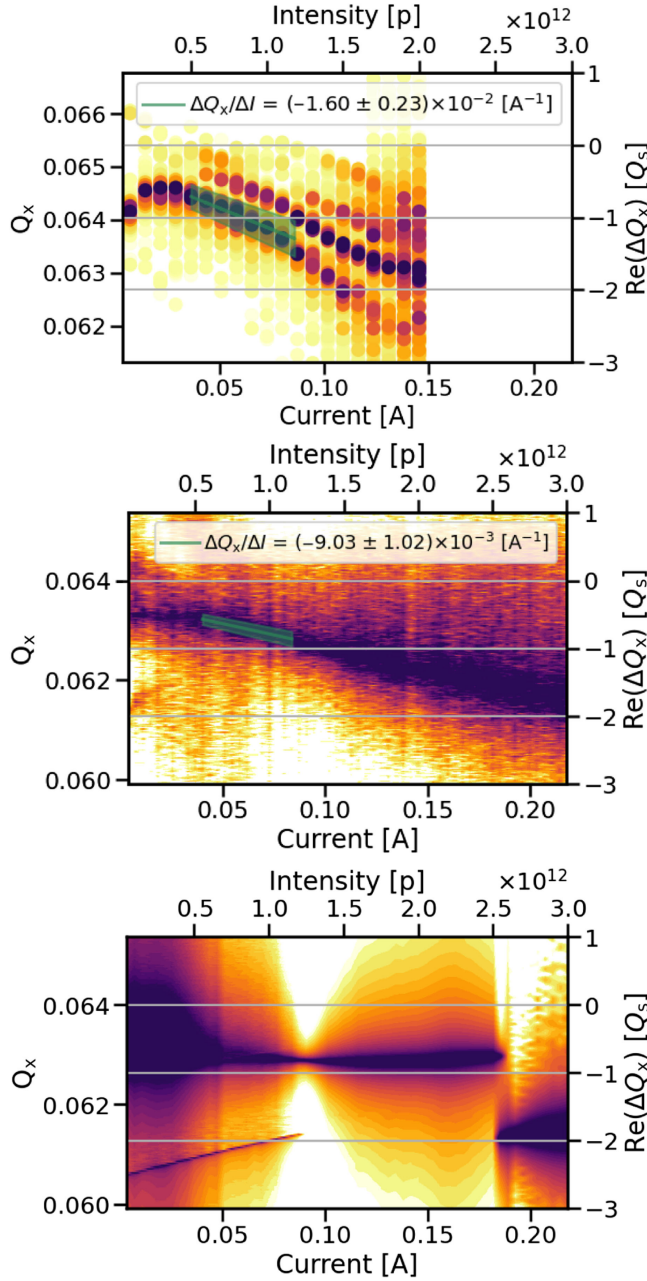


FIG. 9. Horizontal tune shift against beam intensity at injection energy: measurements (top), simulations with space charge (middle), and simulations without space charge (bottom). All simulations include first- and second-order chromaticity and octupolar order nonlinearities. The color scale represents the spectral magnitude, from high (purple) to low (yellow). The fitted tune shift slope and its confidence interval are represented by a green line and envelope.

modes. However, neighboring modes restrict the fitting range of the tune shift slope at high intensities, especially in the horizontal plane, for both measurements and simulations. In the vertical plane, measurements and simulations show good agreement on the tune shift slope. The horizontal plane exhibits a 50% underestimation in simulations, where only

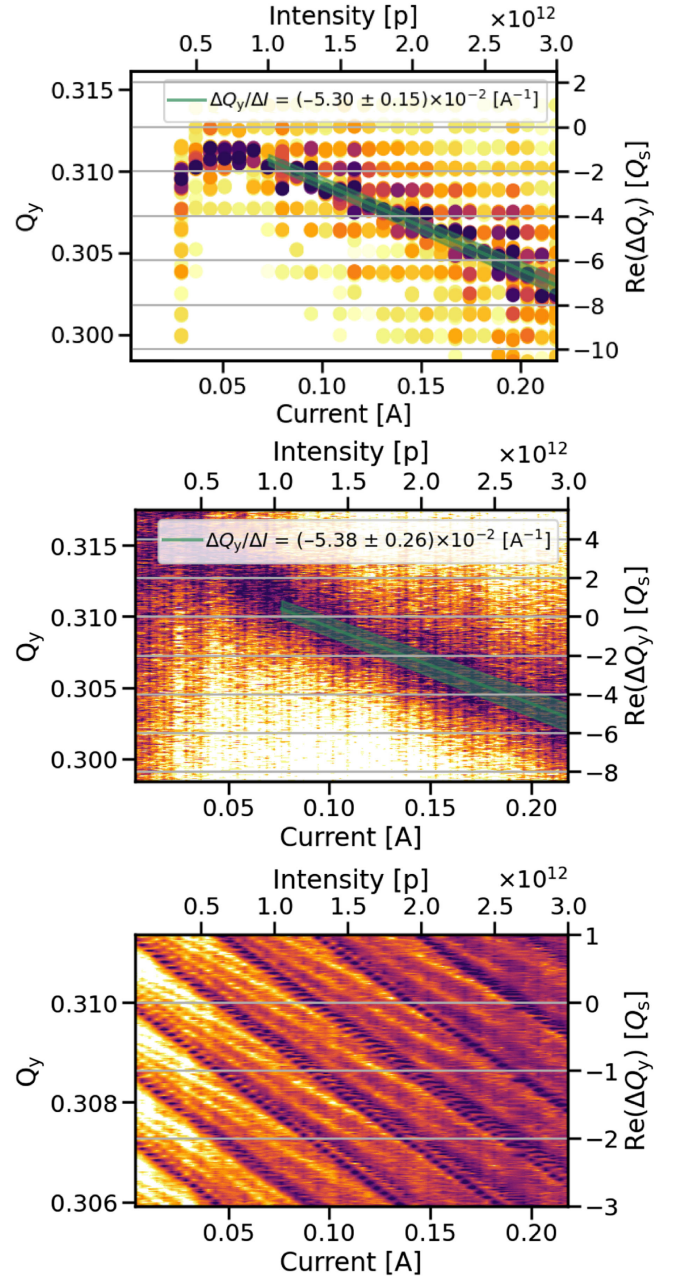


FIG. 10. Vertical tune shift against beam intensity at injection energy: measurements (top), simulations with space charge (middle), and simulations without space charge (bottom). All simulations include first- and second-order chromaticity and octupolar order nonlinearities. The color scale represents the spectral magnitude, from high (purple) to low (yellow). The fitted tune shift slope and its confidence interval are represented by a green line and envelope.

one mode is observed compared to two parallel modes in measurements. The lower line, assumed to be the main mode, was used for the tune shift slope fitting. Finally, at low intensities in both planes, the main mode tune shift exhibits a bend with intensity. This behavior resembles the results from [68], where space charge significantly affects the mode tune

TABLE II. Chromaticities measured at injection and extraction energies during 2021 and 2022 campaigns.

Scenario	Year	Q'_x	Q'_y	Q''_x	Q''_y
Injection	2021	-5.4	-9.6	-206	215
	2022	-5.0	-9.4	89	178
Extraction	2021	2.3	1.7	365	-407
	2022	0.5	1.4	-46	-212

shift when $\Delta Q_{SC} \approx Q_s$. In this intensity range, the transition from the low to intermediate space charge regime may influence both the mode tune shift and Landau damping. As a consequence, the previously dominant mode can become stabilized with a slight increase in intensity, allowing a different mode to become the new dominant one.

The 2022 measurement campaign achieved a major milestone: the clear observation of the main mode tune shift at both injection and extraction energies, with relative uncertainties as low as 5%. This breakthrough was enabled by refining the chirp signal used to excite the beam [43]. The PS impedance model successfully reproduces these measurements when including space-charge effects, particularly in the vertical plane, where simulated and measured tune shift slopes closely match.

VI. VALIDATION OF THE TRANSVERSE IMPEDANCE MODEL

A. Impedance-induced tune shifts

The PS transverse impedance model should be able to accurately reproduce the impact of the impedance on beam dynamics, in particular, the impedance-induced tune shift. Similarly to the previous section, the space charge, octupolar order nonlinearities, first- and second-order chromaticity, and nonlinear synchrotron motion are included in the simulations. The parameters used by the PyHEADTAIL simulations at injection energy can be found in Table V and at extraction energy in Table VI. Measurements from 2021 and 2022 with similar Q' are compared with simulations based on 2022 measurement parameters. The different chromaticities for both sets of measurements at injection and extraction energies can be found in Table II.

The Q'' values measured in 2021 could not be reproduced in 2022, hence, the measured tune shift slopes are expected to be different. The validity of the model has been assessed by comparing the 2022 measurements and the corresponding simulations. The measured and simulated tune shifts are presented in Figs. 11 and 12, with their slopes and confidence intervals summarized in Table III.

Besides the different chromaticities between the 2021 and 2022 measurements, the procedure has been improved

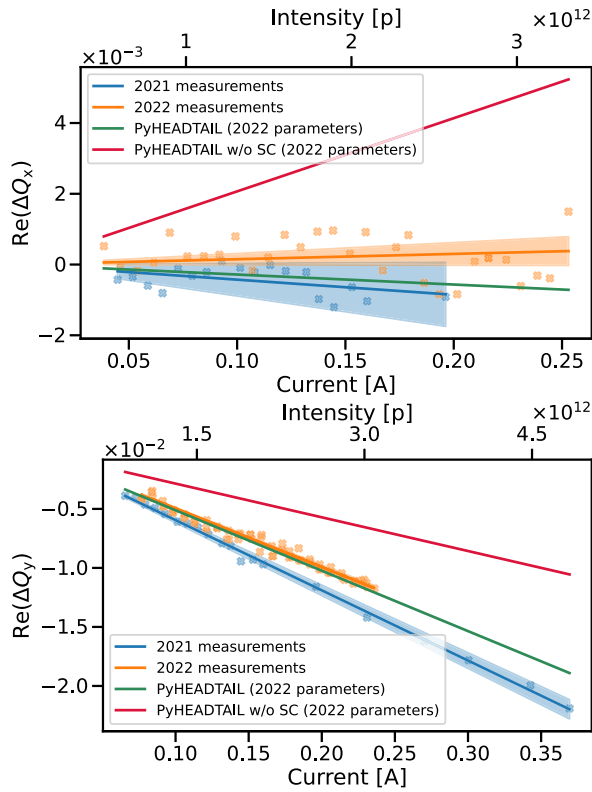


FIG. 11. Comparison between 2021 and 2022 measurements with PyHEADTAIL simulations, at injection energy, in horizontal (top) and vertical (bottom).

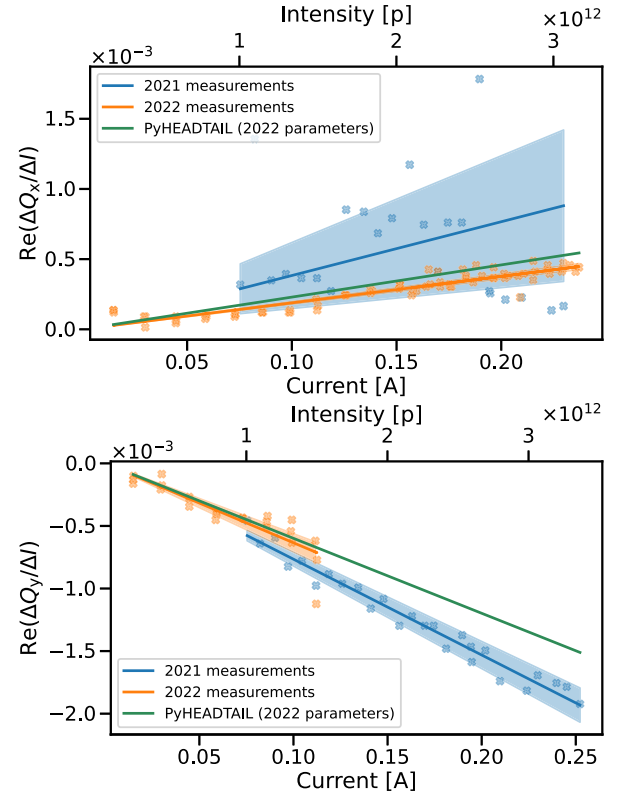


FIG. 12. Comparison between 2021 and 2022 measurements with PyHEADTAIL simulations, at extraction energy, in horizontal (top) and vertical (bottom).

TABLE III. Tune shift slope comparison between 2021 and 2022 measurements, and with PyHEADTAIL simulations (2022 parameters), at injection and extraction energies.

Scenario	Plane	2021, $\Delta Q/\Delta I$ (A^{-1})	2022, $\Delta Q/\Delta I$ (A^{-1})	PyHEADTAIL, $\Delta Q/\Delta I$ (A^{-1})
Injection	x	$(-4.50 \pm 4.86) \times 10^{-3}$	$(1.58 \pm 1.69) \times 10^{-3}$	-2.97×10^{-3}
	y	$(-6.25 \pm 0.23) \times 10^{-2}$	$(-5.22 \pm 0.10) \times 10^{-2}$	-5.38×10^{-2}
Extraction	x	$(3.83 \pm 2.35) \times 10^{-3}$	$(1.89 \pm 0.09) \times 10^{-3}$	2.29×10^{-3}
	y	$(-7.66 \pm 0.54) \times 10^{-3}$	$(-6.34 \pm 0.72) \times 10^{-3}$	-5.99×10^{-3}

TABLE IV. Parameters for the PyHEADTAIL instability growth rate simulations with space charge.

$n_{\text{macroparticles}}$	$n_{\perp}^{\text{cell}}$	Transverse grid span
6×10^6	128	$\pm 8\sigma_{x,y}$
n_{segments}	$n_{\parallel}^{\text{cell}}$	Longitudinal grid span
240	128	$\pm 4\sigma_z$

by refining the chirp signal used to excite the beam [43], leading to significantly reduced uncertainties. At injection energy, while space-charge effects lead to larger uncertainties in the horizontal plane due to a significant tune spread, the vertical tune shift measurements achieve excellent

precision with a confidence interval an order of magnitude smaller than the measured slope.

The tune shifts at extraction energy are smaller than at injection energy due to the $1/\gamma$ scaling and the reduced indirect space charge contribution. However, the shorter bunch length at this energy extends the bunch spectrum to higher frequencies, providing validation of the impedance model over a wider frequency range. Notably, the simulated tune shifts closely match the 2022 measurements, falling within the confidence intervals in both planes.

At both energies, the variation in tune shift slope between 2021 and 2022 measurements is qualitatively consistent with the change in Q'' given in Table II, further validating our understanding of the mechanisms at work.

TABLE V. Machine, bunch, and simulation parameters for the nTOF cycle (injection energy).

Parameter	Value	Unit
Circumference C	628.3185	m
Kinetic energy E_{kinetic}	2.0	GeV
Lorentz factor γ	3.13	
Revolution frequency f_0	452,153	Hz
Horizontal and vertical tune Q_x/Q_y	6.07/6.31	
Horizontal and vertical average beta functions β_x/β_y	16.5/15.8	m^{-1}
Anharmonicity coefficients $a_{xx}/a_{xy}/a_{yy}$	6.11/-53.7/32.6	m^{-1}
Horizontal emittance ϵ_x	$3.82 \times 10^{-18} \times N_{\text{particles}} + 6.90 \times 10^{-7}$	mm mrad
Vertical emittance ϵ_y	$1.32 \times 10^{-18} \times N_{\text{particles}} + 2.50 \times 10^{-8}$	mm mrad
Horizontal and vertical bare first-order chromaticity Q'_x/Q'_y	-5.0/-7.0	
Horizontal and vertical bare second-order chromaticity Q''_x/Q''_y	89/-41	
Momentum compaction factor α_p	0.025	
Bunch length τ_b	131	ns
Longitudinal emittance ϵ_l	1.55	eV s
Synchrotron tune Q_s	1.36×10^{-3}	
Harmonic number h	8	
rf cavity voltage V_{rf}	50	kV
Number of macroparticles	700,000	
Number of slices	1000	
Bunch slicing span	± 4	σ_z
Number of accumulated wake turns	10	turn
Initial transverse kick	1	mm
Smooth optics approximation	Yes	
Synchrotron motion	Nonlinear	

The PS impedance model reproduces the measurements when including all the relevant mechanisms, except in the horizontal plane at the injection energy, where a discrepancy remains due to the large uncertainties and the order of magnitude of the observed tune shift. As the horizontal tune shift is well reproduced at extraction energy, the discrepancy may stem from a missing energy-dependent mechanism or an impedance contribution falling outside the bunch spectrum at extraction; either hypothesis remains to be tested. In addition, the 2022 measurements achieved a sufficient precision to allow the observation of the horizontal main mode tune shift and the determination of its slope with a relative confidence interval of 5%, as it can be seen in Appendix C.

B. Instability growth rates

On top of causing an intensity-dependent tune shift, the transverse impedance is also responsible, in some cases, for transverse instabilities. Instabilities exhibit complex behaviors with Q' [45], Q'' [43,64], and the Landau damping introduced by space charge and lattice nonlinearities [68–70]. However, we focus here on the interplay between space charge and the slow-head-tail instability, and not the fast-head-tail instability as studied in detail in [67,71–76] for hadron accelerators and recently in [7] for fourth generation light sources.

Transverse instabilities can also be used to validate the impedance model. More specifically, reproducing measured growth rates for various chromaticities and intensities in simulations is an adequate way to benchmark the real part of an impedance model. To complement the tune shift measurements in the horizontal plane, where some discrepancies remain, we focus on the horizontal growth rates at injection energy.

For the intensities considered in the following measurements, the PS is in the strong space charge regime with a space charge parameter $q_{SC} = \Delta Q_{SC,x}/2Q_s$ ranging from 12 ($I_b = 5 \times 10^{11}$ p) to 75 ($I_b = 3 \times 10^{12}$ p).

Similarly to the tune shift simulations, octupolar order nonlinearities, first- and second-order chromaticity, and nonlinear synchrotron motion are all included in PyHEADTAIL, as well as direct space charge, using the numerical parameters given in Table IV. The parameters used by the PyHEADTAIL simulations can be found in Table V. Note that the longitudinal resolution of the mesh grid has been increased with respect to the previous simulations, to resolve the transverse oscillations caused by instabilities along the bunch. In addition, the number of macroparticles has been doubled to populate sufficiently the bunch distribution tails. Finally, as the transverse emittance of a bunch increases while an instability is developing, the extent of the mesh grid has been increased to reach $\pm 8\sigma_{x,y}$ to delay the loss of particles as they leave the PIC grid. It translates into turn-by-turn data recorded over a larger number of turns with a preserved beam intensity. For octupolar order nonlinearities, the same

anharmonicity coefficients and intensity dependence on the transverse emittance, as in the tune shift simulations, were used.

In the presence of space charge, instabilities can exhibit multiple behaviors, as illustrated in Fig. 13. They range from a “textbook” exponential growth to a gradual increase in the center-of-mass motion that eventually saturates, or even to successive bursts and decays of the center-of-mass motion. Therefore, an accurate estimation of the growth rate is challenging, as it was already observed in [14].

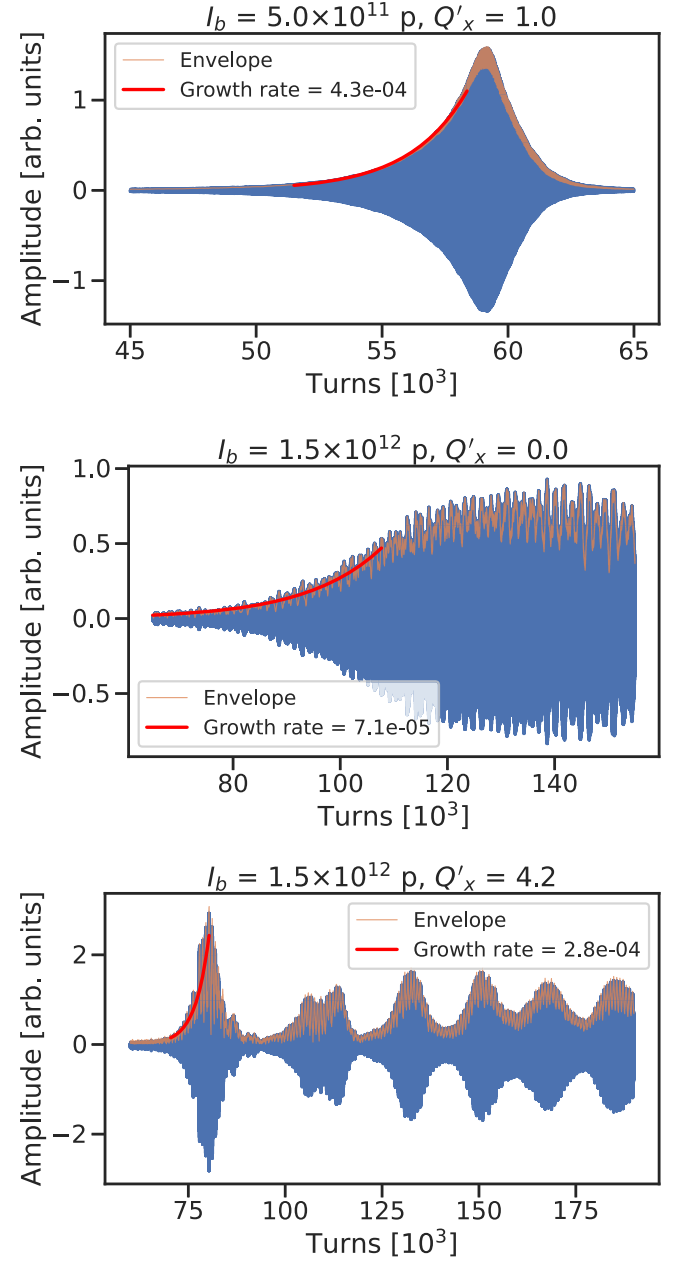


FIG. 13. Examples of BPM data recorded during the measurements shown in Fig. 14, with their respective growth rate fittings.

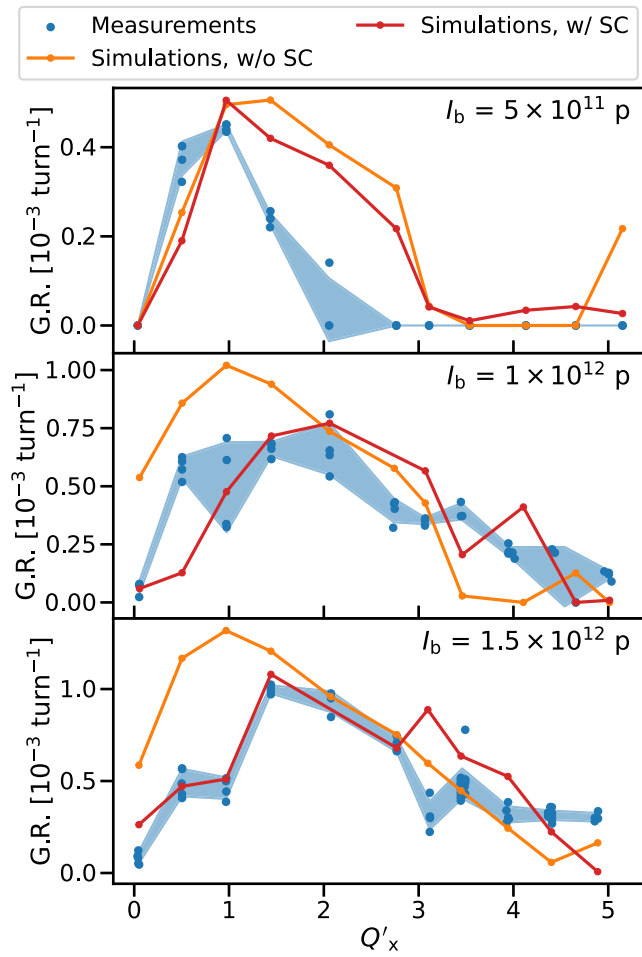


FIG. 14. Comparison between simulated horizontal instability growth rates at injection and those measured during the 2022 campaign, at injection, as a function of first-order chromaticity and for various beam intensities, with and without direct space charge.

The measured growth rates were acquired during the 2022 measurement campaign for a range of Q' and various intensities, as shown in Fig. 14. The growth rate fits were performed on the initial rise of the center-of-mass motion, stopping before the signal reached its maximum amplitude, as described in Sec. III B. The measured turn-by-turn data and analysis script used to generate Fig. 14 can be found in the Supplemental Material [77].

The shaded envelope around the growth rate values indicates the statistical confidence interval, calculated as the standard deviation of the growth rates obtained for each chromaticity value.

Regardless of the beam intensity, the growth rate pattern with Q' follows qualitatively a hump shape. As the intensity increases, both the chromaticity value at which the maximum growth rate occurs and the width of the hump tend to increase. Simulations without space charge reproduce the measured growth rate fairly well, except for low Q' , where

they overestimate the growth rate value. This discrepancy is resolved by including the effect of space charge in simulations, which then reproduces the measured growth rate envelope in most cases.

For the lowest intensity, i.e., 5×10^{11} p, the space-charge force influence on the growth rate is negligible, and a discrepancy between measurements and simulations remains. A possible explanation could be the anharmonicity coefficients used, which may not realistically reflect the real ones. Most importantly, the simulated growth rate values, with and without space charge, remain similar even for scenarios exhibiting a stronger space-charge force. The presence of space charge tends to yield only a slight stabilization of the instability. In the strong space charge regime, the stabilization introduced by space charge tends to saturate when $\Delta Q_{SC} \gg Q_s$ as it was observed in [68,70].

Similarly to the tune shift measurements, the PS impedance model reproduces the instability growth rates to a satisfactory agreement across different beam intensities and a wide range of chromaticities. These measurements also allowed the horizontal components of the impedance to be investigated, completing the validation of the impedance model.

Additional growth rate measurements performed in 2021 at 3×10^{12} p can be found in Appendix D.

VII. CONCLUSION

This paper presented the updated transverse impedance model of the CERN proton synchrotron following the long shutdown 2. It was shown that the most significant change that took place during LIU was the increase of the injection energy, which led to a reduction of the imaginary part of the impedance while leaving its real part largely unchanged. The impedance model includes all known major impedance sources, namely, the vacuum chamber, kicker magnets, vacuum equipment, septa, beam dumps, rf cavities, and beam instrumentation.

To validate this impedance model, two complementary beam observables were used: impedance-induced tune shift and instability growth rates. Their interplay with the first- and second-order chromaticity, and space-charge force was thoroughly investigated in simulations with PyHEADTAIL. The main observations were that Q'' strongly impacts the magnitude of the tune shift and the space charge introduces Landau damping, which needs to be accounted for to accurately reproduce the measured tune shifts and growth rates.

The tune shift measurements, particularly in the vertical plane, provided precise validation of the imaginary part of the impedance. On the other hand, the growth rate measurements complemented this by validating the real part of the impedance, especially in the horizontal plane, where tune shift measurements showed some discrepancies. The PS impedance model could reproduce the vertical tune shifts down to the level of the measurements' confidence interval and the instability growth rates at injection

energy (q_{SC} ranging from 12 to 75) with satisfactory accuracy.

The PS impedance model has been benchmarked against a broad range of beam parameters and shows good agreement with measured impedance-induced tune shifts and instability growth rates when the relevant mechanisms are included in the simulations. This close agreement across different beam parameters and operating conditions demonstrates the reliability of the updated model and provides a solid basis for predicting beam stability in the PS, including scenarios of interest for the Physics Beyond Colliders project and potential future machine upgrades.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data that support the findings of this article are openly available [78].

APPENDIX A: PS TRANSVERSE IMPEDANCE MODEL AT INJECTION

The Post LIU PS transverse impedance model for each component is summarized in Figs. 15–18.

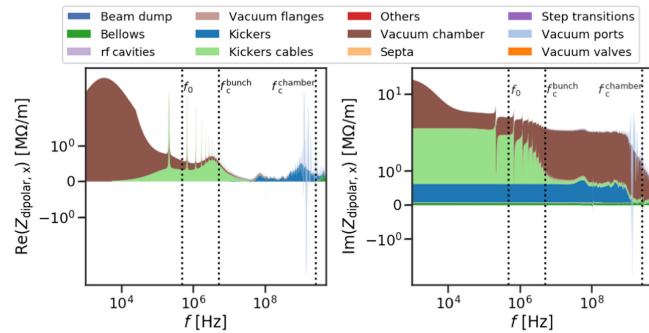


FIG. 15. Representation of the stacked contributions to the real and imaginary parts of the horizontal dipolar impedance accounted for in the Post LIU PS impedance model, at injection energy, in logarithmic scale. The revolution frequency f_0 , the frequency corresponding to the bunch length at injection f_c^{bunch} , and the vacuum chamber cutoff frequency f_c^{chamber} are displayed as dashed black lines.

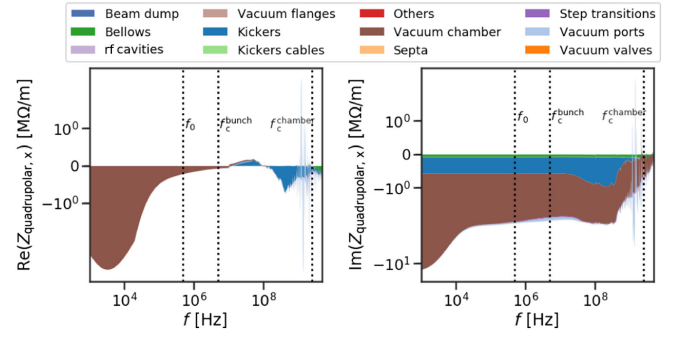


FIG. 16. Representation of the stacked contributions to the real and imaginary parts of the horizontal quadrupolar impedance accounted for in the Post LIU PS impedance model, at injection energy, in logarithmic scale. The revolution frequency f_0 , the frequency corresponding to the bunch length at injection f_c^{bunch} , and the vacuum chamber cutoff frequency f_c^{chamber} are displayed as dashed black lines.

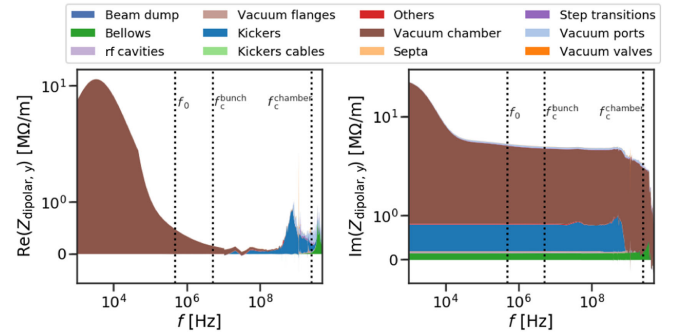


FIG. 17. Representation of the stacked contributions to the real and imaginary parts of the vertical dipolar impedance accounted for in the Post LIU PS impedance model, at injection energy, in logarithmic scale. The revolution frequency f_0 , the frequency corresponding to the bunch length at injection f_c^{bunch} , and the vacuum chamber cutoff frequency f_c^{chamber} are displayed as dashed black lines.

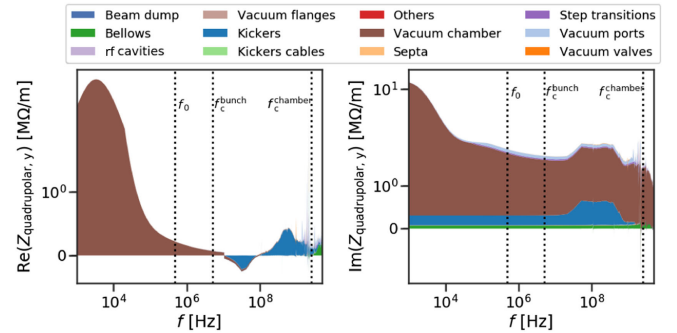


FIG. 18. Representation of the stacked contributions to the real and imaginary parts of the vertical quadrupolar impedance accounted for in the Post LIU PS impedance model, at injection energy, in logarithmic scale. The revolution frequency f_0 , the frequency corresponding to the bunch length at injection f_c^{bunch} , and the vacuum chamber cutoff frequency f_c^{chamber} are displayed as dashed black lines.

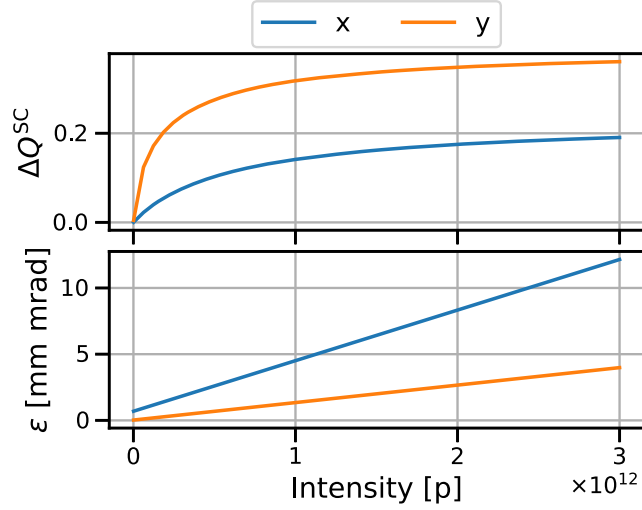


FIG. 19. Space charge tune shift (top) and transverse emittance (bottom) versus intensity for the custom nTOF cycle.

APPENDIX B: PS PARAMETERS AND PyHEADTAIL SETTINGS

All the parameters listed in the following tables were experimentally measured and then used during the PyHEADTAIL simulations. Measurements and simulations

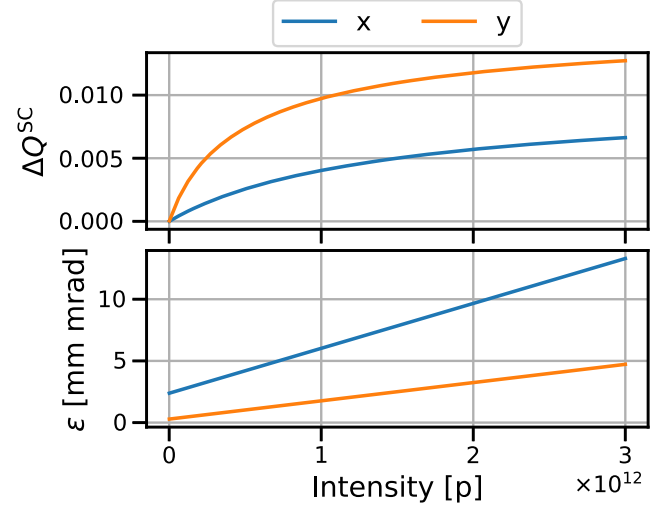


FIG. 20. Space charge tune shift (top) and transverse emittance (bottom) versus intensity for the custom EAST cycle.

are specific to proton beams. Two different custom cycles inspired by operational ones are presented. The custom nTOF cycle was used during the impedance-induced tune shift and instability growth rate measurements at injection energy. Its intensity-dependent space charge tune shift and transverse emittances can be found in Fig. 19 and its

TABLE VI. Machine, bunch, and simulation parameters for the EAST cycle (extraction energy).

Parameter	Value	Unit
Circumference C	628.3185	m
Kinetic energy E_{kinetic}	25.4	GeV
Lorentz factor γ	28.07	
Revolution frequency f_0	476,832	Hz
Horizontal and vertical tune Q_x/Q_y	6.23/6.27	
Horizontal and vertical average beta functions β_x/β_y	16.0/16.0	m^{-1}
Anharmonicity coefficients $a_{xx}/a_{xy}/a_{yy}$	-110/35.6/53.3	m^{-1}
Horizontal emittance ϵ_x	$3.64 \times 10^{-18} \times N_{\text{particles}} + 2.38 \times 10^{-6}$	mm mrad
Vertical emittance ϵ_y	$1.48 \times 10^{-18} \times N_{\text{particles}} + 2.83 \times 10^{-7}$	mm mrad
Horizontal and vertical bare first-order chromaticity Q'_x/Q'_y	0.5/1.4	
Horizontal and vertical bare second-order chromaticity Q''_x/Q''_y	-46/-212	
Momentum compaction factor α_p	0.027	
Bunch length τ_b	42	ns
Longitudinal emittance ϵ_l	2.09	eV s
Synchrotron tune Q_s	4.98×10^{-4}	
Harmonic number h	8	
rf cavity voltage V_{rf}	200	kV
Number of macroparticles	700,000	
Number of slices	1000	
Bunch slicing span	± 4	σ_z
Number of accumulated wake turns	10	turn
Initial transverse kick	1	mm
Smooth optics approximation	Yes	
Synchrotron motion	Nonlinear	

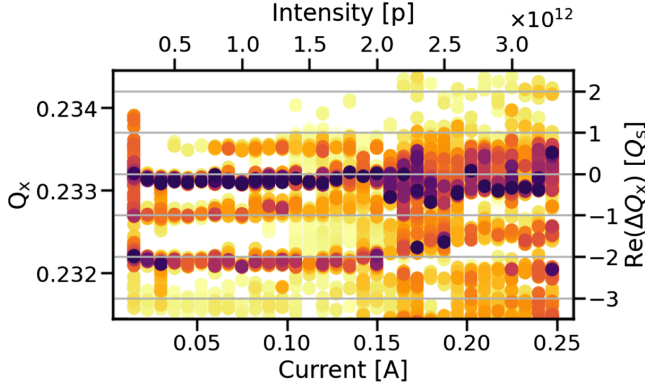


FIG. 21. Measured horizontal tune shift against beam intensity at extraction energy. The color scale represents the spectral magnitude, from high (purple) to low (yellow).

parameters in Table V. The custom EAST cycle was used during the impedance-induced tune shift measurements at extraction energy. Its intensity-dependent space charge tune shift and transverse emittances can be found in Fig. 20 and its parameters in Table VI.

APPENDIX C: HORIZONTAL TUNE SHIFT MEASUREMENTS AT EXTRACTION ENERGY

Measuring the tune shift in the horizontal plane at injection energy has proven to be challenging due to the small magnitude of the tune shift slope with beam intensity (approximately 10^{-3} A^{-1}) and the large tune spread caused by direct space charge, surpassing the impedance-induced tune shift by orders of magnitude. At extraction energy, the tune shift slope magnitude is further reduced due to the reduced indirect space charge contribution. However, the direct space charge tune spread is reduced to a larger extent, thus enabling the observation of not only the main mode but also additional azimuthal modes, as shown in Fig. 21. This acquisition was performed in 2022, with chromaticity measured as $Q'_x = 0.8$ and $Q'_y = -181$.

Thanks to the achieved precision, azimuthal modes from -2 to 1 can be clearly tracked across the entire intensity range. Moreover, a coupling between modes -1 and 0 is observed after 0.15 A .

APPENDIX D: 2021 INSTABILITY MEASUREMENTS AT INJECTION ENERGY

Prior to the measurements performed in 2022, another set of measurements was carried out in 2021 with an intensity of $3 \times 10^{12} \text{ p}$. Contrary to the 2022 measurements, only one acquisition per chromaticity value was done, thus preventing the calculation of a statistical uncertainty for the growth rate values. Moreover, the chromaticity was only controlled through the PFW in 2021, introducing unknown Q'' values significantly larger than in 2022. Unfortunately, the Q'' values were not measured during this measurement

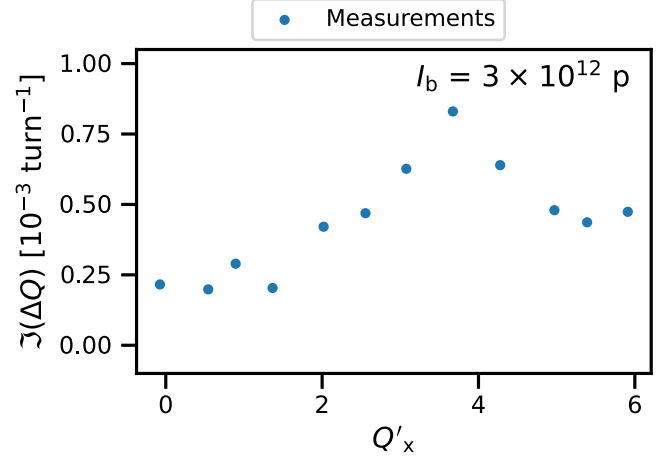


FIG. 22. Measured horizontal instability growth rates at injection against first-order chromaticity for various beam intensities, with and without direct space.

campaign. As a result, the measurements could not be directly compared with simulations. The resulting growth rates can be found in Fig. 22.

The previously observed hump pattern with respect to chromaticity is reproduced with a maximum further pushed to a larger Q' value. In addition, the beam now exhibits an unstable behavior even for $Q'_x = 0$ at this beam intensity.

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