

Noninterferometric method for transverse electron beam size diagnostic with synchrotron radiation at a free-electron laser

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We present a noninterferometric method for measuring the transverse electron beam size at the European XFEL based on synchrotron radiation intensity autocorrelation. The technique enables cell-by-cell diagnostics along the SASE1 undulator using existing hardware: a commissioning monochromator and a spatial x-ray detector. We extend the results of [A. Trebushinin *et al.*, companion paper, First observation of synchrotron radiation spikes for transverse electron beam size measurements at a free-electron laser, *Phys. Rev. Lett.* **135**, 215001 (2025).] focusing on data analysis procedures and technical implementation. Our method allows for extraction of the beam size from noisy measured intensity distributions with a signal-to-noise ratio as low as 0.9 and is capable of distinguishing between the projected and slice sizes. To validate our approach, we introduced a mismatch in the magnetic lattice, inducing a controlled sinusoidal variation in the beam size along the undulator. The corresponding modulation was clearly reproduced in the electron beam size values extracted from the autocorrelation measurements. This method offers a practical solution for transverse electron beam size diagnostics in high-energy linacs, with direct application to the operation and tuning of free-electron laser facilities.

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I. INTRODUCTION

Improving x-ray light sources requires accurate knowledge of the full six-dimensional phase-space distribution of the electron beam. In particular, the performance of free-electron lasers (FEL) strongly depends on maintaining high beam quality—i.e., low electron beam emittance, high current, and low energy spread. In particular, controlling electron beam parameters along the long undulator line, where the FEL lasing process occurs, is essential.

Various techniques are available for measuring the longitudinal phase space of the electron beam. Among these, we cite here transverse deflecting structures [1–4] and spectral analysis of FEL radiation [5,6]. They allow diagnosing longitudinal bunch dynamics, which is critical for advanced lasing modes such as self-seeding [7–18], two-color generation [19–23], and ultrashort pulse production [23–31] among others. However, a better control of FEL performance requires precise control over transverse beam parameters as well—especially for schemes that rely on transverse-longitudinal coupling to produce subfemtosecond pulses [23].

Transverse beam sizes are commonly measured at FEL facilities using destructive diagnostics such as wire scanners [32,33]. While robust, these methods are not suitable for cell-by-cell diagnostics along undulator lines, as they require dedicated hardware installed in many cells, while tracking changes in the transverse electron beam phase space along the undulator is critical [23]. In the case of the SASE1 beamline at the European XFEL [34], this would

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require tens of installations. Moreover, these methods yield only projected sizes, which do not account for the presence of transverse-longitudinal tilts, while transverse size of the electron bunches *slice* may be more important than the projected transverse size of *the entire bunches*, when it comes to beam matching to optimize lasing process.

An alternative approach is to determine source size from the transverse autocorrelation of the far-field radiation, emitted by the electron-beam-in-question. This approach relies on the van Cittert–Zernike theorem [35,36], which relates the transverse distribution of the radiation source to the autocorrelation function of the radiation distribution in the far zone. In an interferometric scheme, this is typically implemented via a double-slit setup (or similar setups), where fringe contrast as a function of slit separation provides access to the correlation function. However, such a scheme also requires the installation of specialized hardware too.

Here we propose to use, instead, a noninterferometric setup, similar to the original Hanbury Brown and Twiss (HBT) setup [37,38], which has since been adapted across many areas of physics [39–42]. Any stochastic radiation source, including undulator light, exhibits fluctuations of electric field distribution, making the HBT concept applicable well beyond thermal light. Notably, synchrotron emission has been used to characterize transverse source sizes and durations of electron beams, as was excellently

demonstrated in [43,44]. Synchrotron radiation, like SASE [45], exhibits random spatiotemporal field fluctuations arising from shot noise in the electron beam [46,47].

We stress that our setup contains no interferometric elements; coherence information is extracted, instead, from the statistical structure of the intensity fluctuations. The intensity autocorrelation function can be directly computed from single-shot images recorded downstream of a monochromator with a properly chosen bandwidth, which enhances the contrast of spikes in the detected transverse distributions (see Fig. 1).

In this paper, we demonstrate the practical applicability of this setup for measuring the averaged transverse size of slices along the electron beam (hereafter referred to as *the averaged transverse slice size*) at the SASE1 undulator of the European XFEL, using only two optical components: a silicon monochromator [48] and an x-ray camera [49]. No additional instrumentation was required, as the proposed setup was readily available—the two elements are already present as commissioning devices at all European XFEL beamlines.

The paper is organized as follows: Section II describes the principles of the method and estimates relevant experimental parameters. Section III presents numerical simulations and outlines the data analysis pipeline, including the distinction between averaged transverse slice size and projected beam size in the presence of spatial tilts.

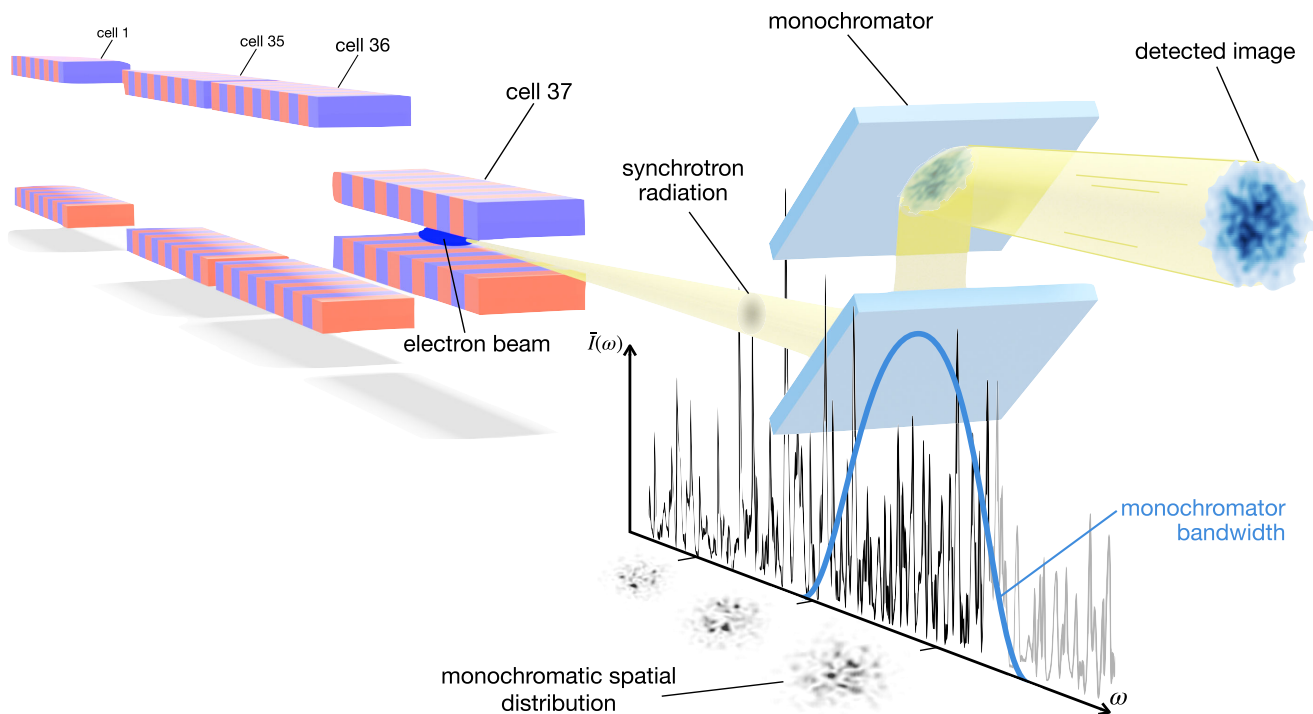


FIG. 1. A schematic of the proposed experimental setup. Radiation is emitted from cell 37 of the SASE1 undulator. The blue distribution, positioned after the monochromator, represents the averaged transverse distribution resulting from multiple longitudinal spikes. The black-and-white spatial distribution near the spectrum illustrates the monochromatic intensity pattern associated with these individual spikes.

Section IV discusses the analysis of real experimental data, including challenges related to signal quality and detector effects. In Sec. V, we present the measured beam sizes along the SASE1 beamline for two different magnetic lattice settings: matched and mismatched electron optics.

II. METHOD

The electromagnetic field of synchrotron radiation exhibits both spatial and temporal fluctuations. In the context of free-electron lasers (FELs), these fluctuations are commonly referred to as “spikes.” They should not be confused with “speckles,” which refer to the stationary characteristic diffraction pattern being a result of interference of coherent light reflected from or transmitted through a static granulated object. The characteristic size of these spikes is typically associated with the coherence time and coherence length of the radiation in the respective domains. In principle, it is possible to directly measure the transverse intensity distribution of synchrotron radiation and detect intensity fluctuations—spikes, if only several longitudinal spikes are present. By collecting a statistical ensemble of such events, one can compute the second-order correlation function, which effectively characterizes the typical shape and distribution of a spike at a given point in the temporal-spectral or spatial-inverse spatial domains.

Furthermore, the correlation function in one domain contains information about the spatial extent of the radiation in the reciprocal domain. This relationship is established by the van Cittert–Zernike theorem for the transverse spatial domain [35,36], and by the Wiener-Khinchin theorem for the longitudinal (temporal or spectral) domain [50,51]. In the case of partially coherent or quasihomogeneous sources, these theorems can be generalized [52,53], with Bochner’s theorem [54] providing a broader theoretical framework in the context of harmonic analysis.

Using the van Cittert–Zernike theorem, one can infer the transverse source size by analyzing the far-zone intensity correlation function. Since the transverse size of the synchrotron radiation source reflects the corresponding phase-space distribution of the electron beam, this method provides access to the electron beam size itself.

A. The generalized van cittert–Zernike theorem

The field autocorrelation in the spatial-spectral domain can be written as

$$\Gamma_\omega(z, \vec{r}_1, \vec{r}_2, \omega_1, \omega_2) = \langle \bar{E}(z, \vec{r}_1, \omega_1) \bar{E}^*(z, \vec{r}_2, \omega_2) \rangle, \quad (1)$$

where $\langle \dots \rangle$ denotes ensemble averaging, and $\bar{E}$ is the electric field in the frequency domain (i.e., the temporal Fourier transform of the time-domain field E). The variables $\omega_{1,2}$ are the angular frequencies of the fields being correlated, $\vec{r}_{1,2}$ are transverse coordinates, and z denotes the longitudinal position along the optical axis, aligned with the direction of radiation propagation.

In synchrotron radiation theory, Γ_ω can be factorized into cross-spectral density function $G(z, \vec{r}_1, \vec{r}_2, \bar{\omega})$ and Fourier transform of the longitudinal bunch profile $\bar{f}(\Delta\omega)$ [46]:

$$\Gamma_\omega(z, \vec{r}_1, \vec{r}_2, \bar{\omega}, \Delta\omega) = G(z, \vec{r}_1, \vec{r}_2, \bar{\omega}) \bar{f}(\Delta\omega), \quad (2)$$

where we introduced a new set of variables: $\bar{\omega} = (\omega_1 + \omega_2)/2$ and $\Delta\omega = \omega_2 - \omega_1$.

The van Cittert–Zernike theorem is formulated for the cross-spectral density function, which is written as the autocorrelation of the field in the following form:

$$G(z, \vec{r}_1, \vec{r}_2, \bar{\omega}) = \langle \bar{E}(z, \vec{r}_1, \bar{\omega}) \bar{E}^*(z, \vec{r}_2, \bar{\omega}) \rangle. \quad (3)$$

One can normalize this function [52,55]:

$$g(z, \vec{r}_1, \vec{r}_2, \bar{\omega}) = \frac{G(z, \vec{r}_1, \vec{r}_2, \bar{\omega})}{\sqrt{\bar{I}(z, \vec{r}_1, \bar{\omega}) \bar{I}(z, \vec{r}_2, \bar{\omega})}}, \quad (4)$$

where, $\bar{I}(z, \vec{r}, \bar{\omega}) = \langle \bar{E}(z, \vec{r}, \bar{\omega}) \bar{E}^*(z, \vec{r}, \bar{\omega}) \rangle$. Then we apply the quasihomogeneous approximation [46,52] and factorize the cross-spectral density function at the source location ($z = 0$) as

$$G(0, \vec{r}, \Delta\vec{r}, \bar{\omega}) = g(0, \Delta\vec{r}, \bar{\omega}) \bar{I}(0, \vec{r}, \bar{\omega}), \quad (5)$$

where we changed the variables again $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$ and $\vec{r} = (\vec{r}_1 + \vec{r}_2)/2$.

This factorization qualitatively implies, loosely speaking, that the spike width, governed by $g(0, \Delta\vec{r}, \bar{\omega})$, is the same across the intensity distribution $\bar{I}(0, \vec{r}, \bar{\omega})$. By propagating $G(0, \vec{r}, \Delta\vec{r}, \bar{\omega})$ to the far field over a distance z , one can derive the following expression for the cross-spectral density function in the far zone, as shown in [52]:

$$\begin{aligned} G(z, \vec{r}, \Delta\vec{r}, \bar{\omega}) &= \frac{e^{i\frac{k_z}{z}\vec{r}\cdot\Delta\vec{r}}}{(\lambda z)^2} \iint_{-\infty}^{\infty} g(0, \Delta\vec{r}', \bar{\omega}) \bar{I}(0, \vec{r}', \bar{\omega}) e^{-i\frac{k_z}{z}(\vec{r}\cdot\Delta\vec{r}' + \vec{r}'\cdot\Delta\vec{r})} d\vec{r}' d\Delta\vec{r}' \\ &= \frac{e^{i\frac{k_z}{z}\vec{r}\cdot\Delta\vec{r}}}{(\lambda z)^2} \int_{-\infty}^{\infty} g(0, \Delta\vec{r}', \bar{\omega}) e^{-i\frac{k_z}{z}\vec{r}\cdot\Delta\vec{r}'} d\Delta\vec{r}' \int_{-\infty}^{\infty} \bar{I}(0, \vec{r}', \bar{\omega}) e^{-i\frac{k_z}{z}\vec{r}'\cdot\Delta\vec{r}} d\vec{r}' \end{aligned} \quad (6)$$

where $k = 2\pi/\lambda$ and λ is the radiation wavelength. This is the main result of the generalized van Cittert–Zernike theorem, elegantly presented in [52]. It is evident from Eq. (6) that $G(z, \vec{r}, \Delta\vec{r}, \bar{\omega})$ —the cross-spectral density function in the far zone—also factorizes, and we can write:

$$g^{(1)}(z, \Delta\vec{r}, \bar{\omega}) = \int_{-\infty}^{\infty} \bar{I}(0, \vec{r}', \bar{\omega}) e^{-ik\vec{r}' \cdot \Delta\vec{r}} d\vec{r}', \quad (7)$$

where we introduced the (1) superscript for g to differentiate between the field autocorrelation and intensity autocorrelation functions. This expression relates the autocorrelation from Eq. (4) to the source intensity distribution via a Fourier transform. By measuring $|g^{(1)}|$, one can retrieve the width of $\bar{I}$.

B. Relation of the experimentally measured correlation with $g^{(1)}$

However, in the experiment, we measure not the field itself but the intensity distributions.

$$g_{T_\omega}^{(2)}(z, \Delta\vec{r}) = \frac{\langle I_1 I_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle}, \quad (8)$$

where subscript T_ω denotes the averaging of the intensity over the monochromator bandwidth, and notation $I_{1,2}$ denotes a pair of points which are separated by distance $|\Delta\vec{r}|$.

For Gaussian processes, one can use the Siegert relation, a property of Gaussian random processes [52,55], to connect the intensity autocorrelation function with the field autocorrelation function:

$$g^{(2)}(z, \Delta\vec{r}, \bar{\omega}) = 1 + |g^{(1)}(z, \Delta\vec{r}, \bar{\omega})|^2. \quad (9)$$

In the case of a finite monochromator bandwidth T_ω , the maximum of $g^{(2)}$ is reduced. In Appendix A, we derive the relation that quantifies the drop in contrast as

$$\frac{\langle I_1 I_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle} = 1 + \frac{\hbar}{\sqrt{2} \Delta E_{T_\omega} \sigma_T} |g^{(1)}(z, \Delta\vec{r}, \bar{\omega}_0)|^2, \quad (10)$$

where ΔE_{T_ω} is the rms monochromator bandwidth, and σ_T is the rms electron beam duration. Equation (10) relates the intensity autocorrelation function with the modulus of $g^{(1)}(z, \Delta\vec{r}, \bar{\omega}_0)$ accounting for the contrast drop. The factor in front of $g^{(1)}$ can be interpreted as the inverse number of longitudinal spikes that fall under the monochromator bandwidth. When deriving Eq. (10), we assumed that the spectral bandwidth of the radiation is much broader than the monochromator transfer function T_ω , and that it varies negligibly over the bandwidth of T_ω .

Taking the inverse Fourier transform of both sides of Eq. (7) and assuming a Gaussian profile for the transverse phase space of the electron beam, one can obtain the following relation between the width of $|g^{(1)}(z, \Delta\vec{r}, \bar{\omega}_0)|^2$ —

σ_{c_x, c_y} —and the standard deviation of the electron beam transverse size [46]:

$$\sigma_{x,y} = \frac{z\lambda}{2\sqrt{2}\pi\sigma_{c_x, c_y}}. \quad (11)$$

This equation, derived as Eq. (92) in [46], assumes the far-zone limit, where the contribution from the radiation divergence can be neglected. As a result, the coherence length of the radiation is not influenced by the angular divergence of the source, which is consistent with the generalized van Cittert–Zernike theorem.

C. Estimation of the experimental parameters

It is possible to detect statistical realizations of I and calculate the autocorrelation $\langle I_1 I_2 \rangle / (\langle I_1 \rangle \langle I_2 \rangle)$ avoiding any interference scheme, which is typically used for coherence measurements. Instead, a simple setup similar to the Hanbury Brown and Twiss experiment could be employed, consisting of a spectral filter and two photon detectors—in our case, it is the whole two-dimensional array. The monochromator extracts several spikes longitudinally, with each spike containing its unique transverse distribution. Upon detection, these longitudinal spikes are averaged, resulting in a lower-contrast image in the transverse dimension. This is depicted in Fig. 1. Once a sufficiently large statistical ensemble of these images is collected, the autocorrelation function can be determined for these events, allowing the second-order correlation function to be calculated.

For a linear accelerator, like at the European XFEL, we have found that the electron beam duration is short enough to use a silicon crystal Bragg monochromator using the Si(111) reflection, along with a present sCMOS camera to conduct such measurements. In fact, the number of longitudinal spikes falling within the bandwidth of the Si(111) commissioning monochromator, for the resonant photon energy of 5500 eV with relative resolution of $(2 \pm 0.2) \times 10^{-4}$ and an electron beam duration of $\delta\tau = 37 \pm 3$ fs (FWHM), is approximately $M_L = 16 \pm 2$, as can

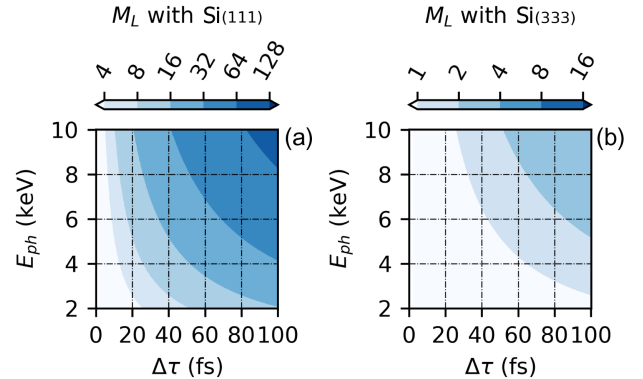


FIG. 2. Color map of the number of spikes M_L extracted after passing the monochromator.

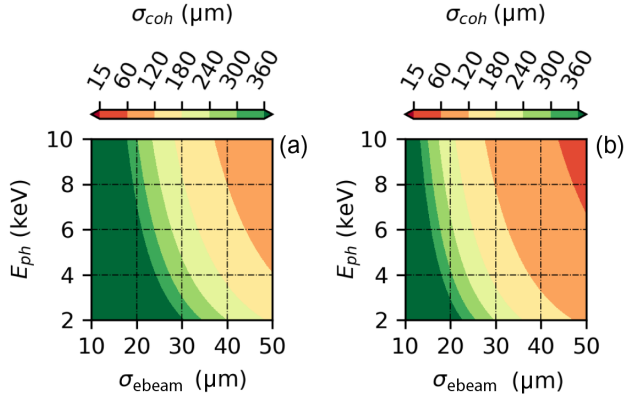


FIG. 3. The radiation coherence length is shown as a color map with respect to the expected electron beam size and photon energy for two cases: panel (a) corresponds to cell 21, and panel (b) to cell 37.

be seen in Fig. 2(a). For this estimation, we measured the electron beam duration using a noninvasive THz spectroscopy method at the European XFEL [56]. This indicates that one can indeed observe intensity fluctuations—the remnants of the spikes—of the synchrotron radiation pulses, as depicted in Fig. 1, with the detected image marked in the inset.

We ensured that the width of the correlation function (that is, basically, the coherence length) is larger than the spatial resolution of the detector system. For the expected electron beam size of 20–30 μm , corresponds approximately to a width of the correlation function of 200–300 μm (standard deviation) of the width of the correlation function as depicted in Fig. 3, which is well within the resolution capabilities of the camera system used at the European XFEL in the spontaneous radiation imager equipped with a yttrium-aluminum garnet screen at European XFEL.

As shown above, this method enables electron beam size measurements using equipment that is either already available at many facilities or easily implemented, relying on a straightforward data analysis procedure. Once the radiation field is filtered by a monochromator, it can be recorded, and the autocorrelation function extracted directly from the intensity distributions. In the following section, we present a more detailed discussion supported by simulations designed to reproduce realistic experimental conditions.

We note that the proposed setup is a natural extension of the HBT experimental scheme for synchrotron radiation sources and has already been demonstrated in [43,44]. In those studies, accessing the correlation function required a monochromator with exceptionally high spectral resolution: relative resolution was of order 10^{-8} – 10^{-9} . In our work, we demonstrate that such measurements can be readily performed at free-electron laser (FEL) facilities using standard components: in particular, Si(111) crystals offer a relative resolution of 2×10^{-4} . Furthermore, with

the use of a two-dimensional detector, we aim to directly visualize the spatial intensity fluctuations of synchrotron radiation.

III. SIMULATION

We reproduced the experimental conditions shown in the previous sections in simulations using the (Synchrotron Emission Rapid VALuator) SERVAL algorithm [57]. This algorithm simulates single instances of partially coherent fields and is fully capable of realistically representing the experimental output with synchrotron radiation. Each statistical realization was simulated separately, representing an ideally monochromatized transverse radiation intensity distribution. Since each spike in the frequency domain contains its own unique transverse distribution, a single recorded event—we denoted as I —can be interpreted as an average of these ideally monochromatized intensity distributions

For this simulation, we utilized the following parameters: the electron beam size $\sigma_x \times \sigma_y = 24 \times 32 \mu\text{m}^2$. This size exceeds the natural size of undulator radiation at the source location, which is given by $\sqrt{2\lambda L_w}/(4\pi) = 3.8 \mu\text{m}$, where $L_w = 5.2 \text{ m}$ and λ is the radiation wavelength corresponding to the undulator radiation resonance photon energy $E_{\text{ph}} = 5.5 \text{ keV}$. So, the electron beam size determines the radiation virtual source size.

The situation is reversed for the divergence, which is dominated by the natural undulator divergence, $\sqrt{\lambda/L_w} = 6.7 \mu\text{rad}$, in contrast to the electron beam divergence, which is approximately $\sqrt{\epsilon/\beta} \approx 1 \mu\text{rad}$. The electron beam emittance and beta function are taken or derived from experimental data: $\epsilon \approx 0.6 \text{ mm mrad}$ and $\beta = 30 \text{ m}$ for an 11.5 GeV electron beam, which we discuss in more detail in Sec. V.

To anticipate the analysis, in the experiment, we averaged over 13 electron bunches for each recorded event to achieve a reasonable observed intensity. Such averaging is equivalent to averaging over different monochromatic

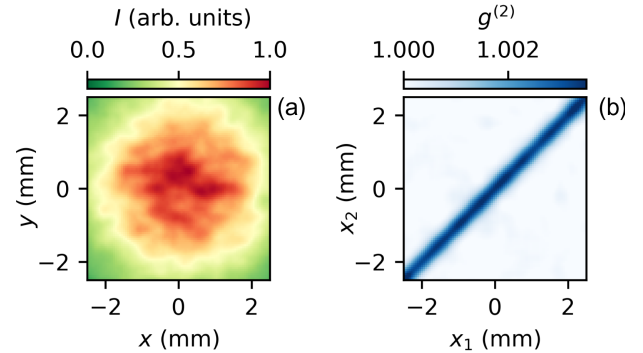


FIG. 4. Panel (a) shows the radiation intensity distribution averaged over 16×13 monochromatic realizations, and the corresponding second-order correlation function is shown in panel (b).

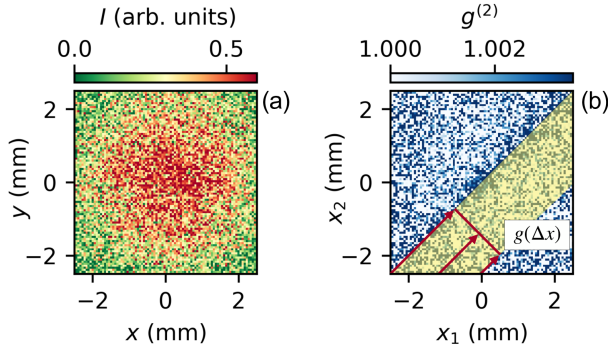


FIG. 5. The same as in Fig. 4 but with added noise. Signal-to-noise ratio is 0.9.

realizations of the field, provided the electron bunches properties remain reproducible. Consequently, in the simulation, we modeled 16×13 instances of the field, which, after averaging, constitute a single detected event with the camera. We then collected statistics to have an ensemble of these events, totaling 1000 simulated occurrences for the demonstration here. The resulting intensity distribution for a single event is depicted in Fig. 4(a). Additionally, we present in Fig. 4(b) the corresponding correlation function for the x_1 and x_2 dimensions, with $y_1 = 0$ and $y_2 = 0$.

The real experimental data, however, contain a very high level of noise, which we modeled as white noise. To effectively extract the signal, we averaged over multiple electron beams. The intensity distribution mixed with noise is shown in Fig. 5.

In the experiment, the signal-to-noise ratio dropped to $S/N \approx 0.9$, which is significantly lower than the threshold defined by the Rose criterion for the features to be distinguishable. Nevertheless, if a single feature remains consistent across different realizations and throughout the image—exhibiting the quasihomogeneity property of the source—even this noise level is acceptable for deduction using advanced correlation analysis.

As shown in Fig. 5(b), the two-point correlation function is strongly affected by detector shot noise, rendering it practically unusable for analysis and retrieval of its distributions. However, we leveraged the property of synchrotron radiation as a quasihomogeneous source, which ensures that the typical spike width remains consistent across the intensity distribution. This property can be effectively utilized to average the correlation function along the anti-diagonal, as illustrated in Fig. 5(b). This approach significantly enhances the statistical quality of the measured correlation function. For example, if 3000 events are collected and averaging is performed over a region of 100×100 pixels², this results in a 2 orders of magnitude increase in statistical significance. We present validity of such “anti-diagonal” averaging in Appendix B.

We present a comparison of the correlation functions in Fig. 6: one obtained from “clean” data and one retrieved by

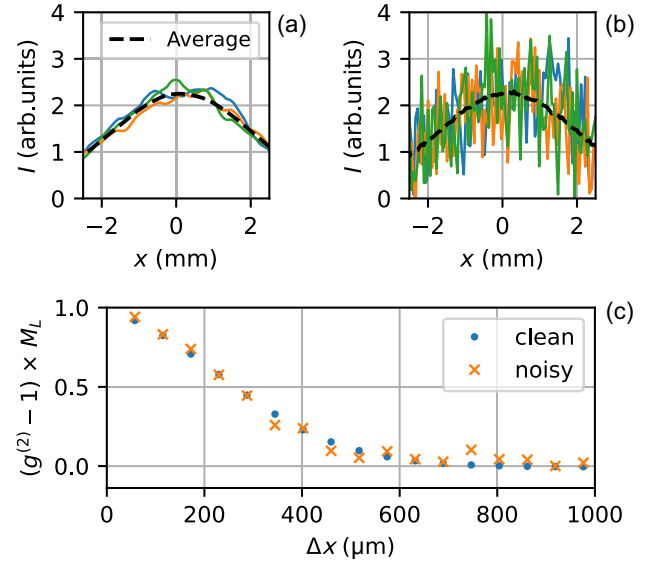


FIG. 6. Comparison of intensity distributions and corresponding second-order correlation functions for clean and noisy signals. (a),(b) Slices of the intensity distribution for individual realizations, with (a) corresponding to clean data and (b) to noisy data. The dashed black line represents the average over all realizations. (c) Retrieved correlation functions $(g^{(2)} - 1)M_L$ as a function of transverse separation Δx for both cases. The correlation is successfully recovered even in the presence of strong noise.

the algorithm we have described. In Fig. 6, slices of the intensity distribution for different statistical realization are shown using colorful lines for both the clean and noisy signals in Figs. 6(a) and 6(b). The corresponding retrieved correlation functions are presented in Fig. 6(c). As can be seen, the correlation function for the noisy data is fully recovered. This algorithm was applied to the analysis of real experimental data, which we present later in this paper.

In this showcased example, we implicitly assumed that the electron beam emitting this signal does not contain any spatiotemporal tilts. However, in reality, spatiotemporal tilts play a crucial role in achieving attosecond-level pulses in advanced SASE FEL schemes.

A. Comparison of slice and projected beam size measurements in the presence of pulse tilts

In the case of tilted bunches—particularly those with linear tilt—two characteristic transverse sizes can be distinguished. One corresponds to the full projection of the electron bunch as viewed along the propagation axis, while the other represents the transverse size of a slice at a specific position along the electron bunch. In this context, the monochromator bandwidth can be exploited to measure and distinguish between these two sizes.

Under conditions of poor monochromatization, when the monochromator bandwidth is much larger than the spectral coherence length, the measured correlation function is

related to the transverse slice size, averaged over the entire electron beam duration. This behavior can be intuitively understood in terms of Fourier domains: a large monochromator bandwidth compared to spectral coherence length in the frequency domain corresponds to a narrow time window. When the pulse is detected, this defines the time window with which the transverse slices are integrated over the full pulse length.

Alternatively, an equation in the frequency domain can be derived for practical calculations (see derivation in Appendix C), where a Gabor-type transformation is applied to the autocorrelation function in the frequency domain:

$$\Gamma_{\omega}(0, \vec{r}, \Delta\vec{r}, \bar{\omega}, \Delta\omega) = |T(\bar{\omega})|^2 g(0, \Delta\vec{r}, \bar{\omega}) \int_{-\infty}^{\infty} f(t') I(0, \vec{r} - \vec{\alpha}t', \bar{\omega}) e^{-it'\Delta\omega} dt'. \quad (12)$$

The presence of the tilt couples the longitudinal profile and the intensity distribution at the source in a convolution-like expression. In addition, because the monochromator bandwidth is large, we can still express Γ_{ω} as a Fourier transform in t' . Then, once the radiation is detected, this effectively corresponds to an integration over frequencies of its modulus squared. Thanks to Parseval's theorem, we can bring the modulus inside of the integral, removing the kernel of the Fourier transform. This swap between operations is crucial, because it determines if and how the temporal integration averages the sizes. In this case, it gives an averaged width. In fact, the measured size, once Γ_{ω} is propagated in the far zone, corresponds to the slice size at each instant of time t' averaged along $f(t')$.

Conversely, when the monochromator bandwidth is sufficiently narrow to resolve individual spectral spikes, the effective time window broadens, encompassing a larger portion of—or even the entire—pulse duration. As a result, measuring the transverse correlation function retrieves the entire projected size of the electron beam. Unlike before, when formulated in the frequency domain, the measured quantity corresponds to the total projected size, weighted over the pulse profile. The recorded quantity is related to the modulus squared of

$$\Gamma_{\omega}(0, \vec{r}, \Delta\vec{r}, \omega_M) = g(0, \Delta\vec{r}, \omega_M) \int_{-\infty}^{\infty} f(t') I(0, \vec{r} - \vec{\alpha}t', \omega_M) dt'. \quad (13)$$

As before, the tilt is the reason behind the coupling of the pulse profile and the intensity distribution, but in this case, no Fourier transform survives. The narrow bandwidth of the monochromator selects a specific frequency, resulting in a simple convolution, and the modulus cannot be brought inside of the integral. The lack of commutativity between this two operations results in the determination of the total width of an averaged quantity.

We present simulations of this effect in Figs. 7 and 8. For these simulations, we employed parameters that are relaxed compared to the actual experimental conditions (that we listed earlier in this article), in order to make the figures more illustrative, while still maintaining realistic values. Specifically, the electron beam duration used in this simulation is $\sigma_T = 15$ fs, and the spectral bandwidth is

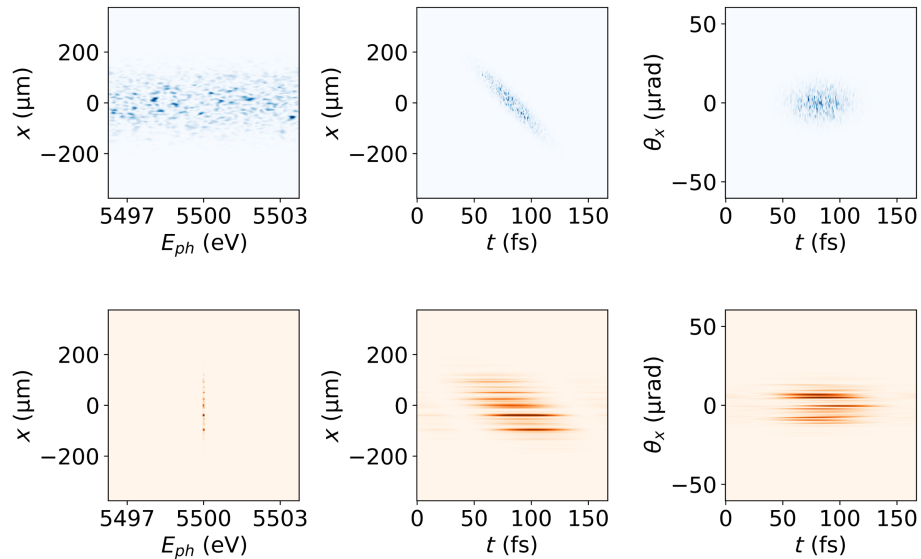


FIG. 7. Two different cases of monochromatization are shown. The upper row represents poor monochromatization, where the monochromator bandwidth is much larger than the spectral correlation length. The lower row corresponds to strong monochromatization, where the monochromator resolves a single spike in the spectral domain. Color here encodes these two different cases. Each column presents, from left to right: the frequency-space domains representation of the radiation, the time-space domains, and the time-inverse-space domains.

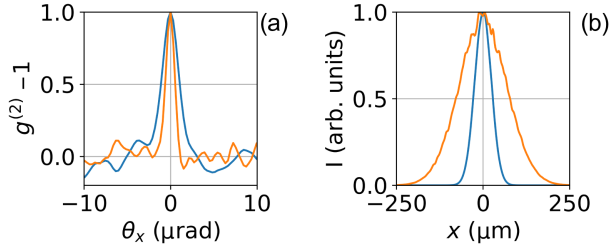


FIG. 8. Subplot (a) correlation functions for poor monochromatization (blue line) and strong monochromatization (orange line). Subplot (b) shows the corresponding effective source sizes: the slice size (blue line) and the projected size (orange line).

42 eV, corresponding to the width of the undulator harmonic.

The first line in Fig. 7 represents radiation with poor monochromatization, where a single spectral spike is not resolved. This case corresponds to the measurements of the averaged transverse slice size, marked with blue lines in Fig. 8. The correlation function shown in Fig. 8(a), when used to retrieve the effective source size, corresponds to the averaged slice size depicted in Fig. 8(b). Conversely, in the case of strong monochromatization, where a single spectral spike is resolved, we observe a narrowing of the correlation function—illustrated by the orange line in Fig. 8(a)—which effectively corresponds to the projected size indicated by the orange line in Fig. 8(b), or more precisely, follows Eq. (13). With this approach, adjusting the monochromator bandwidth relative to the spectral coherence length allows one to distinguish between the two sizes of spatially tilted electron bunches.

IV. EXPERIMENTAL DATA ANALYSIS PROCEDURE

In this section, we describe the data analysis procedures applied to real experimental data, which are affected not only by the detector shot noise but also by more complex noise sources, such as cross-pixel correlations and systematic biases. The methodology presented here is expected to be applicable across different facilities, provided that a similar type of SR imager is employed, as these noise characteristics are commonly encountered. We begin by presenting the measured intensity distributions, which represent the raw recorded data.

A. Spikes of synchrotron radiation in transverse dimension

For these measurements, the radiation intensity from a single electron beam was too low to be detected. To observe a radiation spot on the detector, we combined the signals from 13 successive electron beams. Figure 9 presents a typical detected signal. Panel (a) shows the signal averaged over the entire collected ensemble of 3500 events, while panel (b) displays a single event, which, as noted earlier,

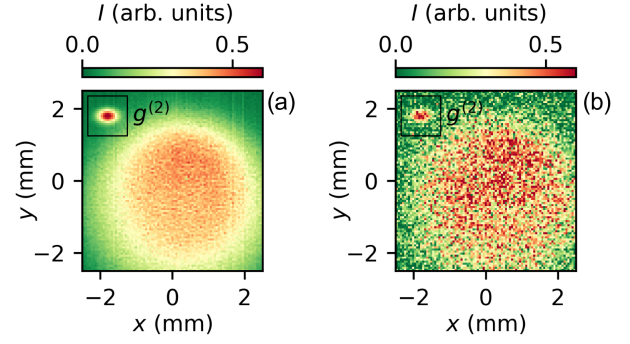


FIG. 9. Radiation intensity distribution averaged over the whole events ensemble (a) and single event intensity distribution (b). The $g^{(2)}$ function is shown in the upper-left corner of each subfigure for feature sizes comparison.

corresponds to the averaged distribution of radiation pulses from 13 electron beams.

Figure 9(b) shows distributions that closely resemble those obtained in our simulations presented in Fig. 5. We expected the recorded images to display characteristic spiky distributions. In Fig. 9(b), the spike variations in the signal are barely noticeable. However, when examining successive events from this run, these characteristic features become more apparent.

In Supplemental Material [58], we provide an animated version of Fig. 9, showing the recorded events, where one can observe event-to-event variations with features comparable in size to the correlation function (depicted for reference in the upper-left corner). To the best of our knowledge, this represents the first direct observation of synchrotron radiation spikes in the transverse domain, which is the highlight in [59].

B. Raw correlation calculations

Then, we analyzed the whole ensemble of events using the autocorrelation method outlined in Sec. III. First, we compute the raw correlation data, as presented in Fig. 10. The distribution in Fig. 10 is presented in the variable pairs

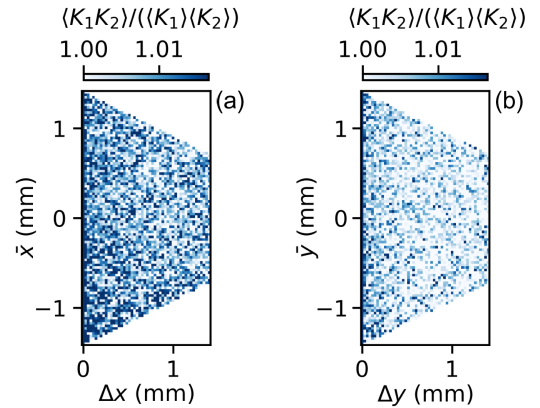


FIG. 10. Experimental raw correlation calculated from *signal* data.

$(\Delta y, \bar{x})$ and $(\Delta y, \bar{y})$, which seems to be more natural variable for calculating autocorrelation numerically and subsequent averaging.

For the analysis, it is essential to subtract the bias from the signal. We subtract the mean value of the dark images—the bias—from the collected signal, assuming that its level remained unchanged throughout the experiment.

$$K = S - \langle D_n \rangle \quad (14)$$

where S is the actual collected signal, and D_n represents the dark images collected in a separate run.

Once we calculated autocorrelation of K , from Fig. 10, one can already discern a significant correlation level, evidenced by a mild gradient in the blue color along the $\Delta x, \Delta y$ direction. Additionally, it is evident that the signal levels differ between the x and y dimensions, indicating the presence of a background correlation component, which is in fact different for each dimension.

We then supposed that K consists of the counts from detecting SR and the detector noise:

$$K = I + N \quad (15)$$

So, we calculate the autocorrelation of K as

$$\frac{\langle K_1 K_2 \rangle}{\langle K_1 \rangle \langle K_2 \rangle} = \frac{\langle (I_1 + N_1)(I_2 + N_2) \rangle}{\langle I_1 + N_1 \rangle \langle I_2 + N_2 \rangle} \quad (16)$$

and simplifying

$$\frac{\langle K_1 K_2 \rangle}{\langle K_1 \rangle \langle K_2 \rangle} = \frac{\langle I_1 I_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle} + \frac{\langle N_1 N_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle}, \quad (17)$$

where we assume that $\langle I_1 N_1 \rangle = 0$ and $\langle I_2 N_2 \rangle = 0$, as usual $\langle \dots \rangle$ denotes ensemble averaging.

To recover the actual correlation, we substitute $\langle N_1 N_2 \rangle$ with $\langle D_1 D_2 \rangle$, where D is the bias subtracted noise. At this step, we assume that N and $D = D_n - \langle D_n \rangle$ share the same statistical properties, as the characteristics of the noise remained unchanged throughout the experiment from one run to another. Thus, we obtain:

$$\frac{\langle K_1 K_2 \rangle}{\langle K_1 \rangle \langle K_2 \rangle} = \frac{\langle I_1 I_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle} + \frac{\langle D_1 D_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle}. \quad (18)$$

We present the distribution of $\langle D_1 D_2 \rangle / (\langle I_1 \rangle \langle I_2 \rangle)$ in Fig. 11.

Analyzing the correlation data from Figs. 10 and 11, we observed few data points with anomalously high correlation levels. Upon diagonal averaging, these points would lead to an artificial increase in the correlation signal at smaller values of $\Delta x, \Delta y$. The presence of such anomalously high correlation levels cannot be explained by any process related to radiation or shot noise of the detector but rather by spontaneous bursts in certain pairs of pixels. Therefore, we refer to these as “hot” correlated pixels and exclude them from further summation.

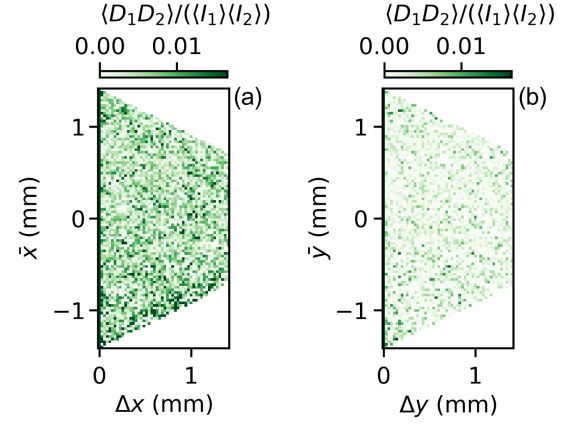


FIG. 11. Experimental raw correlation from *dark* signal.

C. Hot correlated pixels exclusion

To mitigate this effect of hot pixels, we set the exclusion threshold to 6 times the median absolute deviation of the correlation value, as shown in Figs. 12 and 13 with vertical lines. In this case, the standard deviation is not an appropriate measure, as it would overestimate the distribution width due to the presence of anomalously high correlation values. The locations of these hot correlated pixels were not fixed from run to run; they can appear unpredictably in different datasets. Overall, depending on the dataset, we excluded between 800 and 1000 pixel pairs from further analysis.

V. EXPERIMENTAL RESULTS

After the procedures described above, we proceeded to extract the sought correlation function of the radiation, $\langle I_1 I_2 \rangle / (\langle I_1 \rangle \langle I_2 \rangle)$, and calculated the transverse size of the electron beam.

A. Final correlation signal extraction

For this, we express $\langle I_1 I_2 \rangle / (\langle I_1 \rangle \langle I_2 \rangle)$ from Eq. (18):

$$\frac{\langle I_1 I_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle} = \frac{\langle K_1 K_2 \rangle}{\langle K_1 \rangle \langle K_2 \rangle} - \frac{\langle D_1 D_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle}. \quad (19)$$

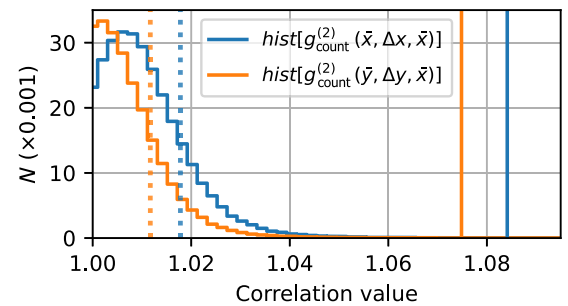


FIG. 12. Histogram of signal correlation values from Fig. 10.

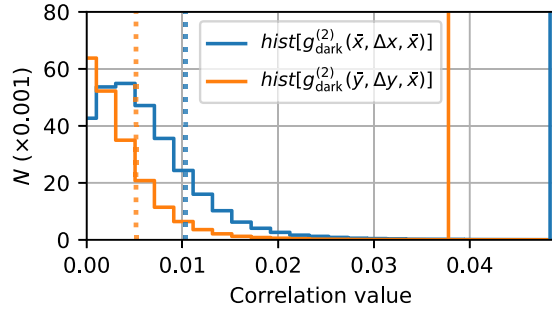


FIG. 13. Histogram of noise correlation values from Fig. 11.

This means that we subtracted the distribution depicted in Fig. 11 from the one in Fig. 10 and then performed averaging along the diagonal, which gives us

$$\frac{\langle I_1 I_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle} = \frac{\langle K_1 K_2 \rangle}{\langle K_1 \rangle \langle K_2 \rangle} - \frac{\langle D_1 D_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle}. \quad (20)$$

We present each distribution corresponding to $\langle K_1 K_2 \rangle / (\langle K_1 \rangle \langle K_2 \rangle)$, $\langle D_1 D_2 \rangle / (\langle I_1 \rangle \langle I_2 \rangle)$, and $\langle I_1 I_2 \rangle / (\langle I_1 \rangle \langle I_2 \rangle)$ in Figs. 14(a)–14(c).

The error estimation is nontrivial, and we determined the uncertainties based on collected statistics. We assumed that each point in Figs. 10 and 11 represents a random variable. Thus, we calculated the standard deviation across the diagonal (in the same manner as we calculated the mean according to Eq. (20), which corresponds to the first two contributions in Eq. (21).

$$\delta_K = \left(\sigma \left(\frac{\langle K_1 K_2 \rangle}{\langle K_1 \rangle \langle K_2 \rangle} \right)^2 + \sigma \left(\frac{\langle N_1 N_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle} \right)^2 + \sigma \left(\frac{1}{\sqrt{B}} \sigma \left(\frac{\langle K_1 K_2 \rangle_b}{\langle K_1 \rangle_b \langle K_2 \rangle_b} \right) \right)^2 \right)^{1/2}, \quad (21)$$

where $\sigma(\cdot)$ denotes calculating standard deviation. The last term in this formula is based on the following considerations: the average level of the signal varied along the ensemble of events due to detector “breathing,” so we decided to collect randomly indexed batches of smaller size within the ensemble, denoted in Eq. (21) as $\langle \cdots \rangle_b$, calculate the correlation once more, estimate the standard deviation, and repeat this procedure a reasonably large number of times (ten iterations) to evaluate the variation and calculate the standard deviation of it. The parameter B in Eq. (21) represents the size ratio between the initial ensemble and the subset, which is used for normalization. This last contribution mainly affects the points at small $\Delta x, \Delta y$, resulting in larger error bars. This behavior is reasonable because we observe an anomalous growth of the signal, which cannot be explained by contributions from synchrotron radiation. A large estimation of the error bars ensures that these points will have a smaller weight during fitting.

Then, we fitted the distribution in Fig. 14(c) with a Gaussian function plus a constant offset and, in this way, measured the width of $|g(\Delta \vec{r}, \bar{\omega}_0)|^2$ and related it to the transverse size of the electron beam using Eq. (11). For this specific measurement, the retrieved electron beam size is $\sigma_x = 16.8 \pm 0.4 \mu\text{m}$ and $\sigma_y = 26 \pm 0.3 \mu\text{m}$. Before moving to the explanation of the measurements of the electron beam size along SASE1 for validation of the method, we first discuss the observation of the contrast level in Fig. 14(c).

As we derived in Eq. (10) and in Appendix B, $\langle I_1 I_2 \rangle / (\langle I_1 \rangle \langle I_2 \rangle)$ is related with $|g^{(2)}|$. The contrast level observed in Fig. 14(c) coincides very well with the one calculated according to Eq. (10). The calculated value, based on an analytical model in Eq. (10) with a pulse duration of 37 ± 3 fs, a relative monochromator bandwidth of $(2 \pm 0.2) \times 10^{-4}$, and averaging over 13 electron beams, is 0.0049(6). This number is inversely proportional to $(16 \pm 2) \times 3$, which corresponds to the estimated number of longitudinal spikes. The values deduced from the experimentally observed correlation in Fig. 14(a) are 0.0048(2) and 0.0044(2) for x and y directions, respectively.

The sensitivity limit of the method can be assessed by comparing the expected contrast level of the correlation

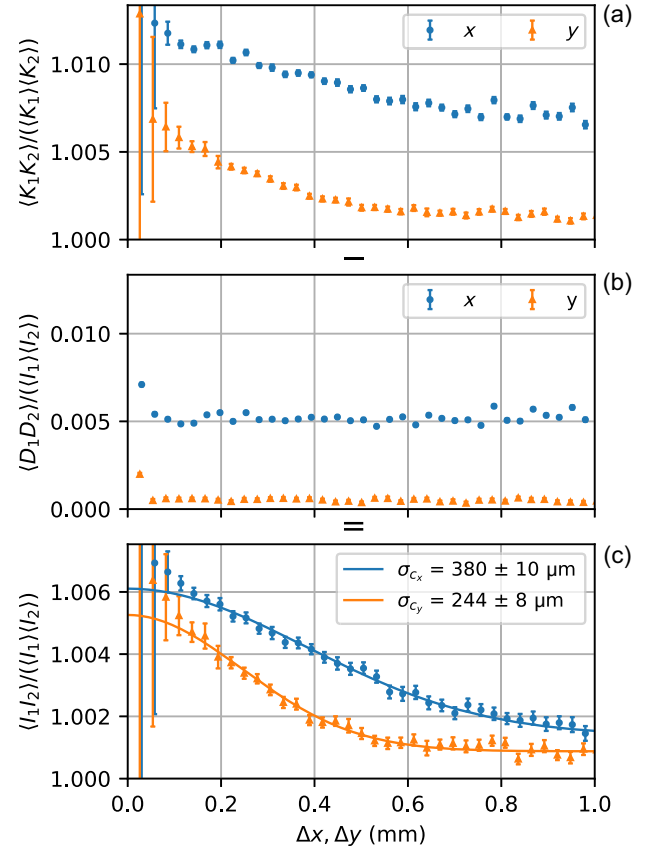


FIG. 14. Calculated correlation function. (a) Raw calculated correlation function. (b) Correlation of the noise normalized to the detected intensity. (c) Result obtained by subtracting the distribution in (b) from that in (a).

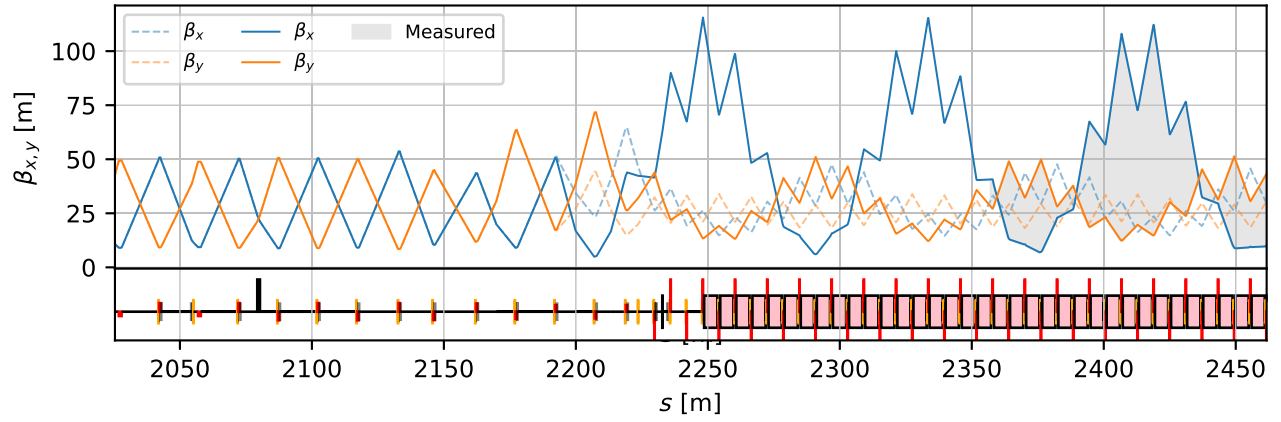


FIG. 15. Magnetic lattice upstream and in SASE1 for the experimental conditions: dashed lines represent the initial optics configuration, while solid lines represent the intentionally introduced mismatch. The gray region indicates the cells where the transverse electron beam size was measured.

function to the statistical uncertainty, as defined in Eq. (21). If the expected contrast falls below the uncertainty level, additional statistics must be acquired to ensure reliable extraction of the beam size.

Accurately determining the uncertainties of the experimentally measured averaged transverse slice size of the electron beam is challenging, as there is no straightforward method aside from relying on the fit, which likely underestimates them. Furthermore, it remains unclear whether systematic errors contribute to the measurements.

For example, a slight discrepancy between the measured correlation functions can be observed in Fig. 14(c), even though both distributions are expected to have the same contrast and converge to the same level. This behavior is observed in some runs, while in others, the two distributions coincide well. Most likely, this discrepancy arises from variations in the overall bias level, leading to fluctuations in the overall detected intensity. However, this variation does not affect the width of the correlation function, which remains the key parameter for calculating the electron beam size.

B. Electron beam size measurements along SASE1 undulator

In the experimental runs, to confirm that the signal observed in Fig. 14 indeed originates from radiation, we deliberately modulated the transverse size of the electron beam along the SASE1 undulator cells. This was achieved by adjusting the quadrupole strength upstream of SASE1, which effectively introduced a mismatch in the FODO lattice. The calculated beta functions along the magnetic lattice, shown in Fig. 15, are based on actual quadrupole readouts and the *projected* Courant–Snyder parameters for the facility.

We performed two sets of measurements along the SASE1 undulator: one using the original FODO lattice configuration present at the beginning of the experiment,

and a second after modifying the quadrupole settings to introduce a mismatch, as illustrated in Fig. 15.

In the initial configuration, we measured the electron beam size over the last eight undulator cells, assuming that the beam was *matched*. This limited measurement range was chosen based on that assumption. However, subsequent data analysis revealed a slight residual mismatch, as indicated by the dashed line in Fig. 15 and confirmed by the beam size measurements shown in Fig. 16.

After adjusting the upstream quadrupole value and introducing the intended mismatch, we measured the electron beam size in cells 21 through 37. As shown in Fig. 15, we expect to observe a modulation of the electron beam, but in the opposite phase compared to the “matched” conditions. This behavior was confirmed experimentally and is presented in Fig. 17.

To provide a reference for these measurements, we calculated the expected variation of the electron beam size

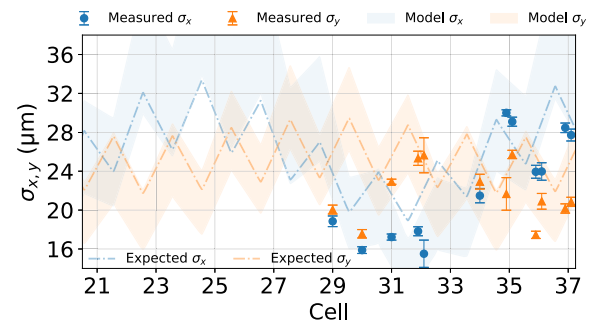


FIG. 16. Measured averaged transverse slice sizes along the SASE1 undulator under *matched* magnetic lattice conditions. The experimental data are shown as blue circles (σ_x) and orange triangles (σ_y). The dashed-dotted line represents the expectation based on an emittance value averaged between the upper and lower bounds, while the shaded regions in the corresponding colors indicate the corrected model within the full emittance range.

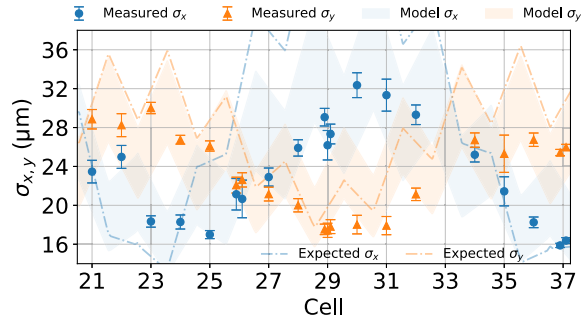


FIG. 17. Measured averaged transverse slice sizes along the SASE1 undulator under *mismatched* magnetic lattice conditions. Same symbols and color coding as in Fig. 16.

along SASE1 in both cases using the European XFEL lattice model. For this estimation, we used the design Courant–Snyder parameters, an estimate of the emittance, and the actual quadrupole readouts from the magnetic lattice. The Courant–Snyder parameters at the upstream location were based on the design optics. Their downstream evolution was then tracked using OCELOT [60], based on the actual magnet settings. The result of this tracking is presented in Fig. 15.

For the beam size estimation, we assumed normalized emittance values in the range of $\epsilon_x = 0.37$ to 0.7 mm mrad and $\epsilon_y = 0.46$ to 0.7 mm mrad. The lower limits are based on emittance measurements performed at the injector immediately following our beam size scan. A value of 0.7 mm mrad was chosen as a conservative upper estimate.

In Figs. 16 and 17, we present two references for the measurements: the dashed-dotted line represents the size estimate based on the actual magnet readouts (the emittance is taken as the average between the two bounds, so as not to overload the plot), and the filled region shows the result after applying a 13% relative reduction to the matching quadrupole current in the tracking model. This correction for a slightly mismatched electron beam allowed us to align the model with the measurements and justifies the need for such a measurement method.

The discrepancy in the uncorrected expectation may originate from several factors: a mismatch already present upstream that is not reflected in the list of quadrupole readouts; possible hysteresis effects in the adjusted quadrupole, since we did not start from freshly cycled magnets; and/or inaccuracies in the assumed design Courant–Snyder parameters upstream. Less likely, the disagreement comes from flaws in the measurements, as the experimental data show a consistent sinusoidal modulation of the electron beam size along SASE1. At the same time, we used several assumptions to construct the reference curves, and the mismatch appears primarily in the amplitude of the variation—not in its period or in the absolute node position (i.e., where the horizontal and vertical beam sizes intersect).

VI. DISCUSSION

Based on the observed variation of the average transverse slice size along SASE1, we conclude that our method is validated and internally consistent. We have demonstrated the simulation framework used to develop the data analysis pipeline and applied it to real, noisy experimental data. The simulated results were then compared to the experimentally observed behavior shown in Figs. 5 and 9, as well as the correlation functions in Figs. 6 and 14. In addition, we conducted independent measurements of the electron bunch duration in order to calculate the expected contrast level, which was then compared to the contrast observed in the measured correlation functions. Finally, we performed a dedicated experiment where we deliberately modified the transverse beam parameters along the beamline and confirmed that the signal responded as expected. In contrast to the projected electron-beam size, which can be measured using, for example, a wire scanner, the average slice size measurements are unique and cannot be independently confirmed by other qualitatively different methods.

This method is experimentally realized with very limited hardware, utilizing equipment that was already available at the facility. As noted, this approach allows for the measurement of the averaged transverse slice size. However, it can be extended to measure the projected size of the entire electron beam in the presence of any spatiotemporal tilts. To achieve this, one would need to use a higher-order reflection of the monochromator crystal, such as Si(333). For this reflection, the total photon flux decreases proportionally to its spectral bandwidth. Further studies on whether the current detector is sufficiently sensitive for these measurements will be the subject of future experiments aimed at refining this method for electron beam size diagnostics.

This type of measurement is not limited to FELs—the principle is general and, as we mentioned, is only constrained by the detection capabilities of x-rays. Therefore, it is worth exploring the application of this method at storage rings. As long as a relatively small number of longitudinal spikes can be extracted, in the conducted experiment, the effective number was $(16 \pm 2) \times 13 = 208 \pm 26$, the primary challenge remains the extraction of data from the collected events. The correlation is inherently present in the detected intensity distributions.

VII. CONCLUSION

In this paper, we describe the complete procedure for measuring the electron beam size using synchrotron radiation at a free-electron laser facility. The method relies solely on a Bragg crystal monochromator with Si(111) reflection and an x-ray imager. We develop a data analysis procedure that effectively retrieves the correlation function from highly noisy data, dominated by shot noise in the x-ray detector. To validate the data analysis procedure, we

simulate the experiment using a Gaussian random field emulator. Additionally, we analyze two types of measurements that can be performed with the given setup: averaged transverse slice size measurements and projected size measurements. In the experiment, we measure the averaged transverse slice size. We further outline the data treatment procedures, including background subtraction and the exclusion of “hot pixels.” We demonstrate that the measured autocorrelation function is directly related to the complex degree of coherence and thus establish a connection between this degree of coherence and the electron beam size based on the van Cittert–Zernike theorem. To further confirm the validity of the method, we modulate the electron beam beta function along the SASE1 beamline and measure the electron beam size at different locations along the beamline, observing the expected variation in size. Moreover, the measured contrast of the correlation function is consistent with theoretical predictions. This proof-of-principle experiment serves as a demonstration of the feasibility of implementing this type of electron beam size diagnostic at free-electron laser facilities. The method requires minimal investment in hardware while providing valuable averaged transverse slice size measurements.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX A: REDUCED CONTRAST OF THE MEASURED AUTOCORRELATION FUNCTION

The measured quantity in the experiment is the intensity of the radiation averaged over the monochromator transfer function $T(\omega)$:

$$\begin{aligned} I &= \int_{-\infty}^{\infty} \bar{E}(\vec{r}, \omega) T(\omega) \bar{E}^*(\vec{r}, \omega) T^*(\omega) d\omega \\ &= \int_{-\infty}^{\infty} \bar{I}(\vec{r}, \omega) |T(\omega)|^2 d\omega, \end{aligned} \quad (\text{A1})$$

where we omitted z dependence for shortens of notation. The radiation is observed in the far zone. Using this representation, we can express the sought-after intensity autocorrelation function:

$$\frac{\langle I_1 I_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle} = \frac{\langle \int_{-\infty}^{\infty} \bar{I}(\vec{r}_1, \omega_1) \bar{I}(\vec{r}_2, \omega_2) |T(\omega_1)|^2 |T(\omega_2)|^2 d\omega_1 d\omega_2 \rangle}{\langle \int_{-\infty}^{\infty} \bar{I}(\vec{r}_1, \omega_1) |T(\omega_1)|^2 d\omega_1 \rangle \langle \int_{-\infty}^{\infty} \bar{I}(\vec{r}_2, \omega_2) |T(\omega_2)|^2 d\omega_2 \rangle} \quad (\text{A2})$$

The two integrals in the denominator of this equation can be simplified if we make the reasonable assumption that the pulse spectral distribution is significantly broader than the monochromator transfer function and varies negligibly over its bandwidth:

$$\begin{aligned} &\int_{-\infty}^{\infty} \langle \bar{I}(\vec{r}_{1,2}, \omega_{1,2}) \rangle |T(\omega_{1,2})|^2 d\omega_{1,2} \\ &\approx \langle \bar{I}(\vec{r}_{1,2}, \omega_{1,2}) \rangle \int_{-\infty}^{\infty} |T(\omega'_{1,2})|^2 d\omega'_{1,2} \end{aligned} \quad (\text{A3})$$

Additionally, we note that the numerator contains an ensemble average over $\langle \bar{I}(\vec{r}_1, \omega_1) \bar{I}(\vec{r}_2, \omega_2) \rangle$. The normal-

ized intensity autocorrelation function for the field at a given frequency is related to the field autocorrelation function via the Siegert relation:

$$\frac{\langle \bar{I}(\vec{r}_1, \omega_1) \bar{I}(\vec{r}_2, \omega_2) \rangle}{\langle \bar{I}(\vec{r}_1, \omega_1) \rangle \langle \bar{I}(\vec{r}_2, \omega_2) \rangle} = 1 + \left| \frac{\Gamma_{\omega}(\vec{r}_1, \vec{r}_2, \omega_1, \omega_2)}{\sqrt{\langle \bar{I}(\vec{r}_1, \omega_1) \rangle \langle \bar{I}(\vec{r}_2, \omega_2) \rangle}} \right|^2. \quad (\text{A4})$$

Following Siegert relation, where $\Gamma_{\omega}(\vec{r}_1, \vec{r}_2, \omega_1, \omega_2) = \langle \bar{E}(\vec{r}_1, \omega_1) \bar{E}^*(\vec{r}_2, \omega_2) \rangle$. Substituting Eq. (A3) in Eq. (A2) and expressing intensity autocorrelation in Eq. (A4) we obtain:

$$\frac{\langle I_1 I_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle} = \frac{\int_{-\infty}^{\infty} \left(1 + \left| \frac{\Gamma_{\omega}(\vec{r}_1, \vec{r}_2, \omega_1, \omega_2)}{\sqrt{\langle \bar{I}(\vec{r}_1, \omega_1) \rangle \langle \bar{I}(\vec{r}_2, \omega_2) \rangle}} \right|^2 \right) \langle \bar{I}(\vec{r}_1, \omega_1) \rangle \langle \bar{I}(\vec{r}_2, \omega_2) \rangle |T(\omega_1)|^2 |T(\omega_2)|^2 d\omega_1 d\omega_2}{\langle \bar{I}(\vec{r}_1, \omega_1) \rangle \langle \bar{I}(\vec{r}_2, \omega_2) \rangle \int_{-\infty}^{\infty} |T(\omega'_1)|^2 d\omega'_1 \int_{-\infty}^{\infty} |T(\omega'_2)|^2 d\omega'_2} \quad (\text{A5})$$

Following the quasistationary approximation, which is applicable to undulator radiation, we can factorize the cross-spectral density function as

$$\Gamma_\omega(\vec{r}_1, \vec{r}_2, \bar{\omega}, \Delta\omega) = G(\vec{r}_1, \vec{r}_2, \bar{\omega})\bar{f}(\Delta\omega) \quad (\text{A6})$$

where $G(\vec{r}_1, \vec{r}_2, \bar{\omega})$ is the cross-spectral density function and $\bar{f}(\Delta\omega)$ is the spectral degree of coherence. As soon as we apply quasistationary approximation, we exchange $\omega_{1,2}$ to $\bar{\omega}$ and obtain:

$$\frac{\langle I_1 I_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle} = 1 + \frac{\iint_{-\infty}^{\infty} \left| \frac{G(\vec{r}_1, \vec{r}_2, \bar{\omega})\bar{f}(\Delta\omega)}{\sqrt{\langle \bar{I}(\vec{r}_1, \bar{\omega}) \rangle \langle \bar{I}(\vec{r}_2, \bar{\omega}) \rangle}} \right|^2 \langle \bar{I}(\vec{r}_1, \bar{\omega}) \rangle \langle \bar{I}(\vec{r}_2, \bar{\omega}) \rangle |T(\bar{\omega})|^4 d\Delta\omega d\bar{\omega}}{\langle \bar{I}(\vec{r}_1, \bar{\omega}_0) \rangle \langle \bar{I}(\vec{r}_2, \bar{\omega}_0) \rangle \int_{-\infty}^{\infty} |T(\bar{\omega}')|^2 d\bar{\omega}' \int_{-\infty}^{\infty} |T(\bar{\omega}'')|^2 d\bar{\omega}''} \quad (\text{A7})$$

With this, we further simplify the expression by introducing the complex degree of coherence, defined as

$$g(\vec{r}_1, \vec{r}_2, \bar{\omega}) = \frac{G(\vec{r}_1, \vec{r}_2, \bar{\omega})}{\sqrt{\langle \bar{I}(\vec{r}_1, \bar{\omega}) \rangle \langle \bar{I}(\vec{r}_2, \bar{\omega}) \rangle}}, \quad (\text{A8})$$

$$\frac{\langle I_1 I_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle} = 1 + \frac{\iint_{-\infty}^{\infty} |g(\vec{r}_1, \vec{r}_2, \bar{\omega})\bar{f}(\Delta\omega)|^2 \langle \bar{I}(\vec{r}_1, \bar{\omega}) \rangle \langle \bar{I}(\vec{r}_2, \bar{\omega}) \rangle |T(\bar{\omega})|^4 d\Delta\omega d\bar{\omega}}{\langle \bar{I}(\vec{r}_1, \bar{\omega}_0) \rangle \langle \bar{I}(\vec{r}_2, \bar{\omega}_0) \rangle \int_{-\infty}^{\infty} |T(\bar{\omega}')|^2 d\bar{\omega}' \int_{-\infty}^{\infty} |T(\bar{\omega}'')|^2 d\bar{\omega}''} \quad (\text{A9})$$

We observe that the integrals in the numerator now factorize, and the radiation-related functions vary slowly with respect to the variable $\bar{\omega}$ compared to the monochromator transfer function. Thus, we can move these functions outside the integral and obtain:

$$\begin{aligned} \frac{\langle I_1 I_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle} &= 1 + \frac{|g(\vec{r}_1, \vec{r}_2, \bar{\omega}_0)|^2 \langle \bar{I}(\vec{r}_1, \bar{\omega}_0) \rangle \langle \bar{I}(\vec{r}_2, \bar{\omega}_0) \rangle \int_{-\infty}^{\infty} |\bar{f}(\Delta\omega)|^2 d\Delta\omega \int_{-\infty}^{\infty} |T(\bar{\omega})|^4 d\bar{\omega}}{\langle \bar{I}(\vec{r}_1, \bar{\omega}_0) \rangle \langle \bar{I}(\vec{r}_2, \bar{\omega}_0) \rangle \int_{-\infty}^{\infty} |T(\bar{\omega}')|^2 d\bar{\omega}' \int_{-\infty}^{\infty} |T(\bar{\omega}'')|^2 d\bar{\omega}''} \\ &= 1 + \frac{\int_{-\infty}^{\infty} |\bar{f}(\Delta\omega)|^2 d\Delta\omega \int_{-\infty}^{\infty} |T(\bar{\omega})|^4 d\bar{\omega}}{(\int_{-\infty}^{\infty} |T(\bar{\omega}')|^2 d\bar{\omega}')^2} |g(\vec{r}_1, \vec{r}_2, \bar{\omega}_0)|^2 \end{aligned} \quad (\text{A10})$$

The prefactor of g represents the expected reduction in the contrast of the autocorrelation function. We estimate this reduction analytically, assuming Gaussian distributions for $\bar{f}$ and T . If $\bar{f}$ is distributed as

$$|\bar{f}(\Delta\omega)| = e^{-\Delta\omega^2/(2\sigma_\omega^2)} \quad (\text{A11})$$

Thus, the integral evaluates to

$$\int_{-\infty}^{\infty} |\bar{f}(\Delta\omega)|^2 d\Delta\omega = \sqrt{\pi}\sigma_\omega. \quad (\text{A12})$$

To relate σ_ω to the pulse duration, we utilize the generalized Wiener-Khinchin theorem, which relates the spectral degree of coherence to the pulse power profile:

$$f(\vec{r}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{f}(\Delta\omega) e^{i\Delta\omega \vec{r}} d\Delta\omega = \frac{\sigma_\omega}{\sqrt{2\pi}} e^{-\vec{r}^2 \sigma_\omega^2/2}. \quad (\text{A13})$$

We obtain $\sigma_T = 1/\sigma_\omega$.

We then calculate the integrals involving the monochromator transfer function $T(\bar{\omega})$, assuming its modulus has the form:

$$T(\bar{\omega}) = e^{-\bar{\omega}^2/(2\sigma_m^2)} \quad (\text{A14})$$

With this, we have

$$\int_{-\infty}^{\infty} |T(\bar{\omega})|^4 d\bar{\omega} = \sqrt{\frac{\pi}{2}} \sigma_m \quad (\text{A15})$$

and

$$\int_{-\infty}^{\infty} |T(\bar{\omega})|^2 d\bar{\omega} = \sqrt{\pi} \sigma_m. \quad (\text{A16})$$

Substituting the integral values from Eqs. (A12), (A15), and (A16) in the expression in Eq. (A10) we obtain:

$$\frac{\langle I_1 I_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle} = 1 + \frac{\sigma_\omega}{\sqrt{2}\sigma_m} |g(\vec{r}_1, \vec{r}_2, \bar{\omega}_0)|^2, \quad (\text{A17})$$

where σ_m can be expressed in terms of the monochromator bandwidth in electron volts as $\sigma_m = \Delta E_{T_\omega}/\hbar$. By recalling the relation $\sigma_T = 1/\sigma_\omega$, we finally arrive at the following expression:

$$\frac{\langle I_1 I_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle} = 1 + \frac{\hbar}{\sqrt{2} \Delta E_{T_\omega} \sigma_T} |g(\vec{r}_1, \vec{r}_2, \vec{\omega}_0)|^2. \quad (\text{A18})$$

This expression describes the reduction in contrast in the measured autocorrelation function compared to the case where a perfect monochromator is used.

APPENDIX B: AVERAGING OVER THE CORRELATION DIAGONAL

The data analysis procedure involved averaging over the correlation diagonal, which helps improve the collected statistics for the measured autocorrelation function $\langle I_1 I_2 \rangle / (\langle I_1 \rangle \langle I_2 \rangle)$. The autocorrelation function is significantly affected by detector noise, and, in essence, the correlation value behaves as a random variable in this context.

In this appendix, we analytically demonstrate that this averaging operation indeed yields the expected autocorrelation function—its expected value—while also improving statistical reliability. First, we note that synchrotron radiation is a quasihomogeneous light source, meaning that the complex degree of coherence depends only on the coordinate difference, as follows:

$$g(\vec{r}_1, \vec{r}_2, \vec{\omega}_0) = g(\Delta \vec{r}, \vec{\omega}_0). \quad (\text{B1})$$

Once we perform the diagonal averaging of Eq. (A18), we obtain:

$$\begin{aligned} & \frac{1}{L^2} \iint_{-L/2}^{-L/2} \frac{\langle I_1 I_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle} d\vec{x} d\vec{y} \\ &= 1 + \frac{1}{L^2} \iint_{-L/2}^{-L/2} \frac{\hbar}{\sqrt{2} \Delta E \sigma_T} |g(\Delta \vec{r}, \vec{\omega}_0)|^2 d\vec{r} \end{aligned} \quad (\text{B2})$$

We find that the averaging yields exactly the same correlation function we are seeking:

$$\frac{\overline{\langle I_1 I_2 \rangle}}{\overline{\langle I_1 \rangle} \overline{\langle I_2 \rangle}} = 1 + \frac{\hbar}{\sqrt{2} \Delta E \sigma_T} |g(\Delta \vec{r}, \vec{\omega}_0)|^2, \quad (\text{B3})$$

With this, we see that averaging $\overline{\langle I_1 I_2 \rangle / (\langle I_1 \rangle \langle I_2 \rangle)}$ does not introduce any modification to the actually measured quantity but helps improve the statistical reliability.

APPENDIX C: AUTOCORRELATION FUNCTION OF THE TILTED PULSES

In this section, we derive Eqs. (13) and (12) from the main text. This derivation is based on the formalism proposed in [46], and the same notation will be used. We want to determine an expression equivalent to Eq. (19) of [46], starting from a more general assumption on the electron distribution. In our case, in fact, the probability density distribution $P(\vec{\eta}_k, \vec{l}_k, t_k)$ in the joint random

variables $\vec{\eta}_k, \vec{l}_k$, and t_k cannot be written as a product of a longitudinal bunch profile and a transverse phase-space distribution. However, we will limit ourselves to consider the simplified case of a bunch linearly tilted in the $(\vec{\eta}_k, t_k)$ domain.

We start from the definition of the mutual coherence function in the time domain,

$$\Gamma_t(z, \vec{r}_1, \vec{r}_2, t_1, t_2) = \langle E(z, \vec{r}_1, t_1) E^*(z, \vec{r}_2, t_2) \rangle, \quad (\text{C1})$$

where $E(z, \vec{r}, t)$ is the electric field generated by the entire bunch and the symbol $\langle \dots \rangle$ indicates the ensemble average. The total field can be defined in the frequency domain as

$$\begin{aligned} \bar{E}(z, \vec{r}, \omega) &= \int_{-\infty}^{\infty} dt E(z, \vec{r}, t) \exp(i\omega t) \\ &= \sum_{k=1}^{N_e} \bar{E}(\vec{\eta}_k, \vec{l}_k, z, \vec{r}, \omega) \exp(i\omega t_k) \end{aligned} \quad (\text{C2})$$

and, plugging this expression in Eq. (C3), we have

$$\begin{aligned} \Gamma_t(z, \vec{r}_1, \vec{r}_2, t_1, t_2) &= \frac{1}{(2\pi)^2} \iint_{-\infty}^{+\infty} \langle \bar{E}(z, \vec{r}_1, \omega_1) \bar{E}^*(z, \vec{r}_2, \omega_2) \rangle \\ &\quad \exp(-i\omega_1 t_1) \exp(i\omega_2 t_2) d\omega_1 d\omega_2. \end{aligned} \quad (\text{C3})$$

By definition, the averaged product of the electric fields in the frequency domain is the second-order field correlation function, Γ_ω . Before conducting the integration in the frequency domain, we can simplify the expression for Γ_ω . As a matter of fact, in our case of interest, the probability density distribution can be expressed as

$$\begin{aligned} P[\vec{\eta}_k, \vec{l}_k(t_k), t_k] &= P(\vec{\eta}_k, \vec{l}_{0k} + \vec{\alpha} t_k, t_k) \\ &= \int d\vec{\zeta}_k \tilde{P}(\vec{\eta}_k, \vec{\zeta}_k, t_k) \delta[\vec{\zeta}_k - (\vec{l}_{0k} + \vec{\alpha} t_k)], \end{aligned} \quad (\text{C4})$$

where $\vec{l}_{0k}$ is the time-independent transverse offset and $\vec{\alpha}$ is the linear chirp coefficient. The integral expression makes t_k statistically independent from the transverse variables and allows to write $\tilde{P}$ as

$$\tilde{P}(\vec{\eta}_k, \vec{\zeta}_k, t_k) = f_\perp(\vec{\eta}_k, \vec{\zeta}_k) f(t_k). \quad (\text{C5})$$

Finally,

$$\begin{aligned} P[\vec{\eta}_k, \vec{l}_k(t_k), t_k] &= \frac{1}{2\pi} \iint f_\perp(\vec{\eta}_k, \vec{\zeta}_k) f(t_k) \\ &\quad \exp\{i\vec{p} \cdot [\vec{\zeta}_k - (\vec{l}_{0k} + \vec{\alpha} t_k)]\} d\vec{p} d\vec{\zeta}_k \end{aligned} \quad (\text{C6})$$

gives a factorization of the kernel that separates the t_k dependence from the other variables. Integration over the

dummy variable p prevents a global factorization. A closer inspection of Eq. (C6) indicates that it can also be written as

$$P[\vec{\eta}_k, \vec{l}_k(t_k), t_k] = \frac{1}{2\pi} \int \exp(-i\vec{p} \cdot \vec{l}_{0k}) \left(\int f_{\perp}(\vec{\eta}_k, \vec{\zeta}_k) \exp(ip\vec{\zeta}_k) d\vec{\zeta}_k \right) f(t_k) \exp(-i\vec{p} \cdot \vec{\alpha} t_k) d\vec{p}, \quad (C7)$$

where the term in brackets is the Fourier transform of the uncorrelated transverse distribution over the offset variables, the integral over $\vec{p}$ is a Fourier antitransformation, and the last factor only depends on t_k . In the absence of tilt, i.e., $\vec{\alpha} = (0, 0)$, this expression coincides with Eq. (5) of [46]. Written in this way, it also becomes clear that P is normalized with respect to the set of variables $(\vec{\eta}_k, \vec{l}_{0k}, t_k)$:

$$\begin{aligned} \iiint P[\vec{\eta}_k, \vec{l}_k(t_k), t_k] d\vec{\eta}_k d\vec{l}_{0k} dt_k &= \frac{1}{2\pi} \iiint \exp(-i\vec{p} \cdot \vec{l}_{0k}) \left(\int d\vec{\zeta}_k f_{\perp}(\vec{\eta}_k, \vec{\zeta}_k) \exp(ip\vec{\zeta}_k) \right) f(t_k) \exp(-ip\alpha t_k) d\vec{\eta}_k d\vec{l}_{0k} dt_k d\vec{p} \\ &= \int \delta(\vec{p}) \left(\int d\vec{\eta}_k d\vec{\zeta}_k f_{\perp}(\vec{\eta}_k, \vec{\zeta}_k) \exp(ip\vec{\zeta}_k) \right) \left(\int dt_k f(t_k) \exp(-i\vec{p} \cdot \vec{\alpha} t_k) \right) d\vec{p} \\ &= \int d\vec{\eta}_k d\vec{\zeta}_k f_{\perp}(\vec{\eta}_k, \vec{\zeta}_k) \int dt_k f(t_k) = 1 \end{aligned}$$

Combining all the definitions,

$$\begin{aligned} \Gamma_{\omega}(z, \vec{r}_1, \vec{r}_2, \omega_1, \omega_2) &= \langle \bar{E}(z, \vec{r}_1, \omega_1) \bar{E}^*(z, \vec{r}_2, \omega_2) \rangle = \left\langle \sum_{m,n=1}^{N_e} \bar{E}(\vec{\eta}_m, \vec{l}_m, z, \vec{r}_1, \omega_1) \bar{E}^*(\vec{\eta}_n, \vec{l}_n, z, \vec{r}_2, \omega_2) \exp[i(\omega_1 t_m - \omega_2 t_n)] \right\rangle \\ &= \sum_{m=1}^{N_e} \langle \bar{E}(\vec{\eta}_m, \vec{l}_m, z, \vec{r}_1, \omega_1) \bar{E}^*(\vec{\eta}_m, \vec{l}_m, z, \vec{r}_2, \omega_2) \exp[i(\omega_1 - \omega_2) t_m] \rangle \\ &\quad + \sum_{m \neq n} \langle \bar{E}(\vec{\eta}_m, \vec{l}_m, z, \vec{r}_1, \omega_1) \exp(i\omega_1 t_m) \rangle \langle \bar{E}^*(\vec{\eta}_n, \vec{l}_n, z, \vec{r}_2, \omega_2) \exp(-i\omega_2 t_n) \rangle \end{aligned}$$

and, in particular, the average quantity in the first term is

$$\begin{aligned} &\langle \bar{E}(\vec{\eta}_m, \vec{l}_m, z, \vec{r}_1, \omega_1) \bar{E}^*(\vec{\eta}_m, \vec{l}_m, z, \vec{r}_2, \omega_2) \exp[i(\omega_1 - \omega_2) t_m] \rangle \\ &= \iiint P[\vec{\eta}_m, \vec{l}_m(t_m), t_m] \bar{E}(\vec{\eta}_m, \vec{l}_{0m}, z, \vec{r}_1, \omega_1) \bar{E}^*(\vec{\eta}_m, \vec{l}_{0m}, z, \vec{r}_2, \omega_2) \exp[i(\omega_1 - \omega_2) t_m] d\vec{\eta}_m d\vec{l}_{0m} dt_m. \end{aligned}$$

Before solving these integrals, it is important to better specify the electric field functional dependence from the electron coordinates and $\vec{r}$. In particular, using the results in [61] or, more generally, the properties of the Green function for the Maxwell equations in the paraxial approximation,

$$\bar{E}(\vec{\eta}_m, \vec{l}_m, z, \vec{r}, \omega) = \bar{E}(\vec{\eta}_m, z, \vec{r} - \vec{l}_m, \omega). \quad (C8)$$

Using the definition in Eq. (C4), we can integrate over $\vec{l}_{0m}$ using the Dirac delta, getting

$$\iiint \tilde{P}(\vec{\eta}_m, \vec{\zeta}_m, t_m) \bar{E}(\vec{\eta}_m, z, \vec{r}_1 - \vec{\zeta}_m + \vec{\alpha} t_m, \omega_1) \bar{E}^*(\vec{\eta}_m, z, \vec{r}_2 - \vec{\zeta}_m + \vec{\alpha} t_m, \omega_2) \exp[i(\omega_1 - \omega_2) t_m] d\vec{\eta}_m dt_m d\vec{\zeta}_m. \quad (C9)$$

If we consider the Fourier transform with respect to the spatial variables of the fields, then

$$\begin{aligned} \hat{\Gamma}_{\omega}(z, \vec{k}_1, \vec{k}_2, \omega_1, \omega_2) &= \sum_m \iiint \tilde{P}(\vec{\eta}_m, \vec{\zeta}_m, t_m) \iint \bar{E}(\vec{\eta}_m, z, \vec{r}_1 - \vec{\zeta}_m + \vec{\alpha} t_m, \omega_1) \bar{E}^*(\vec{\eta}_m, z, \vec{r}_2 - \vec{\zeta}_m + \vec{\alpha} t_m, \omega_2) d\vec{r}_1 d\vec{r}_2 \\ &\quad \times \exp[-i(\vec{k}_1 \cdot \vec{r}_1 - \vec{k}_2 \cdot \vec{r}_2)] \exp[i(\omega_1 - \omega_2) t_m] d\vec{\eta}_m dt_m d\vec{\zeta}_m \end{aligned}$$

and, if we redefine $\vec{r}_i - \vec{\zeta}_m + \vec{\alpha}t_m = \vec{s}_i$,

$$\begin{aligned}
& \sum_m \iiint \tilde{P}(\vec{\eta}_m, \vec{\zeta}_m, t_m) \left(\iint \tilde{E}(\vec{\eta}_m, z, \vec{s}_1, \omega_1) \tilde{E}^*(\vec{\eta}_m, z, \vec{s}_2, \omega_2) \exp[-i(\vec{k}_1 \cdot \vec{s}_1 - \vec{k}_2 \cdot \vec{s}_2)] d\vec{s}_1 d\vec{s}_2 \right) \\
& \exp[-i(\vec{k}_1 - \vec{k}_2) \cdot (\vec{\zeta}_m - \vec{\alpha}t_m)] \exp[i(\omega_1 - \omega_2)t_m] d\vec{\eta}_m dt_m d\vec{\zeta}_m \\
& = \sum_m \iiint f_{\perp}(\vec{\eta}_m, \vec{\zeta}_m) \exp[-i(\vec{k}_1 - \vec{k}_2) \cdot \vec{\zeta}_m] \left(\iint \tilde{E}(\vec{\eta}_m, z, \vec{s}_1, \omega_1) \right. \\
& \left. \tilde{E}^*(\vec{\eta}_m, z, \vec{s}_2, \omega_2) \exp[-i(\vec{k}_1 \cdot \vec{s}_1 - \vec{k}_2 \cdot \vec{s}_2)] d\vec{s}_1 d\vec{s}_2 \right) f(t_m) \exp[i(\omega_1 - \omega_2)t_m] \exp[i(\vec{k}_1 - \vec{k}_2) \cdot \vec{\alpha}t_m] d\vec{\eta}_m dt_m d\vec{\zeta}_m \\
& = \tilde{f}[(\omega_1 - \omega_2) + (\vec{k}_1 - \vec{k}_2) \cdot \vec{\alpha}] \iint \langle \tilde{E}(\vec{\eta}_m, z, \vec{s}_1, \omega_1) \tilde{E}^*(\vec{\eta}_m, z, \vec{s}_2, \omega_2) \rangle \exp[-i(\vec{k}_1 \cdot \vec{s}_1 - \vec{k}_2 \cdot \vec{s}_2)] d\vec{s}_1 d\vec{s}_2 \\
& = \tilde{f}[(\omega_1 - \omega_2) + (\vec{k}_1 - \vec{k}_2) \cdot \vec{\alpha}] \hat{G}(z, \vec{k}_1, \vec{k}_2, \omega_1, \omega_2)
\end{aligned}$$

where we can suppress the dependence from ω_2 in the second factor as explained in [46]. In this case, the last equivalence corresponds to the Fourier transform of Eq. (12) in [46], if there is no tilt. Finally, transforming back to the real space coordinates and changing variables, going from $(\vec{r}_1, \vec{r}_2, \vec{k}_1, \vec{k}_2)$ to $(\Delta\vec{r}, \vec{r}, \Delta\vec{k}, \vec{k})$ where

$$\begin{aligned}
\Delta\vec{r} &= \vec{r}_1 - \vec{r}_2 \\
\vec{r} &= \frac{\vec{r}_1 + \vec{r}_2}{2} \\
\Delta\vec{k} &= \vec{k}_1 - \vec{k}_2 \\
\vec{k} &= \frac{\vec{k}_1 + \vec{k}_2}{2}
\end{aligned} \tag{C10}$$

$$\Gamma_{\omega}(z, \Delta\vec{r}, \vec{r}, \omega_1, \omega_2) = \iint \tilde{f}[(\omega_1 - \omega_2) + \Delta\vec{k} \cdot \vec{\alpha}] \hat{G}(z, \Delta\vec{k}, \vec{k}, \omega_1) \exp[i(\vec{k} \cdot \Delta\vec{r} + \Delta\vec{k} \cdot \vec{r})] d\vec{k} d\Delta\vec{k}.$$

Using the quasihomogeneity condition for $\hat{G}$ in $z = 0$, we can factorize the double integral into

$$\Gamma_{\omega}(0, \Delta\vec{r}, \vec{r}, \omega_1, \omega_2) = \int \tilde{f}[(\omega_1 - \omega_2) + \Delta\vec{k} \cdot \vec{\alpha}] \hat{g}(0, \Delta\vec{k}, \omega_1) \exp[i\Delta\vec{k} \cdot \vec{r}] d\Delta\vec{k} \int \exp[i\vec{k} \cdot \Delta\vec{r}] \hat{I}(0, \vec{k}, \omega_1) d\vec{k}$$

and including the monochromator transfer function

$$\begin{aligned}
\Gamma_{\omega}(0, \Delta\vec{r}, \vec{r}, \omega_1, \omega_2) &= \int d\Delta\vec{k} \tilde{f}[(\omega_1 - \omega_2) + \Delta\vec{k} \cdot \vec{\alpha}] T(\omega_1) T^*(\omega_2) \hat{g}(0, \Delta\vec{k}, \omega_1) \exp[-i\Delta\vec{k} \cdot \vec{r}] \int d\vec{k} \exp[-i\vec{k} \cdot \Delta\vec{r}] \hat{I}(0, \vec{k}, \omega_1) \\
&= \frac{1}{2\pi} g(0, \Delta\vec{r}, \omega_1) \iiint \tilde{f}(q) \exp[ip(q - \Delta\omega + \Delta\vec{k} \cdot \vec{\alpha})] T(\omega_1) T^*(\omega_2) \hat{g}(0, \Delta\vec{k}, \omega_1) \\
&\quad \exp[-i\Delta\vec{k} \cdot \vec{r}] d\Delta\vec{k} dq dp \\
&= T(\omega_1) T^*(\omega_2) g(0, \Delta\vec{r}, \omega_1) \int f(p) I(0, \vec{r} - \vec{\alpha}p, \omega_1) e^{-ip(\omega_1 - \omega_2)} dp.
\end{aligned}$$

In the following, we will consider two cases: large and narrow monochromator bandwidth.

If the monochromator bandwidth is large compared to the width of a spike in the frequency domain,

$$T(\omega_1) T^*(\omega_2) I(0, \vec{r}, \omega_1) g(0, \Delta\vec{r}, \omega_1) \approx |T(\bar{\omega})|^2 I(0, \vec{r}, \bar{\omega}) g(0, \Delta\vec{r}, \bar{\omega})$$

and, consequently,

$$\Gamma_{\omega}(0, \Delta\vec{r}, \vec{r}, \bar{\omega}, \Delta\omega) = |T(\bar{\omega})|^2 g(0, \Delta\vec{r}, \bar{\omega}) \int f(p) I(0, \vec{r} - \vec{\alpha}p, \bar{\omega}) e^{-ip\Delta\omega} dp. \tag{C11}$$

On the other hand, if the monochromator bandwidth is extremely narrow, i.e., $T(\omega) \approx \delta(\omega - \omega_M)$,

$$\Gamma_\omega(0, \Delta\vec{r}, \vec{r}, \omega_M) = g(0, \Delta\vec{r}, \omega_M) \int f(p) I(0, \vec{r} - \vec{\alpha}p, \omega_M) dp. \quad (C12)$$

Since the quasihomogeneity condition is not applicable in the far zone, we resort in a generalized van Cittert–Zernike theorem to obtain:

$$\Gamma_\omega(z, \Delta\vec{r}, \vec{r}, \omega_1, \omega_2) = \left(\frac{k}{2\pi}\right)^2 \frac{e^{i\vec{k}\vec{r}\cdot\Delta\vec{r}}}{z^2} T(\omega_1) T^*(\omega_2) I(z, \vec{r}, \omega_1) \int f(p) g(z, \Delta\vec{r} - \vec{\alpha}p, \omega_1) e^{-ip(\omega_1 - \omega_2)} dp, \quad (C13)$$

where we used the fact that I and g form a Fourier couple. The domain of the Fourier transform in Eq. (C13) is determined by the monochromator bandwidth. Unlike Eq. (A6), the electron longitudinal distribution and the correlation at z are convoluted, and their contributions cannot be factorized, unless the tilt is absent. Therefore, when integrating the squared modulus of Γ_ω (or its normalized form) over the frequency domain, different monochromator bandwidths change the way in which the transverse size is summed over the pulse duration.

Plugging this result in Eq. (A5), for a large monochromator bandwidth, in analogy with the calculation in absence of tilts, we can isolate the term

$$\begin{aligned} & \int \left| \int f(p) g(z, \Delta\vec{r} - \vec{\alpha}p, \bar{\omega}_0) e^{-ip\Delta\omega} dp \right|^2 d\Delta\omega \\ &= \frac{1}{2\pi} \int |f(\bar{t}) g(z, \Delta\vec{r} - \vec{\alpha}\bar{t}, \bar{\omega}_0)|^2 d\bar{t}, \end{aligned} \quad (C14)$$

where the Parseval's theorem has been used. On the other hand, for a narrow bandwidth, the equivalent term becomes

$$\left| \int f(p) g(0, \vec{r} - \vec{\alpha}p, \omega_M) dp \right|^2. \quad (C15)$$

From a mathematical point of view, the fundamental difference between the two expressions is in the order of the operational procedures: integration and modulus. Physically speaking, the first case is, due to the narrow time window (wide frequency window) imposed by the monochromator, the mean value of the width of g along the longitudinal profile; the second case is the width of g after summation over the longitudinal profile, since the latter is encompassed by the wide time window (narrow frequency window) of the monochromator. Finally, and less formally, we can say that the former is an averaged width, while the latter is the width of an average.

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