

Construction of multibend achromat lattices based on their substructures

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This paper presents an alternative approach to designing multibend achromat (MBA) lattices with distributed sextupoles. It is based on the inherent structure of MBA lattices, which are composed of three smaller (recurring) substructures that can be optimized individually. These substructures consist of only a few elements; therefore, detailed parameter studies are feasible, and underlying principles can be revealed. The final linear lattice is a combination of the optimized substructures. Design decisions are mostly dictated by constraints, leaving limited alternatives under the given goals and boundary conditions. The linear lattice is constructed rather than optimized. Thus, it is justified to talk about a deterministic lattice design approach that provides a solid starting point for further fine-tuning and nonlinear optimization. The significance of time- and CPU-intensive optimization algorithms, such as multiobjective genetic algorithms, is much reduced. They can be used for fine-tuning the superperiod and for nonlinear optimization. Although the method optimizes the linear structure, it also considers nonlinear aspects such as chromatic sextupole strength and robustness against deviations from the design parameters. The approach was used to develop a lattice for the fourth-generation synchrotron light source BESSY III, planned at the Helmholtz-Zentrum Berlin. The design criteria and target values were taken from the BESSY III project. Different goals or boundary conditions may lead to alternative decisions or objectives to be considered. Implications of using this approach for higher energy or higher superperiodic lattices are briefly discussed. The resulting solution for BESSY III and initial nonlinear studies are presented.

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I. INTRODUCTION

The worldwide success of multibend achromat (MBA) lattices started in 2016 with the commissioning of the first fourth-generation light source, the 3 GeV storage ring at MAX IV in Lund, Sweden [1]. The concept utilizes the cubic dependence of the emittance on the average bending angle to significantly reduce the emittance compared to third-generation machines.

The arc of the MAX IV lattice consists of seven bends, Fig. 1 (taken from [2]). Five are located at the center of identical and symmetric unit cells (UCs), and two bends terminate the achromat toward the straight section. The elements between the last UC and the straight section belong to the dispersion suppression cell (DSC). The quadrupoles at either end of the straight section adjust the beta functions to insertion devices or injection. Third-generation machines commonly used Chasman-Green lattices double bend achromat or triple bend achromat lattices,

where the number of quadrupoles is low. In MBA lattices, focusing and defocusing quadrupoles and sextupoles are needed to confine the Twiss parameters and compensate for the chromaticity in each UC; the number of magnets increases considerably. With 5 UCs in each of the 20 superperiods (SPs) of MAX IV, standardization of the

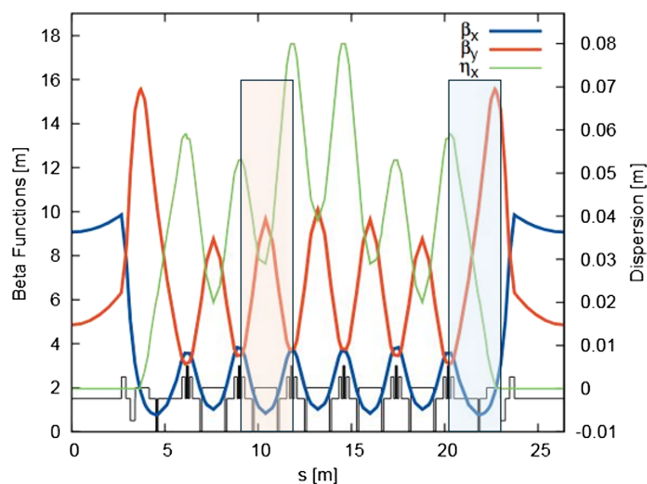


FIG. 1. β -functions and dispersion of one superperiod of MAX IV. The repeating UCs (orange shade) and DSCs (blue shade) are clearly detectable.

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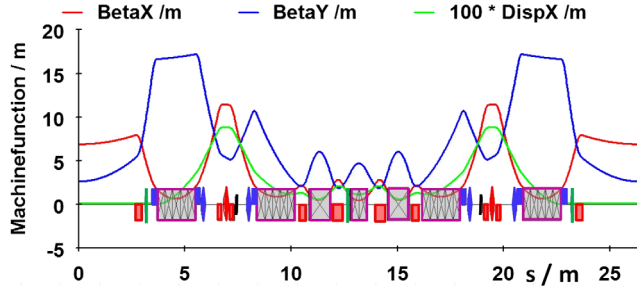


FIG. 2. β -functions and dispersion of one superperiod of the ESRF-EBS lattice. The large dispersion bumps (green) and the sextupoles at the end of the achromatic section characterize the HMBA. Identical UCs are lost.

hardware becomes necessary, leading to repetitive, identical structures.

MAX IV was followed by SIRIUS, the 3 GeV Brazilian light source, in Campinas in 2019 [3]. Today, every upgrade or new machine design of fourth-generation light sources is based on the MBA concept. With increasing interest in low-emittance light sources, different advancements in MBA lattice design have been developed, which are detailed in the following paragraphs.

(i) Hybrid multibend achromat (HMBA) lattices: the upgrade of the ESRF in Grenoble, France (2015–2020), is the first realization of a high-energy (6 GeV) MBA lattice and the first application of the hybrid MBA concept [4], where the chromaticity-compensating sextupoles are concentrated symmetrically at the two ends of the achromatic section, as illustrated in Fig. 2 (taken from [5]). The characteristic dispersion bumps at these locations allow the sextupole strength to be reduced, while the phase advance between the sextupoles is adjusted to $n \cdot \pi$ ($-I$ transformation) to cancel the sextupole-driven resonance driving terms, i.e., the impact of resonances. Note how the repetitive structure of identical UCs is lost due to the need to boost the dispersion.

(ii) Higher-order achromat (HOA): The theoretical background of this approach is treated extensively in [6,7]. In these papers, the term HOA refers to the cancellation of the sextupole-driven resonance driving terms (RDTs) to higher order in sextupole strength in the achromatic section by choosing the right phase advance between the sextupoles, conceptually similar to the $-I$ transformation in the HMBA case.

The repetitive structure of the UCs in MBA lattices simplifies the adjustment of the phase advance to cancel the RDTs. The condition on the phase advance $\phi_{x,y}$ of the UC is

$$\frac{\phi_{x,y}}{2\pi} * n = N, \quad (1)$$

where n is the number of UCs and N is a small integer. The accumulated phase advance then is a multiple of 2π for n

UCs. In case the DSCs include sextupoles, they count as one additional UC equivalent. Thus, in a lattice with $n + 1$ bends, there are n UC equivalents.

Similarly, the phase advance of the full superperiod, $\Phi_{x,y}$, should obey

$$\frac{\Phi_{x,y}}{2\pi} * p = M, \quad (2)$$

where p is the periodicity of the lattice and M is an integer. These conditions lead to an integer working point of the full lattice, which later has to be slightly detuned.

(iii) Reverse bends (RB): To seek minimal emittance, optimal values of the horizontal beta function and the dispersion can be set at the center of the bending magnet of the UC. As described in [8], the simultaneous fitting of the UC symmetry condition for β_x and η_x causes an unacceptably large horizontal phase advance and extended length of the UC, so the optimal values are often compromised. Introducing an RB, a short dipole with an opposite deflection angle, into the UC decouples the dispersion matching from the beta matching, thus relaxing the phase advance and cell length and making even lower emittance feasible. This concept was successfully realized for the first time in the upgrade of the Swiss Light Source [9], SLS2, in operation since 2025.

(iv) Longitudinal gradient bend (LGB): The concept of shaping the dipole field longitudinally has been discussed, among others, in [8,10,11]. The majority of the field is concentrated in short sections at the center of the UC bend, where β_x and η_x are small, whereas positions with less favorable Twiss parameters see lower fields. This way, the emittance can be reduced. At the same time, LGBs can be used to achieve high-field dipoles of up to 3.2 T (realized [3]) and up to 6 T (planned [12]) in the arcs without major harm to the dynamics. The LGB consists of three to several dozen slices with different fields, building a longitudinal field profile optimized toward the specific purpose. The optimization of LGBs is a project in itself and exceeds the scope of this paper. Nevertheless, this concept has to be studied for BESSY III in comparison to the results found in this paper.

The deterministic lattice design approach presented here leads to a physically motivated linear lattice structure that leverages RBs and the HOA principle in nonhybrid MBA lattices. RBs reduce the emittance and the UC length. The HOA condition cancels the RDTs and makes the lattice robust against field errors and misalignments, but it also drastically reduces the accessible parameter space of the UC. Magnets are arranged to reduce the strengths of the chromaticity-compensating sextupole families. However, this does not intrinsically guarantee optimal dynamic aperture (DA) or momentum acceptance (MA), as these quantities strongly depend on tune confinement, which is not addressed in the linear lattice design.

All calculations and graphs presented here are calculated using the lattice design code OPA, developed by Streun at PSI, Switzerland [13]. OPA is an interactive and fast tool that is ideally suited for the kind of investigation presented in this paper.

To avoid getting lost in a too large parameter space, we start with a generic lattice. We use standardized, but not unrealistic, magnet and drift lengths. In particular, drifts are 10 cm long, quadrupoles are 15 cm long, sextupoles are 10 cm long, and the initial dipole length is 1 m, adding up to 2.7 m for the UC. The deflection angles are distributed evenly between the bends, 4.5° for 16-SPs; only the outer bends in the DSC get half that value, 2.25° , as each DSC is considered half a UC. The deflection angle of the reverse bend is weak, $\approx 6\%$ of the main dipole angle; therefore, it is commonly combined with the focusing quadrupole, by offsetting its axis to generate the desired dipolar component. This combination saves up to 10% in circumference due to the high repetition factor of the UC.

The project parameters of BESSY III for which the approach is demonstrated are collected in Sec. II. In Sec. III, we investigate the generic UC to develop an understanding of the underlying principles and limitations of this substructure. We then analyze the DSC, which is constrained by the boundary conditions of the UC on one side, and the need to suppress dispersion on the other side (Sec. IV). The complete lattice, including technically necessary adaptations of the generic structures such as maximal fields, and space for synchrotron radiation absorbers, correctors, and BPMs, is presented in Sec. V. Initial results on the nonlinear performance of the lattice are presented in Sec. VI. Section VII comments on the challenges when extending the approach to lattices with a larger number of SPs or higher energies.

II. THE BESSY III PROJECT

BESSY III is the successor to the 1.7 GeV electron storage ring BESSY II, which has been in operation in Berlin, Germany, since 1998. The demand of users for diffraction-limited radiation at 1 keV from the first undulator harmonics pushes the energy to 2.5 GeV and the emittance down to 100 pm rad. The higher energy and the extremely dense structure of BESSY II inhibit an upgrade in the existing tunnel while maintaining the capacity of 16 straight sections. Unfortunately, the greenfield design is limited by the proposed building site in the vicinity of BESSY II to a maximal circumference of ≈ 350 m. The BESSY III goal parameters are listed in Table I.

An indispensable prerequisite for the deterministic linear lattice design approach is a clear definition of the priorities of the design goals. For BESSY III, these are listed in Table II, where the first column lists the priority ranking. Note that the small emittance, robustness, and compactness belong to the highest priority rank. They provide the major

TABLE I. BESSY III goal parameters.

Parameter	Value
Energy	2.5 GeV
Superperiods/straight sections	≥ 16
Free space in straight sections	5.6 m
Circumference	≈ 350 m
Emittance	100 pm rad
Momentum compaction factor, α_c	$> 1 \times 10^{-4}$
Momentum acceptance	$> 3\%$

TABLE II. Design criteria ranked by priorities, guiding through the deterministic design approach.

	Design criteria	Reason
1	MBA lattice	Emittance
1	HOA, UC phase advance	Robustness
1	Usage of reverse bends	Emittance, compactness
2	Low sextupole strength	Nonlinear behavior
3	Short circumference	Site
4	DSC as close as possible to half UC	Robustness, emittance, momentum compaction

guideline for the development of the lattice substructures described in the following sections.

III. DEPENDENCIES WITHIN AND LIMITATIONS OF THE UC

A. UC and HOA condition

In its most basic form, the UC consists of a central main bend (MB), symmetrically arranged focusing/defocusing quadrupoles (QF and QD), and sextupoles (SF and SD), see Fig. 3. The defocusing gradient might be integrated into the bend. For BESSY III, a separated QD is assumed initially. QF must be the outer focusing element to create separation of the β -functions for chromaticity compensation. The location of the sextupoles is still open. QF can be shifted

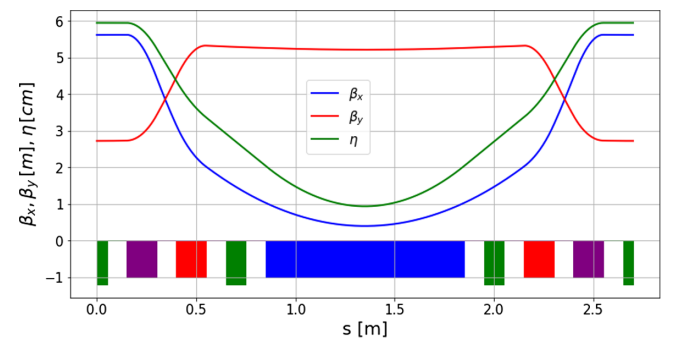


FIG. 3. β -functions and dispersion of a generic UC. The position of the sextupoles (green) has not yet been finalized. The other elements are MB (blue), QD (red), and QF/RB (purple).

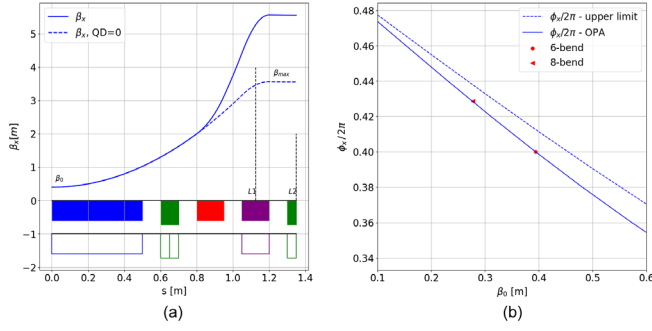


FIG. 4. (a) β_x in half a UC (solid line, upper magnets) and for QD = 0 (dashed line, lower magnets); (b): $\phi_x/2\pi$ over β_0 : analytic estimate (dashed line), OPA phase advance (solid line), options for HOAs (dot, triangle).

transversally off-beam axis to profit from the advantages of an RB. Using the symmetry condition of the UC, the two gradient values of QF and QD, and the RB angle uniquely determine the Twiss parameters in the UC.

Following the list of priorities, we first estimate which horizontal phase advance is accessible in the generic UC and which phase advances correspond to an HOA.

The horizontal β -function and the dispersion need to have a minimum in the central bend of the UC to achieve small emittance. Consequently, most of the horizontal phase advance is concentrated in the dipole. An analytical upper limit of the horizontal phase advance as a function of the minimum of β_x can be derived assuming that QD is zero, Fig. 4(a), dotted line and lower magnets. With QD = 0, β_x develops as in a drift from β_0 at the center of the MB up to the RB, which can be adjusted to keep β_x constant beyond the RB. It is smaller than the original β -function (solid line), therefore:

$$\begin{aligned} \phi_x/2\pi &< 2 \cdot \int_0^{L_1} \frac{\beta_0}{\beta_0^2 + s^2} ds + 2 \cdot \int_{L_1}^{L_2} \frac{1}{\beta_{\max}} ds \\ &= \arctan\left(\frac{L_1}{\beta_0}\right) + \frac{L_2 - L_1}{\beta_{\max}}. \end{aligned} \quad (3)$$

For QD = 0, the first integral represents the phase advance up to the maximum, and the second integral is the phase advance of a constant β_x following the maximum. Figure 4(b) shows the analytically estimated phase advance (dashed line), and the phase advance of the UC calculated by OPA (solid line) as a function of β_0 at the dipole center. The degradation of the estimate for larger β_0 results from the decreasing ratio between the phase advance before and after RB. Small values of β_0 lead to large horizontal chromaticity, ≈ 3 per UC for $\beta_0 = 0.1$ m, e.g., $\gg 200$ for the ring. Large values of β_0 increase the emittance to $\varepsilon \approx 400$ pm rad for the UC at $\beta_0 = 0.6$ m. So for this study, the interval $0.1 \text{ m} < \beta_0 < 0.6 \text{ m}$ represents the relevant range. For all UCs reasonably close to the generic UC, the

TABLE III. HOA horizontal phase advance condition in the UC.

SP	n_B	n	θ_{UC}	N	$\phi_x/2\pi$	β_0
16	8	7	3.214	3	0.429	0.28
	7	6	3.75	2	0.333	0.70
	6	5	4.5	2	0.400	0.39
	5	4	5.625	2	0.500	...
18	8	7	2.857	3	0.429	0.28
	7	6	3.333	2	0.333	0.70
	6	5	4.0	2	0.400	0.39
	5	4	5.0	2	0.500	...
20	8	7	2.571	3	0.429	0.28
	7	6	3.0	2	0.333	0.70
	6	5	3.6	2	0.400	0.39
	5	4	4.5	2	0.500	...

accessible horizontal phase advance lies between 0.36 and 0.48.

On the other hand, the HOA condition on the horizontal phase advance is a strong restriction. Table III lists lattice options depending on the number of superperiods (SPs), the number of bends (n_B) the number of UCs, (n), and the integer in the HOA condition (N) from Eq. (1). $\phi_x/2\pi$ and β_x refer to a UC with QF on axis, e.g., RB angle = 0. For different numbers of bends, the MB angle (θ_B), the phase advance ($\phi_x/2\pi$) necessary to fulfill the HOA condition, and the minimal β -function (β_0) change. They are independent of the number of SP.

Note that: (i) A phase advance of 0.5 is unattainable, it corresponds to $\beta_0 = 0$. On the other hand, $\phi_x/2\pi = 0.3333$ ($\beta_0 \approx 0.7$ m) rules out small ε . (ii) A different arrangement of the sextupoles or moderate variations in the length of the magnets or drift spaces will not essentially change these findings.

Only two options for an HOA fall into the range of accessible horizontal phase advances in the UC, independent of the number of SP: a 6-bend or an 8-bend lattice, indicated in Fig. 4(b). It can be easily estimated that the circumference of an 8-bend would exceed that of a 6-bend by ≈ 90 m for 16-SPs, and that an 18-SP ring is at least 45 m longer than a 16-SP ring. Due to the limitation in circumference, BESSY III will consist of 16-SPs and a 6-BA, see the bold line in Table III.

The vertical phase advance is less restricted, as it does not affect the emittance. $\phi_y/2\pi = 0.1, 0.2, 0.3, \dots$ all fulfill the HOA condition for 5 UCs, as only $2\phi_y/2\pi$ appear in the targeted resonances, e.g., $Q_x \pm 2Q_y = N$. Larger vertical phase advance results in close to parallel shifts of the vertical β -function to lower values. To reduce the overall focusing, e.g., the natural chromaticity, the vertical phase advance of the UC is set to $\phi_y/2\pi = 0.1$.

By skipping the sextupoles in the DSC and adding one UC, a 7-BA HOA can be built with a horizontal phase advance of 0.4 in the UC. But, the larger number of bends,

i.e., smaller dispersion, and the increased sextupole strength to compensate for the chromaticity in the DSC, make the nonlinear optics more challenging. Other projects, such as SLS2 [14], abandon the HOA based on the UC and apply phase compensation, including the harmonic sextupoles in the straight.

B. Emittance reduction in the UC

The natural emittance of a ring results from an equilibrium between quantum excitation and radiation damping and is reached after many turns. Using the radiation integrals, the emittance can be estimated when only the bending radius of the reference trajectory, $\rho(s)$, and the lattice functions are known.

$$\varepsilon_x = \frac{C_q \gamma^2}{J_x} \int_0^C \frac{I_5(s)}{I_2(s)} ds \quad (4)$$

with $C_q = 3.83 \times 10^{-13}$ m, the quantum excitation number, γ the relativistic factor, and J_x the horizontal damping partition number. I_2 and I_5 , both in units of 1/m, are given by

$$I_2 = \int_0^C \frac{1}{\rho^2} ds, \quad I_5 = \int_0^C \frac{\mathcal{H}(s)}{|\rho^3|} ds, \quad (5)$$

with $\mathcal{H}(s) = \beta\eta^2 + 2\alpha\eta'\eta + \gamma\eta^2$, known as the “curly H” function.

The radiation integrals can be used to calculate the emittance contribution of subsections of the lattice, although, strictly speaking, they do not represent emittance in the physical sense. OPA calculates these quantities as a means for lattice developers to estimate the contributions of subparts of the lattice to the overall emittance, momentum compaction factor, α_c , and other parameters.

In this section, we compare different options to reduce the emittance contribution of the generic UC. Introducing an RB into the lattice is the most efficient way, but there exist further options to lower the emittance. For fixed energy, possible “knobs” are J_x and the dipole bending radius ρ , namely, the MB angle (θ) and MB length (L). $\mathcal{H}$ is not a “free” parameter, as it results from fitting the UC to the desired phase advance.

1. Reverse bends

For any ring, the radiation damping partition number, J_x , depends on the radiation integrals I_2 and $I_{4,x}$,

$$J_x = 1 - \frac{I_{4,x}}{I_2}, \quad (6)$$

with

$$I_{4,x} = \int_0^C \left[\frac{\eta_x}{\rho^3} + \frac{2}{\rho} k \eta_x \right] ds. \quad (7)$$

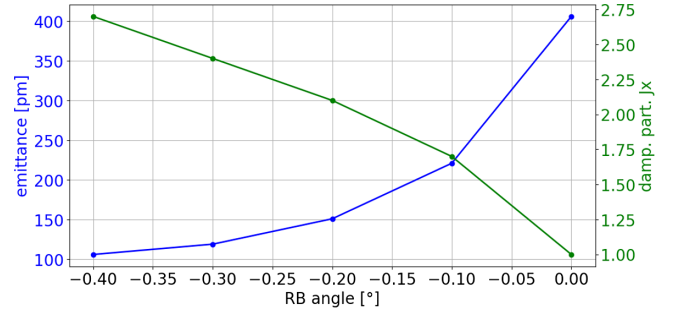


FIG. 5. Emittance (blue) and damping partition number (green) as a function of the RB angle.

where k is the gradient of the bend in [$1/m^2$]. For a *single homogeneous* dipole of length L and constant ρ , Eq. (6) simplifies to:

$$J_x = 1 - \frac{1}{L} \int_0^L \frac{\eta_x}{\rho} ds. \quad (8)$$

For the MB, the integral can be approximated to zero because $\rho \gg \eta_x$.

For a *single* RB realized as a shifted quadrupole of length L_{RB} , $k > 0$, $\rho_{RB} < 0$.

$$J_x = 1 - \frac{1}{\rho_{RB} L_{RB}} \int_0^{L_{RB}} [\eta_x + 2k\eta_x \rho_{RB}^2] ds. \quad (9)$$

While η_x/ρ_{RB} is small also in the RB, the second term in the integral is large, as k and ρ_{RB} are large, and the contribution of the RB to J_x is significant. For the complete UC, the gradient term in I_4 dominates I_2 and J_x increases with larger RB angles.

Figure 5 shows how the emittance and horizontal damping partition number in the UC develop with increasing negative angle of RB, calculated with OPA. The MB angle is adjusted to keep the deflection angle of the UC constant. Within technically realizable limits, the emittance can be reduced by a factor of almost 4 to 106 pm rad. It can be shown that approximately 60% of the reduction in emittance is due to the increase in J_x , 40% results from changes in $\mathcal{H}$ and ρ_{RB} .

Unfortunately, RBs also have drawbacks. They affect the longitudinal plane, particularly the momentum compaction factor α_c , the longitudinal damping time $\tau_z \propto \frac{1}{\alpha_c}$, and the energy spread σ_ϵ . All three quantities are connected to the radiation integrals $I_1, \dots, I_4$, which strongly depend on the RB.

$$\alpha_c = \frac{1}{C} I_1 = \frac{1}{C} \oint \frac{\eta_x(s)}{\rho(s)} ds, \quad (10)$$

$$J_z = 2 + \frac{I_4}{I_2}, \quad \sigma_\epsilon \propto \frac{I_3}{2I_2 + I_4}. \quad (11)$$

The second order momentum compaction factor [15] is also affected, α_1 is given by

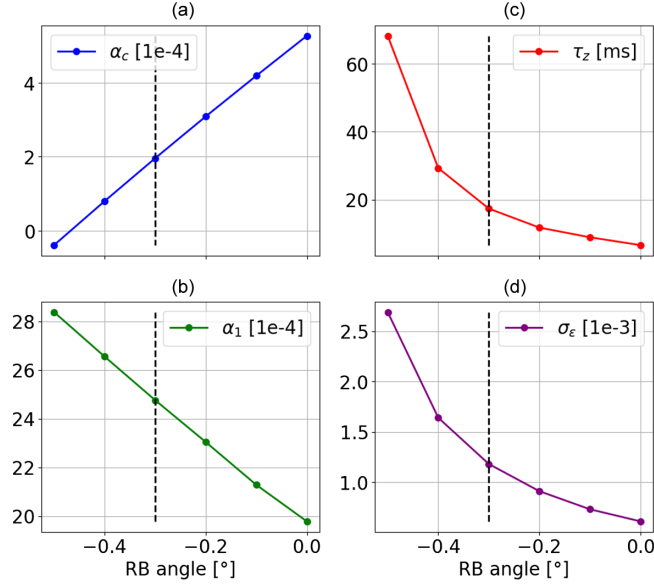


FIG. 6. Graph shows (a) α_c , (b) α_1 , (c) τ_z , and (d) σ_ϵ as a function of increasing RB angle. Black dashed lines: chosen limit for the RB angle.

$$\alpha_1 = \frac{1}{C} \oint \left(\frac{\eta'_x(s)^2}{2} + \frac{\eta_1(s)}{\rho(s)} \right) dx, \quad (12)$$

where

$$\eta'_x = \frac{d\eta_x}{ds}, \quad \eta_1 = \frac{d\eta_x}{d\delta}. \quad (13)$$

The dependence of these quantities on the RB angle in the generic UC is depicted in Fig. 6. Larger negative bending angles of RB lead to a close-to-linear reduction in α_c , Fig. 6(a). The bunch length is reduced accordingly, resulting in higher particle density and a shorter Touschek lifetime. The design value for α_c for BESSY III was set to $\geq 1 \times 10^{-4}$ to achieve reasonable lifetimes. Due to the dependence of α_c on circumference, this translates into a value of $\approx 2 \times 10^{-4}$ for the UC, which corresponds to an absolute RB angle of $< 0.3^\circ$.

The increase in α_1 , Fig. 6(b), leads to an asymmetry in the rf bucket and limits the applicable rf voltage.

The longitudinal damping time, τ_z , Fig. 6(c), and the energy spread, σ_ϵ , Fig. 6(d), increase strongly with larger RB angles, leading to longitudinally unstable bunches.

The limit of $\theta_{RB} = -0.3^\circ$ leads to acceptable values for all four quantities, indicated by black dashed lines in Fig. 6. Therefore, the RB angle is determined: it needs to be large to reduce the emittance, but smaller than 0.3° to keep the longitudinal dynamics and α_c under control.

Alternatively, one could consider shifting QD off axis in the opposite direction of RB, which would lead to an emittance reduction via the same mechanism. In this case, $k < 0$, and α_c increases. The deflection angle of the main

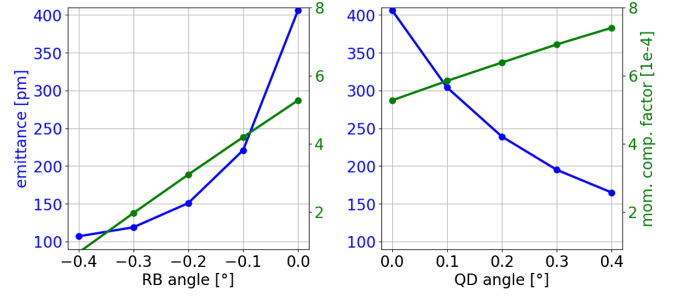


FIG. 7. Emittance (blue) and momentum compaction factor (green) as a function of the QF/RB angle (left) and the QD angle (right).

bend is lowered to keep the overall deflection of the UC. Figure 7 compares both approaches. The gain in emittance is smaller when using QD, due to the smaller Twiss functions. Within technically feasible limits, the goal emittance cannot be reached by shifting QD. Therefore, this alternative does not apply to BESSY III.

2. Gradient main dipole

Similarly to the RB, a gradient in the main dipole would also reduce the emittance via changing J_x . In this case $\rho > 0$ and $k < 0$. Figure 8 compares the effect of a homogeneous dipole and an RB (blue, bottom axis) with that of an increasing gradient in the main dipole without RB (green, top axis). QD is adjusted to supplement the gradient necessary to fit the desired phase advance. The RB is significantly more effective than the dipole gradient. The reason is the gradient-dependent term in J_x . η_x and k are much larger at the RB than at the dipole. For k values $< -1.13 \text{ m}^2$ in the MB, β_y is overfocused, and QD becomes positive.

It is also possible to concentrate the defocusing field completely in the MB and eliminate QD, thereby shortening the UC. The effect on the emittance is indicated by the black dashed line in Fig. 8. The necessary gradient is

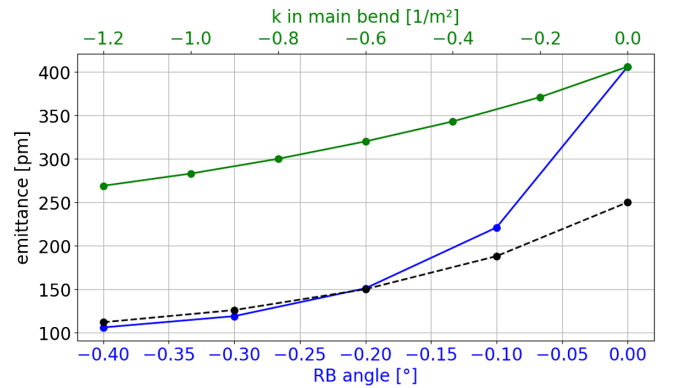


FIG. 8. Emittance reduction for: gradient MB and no RB (green, top axis). RB and homogeneous MB (blue, bottom axis). Gradient MB and no QD (black).

technically feasible with $G = 13$ T/m. The optimal emittance reduction is the same as with QD. The shorter UC leads to smaller minima of β_{x0} and η_0 in the main bend when fitting the phase advance; therefore, this case starts with a highly reduced emittance for zero RB angle. This solution provides $\approx 4\%$ smaller circumference at the cost of 30% lower dispersion and a worse separation of the β -functions at the sextupoles. Therefore, nonlinear aspects of the design will play an increasing role. These aspects are further discussed in Sec. III C. Due to the better nonlinear performance at equal emittance, BESSY III will consist of homogeneous dipoles.

It should be mentioned that the offset of the quadrupole to create the RB field and the gradient in the main dipole both reduce the accessible maximal field strength of the magnets and therefore increase the magnet's length. Furthermore, any combination of gradient and dipole field in a single magnet limits the flexibility of the lattice. The shift of the RB is linked to a careful vacuum design, as the magnet apertures are small in MBA lattices, and later offset adjustments will be limited. Similarly, the gradient in the main bend is determined by the iron yoke.

3. Main bend length

The length of the MB, or equivalently its field (as the angle is fixed), is a parameter that can conveniently change the emittance and the momentum compaction factor in the preferred direction. However, longer bends lead to a larger circumference and might conflict with user demands for dipole radiation. Figure 9 shows the emittance and the momentum compaction factor for a dipole length of 0.6 to 1.4 m and a deflection angle of 4.5° for the generic UC (without RB). Red circles mark the values of the generic 1 m bend. If the circumference permits, the length of the MB could be increased.

Figures 7 (left) and 9 nicely demonstrate that the RB outperforms the TME conditions. The theoretical minimal emittance (TME) of a homogeneous dipole is reached when the minima of β_x and η_x in the dipole obey values that have been derived, among others, in [16]:

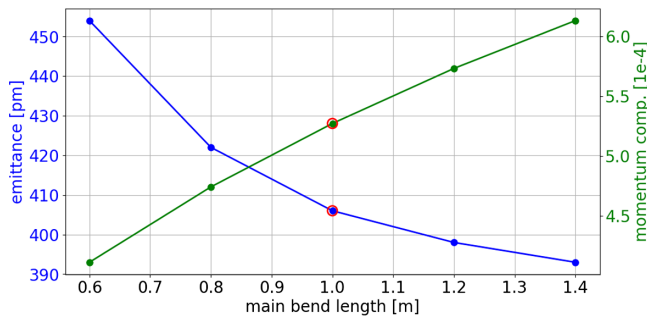


FIG. 9. Emittance (blue), and α_c (green), as a function of the MB length in the UC with RB = 0. Red circles mark the generic values.

$$\beta_{0,\text{TME}} = \frac{L}{2\sqrt{15}}, \quad \eta_{0,\text{TME}} = \frac{\theta L}{24}, \quad (14)$$

where θ and L are the bending angle and the length of a symmetrically placed bend. Shorter dipoles need lower β_0 - and η_0 -values at the center, causing stronger focusing and larger chromaticity. For $L = 1.0$ m, $\theta = 4.5^\circ$, $\beta_{0,\text{TME}} = 0.13$ m, and $\eta_{0,\text{TME}} = 0.0033$ m, corresponding to an emittance of 96 pm rad and a natural horizontal chromaticity of -2.1 . Using an RB with the same dipole, the horizontal chromaticity would be less than 40% of that value for a similar emittance. Alternatively, in the deterministic approach, the minimum of β_x is fixed by the horizontal phase advance to ≈ 0.4 m for $\phi_x = 0.4$. According to the TME conditions, this would correspond to a ≈ 3 m long dipole.

4. Main bend angle

The distribution of the deflection angle between the main bend and the bend in the dispersion suppression cell is a further degree of freedom. The emittance in the complete arc is given by the sum over all emittance contributions, weighted by their length. The lattice utilizes more UCs than DSC; therefore, exploiting the cubic dependence on the bending angle, it makes sense to reduce the UC angle and increase the DSC angle. Furthermore, when the angle of the DSC bend is too low, it becomes difficult to match the DSC to the UC.

OPA allows for a quick scanning of different parameters. For three lengths of the MB, 1.0 m, 1.1 m, and 1.2 m, the MB and RB angle were set to 4.2° , 4.3° , 4.4° , and to -0.3° , -0.28° , and -0.26° , respectively. The 27 resulting ε and α_c values were recorded, where $80 \text{ pm rad} \leq \varepsilon \leq 119 \text{ pm rad}$ and $1.49 \times 10^{-4} \leq \alpha_c \leq 2.75 \times 10^{-4}$. Figure 10 shows the results as a percentage increase or decrease with respect to the range of the results. The origin represents the minimal ε

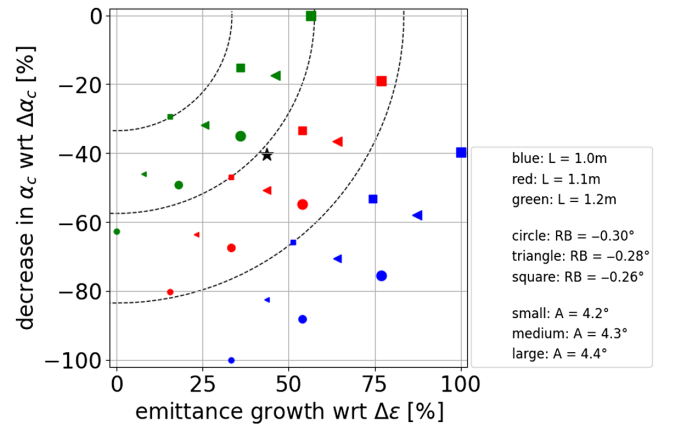


FIG. 10. Percentage decrease in α_c versus percentage increase in ε for 27 combinations of the MB length (color) and angle (symbol size), and RB angle (symbols). The black star indicates the design choice.

in combination with the maximal α_c ; solutions of equal quality, e.g., the same percentage variation distributed between ε and α_c , lie on circles around the origin.

A similar pattern appears for each MB length (color), and the length is the most effective parameter to improve both the emittance and α_c . For each length, the smallest MB angle (smallest symbols) yields the smallest emittance. Smallest RB angles (squares) yield the largest α_c values. The best results for a fixed MB length come from minimizing both the MB and RB angles. The effect of larger angles in the DSC will again be studied in Sec. IV.

Regarding the overall circumference, the following choices were taken: $L_{MB} = 1.1$ m, $\theta_{MB} = 4.25^\circ$, and $\theta_{RB} = -0.26^\circ$ (black star in Fig. 10). This corresponds to $\varepsilon = 97$ pm rad and $\alpha_c = 2.24 \times 10^{-4}$ for the UC. Note: to keep the overall deflection angle of the UC, twice the RB angle has to be added to the MB angle. To stay independent of the RB setting, the MB angle for RB = 0 will be quoted from now on, which is the total bending angle of the UC. The choice balances between ε and α_c and meets the goal parameters for the UC. Thereby, all magnetic parameters of the UC relevant to reaching the expected performance are fixed.

5. Longitudinal gradient bends

It has already been mentioned that LGBs are out of scope of this paper. It should be pointed out, though, that if the phase advance in the UC is fixed, LGBs do not develop their full potential. The emittance reduction of LGBs is caused by reducing β_x^0 and η^0 at the center of the LGB, matched to the high-field section of the magnet. In the deterministic approach, β_x^0 is determined by the HOA condition and is not a free parameter.

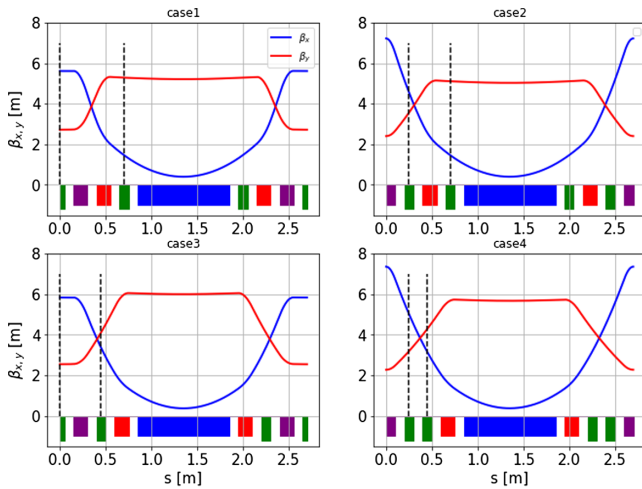


FIG. 11. β -functions for four optional placements of the sextupoles. Dashed black lines indicate the separation of the β -functions at the sextupoles.

TABLE IV. Emittance, natural chromaticity, and sextupole strength in the generic UC for different magnet arrangements and $\xi_{x,y} = 0$.

Case	Magnet arrangement	ε (pm)	ξ_x	ξ_y	SF (1/m ²)	SD (1/m ²)
1	SF/2-RB-QD-SD-B	119	-0.76	-0.27	18.2	-16.4
2	RB/2-SF-QD-SD-B	101	-0.85	-0.23	15.5	-27.4
3	SF/2-RB-SD-QD-B	133	-0.76	-0.28	28.8	-17.2
4	RB/2-SF-SD-QD-B	109	-0.84	-0.24	22.5	-26.2
5	SF/2-RB-SD-B	126	-0.71	-0.35	23.1	-14.6
6	RB/2-SF-SD-B	167	-0.82	-0.27	16.7	-19.8

C. Magnet arrangement inside the UC

The generic UC comprises seven and two half magnets with four optional placements of the sextupoles, as shown in Fig. 11, where all UCs are tuned to the same phase advance ($\phi_x = 0.4$, $\phi_y = 0.1$) and the RB is set to -0.3° .

The RB (purple) or the SF (green) can be the outer elements, and QD (red) or SD (green) can be placed next to the central dipole. The four arrangements lead to minor changes of the Twiss functions, variations in the emittance of $\pm 13\%$, and variations in the natural chromaticity ($\xi_{x,y}$) of $\pm 10\%$. What does change strongly, though, is the separation of the β -functions at the location of the sextupoles, indicated by dashed black lines.

The UC largely determines the nonlinear behavior of the lattice; therefore, the sextupole strength necessary to compensate for the natural chromaticity needs to be compared to distinguish between the four cases. Despite the similar chromaticity, the sextupole strength can vary by almost a factor of 2, see Table IV, depending on the separation of the β -functions at the location of the sextupoles.

The best solution is to place SD next to the main bend and SF at the outer symmetry point (case 1). Despite the lower emittance of case 2 and case 4, the increase of almost 70 % in sextupole strength (SD) was considered more harmful.

In Sec. III B 2, the case is discussed where the defocusing gradient is fully incorporated in the main bend and QD is omitted. In this case, only RB and SF can change places, see cases 5 and 6 in Table IV. Due to the gradient in the bend, the vertical β -function has a maximum at the center of the MB and decreases from there, bringing β_x and β_y closer together at SD. Both combined function dipole cases are less favorable (in emittance and sextupole strength) than the separate function dipole case 1.

D. Summary UC

With the investigations and decisions described in the Secs. III A to III C, the UC is completely determined. The rationale behind the choices made to determine both the location and the parameters of the magnets was clearly

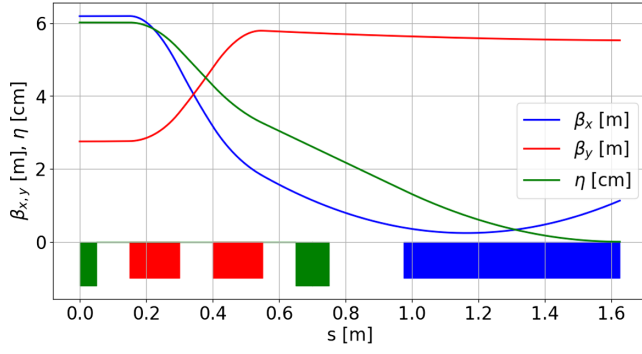


FIG. 12. Picture of β - and η -functions of a generic DSC. Magnets left to right: SF, QF, QD, SD, MB.

articulated. This way, the optimal UC was determined under the boundary conditions and goals of the BESSY III project, although still for generic drifts (0.10 m), quadrupole lengths (0.15 m), and sextupole lengths (0.1 m). After setting up the DSC and the straight section, technical demands and boundary conditions must be implemented.

IV. THE DISPERSION SUPPRESSION CELL

Two dispersion suppression cells (DSC) are located at either end of the four UC blocks. They are needed to suppress the dispersion toward the straight section. Figure 12 shows the downstream DSC, the upstream case being its mirror symmetric counterpart. The basic structure of the DSC is that of half a UC. The higher the similarity, the better the phase advance criterion is satisfied. A slope of the vertical β -function close to zero at the end of the DSC, $\alpha_y^{\text{end}} = 0$, preserves the vertical phase advance. This also facilitates the matching to the center of the straight section. The optimal location of the minimum of β_x lies at $\approx 40\%$ of the DS bend's length from the exit, to seek small emittance. This position should roughly be maintained. Additional focusing or B-field is needed to suppress the dispersion.

Fulfilling all these demands is challenging; to better suppress the dispersion, the length of the outer bend or an offset of either the quadrupoles can be used (B-field), while the two quadrupole fields are adjusted to fit η_x and η_x' to zero. Drift spaces can be modified, if necessary. The DSC is treated as a transfer line for the matching; the optimized UC defines the starting parameters. The bending angle of the DSC is given by the chosen UC angle. To reach 22.5° in the arc with $\theta_{\text{UC}} = 4.25^\circ$, θ_{DSC} has to be 2.75° . The initial structure is generic, as defined in Sec. II.

Figure 13 shows ε , α_c , and β_y^{end} in the generic DSC for a scan of an angle caused by transverse offsets of QF (left) and QD (right) while suppressing η_x and η_x' . A large negative angle in QF would be necessary to maintain the low ε , resulting in $\alpha_c < 0$ (left plot). A moderate positive angle in QD $> 0.2^\circ$ provides the desired ε and α_c values (right plot). Unfortunately, this is accompanied by strong

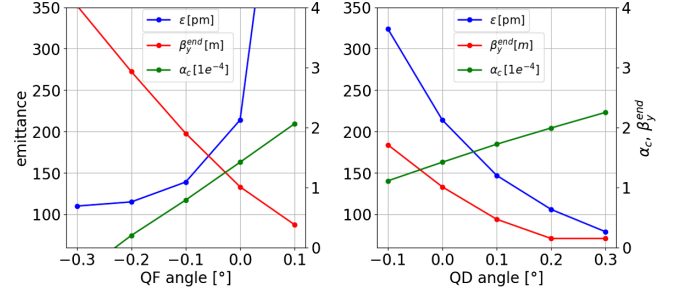


FIG. 13. ε , α_c , and β_y^{end} in the DSC for a scan of the QF (left) and QD angle (right) while suppressing η_x and η_x' .

vertical focusing and, consequently, large vertical chromaticity.

The shorter the DSC, the stronger the necessary gradients (QF and QD) to fit η_x , $\eta_x' = 0$. This leads to a strong vertical focusing. The cure is to lengthen the DSC, either by lengthening the drift next to the bend or the bend itself. In both cases, α_c is almost constant, Fig. 14 (green lines). Enlarging the drift effectively increases β_y^{end} (solid red line), but at the expense of a larger ε (solid blue line). A longer bend, as expected, decreases ε (blue dashed line), though with less effect on β_y^{end} (red dashed line). A combination of both can preserve the ε and α_c while increasing β_y^{end} to the desired value of ≈ 5.5 m.

For BESSY III, QD was shifted to create an additional bend angle of 0.26° , the bend length was increased by 0.15 to 0.65 m, and the drift was lengthened by 0.125 m to 0.225 m. As in the UC, technically necessary adaptations still have to be incorporated.

The question of the distribution of the angle between MB and DSC bend was already raised in Sec. III B 4. As the emittance contribution of the DSC depends not only on the angles but also on the value of the dispersion function at the end of the UC, no simple formula can be given. Again, OPA can help to quickly scan a few cases, as illustrated in Fig. 15.

The dominant contribution to the total emittance (red) comes from the four physical UCs (blue). The UC

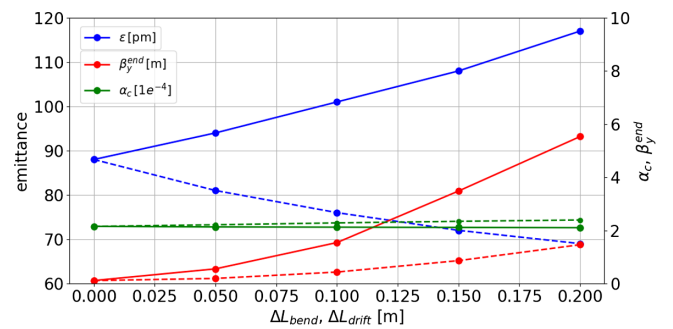


FIG. 14. ε , α_c , and β_y^{end} in the DSC over the length increment of the drift (solid lines) and of the bend (dashed lines) while suppressing η_x and η_x' .

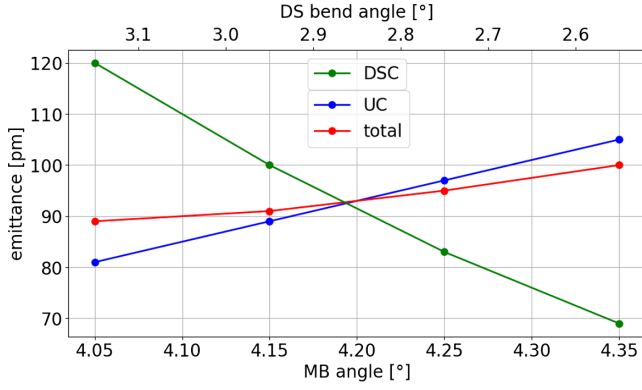


FIG. 15. ε of the UC (blue), the DSC (green), and the complete arc (red) as a function of MB angle and the dependent DS bend angle, keeping the arc's angle constant.

emittance is only slightly modified by the large emittance changes in the DSCs for the complete lattice. Smaller MB angles would cause too short distances between the outer bend and the following sextupole. As the potential gain compared to 4.25° in the MB is less than 5 pm rad of emittance, the chosen angle distribution (Sec. III B 4) is not modified. Table V lists the final angles for the arc.

V. COMPLETE LATTICE AND TECHNICAL ADAPTATIONS

Finally, the matching toward the straight section is needed before the lattice can be assembled using the previously described optimized substructures. The minimum of the β -functions at the center of the 5.6 m magnet-free straight was set to 3 m. It was decided to use four quadrupoles to decouple the insertion device compensation from the arc. Four quadrupoles and two drift lengths were used to reach the desired value of $\beta_{x,y}$ and the desired lattice tune (according to the HOA principle), while keeping symmetry. Finally, when moving the tune away from the integer, minor changes in the minima of $\beta_{x,y}$ have to be accepted. Including the straight section, the lattice can be composed using the optimized subparts.

The next step is the adaptation to technical boundary conditions. Table VI lists the rather conservative limits for

TABLE V. Summary of angles for subcells and magnets in the arc.

Section/magnet	No.	Angle	Total angle
UC	4	4.25°	17°
DSC	2	2.75°	5.5°
Arc			22.5°
Main bend	4	4.77°	
Reverse bend	8	-0.26°	
DS bend	2	2.49°	
Shifted QD	2	0.26°	

TABLE VI. Magnetic field limits for 25 mm bore diameter.

Magnet	Limit
Main bend	1.5 T
Quadrupole	80 T/m
RB-UC	limits depend on the transverse shift
QD-DSC	
Sextupole	4000 T/m ²

the magnetic fields, assuming a round vacuum chamber with a free aperture radius of 9 mm, and a pole tip radius of 12.5 mm. The lower limit on the magnetic length was set to 0.1 m to avoid dominance of fringe fields. Nevertheless, the effect of fringe fields has to be studied in more detail.

The results derived from the generic building blocks did not change significantly with the implementation of the technical limits, and the basic findings persist.

Additional space is required both upstream and downstream of all dipoles for beam radiation absorbers, 0.16 m. To decrease the sextupole strength, the space between QD and RB in the UC was increased to 0.16 m. For the limited circumference, the distance between the sextupoles and quadrupoles was shortened to 0.06 m. Space for an octupole was added in the DSC. Correctors will be integrated into the sextupoles. The drift sections provide space for BPMs. Additional space is needed for two stand-alone fast correctors to guide the beam optimally through the insertion device. With these minor changes, the baseline lattice is defined, Fig. 16. Table VII lists the key parameters.

VI. FIRST TEST ON DA AND MA

The quality of a lattice is reflected in its nonlinear behavior and its robustness toward errors. Although a full discussion of these aspects is beyond the scope of this paper, a few nonlinear features will be presented to corroborate the legitimacy of the choices taken above.

The deterministic construction of the linear lattice aimed at providing a favorable start position for further nonlinear

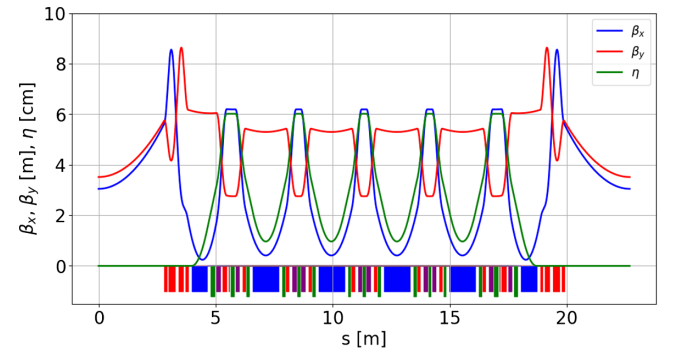


FIG. 16. β -functions (blue, red) and dispersion (green) of the baseline lattice consisting of bends (blue), Qs (red), shifted Qs (purple), and SFs/SDs (green).

TABLE VII. Key lattice parameters.

Parameter	
Circumference	362.88 m
Number of arcs	16
Free space ratio	25%
Emittance	98 pm rad
Momentum compaction factor	1.4×10^{-4}
Natural chromaticity x, y	$-90.5, -34.4$
Damping times x, y, z	9, 21, 31 ms
Energy spread	0.98×10^{-3}
rf voltage	1.6 MV
Bunch length	2.7 mm

optimization. Inevitably, that will lead to deviations from the HOA-principle. The optimization must find a compromise between the excitation of resonances and confining the tune to resonance-free areas. A qualified starting position reduces the effort of the optimization procedure.

During the construction of the linear lattice, only two families of sextupoles were used. By controlling the sextupoles individually in symmetric pairs, the full scope of eight independent sextupole families can be exploited to optimize the DA and MA. Additionally, an octupole was placed in the DSC. The numerical optimization can be carried out using MOGA algorithms.

MOGA calculations as detailed in [17] were performed for the error set in Table VIII. For each run, errors were applied, and the orbit was corrected before optimization; the β -beat was not corrected. The octupole and eight sextupole families were optimized, with zero chromaticity implemented as a constraint. If necessary, the orbit was re-corrected for each new set of multipoles. Full 6D tracking was performed for 1024 turns while physical apertures of ± 9 mm were applied to avoid unphysical results.

With a population of 300 individuals in the design space, each generation produced 100 offsprings, while the 100 worst results were discarded. The optimizations ran for 100 generations. The objectives in the optimization were the limiting momentum acceptance and the area of the DA. The result is displayed in Fig. 17: all solutions provide a large momentum acceptance up to the rf acceptance, indicating long Touschek lifetimes.

In addition, four error seeds (Table VIII) were used to optimize the Touschek lifetime directly using the Fast-

TABLE VIII. Initial standard error set.

Error type	Value
Transverse misalignment (all magnets)	30 μm
Roll (all magnets)	180 μrad
Pitch and yaw—dipoles	60 μrad
Pitch and yaw—other magnets	400 μrad
Calibration	1×10^{-4}

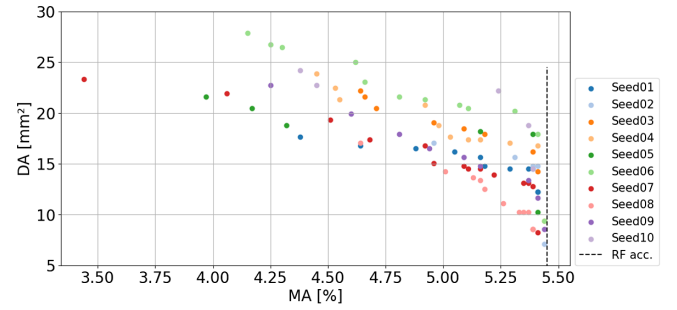


FIG. 17. Pareto fronts for 10 different error seeds, optimizing the area of the DA and the momentum acceptance. The dashed line indicates the rf acceptance.

Touschek-Tracking method [18]. The vertical emittance was assumed to be 10 pm rad, giving an emittance ratio of 10%. In MOGA runs with the Touschek lifetime and the area of the DA as objectives, Touschek lifetimes were determined between 13 and 32 h, depending on the error seed [19]. Landau cavities to lengthen the bunch were not used in either case.

It is almost impossible to compare lifetimes from the literature of different projects, as the energy, circumference, 6D emittance, rf voltage, chromaticity, and others, as well as the errors applied and the tracking parameters used, usually differ in the individual publications. It should therefore only be mentioned that comparable projects such as Diamond2 [20], Soleil2 [21], or SLS2 [22] themselves quote Touschek lifetimes of 2.2, 3, and 7 h, respectively.

To illustrate the stable area in x - y space, Fig. 18 shows the largest DA found on each of the Pareto fronts of the 10 seeds, at the center of the straight section with $\beta_x = 3.05$ m, $\beta_y = 3.52$ m.

The importance of the DA in fourth-generation light sources is solely to enable injection. BESSY III will inject from the inside, so the horizontal acceptance, $A_x = x^2/\beta_x$ for negative x is the relevant number. With a β_x -function of 3.05 m and an average stable amplitude of 3.1 mm, $A_x \approx 3.2$ mm mrad is sufficient for the foreseen multipole kicker injection. The multipole injection kicker will be placed at a

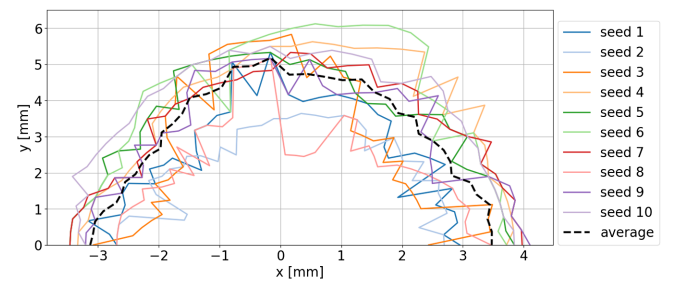


FIG. 18. DAs for 10 different error sets, where the MOGA result with the largest area has been chosen. The black line indicates the average.

position where β_x is large to achieve sufficiently large dynamic aperture.

VII. EXTENSIONS

A. More superperiods

Many storage rings have a higher superperiodicity than BESSY III, e.g., of 20- or 24-SPs, at comparable energy. However, the extension to higher SPs is not straightforward. With increasing superperiodicity, the average MB angle diminishes and the emittance shrinks, but so does α_c and the maximum of the dispersion function, which helps to keep the sextupole strength low.

Initial tests show that to keep $\alpha_c > 2 \times 10^{-4}$ in the generic UC for 24-SPs, only the smallest shifts of the RB can be applied. The emittance reduction expected from the cubic dependence on the average dipole angle (without RB) is more than 70%, ϵ (SP24) = 120 pm rad while ϵ (SP16) = 406 pm rad; however, the additional emittance reduction from the RB is small due to the limited angle, 120 to 80 pm rad, compared to a factor of 4 for 16-SP. The emittance contribution of the DSC is much smaller, though, with 24-SPs and total emittances of 60 to 70 pm rad are achievable with $\alpha_c > 1 \times 10^{-4}$.

The maximum of the dispersion function is reduced by 30% for 24-SPs compared to 16-SPs. This leads to increased sextupole strengths. While the natural chromaticity in a single UC is unchanged (-0.76 and -0.27), the compensating sextupole strengths, $SF = 27.4 \text{ m}^2$ and $SD = -22.1 \text{ m}^{-2}$, increase by 50% and 30%, respectively, requiring highly sophisticated sextupole/octupole compensation schemes.

B. Higher energy

The natural emittance of a lattice is proportional to the square of the Lorentz factor, γ . Going from 2.5 GeV to, e.g., 3 GeV the emittance of the generic UC with $\theta_{MB} = 4.5^\circ$ and $\theta_{RB} = 0$ increases by 44%, ϵ (3 GeV) = 585 pm rad. Both the energy spread and energy loss per turn increase, while the damping times decrease, $\tau_{x,y,z} \propto E^{-3}$. The RB is less effective compared to lower energies. With $\theta_{RB} = -0.3^\circ$, the emittance is 171 pm rad, and α_c slightly smaller than 2×10^{-4} . Further measures, such as longer dipoles, more bends per SP, more SPs, or compromises on the HOA or on $\alpha_c > 1 \times 10^{-4}$, need to be considered to reach a smaller emittance. Using more bending magnets obviously leads to the drawbacks described in the previous section. To cope with the reduced dispersion function, e.g., strong sextupoles, for a larger number of bending magnets, hybrid-MBA lattices with a locally largely increased dispersion function are mostly utilized in machines with energies above 3 GeV and many SPs. The distributed sextupole scheme with phase advance criteria within the UC can no longer be applied. Localized sextupoles at places with increased dispersion are utilized. They must

obey the $-I$ condition on the phase advance to cancel the sextupole-driven resonance driving terms.

VIII. CONCLUSION

In this paper, a new method for developing MBA lattices with distributed sextupoles has been introduced. The approach is suitable for rings with medium to low energy. The separate optimization of substructures enables detailed parameter studies, which help identify the most effective levers to reach the goal parameters, avoiding sophisticated and cost-intensive technical solutions. The decision-making process during the development requires a clear prioritization of the project goals. This process led to a nonstandard lattice, in the sense that homogeneous bending magnets are used rather than combined function bends. The approach to reduce the overall sextupole strength and cancel the resonance driving terms with the linear lattice resulted in a large momentum acceptance, entailing long Touschek lifetimes, and sufficient dynamic acceptance. Modifications due to technical demands were easily incorporated. Initial optimization results of the nonlinear machine performance were presented. Studies adapting the β -functions of one straight to host the injection are underway and indicate stability of lifetime and DA with respect to larger linear symmetry breaks.

This method uses a clear analysis of the parameter interdependence within the substructures of the lattice, as compared to MOGA-optimized results, where valid but somewhat arbitrary solutions are determined.

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