

## Cancelation of coherent synchrotron radiation kicks in chicane bunch compressors

Fancong Zeng,<sup>1</sup> Yi Jiao<sup>1,\*</sup> Weihang Liu<sup>1,2,†</sup> and Cheng-Ying Tsai<sup>3,‡</sup>

<sup>1</sup>*Institute of High Energy Physics, Chinese Academy of Sciences,  
and University of Chinese Academy of Sciences, Beijing 100049, China*

<sup>2</sup>*Spallation Neutron Source Science Center, Dongguan 523803, China*

<sup>3</sup>*School of Electrical and Electronic Engineering, Huazhong University of Science and Technology,  
Wuhan 430074, China*



(Received 2 July 2024; revised 12 November 2024; accepted 20 June 2025; published 22 July 2025)

The most significant advances in accelerator-based light sources (i.e., x-ray free electron lasers) are driven by the production of a kiloampere-level peak current. As a prerequisite to achieve the desired high peak current, multistage symmetric C-chicane compressors are widely utilized due to their simple geometry and effectiveness. However, during the bunch compression process, coherent synchrotron radiation (CSR) effects may lead to transverse emittance dilution, eventually hindering progress toward achieving a higher peak current. Therefore, suppressing or even canceling the CSR effects in chicane compressors is essential. In this paper, we consider a general model of a four-bend chicane and conduct an explicit point-kick analysis of the coherent synchrotron radiation (CSR) effects, based on which we derive the conditions for completely canceling the steady-state CSR (ss-CSR) kicks using analytical formulas. These results yield two novel ss-CSR-immune chicanes that feature nonsymmetric C-shape and S-shape layouts, respectively. We also demonstrate their excellent emittance preservation capabilities with particle tracking simulations. By utilizing the proposed chicanes instead of the widely used C-chicane, the CSR-induced emittance growth can be reduced by about 1 order of magnitude in an almost identical bunch compression scenario.

DOI: [10.1103/2vjn-t4qb](https://doi.org/10.1103/2vjn-t4qb)

### I. INTRODUCTION

The advent of accelerators and accelerator-based light sources has revolutionized advancements in science, industry, medicine, and materials research. In modern accelerators, electron bunch compressors play a pivotal role, with wide applications in linear colliders [1], linacs [2], beam-driven plasma-wakefield accelerators [3,4], and significant roles in x-ray free electron lasers (FELs) [5–12]. Combined with the position-energy correlation provided by the rf cavity, the following dispersive element converts the energy difference into a difference in the time of flight of the particles. This effect causes the particles at the head and tail to become closer, enabling beam compression [13].

Currently, symmetric C-chicanes with four identical bends are the most commonly used bunch compressors in linac-based systems because they are simple, effective,

and dispersion-free at all orders [6–12]. However, due to the high peak current required for FELs, the ability to compress an electron bunch with minimal degradation of beam quality becomes challenging because of the effects of coherent synchrotron radiation (CSR). CSR is emitted for wavelengths longer than or comparable to the length of the electron bunch, and leads to detrimental tail-head interactions in the bends [14]. The shorter and more intense the bunch is, the stronger the CSR fields produced and the more severe the CSR effects become. The emission of CSR results in projected transverse emittance growth [15–18], especially with a peak current of typically 3–5 kA or higher in the final-stage compressor [5–12].

In the past decades, various efforts have been stimulated to suppress CSR-induced emittance growth in chicane compressors, including analytical, numerical, and experimental studies [19–43]. One study exploited the correlation between CSR and the longitudinal distribution of the beam, revealing novel results through a mitigation approach based on longitudinal beam shaping [19,20]. By adding pairs of quadrupoles, skew quadrupoles, and sextupoles into the chicane, the idea of dispersion-based beam-tilt correction was proposed [21] and subsequently experimentally verified [22]. Furthermore, the approach of suppressing the deleterious effects of CSR by manipulating beam optics has sparked continuing research interest.

\*Contact author: [jiaoyi@ihep.ac.cn](mailto:jiaoyi@ihep.ac.cn)

†Contact author: [liuwh@ihep.ac.cn](mailto:liuwh@ihep.ac.cn)

‡Contact author: [jcytsai@hust.edu.cn](mailto:jcytsai@hust.edu.cn)

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The optical balance method was first proposed by Douglas [23] and was further developed by Courant-Snyder (C-S) formalism analysis [24]. Subsequently, an approach was introduced to mitigate emittance dilution in a single achromatic cell by matching the beam envelope to the CSR-induced dispersion using  $R$ -matrix analysis [25,26]. Following the C-S formalism and  $R$ -matrix analyses, an investigation demonstrated that these two analytical methods are equivalent, and indicated that the cumulative CSR effects in a bend can be simplified to a point kick at the center of the bend [27]. The point-kick model works well in transport systems where the bunch length remains constant or changes very little (hereafter referred to as the constant- $\sigma_z$  point-kick model) [28,29]. However, for magnetic bunch compressors, this model lacks self-consistency due to significant variations in bunch length within the bends. Fortunately, a recent study introduced a modified CSR-kick model that accounts for bunch length variations within bends, providing a direct and explicit methodology for determining the CSR cancellation conditions for DBA-based compressors [30]. Similar to the previously proposed and used constant- $\sigma_z$  point-kick model [27–29,39], the point kick still occurs at the center of the bend. However, the expression for the point kick is modified. Inspired by the successful application of this modified CSR-kick model, in this study we will envisage the potential designs for a CSR-immune chicane using the same model.

Existing studies have provided extensive guidelines for chicane design. It is acknowledged that symmetric C-chicanes cannot effectively cancel CSR kicks, which are the primary limiting factor in achieving higher peak currents [31]. To achieve CSR kick suppression or cancellation, several studies have attempted to break the traditional symmetric C-shape geometry and have explored various nonsymmetric chicane layouts with four or more bends [31–37]. For example, one study proposed a nonsymmetric C-chicane based on parameter scans [33,34] and demonstrated its emittance suppression capabilities through numerical simulations. In Sec. III, we develop a solvable model of the CSR effects in a general C-chicane, based on which we find that, in addition to the nonsymmetric geometry, a focusing section (whose transfer matrix is a “negative drift”) between the second and third bends is required to cancel the CSR kicks. With this solvable model, the design and adjustment of such a CSR-immune chicane can be accomplished based on analytical formulas.

In the geometric design of a CSR-immune chicane, the latest advances have successfully applied the constant- $\sigma_z$  point-kick model to chicane compressors [44], and have proposed a four-bend CSR-immune chicane featuring a nonsymmetric S-shape geometry and following parameter scaling. The particle tracking simulations also show excellent emittance preservation capability of such chicane, especially highlighting the robustness in terms of chicane parameters and compression factor. However, using the

constant- $\sigma_z$  model to analyze accumulated CSR kicks, while simple and general, is not self-consistent in systems with significant bunch length variations within bends. This inconsistency limits the accuracy of the derived chicane parameters, resulting in only partial cancelation of CSR kicks rather than full suppression. To this end, in Sec. IV, we employ a modified CSR-kick analysis [30] to develop an accurate and self-consistent solvable model for CSR kick cancelation, while also evaluating the errors in the calculation of chicane parameters compared to the previous results in [44]. To highlight the distinction, we refer to the previous study [44] as a partial-CSR-immune chicane that achieves partial CSR kick cancelation, whereas this study realizes full steady-state CSR cancelation, thus defining it as an ss-CSR-immune chicane.

For the sake of simplicity, we adopt four assumptions. For a review, we refer to the assumptions outlined in the theoretical analysis in [40]. First, we assume that the electron bunch follows a Gaussian distribution. Second, we assume a linear compression process during the electron bunch passing through the rf cavity and the chicane. Third, our analysis of the one-dimensional CSR physical model is restricted to the steady-state regime in free space, without beam pipe shielding, and currently excludes additional effects such as transient CSR and space charge effects. Finally, we ignore the influence of the conducting walls of the vacuum chamber.

## II. FORMALISM

### A. Chicane optics

First, an overview of the chicane linear optics is given. We consider a general chicane model consisting of four bends separated by three adjustable drifts. The length of each bend is  $L_B$ , while the bending angles of the four bends  $\theta_i$  ( $i = 1, 2, 3, 4$ ) are not subject to a specific constraint. The total transfer matrix of such a chicane can be written as

$$M_{\text{tot}} = R_{B4}R_{D3}R_{B3}R_{D2}R_{B2}R_{D1}R_{B1}, \quad (1)$$

where  $R_{B_i}$  ( $i = 1, 2, 3, 4$ ) and  $R_{D_j}$  ( $j = 1, 2, 3$ ) represent the transfer matrices of the four bends and three drifts, respectively. The  $4 \times 4$  transfer matrix of a small angle thin bend  $R_{B_i}$  in the horizontal and longitudinal planes ( $x, x', z, \delta$ ) can be expressed by

$$R_{B_i} = \begin{bmatrix} 1 & L_B & 0 & \frac{L_B\theta_i}{2} \\ 0 & 1 & 0 & \theta_i \\ \theta_i & \frac{L_B\theta_i}{2} & 1 & \frac{L_B\theta_i^2}{6} \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (2)$$

The transfer matrix of a half bend can be obtained by replacing  $(\theta_i, L_B)$  with  $(\theta_i/2, L_B/2)$ . We write the transfer matrix  $R_{D_i}$  of the drift as

$$R_{Di} = \begin{bmatrix} 1 & L_{Di} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (3)$$

where  $L_{Di}$  ( $i = 1, 2, 3$ ) are the drift lengths.

With the total transfer matrix in Eq. (1), the achromatic condition,  $(M_{\text{tot}})_{16} = 0$ ,  $(M_{\text{tot}})_{26} = 0$ , and the first-order longitudinal dispersion  $R_{56}^{s_0 \rightarrow s_f} = (M_{\text{tot}})_{56}$  for the chicane can be calculated as

$$\begin{aligned} (M_{\text{tot}})_{16} &= L_{D1}\theta_1 + L_{D2}(\theta_1 + \theta_2) + L_{D3}(\theta_1 + \theta_2 + \theta_3) \\ &\quad + L_B(3\theta_1 + 2\theta_2 + \theta_3) = 0, \\ (M_{\text{tot}})_{26} &= \theta_1 + \theta_2 + \theta_3 + \theta_4 = 0, \\ R_{56}^{s_0 \rightarrow s_f} &= L_{D1}\theta_1\theta_2 + L_{D3}\theta_3\theta_4 + \frac{L_B}{3}(\theta_1^2 + 4\theta_1\theta_2 + \theta_2^2 + 2\theta_3\theta_4), \end{aligned} \quad (4)$$

where the chicane entrance and exit are denoted by subscripts “ $s_0$ ” and “ $s_f$ ,” respectively. The first two terms in Eq. (4) serve a dual purpose. As a key feature of a lattice consisting only of bends and drifts, the achromatic condition coincides with the beam coaxial condition (also referred to as the “geometric horizontal condition”), which ensures that the horizontal trajectory of the beam remains coaxial before and after passing through the chicane. Consequently, the beam coaxial condition can be achieved naturally.

Dispersion cancellation is essential in chicane design to prevent the dispersion-induced emittance growth caused by momentum dispersive effect. In the following sections, we will focus on the steady-state CSR cancellation conditions for a chicane bunch compressor.

## B. Application of modified CSR-kick model to a four-bend chicane

In this subsection, we derive the analytical expressions of net CSR kick at the chicane exit and further the ss-CSR cancellation conditions using a modified CSR-kick model [30]. In such a modified model, the CSR-induced particle coordinate deviations after passing a bend can be equivalently formulated with a point kick at the center of the bend, while the bunch length variation process can also be involved. After experiencing each CSR kick, the particle will have a horizontal coordinate deviation  $(\Delta x_{ki}, \Delta x'_{ki})^T$  and an additional energy deviation  $\Delta \delta_{ki}$  [30] (under the small bending-angle approximation),

$$\Delta X_{ki} = \begin{bmatrix} \Delta x_{ki} \\ \Delta x'_{ki} \end{bmatrix} = \begin{bmatrix} (2g_{xi} - 3g_{xpi})k_i\theta_i^3\rho_i^{4/3}/12 \\ \theta_i(2\delta_i + g_{xpi}k_i\theta_i\rho_i^{1/3})/2 \end{bmatrix}, \quad (5)$$

and

$$\Delta \delta_{ki} = g_{\delta i}k_i\theta_i\rho_i^{1/3}, \quad (6)$$

where  $\rho_i$  ( $i = 1, 2, 3, 4$ ) is the bending radius,  $\delta_i$  is the energy deviations before experiencing the  $i$ th CSR kick, denoted as

$$\begin{aligned} \delta_1 &= \delta_0, & \delta_2 &= \Delta \delta_{k1}, & \delta_3 &= \sum_{i=1}^2 \Delta \delta_{ki}, \\ \delta_4 &= \sum_{i=1}^3 \Delta \delta_{ki}. \end{aligned} \quad (7)$$

Here  $\delta_0$  is initial energy deviation. The coefficient  $k_i$  in Eqs. (5) and (6) for a Gaussian bunch is of the form

$$k_i = 0.2459 \frac{N_b r_e}{\gamma \sigma_{zi}^{4/3}}, \quad (8)$$

where  $N_b$  is the electron population,  $r_e$  is the classical electron radius,  $\gamma$  is the relativistic Lorentz factor, and  $\sigma_{zi}$  is the rms bunch length at the  $i$ th bend entrance. The correction functions  $(g_{xi}, g_{xpi}, g_{\delta i})$  ( $i = 1, 2, 3, 4$ ) are dimensionless and typically have values close to 1. These functions are related to the compression factor  $C_i$  for the  $i$ th bend and the dispersion function  $(\eta_i, \eta'_i)$  at the entrance of  $i$ th bend [30],

$$\begin{aligned} g_x([a, b], \xi) &= 3 \int_0^1 \left[ 1 + \xi \frac{a\tau + b\tau^2 + \tau^3}{a + b + 1} \right]^{-4/3} (1 - \tau)^2 d\tau, \\ g_{xp}([a, b], \xi) &= 2 \int_0^1 \left[ 1 + \xi \frac{a\tau + b\tau^2 + \tau^3}{a + b + 1} \right]^{-4/3} (1 - \tau) d\tau, \\ g_\delta([a, b], \xi) &= \int_0^1 \left[ 1 + \xi \frac{a\tau + b\tau^2 + \tau^3}{a + b + 1} \right]^{-4/3} d\tau, \end{aligned} \quad (9)$$

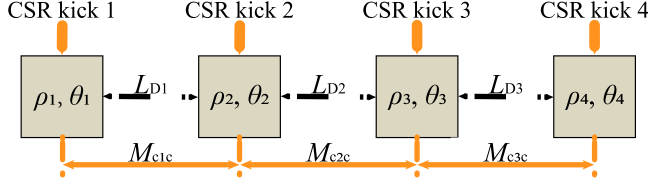


FIG. 1. Schematic of the point-kick analysis of the CSR effects. The arrows point to the centers of the bends.  $M_{c1c}$ ,  $M_{c2c}$ , and  $M_{c3c}$  represent the transfer matrices from the center of first bend to second bend, the center of second bend to third bend, and the center of third bend to fourth bend, respectively.

where the variable  $\xi \equiv \sigma_z(\phi)/\sigma_{z0} - 1$  represents the rate of bunch length variation after passing through the bend, the parameters  $a$  and  $b$  are related to  $\eta_0$  and  $\eta'_0$ , respectively, and can be expressed as functions of  $(a, b, \xi)$  [30],

$$a_i = 6 \frac{\eta_i}{L_B \theta_i^2}, \quad b_i = 3 \frac{\eta'_i}{\theta_i},$$

$$\xi = \frac{\sigma_{zi,f}}{\sigma_{zi,0}} - 1 = \frac{1}{C_i} - 1 = \frac{L_B \theta_i^2}{6} h_i (a_i + b_i + 1). \quad (10)$$

Here  $\sigma_{zi,0}$ ,  $\sigma_{zi,f}$  are the bunch lengths at the entrance and exit of the  $i$ th bend, respectively;  $C_i$  is the compression factor for  $i$ th bend.  $h_i$  is the energy chirp at the bend entrance.

The lattice sections between the center of two adjacent bends, which contain a drift and two half bends, are generally analyzed with three transfer matrices  $M_{cic,2 \times 2}$  ( $i = 1, 2, 3$ ). From the  $2 \times 2$  transfer matrix of the horizontal betatron motion in Eqs. (2) and (3), the transfer matrix  $M_{c1c,2 \times 2}$  between the center of the first two bends is given by

$$M_{c1c,2 \times 2} = (R_{HB2} R_{D1} R_{HB1})_{2 \times 2} = \begin{bmatrix} 1 & L_{D1} + L_B \\ 0 & 1 \end{bmatrix}, \quad (11)$$

where  $R_{HB1}$  and  $R_{HB2}$  are the transfer matrices of the first and second half bends, respectively. Note that Eq. (11) is a universal expression for (half bend) + drift + (half bend) structure. One can obtain the matrices  $M_{c2c,2 \times 2}$  and  $M_{c3c,2 \times 2}$  by substituting  $L_{D1}$  in Eq. (11) with the counterparts  $L_{D2}$  and  $L_{D3}$ , respectively.

To simplify the analysis, we assume a zero initial energy deviation of  $\delta_0 = 0$  and zero initial particle coordinates of  $\Delta X_0 = (\Delta x_0, \Delta x'_0)^T = (0, 0)^T$  at the chicane entrance. After passing through the first CSR kick and the section

from the center of first bend to the center of second bend, the particle experiences the second CSR kick, resulting in particle coordinate deviations  $\Delta X_{s2}$  can be written as

$$\Delta X_{s2} = M_{c1c,2 \times 2} \Delta X_{k1} + \Delta X_{k2}. \quad (12)$$

The idea is illustrated in Fig. 1. Similarly, the particle coordinate deviations after passing through the fourth CSR kick can be written as

$$\begin{aligned} \Delta X_{s4} &= M_{c3c,2 \times 2} \Delta X_{s3} + \Delta X_{k4} \\ &= M_{c3c,2 \times 2} (M_{c2c,2 \times 2} \Delta X_{s2} + \Delta X_{k3}) + \Delta X_{k4} \\ &= M_{c3c,2 \times 2} M_{c2c,2 \times 2} \Delta X_{s2} + M_{c3c,2 \times 2} \Delta X_{k3} + \Delta X_{k4}, \end{aligned} \quad (13)$$

where  $\Delta X_{s3}$  and  $\Delta X_{s4}$  denote the coordinate deviations after passing the center of the third bend and fourth bend, respectively. Finally, the particle coordinate deviations  $\Delta X_f$  at the chicane exit are

$$\Delta X_f = (R_{HB4})_{2 \times 2} \Delta X_{s4} = \begin{bmatrix} \Delta X_{s4}(1, 1) + \frac{L_B}{2} \Delta X_{s4}(2, 1) \\ \Delta X_{s4}(2, 1) \end{bmatrix}, \quad (14)$$

where  $\Delta X_{s4}(1, 1)$ ,  $\Delta X_{s4}(2, 1)$  are the elements of  $\Delta X_{s4}$  in Eq. (13).

Observed at the exit of the chicane the projected transverse emittance growth reads

$$\varepsilon_f^2 = \varepsilon_0^2 + \varepsilon_0 (\beta_x \langle \Delta x_f'^2 \rangle + 2\alpha_x \langle \Delta x_f \Delta x_f' \rangle + \gamma_x \langle \Delta x_f^2 \rangle) + \Delta \varepsilon_0^2, \quad (15)$$

where  $\varepsilon_0$  is the initial projected emittance,  $\Delta x_f$ ,  $\Delta x_f'$  are the elements of  $\Delta X_{s4}$ . Here  $\Delta \varepsilon_0^2 = \langle \Delta x_f^2 \rangle \langle \Delta x_f'^2 \rangle - \langle \Delta x_f \Delta x_f' \rangle^2 = 0$ . Since the particle coordinate deviations  $\Delta x_f$  and  $\Delta x_f'$  from CSR field are correlated, this term goes to 0 [45]. Here  $\varepsilon_0$  is the unperturbed geometric emittance at the entrance  $s_0$ , and  $\alpha_x, \beta_x, \gamma_x$  are the C-S functions at the chicane exit. Equation (15) shows that, in principle, the CSR-induced emittance growth at the chicane exit can be canceled when the particle coordinate deviations satisfy  $\Delta X_f = (0, 0)^T$ , which can also be written as  $\Delta X_{s4} = (0, 0)^T$  according to Eq. (14).

Finally, we derive the analytical net CSR kick for such a general chicane and the corresponding CSR cancelation conditions [see Eq. (16)].

$$\begin{aligned} 0 &= k_1 \theta_1 \rho_1^{1/3} \{ 3g_{xp1} \theta_1 (2L_{D1} + 2L_{D2} + 2L_{D3} + 11L_B) + 2g_{x1} \theta_1 L_B + 12g_{\delta1} [\theta_2 (4L_B + L_{D2} + L_{D3}) + \theta_3 (2L_B + L_{D3})] \} \\ &\quad + k_2 \theta_2 \rho_2^{1/3} [3g_{xp2} \theta_2 (2L_{D2} + 2L_{D3} + 7L_B) + 2\theta_2 g_{x2} L_B + 12g_{\delta2} \theta_3 (2L_B + L_{D3})] \\ &\quad + k_3 \theta_3 \rho_3^{1/3} [6g_{xp3} L_{D3} + (9g_{xp3} + 2g_{x3}) L_B] + k_4 L_B \theta_4^2 \rho_4^{1/3} (2g_{x4} - 3g_{xp4}), \\ 0 &= k_1 \theta_1^2 \rho_1^{1/3} (g_{xp1} - 2g_{\delta1}) + k_2 \theta_2 \rho_2^{1/3} (g_{xp2} \theta_2 + 2g_{\delta2} \theta_3 + 2g_{\delta2} \theta_4) + k_3 \theta_3 \rho_3^{1/3} (g_{xp3} \theta_3 + 2g_{\delta3} \theta_4) + k_4 g_{xp4} \theta_4^2 \rho_4^{1/3}. \end{aligned} \quad (16)$$

The calculated ss-CSR cancellation conditions in Eq. (16) can be used to analyze the factors affecting ss-CSR cancellation. In addition to being dependent on the chicane parameters (chicane geometry) and the compression factor (also mentioned in [44]), Eq. (16) clearly indicates that ss-CSR cancellation in a chicane compressor is also related to the ratio of the bend length, as well as to the dispersion functions at the entrance of each bend (by affecting the correction functions  $g_{xi}, g_{xpi}, g_{\delta i}$ ). In other words, the ss-CSR cancellation conditions derived in Eq. (16) can be reduced to the results reported in Ref. [44] under two approximations. Specifically, one can neglect the lengths of the bends, as they are usually relatively short compared to the drift length in a chicane. Furthermore, one can assume that the bunch length  $\sigma_{zi}$  in Eq. (5) and the corresponding  $k_i$  in Eq. (8) are constant for each CSR kick. The variations in bunch length during each bend can be neglected by setting the correction functions  $(g_{xi}, g_{xpi}, g_{\delta i})$  ( $i = 1, 2, 3, 4$ ) to 1.

In total, it can be observed that there are *seven* variables in a general chicane:  $q_2 = \theta_2/\theta_1, q_3 = \theta_3/\theta_1, q_4 = \theta_4/\theta_1, \ell_2 = L_{D2}/L_{D1}, \ell_3 = L_{D3}/L_{D1}, \ell_B = L_B/L_{D1}$ , and the compression factor  $C$ . Based on these parameters, it is sufficient to derive the expressions for  $k_i/k_1$  ( $i = 2, 3, 4$ ) and the correction functions  $g_{xi}, g_{xpi}, g_{\delta i}$  in Eq. (16), thereby allowing the ss-CSR cancellation conditions to be solved analytically. Additionally, we need not care much about the values of  $\theta_1$  and  $L_{D1}$  here, because the chicane design goals, i.e., the total length  $L_{\text{tot}}$  and longitudinal dispersion  $R_{56}^{s_0 \rightarrow s_f}$  in Eq. (4), impose constraints on these *two* variables. The specific chicane layout can be achieved by adjusting the ratios of other bending angles and drift lengths relative to  $\theta_1$  and  $L_{D1}$ , respectively. Next, we focus on solving for the parameters  $q_2, q_3, q_4, \ell_2, \ell_3, \ell_B, C$ . Here the quantities  $q_4, \ell_3$  can be obtained from the achromatic condition in Eq. (4). Note that there are *five* remaining variables:  $q_2, q_3, \ell_2, \ell_B, C$ , whereas only *two* requirements are specified in ss-CSR cancellation conditions [Eq. (16)]. To obtain the parameter scaling, it is necessary to reduce the *five* variables to *two*. For chicane design, one can fix the

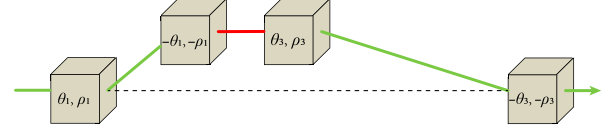


FIG. 2. General geometry of a nonsymmetric C-chicane. The space between the second and third bends is parallel to the entrance or exit of the beam, which is a “negative drift” constructed by several quadrupoles.

compression factor  $C$  and the bend length  $\ell_B$ . For the last degree of freedom, there are many options, and two typical cases are introduced in Secs. III and IV, respectively.

### III. APPLICATION TO FOUR-BEND C-CHICANE

Currently, the symmetric C-chicane is the most commonly used bunch compressor in operation or under construction [5–11,46,47]. Therefore, we aim to discuss the implementation of a chicane based on a symmetric C-chicane as much as possible. At this point, we set the bending angles of the first and second bends to be identical except for their bending direction (denoted as  $\theta_1 = -\theta_2$  or  $q_2 = -1$ ). The schematic layout is shown in Fig. 2. This condition  $q_2 = -1$  preserves the parallelism of the beam orbits between the second and third bends, indicating a similarity with the symmetric C-chicane. Additionally, the condition  $q_2 = -1$  leads to  $\theta_3 = -\theta_4$ . This greatly simplifies the structure of the chicane, reducing the four different bends to only two types. Here, we aim to minimize the number of bend types, thereby reducing both design and production costs. Moreover, the condition  $q_2 = -1$  greatly simplifies the achromatic condition and ss-CSR cancellation conditions. The achromatic condition in Eq. (4) can be rewritten as

$$\ell_3 = -[1 + \ell_B(1 + q_3)]/q_3, \quad q_4 = -q_3. \quad (17)$$

The ss-CSR cancellation conditions in Eq. (16) can be reduced to

$$\begin{aligned} & k_1[2g_{x1}\ell_B + g_{xp1}(33\ell_B + 6 + 6\ell_3 + 6\ell_2) - 12g_{\delta1}(4\ell_B + \ell_3 + \ell_2 - 2\ell_B q_3 - \ell_3 q_3)] \\ & - k_2[2g_{x2}\ell_B + 3g_{xp2}(7\ell_B + 2\ell_2 + 2\ell_3) - 12g_{\delta2}q_3(2\ell_B + \ell_3)] \\ & - k_3q_3^{5/3}[2g_{x3}\ell_B + g_{xp3}(9\ell_B + 6\ell_3)] + k_4q_3^{5/3}\ell_B(2g_{x4} - 3g_{xp4}) = 0, \\ & -\frac{1}{q_3^{5/3}} = \frac{(g_{xp3} - 2g_{\delta3})k_3 - g_{xp4}k_4}{(g_{xp1} - 2g_{\delta1})k_1 - g_{xp2}k_2}. \end{aligned} \quad (18)$$

In order to extract information from Eq. (18), we apply the two mentioned assumptions, i.e., ignoring the bend lengths and the bunch length variation process in each bend. Consequently, the cancellation conditions in Eq. (18) can be simplified to

$$-\frac{1}{q_3^{5/3}} \approx \frac{k_3 + k_4}{k_1 + k_2}, \quad \ell_2 \approx -1 + \frac{1}{q_3} - \frac{k_2 + q_3^{2/3}k_3}{k_1 + k_2}. \quad (19)$$

Note that the second member of Eq. (19) turns out that  $\ell_2 = L_{D2}/L_{D1}$  is negative due to the negative  $q_3$  and the positive  $k_i$  ( $i = 1, 2, 3$ ).

Despite the presence of this “negative drift,” we emphasize that the dispersion and the beam trajectory remain unaffected, as the achromatic condition in Eq. (4) shows the independence of  $L_{D2}$  under the condition  $q_2 = -1$ . Indeed, this “negative drift” is not uncommon in accelerator physics and can be realized by using a focus section with several quadrupoles and drifts [48]. For clarity, in this section we denote  $L_{D2}^{\text{real}}(\ell_2^{\text{real}})$  to represent the actual length of the physical space, and  $L_{D2}^{\text{eff}}(\ell_2^{\text{eff}})$  for a focus section whose transfer matrix is the same as that of a “negative drift.” The actual achievement of this “negative drift” in a physical space  $L_{D2}^{\text{real}}$  will be discussed in detail in Sec. III A.

### A. Parameter calculation of the ss-CSR-immune chicane

Before calculating Eqs. (4) and (18), we need set the ratio between bend length and chicane total length. For the purpose of calculation, the chicane total length excludes the drift between the second and third bends (denoted as “reduced total length”) as this has no effect on the solution. Finally, we set the ratio between the bend length and the reduced total length as  $4L_B : (L_{\text{tot}} - L_{D2}^{\text{real}}) = 4\ell_B : (4\ell_B + 1 + \ell_3) \equiv \Lambda$ . This parameter  $\Lambda$  is usually much smaller for chicane (usually at level of 0.1). Under the condition  $q_2 = -1$  and the achromatic conditions in Eq. (4), we can find that the previously mentioned *seven* variables become *four* variables (i.e.,  $q_3, \ell_3, \ell_2^{\text{eff}}, \Lambda$ , and  $C$ ), and the remaining can be written as functions of these variables as

$$\begin{aligned} q_2 &= -1, & q_4 &= -q_3, & \ell_3 &= \frac{-4 + 3\Lambda - q_3\Lambda}{4q_3 + \Lambda - 3q_3\Lambda}, \\ \ell_B &= \frac{-\Lambda + q_3\Lambda}{4q_3 + \Lambda - 3q_3\Lambda}. \end{aligned} \quad (20)$$

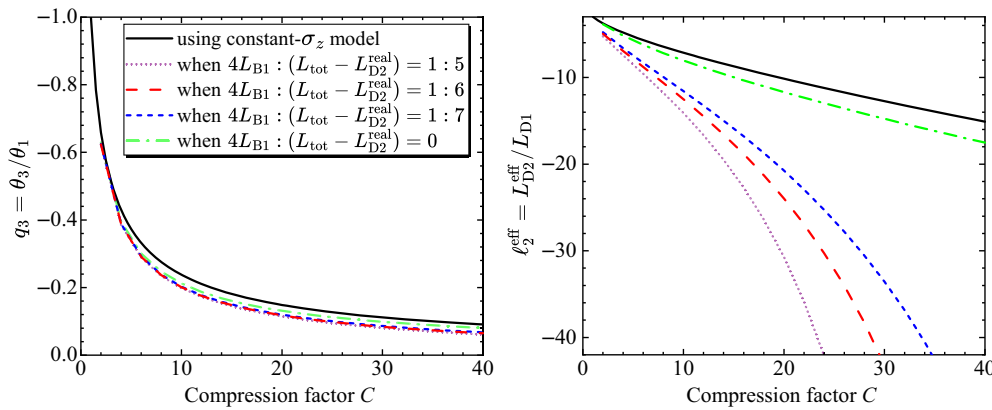


FIG. 3. Required bending angles ratio  $q_3 = \theta_3/\theta_1$  (left) and ratio  $\ell_2^{\text{eff}} = L_{D2}^{\text{eff}}/L_{D1}$  (right) vary with the compression factor  $C$  by solving the ss-CSR cancelation conditions in Eq. (18) under the set condition  $\Lambda = 1:6$ . The results for other values of  $\Lambda = 1:5$ ,  $\Lambda = 1:7$ , and  $\Lambda = 0$  are also included for comparison. Additionally, the results obtained using constant- $\sigma_z$  model are plotted.

Next, we calculate the correction functions ( $g_{xi}, g_{xpi}, g_{\delta i}$ ) ( $i = 1, 2, 3, 4$ ) in Eq. (18). For two special bends, namely the first and fourth bends for a multibend achromat, the corresponding correction functions ( $g_{xi}, g_{xpi}, g_{\delta i}$ ) ( $i = 1, 4$ ) are independent of the chicane parameters while only relying on compression factor  $C_i$  for  $i$ th bend [30]. For the second bend (or the third bend), the corresponding correction functions can be expressed as functions of  $a_2$  and  $\xi_2$  (or  $a_3$  and  $\xi_3$ ) with the parameters  $b_2 = -3$  (or  $b_3 = 0$ ). The detailed expressions of these correction functions are presented in Appendix A.

Based on the above analysis, we can derive an explicit relationship between the chicane parameters and the compression factor. Analytical results of  $(q_3, \ell_2^{\text{eff}})$  vs  $C$  can be obtained for a given value of  $\Lambda$ . Correspondingly, the quantities  $(q_4, \ell_3)$  can also be determined using Eq. (20). Figure 3 presents the chicane parameters  $(q_3, \ell_2^{\text{eff}})$  as functions of  $C$  under the condition  $\Lambda = 1:6$ . These results can be considered generic conditions for ss-CSR-cancelation in a symmetric C-chicane, which impose two design requirements. The first term of Eq. (19) (shown in Fig. 3, left) indicates the requirements for the bending angles, while the second term (Fig. 3, right) relates to the requirements for the focusing section between the second and third bends. In summary, we report for the first time a novel ss-CSR-immune chicane featuring a nonsymmetric C-shape geometry and having a focus section between the second and third bends, along with a specific design criterion for such a chicane (see Fig. 3).

It is worth mentioning that shortening the bend lengths of the last two bends is essential for achieving a ss-CSR-immune chicane (see Fig. 3, left), which is consistent with the conclusions of previous studies (e.g., Refs. [33,34]). This point can be understood qualitatively. Since the first two bends (and the last two bends) have the same bending angles, they can be considered as a single unit, with the two units in a chicane generating opposite CSR kicks [see Eq. (5) for details]. During the compression process, the

bunch length is shorter in the last two bends, resulting in more evident CSR effects and larger CSR kicks compared to the first two bends. To achieve  $\Delta x'_{k,f} = 0$  at the chicane exit, it is essential to generate equivalent CSR kicks in both units, which can be accomplished by reducing the bending angle of the second unit (or increasing the bending angle of the first unit) to an appropriate value (see Fig. 3, left).

Note that the second member of Eq. (18) is a function of  $(q_3, C, \Lambda)$  and is independent of  $\ell_2^{\text{eff}}$ . This indicates that the cancelation of the coordinate deviation  $\Delta x'_{k,f}$  relies only on the chicane parameters (geometry) and is independent of the focus section between the second and third bends. Note that the nonsymmetric C-shape geometries have been proposed previously [33,34]. According to our analysis, these studies can at most cancel the coordinate deviation  $\Delta x'_{k,f}$ , and not completely cancel the CSR kicks.

We also evaluate the impact of two assumptions on the obtained ss-CSR cancelation, i.e., ignoring the bend lengths and the bunch length variation process in each bend. The simplified CSR cancelation conditions are expressed in Eq. (19). In this equation, we should select a constant and lumped bunch length for each bend [i.e., fixed  $\sigma_{zi}$  ( $i = 1, 2, 3, 4$ ) and the corresponding  $k_i$  in Eq. (8) in  $i$ th bend]. Specifically, for the second (third) CSR kick, we use the bunch length at the center of the second (third) bend; and for the first and the fourth kicks, we use the bunch lengths at the entrance and exit of the bends, respectively, i.e.,  $\sigma_{z1} = \sigma_{z0}$  and  $\sigma_{z4} = \sigma_{zf}$ . The detailed expression of the applied bunch length for each bends as functions of chicane parameters are presented in Appendix B. The calculation chicane parameters are plotted in Fig. 3. For these simplified CSR cancelation conditions in Eq. (19), the solved  $q_3$  ( $\ell_2^{\text{eff}}$ ) is equal to  $-1$  ( $-3$ ) for no compression case, i.e.,  $C = 1$ , and decreases as the compression factor  $C$

increases. It can be observed that, although relatively simple expressions have been obtained, there is a significant error in the simplified results, particularly concerning the error in  $\ell_2$  and in the cases with large compression factors  $C$  (see Fig. 3). For this error, next we try to provide a qualitative assessment based on the achromatic condition and the ss-CSR cancelation conditions in Eqs. (17) and (18). Note that the compression process primarily occurs in the second and third bends of the chicane (see Appendix B). Therefore, the correction functions  $g_{x1}, g_{xp1}, g_{\delta1}$  and  $g_{x4}, g_{xp4}, g_{\delta4}$  can be assumed to be 1 since  $\xi_1$  and  $\xi_4$  are very close to 0. From this analysis, it is evident that the second member of Eq. (18) is slightly affected, which corresponds to the essentially unchanged  $q_3$  shown in Fig. 3. In contrast, the first member of Eq. (18) is clearly influenced by the bend length term  $\ell_B$  and correction functions  $g_{xi}, g_{xpi}, g_{\delta i}$ , ultimately resulting in a clear modified  $\ell_2^{\text{eff}}$  (see Fig. 3, right).

To investigate the dominant factor to the mentioned error, we also plot the chicane parameters  $(q_3, \ell_2^{\text{eff}})$  for  $\Lambda = 1:5$  and  $\Lambda = 1:7$ , and  $\Lambda = 0$  (see Fig. 3). Here the case of  $\Lambda = 0$  signifies a neglected bend length, which is used to assess the impact of the bunch length variation process in each bend. For the error of  $q_3$  at different values of  $C$ , one can find that the simplified results show little variation compared to the accurate results, while there is a significant increase in the error of  $\ell_2^{\text{eff}}$  as  $C$  increases. This indicates the necessity for accurate expressions for ss-CSR cancelation, and the mentioned two assumptions cannot be ignored, particularly in cases with large compression factor and in the precise calculation of  $\ell_2^{\text{eff}}$ .

Finally, the chicane design goal,  $R_{56}^{s_0 \rightarrow s_f}$  and  $L_{\text{tot}}$ , can be obtained as

$$\begin{aligned} L_{\text{tot}} &= L_{D1} \left( 1 - \frac{1}{q_3} \right) + 4L_B + L_{D2}^{\text{real}} = 4L_B/\Lambda + L_{D2}^{\text{real}}, \\ R_{56}^{s_0 \rightarrow s_f} &= -L_{D1} \theta_1^2 (1 - q_3) - \frac{\theta_1^2 L_B}{3} (2 - 3q_3 - q_3^2) \\ &= -L_{D1} \theta_1^2 \left( 1 - q_3 + \frac{1}{3} (2 - 3q_3 - q_3^2) \frac{-\Lambda + q_3 \Lambda}{4q_3 + \Lambda - 3q_3 \Lambda} \right), \end{aligned} \quad (21)$$

which indicates that for a given design goal and a larger compression factor  $C$  (that is, a smaller  $|q_3|$ ), there will be a smaller  $L_{D1}$  and a larger  $\theta_1$ , and vice versa.

In principle, the proposed design maintains the same overall structure as the symmetric C-chicane, except (i) the bending angles of the last two bends are weaker than the first two, with a ratio of  $q_3$ . (ii) the distance between the third and fourth bends is increased under the requirement of  $\ell_3$  in Eq. (20). (iii) a focus section, matched to the “negative

drift”  $L_{D2}^{\text{eff}}$ , is provided between the second and third bends, consisting of several quadrupoles.

Sometimes it may need to change the compression factor of a chicane, while keeping the total length unchanged. This can be achieved by programming the driving currents of the bends and quadrupoles, and installing the two inner bends and quadrupoles on remotely movable horizontal stages [49,50]. Then the driving currents and positions can be adjusted according to the values of  $q_3, \ell_3$ , and  $L_{D2}^{\text{eff}}$  for a

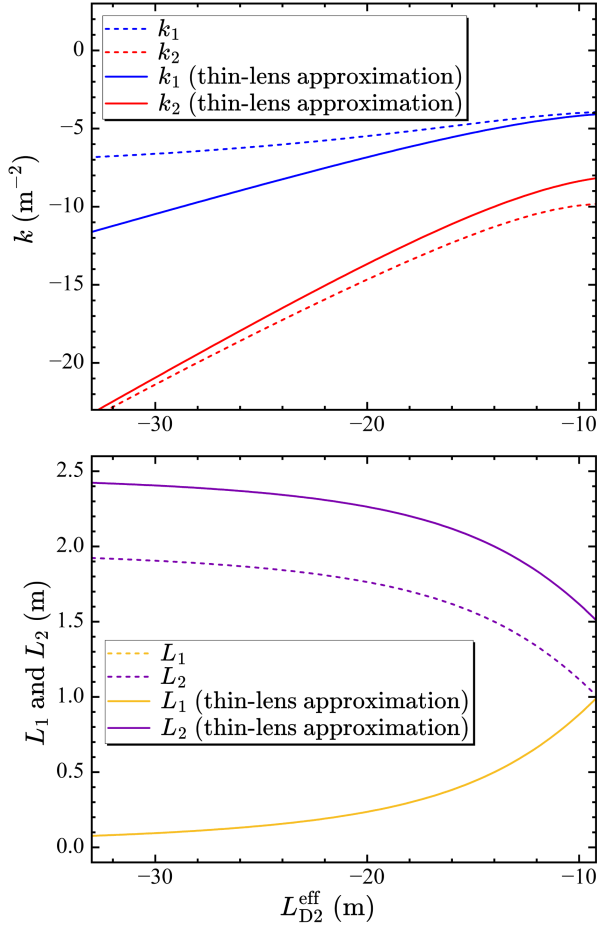


FIG. 4. Required values of  $k_1$ ,  $k_2$ , and  $(L_1, L_2)$  for  $L_{D2}^{\text{real}} = 5$  m and  $L_Q = 0.2$  m, calculated with and without the thin-lens approximation, respectively.

given  $C$  (see Fig. 3) to ensure the CSR cancelation and achromatic conditions. The adjustment of  $L_{D2}^{\text{eff}}$  is achieved by adjusting the position and strength of the quadrupoles. Specific adjustments are described below.

To achieve a negative drift  $L_{D2}^{\text{eff}}$  while avoiding any impact on vertical transport, here five quadrupoles are used to achieve drift sections in both horizontal and vertical directions [48]. The five thin-lens quadrupoles must satisfy the focus strength configuration  $(-k_1, k_1, k_2, k_1, -k_1)$  and satisfy the intervals  $(L_1, L_2, L_2, L_1)$ , where  $k_2 = -2k_1$  and  $L_1 = 1/(L_2 k_1^2 L_Q^2)$  with  $L_Q$  being the quadrupole length. With this arrangement, the resulting negative drift can be expressed as [48]

$$L_{D2}^{\text{eff}} = 2(f_1 - L_1 + L_1^2/f_1) = -\frac{(2f_1 - L_{D2}^{\text{real}})(L_{D2}^{\text{real}} + \sqrt{-16f_1^2 + (L_{D2}^{\text{real}})^2})}{4f_1}, \quad (22)$$

where  $f_1 = 1/(k_1 L_Q)$ . For the mentioned case with  $L_{D2}^{\text{real}} = 5$  m and  $L_Q = 0.2$  m, we solve for the required value of  $k_1$ , and the corresponding  $(k_2, L_1, L_2)$  (see Fig. 4). These results are derived from simple formulas based on the thin-lens approximation, which can be easily extended to other cases. Furthermore, the results can be slightly adjusted when calculated without the thin-lens approximation, as illustrated by the dashed lines in Fig. 4.

## B. Simulation verification of the proposed conditions

Next the ss-CSR cancelation conditions are verified based on a practical chicane compressor through numerical particle tracking simulations using ELEGANT [51,52]. For the initial bunch, a 2 GeV, 300 pC, 100  $\mu\text{m}$  Gaussian

TABLE I. Parameters of the ss-CSR-immune chicane, symmetric, and nonsymmetric C-chicane settings. The simulation results of the projected emittance growth and energy spread are also listed. Here the three chicanes use the same initial projected emittance  $\varepsilon_0 = 1$   $\mu\text{m}$  rad.

|                                                                    | Symbol                                      | ss-CSR-immune chicane | Symmetric C-chicane | Nonsymmetric C-chicane |
|--------------------------------------------------------------------|---------------------------------------------|-----------------------|---------------------|------------------------|
| Total length (m)                                                   | $L_{\text{tot}}$                            | 23                    | 23                  | 23                     |
| First-order longitudinal dispersion (mm)                           | $R_{56}^{s_0 \rightarrow s_f}$              | 35.1                  | 35.1                | 35.1                   |
| Length of the first bend (m)                                       | $L_B$                                       | 0.75                  | 0.75                | 0.75                   |
| Bending angle of the first two bends ( $^\circ$ )                  | $\theta_1,  \theta_2 $                      | 6.14                  | 2.40                | 2.92                   |
| Bending angle of the last two bends ( $^\circ$ )                   | $ \theta_3 , \theta_4$                      | 1.23                  | 2.40                | 1.97                   |
| Length of the first drift (m)                                      | $L_{D1}$                                    | 2.0                   | 9.5                 | 7.52                   |
| Length of the second drift (m)                                     | $L_{D2}^{\text{real}}(L_{D2}^{\text{eff}})$ | $5(-25.02)$           | 1.0                 | 1.0                    |
| Length of the third drift (m)                                      | $L_{D3}$                                    | 13                    | 9.5                 | 11.48                  |
| Peak current (ss-CSR included) (kA)                                | $I_{\text{peak}}$                           | 3.42                  | 3.35                | 3.33                   |
| Energy spread (ss-CSR included)                                    | $\sigma_\delta$                             | 0.110%                | 0.109%              | 0.109%                 |
| Emittance growth (ss-CSR included) ( $\mu\text{m}$ rad)            | $\Delta\varepsilon$                         | 0.0056                | 0.172               | 0.148                  |
| Peak current (more detailed CSR included) (kA)                     | $I_{\text{peak}}$                           | 3.47                  | 3.40                | 3.38                   |
| Energy spread (more detailed CSR included)                         | $\sigma_\delta$                             | 0.116%                | 0.117%              | 0.115%                 |
| Emittance growth (more detailed CSR included) ( $\mu\text{m}$ rad) | $\Delta\varepsilon$                         | 0.077                 | 0.746               | 0.557                  |

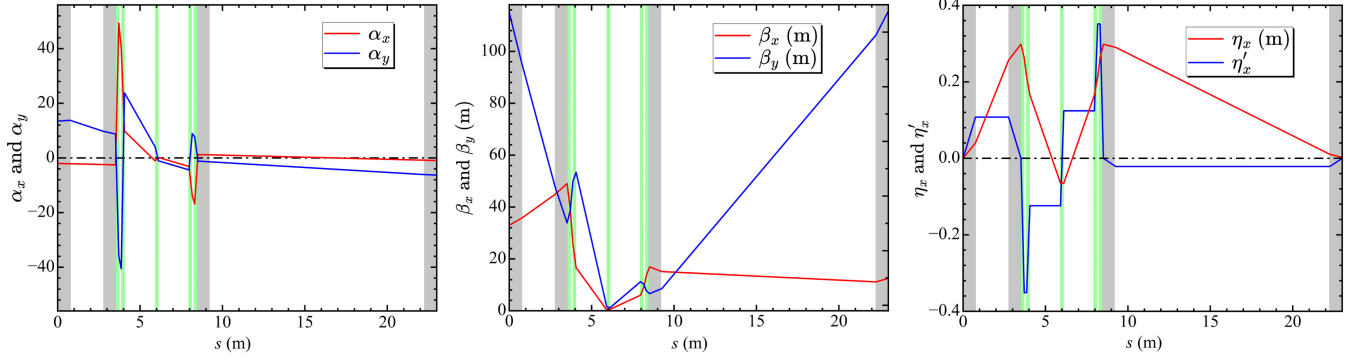


FIG. 5. C-S function and dispersion functions in the  $x$  (red) and  $y$  (blue) planes. The gray areas and green areas in all plots represent bends and quadrupoles, respectively.

bunch is generated with an uncorrelated energy spread of 0.01% and a transverse normalized emittance of 1  $\mu\text{m rad}$ . To achieve a peak current of about 3 kA at the chicane exit, the compression factor is set to 10. Based on the linear compression relation  $h = (1 - C)/(CR_{56}^{s_0 \rightarrow s_f})$ , the initial energy chirp is set to 25.67  $\text{m}^{-1}$ . Such beam parameters and compression scenarios are easily achieved in modern FELs.

From Eqs. (17) and (18) (see Fig. 3, left), we can obtain the theoretical values, denoted as  $(q_3^*, \ell_2^{\text{eff}*}) = (-0.20, -12.51)$ , for a bunch compressed by a factor of 10. We consider a chicane setup with a total length of 23 m, a first-order longitudinal dispersion of  $-35.1$  mm, a bend length of 0.75 m, and a physical length of  $L_{D2}^{\text{real}} = 5$  m designed to accommodate quadrupoles. The specific chicane parameters are calculated using Eq. (21) and listed in Table I. Considering that the net CSR kick may not be exactly zero, initial C-S parameter scans [25] are employed in each case to find the smallest emittance growth at the chicane exit. The simulation results are listed in Table I, and the C-S parameters and dispersion function are also plotted (see Fig. 5). In this scenario, the relative emittance growth is only 0.58%, indicating a successful compensation of the CSR kicks.

To further test the accuracy of the proposed conditions, we conduct a parameters scan of  $(q_2, \ell_2^{\text{eff}})$  within a relative deviation range of  $[-30\%, 30\%]$ . The simulation results for the final emittance are plotted in Fig. 6. Here  $\ell_2^{\text{eff}}$  and  $q_3$  are normalized with respect to  $\ell_2^{\text{eff}*}$  and  $q_3^*$  just for clear comparison. The values of  $(\ell_2^{\text{eff}}/\ell_2^{\text{eff}*}, q_3/q_3^*)$  close to  $(1, 1)$  indicate an accurate calculation by the theoretical model. We also plot the theoretical results for  $\Delta x_f = 0$  and  $\Delta x'_f = 0$ , and compare them with calculation results, as shown in Fig. 6. Note that the region with small simulated emittance aligns well with the curve  $\Delta x'_f = 0$ . This observation can be explained by the fact that this simulation case primarily affects  $\Delta x'_f$ .

However, the numerical simulations and theoretical calculations described above only consider the steady-state CSR effects. To accurately evaluate the CSR effects in practice, it is essential to investigate a more detailed CSR

model that includes steady-state CSR, transient CSR at the edges of the bends, incoherent synchrotron radiation, and the drift spaces following the bends, as well as classical single-particle synchrotron radiation. In this more detailed CSR model, the relative emittance growth for the considered chicane increases to 7.70%. We also conduct a parameters scan of  $(q_3, \ell_2^{\text{eff}})$  within a relative deviation range of  $[-30\%, 30\%]$ . The comparison between the simulation results and the theoretical results is similar to

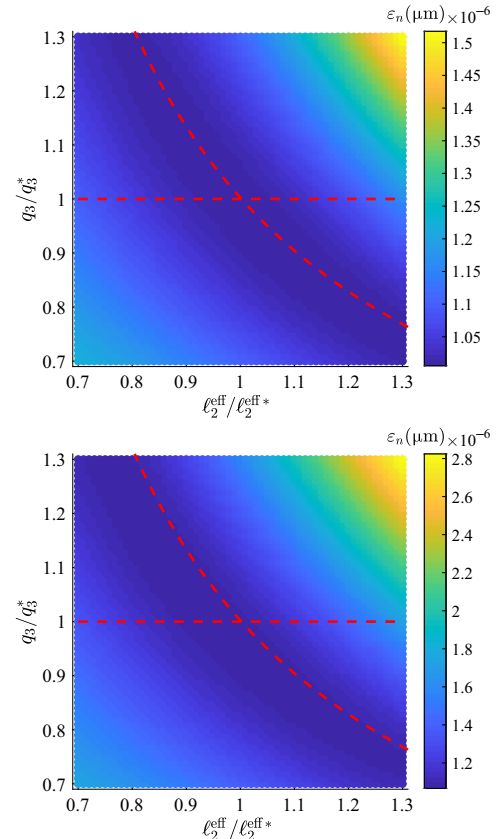


FIG. 6. ELEGANT simulation results of the CSR-induced emittance  $\epsilon_f$  near  $(q_2^*, q_3^*)$  for the proposed C-chicane when considering the ss-CSR (upper) and more detailed CSR (lower).

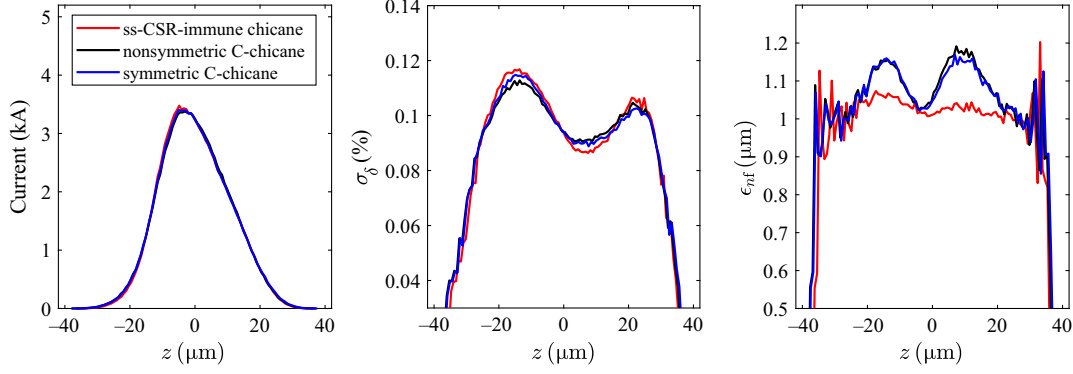


FIG. 7. Simulated distribution of current profile, energy spread, and emittance along the bunch length (considering 100 slices of  $\Delta z$ ).

that observed in the ss-CSR case, see Fig. 6. This indicates that, whether in the ss-CSR case or the detailed CSR case, there is good agreement between the simulation and theoretical results.

We also compare the CSR suppression efficiency of the proposed ss-CSR-immune chicane with the symmetric C-chicane and a nonsymmetric C-chicane proposed in [33,34]. For the convenience of comparison, both chicanes are performed with the same initial beam parameters, and compression target, namely, the same  $L_{\text{tot}}$ ,  $R_{56}^{s_0 \rightarrow s_f}$ , and  $C$ . These enable the derivation of the complete information for these two chicanes, as listed in Table I. Figure 7 presents the current profile, sliced energy spread and emittance at the exit of these chicanes, and the final relative emittance growth and energy spread are listed in Table I. Compared with the

symmetric C-chicane and nonsymmetric C-chicane proposed in Refs. [33,34], it can be seen from Table I that the proposed ss-CSR-immune chicane has a larger total bending angle (in absolute value) and a slight increase in energy spread, suggesting stronger CSR. However, thanks to the CSR compensation, the proposed chicane allows about 1 order of magnitude lower relative emittance growth.

#### IV. APPLICATION TO ASYMMETRIC S-CHICANE

##### A. The error analysis of the chicane parameters using simplified model

By setting  $L_B = 0$  and assuming that all correction functions are equal to 1, the ss-CSR cancelation conditions in Eq. (16) can be reduced to

$$\frac{q_2}{q_3 q_4 \ell_3} = \frac{k_3 \theta_3 \rho_3^{1/3} + k_4 \theta_4 \rho_4^{1/3}}{k_1 \theta_1 \rho_1^{1/3} + k_2 \theta_2 \rho_2^{1/3}},$$

$$\ell_2 = -\frac{1}{q_3} \frac{(q_2 + q_3)(k_1 \theta_1 \rho_1^{1/3} + k_2 \theta_2 \rho_2^{1/3}) + q_3(k_2 \theta_2 \rho_2^{1/3} + k_3 \theta_3 \rho_3^{1/3})}{k_1 \theta_1 \rho_1^{1/3} + k_2 \theta_2 \rho_2^{1/3} + (1 + q_2)(k_2 \theta_2 \rho_2^{1/3} + k_3 \theta_3 \rho_3^{1/3})}, \quad (23)$$

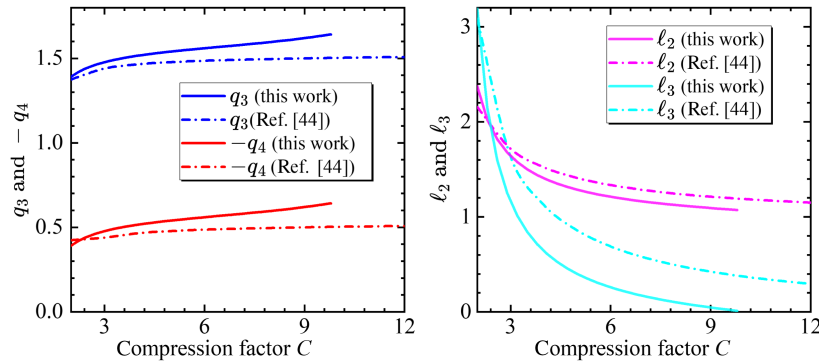


FIG. 8. Calculated  $q_3, q_4, \ell_2, \ell_3$  for  $\Lambda = 0.81:5$  for a partial-CSR-immune chicane and ss-CSR-immune chicane. Note that the  $\ell_3 = 0$  (indicates a zero length of the third drift) when  $C = 9.8$ .

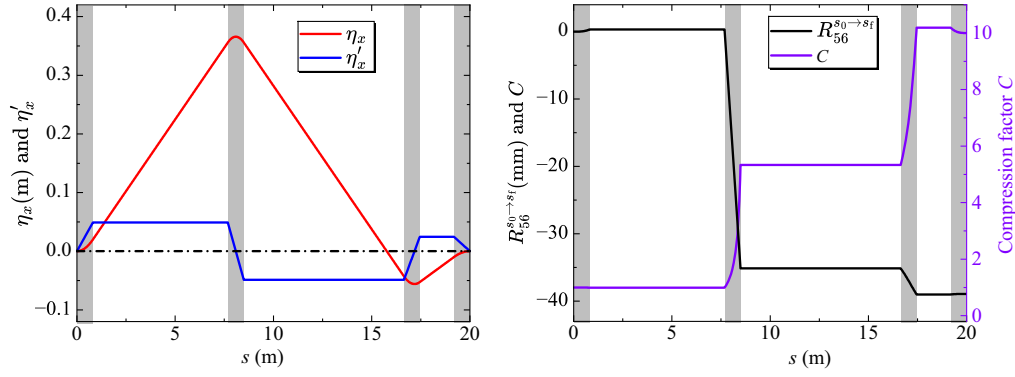


FIG. 9. Dispersion functions  $\eta$ ,  $\eta'$ ,  $R_{56}^{s_0 \rightarrow s_f}$ , and compression factor  $C$  as a function of the position  $s$  for a partial-CSR-immune chicane. The chicane layout are from Ref. [44].

which have been obtained in Ref. [44]. By setting  $\theta_2 = 2\theta_1(q_2 = -2)$ , the other chicane parameters  $q_3$ ,  $q_4$ ,  $\ell_2$ ,  $\ell_3$  as functions of compression factor  $C$  can be determined (see dot-dash curves in Fig. 8), thus achieving a partial-CSR-immune chicane. For a case with  $C = 10$ , Fig. 9 plots the dispersion functions and the bunch compression process. Next, we try to obtain a practical, special, accurate guideline for the ss-CSR-immune chicane by solving the obtained ss-CSR cancelation conditions in Eq. (16).

For an ss-CSR-immune chicane with S-shape, the derivation of the correction functions for the first and last bends is the same as the previous results, as provided in Appendix A. For the second and third bends, the corresponding correction functions can be approximated as functions of  $\xi$  and are independent of the values of  $a_2$ ,  $b_2$  and  $a_3$ ,  $b_3$ . The approximated and accurate results

are comparable over the typical ranges of  $a_2$ ,  $b_2$  and  $a_3$ ,  $b_3$ , respectively (see Appendix C for details).

Using the modified CSR-kick model, the parameters for a ss-CSR-immune chicane are plotted in Fig. 8, compared to the simplified results (dot-dashed curves). It can be observed that these angle parameters  $q_3$ ,  $q_4$  exhibit relatively small errors, while there are relatively larger errors in the drift parameters  $\ell_2$ ,  $\ell_3$ . Apart from this qualitative analysis, we will next provide a qualitative assessment based on the achromatic condition and the ss-CSR cancelation conditions.

Note that the compression process primarily occurs in the second and third bends of the chicane (see Fig. 9, right). Therefore, the correction functions  $g_{x1}$ ,  $g_{xp1}$ ,  $g_{\delta 1}$  and  $g_{x4}$ ,  $g_{xp4}$ ,  $g_{\delta 4}$  can be assumed to be 1 since  $\xi$  is very close to 0. Finally, the ss-CSR cancelation conditions and achromatic conditions can be simplified as

$$\begin{aligned}
 0 &\approx k_1 \theta_1^{5/3} (L_{D1} - 3L_{D2}) - k_1 L_{D3} \theta_1^{2/3} (\theta_1 + 2\theta_4) \\
 &\quad + L_{D3} \theta_3 (g_{xp3} k_3 \theta_3^{2/3} + 2g_{\delta 2} k_2 \theta_2^{2/3}) + g_{xp2} k_2 \theta_2^{5/3} (L_{D2} + L_{D3}) \\
 &\quad + \frac{L_B}{6} [24\theta_3 (k_1 \theta_1^{2/3} + g_{\delta 2} k_2 \theta_2^{2/3}) + k_3 \theta_3^{5/3} (2g_{x3} + 9g_{xp3}) - k_4 \theta_4^{5/3} \\
 &\quad - 61k_1 \theta_1^{5/3} - 2k_2 \theta_2^{2/3} (2g_{x2} + 21g_{xp2})], \\
 0 &\approx k_4 \theta_4^{5/3} - k_1 \theta_1^{5/3} + (g_{xp2} k_2 \theta_2^{5/3} + 2g_{\delta 2} k_2 \theta_2^{2/3} \theta_4 + 2g_{\delta 3} k_3 \theta_3^{2/3} \theta_4) \\
 &\quad + (2g_{\delta 2} k_2 \theta_2^{2/3} \theta_3 + g_{xp3} k_3 \theta_3^{5/3}), \\
 \ell_3 &= \frac{1 - \ell_2}{-1 + q_3} + \ell_B, \\
 q_4 &= -1 - q_2 - q_3.
 \end{aligned} \tag{24}$$

Here, we only focus on the terms related to correction functions and the bend length, as these two factors account for the differences in the chicane parameters. Specifically, the first member of Eq. (24) contains  $L_B$  term, while the coefficient of  $L_B$  can cancel each other, albeit partially.

For the correction functions in the first member of Eq. (24), note that the compression process corresponding to these  $g_{xi}$ ,  $g_{xpi}$ ,  $g_{\delta i}$ , ( $i = 2, 3$ ) is all larger than 1, and their coefficients contain both positive and negative components, which can partially cancel each other out. together with the

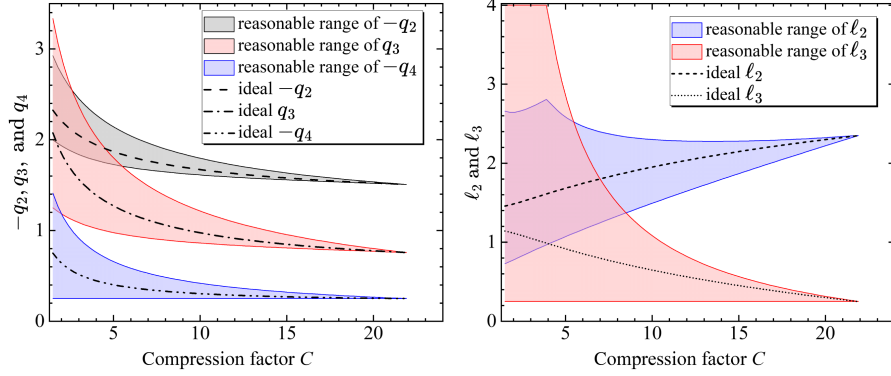


FIG. 10. Allowed parameter regions of  $(q_2, q_3, q_4, \ell_2, \ell_3)$  vs compression factor with all parameters limited within a range of  $(1/4, 4)$  (see the light shaded zones). The ideal parameters are depicted as black curves.

expression for  $q_4$  in Eq. (24), which is independent of correction functions and the bend length, we can conclude that the angle parameters  $(q_3, q_4)$  are basically unaffected by the simplifications (as plotted in Fig 8, left). Moreover, the drift lengths  $\ell_2, \ell_3$  are related to bend length [see the last equation of Eq. (24)]. As a result, the drift parameters  $(\ell_2, \ell_3)$  may exhibit significant modifications (see Fig. 8, right).

### B. Calculation of the reasonable range of parameters $q_2, q_3, q_4, \ell_2, \ell_3$

It can be observed that some of the chicane parameters obtained in Fig. 8 are unreasonable, corresponding to impractical geometries. For example, a larger  $C$  corresponds to a smaller  $\ell_3$ , which indicates a shorter distance between the last two bends. Therefore, in this subsection, we will provide further guidance on chicane design, specifically outlining the reasonable parameter ranges for a CSR-immune chicane.

From a practical point of view, to achieve reasonable chicane geometries, one can limit all the parameters  $(|q_2|, q_3, |q_4|, \ell_2, \ell_3)$  within a range of  $(1/4, 4)$ , albeit this threshold selection may be somewhat arbitrary. With the solvable model, the ranges of  $(|q_2|, q_3, |q_4|, \ell_2, \ell_3)$  vary with the compression factor  $C$  can be calculated, as plotted in Fig. 10. One can see that a larger compression factor  $C$  corresponds to smaller ranges of all parameters. Meanwhile, the allowable compression factors span from 1 to 22, which can meet the requirements for the compression factor of a single chicane in most free electron lasers (FELs) [5–12].

It should be noted that the previously discussed chicane parameters (see Ref. [44]) under the condition  $q_2 = -2$  do not include all allowable ranges, thereby limiting the potential for high-times bunch compression. This raises the question of whether there exists a parameter condition that allows all chicane parameters to cover the entire reasonable range illustrated in Fig. 8. To this end, we find that fixing the position of the second bend at

$(L_{D1} + L_B)/L_{\text{tot}} = 0.27$ , which allows the chicane parameters to span the entire range depicted in Fig. 10. In this case, the longitudinal positions of the first, second, and fourth bends can be fixed when maintaining a constant total length (see Fig. 11). Although not strictly necessary, we impose this position restriction for practical reasons; specifically, adjusting bend positions is generally more challenging than adjusting bending angles.

### C. Simulation verification of the proposed conditions

From the parameter calculations in Sec. IV B, we can obtain the theoretical values, denoted as  $(q_2^*, q_3^*, q_4^*, \ell_2^*, \ell_3^*) = (-1.67, 0.97, -0.30, 1.95, 0.65)$  when the compression factor is set to 10. A similar simulation verification to the previously discussed C-chicane is conducted. The simulation results show that the relative emittance growth is only 1.1% when considering the ss-CSR, and it increases to 5.4% in this more detailed CSR model. Compared to the widely used C-chicane, these results allow for about 1 order of magnitude lower relative emittance growth. We also conduct a parameter scan of  $(q_3, \ell_2)$  within a relative deviation range of  $[-18\%, 18\%]$ . The values of  $(q_3/q_3^*, \ell_2/\ell_2^*) = (0.99, 1.02)$  for ss-CSR and  $(q_3/q_3^*, \ell_2/\ell_2^*) = (1.02, 0.98)$  for the more detailed CSR are presented in Fig. 12. This indicates that there is good agreement between the simulation and theoretical results.

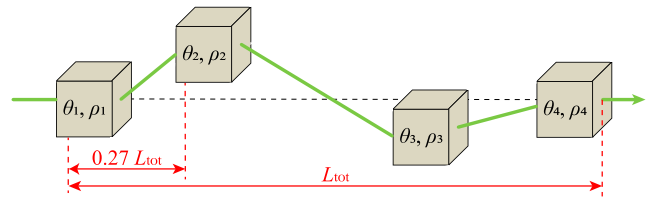


FIG. 11. General geometry of a four-bend ss-CSR-immune chicane with S-shape. The position of the second bend is fixed and accounts for 27% of the total chicane length.

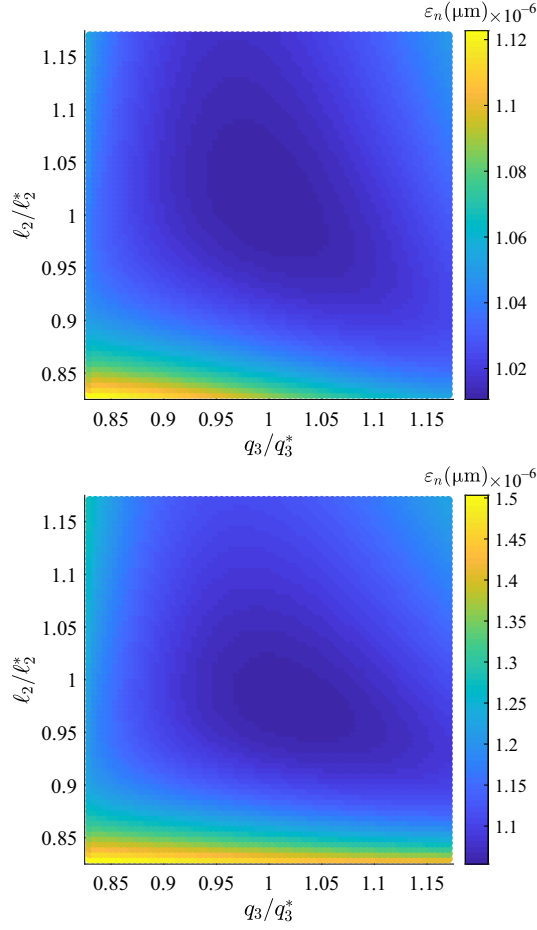


FIG. 12. ELEGANT simulation results of the CSR-induced emittance near  $(q_3^*, \ell_2^*)$  for the proposed C-chicane when considering the ss-CSR (upper) and more detailed CSR (lower).

## V. SUMMARY AND DISCUSSION

This paper presents an explicit point-kick analysis of the CSR effects for a general four-bend chicane. Utilizing the modified CSR-kick model, which accounts for bunch length variations within bends, we derive the conditions for completely canceling the steady-state CSR kicks using analytical formulas. This analysis yields two novel ss-CSR-immune chicanes featuring nonsymmetric C-shape and S-shape layouts, respectively. Both chicanes would not be difficult to practical design and implementation and can be derived from a symmetric C-chicane with just a few modifications. For the former, the nonsymmetric C-shape geometry can be achieved by adjusting the bending angle ratio between the first two bends and the last two bends, as well as the length ratio between the first and third drifts. Additionally, a “negative drift” between the second and third bends is required, which can be achieved by incorporating several quadrupoles. For the latter, the CSR kicks can be canceled without the need for additional components, simply by adjusting the chicane geometry according

to the parameter scaling (see Fig. 10). For both chicanes, the parameter scalings related to the bending angles and drift lengths can be analytically solved to provide direct guidance for practical chicane design.

It can be observed that the key to canceling the CSR kicks in four-bend chicanes is to reduce the strengths (or shorten the lengths) of the last two bends according to specific guidelines. This reduction becomes more pronounced as the compression factor increases. Notably, these features coincide with the nonsymmetric DBA-based bunch compressor introduced in [40], where the last bend is  $C^{-1/3}$  or  $C^{-1/2}$  times weaker than the first bend. An intriguing pattern emerges, indicating that the proposed two ss-CSR-immune chicanes, as well as the DBA-based compressor, share similarities and effectively cancel the CSR kicks during the compression process.

We also demonstrate the excellent emittance preservation capabilities of the proposed two chicanes through particle tracking simulations. By utilizing these chicanes instead of the widely used C-chicane, we can reduce CSR-induced emittance growth by about 1 order of magnitude in an almost identical bunch compression scenario. Additionally, we conduct a parameter scan to verify the proposed CSR cancelation conditions. For both chicanes, the results indicate strong agreement between the simulation and theoretical results.

While the primary focus of this paper lies in analyzing the design principles that effectively suppress CSR-induced emittance growth in the chicane bunch compressors, we also assess the potential microbunching instability (MBI) associated with the lattice and the beam parameters. Using our developed semianalytical Vlasov solver [53,54] enables the fast, efficient evaluation of the various lattice designs. The semianalytical calculations indicate that these chicane designs, which are effective in suppressing CSR-induced emittance growth, also exhibit a well-controlled MBI. Specifically, based on the beam parameters and chicane settings presented in Table I, taking into account both the steady-state and transient CSR and the longitudinal space charge effects, the simulation results for the proposed C-chicane reveal a maximum gain of about 3.5 occurring around an initial modulation wavelength of 50  $\mu\text{m}$ . In the case of the proposed S-chicane, simulation results demonstrate a maximum gain of 4 around a similar modulation wavelength of 50  $\mu\text{m}$ . It is worth mentioning that this paper does not consider other collective effects, such as linac geometric wakefield, and many others. For a physically realistic scenario, these are difficult to cancel out and need to be fully taken into account.

Due to their excellent emittance preservation capability, the proposed ss-CSR-immune four-bend chicanes may be an appealing option for the final-stage compressor, potentially allowing for the production of a higher peak current for FELs. Additionally, the proposed chicane with C-shape

(or S-shape) is suitable for application based on symmetric C-chicanes (or symmetric S-chicanes), requiring just a few modifications for these chicanes that are either in operation or under construction. We hope that our results can serve as a starting point for practical applications, opening up new avenues for the realization of CSR-immune bunch compressors, which will benefit the future development of FELs and other accelerator-based scientific facilities.

### ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China (No. 12275284 and No. 12275094), and the Fundamental Research Funds for the Central Universities (HUST) under Project No. 2021GCRC006,

the National Key Research and Development Program of China (No. 2022YFA1603402), and the National Natural Science Foundation of China (No. 12405176).

### APPENDIX A: CORRECTION FUNCTIONS FOR THE PROPOSED C-CHICANE

For the correction functions in the four bends of a chicane, there are two special cases, as detailed in Ref. [30]. For the first bend, where

$$a_1 = 0, \quad b_1 = 0, \quad \xi_1 = \frac{L_B \theta_1^2}{6} h_1, \quad (\text{A1})$$

the correction functions can be expressed in the following analytical form [30],

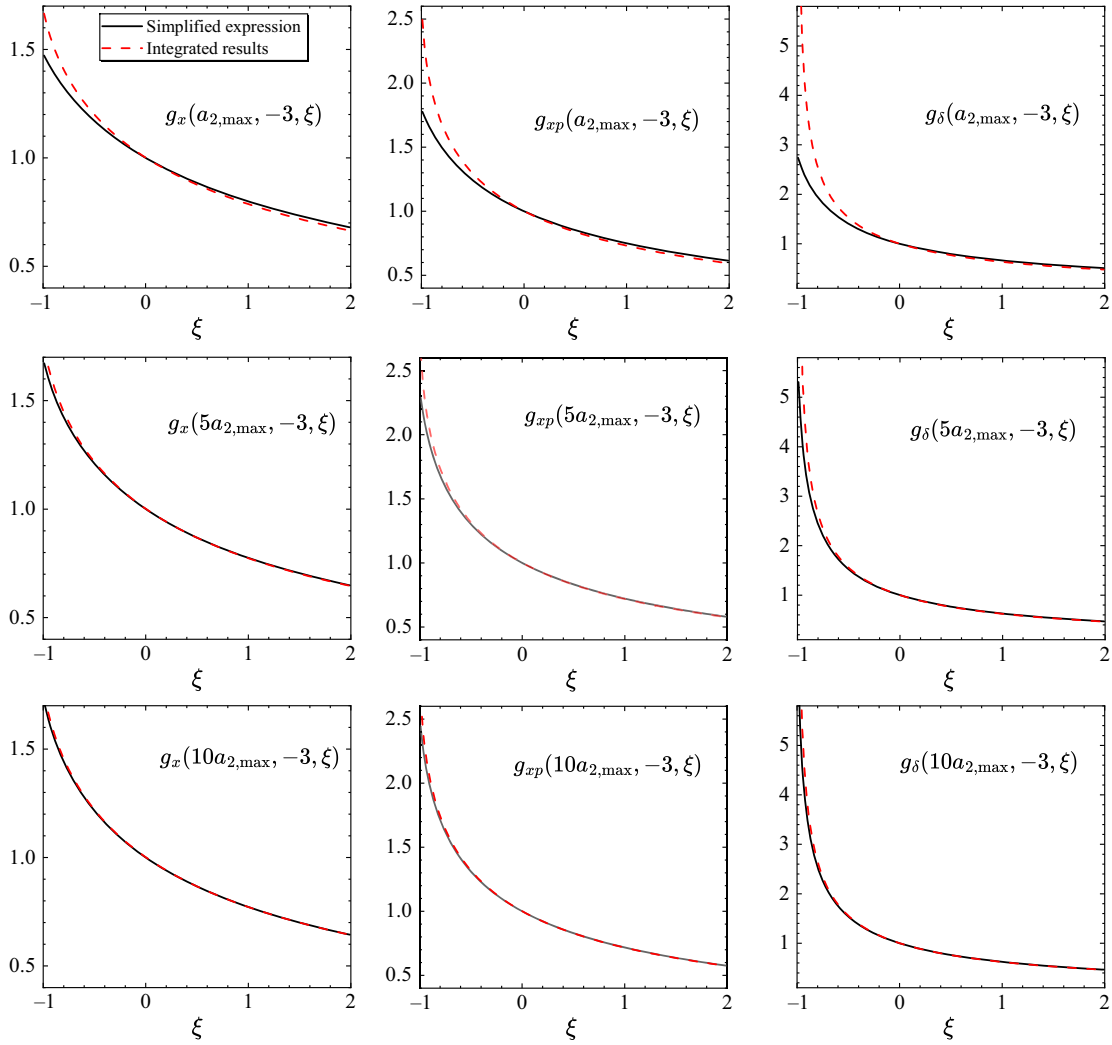


FIG. 13. Comparison of the simplified expressions and integrated results for the cases  $a_2 = a_{2,\max} = -9$ ,  $a_2 = 5a_{2,\max}$ , and  $a = 10a_{2,\max}$  under the case  $b_2 = -3$ .

$$\begin{aligned}
 g_x([0,0],\xi) &= 3 \frac{-1 + (1+\xi)^{1/3} - \xi \left[ {}_2F_1\left(\frac{1}{3}, 1, \frac{5}{3}, -\xi\right) - 1 \right]}{\xi(1+\xi)^{1/3}}, \\
 &\approx 1.12 - \frac{0.12}{(1+\xi)^{0.56}}, \\
 g_{xp}([0,0],\xi) &= \frac{2 - {}_2F_1\left(\frac{1}{3}, 1, \frac{5}{3}, -\xi\right)}{(1+\xi)^{1/3}}, \\
 &\approx 1.5 + \frac{0.5}{(1+\xi)^{0.27}}, \\
 g_\delta([0,0],\xi) &\approx \frac{1}{(1+\xi)^{1/3}}, \tag{A2}
 \end{aligned}$$

where  ${}_2F_1$  denotes the standard hypergeometric function. For the last bend, where

$$\begin{aligned}
 a_4 &= 3, \quad b_4 = -3, \\
 \xi_4 &= \frac{h_1 L_B \theta_1^2 q_3^2 \Lambda}{6\Lambda + h_1 L_B \theta_1^2 (-12q_3(\Lambda - 2) + 2\Lambda + q_3^2 \Lambda)}, \tag{A3}
 \end{aligned}$$

the corresponding correction functions can be expressed analytically as [30]

$$\begin{aligned}
 g_x([3,-3],\xi) &= 3 \frac{(1+\xi)^{1/3} - 1}{\xi(1+\xi)^{1/3}}, \\
 g_{xp}([3,-3],\xi) &\approx \frac{1}{(1+\xi)^{3/4}}, \\
 g_\delta([3,-3],\xi) &\approx \frac{1}{1+\xi}. \tag{A4}
 \end{aligned}$$

For the second and third bends, the corresponding correction functions  $(g_{xi}, g_{xpi}, g_{\delta i})(i = 2, 3)$  cannot be directly determined. For the second bend, the parameter  $a_2, b_2, \xi_2$  can be expressed as

$$\begin{aligned}
 a_2 &= -3 \frac{L_B + 2L_{D1}}{L_B} = \frac{3(q_3(8-5\Lambda) + \Lambda)}{(1-q_3)\Lambda}, \quad b_2 = -3, \\
 \xi_2 &= \frac{h_1 L_{D1} \theta_1^2 [-\Lambda + q_3(-24 + 13\Lambda)]}{(6 - h_1 L_{D1} \theta_1^2) \Lambda + q_3(24 + (-18 + h_1 L_{D1} \theta_1^2) \Lambda)}. \tag{A5}
 \end{aligned}$$

Since  $\Lambda$  is usually small for chicane (on the order of 0.1),  $a_2$  is larger than 1. Therefore, the expression can be simplified as

$$\frac{a_2 \tau + b_2 \tau^2 + \tau^3}{a_2 + b_2 + 1} \approx \frac{a_2 \tau}{a_2 - 2}, \quad \text{for } \tau \in [0, 1]. \tag{A6}$$

Finally, the correction functions in Eq. (9) for this case can be simplified as

$$\begin{aligned}
 g_x([a,-3],\xi) &= 3 \int_0^1 \left[ 1 + \xi \frac{a\tau - 3\tau^2 + \tau^3}{a-2} \right]^{-4/3} (1-\tau)^2 d\tau \\
 &\approx 3 \int_0^1 \left( 1 + \xi \frac{a\tau}{a-2} \right)^{-4/3} (1-\tau)^2 d\tau \\
 &= \frac{9(a-2)}{5a^3 \xi^3 \Psi} \{ 36(-1 + \Psi) - 6a(-6 - 6\xi + 6\Psi + 5\xi\Psi) \\
 &\quad + a^2[9(-1 + \Psi) + x^2(-9 + 5\Psi) + 3x(-6 + 5\Psi)] \}, \\
 g_{xp}([a,-3],\xi) &= 2 \int_0^1 \left[ 1 + \xi \frac{a\tau - 3\tau^2 + \tau^3}{a-2} \right]^{-4/3} (1-\tau) d\tau \\
 &\approx 2 \int_0^1 \left( 1 + \xi \frac{a\tau}{a-2} \right)^{-4/3} (1-\tau) d\tau \\
 &= -\frac{3(a-2)[6 - 6/\Psi + a(-3 - 2\xi + 3/\Psi) + 3\xi/\Psi]}{a^2 \xi^2}, \\
 g_\delta([a,-3],\xi) &= \int_0^1 \left[ 1 + \xi \frac{a\tau - 3\tau^2 + \tau^3}{a-2} \right]^{-4/3} d\tau \\
 &\approx \int_0^1 \left( 1 + \xi \frac{a\tau}{a-2} \right)^{-4/3} d\tau \\
 &= -\frac{3(a-2)(-1 + 1/\Psi)}{a\xi}, \tag{A7}
 \end{aligned}$$

where  $\Psi = \left(\frac{-2+a+a\xi}{a-2}\right)^{1/3}$ . We compare the simplified expressions with the integrated results for the cases  $a_2 = a_{2,\max} = -9$  (assuming the bend length and drift length are identical),  $a_2 = 5a_{2,\max}$  and  $a = 10a_{2,\max}$  (see Fig. 13). Within the  $\xi$  range shown in Fig. 13, the deviation between

the simplified expressions and the integrated results decreases as  $a_2$  increases. For a typical value of  $a_2$ , the approximate expression is sufficiently accurate.

For the third bend, the parameter  $a$  and  $b$  in correction functions ( $g_{x3}$ ,  $g_{xp3}$ ,  $g_{\delta3}$ ) can be expressed as

$$\begin{aligned} a_3 &= 6 \frac{L_B + L_{D1}}{q_3 L_B} = \frac{12(-2 + \Lambda)}{(1 - q_3)\Lambda}, \quad b_3 = 0, \\ \xi_3 &= \frac{h_1 L_{D1} \theta_1^2 q_3^2 (24 + (-13 + q_3)\Lambda)}{2(\Lambda(3 - h_1 L_{D1} \theta_1^2) + q_3[12 - 9\Lambda + (-12 + 7\Lambda)h_1 L_{D1} \theta_1^2])}. \end{aligned} \quad (\text{A8})$$

As  $\Lambda$  usually small for chicane (at level of 0.1),  $a_2$  is large than 1. Thus, the expression can be simplified as

$$\frac{a_3 \tau + b_3 \tau^2 + \tau^3}{a_3 + b_3 + 1} \approx \frac{a_3 \tau}{a_3 + 1}, \quad \text{for } \tau \in [0, 1]. \quad (\text{A9})$$

Finally, Eq. (9) for this case can be simplified as

$$\begin{aligned} g_x([a, 0], \xi) &= 3 \int_0^1 \left[ 1 + \xi \frac{a\tau - 3\tau^2 + \tau^3}{a - 2} \right]^{-4/3} (1 - \tau)^2 d\tau \\ &\approx 3 \int_0^1 \left( 1 + \xi \frac{a_3 \tau}{a_3 + 1} \right)^{-4/3} (1 - \tau)^2 d\tau \\ &= \frac{9(a+1)}{5a^3 \xi^3 \psi} \{ 9(-1 + \psi) + 3a(-6 - 6\xi + 6\psi + 5\xi\psi) \\ &\quad + a^2[9(-1 + \psi) + x^2(-9 + 5\psi) + 3x(-6 + 5\psi)] \}, \\ g_{xp}([a, 0], \xi) &= 2 \int_0^1 \left[ 1 + \xi \frac{a\tau - 3\tau^2 + \tau^3}{a - 2} \right]^{-4/3} (1 - \tau) d\tau \\ &\approx 2 \int_0^1 \left( 1 + \xi \frac{a_3 \tau}{a_3 + 1} \right)^{-4/3} (1 - \tau) d\tau \\ &= \frac{6(a+1)\{3 + 3a + 2a\xi - 3[(a+1)(1 + a + a\xi)^2]^{1/3}\}}{2a^2 \xi^2}, \\ g_\delta([a, 0], \xi) &= \int_0^1 \left[ 1 + \xi \frac{a\tau - 3\tau^2 + \tau^3}{a - 2} \right]^{-4/3} d\tau \\ &\approx \int_0^1 \left( 1 + \xi \frac{a_3 \tau}{a_3 + 1} \right)^{-4/3} d\tau \\ &= \frac{3(a+1)(-(a+1)^{1/3} + (1 + a + a\xi)^{1/3})}{a\xi(1 + a + a\xi)^{1/3}}, \end{aligned} \quad (\text{A10})$$

where  $\psi = \left(\frac{1+a+a\xi}{a+1}\right)^{1/3}$ . We also compare the simplified expressions with the integrated results for the cases  $a = a_{3,\max} = -12$  (assuming the bend length and drift length are identical),  $a = 5a_{3,\max}$  and  $a = 10a_{3,\max}$  (see Fig. 14). Similar to the previous case, for typical values of  $a_3$ , the approximate expression is sufficiently accurate.

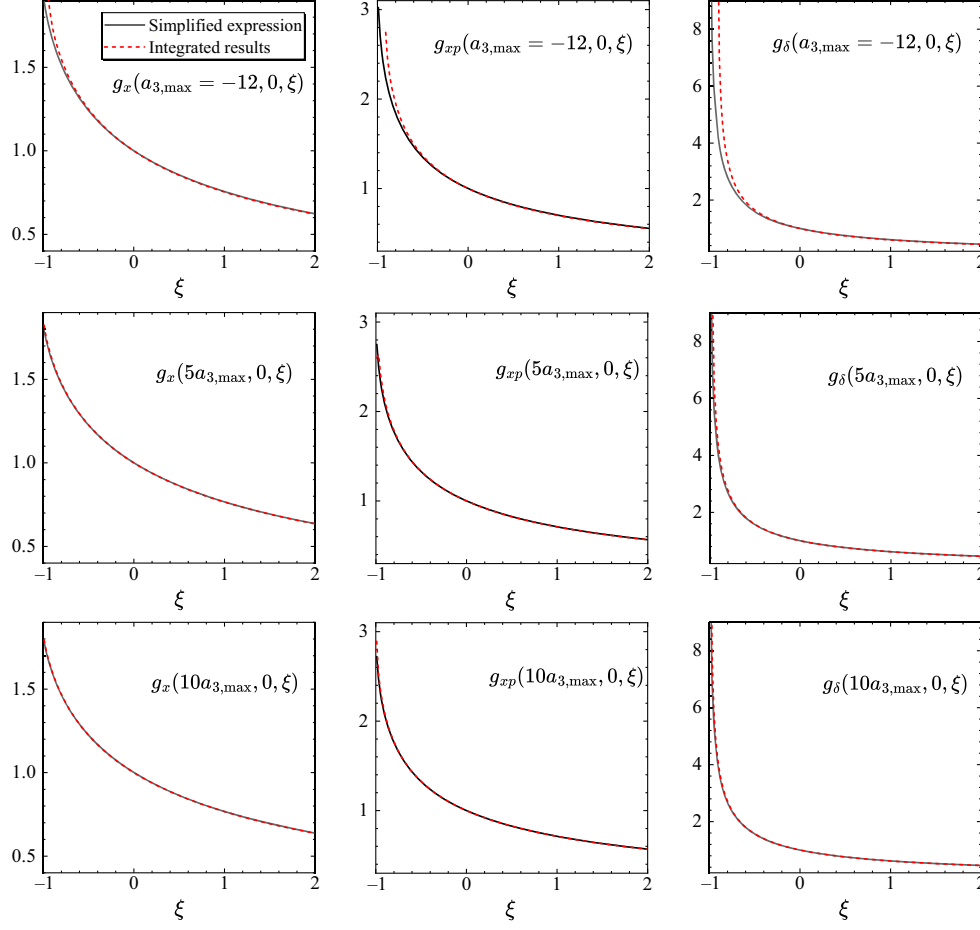


FIG. 14. Comparison of the simplified expressions and integrated results for the cases  $a_3 = a_{3,\max} = -9$ ,  $a_3 = 5a_{3,\max}$ , and  $a = 10a_{3,\max}$  under the case  $b_3 = 0$ .

## APPENDIX B: $k_i$ FOR DIFFERENT BENDS

For the first and the fourth CSR kicks of a general four-bend chicane, we use the bunch lengths at the entrance and exit of the bends, respectively, i.e.,

$$\sigma_{z1} = \sigma_{z0}, \quad \sigma_{z4} = \sigma_{zf} = \sigma_{z0}/C, \quad (\text{B1})$$

where  $\sigma_{z0}$ ,  $\sigma_{zf}$  are the initial and final bunch lengths, respectively. Thus,  $k_1$  and  $k_4$  can be

$$k_1 = 0.2459 \frac{N_b r_e}{\gamma \sigma_{z0}^{4/3}}, \quad k_4 = k_1 \frac{\sigma_{z4}^{-4/3}}{\sigma_{z0}^{-4/3}} = C^{4/3} k_1. \quad (\text{B2})$$

Here  $C = \sigma_{z0}/\sigma_{zf}$  is the bunch compression factor.

The value of  $k_2$  for the second CSR kick is related to  $k_1$  as

$$k_2 = k_1 \frac{\sigma_{z2}^{-4/3}}{\sigma_{z0}^{-4/3}} = \frac{k_1}{(1 + hR_{56}^{s_0 \rightarrow 2\text{ m}})^{4/3}}, \quad (\text{B3})$$

because the bunch length  $\sigma_z \approx \sigma_{z0}(1 + hR_{56}^{s_0 \rightarrow s})$  with

$$h = \frac{1 - C}{CR_{56}^{s_0 \rightarrow s_f}}, \quad R_{56}^{s_0 \rightarrow s_f} = L_{D1} \theta_1^2 (q_2 + \ell_3 q_3 q_4), \quad (\text{B4})$$

here  $h$  is the linear chirp related to rf cavity phase,  $R_{56}^{s_0 \rightarrow s}$  is the longitudinal dispersion at location  $s$ , and the notation “ $s_0 \rightarrow 2\text{ m}$ ” in  $R_{56}^{s_0 \rightarrow 2\text{ m}}$  means transport from the chicane entrance to the midpoint of the second bend, which can be written as

$$R_{56}^{s_0 \rightarrow 2\text{ m}} = \frac{L_{D1} \theta_1^2 q_2}{2}. \quad (\text{B5})$$

The value of  $k_3$  for the third CSR kick is similar to the expression of  $k_2$  as

$$k_3 = k_1 \frac{\sigma_{z3}^{-4/3}}{\sigma_{z0}^{-4/3}} = \frac{k_1}{(1 + hR_{56}^{s_0 \rightarrow 3\text{ m}})^{4/3}}, \quad (\text{B6})$$

with

$$R_{56}^{s_0 \rightarrow 3\text{ m}} = \frac{1}{2} L_{D1} \theta_1^2 [\ell_2(1+q_2)q_3 + (2q_2+q_3)]. \quad (\text{B7})$$

For the proposed C-chicane, the expressions of  $R_{56}^{s_0 \rightarrow 2\text{ m}}$  and  $R_{56}^{s_0 \rightarrow 3\text{ m}}$  in  $k_2$  and  $k_3$  can be simplified as

$$R_{56}^{s_0 \rightarrow 2\text{ m}} = -\frac{\theta_1^2}{2} L_{D1}, \quad (\text{B8})$$

$$R_{56}^{s_0 \rightarrow 3\text{ m}} = -L_{D1} \theta_1^2 \left(1 + \frac{1}{2} \ell_3 q_3^2\right). \quad (\text{B9})$$

### APPENDIX C: CORRECTION FUNCTIONS FOR THE PROPOSED S-CHICANE

For the proposed S-chicane, the derivation of the correction functions for the first and last bends is provided in Eqs. (A2) and (A4). For the second bend, we have

$$a = -\frac{3(L_B + 2L_{D1})}{2L_B}, \quad b = \frac{3\theta_1}{\theta_2} = -\frac{3}{2}. \quad (\text{C1})$$

Note that the absolute value  $|a|$  is much larger than  $b+1$  as the drift length usually much larger than the bend length for applying enough  $R_{56}^{s_0 \rightarrow s_f}$ . Therefore, we can write

$$\frac{a\tau + b\tau^2 + \tau^3}{a+b+1} \approx a, \quad \text{for } \tau \in [0, 1]. \quad (\text{C2})$$

Finally, Eq. (9) for this case can be simplified as

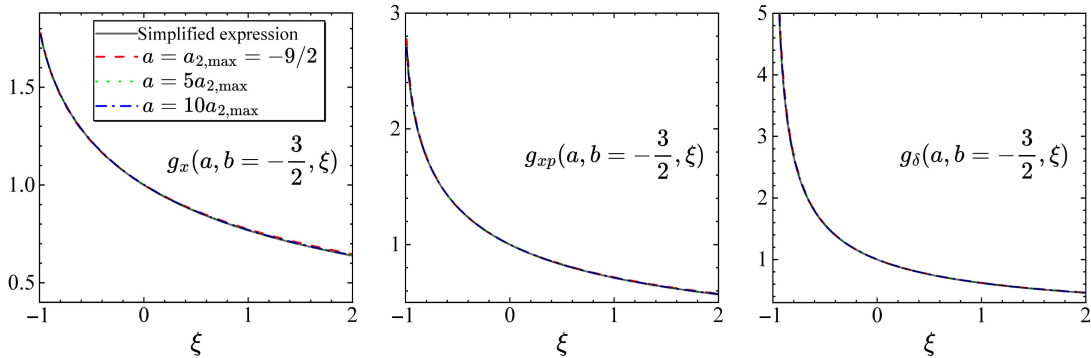


FIG. 15. Comparison of the simplified expressions and integrated results for the cases  $a_2 = a_{2,\text{max}} = -9/2$ ,  $a_2 = 5a_{2,\text{max}}$ , and  $a = 10a_{2,\text{max}}$  under the case  $b_2 = -3/2$ .

$$\begin{aligned} g_x([a, b], \xi) &= 3 \int_0^1 \left[ 1 + \xi \frac{a\tau + b\tau^2 + \tau^3}{a+b+1} \right]^{-4/3} (1-\tau)^2 d\tau \\ &\approx 3 \int_0^1 (1+\xi\tau)^{-4/3} (1-\tau)^2 d\tau \\ &= \frac{9}{5} \times \frac{9-9(1+\xi)^{2/3} + \xi(15+5\xi-9(1+\xi)^{2/3})}{\xi^3}, \\ g_{xp}([a, b], \xi) &= 2 \int_0^1 \left[ 1 + \xi \frac{a\tau + b\tau^2 + \tau^3}{a+b+1} \right]^{-4/3} (1-\tau) d\tau \\ &\approx 2 \int_0^1 (1+\xi\tau)^{-4/3} (1-\tau) d\tau \\ &= \frac{9+6\xi-9(1+\xi)^{2/3}}{\xi^2}, \\ g_\delta([a, b], \xi) &= \int_0^1 \left[ 1 + \xi \frac{a\tau + b\tau^2 + \tau^3}{a+b+1} \right]^{-4/3} d\tau \\ &\approx \int_0^1 (1+\xi\tau)^{-4/3} d\tau \\ &= 3 \frac{(1+\xi)^{1/3} - 1}{\xi(1+\xi)^{1/3}}, \end{aligned} \quad (\text{C3})$$

We also compare with the integrated results by setting  $a_2 = a_{2,\text{max}} = -9/2$  (assuming the bend length and drift length are identical),  $a = 5a_{2,\text{max}}$ , and  $a = 10a_{2,\text{max}}$ . One can find that a relatively small difference can be observed between the approximate expressions and the integrated results (see Fig. 15).

For the third bend, we should evaluate the values of  $a_3$  and  $b_3$ . Note that the dispersions at the second and third bends are opposite and their absolute values have the same order. The bending angle of the third bend  $q_3$  is usually at the range of  $(1, 2)$ . Finally, we can assume

$$\begin{aligned} a_3 &= 6 \frac{\eta_3}{L_B \theta_3} \sim -6 \frac{\eta_2}{L_B \theta_3} = \frac{2a_2}{q_3} \in (a_2, 2a_2), \\ b_3 &= 3 \frac{\eta'_3}{\theta_3} = -\frac{3}{q_3} \in \left(-3, -\frac{3}{2}\right). \end{aligned} \quad (\text{C4})$$

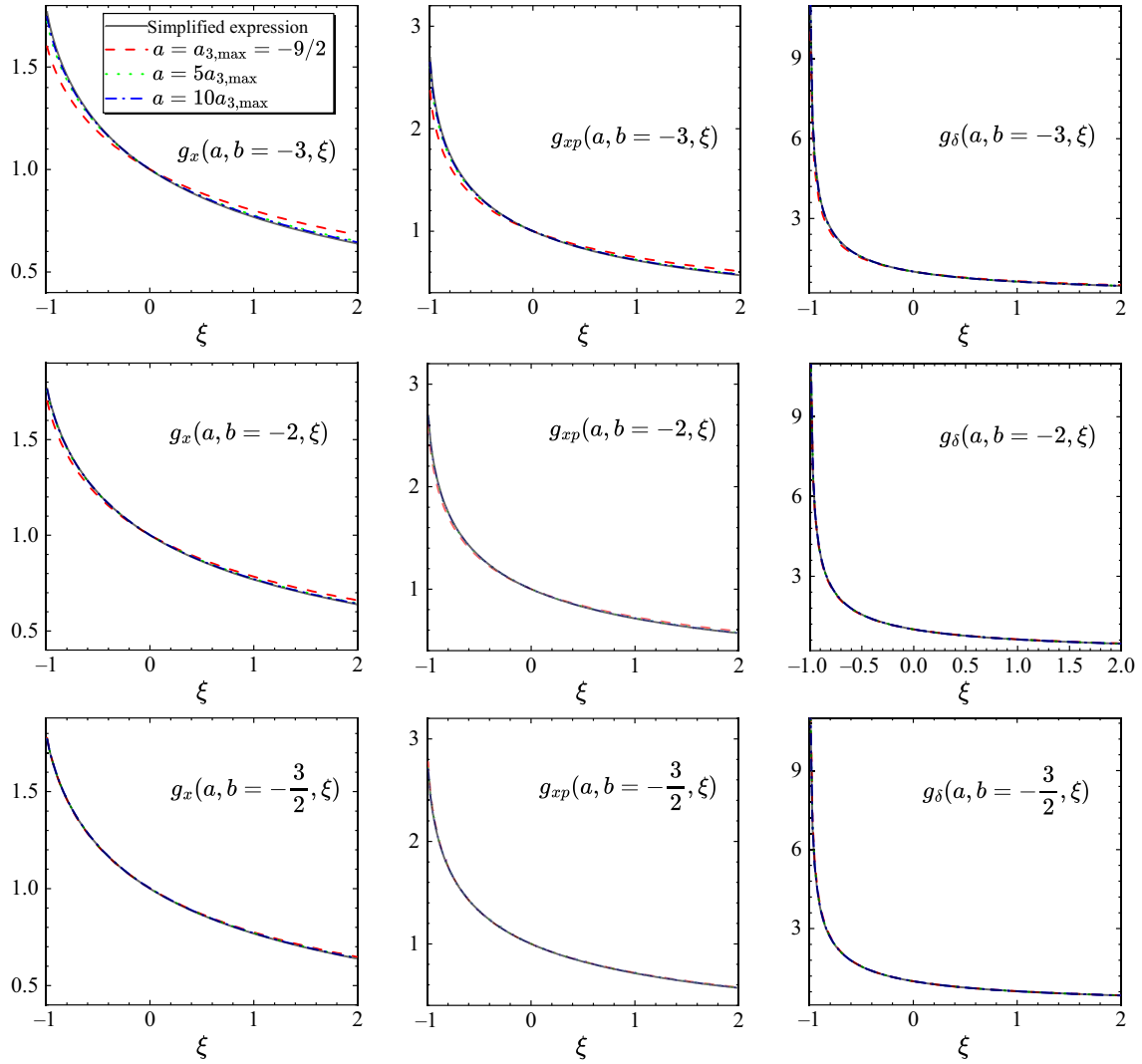


FIG. 16. Comparison of the simplified expressions and integrated results for the cases  $a_3 = a_{3,\max} = -9$ ,  $a_3 = 5a_{3,\max}$ , and  $a = 10a_{3,\max}$  under the case  $b_3 = -3$ ,  $b_3 = -2$ , and  $b_3 = -3/2$ .

One can find that the absolute value  $|a_3|$  is much larger than  $b_3 + 1$ , the same as the case of  $|a_2|$  for the second bend. So the same simplification can be used as the above case [Eq. (C3)]. Figure 16 shows the simplified expressions [Eq. (C3)] and the integrated results for different  $a$  and  $b$ . Here the parameter  $a$  is selected for  $a = a_{2,\max} = -9/2$ ,  $a = 5a_{2,\max}$ ,  $a = 10a_{2,\max}$ , and  $b$  is selected for  $b = -3/2$ ,  $b = -2$ , and  $b = -3$ . One can find that, even for the most extreme case  $a = a_{2,\max}$ ,  $b = -3$ , a relatively small difference can be observed between the approximate expressions and the integrated results.

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