

Strongly tapered field generation by beat frequency undulator

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Undulator tapering is an important technique in free-electron lasers and synchrotron radiation sources, used to enhance radiation efficiency, broaden spectral bandwidth, and perform other related functions. However, conventional tapering methods, such as gap variation or electromagnetic control of individual magnets, involve complex mechanical structures or power supply systems, posing challenges in tuning the taper profile while maintaining good field integral performance in practice. This paper presents a beat frequency undulator that enables dynamic control of undulator tapering over a wide range. The concept employs two permanent magnet arrays with slightly different periods, whose magnetic field superposition generates a longitudinal beat frequency envelope, creating a tunable taper profile without gap adjustment or electromagnetic tuning. By longitudinally shifting one array, the taper strength and direction can be reconfigured in real time. This approach offers inherent advantages in mechanical simplicity and maintains a field integral comparable to that of a standard planar undulator, providing a robust and flexible solution for undulator tapering.

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I. INTRODUCTION

Undulator tapering is a general technique in synchrotron radiation (SR) sources and free-electron lasers (FELs). In SR facilities, tapering is commonly employed to broaden the spectral bandwidth, while in FELs, it is typically applied to maintain the resonance condition, thereby enhancing energy extraction efficiency [1,2]. In particular, a weak taper is essential in a long undulator line to compensate for electron energy losses, preserving electron bunching and FEL gain [3]. The weak taper addresses bandwidth control and FEL gain, while the strong taper is critical for a range of advanced schemes [4–11]. Strongly tapered undulators have been experimentally used to achieve high-efficiency energy extraction, with up to 10% and 30% energy extraction demonstrated using undulator strength parameter K variation ranging from -0.62 m^{-1} to -1.72 m^{-1} [9–11].

Beyond efficiency enhancement, strong reverse taper is also instrumental in inverse free-electron laser accelerators, enabling substantial energy gains with high average accelerating gradients [4,7], generating attosecond x-ray pulses [5], and strengthening the beam bunching process [6].

Direct adjustment of the undulator girder typically achieves only a small taper range. Strong taper has been realized through gap adjustment with progressive variation of the undulator period [12–15]. Advanced designs, such as the variable longitudinal profile undulator developed by SPring-8, illustrate the high mechanical complexity required for maximum flexibility [16]. Although mechanical tapering schemes have proven successful, there remains a strong demand to reduce engineering challenges and costs. On the other hand, electromagnetic undulators, while offering fast dynamic control, are generally limited in peak field strength and suffer from high power consumption and field stability challenges. Therefore, a simple, stable, and cost-effective strong-tapered undulator with field performance comparable to that of a standard planar undulator is still in demand.

In this paper, we propose a simple and novel tapering scheme based on a BFU, an adjustable-phase two-frequency undulator inspired by the asymmetric magnet pole undulator, which was initially proposed for suppressing high-order harmonics or generating variable-polarization radiation with low on-axis heat load [17,18]. The core concept leverages the superposition of magnetic fields from two magnet arrays with slightly different periods, as shown in Fig. 1. This superposition generates a longitudinal beat frequency envelope, imposing a slowly varying amplitude modulation on the fundamental magnetic field, thereby creating an effective taper without mechanical gap changes. The key advantage of this scheme is its dynamic controllability: the taper rate, field

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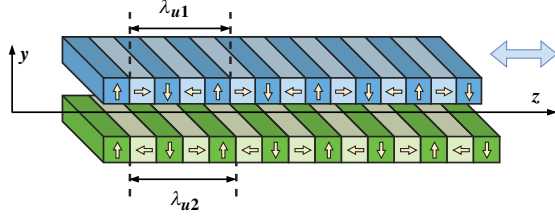


FIG. 1. Schematic of beat frequency undulator (BFU).

strength, position, and direction can be adjusted in real time via simple longitudinal translation of one magnet array. This paper provides an in-depth analysis of the theoretical model and basic performance of the BFU scheme, along with a brief discussion of extended applications.

II. PRINCIPLES

The principle of the BFU is to generate a tapered field by spatially synthesizing an effective undulator field with a slowly varying amplitude. The on-axis field of the BFU can be expressed as the sum of two sinusoidal components:

$$B_y(z) = B_1 \sin(k_1 z + k_1 s) + B_2 \sin(k_2 z), \quad (1)$$

where $k_1 = 2\pi/\lambda_{u1}$ and $k_2 = 2\pi/\lambda_{u2}$ are the wave numbers of the magnetic fields from the upper and lower magnet arrays, B_1 and B_2 are the peak field amplitudes, and s denotes the longitudinal displacement of the upper magnet array relative to the lower one.

Assuming $k_1 > k_2$ and defining $\Delta k = k_1 - k_2$ and $k_0 = (k_1 + k_2)/2$, we obtain:

$$B_y(z) = A(z) \sin[k_0 z + \phi(z)], \quad (2)$$

with

$$A(z) = \sqrt{B_1^2 + B_2^2 + 2B_1 B_2 \cos(\Delta k \cdot z + k_1 s)}$$

$$\phi(z) = \arctan \frac{B_1 \sin(\frac{\Delta k}{2} z + k_1 s) - B_2 \sin(\frac{\Delta k}{2} z)}{B_1 \cos(\frac{\Delta k}{2} z + k_1 s) + B_2 \cos(\frac{\Delta k}{2} z)}. \quad (3)$$

This result is analogous to the “beat frequency” phenomenon in acoustics, where two tuning forks with similar frequencies produce a composite sound with periodically varying volume. The resulting magnetic field consists of a high-frequency carrier wave modulated by a low-frequency envelope $A(z)$, whose spatial variation is governed by the wave number difference Δk . Since Δk is small, the envelope period $\lambda_{en} = 2\pi/|\Delta k|$ can be very long. Additionally, Eqs. (2) and (3) show that the phase shift does not alter the envelope shape but acts as a phase offset, translating the entire envelope along the z axis. This effect can be quantified by solving for the position of the envelope maximum:

$$z_{\max} = \frac{2n\pi - k_1 s}{\Delta k}. \quad (4)$$

By adjusting the phase shift s , the position of the magnetic field envelope peak can be linearly controlled. This allows any desired portion of the envelope—such as its peak, valley, or a segment with a specific slope—to be positioned within the effective undulator length simply by varying s .

For the common case where $B_1 \approx B_2 = B_0$, we obtain:

$$\frac{dA(z)}{dz} = B_0 \Delta k \sin\left(\frac{\Delta k z + k_1 s}{2}\right). \quad (5)$$

This indicates that the taper strength is proportional to the wave number difference Δk and the phase term. A small Δk yields a long beat wavelength and a shallow, highly linear taper gradient. Operating near the envelope peak provides a nearly uniform field, similar to a standard planar undulator. A larger Δk produces a stronger taper gradient but introduces increased nonlinearity near the envelope peak and trough, along with a reduced effective tapering length. The phase shift s allows selection of the most linear segment for interaction. Thus, selecting Δk involves balancing taper strength against profile linearity and interaction length. Notably, the capability to generate a strong-tapered field is determined by the period difference $\Delta\lambda_u$ between the two magnet arrays, rather than the specific period lengths, as indicated by Eq. (5). The BFU concept can be implemented with various period lengths to satisfy different resonant wavelength requirements.

III. UNDULATOR DESIGN

Based on the principles described above, a BFU model was designed using *RADIA* [19,20], with main parameters summarized in Table I. Magnet blocks were selected based on general parameters. The upper and lower magnet arrays have period lengths of 59.9 and 60.1 mm, respectively. The period difference $\Delta\lambda_u = 0.2$ mm, corresponding to approximately 0.33% of the mean period, results in a beating wavelength of $\lambda_{en} \approx 18$ m. Figure 2 shows the normalized envelope over a full beating period, demonstrating that any segment of this amplitude modulation can be utilized as the effective field. Distinct regions of the envelope correspond to specific operational modes: a

TABLE I. Undulator parameters.

Undulator type	Pure permanent magnet
Period length	59.9 mm/60.1 mm
Period number	40
Dimension of magnetic blocks	50 mm × 36 mm
Minimum working gap	20 mm
Remanent magnetization of NdFeB	1.26 T

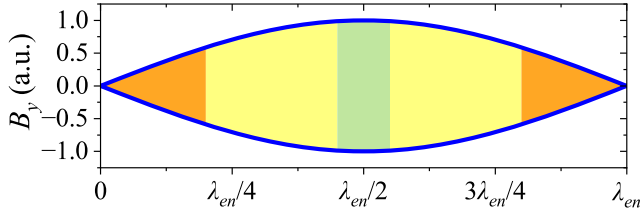


FIG. 2. Normalized envelope over a full beating period (blue curve) of the BFU. The shaded regions highlight distinct configurations: linear taper (orange), quasi-uniform field (green), and nonlinear taper (yellow).

nearly linear taper on the ascending or descending slopes, a quasi-uniform field near the extremum, or a nonlinear taper. For a typical undulator length of about 2.5 m (approximately 14% of λ_{en}), various operational modes become accessible through phase shift.

The envelope position is highly sensitive to phase shift, characterized by a scaling factor $|dz_{\max}/ds| = k_1/\Delta k \approx \lambda_{en}/\lambda_{u1}$. For the proposed parameters, this factor is approximately 300, meaning a longitudinal translation of about one mean period length (~ 60 mm) is sufficient to shift the envelope over a full beating wavelength. The taper profile for different modes, depending on phase shift, can be well predicted by Eq. (4). In the following discussion, we use $z_0 = s - s_q$, where $s_q = 4$ mm is the absolute displacement that places the envelope crest near the center of the present device. In this BFU case, $z_0 = 0$ mm corresponds to the quasi-uniform mode. In Fig. 3, two representative examples are shown. For $z_0 = -16$ mm, the ascending-descending slopes near the envelope nodes yield a strong taper with a highly linear K gradient. The taper rate along the undulator axis is about -0.1 T/m, corresponding to a K variation of -0.55 m $^{-1}$. In contrast, for $z_0 = 0$ mm, alignment with the envelope

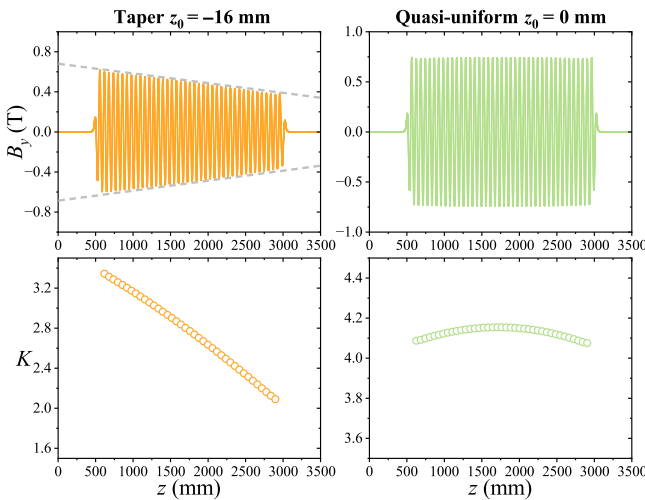


FIG. 3. Two examples of magnetic field profiles and corresponding K values at different phase shifts.

crest produces a quasi-uniform field, where the K variation is minimal and nonlinear.

According to Eq. (5), a larger $\Delta\lambda_u$ yields a stronger gradient but introduces significant nonlinearity near the envelope peak. Conversely, the small period difference adopted here prioritizes better uniformity at the expense of a limited maximum gradient. In the quasi-uniform mode, field reduction at the undulator ends is minimized. Nevertheless, the residual envelope roll-off near the ends still has an important influence on the field performance, such as the nonuniformity distribution for K and phase error, which might broaden the bandwidth in SR sources and degrade FEL gain. To directly investigate the basic undulator radiation performance of the BFU, the flux spectra for four typical undulator configurations are presented in Fig. 4, based on *SPECTRA* [21] and parameters of the Hefei Advanced Light Facility, a 2.2-GeV fourth-generation light source under construction in China [22]. To ensure comparison at the same resonant wavelength, working gaps for the quasi-uniform and taper modes were increased to compensate for peak field reduction and resonant wavelength shift during tapering, with a reference $K = 3.34$.

The quasi-uniform mode exhibits a slightly broader bandwidth due to minor field variations at the envelope extremities. Its fundamental peak flux remains close to that of a normal planar undulator, while the harmonics show noticeable peak-flux reductions. According to the analytical expression derived in Appendix A, the rms phase error of the quasi-uniform mode is estimated to be 12.8° for a gap of 24.1 mm, which is relatively large compared with that of a well-optimized planar undulator. Since the phase error is mainly caused by the residual envelope roll-off near the ends, it is reduced when only the central effective periods are considered; for example, the rms phase error decreases to 2° for the central 20 periods and to 0.7° for the central 10 periods. If a stricter phase quality is required in

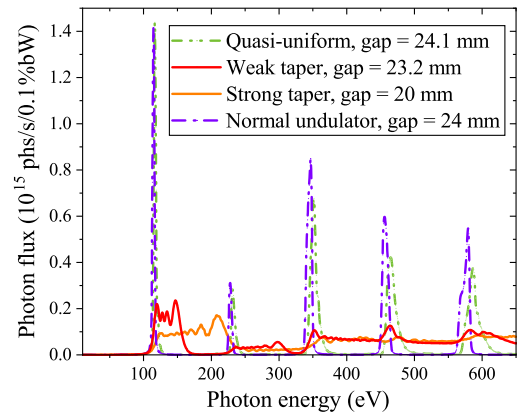


FIG. 4. Flux spectra for a normal undulator, the quasi-uniform mode, and two taper modes of the BFU. The acceptance angle is kept at two multiples of divergence at a photon energy of 100 eV (0.1 mrad) for all cases.

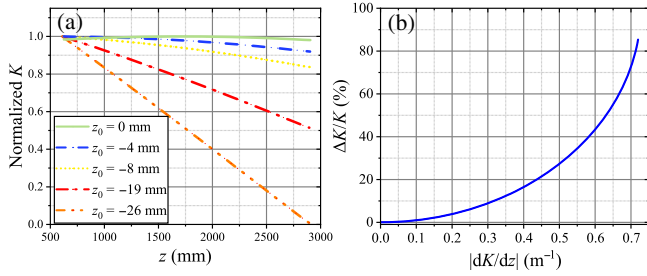


FIG. 5. (a) Normalized K value profile along the longitudinal axis for different phase shift z_0 of the BFU. (b) Relative entrance K value loss as a function of the fitted linear K variation calculated from Eq. (B4).

applications, the length of a single BFU must not be too long. As expected for taper modes, peak-flux reduction is accompanied by spectral bandwidth expansion. The full width at half maximum (FWHM) bandwidth of the fundamental energy increases moderately from 2.35 eV for the normal undulator to 3.5 eV for the quasi-uniform mode. In tapered configurations, this broadening is dramatic, reaching approximately 40 and 100 eV for weak and strong taper modes, respectively. Consequently, discrete harmonics evolve into smooth and continuous spectra.

A defining feature of the BFU is its dynamic tunability. Figure 5(a) illustrates the undulator taper profiles for different phase shifts, demonstrating continuous control over the taper rate. The whole K variation shifts from 0 to $-0.68 m^{-1}$ as the phase shift increases from 0 to -26 mm. The nonlinear component of the taper profile gradually reduces with increasing phase shift. Notably, the device supports reverse taper configurations: by adjusting the phase shift to sample the envelope's ascending slope (e.g., $z_0 \approx 26$ mm), the taper gradient inverts, generating an adjustable reverse taper profile. However, the stronger taper is accompanied by a reduction of the available K value at a fixed gap, which can be estimated analytically in the linear taper regime, as given in Appendix B. The resulting relation between the relative entrance K loss and the fitted linear K variation is shown in Fig. 5(b). For the strong taper case, the K variation is $-0.55 m^{-1}$, giving an entrance K loss of approximately 34.5%. For the maximum linear taper case, the K variation is $-0.68 m^{-1}$, giving an entrance K loss of approximately 65%.

A critical objective in undulator design is to maintain a small field integral across different working states. In the proposed design, the undulator termination is designed with static magnetic structures integrated into the magnet arrays. Figure 6 presents the field integrals for two different modes shown in Fig. 3. The field exhibits good symmetry for both first and second field integrals, returning to zero at the exit. As confirmed by previous studies on the adjustable-phase undulator [23,24], the effectiveness of end correction is independent of phase shift. This demonstrates that the same termination

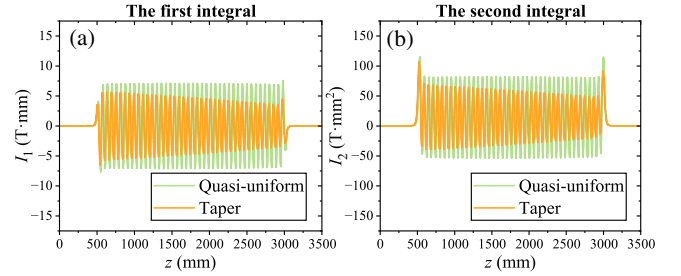


FIG. 6. First and second field integrals for two different modes: the strong taper mode and the quasi-uniform mode from Fig. 3.

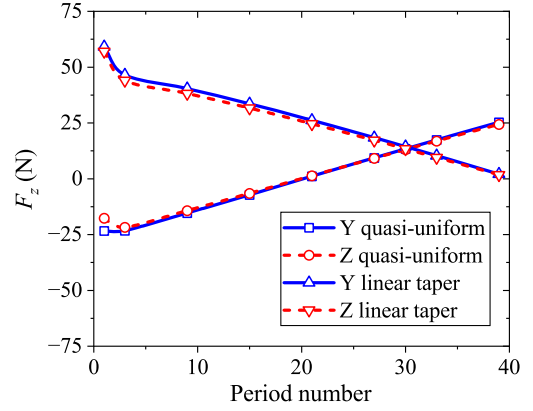


FIG. 7. Calculated longitudinal force F_z on representative magnet blocks in the single array for the quasi-uniform mode and the linear taper mode. Y and Z denote vertically and longitudinally magnetized blocks, respectively.

structure can maintain small field integrals in different phase-shift states. Therefore, the field integral correction does not need to be reconfigured when adjusting the taper profile, even across a large taper range.

Unlike a conventional adjustable-phase undulator, the BFU has two magnet arrays with slightly different periods; hence, the longitudinal magnetic force depends not only on the phase shift z_0 but also on the longitudinal position. Figure 7 shows the calculated longitudinal force F_z on representative magnet blocks for the quasi-uniform and linear taper modes. Both the vertically magnetized blocks and the longitudinally magnetized blocks exhibit a clear position-dependent force distribution. In the linear taper mode, the longitudinal force decreases from about 60 N near the upstream high-field region to nearly zero near the downstream low-field region. The maximum longitudinal force for BFU is at the same level as the conventional adjustable-phase undulator. Therefore, it should be acceptable for a general mechanical structure.

IV. EXTENDED APPLICATIONS

Having demonstrated the features of the BFU concept using basic undulator radiation as an example, we now

discuss its applications in FEL facilities. In FELs, undulator tapering is primarily employed to enhance energy efficiency, for which various taper strategies have been developed. Both linear and nonlinear tapers are known to be effective in high-efficiency FELs. The BFU addresses this need by enabling dynamic control over the taper profile, transitioning from quasi-uniform to nonlinear, and further to linear taper with high taper rates. To quantify the tunability of taper profiles, the normalized K values in Fig. 5(a) were analyzed using a second-order polynomial fit $K_{\text{norm}}(z) \approx C + \alpha z + \beta z^2$. For the strong taper mode at $z_0 = -26$ mm, the profile exhibits high linearity with a dominant linear rate $|\alpha| \approx 0.42 \text{ m}^{-1}$ and negligible quadratic term $|\beta| \approx 4.6 \times 10^{-3} \text{ m}^{-2}$. Shifting to $z_0 = 0$ mm increases curvature to $|\beta| \approx 0.014 \text{ m}^{-2}$. The nonlinearity in this specific design is small due to the minor period difference selected for optimizing field uniformity. A strong quadratic taper can be achieved by increasing $\Delta\lambda_u$ with a suitable phase shift, as indicated by Eq. (5). For instance, increasing $\Delta\lambda_u$ from 0.2 to 0.8 mm, with a phase shift z_0 of 0 mm and other parameters unchanged, yields a larger quadratic term $|\beta| \approx 0.23 \text{ m}^{-2}$, accompanied by a synchronous increase in the linear term $|\alpha| \approx 0.8 \text{ m}^{-1}$, maintaining the ratio between linear and quadratic components. These tunable taper ranges can match typical linear and nonlinear taper profiles used in high-efficiency FEL operations, as demonstrated in related experiments [10,25–27]. Since a strong reverse taper is available in the BFU, it also has potential for inverse free-electron laser acceleration. Additionally, benefiting from its inherent advantage in field integral control, the complexity of related FEL online commissioning can be reduced, especially for a long undulator line with a high taper rate.

Building upon the core BFU concept, its flexibility can be further extended to additional applications in SR sources. For example, achieving a large and symmetrical FOV for synchrotron radiation imaging has attracted increasing attention recently. The Mango wiggler has been developed to increase FOV by employing two orthogonal wigglers with a small period difference [28]. The BFU provides a more dynamic method to achieve similar, if not superior, outcomes. By arranging two BFU devices orthogonally, the phase shift in each can be adjusted independently, enabling continuous manipulation of electron trajectories from linear to circular or spiral motion, effectively replicating the principles of the Mango wiggler and other proposed schemes.

To demonstrate this, we consider a DELTA-type structure comprising two orthogonal 18-period BFUs. Design parameters include an undulator gap of 14 mm, period lengths of 119 and 121 mm, and a remanent magnetization of 1.32 T. Figure 8 illustrates magnetic fields and corresponding electron trajectories for two representative configurations. The first case uses a phase difference of $\pi/2$,

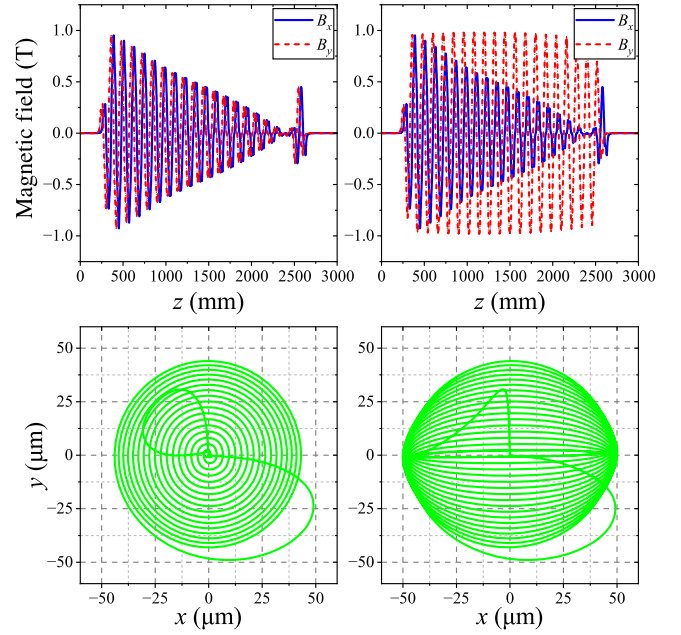


FIG. 8. Magnetic fields and corresponding electron trajectories for generating a large field of view (FOV). The left column shows a helical mode, and the right column shows a Mango mode.

with both B_x and B_y tapered from maximum to zero, yielding a helical-like trajectory, as shown in Figs. 8(a) and 8(c). The second case, with a $3\pi/4$ phase difference, maintains uniform B_y while varying B_x from maximum to zero, producing a “mangolike” trajectory, as shown in Figs. 8(b) and 8(d). The capability to synthesize such complex trajectories demonstrates the potential of the BFU to provide a large, symmetrical FOV for SR imaging.

V. SUMMARY

In this paper, we proposed and theoretically analyzed a novel concept based on the beat frequency undulator. It can generate a strongly tapered magnetic field by leveraging the beat frequency envelope produced by the superposition of fields from magnet arrays with slightly different period lengths. Different taper profiles, including positive, reverse, and quasi-uniform field configurations, can be dynamically selected via simple longitudinal translation. The BFU scheme allows for a simple structural design and presents a flexible alternative to conventional tapering technologies, ensuring good operational robustness. It eliminates the need for complex, high-precision control and optimization associated with gap adjustment or power supply systems, thereby mitigating engineering challenges related to stringent stability requirements. In summary, the BFU represents an attractive solution for undulator tapering in related FEL and SR applications.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: PHASE ERROR IN QUASI-UNIFORM MODE

The on-axis radiation phase corresponding to the reference undulator parameter K can be written as [29,30]:

$$\Phi(z; K) = \frac{2\pi}{\lambda_u(1 + K^2/2)} \times \left[z + \left(\frac{e}{m_0 c} \right)^2 \int_{z_s}^z \left(\int_{z_s}^{z'} B_y(z'') dz'' \right)^2 dz' \right], \quad (\text{A1})$$

where λ_u is the undulator period and z_s is chosen sufficiently far upstream of the magnetic field. For an ideal undulator, the phase advance over one period is 2π . For the BFU operated in the quasi-uniform mode, the residual phase error originates from the weak roll-off of the beat envelope near the two ends. For the symmetric case $B_1 \approx B_2 = B_0$, the envelope near its maximum can be written as:

$$A(\xi) = 2B_0 \cos\left(\frac{\Delta k}{2}\xi\right), \quad (\text{A2})$$

where $\xi = z - z_{\max}$ is the longitudinal position relative to the envelope maximum. The local undulator parameter in the quasi-uniform mode can be approximated as:

$$K_i = K_{\max} \cos\left(\frac{\Delta k}{2}\xi_i\right), \quad (\text{A3})$$

where $\xi_i = z_i - z_{\max}$ is the position of the i -th period relative to the envelope maximum and $K_{\max} = e(2B_0)/m_0 c k_0$ is the max undulator parameter. Substituting Eq. (A3) into Eq. (A1), the phase error over the i -th period is:

$$\delta\Phi_{\text{per},i} = 2\pi \frac{K_{\max}^2 \cos^2(\Delta k \xi_i/2) - K^2}{2 + K^2}. \quad (\text{A4})$$

According to the Eqs. (A3) and (A4), the phase error and K highly depend on Δk and the position ξ_i of the i -th period. Therefore, in the quasi-uniform mode, the rms phase error can be kept at a moderate level with a short total undulator length. In the strong taper mode, by

contrast, the K variation inevitably gives rise to a large phase error.

The cumulative phase error at the beginning of the j -th period is then obtained by summing the period-by-period phase errors:

$$\Psi_j = \sum_{i=1}^{j-1} \delta\Phi_{\text{per},i}, \quad j = 1, 2, \dots, N. \quad (\text{A5})$$

Here N is the number of periods included in the phase-error evaluation. The rms phase error is calculated after subtracting the mean cumulative phase error:

$$\sigma_\Phi = \sqrt{\frac{1}{N-1} \sum_{j=1}^N (\Psi_j - \bar{\Psi})^2}. \quad (\text{A6})$$

APPENDIX B: RELATION BETWEEN TAPER STRENGTH AND K VALUE LOSS

For a simple analytical estimate, we consider the linear taper regime, where the entrance value is the maximum K value within the tapered section. For the symmetric case $B_1 \approx B_2 = B_0$, the local K parameter is proportional to the beat envelope and can be written as:

$$K(z) = K_{\max} \left| \cos\left(\frac{\Delta k z + k_1 s}{2}\right) \right|, \quad (\text{B1})$$

where $K_{\max}$ is the peak K value at the envelope maximum, corresponding to the maximum available K in the quasi-uniform mode at the same gap. Taking the entrance of the tapered section as the reference position and defining the entrance envelope phase as φ_{in} , the entrance K value is $K_{\text{in}} = K_{\max} |\cos(\varphi_{\text{in}})|$. The taper strength can be estimated by the fitted linear K variation:

$$G_K = \left| \frac{dK}{dz} \right| \approx K_{\max} \cdot \frac{|\Delta k \cdot \sin \varphi_{\text{in}}|}{2}. \quad (\text{B2})$$

Eliminating φ_{in} gives the relation between the entrance K loss and the K variation:

$$\left(\frac{K_{\text{in}}}{K_{\max}} \right)^2 + \left(\frac{2G_K}{K_{\max} |\Delta k|} \right)^2 = 1, \quad (\text{B3})$$

or

$$\frac{\Delta K}{K_{\max}} = 1 - \sqrt{1 - \left(\frac{2G_K}{K_{\max} |\Delta k|} \right)^2}, \quad (\text{B4})$$

where $\Delta K = K_{\max} - K_{\text{in}}$. Equation (B4) indicates that the relative K loss can be increased if a large gradient is selected.

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