

Intrabeam scattering studies with large-emittance-ratio ion beams in the relativistic heavy ion collider and implications for the electron-ion collider

Y. Luo^{✉,*}, B. Lepore, K. Mernick[✉], W. Bergan[✉], and S. Nagaitsev[✉]

Brookhaven National Laboratory, Upton New York, 11973, USA



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The electron-ion collider (EIC), to be constructed at Brookhaven National Laboratory, will collide polarized, high-energy electron beams with hadron beams, achieving peak luminosities of up to $1.0 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$. To reach such high luminosity, the EIC will employ flat-beam collisions at the interaction point. The design transverse emittance ratio will be about 10:1 in the hadron storage ring. Thanks to stochastic cooling and precise decoupling, we successfully generated and accelerated gold ion beams with a large emittance ratio of 11:1 in the relativistic heavy ion collider. In this article, we present results of intrabeam scattering (IBS) measurements and modeling for large-emittance-ratio gold ion beams, both without and with controlled betatron coupling. To model the IBS growth, we use the formulas developed by Lebedev and Nagaitsev.

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I. INTRODUCTION

The electron-ion collider (EIC), to be constructed at Brookhaven National Laboratory, will collide polarized, high-energy electron beams with hadron beams, achieving peak luminosities of up to $1.0 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ over a center-of-mass energy range of 20–140 GeV [1]. The EIC consists of two storage rings: the hadron storage ring (HSR) and the electron storage ring, both of which will be housed in the existing tunnel of the relativistic heavy ion collider (RHIC) [2]. The HSR will reuse the arcs of the RHIC yellow ring and connect them with new straight sections. RHIC was decommissioned in Feb. 2026.

To reach high luminosities, the EIC will employ flat beams at the interaction point (IP). Flat ion beams can be realized through unequal transverse emittances and β functions at the IP, with ϵ_x/ϵ_y and β_x^*/β_y^* , both on the order of 10 [1]. Owing to stochastic cooling and precise decoupling, we demonstrated a transverse emittance ratio of 11:1 in RHIC in 2023 using gold ion beams [3,4]. In 2025, we further demonstrated acceleration of large-emittance-ratio ion beams from 31 to 100 GeV/nucleon [5]. These results validate the HSR design assumption that large-emittance-ratio hadron beams can be generated at injection and preserved during acceleration to collision energy.

The availability of flat beams in RHIC enables a range of EIC-relevant beam physics studies. In this work, we present intrabeam scattering (IBS) measurements and modeling for gold ion beams with large transverse emittance ratios, both without and with controlled betatron coupling. This is an experimental study benchmarking IBS growth for hadron beams with large emittance ratios, as well as to include deliberately introduced betatron coupling.

The theory of IBS in particle accelerators was first developed by Piwinski [6] and later extended by Martini [7] to account for variations in the optical parameters around the ring. An alternative approach was developed by Bjorken and Mtingwa (B–M) using quantum mechanics [8]. Over the years, several high-energy approximations to the full theory have been developed to speed up IBS growth rate computing [9–12]. Nagaitsev subsequently developed a fast numerical method for calculating IBS growth rates based on the B–M formalism [13]. In Ref. [14], Wilson Prize recipients Piwinski, Bjorken, and Mtingwa presented a comprehensive review of the theoretical developments in IBS.

Both the Piwinski and B–M approaches solve the local two-particle Coulomb scattering problem for six-dimensional, uncoupled Gaussian beams. More recently, Lebedev and Nagaitsev developed an alternative set of formulas based on the Landau collision integral from plasma physics [15]. Combined with the coupling parameterization developed by Lebedev and Bogacz [16], these formulas enable IBS calculations that include betatron coupling. In the uncoupled limit, these formulas reduce to the B–M expressions but are expressed in terms of the elliptic integral of the second kind, which can be evaluated efficiently using the method described in Refs. [13,17,18].

*Contact author: yluo@bnl.gov

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Although Piwinski also derived IBS growth rate expressions, including betatron coupling [19], their practical use is limited by the complexity of the eigenvectors and the double integrals involved. For electron storage rings with synchrotron radiation, Kubo and Oide proposed a beam-envelope approach [20] that accounts for betatron coupling, although it does not explicitly provide the IBS diffusion matrix.

For high-energy hadrons (e.g., 100 GeV/nucleon in RHIC), IBS growth is usually dominated by the longitudinal and horizontal planes, with horizontal growth driven by dispersion along the ring [9,10]. In contrast, direct vertical IBS growth is negligible due to the small vertical dispersion; instead, vertical emittance growth arises primarily through betatron coupling.

Consequently, many previous IBS studies for hadron beams assumed either the same transverse beam emittances or equal transverse IBS growth rates (“fully coupled” conditions) [21–24]. For beams with large emittance ratios, however, such assumptions are no longer valid. In our experiments, the horizontal emittance exceeds the vertical by approximately an order of magnitude. To generate and maintain such beams, residual coupling must be carefully minimized.

Under these conditions, horizontal and vertical IBS growth must be treated independently. Experimentally, we observe distinct growth rates in the two transverse planes, which also depend on the ion beam energy. Accordingly, we abandon the fully coupled approximation. In this work, we employ the Lebedev–Nagaitsev formulas to treat the growth of transverse eigenmodes separately and consistently, both without and with controlled betatron coupling.

The paper is organized as follows. We first review betatron coupling theory, the Lebedev–Nagaitsev IBS formulas, and the experimental setup. We then present IBS measurements and modeling for well-decoupled beams with large transverse emittance ratios at 31 and 100 GeV/nucleon. Next, we analyze IBS growth of ion beams with large emittance ratios under controlled weak coupling at 31 GeV/nucleon. Finally, we estimate IBS growth times in the EIC HSR at various beam energies with different flatness and weak betatron coupling.

II. THEORIES AND EXPERIMENT SETUP

A. Betatron coupling

There are two main factors affecting the preservation of a large transverse emittance ratio in the HSR EIC: betatron coupling and IBS. Betatron coupling resonance couples the horizontal and vertical motions of particles, leading to changes in the betatron tunes and the mixing of transverse emittances [25,26]. The sources of betatron coupling include detector solenoids, skew quadrupoles, and residual coupling sources such as quadrupole roll errors and vertical closed orbits in sextupoles.

For random sources of betatron coupling in a ring, Hamiltonian perturbation theory is more suitable for analytical estimates than the strict matrix-based approach. In the following, we use the amplitude of the coupling coefficient defined in Hamiltonian perturbation theory to measure the strength of global coupling, which is defined as [26]:

$$C^- = \frac{1}{2\pi} \oint \sqrt{\beta_x \beta_y} \left[k_{1s} + k_s \left(\frac{\alpha_x}{\beta_x} - \frac{\alpha_y}{\beta_y} \right) - ik_s \left(\frac{1}{\beta_x} + \frac{1}{\beta_y} \right) \right] e^{i(\Psi_x - \Psi_y)} dl. \quad (1)$$

Here k_{1s} and k_s are the strengths of skew quadrupoles and solenoids, respectively, $\beta_{x,y}$ and $\alpha_{x,y}$ are the unperturbed Twiss parameters, and $\Psi_{x,y}$ are the unperturbed betatron phase advances.

With betatron coupling, both horizontal and vertical motions are composed of two transverse eigenmodes, with eigentunes $Q_{1,2}$ and eigen-emittances $\epsilon_{1,2}$. For weak coupling, eigenmode 1 is typically associated with the horizontal plane and eigenmode 2 with the vertical plane.

The eigentunes can be analytically evaluated through perturbation theory:

$$Q_1 = Q_{x,0} - \frac{\Delta}{2} + \frac{1}{2} \sqrt{\Delta^2 + |C^-|^2}, \quad (2)$$

$$Q_2 = Q_{y,0} + \frac{\Delta}{2} - \frac{1}{2} \sqrt{\Delta^2 + |C^-|^2}. \quad (3)$$

Here $Q_{x,y,0}$ are the fractional uncoupled tunes and $\Delta = Q_{x,0} - Q_{y,0}$. The fractional eigentune split with coupling is

$$|Q_1 - Q_2| = \sqrt{\Delta^2 + |C^-|^2}. \quad (4)$$

The minimum tune split is equal to the amplitude of the coupling coefficient $|C^-|$ when $\Delta = 0$. $\Delta = 0$ corresponds to the difference coupling resonance. In some literature, $|C^-|$ is referred to as the minimum tune split $|\Delta Q_{\min}|$.

Betatron coupling also modifies transverse beam sizes along the ring. Knowing the eigen-emittances $\epsilon_{1,2}$, the beam size matrix with coupling can be calculated as [27,28]:

$$\Sigma_X = \mathbf{P} \begin{pmatrix} \epsilon_{1,\text{rms}} & 0 & 0 & 0 \\ 0 & \epsilon_{1,\text{rms}} & 0 & 0 \\ 0 & 0 & \epsilon_{2,\text{rms}} & 0 \\ 0 & 0 & 0 & \epsilon_{2,\text{rms}} \end{pmatrix} \mathbf{P}', \quad (5)$$

where $\mathbf{P}$ contains Edwards–Teng’s Twiss and coupling parameters:

$$\mathbf{P} = \begin{pmatrix} r\sqrt{\beta_1} & 0 & c_{11}\sqrt{\beta_2} - \frac{c_{12}\alpha_2}{\sqrt{\beta_2}} & \frac{c_{12}}{\sqrt{\beta_2}} \\ -\frac{\alpha_1 r}{\sqrt{\beta_1}} & \frac{r}{\sqrt{\beta_1}} & c_{21}\sqrt{\beta_2} - \frac{c_{22}\alpha_2}{\sqrt{\beta_2}} & \frac{c_{22}}{\sqrt{\beta_2}} \\ -\frac{c_{12}\alpha_1}{\sqrt{\beta_1}} - c_{22}\sqrt{\beta_1} & \frac{c_{12}}{\sqrt{\beta_1}} & r\sqrt{\beta_2} & 0 \\ \frac{c_{11}\alpha_1}{\sqrt{\beta_1}} + c_{21}\sqrt{\beta_1} & -\frac{c_{11}}{\sqrt{\beta_1}} & -\frac{\alpha_2 r}{\sqrt{\beta_2}} & \frac{r}{\sqrt{\beta_2}} \end{pmatrix}. \quad (6)$$

In the betatron parameterization by Lebedev and Bogacz [16], $\mathbf{P}$ is denoted as $\mathbf{V}$, since it can be directly constructed from the eigenvectors of the one-turn map, too. With betatron coupling, the eigen-emittances cannot be directly measured. In the following experiments, knowing the eigen-emittances from modeling, we can use Eq. (6) to calculate transverse beam sizes under coupled conditions.

B. Lebedev–Nagaitsev formulas

Both Piwinski and Bjorken–Mtingwa derived IBS formulas starting from first principles of two-body Coulomb collisions. Recently, Lebedev and Nagaitsev developed another set of IBS formulas based on the Landau collision integral from plasma physics [15]. For a uniform, non-relativistic plasma with a Gaussian distribution, the time derivatives of rms velocities in the beam frame are

$$\frac{d}{dt} \begin{pmatrix} \sigma_{vx}^2 \\ \sigma_{vy}^2 \\ \sigma_{vz}^2 \end{pmatrix} = \frac{(2\pi)^{3/2} e^4 n L_c}{m^2 \sqrt{\sigma_{vx}^2 + \sigma_{vy}^2 + \sigma_{vz}^2}} \times \begin{pmatrix} \Psi(\sigma_{vx}, \sigma_{vy}, \sigma_{vz}) \\ \Psi(\sigma_{vy}, \sigma_{vz}, \sigma_{vx}) \\ \Psi(\sigma_{vz}, \sigma_{vx}, \sigma_{vy}) \end{pmatrix}, \quad (7)$$

with

$$\Psi(x, y, z) = \frac{\sqrt{2}r}{3\pi} (y^2 R_D(z^2, x^2, y^2) + z^2 R_D(x^2, y^2, z^2) - 2x^2 R_D(y^2, z^2, x^2)), \quad (8)$$

$$R_D(x, y, z) = \frac{3}{2} \int_0^\infty \frac{dt}{\sqrt{(t+x)(t+y)(t+z)}}, \quad (9)$$

$$r = \sqrt{x^2 + y^2 + z^2}. \quad (10)$$

Here n is the plasma density, m and e are the charged particle's mass and charge, and L_c is the Coulomb logarithm. $R_D(x, y, z)$ is the elliptic integral of the second kind.

To apply the above equations to IBS growth in accelerators, we express the rms velocities in the beam frame in terms of the rms momenta (p_x , p_y , and p_z) in the laboratory frame, from which the derivatives of the eigen-emittances can be derived. Using the 4D betatron

coupling parameterization by Lebedev and Bogacz, Lebedev and Nagaitsev obtained:

$$\frac{d\epsilon_q}{dt} = \frac{Nr_0^2 c}{3\sqrt{\pi}\beta^3\gamma^4 C\epsilon_1\epsilon_2\sigma_p} [L_c\sigma_1\sigma_2\sigma_3 \times (\sigma_1^2 R_D(\sigma_2^2, \sigma_3^2, \sigma_1^2)(\text{Trace}(\mathbf{a}^q) - 3[\mathbf{T}'\mathbf{a}^q\mathbf{T}]_{11}) + \sigma_2^2 R_D(\sigma_3^2, \sigma_1^2, \sigma_2^2)(\text{Trace}(\mathbf{a}^q) - 3[\mathbf{T}'\mathbf{a}^q\mathbf{T}]_{22}) + \sigma_3^2 R_D(\sigma_1^2, \sigma_2^2, \sigma_3^2)(\text{Trace}(\mathbf{a}^q) - 3[\mathbf{T}'\mathbf{a}^q\mathbf{T}]_{33}))], \quad (11)$$

$$\frac{d\sigma_p^2}{dt} = \frac{Nr_0^2 c}{3\sqrt{\pi}\beta^3\gamma^4 C\epsilon_1\epsilon_2\sigma_p} [L_c\sigma_1\sigma_2\sigma_3 \times (\sigma_1^2 R_D(\sigma_2^2, \sigma_3^2, \sigma_1^2)(\text{Trace}(\mathbf{a}^s) - 3[\mathbf{T}'\mathbf{a}^s\mathbf{T}]_{11}) + \sigma_2^2 R_D(\sigma_3^2, \sigma_1^2, \sigma_2^2)(\text{Trace}(\mathbf{a}^s) - 3[\mathbf{T}'\mathbf{a}^s\mathbf{T}]_{22}) + \sigma_3^2 R_D(\sigma_1^2, \sigma_2^2, \sigma_3^2)(\text{Trace}(\mathbf{a}^s) - 3[\mathbf{T}'\mathbf{a}^s\mathbf{T}]_{33}))]. \quad (12)$$

Equations (11) and (12) are reproduced here from Eqs. (53) and (54) in Ref. [15]. Here N is the particle population per bunch, r_0 is the classical radius of the charged particle, and c is the speed of light. β and γ are relativistic parameters. ϵ_q , with $q = 1, 2$, are the transverse eigen-emittances, and σ_p is the rms momentum spread. $\sigma_{1,2,3}$ are uncoupled rms momenta in the beam frame. For definitions of $\mathbf{T}$, $\mathbf{a}^q$, and $\mathbf{a}^s$, see Ref. [15]. Note that the matrix $\mathbf{a}^q$ has the dimension of *length*, while both $\mathbf{T}$ and $\mathbf{a}^s$ are dimensionless. These quantities are expressed in terms of coupled optics parameters, which connect eigen-emittances to rms values of momenta.

To obtain Eqs. (11) and (12), a Gaussian distribution in all six dimensions is assumed. Typically, for weakly coupled beams in hadron accelerators, the distribution remains close to Gaussian, which justifies the use of these formulas in the present study. As mentioned earlier, in the absence of coupling, the Lebedev–Nagaitsev IBS formulas reduce to the Bjorken–Mtingwa formulas but expressed in terms of R_D , which accelerates the evaluation of IBS growth rates.

C. Experiment setup

In RHIC, the transverse emittances were measured using ionization profile monitors (IPMs) [29,30], one for the horizontal plane and one for the vertical plane. They were installed at different locations in the ring, with horizontal and vertical β -functions of approximately 200 m, respectively. The IPMs directly measure the transverse beam profiles and fit them to Gaussian distributions to derive the rms beam sizes. Together with the model β -functions at the IPM locations, the transverse emittances are reported. The main uncertainty in the IPM emittance measurement arises from the uncertainty in the β -functions at the IPMs. There is approximately a 10%

β -beat uncertainty from the lattice model in RHIC. In this article, 95% normalized transverse emittances are used for experimental data, which is

$$\epsilon_{95\%,\text{norm}} = 6\epsilon_{\text{norm}} = 6\beta\gamma\epsilon_{\text{rms}}. \quad (13)$$

Here, the rms emittance $\epsilon_{\text{rms},x,y} = \sigma_{x,y}^2/\beta_{x,y}$, with $\sigma_{x,y}$ and $\beta_{x,y}$ as transverse beam size and β -function.

The bunch length was measured using a wall current monitor (WCM) in RHIC. In the following experiments, we used 28 equally spaced gold ion ($^{197}\text{Au}^{79+}$) bunches to improve the quality of the IPM emittance measurements. The longitudinal profile of each bunch can be directly measured. In RHIC, the bunch length is typically reported using the full width at half maximum (FWHM). In the following experiments, a Gaussian distribution is assumed, for which the FWHM bunch length is 2.355 times the rms bunch length. According to Ref. [22], the uncertainty in the RHIC bunch length measurement is about 0.25 ns, or 7.5 cm. The typical rms ion bunch length in these experiments is greater than 1.4 m.

In 2006, we successfully demonstrated and implemented a global decoupling feedback system at RHIC [31,32]. This system relies on continuous measurement of the coupling coefficient, achieved through a baseband phase-locked loop tune meter. The coupling coefficient is derived from the measured eigenmode amplitude projections onto the transverse axes. Since the coupling coefficient is a complex number, we sorted the existing skew quadrupoles at RHIC into two orthogonal families to effectively correct the global coupling. Since then, decoupling feedback has become an integral part of routine RHIC operation, applied during injection, at the store, and during acceleration. In RHIC, after applying decoupling feedback, the amplitude of the coupling coefficient $|C^-|$ is typically about 0.001.

To generate large emittance ratios for IBS studies, stochastic cooling was used in our experiments. Operational longitudinal cooling of bunched ion beams was first achieved in RHIC in 2007 [33], followed by vertical cooling in 2011. In 2012, with the addition of the horizontal plane, full three-dimensional stochastic cooling became operational [34]. To maximize the transverse emittance ratio, horizontal-plane cooling is turned off while vertical-plane cooling is maintained. In this configuration, IBS increases the horizontal emittance while stochastic cooling suppresses the vertical emittance, thereby maximizing the transverse emittance ratio [4].

III. IBS FOR WELL-DECOUPLED BEAMS

For the well-decoupled case, we use the measurement data from a recent beam experiment of accelerating large-emittance ion beams in RHIC, conducted on July 23, 2025. Figure 1 shows the measured transverse emittances $\epsilon_{x,y}$ and the emittance ratio $r = \epsilon_x/\epsilon_y$ during the entire process.

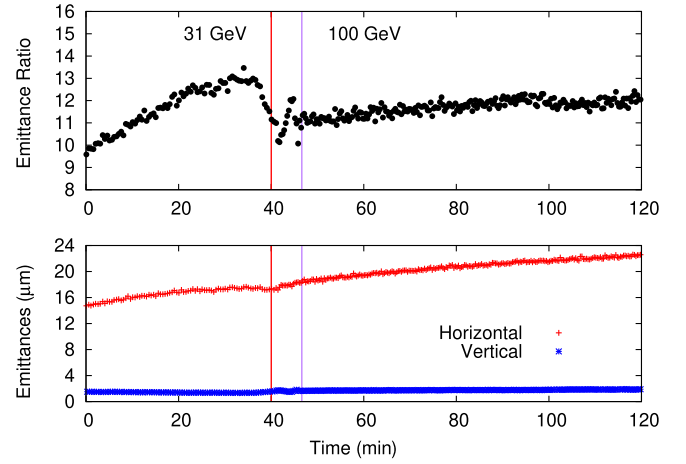


FIG. 1. Measured transverse emittances in the bottom plot and the transverse emittance ratio in the top plot for the experiment conducted on July 23, 2025.

Between the two vertical lines, the beam was accelerated from 31 to 100 GeV/nucleon.

In this experiment, 28 gold ion bunches were used in the yellow ring of RHIC. The average ion population was about 0.6×10^9 per bunch prior to acceleration. At 31 GeV/nucleon, we first generated a large emittance ratio by applying only vertical stochastic cooling to reduce the vertical emittance while allowing the horizontal emittance to grow due to IBS. In the following experiments, it typically takes about 2.5 h to reach a transverse emittance ratio of approximately 10:1.

When the emittance ratio reached about 11:1, the beam was accelerated to 100 GeV/nucleon. With high-performance control of orbit, tune, and coupling during the energy ramp, the large emittance ratio was successfully preserved. At 100 GeV/nucleon, the large-emittance-ratio beam was maintained for approximately 2 h.

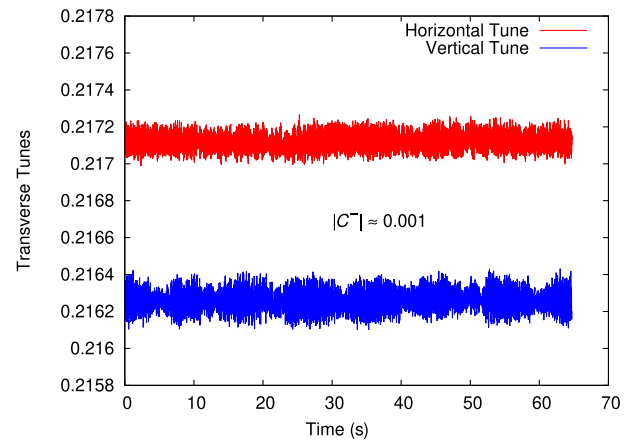


FIG. 2. Betatron tunes during measurement of the minimum tune split, or the amplitude of coupling coefficient $|C^-|$, at 100 GeV/nucleon.

The transverse fractional tunes were kept at (0.238, 0.212) at 31 GeV/nucleon and during the acceleration to 100 GeV/nucleon. At store, these tunes were maintained for the first 50 min. The tunes were then moved to the HSR design values (0.238, 0.210), where the beam was kept for approximately another hour.

Throughout the experiment, betatron coupling was corrected using decoupling feedback. In this experiment, we measured the minimum tune split, i.e., the amplitude of the coupling coefficient $|C^-|$, at 100 GeV/nucleon before the end of this experiment, as shown in Fig. 2. Since the tune split was 0.026 during cooling, acceleration, and at the beginning of store, which was more than an order of magnitude larger than the residual coupling amplitude, we assumed that the yellow ring was well decoupled during this experiment.

A. Modeling at 31 GeV/nucleon

Just before accelerating the large-emittance-ratio beam, we intentionally paused for about 5 min without any cooling. The initial purpose was to ensure that stochastic cooling was completely switched off. Here, we model the evolution of the transverse emittances and bunch length due to IBS during this period. Table I lists the initial measured beam parameters used for IBS modeling, along with the IBS growth times obtained from both measurement and modeling.

For IBS modeling, the rms momentum spread is required but cannot be measured directly. In this experiment, 400 kV of 28 MHz rf cavities were used. Given the measured rms bunch length and the lattice model, the rms momentum spread can be determined via single-particle tracking around the ring with the rf cavities on.

TABLE I. Initial beam parameters for IBS modeling and IBS growth times from both measurement and modeling at 31 GeV/nucleon.

Quantity	Unit	Value
Initial parameters:		
Bunch intensity	10^9	0.57
Horizontal emittance	μm	17.3
Vertical emittance	μm	1.33
Bunch length	m	1.47
Momentum spread	10^{-3}	1.30
IBS growth times from measurement:		
Horizontal emittance	h	-10.0
Vertical emittance	min	25.1
Bunch length	h	239
IBS growth times from modeling:		
Horizontal emittance	h	-167
Vertical emittance	min	25.7
Bunch length	h	31.6

During these 5 min at 31 GeV/nucleon, we observed about a 19.1% increase in the vertical emittance, while the horizontal emittance and bunch length remained nearly unchanged. The transverse emittance ratio decreased from 13.5:1 to about 11:1. Figure 3 shows the average bunch intensity, transverse emittances, and bunch length. The measurement data are shown in red, and the IBS modeling results are shown in blue.

In these experiments, we do not attempt to compare instantaneous IBS growth times between measurement and modeling, as such quantities cannot be determined precisely and typically carry large uncertainties. Instead, we focus on the net changes over a finite observation period. For simplicity, we define an average growth time $\bar{\tau}$ using the initial and final values of a quantity as:

$$y_1 = y_0 \exp(\Delta t / \bar{\tau}), \quad (14)$$

where y_0 and y_1 are the initial and final values and Δt is the duration of the observation period. From the measurements, the average growth time for the vertical emittance is 25.1 min. The growth times for the horizontal emittance and bunch length are -10 and +239 h, respectively, and can be considered effectively infinite.

As shown in Fig. 3, there is negligible bunch intensity loss during these 5 min, and it is therefore neglected in the IBS modeling. In the simulation, the IBS growth rates, emittances, bunch length, and momentum spread are updated every second. The modeling reproduces the measurements well. From the simulation, the average growth time for the vertical emittance is about 25.7 min, in good agreement with the measured value. The growth times for the horizontal emittance and bunch length are effectively negative and positive infinity, respectively. Table I summarizes the average IBS growth times from both measurement and modeling.

B. Modeling at 100 GeV/nucleon

Next, we model the IBS measurement data at 100 GeV/nucleon. Small discontinuities are observed in the measured transverse emittances and bunch length during the tune change from (0.238, 0.212) to (0.228, 0.210). For consistency, we use only the data from the first 40 min, during which the tunes were kept at (0.238, 0.212).

Figure 4 shows the measured bunch intensity, transverse emittances, and bunch length as red dots, together with the IBS modeling results in blue. Unlike the 31 GeV/nucleon case, clear growth is observed in both transverse emittances and the bunch length. The average growth times for the horizontal and vertical emittances are 4.9 and 11.1 h, respectively, indicating that the horizontal emittance grows roughly twice as fast as the vertical. The average growth time for the bunch length is 4.8 h, comparable to that of the horizontal emittance.

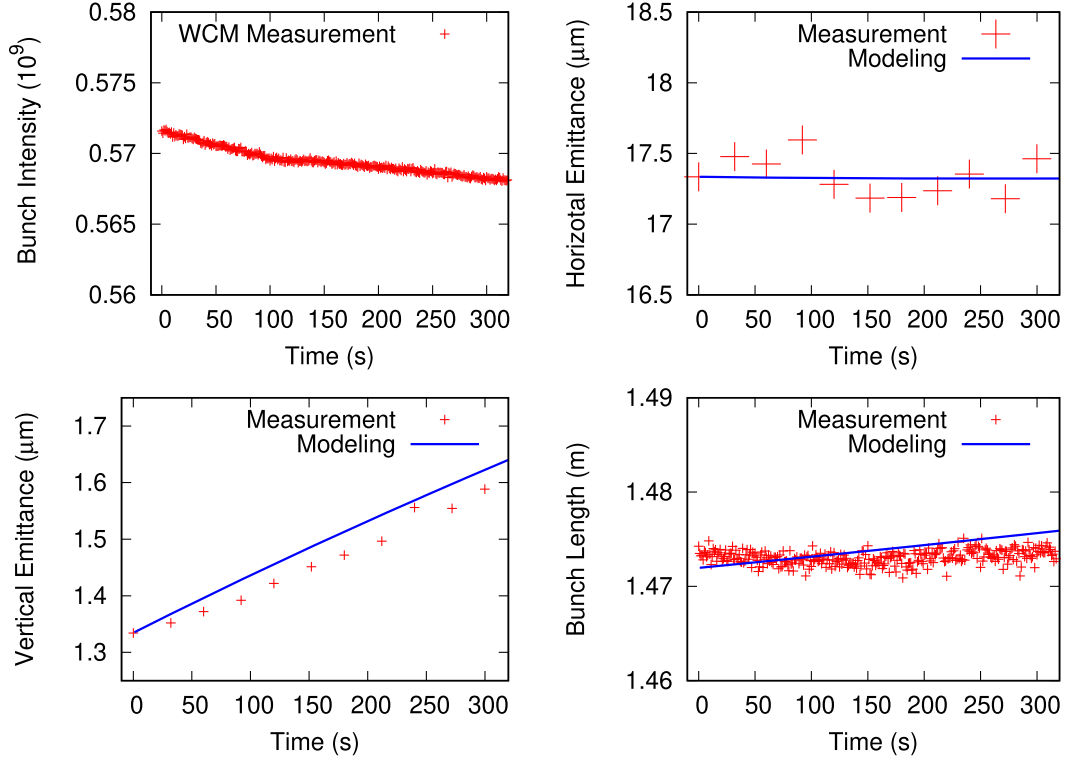


FIG. 3. Emittances and bunch length from measurement and IBS molding at 31 GeV/nucleon. Top-left: Bunch intensity. Top-right: Horizontal emittance. Bottom-left: Vertical emittance. Bottom-Right: RMS bunch length.

During this 40 min, the bunch intensity decreased by about 6%, as measured by the WCM, while the beam current measured by the direct current current transformer remained nearly constant. This indicates that the beam loss was primarily due to ions leaking out of the longitudinal rf bucket. For IBS modeling, the WCM bunch intensity is fitted with a piecewise exponential function before and after 1440s. In the simulation, the IBS growth rates, emittances, bunch length, and momentum spread are updated every 10 s.

Table II lists the initial beam parameters used for IBS modeling and the average growth times from both measurement and simulation. As shown in Fig. 4, the IBS model reproduces the measured evolution of the emittances and bunch length reasonably well. The modeled average growth times for the horizontal and vertical emittances are 4.3 and 13.2 h, respectively, while the bunch length growth time is 4.2 h, somewhat shorter than the measured 4.8 h. This discrepancy may be attributed to ions leaking out of the rf bucket, which effectively slows the actual bunch length growth. In Sec. VA, we will give an additional discussion of the discrepancy in bunch lengths between measurement and modeling.

IV. IBS FOR WEAKLY COUPLED BEAMS

In this section, we consider growth of transverse emittances and bunch length due to IBS under weakly

coupled conditions. For this purpose, a dedicated beam experiment was performed on Nov. 14, 2025. To avoid ion leakage from the rf buckets observed at 100 GeV/nucleon, the experiment was conducted at 31 GeV/nucleon, where the bunch length growth is negligible, as discussed above.

In this experiment, 28 gold ion bunches were stored in the yellow ring at an average bunch intensity of 0.7×10^9 . Prior to cooling, the minimum tune split—corresponding to the amplitude of the betatron coupling coefficient $|C^-|$ —was measured to verify effective decoupling. The betatron tunes were brought as close as possible by adjusting the arc quadrupole strengths. As shown in Fig. 5, a minimum tune split of approximately 4×10^{-4} was achieved, indicating a well-decoupled lattice. During the subsequent IBS experiment, the uncoupled betatron tunes were set to (0.235, 0.215).

Measurements of emittances and bunch length due to IBS were performed under three conditions: well-decoupled, $|C^-| = 0.01$, and $|C^-| = 0.02$. For each case, the vertical emittance was first cooled to obtain an emittance ratio of approximately 10:1 under well-decoupled conditions. Cooling was then turned off, and coupling was introduced by ramping the skew-quadrupole currents. The beam evolution was recorded for approximately 10 min. After each measurement, the skew quadrupoles were reset, and the beam was recooled to prepare for the next condition.

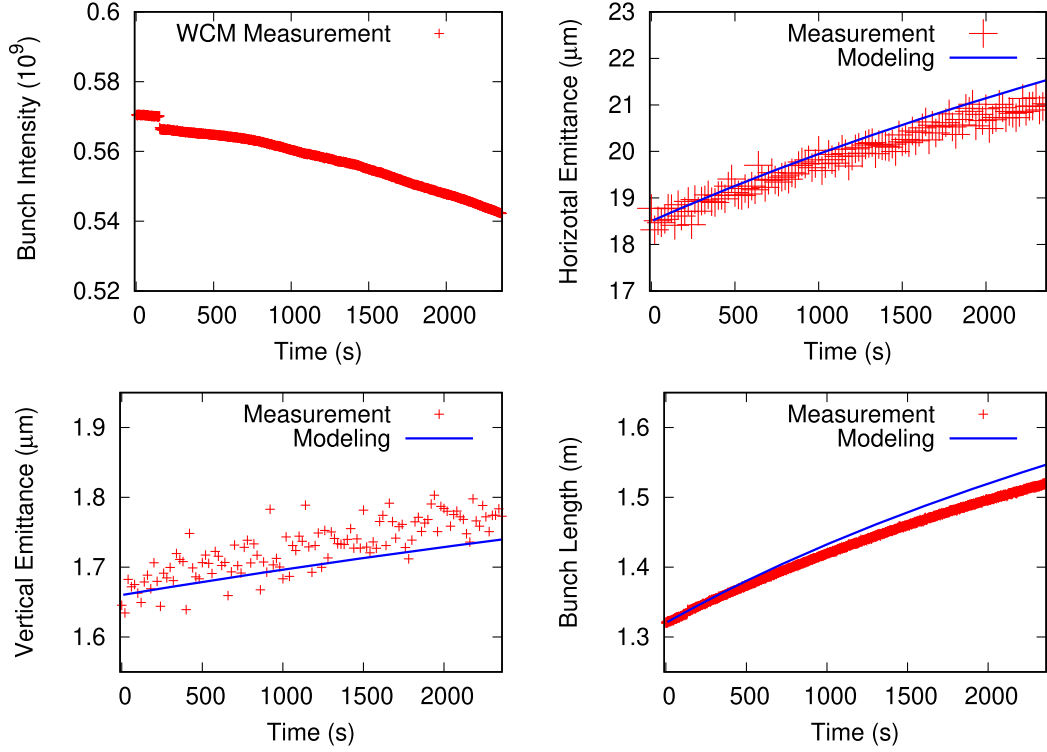


FIG. 4. Emittances and bunch length from measurement and IBS molding at 100 GeV/nucleon. Top-left: Bunch intensity. Top-right: Horizontal emittance. Bottom-left: Vertical emittance. Bottom-Right: RMS bunch length.

Each RHIC ring contains three families of skew quadrupoles (F1, F2, and F3), each powered by four independent power supplies. In this experiment, the F1 and F3 families were used to introduce controlled coupling on top of a well-decoupled lattice.

In the absence of coupling, the IPMs directly measure the horizontal and vertical emittances, or eigen-emittances.

TABLE II. Initial beam parameters for IBS modeling and IBS growth times from both measurement and modeling at 100 GeV/nucleon.

Quantity	Unit	Value
Initial parameters:		
Bunch intensity	10^9	0.57
Horizontal emittance	μm	18.50
Vertical emittance	μm	1.66
Bunch length	m	1.32
Momentum spread	10^{-4}	5.35
IBS time from measurement:		
Horizontal emittance	h	4.9
Vertical emittance	h	11.1
Bunch length	h	4.8
IBS time from modeling:		
Horizontal emittance	h	4.3
Vertical emittance	h	13.2
Bunch length	h	4.2

Under coupled conditions, however, the IPMs measure rms beam sizes, and the emittances are still inferred using the uncoupled lattice functions $\beta_{x,y,0}$. These inferred quantities do not correspond to eigen-emittances and are therefore referred to here as quasi-emittances. With quasi-emittances and model $\beta_{x,y,0}$, we could reconstruct the measured beam sizes $\sigma_{x,y}$ at the IPMs.

Although the eigen-emittances cannot be directly measured during the coupled periods, their changes due to IBS and coupling can be inferred from the difference between the eigen-emittances measured immediately before coupling is introduced and after it is removed.

Figure 6 shows the evolution of the quasi-emittances over the full experiment, with the three coupling conditions indicated. The quasi-horizontal emittance remains nearly constant, except in the case with $|C^-| = 0.02$, which will be discussed below. In contrast, the quasi-vertical emittance exhibits an immediate increase upon the introduction of coupling, with larger increases observed for stronger coupling. These instantaneous changes arise from coupling effects and have been discussed in detail in Ref. [4]. In the following, we focus on the growth of the eigen-emittances and the bunch length in the presence of coupling.

The rms bunch length is shown in Fig. 7. Over approximately 1 h, the bunch length increases by only about 3%, confirming that longitudinal growth remains weak at this energy. A modest increase in the growth rate is

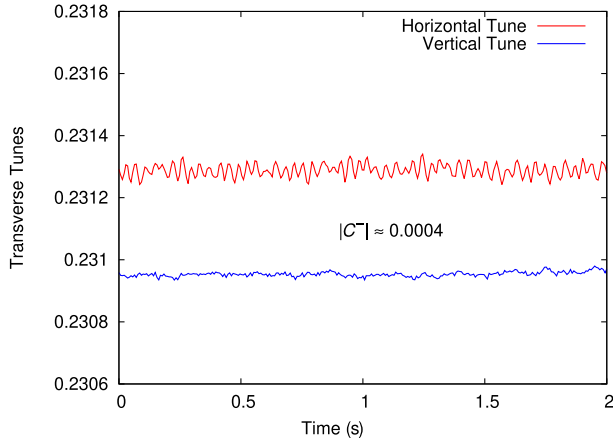


FIG. 5. Betatron tunes during measurement of the minimum tune split, or the amplitude of coupling coefficient $|C^-|$, at 31 GeV/nucleon during the experiment conducted on Nov. 14, 2025.

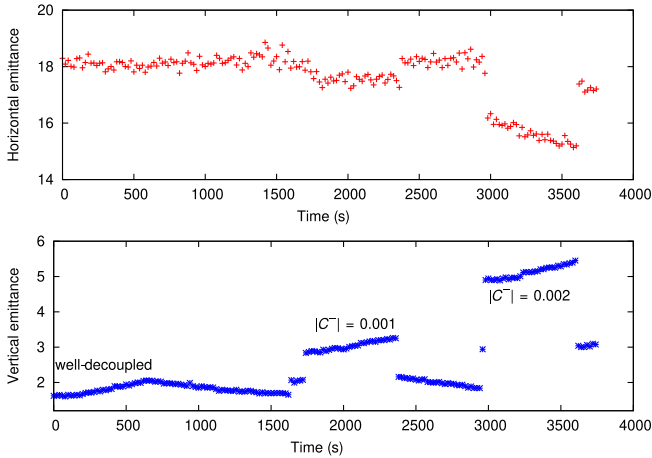


FIG. 6. IPM-measured transverse emittances for the experiment conducted on Nov. 14, 2025.

observed around 3000 s, coinciding with the ramp-up of the skew-quadrupole currents for the case with $|C^-| = 0.02$.

For completeness, IBS modeling was also performed for the well-decoupled condition. The measured and modeled vertical emittance growth times are 36 and 35 min, respectively, in excellent agreement. The corresponding growth times for the horizontal emittance and the bunch length are effectively infinite.

A. With $|C^-| = 0.01$

In this case, coupling was introduced by changing the skew-quadrupole strengths of the F1 and F3 families, with $\Delta k_{1s}l = 0.1 \times 10^{-3}$. The total change was applied in two equal steps. The amplitude of the coupling coefficient is

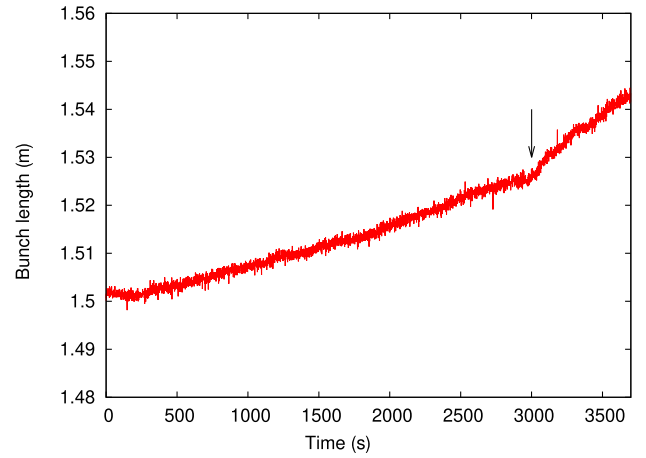


FIG. 7. Measured bunch length for the experiment conducted on Nov. 14, 2025.

about an order of magnitude larger than the typical values after applying global decoupling feedback.

Before introducing coupling, the measured fractional tunes were (0.235, 0.215). After coupling was applied, the tunes shifted to (0.236, 0.214). As expected, the betatron tune separation increased with coupling. Using the model lattice, applying the same skew-quadrupole strength changes to the uncoupled lattice yields tunes of (0.2362, 0.2138), in good agreement with the measurements.

With coupling applied, the bunch length change remained negligible. Figures 8 and 9 show the measured quasi-horizontal and quasi-vertical emittances (red dots). The eigen-emittances immediately before coupling was introduced were (18.07, 1.65 μm) and those measured immediately after coupling was removed were (18.29, 2.16 μm). The mode I eigen-emittance remained nearly constant over the ~ 10 -min observation period. The mode II eigen-emittance increased by approximately 31%, corresponding to an average growth time of 38 min.

The initial bunch length and momentum spread for IBS modeling are 1.51 m and 1.33×10^{-3} , respectively. The average ion population is 0.696×10^9 per bunch. To model IBS-induced emittance and bunch length evolution, we use the IBS formulas of Lebedev–Nagaitsev. In the simulation code, the IBS growth rates, eigen-emittances, bunch length, and momentum spread are updated every 1 s.

For comparison with measurements, the modeled eigen-emittances are also shown as blue curves in Figs. 8 and 9. The final eigen-emittances from the modeling are (18.17, 2.18 μm), in good agreement with IPM measurements (18.29, 2.16 μm) immediately after coupling was removed. From the modeling, the mode II eigen-emittance increases by about 32% over 10 min, corresponding to an average growth time of 37 min.

Using the modeled eigen-emittances $\epsilon_{I,II}$ during the coupled period, the beam sizes at the IPMs and the corresponding quasi-emittances can be reconstructed; these are also shown as green curves in Figs. 8 and 9.

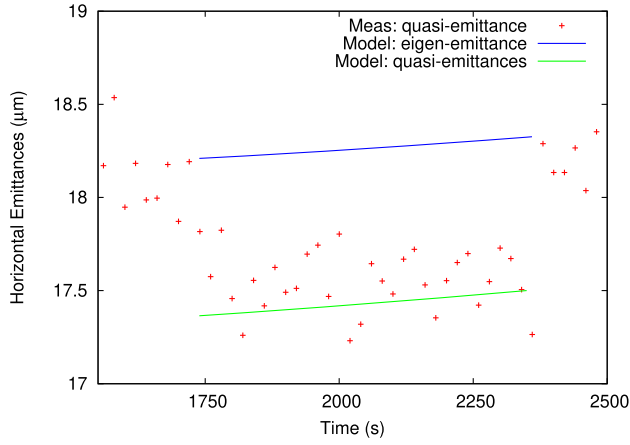


FIG. 8. Quasi-horizontal emittance and eigenmode I's emittance with $|C^{-1}| = 0.01$.

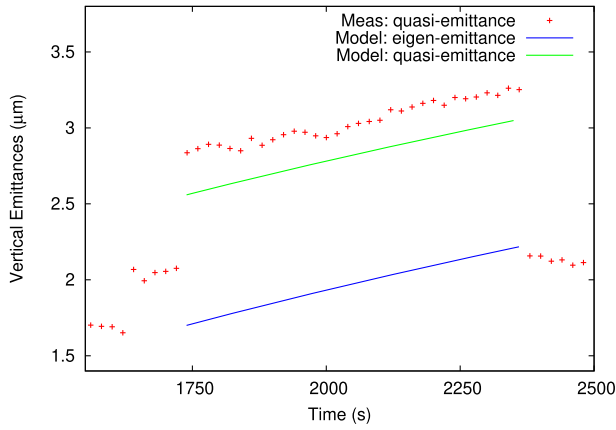


FIG. 9. Quasi-vertical emittance and eigenmode II's emittance with $|C^{-1}| = 0.01$.

The relative differences between the measured and modeled quasi-emittances are about 1.5% and 10% in the horizontal and vertical planes.

For comparison, we also modeled eigen-emittance growth assuming no coupling in the ring. In that case, the mode II eigen-emittance would increase to 2.21 μm , corresponding to an increase of about 34% over 10 min. The growth time for eigenmode II without coupling is 35 min. Compared to 37 min with betatron coupling $|C^{-}| = 0.01$, coupling in the ring slows down the eigenmode II's emittance growth.

In this experiment with betatron coupling $|C^{-}| = 0.01$, the most significant effect of coupling is the increase of the quasi-vertical emittance by about 71%, and the ratio of quasi-emittances changes from about 11:1 to 6:1.

B. With $|C^{-1}| = 0.02$

For this case, stronger transverse coupling was introduced in the ring. The resulting coupling coefficient was

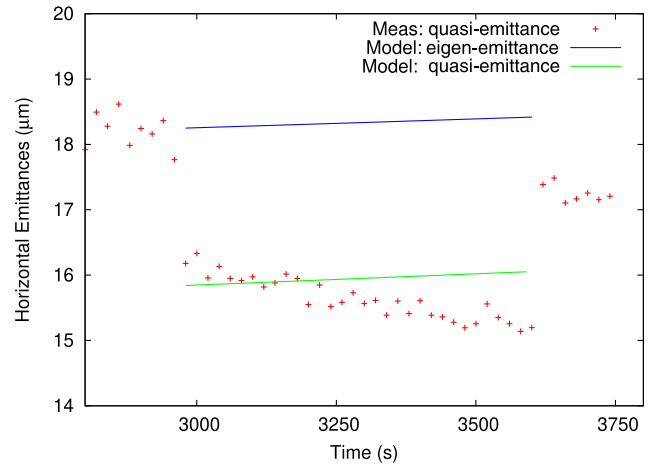


FIG. 10. Quasi-horizontal emittance from measurement and eigenmode I's emittance from modeling with $|C^{-1}| = 0.02$.

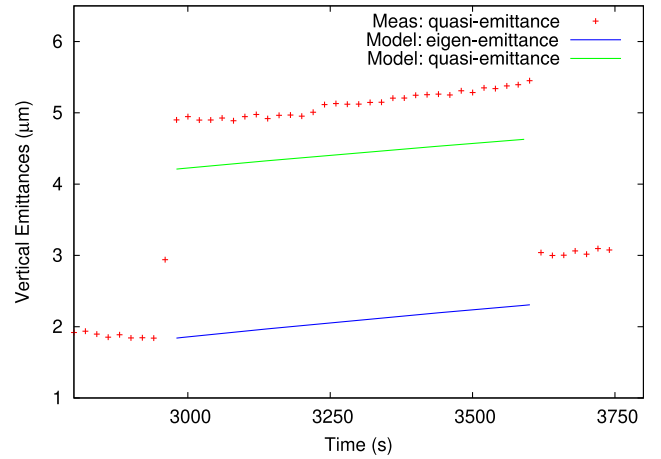


FIG. 11. Quasi-vertical emittance from measurement and eigenmode II's emittance from modeling with $|C^{-1}| = 0.02$.

approximately 20 times larger than the typical residual coupling after applying decoupling feedback. The skew-quadrupole strength changes in the F1 and F3 families were $\Delta k_{1s}l = 0.2 \times 10^{-3}$, also applied in two equal steps.

Prior to introducing coupling, the measured betatron tunes were (0.235, 0.215). After the coupling was applied, the tunes shifted to (0.238, 0.212). The lattice model predicts tunes of (0.2391, 0.2108) for the same skew-quadrupole settings, in good agreement with the measurements.

Figures 10 and 11 show the measured quasi-horizontal and quasi-vertical emittances in red. The eigen-emittances immediately before coupling was introduced were (18.25, 1.84 μm), while those immediately after coupling was removed were (17.38, 3.04 μm). The eigen-emittance of mode I (mostly associated with horizontal emittance) decreased by about 5%, whereas that of mode II (mostly associated with vertical emittance) increased by about 65%.

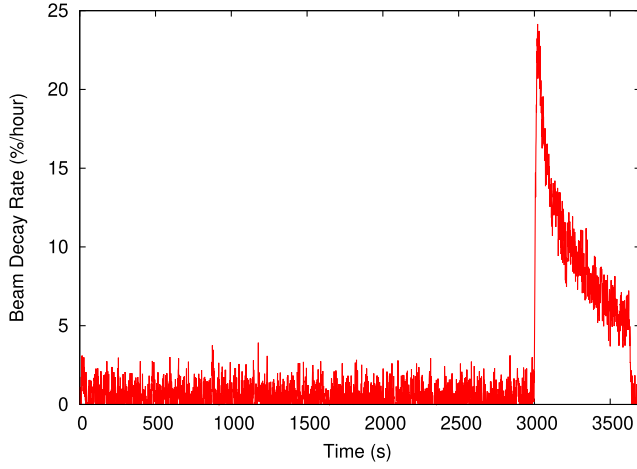


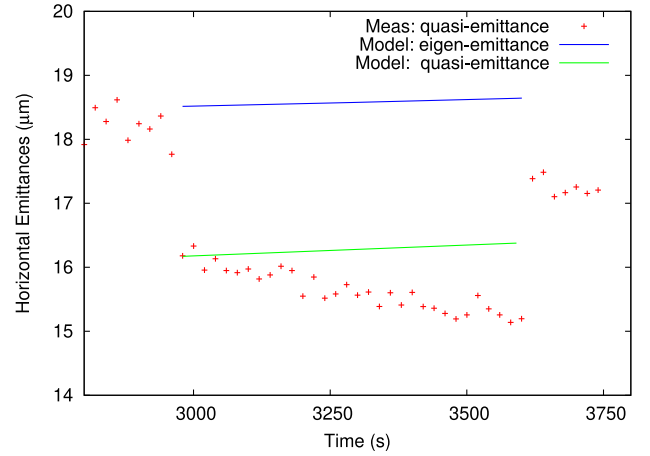
FIG. 12. Beam decay rate in the whole beam experiment.

The initial bunch length and momentum spread for IBS modeling are 1.52 m and 1.33×10^{-3} , respectively. The ion population is 0.686×10^9 per bunch. Using these initial beam parameters, we modeled the evolution of eigen-emittances using the IBS formulas by Lebedev and Nagaitsev. In the simulation, the growth rates, eigen-emittances, bunch length, and momentum spread were updated at 1 s intervals. The resulting evolution of the eigen-emittances is shown by the blue curves in Figs. 10 and 11.

From the model, over about 10 min with coupling, the eigen-emittances of mode I and mode II would be (18.42, 2.31 μm), with relative changes of about 0.9% and 25%, respectively. The eigen-emittances do not agree with the emittance measurements (17.38, 3.04 μm) right after coupling was removed. Also, significant discrepancies are noticed between the measured and modeled quasi-emittances during the period with coupling.

Figure 12 shows the recorded beam loss rate throughout the experiment. The loss rate remained below 2%/h until the skew-quadrupole currents were ramped up for the case with $|C^-| = 0.02$, around 3000 s in the plot. At that point, the loss rate peaked at approximately 25%/h and then gradually decreased. After the coupling was removed, the beam loss rate returned to its nominal level.

We attribute the observed reduction in the IPM-measured quasi-horizontal emittance to the loss of large-amplitude particles in the horizontal plane. One possible explanation is that strong coupling with $|C^-| = 0.02$ pushed the eigentune of mode I close to the fourth-order resonance at 0.25. A similar effect has been consistently observed during the early phase of physics stores in proton-proton collisions at RHIC, where large-amplitude particles are lost due to reduced momentum aperture in the presence of beam-beam interactions. There, we also observed IPM-measured transverse emittance reduction in the first hour of stores [35]. In Appendix A, we will provide additional evidence supporting our hypothesis that

FIG. 13. Quasi-horizontal emittance from measurement and eigenmode I's emittance from IBS modeling with $|C^-| = 0.02$. Here, adjusted initial eigen-emittances are used for IBS modeling.

the beam loss observed with coupling coefficient $|C^-| = 0.02$ was caused by the fourth-order betatron resonance at 0.25.

Comparing the eigenmode II emittances from measurement and modeling after coupling was removed, we infer that additional emittance increase occurred during the ramp-up of the skew-quadrupole currents. Relative to the case with $|C^-| = 0.01$, the ramp rate was approximately twice as fast, which may have induced nonadiabatic eigen-emittance increase.

In addition, small discontinuities in the measured quasi-emittances are observed around 3200 s in Figs. 10 and 11, which may have contributed to additional increases in the eigen-emittances. At 3200 s, an unplanned automatic store orbit correction was triggered.

To better represent the initial beam parameters for IBS modeling with coupling $|C^-| = 0.02$, we adjusted the initial eigen-emittances to match the IPM-measured quasi-emittances of (16.20, 4.85 μm) immediately after the introduction of coupling. An optimization procedure was used to obtain the corresponding eigen-emittances of (18.51, 2.54 μm). Compared to the measured values prior to coupling (18.25, 1.87 μm), this implies an increase of approximately 36% in the eigenmode II emittance. These results suggest that slower ramping of the skew-quadrupole currents is required to preserve the eigen-emittances.

Using these adjusted initial eigen-emittances (18.51, 2.54 μm), we remodeled the evolution of the eigen-emittances and bunch length for the coupling case $|C^-| = 0.02$, as shown in Figs. 13 and 14. With the updated initial conditions, the IBS model reproduces the IPM-measured eigenmode II emittance immediately after coupling removal, with a relative error of 4%. The modeled quasi-vertical emittance also agrees well with the measurements, although a small increase is observed

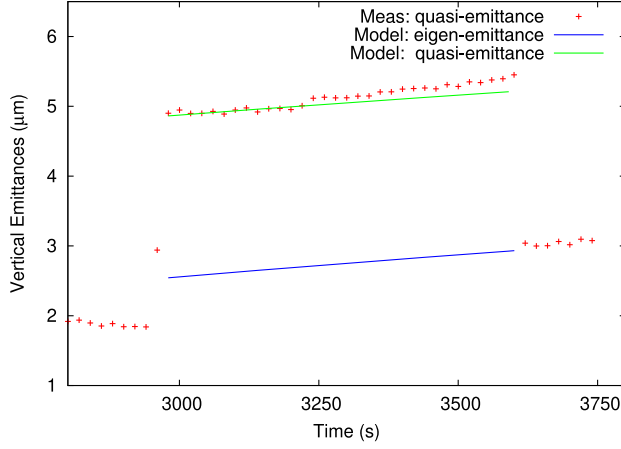


FIG. 14. Quasi-vertical emittance from measurement and eigenmode II's emittance from IBS modeling with $|C^-| = 0.02$. Here, adjusted initial eigen-emittances are used for IBS modeling.

around 3200 s due to the automatic store orbit correction. As noted above, the decrease in eigenmode I emittance is attributed to particle loss in the horizontal plane and is therefore not captured by the model.

For comparison, with initial eigen-emittances of (18.25, 1.84 μm), the modeled growth time for eigenmode II is about 46 min. Compared to the 37-min growth time obtained in the case above with $|C^-| = 0.01$, the emittance growth of eigenmode II is slower. Therefore, stronger coupling in the ring further slows the IBS-induced emittance growth of eigenmode II.

V. FURTHER DISCUSSION

In this section, we present additional discussion to address the discrepancy between the measured and modeled bunch lengths, the ion loss caused by the limited rf bucket height, and the uncertainty in the transverse emittance measurement and its impact on the IBS modeling results. In Appendix B, we will review the definition of the Coulomb logarithm and its impact on the IBS growth rate calculation with EIC HSR proton lattice.

A. Discrepancy between measured and modeled bunch length

In the IBS modeling of the experimental data, we found that the measured bunch length was consistently smaller than the value predicted by IBS modeling, as shown in Figs. 3 and 4. We attribute this to the limited rf bucket height, or limited rf maximum momentum acceptance. In the IBS experiment performed at 31 GeV/nucleon with a well-decoupled ring, the bunch length increased by 0.1% and the bunch intensity decreased by 0.6% over approximately 300 s. During the experiment at 100 GeV/nucleon with a well-decoupled ring, the bunch length increased by 14% and the bunch intensity decreased by 5% over

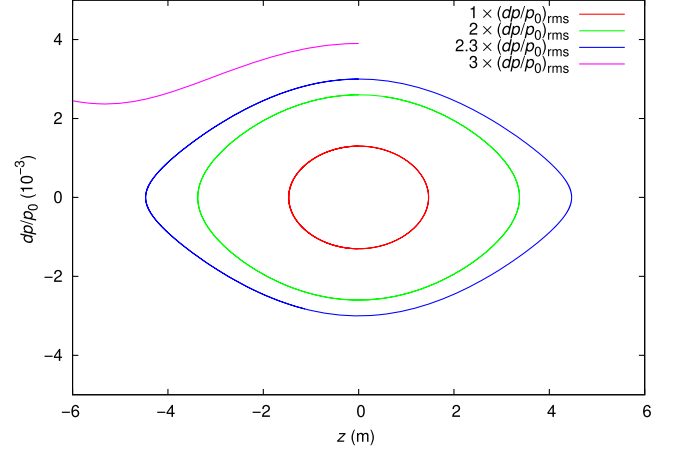


FIG. 15. Longitudinal phase space of the 31 GeV gold ion beam in the experiment. The rf bucket height is approximately 2.3 times the beam rms relative momentum spread.

approximately 300 s. The relative changes in bunch length and bunch intensity were of a similar order of magnitude.

In our beam experiments, only the 400 MV 28 MHz rf cavities were used. As an example, Fig. 15 shows the longitudinal phase space of the 31 GeV gold ion beam. The rf bucket height is 3.3×10^{-3} at 31 GeV and 1.28×10^{-3} at 100 GeV. As mentioned above, the bunch rms relative momentum spread was 1.30×10^{-3} at 31 GeV and 5.5×10^{-4} at 100 GeV. Therefore, the rf bucket height is only about 2.3 times the bunch rms relative momentum spread. For an initial Gaussian distribution, particles with relative momentum deviations exceeding the rf bucket height would leak out of the rf bucket, become unbunched, and eventually be lost. Therefore, changes in the bunch length due to IBS are strongly suppressed.

To illustrate the bunch length evolution due to IBS, we adopt a simple one-dimensional diffusion model with a constant diffusion coefficient in a harmonic potential $U = \frac{1}{2}kz^2$. The corresponding Fokker-Planck equation governing the evolution of the particle distribution is

$$\frac{\partial f(z, t)}{\partial t} = D \frac{\partial^2 f(z, t)}{\partial z^2} + \beta \frac{\partial}{\partial z} (zf(z, t)). \quad (15)$$

Here, $f(z, t)$ is the longitudinal density distribution, D is the diffusion coefficient related to IBS, and β is the drift coefficient due to the harmonic potential, with $\beta \propto k$.

Assuming an initial Gaussian distribution, the bunch distribution remains Gaussian during the diffusion process. The rms bunch length evolves as:

$$\sigma_t^2 = \sigma_0^2 \exp(-2\beta t) + \frac{D}{\beta} (1 - \exp(-2\beta t)). \quad (16)$$

Here, σ_0 is the initial rms bunch length. For a short time period with $\beta t \rightarrow 0$, we have

$$\sigma_t^2 \approx \sigma_0^2 + 2Dt, \quad (17)$$

which corresponds to the solution of a one-dimensional free diffusion process. For a sufficiently long time, $t \rightarrow +\infty$, the system reaches an equilibrium state where diffusion balances confinement,

$$\sigma_{+\infty} = \sqrt{\frac{D}{\beta}}, \quad (18)$$

which is independent of the initial bunch length σ_0 .

If an aperture is located at $\pm W$, due either to a limited momentum dynamic aperture or the rf bucket height, the number of particles remaining inside the aperture is

$$N_{\text{inside}}(t) = N_0 \operatorname{erf}\left(\frac{W}{\sqrt{2\sigma_t^2}}\right). \quad (19)$$

Here, N_0 is the total number of ions in the initial Gaussian distribution without an aperture and erf is the error function.

For a more rigorous treatment of longitudinal distribution evolution and beam loss in the presence of IBS and stochastic cooling for a non-Gaussian beam, please refer to Ref. [36].

B. Uncertainty in transverse emittance measurement

During our experiment, the uncertainty in the IPM emittance measurement contributed most significantly to the uncertainty in our IBS growth benchmarking. The IPM measures the distribution of free electrons generated by beam ionization of the residual gas. The electrons are swept away from the beamline by a transverse electric field, amplified by a microchannel plate (MCP), and collected on a circuit board with strip anodes oriented parallel to the beam axis. A uniform magnetic field, parallel to the sweep electric field, compensates for the defocusing effects caused by space charge and recoil momentum.

There are two possible sources of errors in the emittance measurement with IPMs: hardware-related errors and data-processing errors [37]. Hardware-related errors mainly include poorly calibrated channels, MCP aging, detector and readout malfunctions, and other related effects. The second category includes uncertainties associated with Gaussian fitting of the raw data and uncertainties in the local β -function at the IPM location. For modern IPMs, the uncertainty in beam size measurement is typically within 5%. As mentioned above, the typical β -beat from the model in RHIC was about 10%.

We could not directly estimate the absolute errors in the IPM emittance measurement in our experiment. Instead, we classify the uncertainty in the IPM emittance measurement into two categories: systematic and random

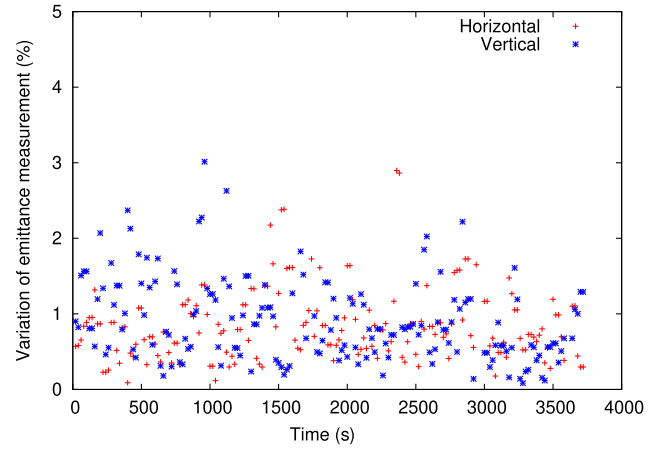


FIG. 16. Percentage variation of IPM emittance measurements during the experiment with weak coupling.

errors. The systematic error in the IPM emittance measurement can be corrected by applying a scaling factor to the measured emittance. In RHIC, IPM emittance measurements can be benchmarked against luminosity models and polarimeter measurements during proton operation.

Random errors cause variations among multiple measurements under the same beam conditions. In the following, we estimate the variation using several consecutive measurements. In our experiment, one measurement was typically taken every 30 s due to the relaxation time limitation of the IPM. We define the percentage variation as:

$$\eta = \frac{\epsilon_{\max} - \epsilon_{\min}}{2\epsilon_{\text{mean}}} \times 100\%. \quad (20)$$

Here, $\epsilon_{\max}$, $\epsilon_{\min}$, and ϵ_{mean} are the maximum, minimum, and mean values of three consecutive emittance measurements, respectively. Figure 16 shows the percentage variation of the IPM emittance measurement during the experiment with weak coupling. The typical variation in the measured emittance was below 2% for both transverse planes. It was noticed that the variation in the horizontal emittance measurement with coupling coefficient $|C^-| = 0.01$, as shown in Fig. 8, was slightly larger than that with coupling coefficient $|C^-| = 0.02$, as shown in Fig. 10. The reason for this difference is not clear to us. We noticed that there was one channel in the horizontal IPM with clear miscalibration or malfunction during this experiment.

To estimate the impact of uncertainty in the initial emittance on the IBS modeling results, we varied the initial rms emittance inputs by $\pm 10\%$ for both transverse emittances. As an example, for the IBS study without coupling at 31 GeV/nucleon, Fig. 17 shows the evolution of the vertical beam rms emittance with a $\pm 10\%$ uncertainty in the initial transverse rms emittances. For this experiment, vertical emittance growth was much more profound than in

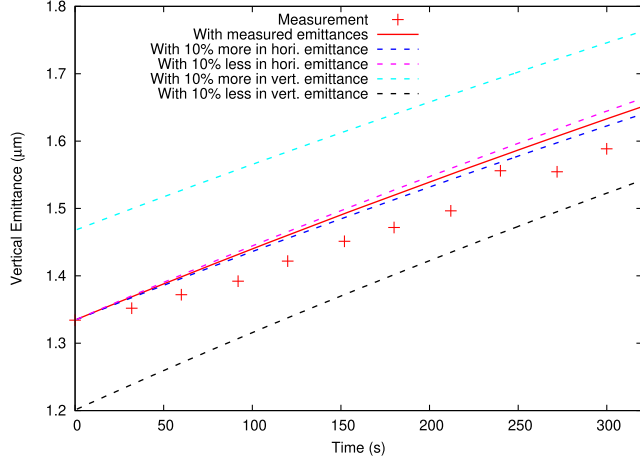


FIG. 17. Evolution of vertical emittances from measurement and modeling for the IBS study at 31 GeV. A $\pm 10\%$ uncertainty in the initial transverse beam emittances is assumed.

the horizontal plane. It can be seen that the uncertainty in the horizontal emittance has little effect on the final vertical emittance prediction. Even though a $\pm 10\%$ variation in the initial vertical emittance shifts the absolute value of the modeled vertical emittance, the average vertical emittance growth time from IBS modeling is barely affected.

Figure 18 shows the evolution of the horizontal rms beam emittance with a $\pm 10\%$ uncertainty in the initial rms emittance for the IBS study without coupling at 100 GeV/nucleon. For this experiment, we focus on the comparison of the horizontal emittance between measurements and modeling. It can be seen that a 10% variation in the initial vertical emittance has little effect on the horizontal emittance evolution. Even though a $\pm 10\%$ variation in the initial horizontal emittance shifts the absolute value of the modeled horizontal emittance, the average horizontal emittance growth time from modeling is barely affected.

For our experiments with large-emittance-ratio ion beams, the discrepancy in the IBS growth time between measurement and modeling was typically less than 15%, except for the case with coupling coefficient $|C^-| = 0.02$. In previous experiments performed at other hadron accelerators with the same transverse emittances, the reported discrepancy between measured IBS growth rates and theoretical predictions was approximately 10%–20% [38,39].

VI. IMPLICATIONS FOR THE EIC

In this section, we first evaluate the IBS growth times for the EIC HSR at various beam energies. We then study the impact of the transverse emittance ratio and betatron coupling on the IBS growth time. The IBS growth times are calculated using the latest proton design lattices, and the Lebedev–Nagaitsev formalism is used.

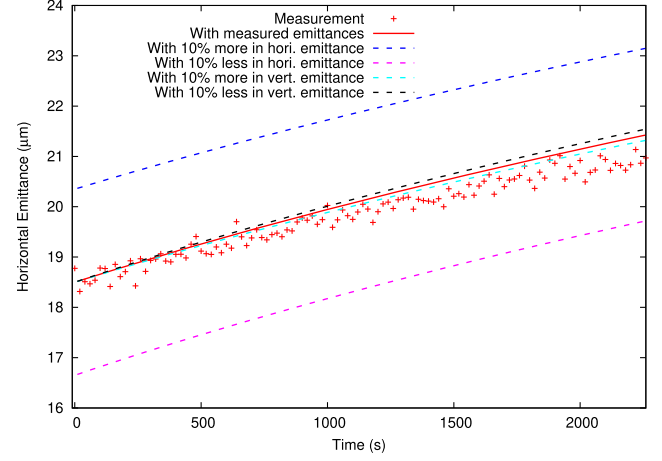


FIG. 18. Evolution of horizontal emittances from measurement and modeling for the IBS study at 100 GeV. A $\pm 10\%$ uncertainty in the initial transverse beam emittances is assumed.

The IBS growth rate scaling for high-energy flat beams in the very large hadron collider (VLHC) was studied by Wei and Parzen [40]. For the VLHC design, the proton beam energy is 50 TeV, and the design transverse emittance ratio is approximately 10:1, which can be achieved through proton synchrotron radiation damping [41]. Starting from their analytical formulae for IBS at high energies [9,10], they found that horizontal and longitudinal growth rates are proportional to each other, and vertical damping occurs when $e_y/e_x \geq \frac{1}{4}$.

A. IBS growth times for the HSR

The EIC HSR will reuse the arcs of the yellow ring of RHIC and connect them with new straight sections. The hadron injection accelerators for RHIC will be retained for the EIC HSR. The typical 95% normalized transverse emittance for the RHIC injection accelerators is about 15 μm .

For proton-electron collisions in the EIC, the ratio of transverse beam sizes at the IP is about 10:1 to achieve high peak luminosity. The proton beam will first be cooled at the injection energy of about 23.8 GeV using conventional electron cooling [42]. The transition energy for the injection lattice is 23.5 GeV. After a large emittance ratio is achieved, the flat proton beam will be accelerated to various collision energies.

Table III lists the design parameters at the injection energy and at two collision energies, 100 and 275 GeV. For consistency throughout this paper, we use 95% normalized transverse emittances in this table. The parameters at injection correspond to the cooled proton beam. The IBS growth times $\tau_{x,y,z}$ correspond to the transverse and longitudinal emittances, respectively.

The proton bunch intensity at injection energy is 28×10^{10} . This bunch will be split into four bunches at

TABLE III. Design parameters for proton beams and calculated IBS growth times.

Quantities	Unit	23.8 GeV	100 GeV	275 GeV
γ	[...]	25.4	106.6	293.1
Bunch intensity	10^{10}	28	6.9	6.9
Hori. emittance	μm	18.7	19.2	19.9
Vert. emittance	μm	1.7	1.72	1.76
Bunch length	m	1	0.07	0.06
Momentum spread	10^{-4}	6	9.7	6.8
IBS τ_x	h	-6.8	1.7	2.1
IBS τ_y	h	0.3	4.1	+INF
IBS τ_z	h	0.67	3.0	3.9

collision energies. The IBS growth rate is proportional to the bunch intensity and also strongly depends on the particle energy. As shown in Table III, the IBS growth time for vertical emittance is only 18 min for the cooled beam. It is crucial to accelerate the beam immediately after the design emittances are reached.

From Table III, with the increase in the proton energy to collision energies, the growth time for the vertical emittance gradually increases and approaches infinity at 275 GeV. The reason is that there is no vertical dispersion by design for the EIC HSR. The vertical emittance increase with IBS will come from the horizontal emittance growth due to betatron coupling.

B. IBS vs transverse emittance ratio

For electron cooling at the injection energy, it is more efficient to cool both transverse planes simultaneously rather than cooling only the vertical plane. It takes about 30 min to cool the transverse emittances to (0.5, 0.3 μm). Table IV compares the IBS growth times for different transverse emittance ratios κ . In this calculation, the vertical emittance is kept constant. The transverse emittance ratio is changed by adjusting horizontal emittance.

From Table IV, a larger initial horizontal emittance or a larger transverse emittance ratio at injection leads to slightly slower growth times for the vertical emittance and the bunch length. Based on simulation, the horizontal emittance is reduced when the emittance ratio is larger than 4:1. The emittance ratio 1.7:1 or 5:3 was proposed by the cooling design team. If we cool the proton beams in both transverse planes, the horizontal emittance needs to blow up at the collision energy to match the design emittance ratio for collision.

At the collision energy, a larger transverse emittance ratio leads to a higher peak luminosity. However, beam-beam interaction simulations for the EIC indicate that the proton beam will have a larger vertical emittance growth rate and a smaller dynamic aperture. Therefore, it is necessary to balance the transverse emittance ratio (peak

TABLE IV. IBS growth times with different transverse emittance ratios at the proton's injection energy. For this calculation, the vertical emittance is kept constant.

$\kappa = \epsilon_x/\epsilon_y$	IBS τ_x (h)	IBS τ_y (h)	IBS τ_z (h)
11	-6.8	0.30	0.67
8	-5.5	0.26	0.60
6	-5.2	0.24	0.56
4	-14.0	0.22	0.50
1.7	0.4	0.24	0.41

TABLE V. IBS growth times with different transverse emittance ratios at the proton's 275 GeV. For this calculation, the horizontal emittance is kept constant.

$\kappa = \epsilon_x/\epsilon_y$	IBS τ_x (h)	IBS τ_y (h)	IBS τ_z (h)
11	2.1	+INF	3.9
8	2.5	+INF	4.7
6	3.0	+INF	5.6
4	3.8	+INF	7.1
3	4.5	-INF	8.4
2	5.8	-INF	10.9
1	9.2	-INF	17.0

luminosity) and beam lifetime to maximize the integrated luminosity.

Table V compares the calculated IBS growth times at 275 GeV for transverse emittance ratios ranging from 11:1 to 1:1. In these calculations, the horizontal emittance is kept constant, while the transverse emittance ratio is varied by adjusting the vertical emittance.

The results show that reducing the transverse emittance ratio increases the IBS growth times of the horizontal and longitudinal emittances. We numerically confirm Wei and Parzen's earlier finding that when $\epsilon_y/\epsilon_x \geq 1/4$, vertical damping occurs. However, the vertical IBS growth (or damping) time in Table V remains effectively infinite (greater than 1000 h), regardless of the transverse emittance ratio. During operation, vertical emittance growth at 275 GeV is expected to arise primarily from betatron coupling.

C. IBS vs betatron coupling

The sources of betatron coupling in the HSR include the detector solenoid, quadrupole roll errors, and vertical closed-orbit distortions at the arc sextupoles. The EIC detector solenoid in interaction region 6 (IR6) has a total length of 4 m and a design longitudinal magnetic field of approximately 2 T. The effect of the detector solenoid scales inversely with the particle energy. Its contribution to

the global coupling coefficient is 0.053 at 23.8 GeV and 0.0053 at 275 GeV.

For a random distribution of quadrupole roll angles along the ring, the amplitude of the coupling coefficient can be estimated as:

$$|C^-|^2 = \frac{1}{\pi^2} \left(\sum \beta_x \beta_y (k_1 l)^2 \right) \langle \theta_{\text{roll}}^2 \rangle. \quad (21)$$

Here $\langle \theta_{\text{roll}} \rangle$ is the rms of the quadrupole roll errors. Its contribution is not sensitive to particle energy. For example, using the latest 275 GeV HSR store lattice and assuming $\langle \theta_{\text{roll}} \rangle = 100 \mu\text{rad}$, the resulting $|C^-|$ is 0.0054.

There are two groups of skew quadrupoles for coupling compensation: global and local skew quadrupoles. The local skew quadrupoles are located in the interaction region. At store energies, they are used to compensate the detector solenoid's effects of coupling and vertical crab dispersion at IP, while the global skew quadrupoles are used for residual coupling correction. At injection, we can use either global or local skew quadrupoles for global coupling correction. We are evaluating the IBS rates with different decoupling ways.

As discussed in Ref. [4], the amplitude of the coupling coefficient should be about one order of magnitude smaller than the transverse tune split. For example, for the HSR design tunes (0.228, 0.210) at collision energy, $|C^-|$ should be less than 0.0018. We will employ the same RHIC decoupling feedback for the EIC HSR. As mentioned earlier, the typical coupling amplitude in RHIC is about 0.001 after applying decoupling feedback. During cooling at injection and acceleration, the tune split can be increased, provided that the beam lifetime remains acceptable.

Here we simulate the evolution of eigen-emittances in the presence of residual weak betatron coupling. Using the 275 GeV HSR store lattice, coupling is introduced through quadrupole rolls. The transverse tunes without coupling are set to (0.228, 0.210). In the simulation, the IBS growth rates and eigen-emittances are updated every 10 s. Figures 19 and 20 show the eigen-emittances of mode I and mode II under four different coupling conditions: no-coupling, $|C^-| = 0.001$, $|C^-| = 0.005$, and $|C^-| = 0.01$. The simulation is performed over a real beam time of 4 h.

From the plots, as the betatron coupling in the ring increases, the IBS growth rate for eigenmode II increases, while the IBS growth rate for eigenmode I decreases. Comparing the cases without and with coupling, the relative change in the eigen-emittance of mode I is much smaller than that of mode II. There is about a 5% increase in the eigen-emittance of mode II over 4 h if there is no coupling in the ring. There is no significant difference in the evolution of eigen-emittances between the cases without coupling and $|C^-| = 0.001$. For $|C^-| = 0.005$, the eigen-emittance of mode II increases by about 32% over

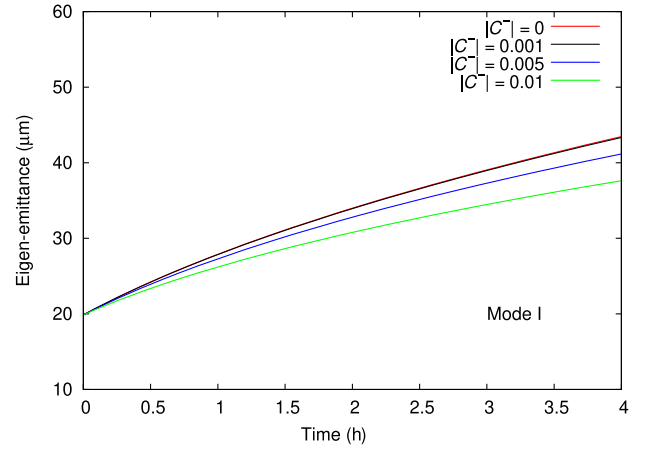


FIG. 19. Evolution of eigen-emittance of mode I with different coupling for proton beam at 275 GeV.

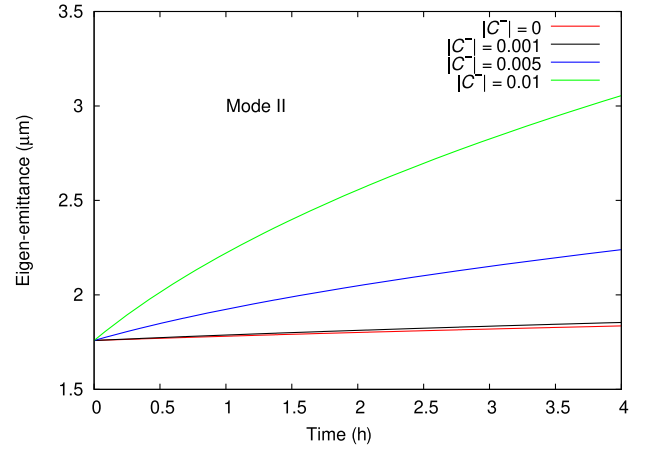


FIG. 20. Evolution of eigen-emittance of mode II with different coupling for proton beam at 275 GeV.

the course of the simulation. The enlarged eigen-emittance of mode II increases the vertical beam size at IP and therefore reduces luminosity.

In our beam experiment at 31 GeV/nucleon, we observed a slower emittance growth rate for eigenmode II after introducing coupling in the ring. The reason is that, at low energy, the dominant IBS growth occurs in the vertical plane due to the large transverse emittance ratio, while horizontal IBS growth is negligible. When coupling is introduced, the IBS-driven emittance growth of eigenmode II is partially transferred to eigenmode I. In contrast, for a 275 GeV proton beam, the dominant IBS-induced emittance growth occurs in the horizontal plane, with negligible vertical growth. After coupling is introduced, the IBS-driven emittance growth of eigenmode I is partially transferred to eigenmode II. The main impact of coupling on IBS is to mix or redistribute the emittances and their growth.

As discussed in Ref. [4], emittance growth from IBS in the presence of coupling cannot be simply interpreted as a process in which diffusion occurs only in one transverse plane (horizontal or vertical), while the emittance growth in the other plane is entirely transferred from the IBS-dominant plane through coupling. In reality, coupling modifies the transverse particle distribution, which in turn affects the IBS growth rates.

VII. SUMMARY

In this article, we benchmark IBS-induced emittance growth for beams with large transverse emittance ratios in RHIC, both without and with betatron coupling, using the Lebedev–Nagaitsev formalism. The simulations reproduce the observations for uncoupled cases at 31 GeV/nucleon and 100 GeV/nucleon, as well as for the weakly coupled case with $|C^-| = 0.01$.

For $|C^-| = 0.02$, strong coupling pushes the eigenmode I tune close to a fourth-order resonance, resulting in particle loss and a reduction of the eigenmode I emittance. The eigenmode II emittance increased as the skew-quadrupole strength was ramped. By adjusting the initial eigen-emittances in the IBS model to match the quasi-emittances, the observed evolution of the eigenmode II emittance can be reproduced.

IBS growth times are also evaluated for proton beams in the EIC HSR over a range of energies. To achieve and maintain large emittance ratios for high-luminosity collisions, betatron coupling must be minimized. For the EIC HSR, RHIC-like decoupling feedback will be employed to suppress the coupling amplitude to below 0.002.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: REASONS FOR BEAM LOSS WITH COUPLING COEFFICIENT $|C^-| = 0.02$

In the IBS beam experiment with coupling coefficient $|C^-| = 0.02$, we observed beam loss accompanied by a

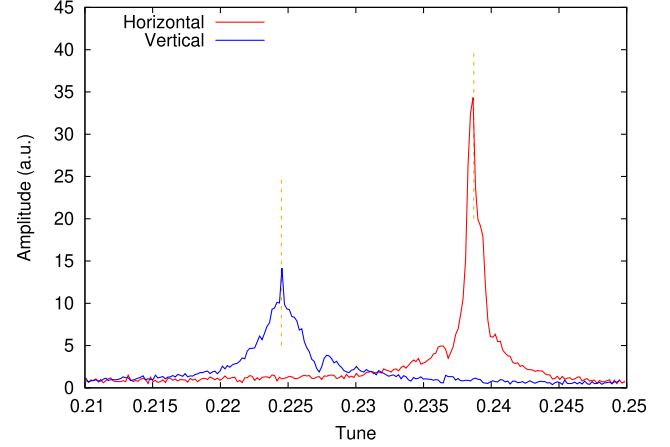


FIG. 21. Example of a BTF measurement obtained a few fills after the experimental session.

decrease in the horizontal emittance. We hypothesized that the coupling moved up the horizontal tune close to the fourth-order resonance at 0.25. In this section, we present additional evidence supporting this hypothesis.

During the experiment, the tune meter reported transverse tunes of (0.235, 0.215) without coupling and (0.238, 0.212) with coupling. The BBQ tune meter measures the tune corresponding to the maximum coherent response of the beam while the excitation frequency is swept over a specified tune range. In contrast, the beam has an incoherent tune spread arising from the distribution of individual particles. As an example, Fig. 21 shows a beam transfer function (BTF) measurement obtained only a few fills after the experimental session. The measured horizontal tune was 0.2387, while the incoherent tune spread extended to approximately 0.242.

The incoherent tune spread arises from both momentum spread and transverse amplitudes of individual particles within the bunch. During the experiment, the linear chromaticities were estimated to be larger than 2 because they had to be increased to maintain beam stability at this relatively low beam energy. Figure 22 shows the off-momentum tunes as functions of the relative momentum deviation dp/p_0 . The rms relative momentum spread of the beam is $(dp/p_0)_{\text{rms}} = 1.3 \times 10^{-3}$. For a bunch-core horizontal tune of 0.238, ions with $dp/p_0 = 2(dp/p_0)_{\text{rms}}$ can reach a horizontal tune of approximately 0.242.

In addition to the tune shift induced by chromaticity, the transverse-amplitude-dependent tune shift (detuning) also contributes to the incoherent tune spread. The primary sources of detuning are nonlinear field errors in the superconducting magnets located in the IRs and second-order effects of the chromatic sextupoles. In the experiment, the octupoles in the arcs were not powered. Figure 23 shows the tune footprint of the 31 GeV lattice. Particles with large transverse amplitudes exhibit higher horizontal tunes than those in the bunch core.

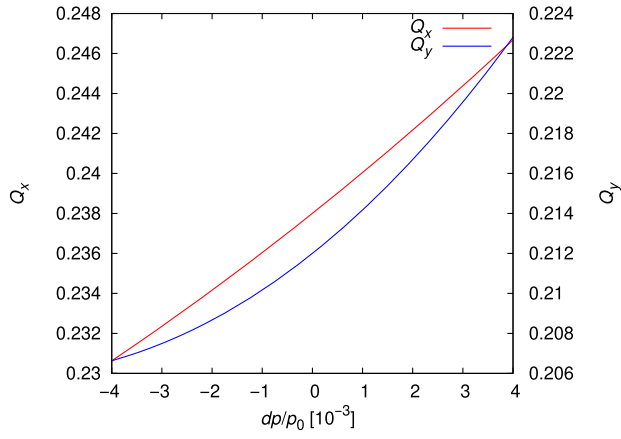


FIG. 22. Off-momentum tunes for the 31 GeV lattice. The rms relative momentum deviation is $(dp/p_0)_{\text{rms}} = 1.3 \times 10^{-3}$.

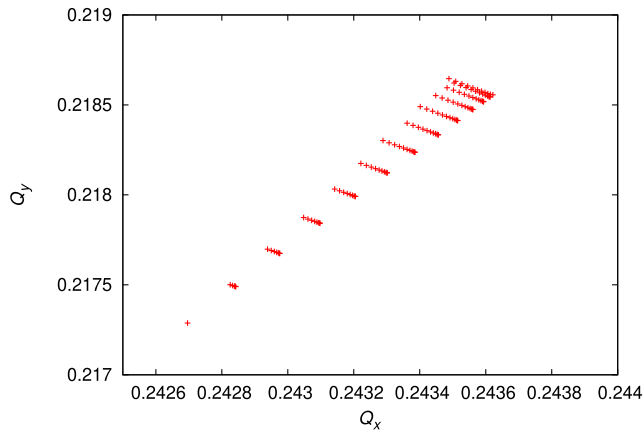


FIG. 23. Tune footprint for the 31 GeV lattice for particles with transverse amplitudes from 0 to 6σ . In this simulation, all test particles have initial $(dp/p_0) = 2(dp/p_0)_{\text{rms}}$.

Dynamic aperture is a useful simulation metric for quantifying beam lifetime. Unfortunately, an IR nonlinear field model is not available for the 31 GeV RHIC lattice. Instead, we performed a tune scan along the diagonal in tune space using the 100 GeV gold ion lattice, which includes the measured IR nonlinear field errors. The beam-beam interaction was excluded. Figure 24 shows the calculated dynamic aperture as a function of the fractional horizontal tune. The fractional vertical tune is fixed at 0.026 below the horizontal tune, matching the tune split used in the beam experiment. The dynamic aperture begins to decrease once the horizontal tune exceeds 0.24. This result is consistent with the operational experience that the preferred working point for RHIC ion operation is around (0.235, 0.215).

Consistent with this interpretation, several increases in the beam decay rate were observed during the same experimental fill when the yellow-ring horizontal tune approached 0.24. Figure 25 shows the beam decay, in units

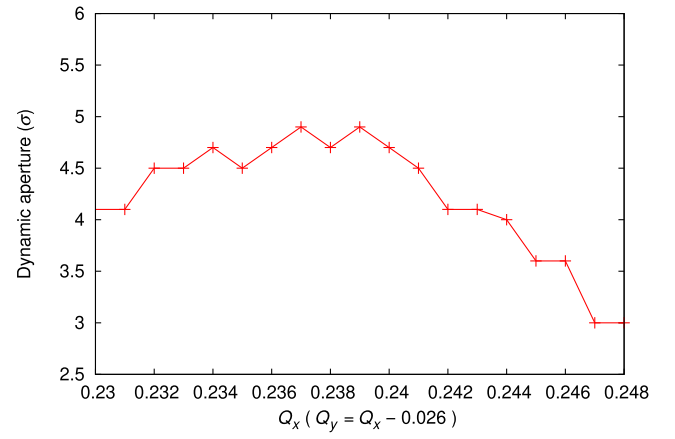


FIG. 24. Calculated dynamic aperture as a function of tune. The calculation uses the 100 GeV gold ion lattice, including the IR nonlinear field errors.

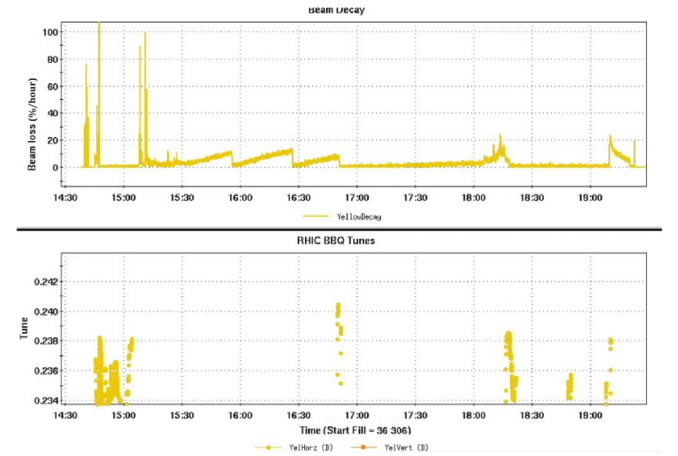


FIG. 25. Measured beam decay rate and yellow-ring horizontal tune during the experimental session with weak coupling.

of %/hour, together with the yellow-ring horizontal tune. To minimize any potential impact on the beam emittance, the tune meter was operated only intermittently for tune verification. As shown in Fig. 25, enhanced beam decay was observed at approximately 15:00, 18:15, and 19:15. The beam loss at 19:15 corresponds to the measurement with coupling coefficient $|C^-| = 0.02$. The first two beam decay peaks at approximately 14:45 occurred during energy ramping. The sawtooth pattern in the beam decay between 15:30 and 17:00 resulted from stochastic cooling tuning and chromaticity adjustments.

APPENDIX B: ABOUT THE COULOMB LOGARITHM

For intrabeam scattering, the Coulomb logarithm is determined by the range of impact parameters that contribute to binary Coulomb scattering within the beam. Although its exact definition depends on the specific IBS

formalism, the underlying idea is the same:

$$L_c = \ln\left(\frac{b_{\max}}{b_{\min}}\right). \quad (\text{B1})$$

The maximum impact parameter, $b_{\max}$, is usually taken to be the smallest characteristic beam dimension, since collisions at larger distances are effectively screened by the finite beam size. A common choice is

$$b_{\max} = \min(\sigma_x, \sigma_y, \gamma\sigma_z), \quad (\text{B2})$$

where σ_x and σ_y are the rms transverse beam sizes, σ_z is the rms bunch length in the laboratory frame, and $\gamma\sigma_z$ is the bunch length transformed into the beam rest frame. For many accelerators, $b_{\max} = \sigma_y$, since the vertical beam size is usually the smallest, which is true for the flat beams in the EIC HSR.

The minimum impact parameter is normally defined as the larger of the classical (large-angle scattering) limit b_c and the quantum diffraction limit b_q ,

$$b_{\min} = \max(b_c, b_q), \quad (\text{B3})$$

and

$$b_c = \frac{r_0 c^2}{v_r^2}, \quad (\text{B4})$$

$$b_q = \frac{\hbar}{m_0 v_r}, \quad (\text{B5})$$

where r_0 is the charge particle's classical radius, $r_0 = Z^2 e^2 / (4\pi\epsilon_0 m_0 c^2)$, Ze is the particle's charge, m_0 is the particle's rest mass, v_r is the relative velocity in the beam frame, and $\hbar$ is the reduced Planck constant.

For proton and heavy-ion rings, v_r is typically in the range of 10^4 – 10^6 m/s, so that $b_c > b_q$, and the classical cutoff dominates. For electron storage rings, however, the much smaller electron mass leads to $b_q > b_c$, so the quantum cutoff dominates. A detailed discussion of applying a tail-cut procedure to account for synchrotron radiation effects in IBS growth rate calculations for electron storage rings can be found in Refs. [43,44].

For high-energy proton and heavy-ion accelerators, the Coulomb logarithm is typically between 15 and 25. The Lebedev–Nagaitsev formalism uses Eqs. (B2) and (B4) to evaluate the local Coulomb logarithm around the ring. For our studies of gold ion beams in RHIC, the Coulomb logarithm is approximately 18.

For the 275 GeV proton beam in the EIC HSR, the Coulomb logarithm is approximately 24.5 for equal horizontal and vertical emittances. For a flat proton beam with an 11:1 transverse emittance ratio, the typical Coulomb logarithm is about 22. Thus, the Coulomb

logarithm differs by approximately 10% between the round- and flat-beam cases.

Since the IBS growth rates from theory are directly proportional to the Coulomb logarithm, we noticed that different definitions of the Coulomb logarithm can result in differences of approximately 5% in the predicted IBS growth rates.

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