

Impact of electron-beam energy spread on the generation of high harmonics in seeded free-electron lasers

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(Received 16 June 2025; accepted 20 February 2026; published 6 April 2026)

In seeded free-electron lasers (FELs), harmonic up-conversion of an external seed laser can be used to generate coherent emission at short wavelengths. The harmonic generation process is sensitive to the electron beam slice energy spread, which limits the highest harmonic conversion factor. We present here an experimental study of the impact of the slice energy spread on the FEL performance in two different harmonic generation schemes: high-gain harmonic generation and echo-enabled harmonic generation (EEHG). The slice energy spread is controlled by means of a laser heater installed in the low energy region of the linear accelerator that drives the FEL. The measurements of the FEL pulse energy as a function of the electron beam slice energy spread were performed at different harmonics. Our results confirm the robustness of EEHG to the slice energy spread and open the way toward fully coherent FEL pulses in the soft x-ray domain.

DOI: [10.1103/13z8-2ktg](https://doi.org/10.1103/13z8-2ktg)

I. INTRODUCTION

The recent generation of extreme ultraviolet (EUV) and x-ray light sources based on high-gain single-pass free-electron lasers (FELs) [1] has paved the way for a wide class of experiments in physics, chemistry, and other fields, such as biology [2]. Most operational X-FEL facilities [3–7] are based on self-amplified spontaneous emission (SASE) [8]. In SASE, the chaotic spontaneous emission arises from shot noise in the electron bunch density and leads to a poor longitudinal coherence of the FEL pulses characterized by a relatively broad and noisy spectrum. Self-seeding techniques [9–11] can be used to improve the spectral quality of the FEL pulses. The aforementioned limitations can also be overcome by expediting the FEL amplification process with a stable external seed laser. However, direct amplification of an external seed [12–14] is typically limited to the EUV domain (above several tens

of nanometers) due to the lack of powerful coherent sources in the x-ray spectral region. Operation at shorter wavelengths requires a frequency up-conversion strategy such as high-gain harmonic generation (HGHG) [15,16], echo-enabled harmonic generation (EEHG) [17,18], or variants of those seeding schemes [19–21]. These schemes rely on one or several seed laser-electron interactions to generate bunching at high harmonics. These methods are likely to be sensitive to the initial electron beam properties such as slice energy spread, which can strongly affect the efficiency of the harmonic conversion process.

High-gain seeded FELs require high-brightness electron beams with high peak current, low transverse emittance, and low energy spread [22]. The high current is obtained by compressing the electron bunch in one or several magnetic chicanes installed along the accelerator. However, collective effects, like coherent synchrotron radiation [23] and longitudinal space charge [24] developing in bunch compressors and in the accelerator, drive the formation of stochastic modulations at the micrometer scale in the longitudinal electron bunch phase space (i.e., in energy and current) [25]. This so-called microbunching instability (MBI) has been observed in many FEL facilities [26–28] and is known to spoil the electron beam quality by increasing the slice energy spread in a range that impacts

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on the FEL amplification process [29,30]. A device called the laser heater (LH) [24] is commonly used to counteract the MBI growth by introducing incoherent energy spread in a controlled manner via a resonant laser-electron interaction in an undulator located in the low energy part of the accelerator [29–31]. A careful control of MBI and increase of energy spread are particularly useful for seeding schemes in order to reduce spurious pedestal contents in the FEL spectra [32] at the cost of a reduction of the bunching at high harmonics. The laser heater power must be carefully limited to avoid degradation of the free-electron laser (FEL) gain. Under certain operating conditions, this constraint leads to residual microbunching. When a highly narrow FEL bandwidth is required, the laser heater power is typically increased, sacrificing output intensity for spectral purity. Therefore, optimizing the laser heater power in a seeded FEL involves a trade-off between achieving high spectral purity and maintaining maximum pulse energy. Nonetheless, the slice energy spread is expected to have a smaller impact on the degradation of the harmonic bunching in the EEHG scheme compared to HGHG, due to the specific phase-space shaping using two seed lasers [33,34]. Consequently, EEHG offers a more flexible optimization of the laser heater power, satisfying experimental requirements that demand both high spectral purity and high pulse energy at high harmonics, where sensitivity to energy spread and phase-space imperfections is more pronounced. The microbunching instability is related to the linac configuration and the properties of the electron beam delivery system (bunch charge, compression factor, accelerating gradients, optics functions along the machine, and transport matrix elements), combined with possible initial sources of the microbunching (shot noise and imperfections in the temporal intensity profile of the photoinjector laser) and determines the final beam energy spread. The instability, being very sensitive to the slice energy spread at the very low energy part of the linac, can also depend on other processes such as intrabeam scattering that in some cases can significantly increase the energy spread [35,36]. For this reason, the resilience of the EEHG on the electron-beam energy spread is highly valuable and makes it suitable to operate below 4 nm [37]. In this work, we report on the impact of the slice energy spread on the performance of the HGHG and EEHG schemes. The experimental study is carried out at the FERMI FEL, where HGHG is extensively used [38,39] and EEHG in the soft x-ray domain has been recently demonstrated with a temporary setup [40]. A description of the two configurations used at FERMI is introduced in Sec. II together with the analytical framework. A thorough experimental characterization of the EEHG scheme at FERMI is then presented in Sec. III with comparison to HGHG. The slice energy spread is controlled using an LH, and the FEL properties are characterized, both in HGHG and EEHG configurations, at different harmonics. The study is

completed by an analysis of the sensitivity to energy spread for several settings of the EEHG parameters.

II. MODEL AND EXPERIMENTAL SETUP

A. High-gain harmonic generation

In HGHG, an external seed laser is used to modulate the electron beam energy in a first undulator, called the modulator. Then, the electron beam passes through a dispersive section (typically a four-dipole magnetic chicane) that converts the periodic energy modulation at the seed wavelength into a current density modulation containing Fourier components at the harmonics of the seed laser. Assuming an initial Gaussian beam energy distribution with a mean energy E and a root mean square (rms) energy spread σ_E , the bunching factor at the harmonic a_E of the seed can be written as [15]:

$$b_{a_E} = \left| \exp \left[-\frac{(a_E \sigma_E D)^2}{2} \right] J_{a_E}(a_E \Delta E D) \right|, \quad (1)$$

where $D = \frac{2\pi R_{56}}{\lambda E}$, with R_{56} the strength of the dispersive section and λ the seed laser wavelength. J_{a_E} is a Bessel function of the first kind of order a_E , and ΔE is the energy modulation amplitude induced by the seed laser.

This bunched beam is then injected into a second undulator, called the radiator, tuned to the harmonic a_E of the seed wavelength, where the coherent emission starts and is amplified by the FEL process. In order to produce a significant bunching at the a_E th harmonic, the induced energy modulation (ΔE) has to be $\sim a_E$ times larger than the natural slice energy spread of the beam (σ_E) while remaining smaller than the Pierce parameter to sustain FEL amplification [8]. This trade-off [15,16] limits the highest harmonic number achievable in one stage frequency up-conversion to about 13–15 (note that a non-Gaussian energy distribution increases this harmonic number [41]). Hence, for an efficient FEL amplification in the final radiator, the relative slice energy spread σ_E/E has to be a_E times smaller than the FEL parameter ρ at the entrance of the seeding section.

To overcome this limitation, the fresh-bunch HGHG double-stage cascade scheme uses the light emitted by a first HGHG FEL to seed a second FEL stage in a fresh portion of the electron beam [42]. Such a multiple-stage approach permits extending the achievable harmonic range up to $\sim 60 - 70$, thus enabling generation of coherent soft x-ray FEL pulses in the few-nanometer range from an ultraviolet seed laser [39]. Though simple and effective, the performance of the HGHG, and more particularly of the cascaded configuration, is quite sensitive to the electron beam properties.

The FERMI FEL in Trieste, Italy, operates two FEL lines based on HGHG scheme. FEL-1 is based on a single HGHG stage that can deliver coherent pulses at

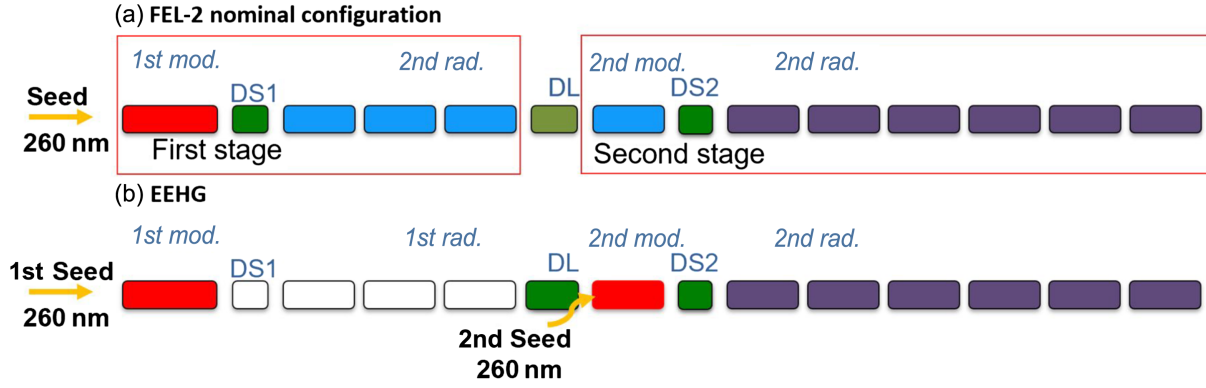


FIG. 1. FERMI FEL-2 layout. (a) Nominal layout of the double-stage HGHG configuration. A first HGHG FEL seeded by an external laser is used to seed a second HGHG FEL stage. The interaction in the second stage is delayed toward the head of the electron bunch by means of a magnetic delay line (DL). (b) Temporary EEHG layout implemented on FEL-2. The first dispersive section (DS1) is set to zero and the radiators of the first stage are fully opened and act as a drift section. The delay line is used to produce the strong dispersion needed for EEHG. A second external seed laser together with a second modulator resonant at the seed wavelength is used in the HGHG-like second stage.

wavelengths down to 20 nm [38]. FEL-2 consists of two-stage HGHG [Fig. 1(a)] and can operate down to 4 nm [39].

B. Echo-enabled harmonic generation

Some of the limitations of the two-stage HGHG can be overcome using the EEHG technique that uses two external seed lasers to precisely tailor the electron bunch distribution and generate very high harmonics in a single stage through a highly nonlinear harmonic frequency up-conversion process. As in HGHG, the electron beam is first modulated by an external seed laser in the first modulator. After passing through a strong dispersive section, the energy modulation is rotated such that it is folded in the longitudinal phase space resulting in a series of stripes, each stripe characterized by a reduced energy spread. After this first modulating section, the electron beam enters a single-stage HGHG where a second external seed laser and a second small dispersive section are used to modulate the striped electron bunch. As a result, the electron beam after the second chicane is density modulated at wave numbers k_E given by $k_E = nk_1 + mk_2$, where n and m are nonzero integers and $k_{1,2} = 2\pi/\lambda_{1,2}$ are the wave numbers of the two seed lasers operating at wavelengths $\lambda_{1,2}$. In the experiment reported here, the two seed lasers have the same wavelengths $\lambda_1 = \lambda_2 = \lambda$, so the output wave number is simply $k_E = (n + m)k = a_E k$ (or $\lambda_E = \lambda/a_E$), where $a_E = n + m$ is the harmonic number. In this case, the bunching factor at the harmonic $a_E = n + m$ is given by [17,18]:

$$b_{a_E} = \left| \exp \left[-\frac{(\xi_E \sigma_E)^2}{2} \right] J_n(-\xi_E \Delta E_1) J_m(-a_E \Delta E_2 D_2) \right|, \quad (2)$$

where $\Delta E_{1,2}$ are the energy modulations induced by the two seed lasers, $\xi_E = nD_1 + a_E D_2$ is the so-called EEHG

scaling factor, and $D_{1,2} = \frac{2\pi R_{56}^{(1,2)}}{\lambda E}$ with $R_{56}^{(1,2)}$ the strengths of the two dispersive sections.

Because of the reduced slice energy spread of the stripes, it is possible to produce a significant bunching at very high harmonics of the seed in a single stage using a relatively small energy modulation in the second modulator. Due to the specific highly nonlinear shaping of the electron beam longitudinal phase space of the EEHG scheme, the performance of EEHG is known to be much less sensitive to the electron beam properties compared to HGHG [33]. As a consequence, EEHG permits reaching higher harmonics than HGHG, as demonstrated at FERMI [40]: starting from a seed at 260 nm, it was possible to measure coherent emission at a harmonic order as high as 101 (i.e., 2.6 nm wavelength or 474 eV photon energy).

The FERMI FEL-2 line was temporarily modified to demonstrate EEHG in the soft x-ray domain [40,43,44], as illustrated in Fig. 1(b). The first dispersive section of the two-stage HGHG is switched off, and the first radiator segments are opened at a large gap to provide an almost null on-axis field. The current of the magnetic delay line is increased to provide a sufficiently high dispersion for EEHG. Finally, an HGHG-like second stage is composed of a second external seed laser used to modulate the beam in a second modulator and a second small dispersive section. After the successful operation of EEHG with the temporary EEHG + HGHG setup reported here, FERMI FEL-1 has been recently modified to accommodate an EEHG line for users [45].

C. Sensitivity to energy spread

The bunching factor is usually optimized by tuning the dispersion and the energy modulation amplitude. However, as seen in the expressions of the bunching factor

[Eqs. (1) and (2)], the energy spread σ_E plays a role in the harmonic generation efficiency.

In both the HGHG and EEHG schemes, the bunching factor exhibits an exponential decay proportional to the energy spread. For HGHG, as indicated by Eq. (1), this decay accelerates with increasing harmonic order, leading to a more rapid suppression of bunching. In contrast, the exponential decay in the EEHG bunching factor, as shown in Eq. (2), is governed by the EEHG scaling factor ξ_E . By minimizing this scaling factor, e.g., by choosing a small negative value for n [18], the sensitivity to energy spread can be significantly reduced compared to the pure exponential decay observed in HGHG. This behavior is shown in Fig. 2, which compares the bunching factor as a function of energy spread for HGHG (solid lines) and EEHG (dashed lines) using realistic parameters. To ensure a fair comparison between HGHG and EEHG at high harmonics, a fresh bunch double-stage cascade (HGFG-FB) has been considered for HGHG at harmonic 60 (Fig. 2, dash-dotted lines). This arises from the limited efficiency of HGHG's harmonic conversion process.

For all schemes, we assumed that the total energy modulation induced by the seeding does not exceed 1.5 MeV. For the parameter range considered here (beam

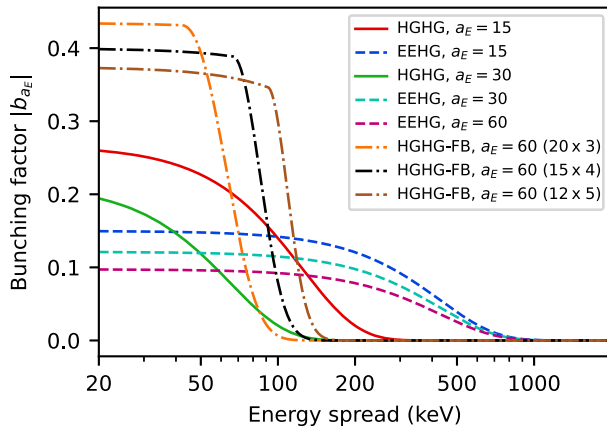


FIG. 2. Comparison of the bunching factors as a function of the energy spread for single-stage HGHG (solid lines), fresh-bunch double-stage HGHG (dash-dotted lines), and EEHG (dashed line), for different harmonic conversion ($a_E = 15, 30$, and 60). The HGHG parameters are: $E = 1$ GeV, $\lambda = 260$ nm, $\Delta E = 1.5$ MeV, $R_{56} = 31.3$ μm for $a_E = 15$ (red), $R_{56} = 29.9$ μm for $a_E = 30$ (green). For the double-stage HGHG-FB, the parameters are: $R_{56}^{(1)} = 30.6$ μm and $R_{56}^{(2)} = 1.9$ μm for $a_E = 20 \times 3$ (orange), $R_{56}^{(1)} = 31.3$ μm and $R_{56}^{(2)} = 2.4$ μm for $a_E = 15 \times 4$ (black), $R_{56}^{(1)} = 31.9$ μm and $R_{56}^{(2)} = 2.9$ μm for $a_E = 12 \times 5$ (brown). The EEHG parameters are: $E = 1$ GeV, $\lambda_1 = \lambda_2 = 260$ nm, $n = -1$, $\Delta E_1 = 0.5$ MeV, $R_{56}^{(1)} = 2$ mm and $\Delta E_2 = 0.35$ MeV, $R_{56}^{(2)} = 142.4$ μm for $a_E = 15$ (blue), $\Delta E_2 = 0.65$ MeV, $R_{56}^{(2)} = 71.2$ μm for $a_E = 30$ (cyan), $\Delta E_2 = 1.24$ MeV, $R_{56}^{(2)} = 35.6$ μm for $a_E = 60$ (magenta).

energy of 1.0 GeV), exceeding this threshold would degrade the FEL amplification process, as the total relative energy spread would surpass the FEL Pierce parameter ($\rho \sim 10^{-3}$). Modeling the two-stage HGHG-FB using simple bunching equations requires some level of approximation. For the data reported in Fig. 2, we assumed that the setup requires a 20% bunching from the first stage to generate a 1.5 MeV energy modulation in the second modulator. The model further assumes that for bunching factors exceeding 20% in the first stage, the second-stage modulation remains at 1.5 MeV while for lower bunching values of the first stage, the second-stage modulation scales with the square of the bunching factor. This choice determines the results of the sensitivity of HGHG-FB and is based on the FERMI setup. A different setup (i.e., with longer first stage radiators) could show slightly different results. However, we consider this to be a fair comparison and close to the ones experimentally investigated. For the HGHG-FB scheme, a final harmonic of $a_e = 60$ can be achieved by distributing harmonics between the first and second stages. Our results indicate that higher harmonics in the first stage are preferable for low energy spread but are also more vulnerable to increases in energy spread.

Figure 2 clearly shows that significant bunching can still be achieved at higher energy spread levels for EEHG compared to HGHG. The fact that the absolute maximum value of EEHG bunching for low energy spread is lower than that of HGHG is not an issue since optimal FEL operation requires an initial bunching in the range of 0.05–0.1 to allow for substantial amplification of the FEL in the downstream radiator.

The impact of the EEHG scaling factor on the sensitivity to energy spread is presented in Fig. 3. Figure 3(a) shows the EEHG bunching factor at harmonic 15 as a function of the second dispersive section, which is one of the parameters to adjust the scaling factor, and as a function of the energy spread. When the scaling factor increases to reach the value that maximizes the bunching factor, the sensitivity to the energy spread grows too. Figure 3(b) shows simultaneously the EEHG bunching factor and the scaling factor as a function of the second dispersive section. The behavior of the bunching factor as a function of the energy spread for three values of the scaling factor is shown in Fig. 3(c). As long as the absolute value of the scaling factor increases, the bunching factor sensitivity to the energy spread increases.

III. EXPERIMENTAL RESULTS

Taking advantage of the temporary EEHG scheme implemented at FERMI FEL-2 [40,43] (Fig. 1), we have done an experiment to characterize the sensitivity of the EEHG to the slice energy spread. Thanks to the careful control of the slice energy spread through the laser heater, we investigated the impact on the bunching factor and the

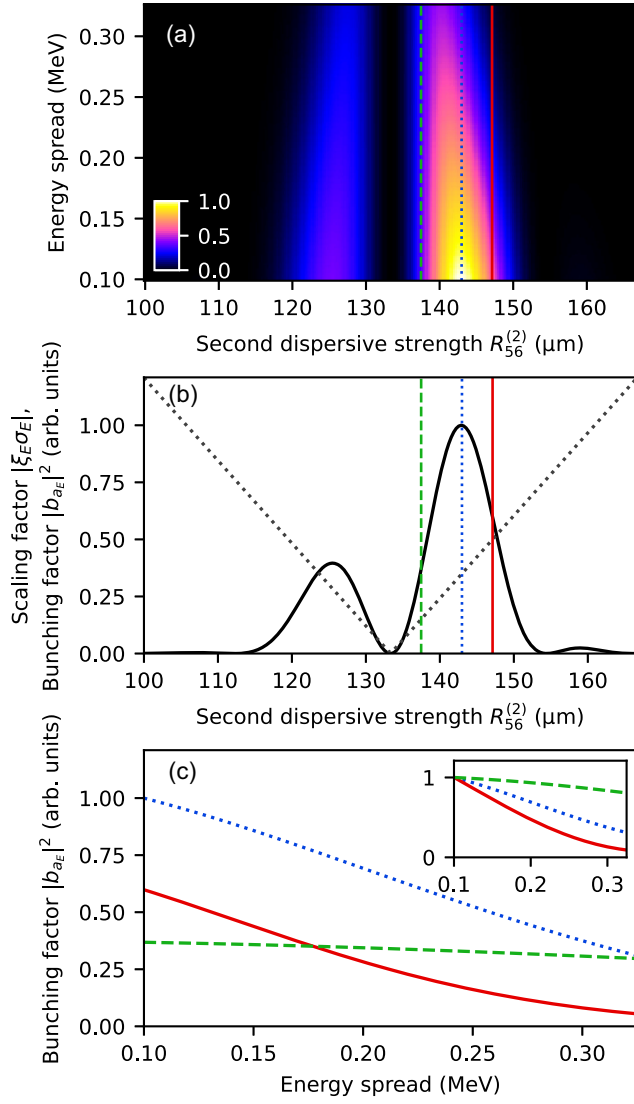


FIG. 3. (a) Square of the EEHG bunching factor (normalized to the maximum value) as a function of the second dispersive section $R_{56}^{(2)}$ and of the energy spread σ_E . (b) Square of the bunching factor (black line) and scaling factor $|\xi_E \sigma_E|$ (black dotted line) as a function of the second dispersive section for an energy spread $\sigma_E = 100$ keV. (c) Square of the bunching factor versus energy spread for different values of the EEHG scaling factor: $|\xi_E \sigma_E| = 0.15$ or $R_{56}^{(2)} = 137.5$ μm (green line), $|\xi_E \sigma_E| = 0.35$ or $R_{56}^{(2)} = 143$ μm (blue line), $|\xi_E \sigma_E| = 0.5$ or $R_{56}^{(2)} = 147.1$ μm (red line). The three curves correspond to the vertical linecuts in (a) and are associated with the scaling factor $|\xi_E \sigma_E|$ at the intersection with the three colored vertical lines in (b). Inset: same as in (c) with the bunching factor normalized to its maximum value. EEHG at harmonic $a_E = 15$ ($n = -1$) with the same parameters as in Fig. 2.

FEL pulse energy in various conditions. In a first step, we present a comparison between the EEHG setup and the two-stage HGHG configuration. Then, we focus on the EEHG behavior as a function of the energy spread for different harmonics. Finally, the impact of the EEHG

parameters, i.e., dispersive strength and EEHG n -number setting, on the sensitivity of EEHG to the slice energy spread is investigated. We measured the energy spread of the electron beam using the transverse deflector and the dipole spectrometer located at the end of the LINAC [29,30]. The analysis was performed using data collected in the high-energy-spread regime added by the laser heater. Our objective is to quantitatively investigate the behavior of the FEL in this regime by directly correlating the measured FEL pulse energy with the beam energy spread. In particular, we aim to demonstrate the reduced sensitivity of the EEHG process to large energy spreads compared to HGHG. Note that for energy spreads below a certain threshold, the measurements obtained with the deflector become unreliable due to the limited resolution of the diagnostic setup (approximately 100 keV). To be able to operate over a large range of harmonics, the electron beam energy has to be varied experimentally to match the resonant condition of the undulators. The various experimental configurations are reported in Table I.

A. High-gain harmonic generation versus echo-enabled harmonic generation

We have shown that the sensitivity of the bunching factor to the energy spread depends on the different harmonic generation schemes. Figure 4 shows the evolution of the FEL intensity as a function of the LH intensity for HGHG and EEHG. Experimentally, when far from saturation, the FEL intensity is approximately proportional to the square of the bunching factor. The slice energy spread is controlled by scanning the LH intensity. The measurements were performed at the 18th and 30th harmonics of the seed, which correspond to FEL emission at 14.7 and 8.6 nm, respectively.

For very low LH intensities, the drop in FEL intensity is associated with the presence of strong MBI that leads to the formation of microstructures in the longitudinal phase space and affects the FEL gain. In the configuration of the reported experiment, the minimum heating level required to limit MBI growth and optimize the FEL was achieved with the laser heater set at 1 μJ , resulting in a minimum slice energy spread slightly above 50 keV that was measured for settings not affected by MBI [46,47]. In case of HGHG, around 1 μJ LH pulse energy is required to suppress the MBI gain in the FERMI accelerator and to improve the FEL performances (Fig. 4, blue curves). Above this optimum value, the FEL intensity drops as long as the slice energy spread is increased due to the reduced bunching efficiency [cf. Eq. (1)]. Note that the presence of a second local maximum at the 18th harmonic [Fig. 4(a)] implies a non-Gaussian energy distribution induced by the LH [41].

The optimum LH pulse energy that maximizes the FEL intensity appears to be significantly higher in case of EEHG than in HGHG (Fig. 4, red curves). This behavior may be explained by the very large first dispersive strength required

TABLE I. EEHG and HGHG experimental parameters. Note that for the HGHG configuration, $R_{56}^{(1)}$ and $R_{56}^{(2)}$ refer to the first and second stage dispersive sections, respectively.

Configuration	Harmonic	FEL wavelength (nm)	Maximum FEL pulse energy (μ J)	e-beam energy (GeV)	$R_{56}^{(1)}$ (mm)	$R_{56}^{(2)}$ (μ m)
EEHG	18 ($n = -2$)	14.7	48	0.9	2.04	226
EEHG	30 ($n = -1$)	8.8	5.6	1.1	2.26	79
EEHG	37 ($n = -2$)	7.1	7.3	1.3	2.03	113–119
Two-stage HGHG	18 (3×6)	14.7	61	0.9	0.028	24
Two-stage HGHG	30 (5×6)	8.8	15	1.1	0.055	20

in EEHG, which can further drive the MBI. Consequently, a higher heating level is required to suppress the MBI and to optimize the slice energy spread at the entrance of the second dispersive section where the harmonic bunching is produced. However, as expected from the expression of the bunching factor [cf. Eq. (2)], the increase of the LH intensity above its optimum value induces a decrease of the EEHG FEL intensity, although at a lower rate compared to HGHG. The reduced sensitivity of the EEHG scheme to the slice energy spread with respect to HGHG permits to operate the FEL with stronger LH intensity. This is even more evident in the case of harmonic 30 [see Fig. 4(b)]. This is already impressive evidence that EEHG's reduced sensitivity to slice energy spread—compared to HGHG—

allows operation with higher levels of LH energy heating. This is crucial for suppressing higher microbunching associated with specific experimental conditions. However, this advantage comes at the expense of the FEL pulse energy, as the increased energy spread required to damp microbunching instability leads to the FEL saturation at lower intensity (see maximum FEL pulse energy in Table I).

We can also analyze how the spectral properties vary with beam energy spread for the two seeding mechanisms. Figure 5 shows the evolution of the relative spectral bandwidth (FWHM) as a function of LH intensity for both HGHG and EEHG. While previous studies [40] suggest that the EEHG seeding scheme is generally less sensitive to imperfections in the electron beam phase space, the results in Fig. 5 reveal some exceptions. At lower LH energies, the EEHG bandwidth is significantly broader, and the spectral stability is reduced due to the dominance of MBI. This effect broadens the FEL spectra and increases the fluctuations, as indicated by the larger error bars. HGHG even exhibits a narrower bandwidth compared to EEHG. However, when MBI is effectively suppressed at higher LH energies, EEHG demonstrates superior spectral properties, with its bandwidth remaining less affected by increases in beam energy spread due to the laser heater. Under these conditions, the EEHG bandwidth is consistently similar to or lower than that of HGHG. The data in Fig. 5 were collected simultaneously with those in Fig. 4, ensuring identical configurations and experimental conditions.

B. Echo-enabled harmonic generation versus harmonic order

Even though EEHG is less sensitive to the energy spread than HGHG, the sensitivity of the bunching factor depends on the frequency up-conversion order. In the following section, we have experimentally characterized the EEHG FEL intensity as a function of the slice energy spread at several harmonics (Fig. 6). We focus our study on LH intensity greater than the optimum in order to get rid of the MBI [48]. In this regime, the slice energy spread can be estimated from the LH intensity, the former scaling with the square root of the latter [29].

Figure 6 shows the evolution of the measured FEL intensity as a function of the slice energy spread at the 18th, 30th, and 37th harmonics of the seed. The experimental

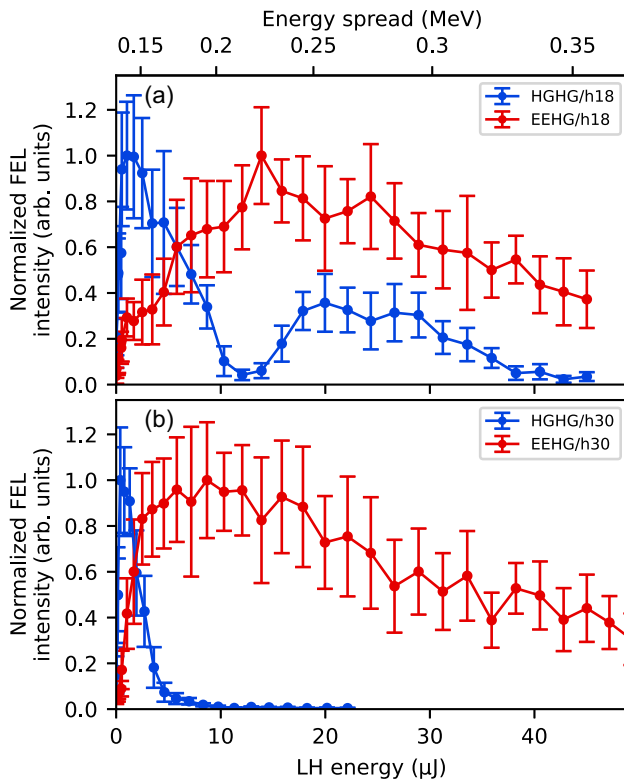


FIG. 4. Experimental FEL intensity (normalized to its maximum value) as a function of the LH energy (i.e., the slice energy spread) at the 18th (a) and 30th (b) harmonics for the EEHG (red) and the two-stage HGHG (blue) schemes.

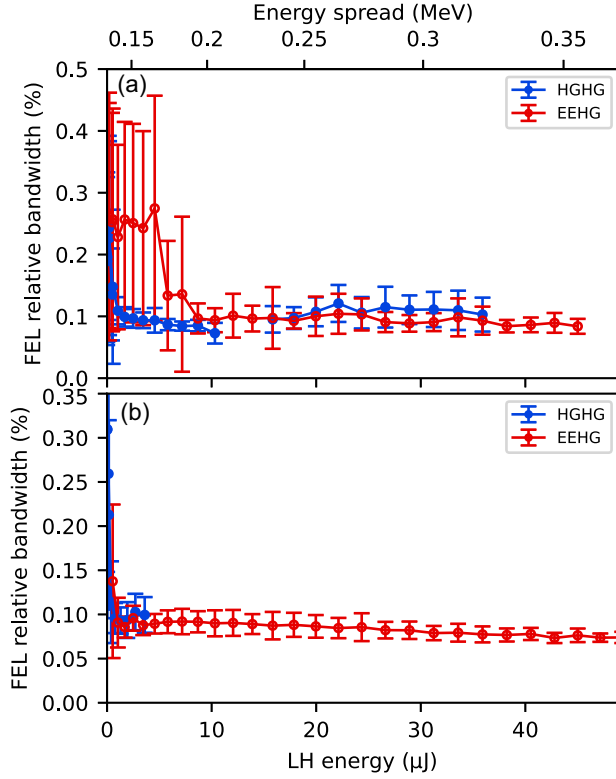


FIG. 5. Relative spectral bandwidth (FWHM) of HGHG (blue curves) and EEHG (red curves) as a function of the LH energy at the 18th (a) and 30th (b) harmonics. When the FEL pulse energy is too low, the signal level at the spectrometer becomes insufficient to accurately determine the bandwidth. Therefore, bandwidth calculations are only performed for FEL intensity levels exceeding 10% of the maximum value.

observations are in good agreement with the behavior of the square of the bunching factor [Eq. (2)], as reported by the dashed lines in Fig. 6. For higher harmonic order, the FEL intensity decreases at a higher rate as a function of the slice energy spread.

Moreover, one can notice that the maximum FEL intensity is reached for larger LH intensity when operating at a lower harmonic order. As reported in Table I, the measurements at low harmonics were performed at a lower electron beam energy, where microbunching effects are expected to be stronger because of the reduced beam rigidity. The larger heating required at the lower harmonic order can be explained by a higher level of microbunching.

C. Impact of the echo-enabled harmonic generation scaling factor

In the measurements reported above, the EEHG parameters were optimized to obtain the best FEL performance in terms of pulse energy and spectral quality. In particular, the value of the second dispersive section ($R_{56}^{(2)}$) was set to give the maximum bunching factor, as shown in Fig. 7(a). We have seen that the sensitivity to the slice energy spread

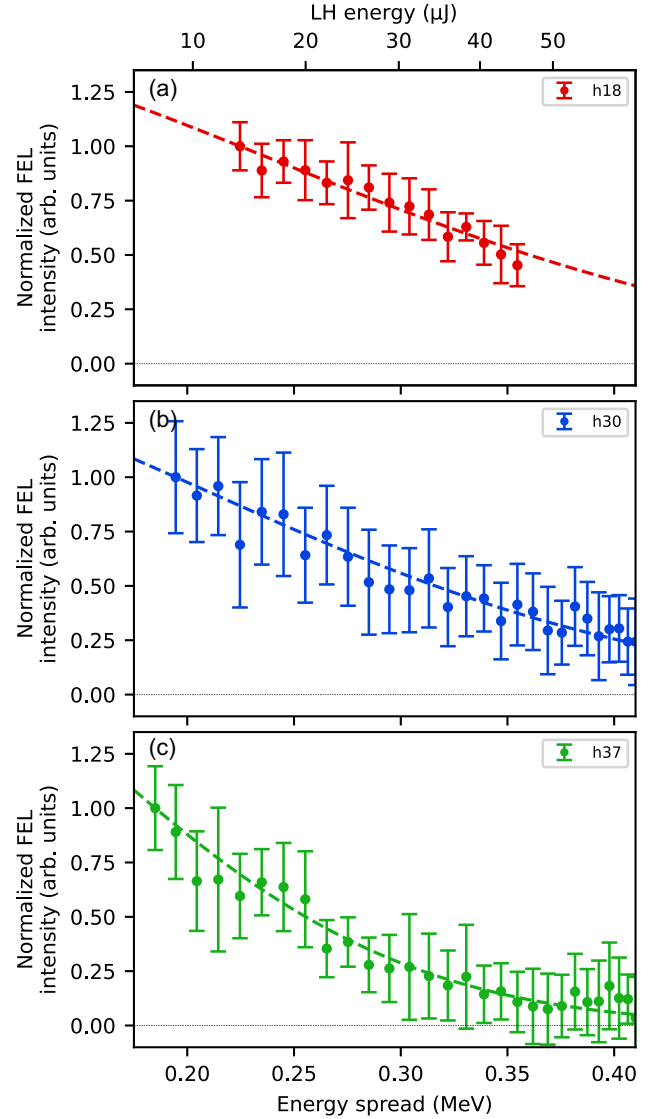


FIG. 6. EEHG experimental (dots) FEL intensity (normalized to its maximum value) and square of the EEHG theoretical bunching factor (dashed lines) (normalized to its maximum value) as a function of the LH energy (i.e., slice energy spread) at the 18th (a), 30th (b), and 37th (c) harmonics.

depends on the value of the scaling factor [cf. Eq. (2)], which can be tuned with the strength of the second dispersive section.

Figure 7 shows measurements taken at the 37th harmonic of the seed. (Note that the measurements were repeated for different harmonics.) The optimization of the FEL intensity as a function of the second dispersive section is represented together with the evolution of the absolute value of the scaling factor in Fig. 7(a). We sample three values of the scaling factor around the optimum value that maximizes the FEL intensity. The evolution of the FEL intensity for these three cases is shown in Fig. 7(b). During the measurements, all parameters of the EEHG setup were held constant except for the energy spread and the strength

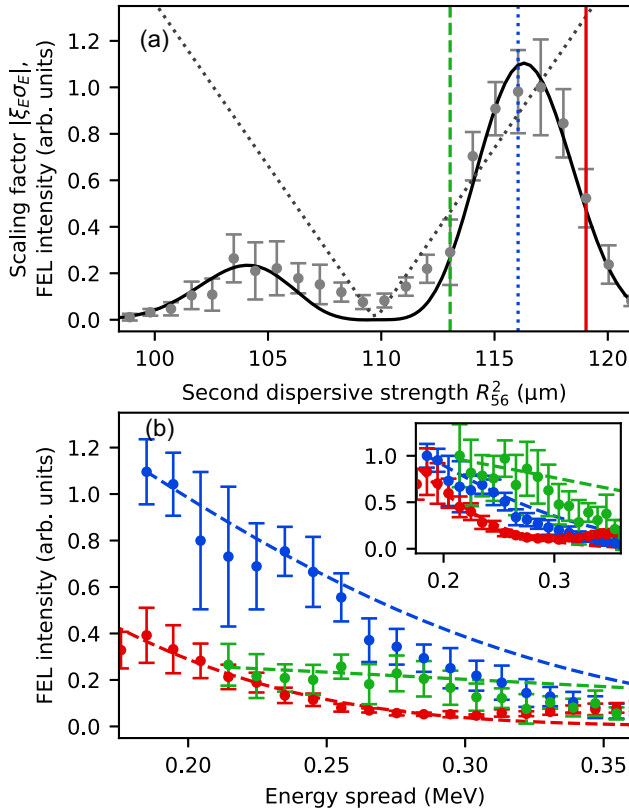


FIG. 7. (a) Theoretical (black line) and experimental (dots) EEHG FEL intensity (normalized to its maximum value) as a function of the second dispersive section and EEHG scaling factor (dotted line) for an energy spread of $\sigma_E = 204$ keV. (b) FEL intensity as a function of the energy spread for three values of the second dispersive section (corresponding to the vertical lines in (a): $113 \mu\text{m}$ (green), $116 \mu\text{m}$ (blue), and $119 \mu\text{m}$ (red). Inset: same as in (b), with the FEL intensity normalized to its maximum value.

of the second dispersive section. As expected from the square of the bunching factor, the experimental measurements confirm that, for small values of the scaling factor, the sensitivity of the bunching factor to the energy spread is strongly reduced. Indeed, the FEL intensity is decreasing at a lower rate in the case of $R_{56}^{(2)} = 113 \mu\text{m}$ [Fig. 7(b), green curve] than in the case of $R_{56}^{(2)} = 119 \mu\text{m}$ [Fig. 7(b), red curve], and the experimental observations are in good agreement with the theoretical predictions.

The optimal operating point is determined by evaluating FEL intensity, stability, and spectral performance, specifically focusing on bandwidth and stability to meet experimental requirements. During the experimental period when the reported data were collected, the spectral quality was inherently stable and favorable for the energy spread and strength of the second dispersive section that maximized intensity. These parameters were thus selected as the baseline for the setup. A key question is whether shorter wavelengths can be achieved while maintaining a

high-quality spectrum, without significantly compromising pulse energy. This could involve increasing the energy spread to better suppress microbunching and adjusting the R_{56} of the second dispersive section to operate with a smaller scaling factor, thereby reducing sensitivity to energy spread.

IV. CONCLUSION

We have evaluated the impact of the slice energy spread on the performance of HGHG and EEHG by measuring the FEL pulse energy as a function of the electron beam slice energy spread. We conducted a systematic investigation of the dependence on energy spread and scaling factor for several harmonic settings. Our experimental observations confirm that EEHG is less affected by slice energy spread compared to HGHG, aligning well with the analytical modeling of the bunching factor.

A key advantage of the EEHG seeding scheme is its reduced sensitivity to the electron-beam energy spread, which allows the use of a stronger laser heater. This capability is particularly beneficial for operation with highly compressed electron beams that provide higher peak currents but are more susceptible to microbunching instabilities. The application of a stronger laser heater can effectively suppress these instabilities while preserving the beam brightness required for high-gain FEL operation. This aspect becomes especially critical when extending the EEHG scheme to shorter wavelengths, where even a modest enhancement of the FEL gain can be decisive in reaching saturation within a practical undulator length.

ACKNOWLEDGMENTS

The authors would like to thank G. Stupakov, W. Fawley, H. Braun, C. Callegari, V. Grattoni, Z. Huang, G. Penn, M. Pop, E. Prat, and S. Reiche for useful discussions. The authors also acknowledge the continuous support of R. Godnig, R. Bracco, R. Visintini, and the machine operators of Elettra-Sincrotrone Trieste during the experiment. E. R. has been partially supported by the LABEX CEMPI (ANR-11-LABX-0007) as well as by the Ministry of Higher Education and Research, Hauts de France council and European Regional Development Fund (ERDF) through the Contrat de Projets Etat-Region (CPER) Photonics for Society (P4S).

DATA AVAILABILITY

The data that support the findings of this article are openly available [49].

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