

## Erratum: Gauge-invariant theory of truncated quantum light-matter interactions in arbitrary media [Phys. Rev. A **107**, 013722 (2023)]

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In our original paper, we made a claim about the unique correctness of the Coulomb gauge when deriving system-reservoir models (e.g., of loss from a quantized cavity mode) from a rigorous open-system perspective. The argument in the paper relied on the separability of the initial reservoir state in the Coulomb gauge, as the Coulomb gauge is the unique gauge in which the transverse electromagnetic fields can be written entirely in terms of bosonic polariton operators.

It has come to our attention that this argument is incorrect because under the assumption of weak coupling that is inherent to the Markov approximation, any entanglement present in an initial state due to a choice of gauge will be negligible to the same degree as correlations induced over time neglected by, for example, the second-order Born-Markov master equation. Indeed, recent work by some of us has shown that either Coulomb or dipole gauges can be used to rigorously describe phenomenological dissipation from quantized cavities [1], as well as *ab initio* dissipation models using quantized quasinormal modes [2]. Additionally, either gauge can be used to rigorously describe loss from a two-level system undergoing internal dynamics in general photonic environments [3]. In all cases, the choice of gauge can yield different *spectral densities* of the system-reservoir interaction, but leads to identical observable predictions.

As such, we retract the finding (i) we listed in Sec. I of our paper, the paragraph that begins “To be concrete, consider the case of a cavity resonator . . .” in Sec. IV C, and related claims made in the conclusions, Sec. VIII.

The remainder of Sec. IV C after the aforementioned paragraph requires some discussion. The main argument of this section was in response to Ref. [4] (Appendix D), which itself was a response to an earlier paper by some of us, Ref. [5].

In Ref. [5] we made an assumption about the form of our system-reservoir coupling Hamiltonian that the system operators which couple to the cavity reservoir should correspond to the same operators in which the transverse electromagnetic fields are expanded, i.e., in the Coulomb gauge, bosonic cavity operators. This was not central to the findings of our paper (only mentioned in the Appendixes), but rather a concrete and plausible guess as to the form of the system-reservoir Hamiltonian, in the absence of a more rigorous *ab initio* theory. Reference [4] claims that, in general, physical models, i.e., *ab initio* approaches, would be required to definitely determine the proper form of the system-reservoir interaction, which we certainly agree with.

However, Ref. [4] made a claim that if one were to assume that the operators corresponding to the transverse cavity fields (in our considered case, the electric field) were to couple to the cavity reservoir, then our approach would be incorrect as we did not account for the different representations of the transverse electric field at and away from the dipole location. As we mentioned in our paper, this criticism is invalid because a correct truncation to a single-mode representation of the cavity fields collapses this spatially dependent distinction (which is only resolved as a  $\delta$ -function difference in the limit of a complete set of modes). Our response to the criticism of Ref. [4] remains correct, and this was also previously pointed out in Ref. [6].

However, as we have shown in recent works [1,3], the question of which operators to couple to an external reservoir to obtain rigorous and accurate results is not unique. As we show in Ref. [2], a projection into resonant cavity components and residual reservoir fields can be made in any gauge, regardless of the representation of light and matter degrees of freedom in said gauge; what is important is simply that the coupling induced between the two subsystems as a result of the partitioning is weak. Any effect of the choice of gauge is then reflected in the spectral density, but if treated consistently, correct results can be obtained.

In conclusion, we retract our statements that the Coulomb gauge is uniquely correct for rigorous open-system models of lossy cavity systems. Our response to the criticism of Ref. [4], as well as the rest of our paper, remains valid.

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