

Influence of frequency detuning on a four-level heterodyne Rydberg-atom receiver

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In recent years, quantum reception technology has achieved significant progress in microwave detection. However, most existing studies focus on energy-level resonance conditions, while research on the influence of frequency detuning is relatively scarce. Based on the four-level heterodyne Rydberg-atom receiver system, this paper establishes a theoretical model using the two-dimensional Fourier transform to analyze the correlation between frequency detuning and system response. Through theoretical simulation, the system response under different detuning conditions is quantitatively obtained, and the model is verified by experiment. This study expands the understanding of frequency detuning in Rydberg-atom receivers and provides guidance for designing their parameters in complex environments.

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I. INTRODUCTION

With the rapid development of quantum technology, Rydberg atoms, owing to their advantages such as high controllability, large electric dipole moment, and strong coupling with external electromagnetic fields [1], find applications in fields including antenna calibration [2] and terahertz wave sensing [3]. Moreover, relying on their outstanding performance in International System of units traceability, ultrawideband response, and ultrahigh sensitivity, Rydberg atoms exhibit unique advantages in measuring microwave electric fields.

In a typical four-level heterodyne Rydberg-atom receiver, multiple laser beams must interact precisely to prepare Rydberg atoms. The laser that excites atoms from the ground state to an intermediate level is called the probe laser, while the laser that excites atoms from the intermediate state to the Rydberg state is the coupling laser. During heterodyne measurement, the microwave electric field used for coherent mixing with the field being measured (enabling signal frequency conversion and amplification) is termed the local oscillator (LO) electric field. Most work on four-level heterodyne Rydberg-atom receivers assumes that the probe laser, coupling laser, and LO electric field are all resonant [2,4–13]. For example, under resonance, Jing *et al.* measured microwave electric fields with a Rydberg-atom receiver employing a superheterodyne structure, achieving a sensitivity of $55 \text{ nV cm}^{-1} \text{ Hz}^{-1/2}$ [4]. In contrast, relatively few studies examine the effect of detuning on atomic receivers [14–19]. Simons *et al.* showed that a weakly detuned radio-frequency

(rf) field can enhance electric-field intensity measurement sensitivity compared to resonance [14]; Cai *et al.* received amplitude-modulated signals under an rf field detuned by -6 MHz , achieving sensitivity of $35.75(51) \text{ nV cm}^{-1} \text{ Hz}^{-1/2}$ [16]; Yang *et al.* used the Mach-Zehnder interferometer method to measure orthogonal components. When the probe laser was detuned at -6 MHz , the signal-to-noise ratio increased by 12 dB compared to the resonance frequency [19]. These results indicate that detuning has a significant impact on system performance. However, most research on detuning examines only one excitation source and lacks a systematic discussion of the impact of detuning on system response.

To address the above issues, this paper, based on the four-level heterodyne Rydberg-atom receiver system, not only analyzes the detuning of a single-excitation source, but also studies the influence of detuning under locked conditions on the system. By applying the two-dimensional Fourier transform method to calculate the elements of the density matrix, this paper establishes a system response equation that simultaneously incorporates the frequency detuning of the probe laser, the coupling laser, and the LO electric field. Based on this, through theoretical simulation of the established response equation, the laws governing the influence of detuning on the system response are obtained under the conditions of single-excitation source detuning and locked detuning, respectively. Furthermore, a four-level heterodyne Rydberg-atom receiver model is built to verify the correctness of the theory. This study provides a crucial theoretical foundation for the application of Rydberg-atom receivers.

II. SYSTEM RESPONSE MODEL UNDER THE EFFECT OF FREQUENCY DETUNING

Figure 1(a) shows a schematic diagram of a four-level heterodyne Rydberg-atom receiver. The yellow box in the figure represents an atomic vapor cell filled with an atomic medium. Two laser beams overlap and pass through the vapor cell to generate Rydberg atoms. The probe laser and

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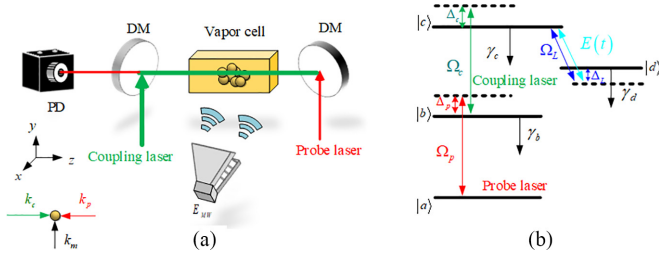


FIG. 1. Schematic diagram and energy-level diagram of the Rydberg-atom receiver. (a) Structural diagram of the four-level heterodyne Rydberg-atom receiver, where DM is a dichroic mirror and PD is a photodetector. (b) Energy-level diagram of the receiver, where $|a\rangle$ is the ground state, $|b\rangle$ is the intermediate state, and $|c\rangle$ and $|d\rangle$ are two Rydberg states.

coupling laser counterpropagate along the z axis. The microwave electric field propagates along the x axis, with k_p , k_c , and k_m representing the wave vectors of the probe laser, the coupling laser, and the microwave electric field, respectively. The microwave electric field acts on the vapor cell through a horn antenna. E_{MW} denotes the electric field acting on the vapor cell, which consists of a local oscillator electric field and a microwave electric field to be measured. The amplitude of the microwave electric field to be measured is much smaller than that of the LO electric field. Figure 1(b) is the energy-level diagram of the four-level heterodyne Rydberg-atom receiver. Here the Rabi frequencies of the probe laser, the coupling laser, and the LO electric field are Ω_p , Ω_c , and Ω_L , respectively. Their detunings are Δ_p , Δ_c , and Δ_L , respectively. The decay rates of the energy levels $|b\rangle$, $|c\rangle$, and $|d\rangle$ are γ_b , γ_c , and γ_d , respectively. The expressions of the probe laser, coupling laser, and LO electric field are

$$E_i(t) = E_i \cos(\omega_i t + \Delta_i t), \quad i = p, c, L, \quad (1)$$

where t is time; ω_p , ω_c , and ω_L are the resonant frequencies between energy levels $|a\rangle$ and $|b\rangle$, $|b\rangle$ and $|c\rangle$, and $|c\rangle$ and $|d\rangle$, respectively; and E_p , E_c , and E_L are the amplitudes of the probe laser, the coupling laser, and the LO electric field, as indicated in the diagram.

Lasers and electric fields interact, allowing the probe laser to pick up information about the microwave electric field as it passes through the atomic vapor cell. During this step, the four-level heterodyne Rydberg-atom receiver system senses the microwave signal by changing the medium's electric susceptibility. Correspondingly, the optical field of the probe laser after passing through the vapor cell, as depicted in Fig. 1(a), can be expressed as

$$S(t) = E_p e^{-k_p \chi_{im}(t)L/2} \cos\left(\omega_p t + \Delta_p t - k_p \frac{\chi_{re}(t)}{2} L + \varphi_0\right), \quad (2)$$

where L is the path length of the probe laser through the vapor cell; $\chi_{im}(t)$ is the imaginary part of the susceptibility $\chi(t)$ of the atomic medium, representing its absorption of the probe laser; $\chi_{re}(t)$ is the real part of the susceptibility $\chi(t)$, describing the dispersion effect imparted by the atomic medium on the probe laser; and φ_0 is the constant phase of the optical field. For thermal atoms, the susceptibility can be

expressed as [20]

$$\begin{aligned} \chi(t) &= \int N(v) \chi_v(v) dv \\ &= \frac{-2g_{ab}^2}{N\epsilon_0\hbar\Omega_p} \int_{-\infty}^{+\infty} N(v) \int_0^L \rho_{ba}(z, v, t) U_n^*(z) dz dv. \end{aligned} \quad (3)$$

The normalization factor in the formula is given by $N = \int_0^L |U_n(z)|^2 dz$, where $U_n(z) = e^{ik_p z}$. The function $N(v) = N_0 f(v)$ describes the atomic velocity distribution, where $f(v)$ is the Maxwell-Boltzmann distribution, N_0 is the atomic concentration in the vapor cell, and v is the atomic velocity. Due to the counterpropagating geometry of the probe and coupling lasers along the z axis, the Doppler shift depends only on the atomic velocity component $v = v_z$. Therefore, only this velocity component is considered in the model. The transition dipole moment between energy levels $|a\rangle$ and $|b\rangle$ is g_{ab} , while ϵ_0 and $\hbar$ are the vacuum permittivity and reduced Planck constant, respectively. From Eq. (3), susceptibility $\chi(t)$ depends on the density-matrix element ρ_{ba} , which carries the microwave signal to be measured. Thus, solving for ρ_{ba} is necessary.

At room temperature, the velocity of atoms in the vapor cell is $\mathbf{v}$. At this time, the density matrix ρ depends on both time and position. The Lindblad master equation of the density matrix can be expressed as [21]

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla\right) \rho = -\frac{i}{\hbar} [\mathbf{H}, \rho] + \mathbf{L}, \quad (4)$$

where ∇ is the total differential operator, $\mathbf{H}$ is the Hamiltonian matrix of the atomic system, and $\mathbf{L}$ is the decay matrix. By applying the two-dimensional Fourier transform, the density-matrix element $\rho_{ba}(t, z)$ is obtained (see the Appendix for a detailed derivation),

$$\begin{aligned} \rho_{ba}(t, z) &= \rho_{ba}^0 e^{ik_p z} + h_{1v}(t) \otimes \Omega_s(t) e^{ik_p z} \\ &\quad + h_{2v}(t) \otimes \Omega_s^*(t) e^{ik_p z}. \end{aligned} \quad (5)$$

In the above formula, $h_{1v}(t)$ and $h_{2v}(t)$ are the inverse Fourier transforms of $H_{1v}(\omega)$ and $H_{2v}(\omega)$, and $h_{1v}(t)$ and $h_{2v}(t)$ represent the response functions of the four-level heterodyne Rydberg-atom receiver, where $\otimes$ denotes convolution. According to the relationship with the microwave electric field to be measured, ρ_{ba} can be divided into two parts. One part is ρ_{ba}^0 , which is irrelevant to the microwave electric field to be measured and is only related to the probe laser, coupling laser, and local oscillator electric field. It can be obtained via the exact Lindblad master equation. The other part is related to the measured microwave electric field. This is the result of the combined action of the measured microwave electric field and its conjugate. When receiving the microwave electric field to be measured, the primary focus is on the reception of this component. By substituting Eq. (5) into Eq. (3), the susceptibility of the system is obtained as

$$\chi(t) = C_1 [\overline{\rho_{ba}^0} + h_1(t) \otimes \Omega_s(t) + h_2(t) \otimes \Omega_s^*(t)]. \quad (6)$$

Here $C_1 = \frac{2N_0 g_{ab}^2}{\epsilon_0 \hbar \Omega_p}$, $\overline{\rho_{ba}^0} = \int \rho_{ba}^0 f(v_z) dv_z$, $h_1(t) = \int h_{1v}(t) f(v_z) dv_z$, and $h_2(t) = \int h_{2v}(t) f(v_z) dv_z$.

In addition,

$$\begin{aligned}\chi_{\text{im}}(t) &= C_1 \text{Im}[\bar{\rho}_{ba}^0 + h_1(t) \otimes \Omega_s(t) + h_2(t) \otimes \Omega_s^*(t)], \\ \chi_{\text{re}}(t) &= C_1 \text{Re}[\bar{\rho}_{ba}^0 + h_1(t) \otimes \Omega_s(t) + h_2(t) \otimes \Omega_s^*(t)].\end{aligned}\quad (7)$$

Substituting Eq. (6) into Eq. (2), the transmitted optical power can be obtained as

$$P(t) = \bar{P} e^{C \text{Im}[h_1(t) \otimes \Omega_s(t) + h_2(t) \otimes \Omega_s^*(t)]}, \quad (8)$$

where $C = \frac{2k_p L N_0 \Omega_{ab}^2}{\varepsilon_0 \hbar \Omega_p}$ and $\bar{P} = P_0 e^{C \text{Im}(\bar{\rho}_{ba}^0)}$. Here we take a single-frequency electric field as an example to discuss the influence of detuning on the system. Let the microwave signal to be measured be $E_s(t) = E_s e^{i\Delta_s t}$, where Δ_s is the frequency difference between the signal field to be measured and the local oscillator electric field. Then Eq. (8) can be transformed into

$$P(t) = \bar{P} e^{C(E_s \Omega_{cd} / \hbar) \text{Im}[H_1(\Delta_s) e^{i\Delta_s t} + H_2(-\Delta_s) e^{-i\Delta_s t}]}. \quad (9)$$

Here $H_1(\omega)$ and $H_2(\omega)$ are the Fourier transforms of $h_1(t)$ and $h_2(t)$, and $H_1(\omega)$ and $H_2(\omega)$ can be written as

$$\begin{aligned}H_1(\omega) &= H_{1r}(\omega) + iH_{1i}(\omega), \\ H_2(\omega) &= H_{2r}(\omega) + iH_{2i}(\omega),\end{aligned}\quad (10)$$

where $H_{1r}(\omega)$ and $H_{2r}(\omega)$ are the real parts of $H_1(\omega)$ and $H_2(\omega)$, and $H_{1i}(\omega)$ and $H_{2i}(\omega)$ are the imaginary parts of $H_1(\omega)$ and $H_2(\omega)$. The amplitude of the LO electric field is much larger than that of the microwave signal to be measured. After the probe laser passes through the vapor cell, the beam power divides into two parts. One part, unrelated to the microwave signal and only dependent on the probe laser, coupling laser, and LO field of the four-level system, is called the average power $\bar{P}$. The other part is related to the input microwave signal to be measured and is referred to as the effective optical power P_{eff} . Then Eq. (9) can be transformed into

$$P(t) \approx \bar{P} + P_{\text{eff}} \sin(\Delta_s t + \phi), \quad (11)$$

where

$$\bar{P} = P_0 e^{C \text{Im}(\bar{\rho}_{ba}^0)}, \quad (12)$$

$$P_{\text{eff}} = \bar{P} C \frac{E_s \Omega_{cd}}{\hbar} \sqrt{[H_{1i}(\Delta_s) + H_{2i}(-\Delta_s)]^2 + [H_{1r}(\Delta_s) - H_{2r}(-\Delta_s)]^2}. \quad (13)$$

The phase ϕ satisfies the formulas

$$\cos \phi = \frac{H_{1r}(\Delta_s) - H_{2r}(-\Delta_s)}{\sqrt{[H_{1i}(\Delta_s) + H_{2i}(-\Delta_s)]^2 + [H_{1r}(\Delta_s) - H_{2r}(-\Delta_s)]^2}}, \quad (14)$$

$$\sin \phi = \frac{H_{1i}(\Delta_s) + H_{2i}(-\Delta_s)}{\sqrt{[H_{1i}(\Delta_s) + H_{2i}(-\Delta_s)]^2 + [H_{1r}(\Delta_s) - H_{2r}(-\Delta_s)]^2}}. \quad (15)$$

III. SIMULATION ANALYSIS UNDER DIFFERENT DETUNING CONDITIONS

In a room-temperature environment, the differing wavelengths of the probe and coupling lasers lead to a Doppler mismatch. In order to avoid the influence of this detuning on the system, in the case of counterpropagation, the detuning of the probe laser and the detuning of the coupling laser should satisfy the formula

$$\Delta_c = -\frac{\lambda_p}{\lambda_c} \Delta_p, \quad (16)$$

where λ_p is the wavelength of the probe laser and λ_c is the wavelength of the coupling laser. The above formula defines the locking relationship. When discussing the effect of detuning on the Rydberg-atom receiver system, this paper analyzes two cases. The first is detuning of a single-excitation source: The probe laser, coupling laser, or LO electric field is detuned. The second is cooperative detuning under the locking relationship: The probe and coupling lasers are detuned simultaneously, with their detuning amounts constrained as in Eq. (16). For comparison, the simulation analysis below assumes a constant Rabi frequency and a uniform system attenuation coefficient.

A. Influence of single detuning on system response

In the theoretical analysis of single detuning, we compare the variation laws of the system response functions H_1 and H_2 with the frequency difference Δ_s between the signal to be measured and the local oscillator electric field under the conditions of no detuning ($\Delta_{p,c,L} = 0$) and single detuning (Δ_p , Δ_c , or Δ_L is set to $2\pi \times 1$ MHz), as shown in Fig. 2. Figures 2(a) and 2(b) show the imaginary and real parts of H_1 and Figs. 2(c) and 2(d) show the imaginary and real parts of H_2 . It can be seen from Fig. 2 that under the condition of no detuning, the real-part curves of H_1 and H_2 show typical odd symmetry, and the zero-crossing points are located at the center frequency; their imaginary-part curves show typical even symmetry, and the peak points are located at the center frequency. When a certain optical field is detuned, the response curves of H_1 and H_2 change significantly and the parity with respect to Δ_s no longer exists, causing their peak points and zero-crossing points to shift accordingly. Moreover, it can be seen from the figure that under the same detuning amount, the detuning of the probe laser has the greatest influence on the real and imaginary parts of H_1 and H_2 . The detuning of the probe laser alter the velocity group of the excited Rydberg atoms, which plays a dominant role in determining the response. In the four-level heterodyne Rydberg-atom receiver

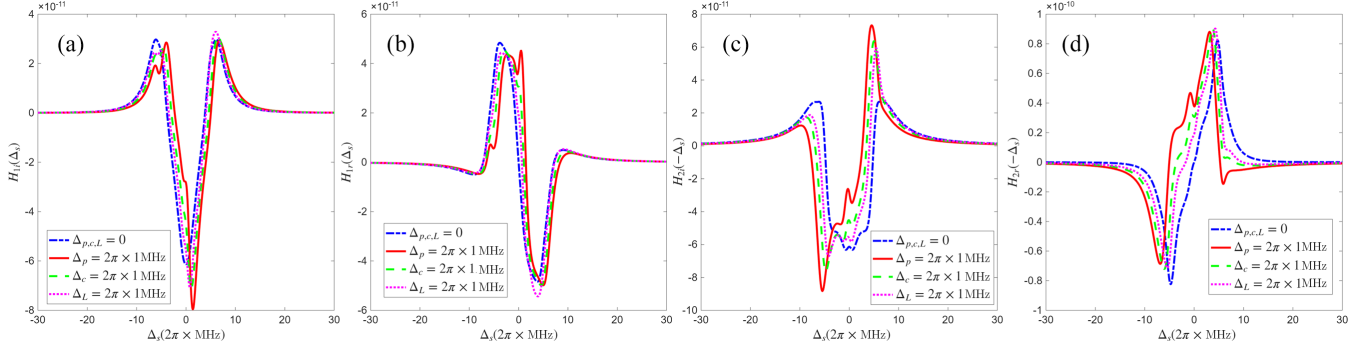


FIG. 2. Real and imaginary parts of H_1 and H_2 under single detuning: (a) and (b) imaginary and real parts of H_1 and (c) and (d) imaginary and real parts of H_2 . The blue curve corresponds to the real and imaginary parts of H_1 and H_2 under the resonance condition of all fields; the red, green, and magenta curves represent the real and imaginary parts of H_1 and H_2 when the probe laser, coupling laser, and local oscillator field are detuned by $2\pi \times 1$ MHz, respectively, with the other two fields remaining in resonance in each case. The simulation parameters are configured as follows: The Rabi frequencies of the probe laser, coupling laser, and LO field are $\Omega_p = 2\pi \times 2$ MHz, $\Omega_c = 2\pi \times 7$ MHz, and $\Omega_L = 2\pi \times 6$ MHz, respectively, and the decay rates of energy levels $|b\rangle$, $|c\rangle$, and $|d\rangle$ are $\gamma_b = 2\pi \times 5.2$ MHz, $\gamma_c = 2\pi \times 3.9$ kHz, and $\gamma_d = 2\pi \times 2.5$ kHz, respectively.

system, the electric-field signal to be measured is obtained by detecting the effective optical power. Substituting the real and imaginary parts of $H_1(\Delta_s)$ and $H_2(-\Delta_s)$ into Eq. (13) gives the effective optical power. The relationship between P_{eff} and Δ_s under different detuning is shown in Fig. 3. Under resonance conditions, the effective optical power of the system is an even function, with a bandwidth of 13.75 MHz, and the in-band system response, i.e., the receiver's response to signals falling within its operational frequency band) [22], is

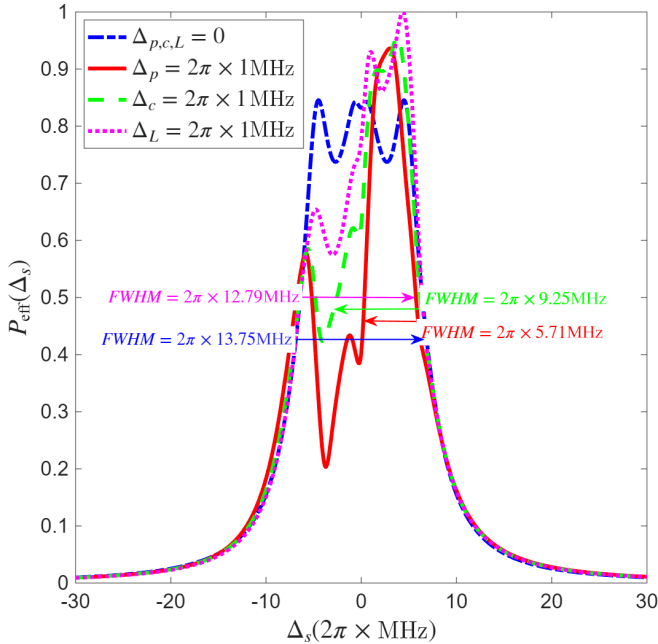


FIG. 3. Variation of the effective optical power with Δ_s under single detuning. The effective optical powers are normalized using the maximum effective optical power as the standard. The simulation parameters are configured as follows: The Rabi frequencies of the probe laser, coupling laser, and LO field are $\Omega_p = 2\pi \times 2$ MHz, $\Omega_c = 2\pi \times 7$ MHz, and $\Omega_L = 2\pi \times 6$ MHz, respectively, and the decay rates of energy levels $|b\rangle$, $|c\rangle$, and $|d\rangle$ are $\gamma_b = 2\pi \times 5.2$ MHz, $\gamma_c = 2\pi \times 3.9$ kHz, and $\gamma_d = 2\pi \times 2.5$ kHz, respectively.

relatively flat. The system bandwidth decreases under excitation source frequency detuning, particularly under probe laser detuning, where a drop to 5.71 MHz is observed. However, compared with P_{eff} at the resonance frequency, P_{eff} under single-detuning conditions is improved. Specifically, under LO electric-field detuning, P_{eff} increases the most, but the fluctuation of the system response is relatively large. From the above analysis, it can be concluded that when the system is in the state of single-excitation source detuning, the intensity of P_{eff} is slightly enhanced compared with the resonance condition. In some specific application scenarios, this gain is regarded as a potential performance optimization point. However, in-depth analysis reveals that detuning a single-excitation source will lead to a significant narrowing of the system bandwidth. As a key indicator for measuring the system's ability to respond to signals of different frequencies, the narrowing of the system bandwidth means that the frequency range of signals the system can process becomes narrower. For application scenarios that require handling wideband signals, this will undoubtedly significantly limit the system's practicality. At the same time, the system response flatness is also significantly affected, as evidenced by large fluctuations in the amplitude of the output signal within the frequency range where the system should maintain a stable response. Such fluctuations will introduce additional signal distortion, reducing the system's accuracy in signal restoration. In conclusion, although single-excitation source detuning can introduce gain to the system, it must be at the cost of a sharp narrowing of the system bandwidth and uneven in-band response, while the gain that this detuning can bring to the system is extremely limited. From the perspective of the system's overall performance, the negative impact of single detuning on the system is far greater than the value of the gain it provides. Therefore, for the quantum receiver system, detuning of the single-excitation source cannot achieve optimal system performance.

B. Influence of detuning on system response under the locking relationship

Under the locking relationship, assuming the wavelengths of the probe laser and the coupling laser are 852 and 510 nm,

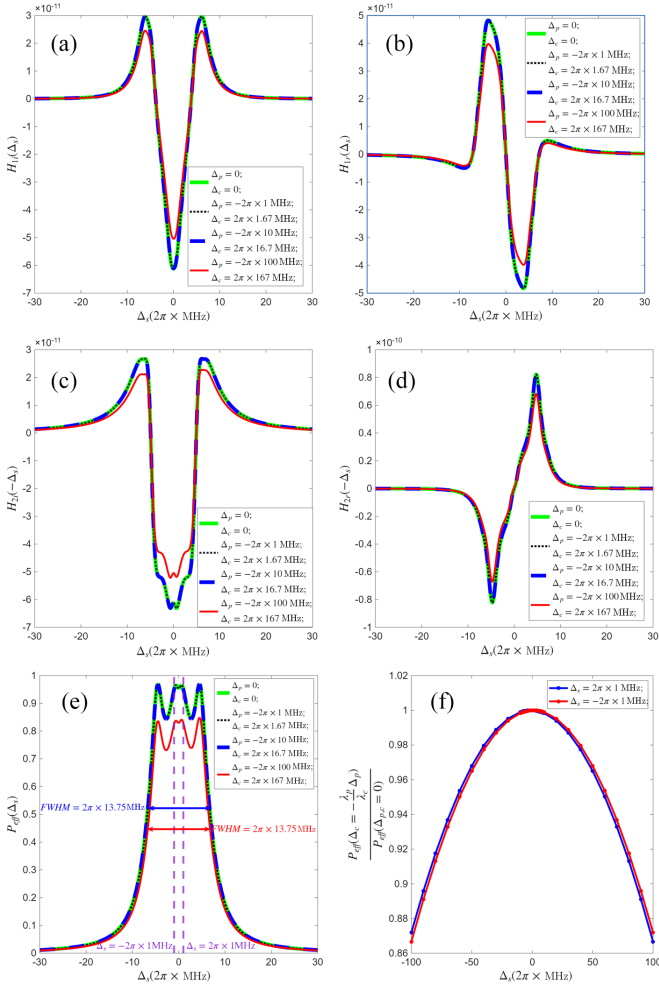


FIG. 4. Real and imaginary parts of H_1 and H_2 and effective optical power under different detunings in the locking relationship: (a) and (b) imaginary and real parts of H_1 under different detunings, (c) and (d) imaginary and real parts of H_2 under different detunings, (e) normalized result of effective optical power under different detunings with the maximum effective optical power at the resonance frequency as the standard, and (f) ratio of the effective optical power at $\Delta_s = 2\pi \times 1$ and $-2\pi \times 1$ MHz under different detunings compared with that at the resonance frequency. The simulation parameters are configured as follows: The Rabi frequencies of the probe laser, coupling laser, and LO field are $\Omega_p = 2\pi \times 2$ MHz, $\Omega_c = 2\pi \times 7$ MHz, and $\Omega_L = 2\pi \times 6$ MHz, respectively, and the decay rates of energy levels $|b\rangle$, $|c\rangle$, and $|d\rangle$ are $\gamma_b = 2\pi \times 5.2$ MHz, $\gamma_c = 2\pi \times 3.9$ kHz, and $\gamma_d = 2\pi \times 2.5$ kHz, respectively.

respectively, we set the probe laser detuning to 0, -1 , -10 , and -100 MHz and the coupling laser detuning to 0, 1.67, 16.7, and 167 MHz. The simulation results of studying the system responses under different detuning conditions are presented in Fig. 4. Here the probe laser detuning and the coupling laser detuning satisfy Eq. (16); for the convenience of discussion, hereinafter we refer to these detunings as 0, 1, 10, and 100 MHz, respectively.

As shown in Fig. 4, under the locking relationship, when the detuning is less than 10 MHz, the shapes and sizes of the real and imaginary parts of H_1 and H_2 remain basically unchanged. When the detuning is 100 MHz, the real and

imaginary parts of H_1 and H_2 will change significantly, yet their shapes still do not change obviously. Substituting the real and imaginary parts of $H_1(\Delta_s)$ and $H_2(-\Delta_s)$ into Eq. (13), the effective optical power can be obtained, as shown in Fig. 4(e). When the detuning is less than 10 MHz, the effective optical power changes little; when the detuning reaches 100 MHz, the effective optical power decreases significantly. However, for the system bandwidth, the magnitude of detuning does not affect the bandwidth, that is, when the detuning reaches 100 MHz, the bandwidth is still 13.75 MHz. To further study the influence of detuning on the system response under the locking relationship, we take the effective optical powers at frequency differences $\Delta_s = 2\pi \times 1$ and $-2\pi \times 1$ MHz under different detunings and compare the obtained powers with the effective optical power under the resonance condition to get the results shown in Fig. 4(f). It can be seen from the figure that when the detuning is less than 20 MHz, the attenuation of the effective optical power under detuning compared with that under resonance is less than 1%, which is basically negligible. At this time, the system can be regarded as even symmetric with respect to Δ_s . It can be seen from the figure that the influence of detuning on the positive frequency Δ_s and the negative frequency Δ_s is different. When the probe laser detuning Δ_p is positive, Δ_s at negative frequency is larger than that at positive frequency; when the probe laser detuning Δ_p is negative, Δ_s at negative frequency is smaller than that at positive frequency. Moreover, these changes are relatively small and do not affect the flatness within the system bandwidth. However, as the detuning continues to increase, these changes also increase, and at this time, the even symmetry of the effective optical power with respect to Δ_s will no longer exist.

It can be concluded that when the locking relationship is satisfied, small detuning amounts have a negligible impact on the system response. This is valuable for the practical application of Rydberg-atom receivers. In engineering, it is unnecessary to strictly calibrate the probe laser and coupling laser frequencies to the resonance. Ensuring the locking relationship between the two lasers is sufficient. This greatly reduces system construction difficulty and supports the promotion and application of Rydberg-atom receivers. Furthermore, the above analysis is based on the thermal vapor condition. Under this condition, the broad Doppler profile ensures that a sufficiently large number of atoms satisfy the locking condition, so small laser detunings can be neglected. However, as the temperature decreases, the atomic velocity distribution becomes narrower, and the number of atoms available for excitation to the Rydberg state decreases significantly when the detuning is large. Therefore, the conclusion that the effect of small detunings on the system is negligible is mainly applicable to receivers based on thermal vapor.

To summarize, based on the above research on the impact of detuning on quantum systems, detuning a single-excitation source requires exchanging a small amount of gain at the cost of a sharp drop in system bandwidth and severe fluctuations in response. This detuning method, which trades significant performance loss for limited gain, is not suitable for system optimization design. Under the constraint of the locking relationship, although a small detuning fails to bring additional gain to the system response, it reduces engineering difficulty

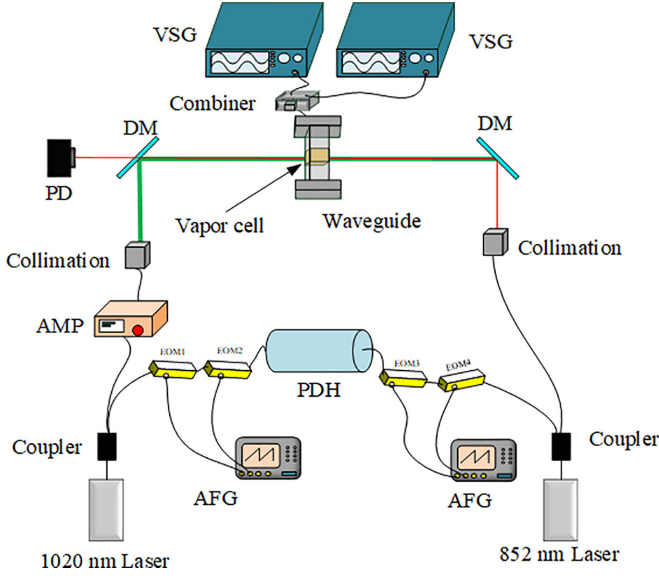


FIG. 5. Schematic diagram of the four-level heterodyne Rydberg-atom receiver device: AMP, laser frequency doubling amplifier; EOM, electro-optic modulator, where EOM1 and EOM4 are used for frequency shifting of frequency-locking signals and EOM2 and EOM3 are used for laser modulation; PDH, Pound-Drever-Hall frequency stabilization device; AFG, arbitrary function generator; PD, photoelectric detector; VSG, vector signal generator; and Coupler, 1×2 fiber coupler.

of the original basis. More specifically, in the process of constructing an actual quantum system, there is no need to strictly pursue energy-level resonance, which provides an important theoretical basis for the large-scale promotion and practical application of quantum reception technology.

IV. EXPERIMENTAL VERIFICATION OF THE INFLUENCE OF THE DETUNING EFFECT ON SYSTEM RESPONSE

According to Fig. 1, an experimental setup diagram of a four-level heterodyne Rydberg-atom receiver is constructed, as shown in Fig. 5. Here, at a temperature of 26°C , we use 852- and 510-nm lasers to excite cesium atoms in the vapor cell for preparing Rydberg-state atoms. Among them, the vapor cell has a length of 15 mm, and the 852-nm laser (Moglabs, ECDL) serves as the probe laser, which is used to excite cesium atoms in the ground state $6S_{1/2}$ to the intermediate state $6P_{3/2}$, while the 510-nm laser is the coupling laser, used to excite atoms in the $6P_{3/2}$ state to the Rydberg state $47D_{5/2}$. Due to the small transition dipole moment between $6P_{3/2}$ and $47D_{5/2}$, a 1020-nm seed laser (Moglabs, ECDL) combined with a second-harmonic generator (Precilasers, YFA-SSHG-509) is employed to generate a high-power 510-nm laser beam. To ensure the accuracy and stability of the laser frequency, a dual-wavelength ultrastable cavity (SLS, VH-6020-4, finesse of 200 000) is employed here to achieve frequency locking of the probe laser and coupling laser. Additionally, two electro-optic modulators are used to modulate and frequency shift the lasers, thereby realizing the locking of any laser frequency and ensuring that the detuning amounts

of the probe laser and coupling laser frequencies can be accurately regulated. In addition, in the experiment, to accurately obtain the electric-field intensity acting in the vapor cell, the vapor cell is placed in a waveguide (WR-137), and two small holes with a diameter of 2 mm are made on the waveguide wall to ensure that the laser can pass through the waveguide smoothly. Two vector signal generators (R&S, SMCV100B) are used to respectively generate the local oscillator electric field and the microwave signal field to be measured required for the experiment, which are used to couple the two Rydberg energy levels $47D_{5/2}$ and $48P_{3/2}$. The two electric fields are combined by a combiner (ZN2PD-9G-S+, MINI circuits) and input into the waveguide. Finally, the transmitted laser is detected by a photodetector (Thorlabs, APD410A, conversion gain of $M \times 125$ kV/W, where $M = 10$). In the experiment, the probe laser power is $1 \mu\text{W}$, focused to a full width at half maximum (FWHM) of 1 mm, with a Rabi frequency of $2\pi \times 1.1$ MHz; the coupling laser power is 150 mW, focused to a FWHM of 1 mm, with a Rabi frequency of $2\pi \times 3.98$ MHz; the local oscillator electric field is 0.1664 V/m, with a Rabi frequency of $2\pi \times 3.07$ MHz and an electric-field frequency of 6.9467 GHz; and the intensity of the microwave electric field to be measured is 41.8 mV/m, with a Rabi frequency of $2\pi \times 0.77$ MHz. The measured influence of detuning on the effective optical power is shown in Fig. 6. Figure 6 shows the relationship between P_{eff} and Δ_s when different excitation sources (probe laser, coupling laser, and local oscillator electric field) are detuned by 1 MHz: The solid lines represent the simulation results, where the three curves correspond to the variation of P_{eff} with Δ_s when the probe laser, coupling laser, and local oscillator electric field, respectively, are detuned by 1 MHz. It can be seen that the curve corresponding to the detuning of the local oscillator electric field has the highest peak; the detuning of both the probe laser and the coupling laser will cause the curve to have a depression near a specific frequency. By comparing the curves in the figure, it is found that the curve corresponding to the detuning of the probe laser has the lowest peak and the largest depression at a specific frequency, which reflects that the detuning of the probe laser destroys the coherent matching with the coupling laser, and the coherent superposition of optical fields changes from enhancement to partial cancellation. The curve corresponding to the detuning of the coupling laser also shows a similar trend. However, compared with the detuning of the probe laser, under the same detuning, the detuning of the coupling laser has a smaller impact. This is because the wavelength of the coupling laser is shorter than that of the probe laser; therefore, the peak of the curve corresponding to the detuning of the coupling laser is higher than that of the probe laser detuning, and the depression is shallower. The dashed lines represent the experimental results, whose overall trend is consistent with the simulation; however, there are quantitative differences: The depression depth of the curve corresponding to the probe laser detuning is shallower and the measured curve exhibits slight fluctuations. However, these do not affect the trend of the curve's variation. Thus, it can be concluded that under the detuning of a single-excitation source, the variation trend of the effective optical power conforms to the theoretical prediction, which verifies the rationality of the model. Under the locking relationship, the variation law of

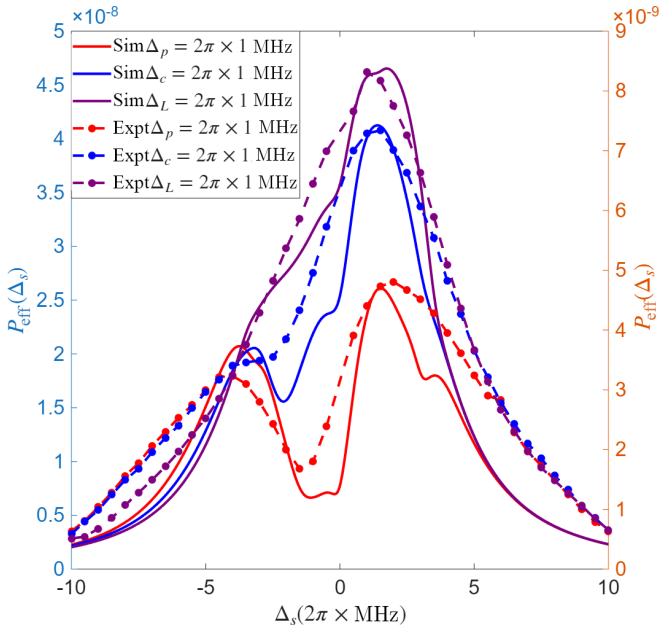


FIG. 6. Influence of single detuning on effective optical power. The solid lines represent the simulation results and the dashed lines represent the experimental results. The abscissa represents the frequency difference of the signal to be measured relative to the local oscillator electric field. The left ordinate denotes the effective optical power from the simulation and the right ordinate denotes the effective optical power from the experimental measurements. The parameters of simulation and measurement are configured as follows: The Rabi frequencies of the probe laser, coupling laser, LO field, and microwave electric field to be measured are $\Omega_p = 2\pi \times 1.1$ MHz, $\Omega_c = 2\pi \times 3.98$ MHz, $\Omega_L = 2\pi \times 3.07$ MHz, and $\Omega_s = 2\pi \times 0.77$ MHz, respectively, and the decay rates of energy levels $|b\rangle$, $|c\rangle$, and $|d\rangle$ are $\gamma_b = 2\pi \times 5.2$ MHz, $\gamma_c = 2\pi \times 3.9$ kHz, and $\gamma_d = 2\pi \times 2.5$ kHz, respectively.

P_{eff} with Δ_s under different detuning conditions is shown in Fig. 7, where $\text{Sim}\Delta_{\text{locking}}$ are the simulation results and $\text{Expt}\Delta_{\text{locking}}$ are the experimental results. It can be seen from the simulation result that in the locked state, the curve of P_{eff} under small detuning has the same shape as that under the resonance condition, indicating that the system response remains stable within this detuning range without obvious distortion; as the detuning amount gradually increases, the system response begins to change. When the detuning amount increases to 100 MHz, the peak value of the effective optical power decreases significantly and the curve separates significantly from the resonance curve. This phenomenon intuitively reflects the suppression effect of large detuning on the system response. Further measurement of the FWHM of the curves under different detuning conditions reveals that their values remain stable 7.8 MHz, indicating that the frequency selectivity of the system remains consistent under this simulation condition. The experimental results indicate that the variation trend of the system response under different detuning parameters is highly consistent with the simulation results, further verifying the reliability of the theoretical model. Specifically, within the small detuning range (typically less than 10 MHz), the peak value of the effective optical power remains stable, with

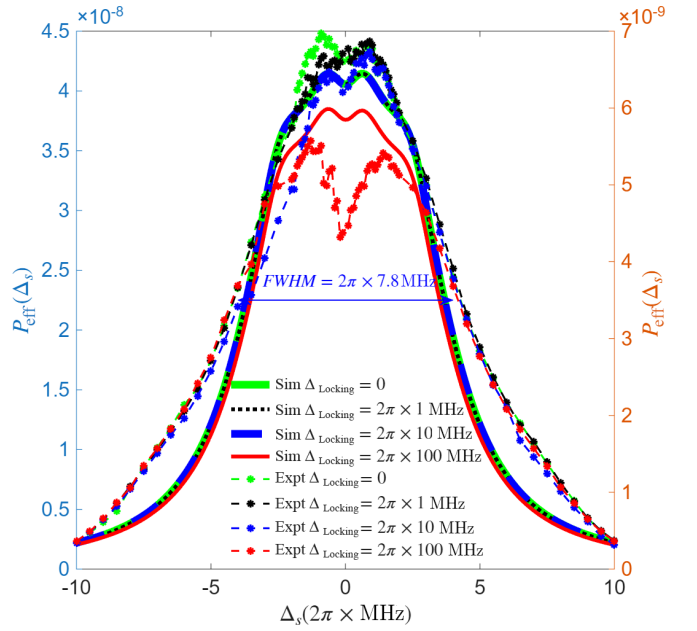


FIG. 7. Variation of P_{eff} with Δ_s under the locking relationship. The $\text{Sim}\Delta_{\text{locking}}$ are the simulation results and the $\text{Expt}\Delta_{\text{locking}}$ are the experimental results. The abscissa represents the frequency difference of the signal to be measured relative to the local oscillator electric field. The left ordinate denotes the effective optical power from the simulation and the right ordinate denotes the effective optical power from the experimental measurements. The blue double arrows denote the FWHM of the system. The parameters of simulation and measurement are configured as follows: The Rabi frequencies of the probe laser, coupling laser, LO field, and microwave electric field to be measured are $\Omega_p = 2\pi \times 1.1$ MHz, $\Omega_c = 2\pi \times 3.98$ MHz, $\Omega_L = 2\pi \times 3.07$ MHz, and $\Omega_s = 2\pi \times 0.77$ MHz, respectively, and the decay rates of energy levels $|b\rangle$, $|c\rangle$, and $|d\rangle$ are $\gamma_b = 2\pi \times 5.2$ MHz, $\gamma_c = 2\pi \times 3.9$ kHz, and $\gamma_d = 2\pi \times 2.5$ kHz, respectively.

the system bandwidth consistently at 8 MHz. Although the measured bandwidth slightly broadens compared to the ideal simulation results, and the peak power fluctuates slightly due to the influence of the detector's gain and noise floor, the response law of the system under small-detuning conditions remains consistent with the simulation, and the core physical mechanism remains unchanged.

As the detuning amount further increases, the system response gradually deviates from that under the resonance state. When the detuning amount reaches 100 MHz, the effective optical power decreases significantly, showing the suppression effect of large detuning on the system response. Notably, even under this detuning condition, the system bandwidth remains at approximately 8 MHz without any obvious changes. This result indicates that the system bandwidth maintains good stability under large detuning amounts. This measured phenomenon is highly consistent with the simulation results, collectively confirming that large detuning suppresses the effective optical power but has little impact on the system bandwidth. In addition, we characterized the response of the system under both resonant and nonresonant conditions, with the results presented in Fig. 8. When the spectrum analyzer's

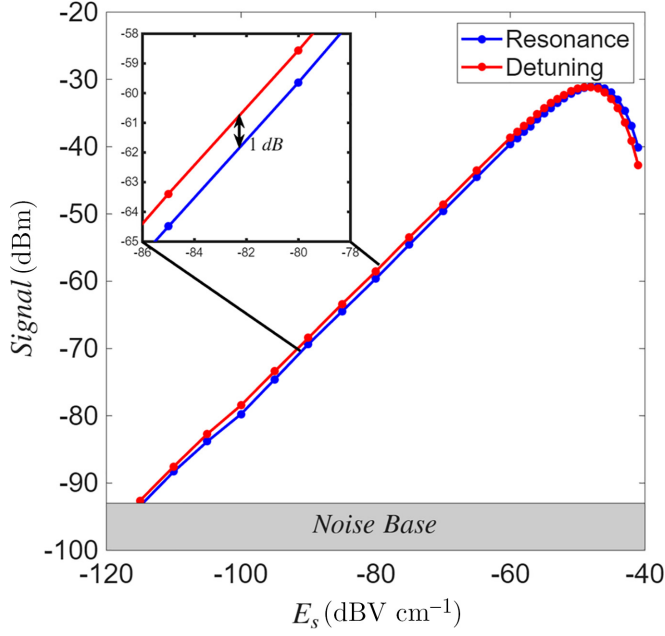


FIG. 8. Measurement of electric-field detection sensitivity. The blue curve represents the sensitivity curve under resonance conditions and the red curve denotes that under detuning conditions. The parameters of simulation and measurement are configured as follows: The Rabi frequencies of the probe laser, coupling laser, and LO field are $\Omega_p = 2\pi \times 1.1$ MHz, $\Omega_c = 2\pi \times 3.98$ MHz, and $\Omega_L = 2\pi \times 3.07$ MHz, respectively, and the decay rates of energy levels $|b\rangle$, $|c\rangle$, and $|d\rangle$ are $\gamma_b = 2\pi \times 5.2$ MHz, $\gamma_c = 2\pi \times 3.9$ kHz, and $\gamma_d = 2\pi \times 2.5$ kHz, respectively.

resolution bandwidth was set at 100 Hz and the video bandwidth at 1 Hz, the minimum detectable power of the system was measured at -112 dBm. Normalizing the resolution bandwidth to 1 Hz, we obtained an electric-field sensitivity of $14.8 \mu\text{V m}^{-1} \text{Hz}^{-1/2}$ for the system under resonant conditions. According to our study, the system response gain reaches its maximum at the detuning of the local oscillator electric field. Here we set the frequency detuning of the microwave electric field to be 1 MHz. Under this condition, the system response increased by 1 dB compared to the resonant condition, corresponding to an improved electric-field sensitivity of $13.2 \mu\text{V m}^{-1} \text{Hz}^{-1/2}$.

To summarize, this study has experimentally verified the mechanism of action and influence law of single-excitation

source detuning and detuning under the locking relationship on the Rydberg-atom receiver system and based on this proposed an optimization strategy for the atomic system. Specifically, under the locking relationship, the introduction of detuning makes the laser frequency no longer strictly adhere to energy-level resonance; it allows the probe laser and coupling laser to break free from the constraints of complete resonance, thereby greatly relaxing the requirements for laser frequency accuracy, simplifying the system, and providing greater flexibility for engineering implementation. Notably, this optimization method not only maintains system performance but also provides a solution for its stable operation in complex environments, laying an important foundation for subsequent engineering applications and system design.

V. CONCLUSION

This study systematically investigated the impact of detuning on a four-level heterodyne Rydberg-atom receiver. Based on the derived theoretical framework, we classified detuning into two scenarios, single-excitation source detuning and detuning under the locking relationship, to systematically examine their respective influences on the system response. Single-source detuning generates weak gain at the severe cost of bandwidth narrowing and flatness deterioration. In contrast, detuning under the locking relationship has no significant impact on the system response within a small range. This characteristic greatly reduces implementation difficulty and facilitates system construction. Experimental results validate our theoretical model and analysis. These findings not only clarify the influence of detuning but also provide key theoretical support for the practical design of Rydberg-atom receivers, paving the way for their application in quantum sensing and precision measurement.

DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX: DETAILED DERIVATION PROCESS OF DENSITY-MATRIX ELEMENTS ρ_{ba}

Figure 1(a) shows the probe and coupling laser propagating in opposite directions along the z axis, while the local oscillator electric field travels along the x axis. Because the beam diameter is small, $e^{-ik_m x} \approx 1$. This simplification allows us to express the system with the Hamiltonian matrix

$$\mathbf{H} = \frac{\hbar}{2} \begin{bmatrix} 0 & \Omega_p e^{i\Delta_p t} e^{-ik_p z} & 0 & 0 \\ \Omega_p e^{-i\Delta_p t} e^{ik_p z} & 0 & \Omega_c e^{i\Delta_c t} e^{ik_c z} & 0 \\ 0 & \Omega_c e^{-i\Delta_c t} e^{-ik_c z} & 0 & [\Omega_L + \Omega_s(t)] e^{i\Delta_L t} \\ 0 & 0 & [\Omega_L + \Omega_s^*(t)] e^{-i\Delta_L t} & 0 \end{bmatrix}, \quad (\text{A1})$$

$$\mathbf{L} = \begin{bmatrix} \gamma_b \rho_{bb} + \gamma_d \rho_{dd} - \gamma \rho_{aa} + \gamma & -\frac{\gamma_b}{2} \rho_{ab} - \gamma \rho_{ab} & -\frac{\gamma_c}{2} \rho_{ac} - \gamma \rho_{ac} & -\frac{\gamma_d}{2} \rho_{ad} - \gamma \rho_{ad} \\ -\frac{\gamma_b}{2} \rho_{ba} - \gamma \rho_{ba} & -\gamma_b \rho_{bb} + \gamma_c \rho_{cc} - \gamma \rho_{bb} & -\frac{\gamma_{bc}}{2} \rho_{bc} - \gamma \rho_{bc} & -\frac{\gamma_{bd}}{2} \rho_{bd} - \gamma \rho_{bd} \\ -\frac{\gamma_c}{2} \rho_{ca} - \gamma \rho_{ca} & -\frac{\gamma_{bc}}{2} \rho_{cb} - \gamma \rho_{cb} & -\gamma_c \rho_{cc} - \gamma \rho_{cc} & -\frac{\gamma_{cd}}{2} \rho_{cd} - \gamma \rho_{cd} \\ -\frac{\gamma_d}{2} \rho_{da} - \gamma \rho_{da} & -\frac{\gamma_{bd}}{2} \rho_{db} - \gamma \rho_{db} & -\frac{\gamma_{cd}}{2} \rho_{dc} - \gamma \rho_{dc} & -\gamma_d \rho_{dd} - \gamma \rho_{dd} \end{bmatrix}, \quad (\text{A2})$$

where $\Omega_L = \frac{\mathcal{E}_{dL} E_L}{\hbar}$ is the Rabi frequency of the local oscillator microwave electric field, $\Omega_s = \frac{\mathcal{E}_{dL} E_s(t)}{\hbar}$ is the Rabi frequency of the microwave signal field to be measured, $\mathcal{E}_{dL}$ is the transition dipole moment between energy level $|c\rangle$ and energy level $|d\rangle$, $\gamma_{ij} = \gamma_i + \gamma_j$ ($i, j = b, c, d$) is the decay rate, and γ is the transit relaxation rate [23]

$$\gamma = \sqrt{\frac{8k_B T}{\pi m}} \frac{1}{w\sqrt{2\ln(2)}}, \quad (\text{A3})$$

where k_B is the Boltzmann constant, T is the temperature, and w is the $1/e^2$ radius. The density-matrix equation system is obtained by substituting Eqs. (A1) and (A2) into Eq. (4). Here we take the lower triangular part of the matrix to solve the density-matrix element ρ_{ba} :

$$2i\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla\right)\rho_{ba} = \Omega_p e^{-i\Delta_p t} e^{ik_p z} \rho_{aa} + \Omega_c e^{i\Delta_c t} e^{ik_c z} \rho_{ca} - \Omega_p e^{-i\Delta_p t} e^{ik_p z} \rho_{bb} - i\gamma_b \rho_{ba} - 2i\gamma \rho_{ba}, \quad (\text{A4})$$

$$2i\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla\right)\rho_{ca} = \Omega_c e^{-i\Delta_c t} e^{-ik_c z} \rho_{ba} - \Omega_p e^{-i\Delta_p t} e^{ik_p z} \rho_{cb} - i\gamma_c \rho_{ca} - 2i\gamma \rho_{ca} + [\Omega_L + \Omega_s(t)] e^{i\Delta_L t} \rho_{da}, \quad (\text{A5})$$

$$2i\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla\right)\rho_{cb} = [\Omega_L + \Omega_s(t)] e^{i\Delta_L t} \rho_{db} + \Omega_c e^{-i\Delta_c t} e^{-ik_c z} (\rho_{bb} - \rho_{cc}) - \Omega_p e^{i\Delta_p t} e^{-ik_p z} \rho_{ca} - (i\gamma_{bc} + 2i\gamma) \rho_{cb}, \quad (\text{A6})$$

$$2i\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla\right)\rho_{da} = -\Omega_p e^{-i\Delta_p t} e^{ik_p z} \rho_{db} - i\gamma_d \rho_{da} - 2i\gamma \rho_{da} [\Omega_L + \Omega_s^*(t)] e^{-i\Delta_L t} \rho_{ca}, \quad (\text{A7})$$

$$2i\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla\right)\rho_{db} = [\Omega_L + \Omega_s^*(t)] e^{-i\Delta_L t} \rho_{cb} - i\gamma_{bd} \rho_{db} - 2i\gamma \rho_{db} - (\Omega_p e^{i\Delta_p t} e^{-ik_p z} \rho_{da} + \Omega_c e^{-i\Delta_c t} e^{-ik_c z} \rho_{dc}), \quad (\text{A8})$$

$$2i\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla\right)\rho_{dc} = [\Omega_L + \Omega_s^*(t)] e^{-i\Delta_L t} (\rho_{cc} - \rho_{dd}) - \Omega_c e^{i\Delta_c t} e^{ik_c z} \rho_{db} - (i\gamma_{cd} + 2i\gamma) \rho_{dc}. \quad (\text{A9})$$

We perform a transformation on the density-matrix elements with respect to the position z ,

$$\begin{aligned} \rho_{ba} &= \rho_{ba}^0 e^{-i\Delta_p t} e^{ik_p z}, \\ \rho_{ca} &= \rho_{ca}^0 e^{-i(\Delta_p + \Delta_c)t} e^{i(k_p - k_c)z}, \\ \rho_{cb} &= \rho_{cb}^0 e^{-i\Delta_c t} e^{-ik_c z}, \\ \rho_{da} &= \rho_{da}^0 e^{-i(\Delta_p + \Delta_c + \Delta_L)t} e^{i(k_p - k_c)z}, \\ \rho_{db} &= \rho_{db}^0 e^{-i(\Delta_c + \Delta_L)t} e^{-ik_c z}, \end{aligned} \quad (\text{A10})$$

where ρ_{ij}^0 ($i, j = a, b, c, d$) represents the z -independent part of ρ_{ij} .

Furthermore, due to the weak influence of the microwave electric field to be measured on the atomic population distribution, the probabilities of atoms occupying different energy levels can be approximated as constants:

$$\rho_{ii}^0 = \rho_{ii}, \quad i = a, b, c, d. \quad (\text{A11})$$

By applying a two-dimensional Fourier transform with respect to time t and position z to Eqs. (A4)–(A9), we obtain

$$-2(\omega + v_z \omega_z) \rho_{ba}(\omega, \omega_z) = \Omega_c \rho_{ca}(\omega - \Delta_c, \omega_z - k_c) + \Omega_p (\rho_{aa}^0 - \rho_{bb}^0) 4\pi^2 \delta(\omega + \Delta_p, \omega_z - k_p) - (i\gamma_b + 2i\gamma) \rho_{ba}(\omega, \omega_z), \quad (\text{A12})$$

$$\begin{aligned} -2(\omega + v_z \omega_z) \rho_{ca}(\omega, \omega_z) &= \Omega_c \rho_{ba}(\omega + \Delta_c, \omega_z + k_c) + \Omega_s(\omega + \Delta_p + \Delta_c) 2\pi \delta(\omega_z - (k_p - k_c)) \rho_{da}^0 \\ &\quad + \Omega_L \rho_{da}(\omega - \Delta_m, \omega_z) - \Omega_p \rho_{cb}(\omega + \Delta_p, \omega_z - k_p) - (i\gamma_c + 2i\gamma) \rho_{ca}(\omega, \omega_z), \end{aligned} \quad (\text{A13})$$

$$\begin{aligned} -2(\omega + v_z \omega_z) \rho_{cb}(\omega, \omega_z) &= \Omega_s(\omega + \Delta_c) 2\pi \delta(\omega_z + k_c) \rho_{db}^0 + \Omega_c 4\pi^2 \delta(\omega + \Delta_c, \omega_z + k_c) (\rho_{bb}^0 - \rho_{cc}^0) \\ &\quad - (i\gamma_{bc} + 2i\gamma) \rho_{cb}(\omega, \omega_z) + \Omega_L \rho_{db}(\omega - \Delta_L, \omega_z) - \Omega_p \rho_{ca}(\omega - \Delta_p, \omega_z + k_p), \end{aligned} \quad (\text{A14})$$

$$\begin{aligned} -2(\omega + v_z \omega_z) \rho_{da}(\omega, \omega_z) &= \Omega_L \rho_{ca}(\omega + \Delta_L, \omega_z) - \Omega_p \rho_{db}(\omega + \Delta_p, \omega_z - k_p) - (i\gamma_d + 2i\gamma) \rho_{da}(\omega, \omega_z) \\ &\quad + \Omega_s^* [-(\omega + \Delta_p + \Delta_c + \Delta_L)] \rho_{ca}^0 2\pi \delta(\omega_z - (k_p - k_c)), \end{aligned} \quad (\text{A15})$$

$$\begin{aligned} -2(\omega + v_z \omega_z) \rho_{db}(\omega, \omega_z) &= -(i\gamma_{bd} - 2i\gamma) \rho_{db}(\omega, \omega_z) - \Omega_p \rho_{da}(\omega - \Delta_p, \omega_z + k_p) - \Omega_c \rho_{dc}(\omega + \Delta_c, \omega_z + k_c) \\ &\quad + \Omega_s^* [-(\omega + \Delta_c + \Delta_L)] 2\pi \delta(\omega_z + k_c) \rho_{cb}^0 + \Omega_L \rho_{cb}(\omega + \Delta_L, \omega_z), \end{aligned} \quad (\text{A16})$$

$$\begin{aligned} -2(\omega + v_z \omega_z) \rho_{dc}(\omega, \omega_z) &= \Omega_L (\rho_{cc}^0 - \rho_{dd}^0) 4\pi^2 \delta(\omega + \Delta_L, \omega_z) - \Omega_c \rho_{db}(\omega - \Delta_c, \omega_z - k_c) - (i\gamma_{cd} + 2i\gamma) \rho_{dc}(\omega, \omega_z) \\ &\quad + \Omega_s^* [-(\omega + \Delta_L)] 2\pi \delta(\omega_z) (\rho_{cc}^0 - \rho_{dd}^0). \end{aligned} \quad (\text{A17})$$

By combining Eqs. (A12)–(A17), the relationship between detuning and the density-matrix element ρ_{ba} can be expressed as

$$\rho_{ba}(\omega - \Delta_p, \omega_z + k_p) = \rho_{ba}^0(\omega, \omega_z) 4\pi^2 \delta(\omega, \omega_z) + H_{1v}(\omega) \Omega_s^*[-(\omega)] 2\pi \delta(\omega_z) + H_{2v}(\omega) \Omega_s(\omega) 2\pi \delta(\omega_z), \quad (\text{A18})$$

where

$$\begin{aligned} \rho_{ba}^0(\omega, \omega_z) = & \Omega_p \left\{ (\rho_{aa}^0 - \rho_{bb}^0) + \frac{\Omega_c}{E(-\Delta_p - \Delta_c, k_p - k_c)} \left[\frac{\Omega_L^2}{A(-\Delta_L - \Delta_c, -k_c) D(-\Delta_p - \Delta_c - \Delta_L, k_p - k_c)} \right. \right. \\ & \times \left(1 + \frac{\Omega_p^2}{A(-\Delta_c - \Delta_L, -k_c) B(-\Delta_c, -k_c)} \right) \left(\frac{\Omega_c(\rho_{cc}^0 - \rho_{dd}^0)}{(i\gamma_{cd} + 2i\gamma + 2\Delta_L)} - C_0 \right) - C_0 \left. \right\} \\ & \times \left(i\gamma_b + 2i\gamma - 2(-\Delta_p + k_p v_z) - \frac{\Omega_c^2}{E(-\Delta_p - \Delta_c, k_p - k_c)} \right)^{-1}, \end{aligned} \quad (\text{A19})$$

$$\begin{aligned} H_{1v}(\omega) = & \Omega_c \Omega_L \left\{ \frac{\rho_{ca}^0}{D(\omega - \Delta_p - \Delta_c - \Delta_L, k_p - k_c)} \left(1 + \frac{\Omega_p^2}{A(\omega - \Delta_c - \Delta_L, -k_c) B(\omega - \Delta_c, -k_c)} \right) \right. \\ & + \frac{\Omega_p}{A(\omega - \Delta_c - \Delta_L, -k_c)} \left(\frac{\Omega_c(\rho_{cc}^0 - \rho_{dd}^0)}{i\gamma_{cd} + 2i\gamma - 2(\omega - \Delta_L)} - \rho_{cb}^0 \right) \\ & \times \left[\frac{1}{B(\omega - \Delta_c, -k_c)} + \frac{1}{D(\omega - \Delta_p - \Delta_c - \Delta_L, k_p - k_c)} \right. \\ & \times \left(1 + \frac{\Omega_p^2}{A(\omega - \Delta_c - \Delta_L, -k_c) B(\omega - \Delta_c, -k_c)} \right) \left. \left(1 + \frac{\Omega_L^2}{A(\omega - \Delta_c - \Delta_L, -k_c) B(\omega - \Delta_c, -k_c)} \right) \right] \left. \right\} \\ & \times \left[E(\omega - \Delta_p - \Delta_c, k_p - k_c) \left(i\gamma_b + 2i\gamma - 2(\omega - \Delta_p + k_p v_z) - \frac{\Omega_c^2}{E(\omega - \Delta_p - \Delta_c, k_p - k_c)} \right) \right]^{-1}, \end{aligned} \quad (\text{A20})$$

$$\begin{aligned} H_{2v}(\omega) = & \Omega_c \left\{ \rho_{da}^0 - \frac{\Omega_p \rho_{db}^0}{B(\omega - \Delta_c, -k_c)} \left[1 + \frac{\Omega_L^2}{A(\omega - \Delta_c - \Delta_L, -k_c) D(\omega - \Delta_p - \Delta_c - \Delta_L, -k_c + k_p)} \right. \right. \\ & \times \left(1 + \frac{\Omega_p^2}{A(\omega - \Delta_c - \Delta_L, -k_c) B(\omega - \Delta_c, -k_c)} \right) \left. \right] \left. \right\} \\ & \times \left[E(\omega - \Delta_p - \Delta_c, k_p - k_c) \left(i\gamma_b + 2i\gamma - 2(\omega - \Delta_p + k_p v_z) - \frac{\Omega_c^2}{E(\omega - \Delta_p - \Delta_c, k_p - k_c)} \right) \right]^{-1}, \end{aligned} \quad (\text{A21})$$

with

$$A(\omega, \omega_z) = i\gamma_{bd} - 2i\gamma - 2(\omega + v_z \omega_z) - \frac{\Omega_c^2}{i\gamma_{cd} + 2i\gamma - 2[\omega + \Delta_c + v_z(\omega_z + k_c)]}, \quad (\text{A22})$$

$$B(\omega, \omega_z) = i\gamma_{bc} + 2i\gamma - 2(\omega + v_z \omega_z) - \frac{\Omega_L^2}{A(\omega - \Delta_L, \omega_z)}, \quad (\text{A23})$$

$$C_0 = \frac{\Omega_c(\rho_{bb}^0 - \rho_{cc}^0)}{B(-\Delta_c, -k_c)} - \frac{\Omega_c \Omega_L^2(\rho_{cc}^0 - \rho_{dd}^0)}{A(-\Delta_c - \Delta_L, -k_c) B(-\Delta_c, -k_c) (i\gamma_{cd} + 2i\gamma + 2\Delta_L)}, \quad (\text{A24})$$

$$\begin{aligned} D(\omega, \omega_z) = & i\gamma_d + 2i\gamma - 2(\omega + v_z \omega_z) - \frac{\Omega_p^2}{A(\omega + \Delta_p, \omega_z - k_p)} \\ & - \frac{\Omega_p^2 \Omega_L^2}{A(\omega + \Delta_p, \omega_z - k_p) A(\omega + \Delta_p, \omega_z - k_p) B(\omega + \Delta_p + \Delta_L, \omega_z - k_p)}, \end{aligned} \quad (\text{A25})$$

$$\begin{aligned} E(\omega, \omega_z) = & i\gamma_c + 2i\gamma - 2(\omega + v_z \omega_z) - \frac{\Omega_p^2}{B(\omega + \Delta_p, \omega_z - k_p)} \\ & - \frac{\Omega_L^2}{D(\omega - \Delta_L, \omega_z)} \left(1 + \frac{\Omega_p^2}{A(\omega + \Delta_p - \Delta_L, \omega_z - k_p) B(\omega + \Delta_p, \omega_z - k_p)} \right)^2. \end{aligned} \quad (\text{A26})$$

Performing the inverse Fourier transform on Eq. (A15) yields

$$\rho_{ba}(t, z) = \rho_{ba}^0 e^{-i\Delta_p t} e^{ik_p z} + h_{1v}(t) \otimes \Omega_s^*(t) e^{-i\Delta_p t} e^{ik_p z} + h_{2v}(t) \otimes \Omega_s(t) e^{-i\Delta_p t} e^{ik_p z}, \quad (\text{A27})$$

where $h_{1v}(t)$ and $h_{2v}(t)$ are the inverse Fourier transforms of $H_{1v}(\omega)$ and $H_{2v}(\omega)$, respectively. By applying a unitary transformation to Eq. (A27), it can be simplified as

$$\rho_{ba}(t, z) = \rho_{ba}^0 e^{ik_p z} + h_{1v}(t) \otimes \Omega_s(t) e^{ik_p z} + h_{2v}(t) \otimes \Omega_s^*(t) e^{ik_p z}. \quad (\text{A28})$$

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