

Resonant photon scattering in the presence of external fields and its applications for the CERN Gamma Factory project

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(Received 11 February 2025; accepted 28 May 2025; published 24 June 2025)

We present a theoretical study of resonant elastic photon scattering by highly charged ions in the presence of external electric and magnetic fields. Special emphasis is placed on head-on collisions of relativistic ion beams and counterpropagating laser photons, as can be observed in ion storage ring experiments. If the collision zone for such a scenario is exposed to a moderate field of a dipole magnet, a fast-moving ion will be subjected to strong electric and magnetic fields in its rest frame due to the Lorentz transformation. To investigate the effect of these fields we performed detailed calculations for the elastic photon scattering by He-like Ca ions moving with the Lorentz factor $\gamma_L = 2395$. Such a scenario is of particular interest for the Gamma Factory project which is supported by the CERN Physics Beyond Colliders framework. Based on the results of our calculations, we argue that even moderate magnetic fields significantly modify the rate of detected photons as well as their emission pattern and polarization. This sensitivity to external fields opens interesting avenues for diverse applications for the Gamma Factory project, such as resonance condition tuning, beam cooling, and polarization control.

DOI: [10.1103/yysz-444v](https://doi.org/10.1103/yysz-444v)

I. INTRODUCTION

Elastic photon scattering by atoms and ions is one of the most fundamental processes of light-matter interaction and has been the subject of numerous experimental and theoretical studies [1–7]. Special attention in these studies is given to the scattering of photons, whose energy is close to the transition energy between the ground and some excited atomic state. This so-called resonant photon scattering is of interest for various applications such as, for example, measurements of oscillator strengths and lifetimes of highly charged ions [8–10], investigations of spacial modulations and electronic structure of complex materials [11], and the production of high-intensity γ rays. The latter case is expected to be realized by the Gamma Factory project at CERN [12], which is supported by the Physics Beyond Colliders framework [13]. The key principle of the Gamma Factory is rooted in the resonant scattering of relativistic ion beams and counterpropagating laser photons involving a strong enhancement of the photon frequency caused by the Doppler boost. The use of Doppler

boosted laser photons is well established in atomic physics experiments and was successfully employed already in the 1970s [14,15]. For ultrarelativistic energies of the projectile ions, the resonant scattering can produce small-divergence, high-energy photon beams with tremendous potential for a wide range of experiments [16–27].

A deep understanding of the resonant photon scattering is needed for planning and guidance of future Gamma Factory experiments. A number of theoretical studies have been reported recently, therefore, that analyzed the angular distribution and polarization of scattered photons [5–7]. Those studies have been performed within the fully relativistic framework and by taking into account the electron-photon interaction beyond the dipole approximation, but in the absence of external (static) electric and magnetic fields. Thus, the previous theoretical analysis can be used to describe a typical Gamma Factory setup where the light-ion interaction takes place in the field-free zone of the storage ring. In this contribution, we present an extension of previous theoretical studies toward resonant photon scattering in the presence of external fields. These fields, as seen in the rest frame of the ion, can be introduced by a dipole magnet installed in the collision zone. Due to the Lorentz boost, already a weak laboratory magnetic field can have a strong impact on the scattering process. Such Lorentz transformations of external fields have been successfully applied in the past for beam energies much smaller than those available at the large hadron collider (LHC) [15,28,29]. In the present study we will deal with projectile

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velocities approaching the speed of light, leading to strong Lorentz-boosted fields that open up new possibilities for the Gamma Factory project.

To analyze the effects of external fields on the scattering process, one needs to specify the geometry of the “ion *plus* light” system exposed to the external electric and magnetic fields in both the laboratory and the ion frame. The geometry is discussed in Sec. II, together with basic parameters used in our study. After the geometry is established, we turn to the analysis of the influence of external fields both on the structure and dynamics of ions. In Sec. III, for example, we briefly recall Zeeman and Stark shifts that modify the electronic levels of an ion. The scattering amplitude, which is the main “building block” in the theory of resonant scattering, is obtained later in Sec. IV by taking the effects of external fields into account. In Sec. V we apply the derived theory to the description of the $1s^2\ ^1S_0 \rightarrow 1s2p\ ^1P_1 \rightarrow 1s^2\ ^1S_0$ photon scattering by He-like Ca ions. Here, we pay special attention both to the reaction rate and to the angular distribution and polarization of scattered photons. We argue that even moderate fields of a dipole magnet (in the laboratory frame) can significantly modify these properties. Besides the fundamental interest, the high sensitivity of resonant photon scattering to external fields offers many interesting applications for the Gamma Factory project. These applications, such as resonance condition tuning, beam cooling, and polarization control, are discussed in Sec. VI. Finally, the most important aspects of the present study will be summarized in Sec. VII.

II. GEOMETRY AND BASIC PARAMETERS

Before we delve into the theory of resonant scattering we have to discuss the geometry of this process. For the case of a head-on collision between photons and a relativistic ion beam, which is a typical scenario for storage ring experiments, the geometry is shown in Fig. 1. Here, the ions move along the y axis while the photons counterpropagate in the negative y direction. The direction of the emitted photon is characterized by the polar scattering angle θ_f and the azimuthal angle ϕ_f which are defined with respect to the y axis and the y - z plane, respectively.

Taking into account the relativistic motion of the ions, the scattering angle depends on the choice of a particular reference frame. In this study, two reference frames are considered: the laboratory frame and the ion rest frame. All calculations are performed in the latter, and the results are then Lorentz transformed into the laboratory frame, where all measurements take place. This transformation can be easily carried out by using the basic relations for the scattering solid angles:

$$\cos \theta_f^{(\text{ion})} = \frac{\cos \theta_f^{(\text{lab})} - \beta}{1 - \beta \cos \theta_f^{(\text{lab})}} \quad (1)$$

and

$$\frac{d\Omega^{(\text{ion})}}{d\Omega^{(\text{lab})}} = \frac{1 - \beta^2}{(1 - \beta \cos \theta_f^{(\text{lab})})^2}, \quad (2)$$

with $\beta = v/c$ and the ion beam velocity v [30].

In the present study, special attention will be paid to the effect of external electromagnetic fields on the scattering

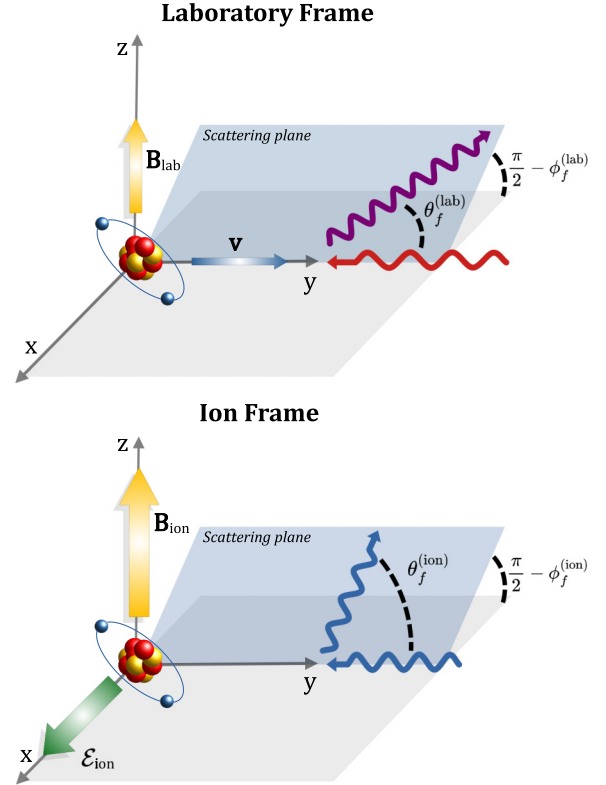


FIG. 1. Elastic photon scattering by highly charged ions in the laboratory (upper panel) and ion rest frame (lower panel). The direction of the ion beam propagation in the laboratory frame defines the y axis which together with the magnetic field spans the y - z plane. While the incident photons counterpropagate to the ion beam, the emission direction of the outgoing photons is characterized by the angles θ_f and ϕ_f . Due to Lorentz transformation an electric field emerges in the x direction in the ion frame.

process. Below, we will assume that the ion beam in the laboratory frame is exposed to a static external magnetic field orthogonal to the ion velocity (see upper panel of Fig. 1). This corresponds to a scenario in which the ion beam propagates through the straight collision zone of the storage ring and is exposed to the field of a dipole magnet. For the entirety of this paper, the direction of this magnetic field defines the z (quantization) axis. Due to the relativistic motion of the ions, the laboratory magnetic field $\mathbf{B}_{\text{lab}}$ gives rise to both magnetic and electric fields in the ion frame:

$$\mathbf{B}_{\text{ion}} = \gamma_L \mathbf{B}_{\text{lab}}, \quad (3a)$$

$$\mathbf{E}_{\text{ion}} = \gamma_L (\mathbf{v} \times \mathbf{B}_{\text{lab}}). \quad (3b)$$

As seen from Eq. (3a), the direction of the magnetic field does not change under transformation while its strength is enhanced by the Lorentz factor $\gamma_L = 1/\sqrt{1 - \beta^2}$. Moreover, in the ion frame an electric field emerges in the x direction with a field strength also proportional to γ_L . This configuration of the fields is shown in the lower panel of Fig. 1.

For further discussions, it is helpful to gain an understanding of the magnitude of the field strengths within the ion frame. In Table I we display B_{ion} and E_{ion} for the case of $B_{\text{lab}} = 1$ T and different Lorentz factors

TABLE I. Electric and magnetic field strengths in the ion frame for a laboratory magnetic field strength of $B_{\text{lab}} = 1$ T.

	B_{ion} (T)	$\mathcal{E}_{\text{ion}}$ (Vm ⁻¹)
$\gamma_L = 15$	15	4×10^9
$\gamma_L = 100$	100	3×10^{10}
$\gamma_L = 3000$	3000	9×10^{11}

achievable in the super proton synchrotron (SPS) and LHC [20]. It is informative to compare the resulting values of $\mathcal{E}_{\text{ion}}$ with the electric field strengths experienced by an electron in the ground state of a neutral hydrogen atom ($\mathcal{E} \approx 5 \times 10^{11}$ Vm⁻¹) and a hydrogen-like calcium ion ($\mathcal{E} \approx 4 \times 10^{15}$ Vm⁻¹). As seen from this comparison, the Lorentz-boosted external electric field is more than a thousand times weaker compared to the Coulomb field in the calcium ion, which will be employed as a test bed in our study. In order to question the strength of the external magnetic field it is natural to compare the fine-structure splitting and Zeeman shift of ionic levels. For the H-like Ca ion these are $\Delta E_{2p_{1/2}-2p_{3/2}} \approx 7$ eV and $\Delta E_Z \sim \mu_B B_{\text{ion}} \approx 0.2$ eV for $B_{\text{lab}} = 1$ T and $\gamma_L = 3000$, where μ_B denotes the Bohr magneton. We can conclude, therefore, that effects of external electromagnetic fields on medium- and high- Z highly charged ions can be treated perturbatively.

To describe a realistic scenario, we will assume moreover that the ions exhibit a finite spread of longitudinal velocity and hence momentum. As usual in accelerator physics, this distribution is characterized by the parameter $\Delta p/p$, where Δp is the uncertainty of the momentum. In the analysis below, we will consider two scenarios, which correspond to an ion beam injected without beam cooling and one exposed to additional laser cooling [31–33] (see Sec. VIB 4). These two scenarios will be referred to as uncooled and cooled beams and are characterized by the values $\Delta p/p \approx 2 \times 10^{-4}$ and $\Delta p/p \approx 1 \times 10^{-5}$, respectively [32].

III. ENERGY SHIFTS INDUCED BY EXTERNAL FIELDS

In order to analyze the effect of external electromagnetic fields on resonant photon scattering, one has to understand how such fields affect the electronic structure of an ion.

For an ion exposed to a strong magnetic field, the linear and quadratic Zeeman shifts have to be taken into account. If, moreover, an electric field is applied, the ionic energy levels are altered by the quadratic Stark shift. Below, we will briefly present the basic equations of the Zeeman and Stark shifts.

A. Zeeman shift

The interaction of a bound electron, carrying a magnetic moment $\boldsymbol{\mu}$, with an external magnetic field $\mathbf{B}$ is described by the operator

$$\hat{H}_Z = -\boldsymbol{\mu} \cdot \mathbf{B}. \quad (4)$$

In first-order perturbation theory, this interaction leads to a shift of atomic energy levels which is linear in the magnetic

field strength $B = |\mathbf{B}|$:

$$\Delta E_Z^{(1)} = \mu_B g B M_J. \quad (5)$$

Here, M_J is the projection of the atomic state angular momentum in the z direction and g is the Landé g factor. Equation (5) leads to the well-known splitting of a state with total angular momentum J into $2J + 1$ magnetic sublevels.

Treating the Hamiltonian from Eq. (4) in second-order perturbation theory, one obtains the additional quadratic Zeeman shift

$$\Delta E_Z^{(2)} = C_2 B^2, \quad (6)$$

which is proportional to B^2 . For more details about the calculation of the quadratic Zeeman shift coefficient C_2 , see Refs. [34,35].

B. Stark shift

Similar to the Zeeman effect, the interaction of a bound electron with an external electric field $\mathcal{E}$, well known as the Stark effect, is described by the operator

$$\hat{H}_S = e \mathbf{r} \cdot \boldsymbol{\mathcal{E}}. \quad (7)$$

As shown in Refs. [36,37], the quadratic Stark shift for a state with total angular momentum J and projection M_J along the quantization z axis is given by

$$\Delta E_S = -\frac{1}{2} \alpha_0 \mathcal{E}^2 - \frac{1}{4} \alpha_2 \frac{3M_J^2 - J(J+1)}{J(2J-1)} (3\mathcal{E}_z^2 - \mathcal{E}^2). \quad (8)$$

Here, α_0 and α_2 are the scalar and tensor polarizabilities which depend on the atomic state but not on its angular momentum projection M_J . Hence, only the second term in Eq. (8) introduces a splitting of sublevels with different values of M_J^2 . For a more thorough analysis of the scalar and tensor polarizabilities see Refs. [36,37].

IV. RESONANT SCATTERING

This section aims to revisit the basic theory of resonant elastic photon scattering, whose detailed analysis is given in Refs. [5–7]. First, we will discuss the scattering amplitude and pay special attention to the effects of the Zeeman and Stark shifts. By making use of this amplitude and the well-known density-matrix approach, we evaluate the scattering cross section and polarization properties of the emitted photons.

A. Scattering amplitude

For low intensities, the coupling between light and an ion is usually treated within perturbation theory. For the process of photon scattering, this leads to the well-known second-order scattering amplitude, thoroughly discussed in the literature [1,3–5,38]. In the resonant case, when the photon energy ω_i approaches the excitation energy of some intermediate ionic state, see Fig. 2, the calculations can be significantly simplified. In this resonant approximation, $\omega_i \approx E_v - E_i$, the scattering amplitude takes the form

$$\mathcal{M}_{M_f, M_i}^{\text{res}} = r_0 m c^2 \sum_{M_v} \frac{\langle f | \hat{\mathcal{R}}^\dagger(\mathbf{k}_f, \boldsymbol{\epsilon}_f) | v \rangle \langle v | \hat{\mathcal{R}}(\mathbf{k}_i, \boldsymbol{\epsilon}_i) | i \rangle}{E_i - E_v + \omega_i + i\Gamma_v/2}, \quad (9)$$

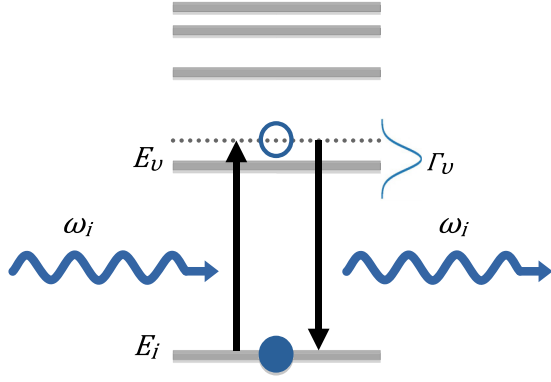


FIG. 2. Schematic illustration of the resonant scattering process. The electron is excited from the ground state to the intermediate state with the natural width Γ_v by absorbing the incoming photon with an energy of $\omega_i \approx E_v - E_i$. Afterward a second photon is emitted with $\omega_i = \omega_f$.

with $|i\rangle = |\xi_i J_i M_i\rangle$, $|v\rangle = |\xi_v J_v M_v\rangle$, and $|f\rangle = |\xi_f J_f M_f\rangle$ denoting the initial, intermediate, and final ionic states. These states are defined by their total angular momentum J , its projection M , and ξ , which accounts for all other quantum numbers necessary for a unique identification of the states. Moreover, Γ_v is the natural width of the intermediate state and r_0 and m are the classical electron radius and electron mass. The sum in Eq. (9) runs over the degenerate sublevels of the excited state with different angular momentum projections M_v [5–7]. The photon absorption and emission operators, $\hat{\mathcal{R}}$ and $\hat{\mathcal{R}}^\dagger$, can be expressed as a sum of their single-particle counterparts, written here in Coulomb gauge,

$$\hat{\mathcal{R}}(\mathbf{k}, \epsilon) = \sum_q \alpha_q \cdot \epsilon e^{i\mathbf{k}\mathbf{r}_q}, \quad (10)$$

where the index q refers to the q th electron. Here, α is the vector of Dirac matrices and ϵ and $\mathbf{k}$ are the photon polarization and momentum vectors, respectively. The matrix elements of these operators are usually evaluated by using a multipole expansion and the Wigner-Eckart theorem [5–7].

Amplitude (9) was frequently used to describe the process of resonant elastic photon scattering in the field-free case [5–7]. It can be extended, however, for a target atom exposed to external electric and magnetic fields [39]. The effect of these external fields can also be treated within perturbation theory, leading to the modification of the ionic levels:

$$E_i = E_i^{(0)} + \Delta E_{Z,i} + \Delta E_{S,i} \quad (11a)$$

$$E_v = E_v^{(0)} + \Delta E_{Z,v} + \Delta E_{S,v}, \quad (11b)$$

and generally of the ionic wave functions. These wave functions and energies can be used to describe the coupling to the incoming laser field, which again can be treated perturbatively. In this approach, the amplitude can still be written in the form of Eq. (9), where the energies are given by Eq. (11).

One has to note here that the influence of the external fields on the wave functions is neglected in the present work. Indeed, the external electric field also leads to the mixing of opposite parity states, which results in additional interference effects [40]. However, as we focus

in this study on $J_i = 0 \rightarrow J_v = 1 \rightarrow J_f = 0$ electric dipole transitions, the electric field induced state mixing leads to additional magnetic dipole transitions which are generally much smaller and hence can be neglected.

B. Cross section and polarization

With the help of the scattering amplitude (9) and the energies (11a) and (11b), one can evaluate the basic properties of the scattering process, such as the cross section and polarization of the emitted photons. Most naturally, this can be done within the framework of the density-matrix approach [6,7,41]. Within this theory the density matrix of scattered photons can be expressed as

$$\begin{aligned} \langle \mathbf{k}_f, \lambda_f | \hat{\rho}_f | \mathbf{k}_f, \tilde{\lambda}_f \rangle &= \frac{1}{2J_i + 1} \sum_{\lambda_i, \tilde{\lambda}_i} \sum_{M_i M_f} \langle \mathbf{k}_i, \lambda_i | \hat{\rho}_i | \mathbf{k}_i, \tilde{\lambda}_i \rangle \\ &\times \mathcal{M}_{M_f, M_i}^{\text{res}}(\lambda_i, \lambda_f) \mathcal{M}_{M_f, M_i}^{\text{res}*}(\tilde{\lambda}_i, \tilde{\lambda}_f). \end{aligned} \quad (12)$$

Here, $\mathcal{M}_{M_f, M_i}^{\text{res}}(\lambda_i, \lambda_f)$ is the scattering amplitude (9) written for a special case of circularly polarized incoming and outgoing photons with the helicity $\lambda_{i,f}$, and $\langle \mathbf{k}_i, \lambda_i | \hat{\rho}_i | \mathbf{k}_i, \tilde{\lambda}_i \rangle$ is the density matrix of the incident photons. Moreover, in Eq. (12) the target atoms are assumed to be unpolarized. By making use of the photon density matrix, one can easily access the parameters of the scattering process as, for instance, the differential cross section:

$$\frac{d\sigma}{d\Omega}(\theta_f, \phi_f, \omega_i) = \sum_{\lambda_f} \langle \mathbf{k}_f \lambda_f | \hat{\rho}_f | \mathbf{k}_f \lambda_f \rangle. \quad (13)$$

Moreover, the polarization of the photons can be naturally expressed in terms of the three Stokes parameters, which can be related to the density matrix by

$$\langle \mathbf{k} \lambda | \hat{\rho} | \mathbf{k} \tilde{\lambda} \rangle = \mathcal{N} \begin{pmatrix} 1 + P_3 & -P_1 + iP_2 \\ -P_1 - iP_2 & 1 - P_3 \end{pmatrix}, \quad (14)$$

where $\mathcal{N} = 1$ describes the incoming radiation and $\mathcal{N} = d\sigma/d\Omega$ the outgoing radiation. Here, the first two Stokes parameters describe linear polarization, either within the scattering plane or perpendicular to it, in the case of P_1 , or at angles of 45° or 135° , in the case of P_2 . The parameter P_3 characterizes circular polarization.

V. SCATTERING OFF He-LIKE IONS IN EXTERNAL FIELDS

The theory introduced in the previous sections is general and can be applied to any ion. In what follows, we will consider the particular case of $1s^2 \ ^1S_0 \rightarrow 1s2p \ ^1P_1$ scattering of photons by He-like Ca ions. This process will be discussed for typical parameters of the Gamma Factory project at CERN. In particular, in the analysis below we assume the Lorentz factor of $\gamma_L \approx 2395$, initial photons with energy $\omega_{\text{lab}} \approx 0.815$ eV which can be delivered by commercially available erbium-doped fiber lasers [19] and external magnetic field strengths up to $B_{\text{lab}} = 3$ T.

TABLE II. The $1s^2\ ^1S_0 \rightarrow 1s2p\ ^1P_1$ transition energies, the scalar and tensor polarizabilities (in atomic units), and the lifetimes of the 1P_1 state for neutral He and He-like Ca. Our results, based on CI calculations using the AMBIT code [43], agree well with available data from Refs. [44–46,48–50].

	$E_{1P_1}^{(0)} - E_{1S_0}^{(0)}$ (eV)	$\alpha_0(^1S_0)$ (a.u.)	$\alpha_0(^1P_1)$ (a.u.)	$\alpha_2(^1P_1)$ (a.u.)	$\tau(^1P_1)$ (s)
He	21.212	1.36	-5.99×10^1	2.23×10^2	5.57×10^{-10}
	21.218 [44]	1.38 [45]	-6.00×10^1 [46]	2.24×10^2 [47]	5.51×10^{-10} [48]
Ca ¹⁸⁺	39.039×10^2	5.67×10^{-5}	-2.27×10^{-2}	2.38×10^{-2}	6.11×10^{-15}
	39.024×10^2 [49]				6.09×10^{-15} [50]

A. Energy shifts of the $1s^2\ ^1S_0$ and $1s2p\ ^1P_1$ states

As discussed already in Secs. III and IV, the splitting and shift of ionic levels due to Zeeman and Stark effects may affect the scattering process and hence have to be well understood. In Fig. 3, therefore, we present a schematic illustration of how the $1s^2\ ^1S_0$ and $1s2p\ ^1P_1$ magnetic sublevels are affected by external electric and magnetic fields in the ion rest frame. As seen in this figure, the Stark shift ($\mathcal{E} \neq 0$) leads to a shift of the $|^1S_0, M_i = 0\rangle$ sublevel and a splitting of the $|^1P_1, M_v = 0\rangle$ and $|^1P_1, M_v = \pm 1\rangle$ sublevels. This prediction can be well understood based on Eq. (8), which predicts that the quadratic Stark shift depends on M_J^2 . If in addition to the electric field an external magnetic field is applied, the ionic levels also exhibit first- and second-order Zeeman shift. While the second-order Zeeman shift is usually small and depends on M_J^2 , the first-order correction is linear in M_J and leads to a splitting of the $|^1P_1, M_v = +1\rangle$ and $|^1P_1, M_v = -1\rangle$ sublevels. The combined effect of electric and magnetic fields on the magnetic sublevels is displayed in the right column of Fig. 3.

So far we have only presented the qualitative picture of the splitting of 1S_0 and 1P_1 sublevels for ions exposed to external electromagnetic fields. Using the equations discussed in the previous sections, one can also derive formulas for a quantitative analysis. Indeed, by combining Eqs. (3), (5), (8), and (11) we obtain

$$\Delta E_{\text{tot}}(^1S_0) = -\frac{1}{2}\alpha_0(^1S_0)\gamma_L^2 v^2 B_{\text{lab}}^2, \quad (15a)$$

$$\Delta E_{\text{tot}}(^1P_1) = -\frac{1}{2}\alpha_0(^1P_1)\gamma_L^2 v^2 B_{\text{lab}}^2 + \frac{3M_v^2 - 2}{4}\alpha_2(^1P_1) \times \gamma_L^2 v^2 B_{\text{lab}}^2 + M_v g \mu_B \gamma_L B_{\text{lab}}, \quad (15b)$$

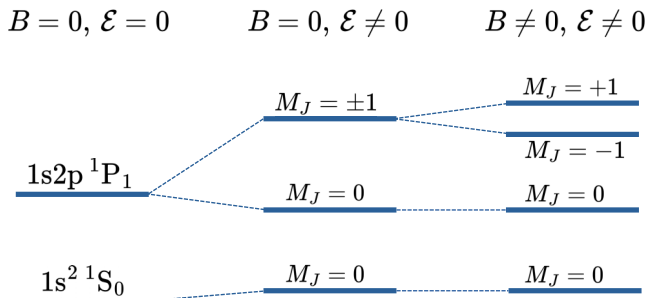


FIG. 3. Schematic illustration of the Zeeman and Stark shift on the $1s^2\ ^1S_0$ and $1s2p\ ^1P_1$ states.

where $\Delta E_{\text{tot}} = \Delta E_Z + \Delta E_S$ is the sum of the Zeeman and Stark shifts, and α_0 and α_2 are the scalar and tensor polarizabilities. Moreover, in Eq. (15) the second-order Zeeman effect is neglected since, according to our calculations, it is much smaller than $\Delta E_Z^{(1)}$. Indeed, for Ca¹⁸⁺ ions moving with $\gamma_L = 2395$ in the presence of the external magnetic field with $B_{\text{lab}} = 1$ T we found $\Delta E_Z^{(1)} \approx 0.1$ eV and $\Delta E_Z^{(2)} \approx 10^{-5}$ eV.

As seen from Eq. (15), the calculations of the energies of the $|^1S_0\rangle$ and $|^1P_1, M_v\rangle$ sublevels for the scenario of interest requires evaluations of the g factor and the polarizabilities α_0 and α_2 . While the g factor of the 1P_1 state is given as $g \approx 1$ [42], the polarizabilities are calculated using the configuration interaction method implemented in the AMBIT code, thoroughly explained in Ref. [43]. The polarizabilities and the transition energies for the unperturbed case as well as the lifetimes of the excited 1P_1 state are shown in Table II. By making use of these data and Eq. (15) one can finally calculate the Zeeman and Stark shift of the ground and excited ionic states. This calculation indicates that $\Delta E_{\text{tot}}(^1S_0)$ is much smaller than $\Delta E_{\text{tot}}(^1P_1)$. In Table III, therefore, we present the energy shifts for the excited sublevels $|^1P_1, M_v\rangle$ only. From this table one can conclude that the Stark shift is the dominant effect for the chosen experimental parameters and He-like Ca projectiles.

B. External field effects on resonant scattering

In the previous subsection, we have briefly discussed how external electric and magnetic fields modify the sublevels $|^1S_0, M_i\rangle$ and $|^1P_1, M_v\rangle$ of He-like Ca ions. Now we are ready to explore how such fields affect the properties of the $^1S_0 \rightarrow ^1P_1$ resonant photon scattering. In the next subsection, for instance, we will discuss the scattering rate of photons as observed in the foreseen Gamma Factory setup. Their angular distribution and polarization for the case of initially circularly polarized light will be studied then in Sec. VB 2. The presented formalism can be applied to an arbitrary polarization of the laser photons. The choice of circularly polarized photons is related to their important role for the Gamma

TABLE III. Resulting Zeeman and Stark shifts of the $1s2p\ ^1P_1$ state in He-like Ca for a magnetic field strength in the laboratory frame of $B_{\text{lab}} = 1$ T and a Lorentz factor of $\gamma_L = 2395$.

M_J	ΔE_S (eV)	$\Delta E_Z^{(1)}$ (eV)	ΔE_{tot} (eV)
0	-2.92×10^{-2}	0	-2.92×10^{-2}
1	9.18×10^{-1}	1.39×10^{-1}	10.57×10^{-1}
-1	9.18×10^{-1}	-1.39×10^{-1}	7.79×10^{-1}

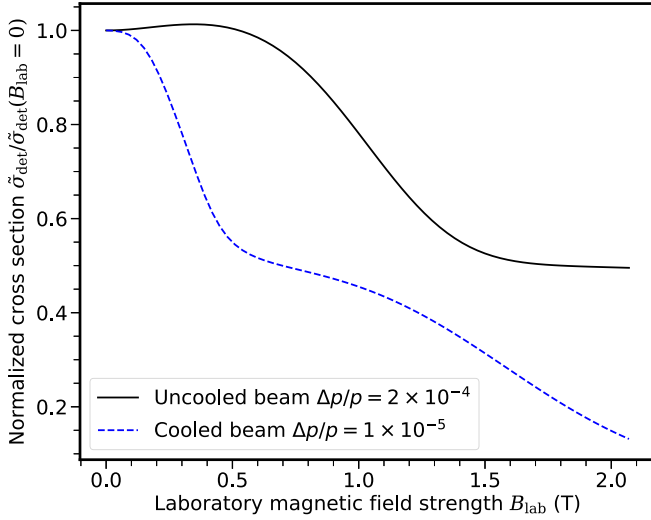


FIG. 4. The normalized cross section of photons scattered in a finite solid angle in the y direction with an opening angle of 1 mrad in the laboratory frame for He-like Ca ions. The cross section is shown as a function of the magnetic field strength in the laboratory frame. The black solid line is calculated for the uncooled beam scenario, $\Delta p/p = 2 \times 10^{-4}$, and the blue dashed line represents the cooled beam scenario, $\Delta p/p = 1 \times 10^{-5}$.

Factory project, e.g., for investigations of atomic parity violation [40] or the production of longitudinally polarized muon and positron beams [16].

1. Rate of scattered photons

Photons resonantly scattered by ultrarelativistic projectiles are emitted in the laboratory frame within a small solid angle around the ion beam propagation direction. For the typical parameters of the Gamma Factory, for example, the majority of the photons will be scattered within an opening angle $\theta_f^{(\text{lab})} \leq 1$ mrad. Therefore, detectors covering a comparable solid angle range will detect most of the scattered radiation. In order to investigate how the count rate observed by such detectors will depend on the strength of the laboratory magnetic field, we have performed calculations, whose results are summarized in Fig. 4. In this figure, we display the differential cross section, integrated over the solid angle of the detector ($\theta_f^{(\text{lab})} \leq 1$ mrad),

$$\sigma_{\text{det}}(\omega_i, B_{\text{lab}}) = \int_{\Omega_{\text{det}}} \frac{d\sigma}{d\Omega_f}(\theta_f, \phi_f, \omega_i, B_{\text{lab}}) d\Omega_f, \quad (16)$$

normalized to the predictions for $B_{\text{lab}} = 0$. For the latter case and for zero detuning, the integrated cross section (16) can be estimated as $\sigma_{\text{det}}(\omega_i = E_\nu - E_i, B_{\text{lab}} = 0) \approx 4 \times 10^8$ barn. Moreover, we have taken into account the momentum spread of the ion beam, which leads to the fact that the laser radiation, which is assumed to be monochromatic in the laboratory frame, is Doppler broadened in the ion frame. To account for this broadening, the cross section is convoluted with a Gaussian frequency distribution:

$$\tilde{\sigma}_{\text{det}}(B_{\text{lab}}) = \int \sigma_{\text{det}}(\omega_i, B_{\text{lab}}) \frac{1}{\sqrt{2\pi} \Delta\omega^2} e^{-\frac{(\omega - \omega_{\text{ion}})^2}{2\Delta\omega^2}} d\omega_i, \quad (17)$$

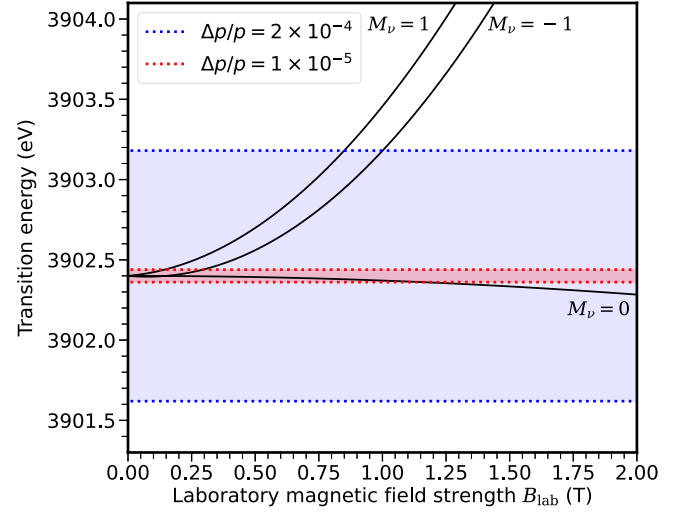


FIG. 5. The transition energies from the ground state to the magnetic $|^1P_1, M_v\rangle$ sublevels as a function of the laboratory magnetic field strength. The red and blue dotted lines represent the frequency width of the incoming radiation in the ion frame for both scenarios of uncooled (blue) and cooled (red) ion beams. The natural line width $\Gamma_\nu = \hbar/\tau$ is neglected in this visualization.

where $\omega_{\text{ion}} = 2\gamma_L \omega_{\text{lab}}$ is the mean laser frequency in the ion frame, and $\Delta\omega$ is its width due to the ion beam's momentum spread. As mentioned already in Sec. II, scenarios of uncooled and cooled ion beams are considered, corresponding to relative widths of $\Delta\omega/\omega_{\text{ion}} = 2 \times 10^{-4}$ and $\Delta\omega/\omega_{\text{ion}} = 1 \times 10^{-5}$, respectively. The results for these scenarios are depicted by the solid and dashed lines in Fig. 4. As seen from this figure, the normalized rate of detected photons is very sensitive to the laboratory magnetic field strength. The effect of the magnetic field is strongest for the cooled ion beam, where the normalized rate falls to approximately $\tilde{\sigma}_{\text{det}}(B_{\text{lab}})/\tilde{\sigma}_{\text{det}}(0) \approx 0.15$ as B_{lab} increases from zero to 2 T. In the uncooled beam scenario, $\tilde{\sigma}_{\text{det}}(B_{\text{lab}})/\tilde{\sigma}_{\text{det}}(0)$ also declines with increasing B_{lab} , though this effect is less pronounced. Additionally, for the uncooled beam, the count rate stabilizes around $\tilde{\sigma}_{\text{det}}(B_{\text{lab}})/\tilde{\sigma}_{\text{det}}(0) \approx 0.5$ for $B_{\text{lab}} > 1.5$ T.

In order to gain a qualitative understanding of this behavior, we inspect Fig. 5. This figure shows the transition energies from the ground state to the different magnetic sublevels $|^1P_1, M_v\rangle$ as a function of the magnetic field strength, compared to the width of the frequency distribution of the incident radiation as seen in the ion frame. This figure reveals that $M_v = \pm 1$ sublevels are quickly shifted out of resonance in the cooled beam scenario, leading to the rapid drop of the rate of detected photons. For the uncooled beam, however, the rate begins to decrease not until $B_{\text{lab}} > 0.5$ T because of the larger frequency width. Moreover, the $M_v = 0$ sublevel stays in resonance at higher B_{lab} values in the uncooled beam scenario, leading to the observed stagnation of the count rate at $\tilde{\sigma}_{\text{det}}(B_{\text{lab}})/\tilde{\sigma}_{\text{det}}(0) \approx 0.5$.

The qualitative understanding gained from Fig. 5 can be extended to a quantitative analysis by studying the relative contributions of the different $|^1P_1, M_v\rangle$ sublevels to the scattering process. By using the photon absorption operator (10) and some angular momentum algebra [51,52], it can be shown that

the photoexcitation rate from the $|^1S_1, M_i = 0\rangle$ ground state to one excited $|^1P_1, M_v\rangle$ sublevel is proportional to the absolute square of the small Wigner- d matrix:

$$W_{|^1S_0, M_i=0\rangle \rightarrow |^1P_1, M_v\rangle} \propto \left| d_{M_v, \lambda}^1 \left(\theta_i - \frac{\pi}{2} \right) \right|^2 |a_{E1}|^2. \quad (18)$$

Here, a_{E1} is the reduced transition matrix element and $\theta_i - \pi/2 = \pi/2$ is the angle between the initial photon propagation direction and the quantization axis. From Eq. (18), it follows immediately that the $M_v = \pm 1$ sublevels each contribute 25% to the total excitation rate, while the remaining 50% are provided by the $M_v = 0$ sublevel. Based on this observation, one can easily understand why the rate of detected photons reaches a steady value around $\tilde{\sigma}_{\text{det}}(B_{\text{lab}})/\tilde{\sigma}_{\text{det}}(0) \approx 0.5$ for high magnetic fields in the uncooled beam scenario.

So far, our analysis was focused on a particular Lorentz factor $\gamma_L = 2395$. We now extend this study to a variable Lorentz factor, as it is used in the Gamma Factory setup to scan over the photon frequency in the ion frame. Therefore, we calculate the normalized count rate of detected photons as a function of the Lorentz factor by varying $\omega_{\text{ion}} = 2\gamma_L\omega_{\text{lab}}$ in Eq. (17). The resulting spectrum, as it could be observed in a Gamma Factory experiment, is shown in Fig. 6 for three different laboratory magnetic field strengths. This figure demonstrates that the Stark splitting between the $|^1P_1, M_v = 0\rangle$ and $|^1P_1, M_v = \pm 1\rangle$ sublevels can be observed in both uncooled and cooled beam settings, whereas the smaller Zeeman splitting of the $|^1P_1, M_v = \pm 1\rangle$ sublevels is resolved only in the cooled beam scenario.

2. Angular distribution and polarization

Alongside the modified rate of detected photons, which was discussed in the previous subsection, a laboratory magnetic field can have a notable impact on the angular and polarization properties of scattered radiation. This impact of the B field can also be attributed to the mutual shift of the magnetic sublevels $|^1P_1, M_v\rangle$ and is related to the well-known Hanle effect [9,10,53,54].

To discuss how the laboratory magnetic field can influence the angular distribution of emitted photons, we review first the well-established results for the field-free case, $B_{\text{lab}} = 0$,

$$W(\theta_f^{(\text{ion})}, \phi_f^{(\text{ion})}, B_{\text{lab}} = 0) = \frac{W_0}{4} (3 + \cos 2\theta_f^{(\text{ion})}). \quad (19)$$

This formula is derived in the ion rest frame, for the case of a $J_i = 0 \rightarrow J_v = 1 \rightarrow J_f = 0$ transition and circularly polarized incident light [6]. To apply Eq. (19) to describe an experiment, one would need to average it over the photon frequency distribution in the ion frame, similar to what is done in Eq. (17). However, since for the field-free case (19) all magnetic sublevels are degenerate and, hence, the energy dependence only appears in the prefactor W_0 , the shape of the angular distribution is not affected by the frequency averaging. As seen from Eq. (19) and Fig. 7, the angular distribution of emitted photons for the field-free case is independent on the azimuthal angle $\phi_f^{(\text{ion})}$ and is symmetric with respect to $\theta_f^{(\text{ion})} = \pi/2$.

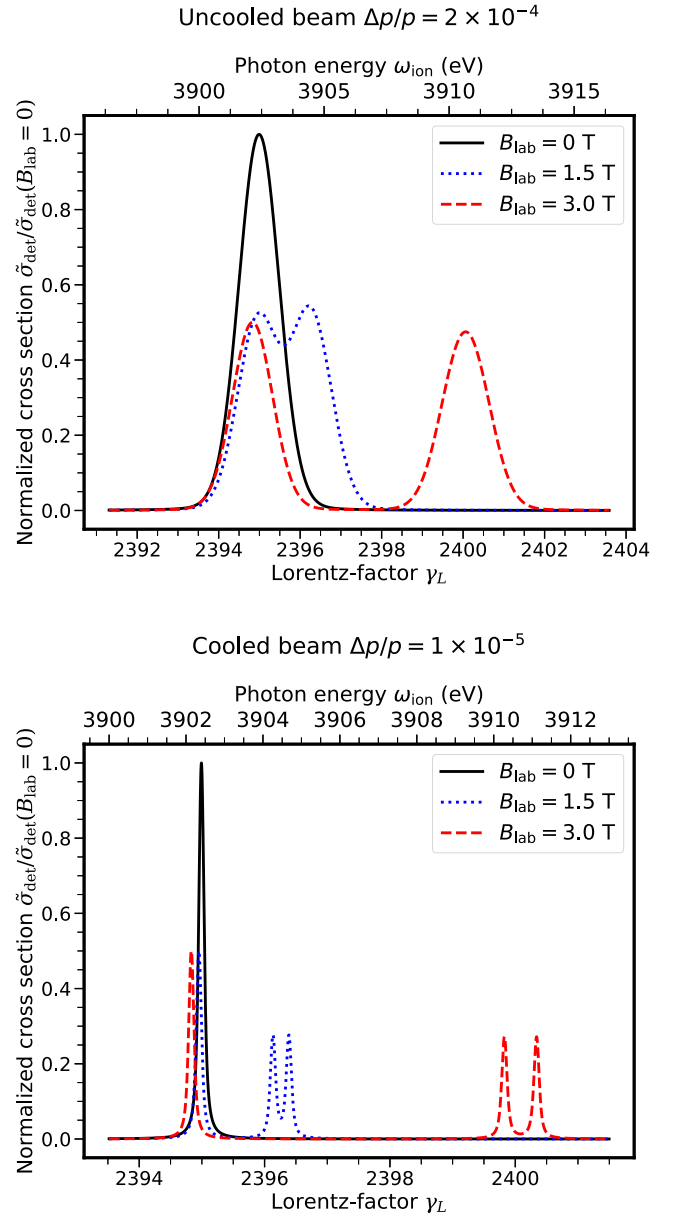


FIG. 6. The normalized cross section of photons scattered in a finite solid angle in the y direction with an opening angle of 1 mrad in the laboratory frame for He-like Ca ions. The cross section is shown as a function of the Lorentz factor γ_L and the corresponding laser photon energy as seen in the ion frame. The upper plot is calculated for the uncooled beam scenario, $\Delta p/p = 2 \times 10^{-4}$, and the lower plot represents the cooled beam scenario, $\Delta p/p = 1 \times 10^{-5}$.

In order to explore, how the angular distribution (19) is altered if an external magnetic field is applied, we employ the matrix element (9), the modified energies (11) and the general expression for the angle-differential cross section (13), averaged over the photon frequency distribution. The resulting emission patterns in the ion frame, obtained for the uncooled beam scenario, are displayed in the upper panels of Fig. 7. One can see from this figure that the angular distribution of emitted photons is very sensitive to B_{lab} . In particular, $W(\theta_f^{(\text{ion})}, \phi_f^{(\text{ion})}, B_{\text{lab}} \neq 0)$ has a pronounced dependence on

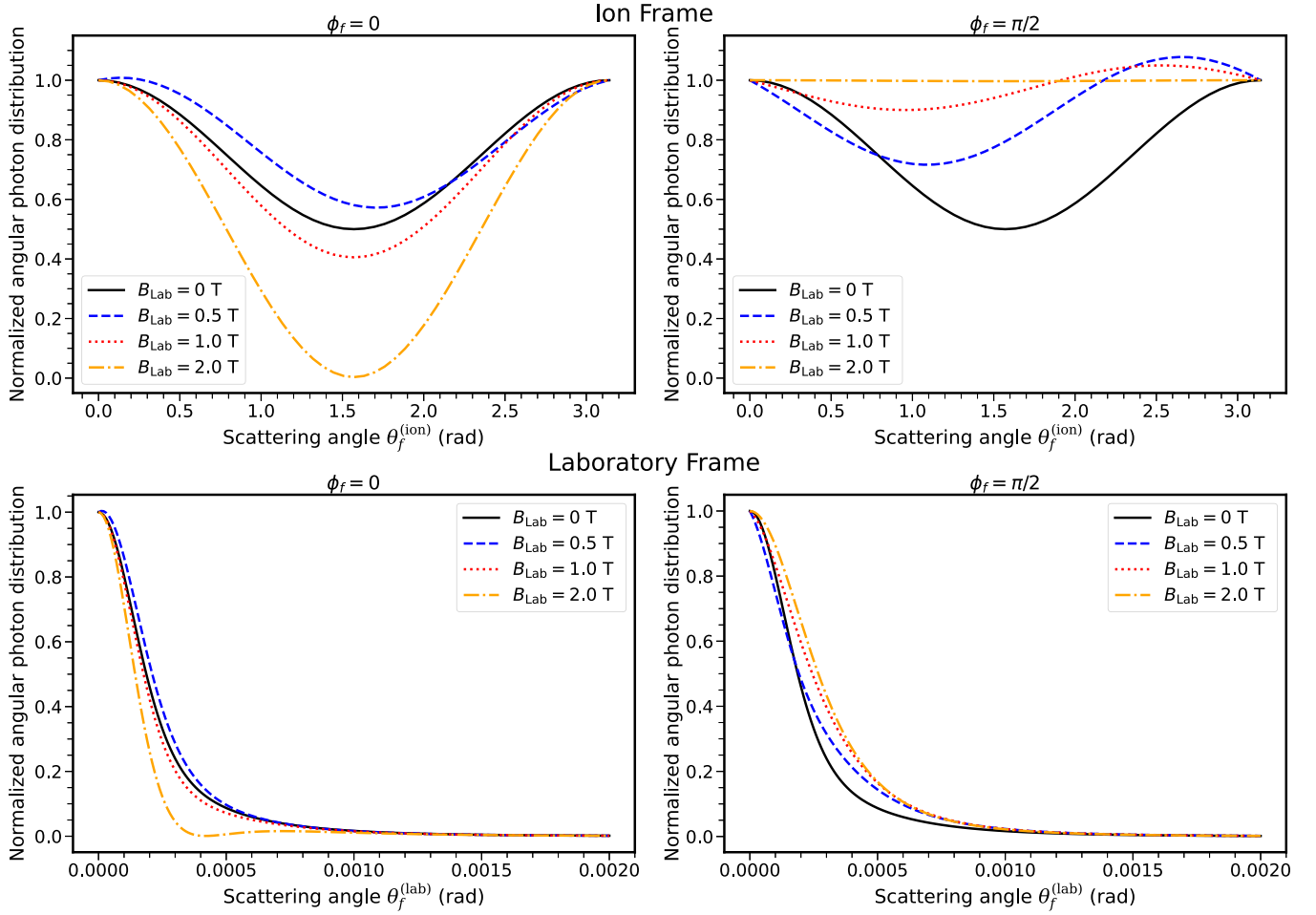


FIG. 7. The angular distribution of the emitted photons in the $1s^2\ ^1S_0 \rightarrow 1s2p\ ^1P_1$ scattering process in He-like calcium for $\gamma_L = 2395$ and different laboratory magnetic field strengths. For each value of B_{lab} the angular distribution is normalized by its value for $\theta_f = 0$. The upper panels show the result in the rest frame of the ion while the lower panels show the Lorentz transformed result in the laboratory frame.

the azimuthal angle ϕ_f^{ion} that can be attributed to the fact that electric and magnetic fields, as seen by the ion, break the azimuthal symmetry of the system. Moreover, for the case of $\phi_f^{\text{ion}} = \pi/2$ the original symmetry to the polar angle $\theta_f^{\text{ion}} = \pi/2$ is clearly broken already for smaller magnetic fields, $B_{\text{lab}} \approx 0.5$ T.

It is interesting to note that the angular distribution of scattered photons responds differently to weak and strong magnetic fields. Most clearly it can be seen for $\phi_f^{\text{ion}} = 0$. For this geometry, the emission pattern becomes more isotropic as the magnetic field strength increases from $B_{\text{lab}} = 0$ to $B_{\text{lab}} \approx 0.5$ T. Further enhancement of B_{lab} , however, leads to a more pronounced anisotropy of W . This irregular behavior of the angular distribution can be attributed to an interplay of two different effects. Indeed, for small B_{lab} the Stark and Zeeman shifts remain small compared to the width of the Doppler-broadened frequency distribution, but are comparable to the natural width Γ_ν . In this regime, the behavior of the angular distribution is governed by the Hanle effect, which reduces the anisotropy of W . In contrast, for $B_{\text{lab}} \geq 1$ T, the $|^1P_1, M_\nu = \pm 1\rangle$ sublevels are shifted out of resonance, see Fig. 5. Hence, the scattering proceeds via a single sublevel

$|^1P_1, M_\nu = \pm 0\rangle$, resulting in a strong asymmetry of the angular distribution.

So far we have only discussed the angular distribution of emitted photons in the ion frame. However, since all measurements are performed in the laboratory frame, a transformation of our results to this frame is required, which can be easily executed by using Eqs. (1) and (2). The resulting angular distributions of scattered photons in the laboratory frame are depicted in the lower panels of Fig. 7. This figure demonstrates the well-known effect of a focusing of scattered radiation along the direction of the ion beam propagation, $\theta_f^{\text{(lab)}} \leq 1$ mrad, due to the ultrarelativistic movement of the ions. Despite this focusing, the influence of the external magnetic field can be observed even in the laboratory frame. In particular in the range $0.3 \text{ mrad} \leq \theta_f^{\text{(lab)}} \leq 0.6 \text{ mrad}$ the applied magnetic field leads to strong deviations from the field-free case. For $\phi_f^{\text{(lab)}} = 0$, for example, a laboratory magnetic field of $B_{\text{lab}} = 2$ T results in an even stronger focusing of the emitted photons along the ion beam propagation direction compared to the field-free scenario. In contrast, for $\phi_f^{\text{(lab)}} = \pi/2$, the magnetic field slightly broadens the angular distribution. Besides the angular distribution, the polarization

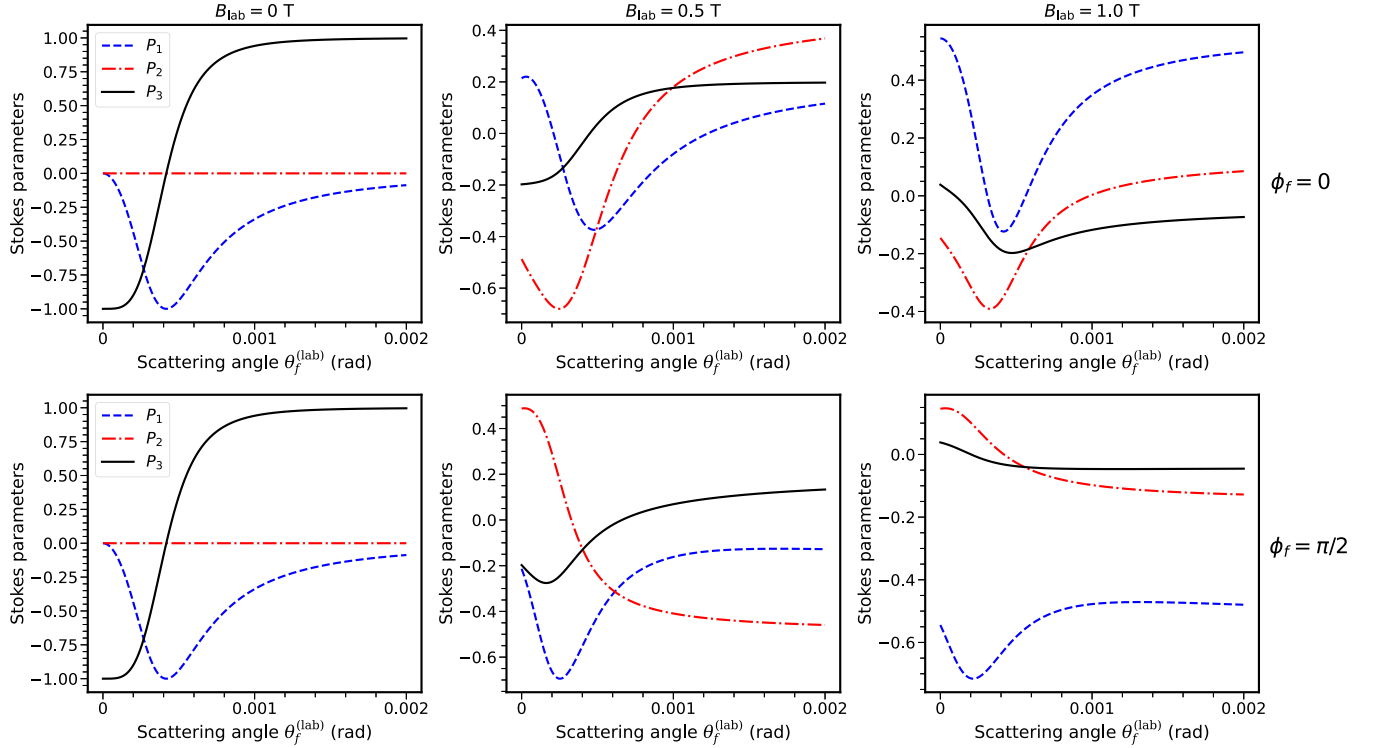


FIG. 8. The three Stokes parameters $P^{(f),(lab)}$ of the emitted photons as a function of the scattering angle in the laboratory frame for different magnetic field strengths.

of scattered photons can also be strongly affected by the external magnetic field. Similarly to before, we start the discussion of this B dependence with a short reminder of the field-free case. Namely, for the here considered scenario of a $J_i = 0 \rightarrow J_v = 1 \rightarrow J_f = 0$ transition and circularly polarized incident radiation, the polarization of scattered light is described by three Stokes parameters [6]:

$$P_1^{(f),(ion)}(\theta_f^{(ion)}, \phi_f^{(ion)}) = \frac{-2 \sin^2 \theta_f^{(ion)}}{3 + \cos 2\theta_f^{(ion)}}, \quad (20a)$$

$$P_2^{(f),(ion)}(\theta_f^{(ion)}, \phi_f^{(ion)}) = 0, \quad (20b)$$

$$P_3^{(f),(ion)}(\theta_f^{(ion)}, \phi_f^{(ion)}) = \frac{-4 \cos \theta_f^{(ion)}}{3 + \cos 2\theta_f^{(ion)}}. \quad (20c)$$

While these expressions are written in the ion rest frame, their transformation to the laboratory frame follows the simple relation

$$P_{1,2,3}^{(f),(ion)}(\theta_f^{(ion)}, \phi_f^{(ion)}) = P_{1,2,3}^{(f),(lab)}(\theta_f^{(lab)}, \phi_f^{(lab)}), \quad (21)$$

where $\phi_f^{(ion)} = \phi_f^{(lab)}$ and the transformation of θ_f is given by Eq. (1).

Utilizing the theory presented in Sec. IV and performing again the averaging over the frequency distribution, we can also calculate the Stokes parameters of light scattered on an ion exposed to external electromagnetic fields. These Stokes parameters, as observed in the laboratory frame in the uncooled beam scenario, are presented in Fig. 8 for three different external magnetic field strengths, $B_{lab} = 0$ (left column), $B_{lab} = 0.5$ T (middle column), and $B_{lab} = 1$ T (right

column). Moreover, we investigate the cases when the photons are emitted within the y - z plane ($\phi_f = 0$) or perpendicular to it ($\phi_f = \pi/2$), see Fig. 1. For the field-free case, $B_{lab} = 0$, the result of our calculations perfectly reproduces the analytical expression (20). However, the growth of B_{lab} leads to a significant modification of the Stokes parameters.

A very pronounced example of the external magnetic field effect can be observed for the backscattering of photons, $\theta_f^{(ion)} = \theta_f^{(lab)} = 0$. We remind here that the polar angles θ_i and θ_f , describing the propagation of incident and scattered photons, are defined with respect to the ion beam propagation direction (y axis). Thus, backscattering corresponds to the case of $\theta_i = \pi$ and $\theta_f = 0$, see Fig. 1. For this case, the field-free formula (20c) predicts a complete polarization transfer from initial to final photons, $P_3^{(i)} = 1$ to $P_3^{(f)} = -1$. This transfer of polarization can serve as a promising tool for generating circularly polarized γ rays at the Gamma Factory, which are important, for example, for the production of longitudinally polarized muon and positron beams [16]. However, as seen from the middle and right panels of Fig. 8, the external magnetic field leads to the reduction of $|P_3^{(f)}|$, thus resulting in partial polarization transfer. In contrast, the linear polarization of backscattered photons becomes nonzero with increasing B_{lab} and can reach quite high values.

The B_{lab} dependence of the Stokes parameters of backscattered light with $\theta_f^{(ion)} = \theta_f^{(lab)} = 0$ is additionally visualized in Fig. 9. As seen from this figure, the degree of circular polarization $|P_3^{(f)}|$ rapidly drops with the increase of B_{lab} , while, in turn, the linear polarization parameters $P_1^{(f)}$ and $P_2^{(f)}$ become nonvanishing. In particular, the Stokes parameter

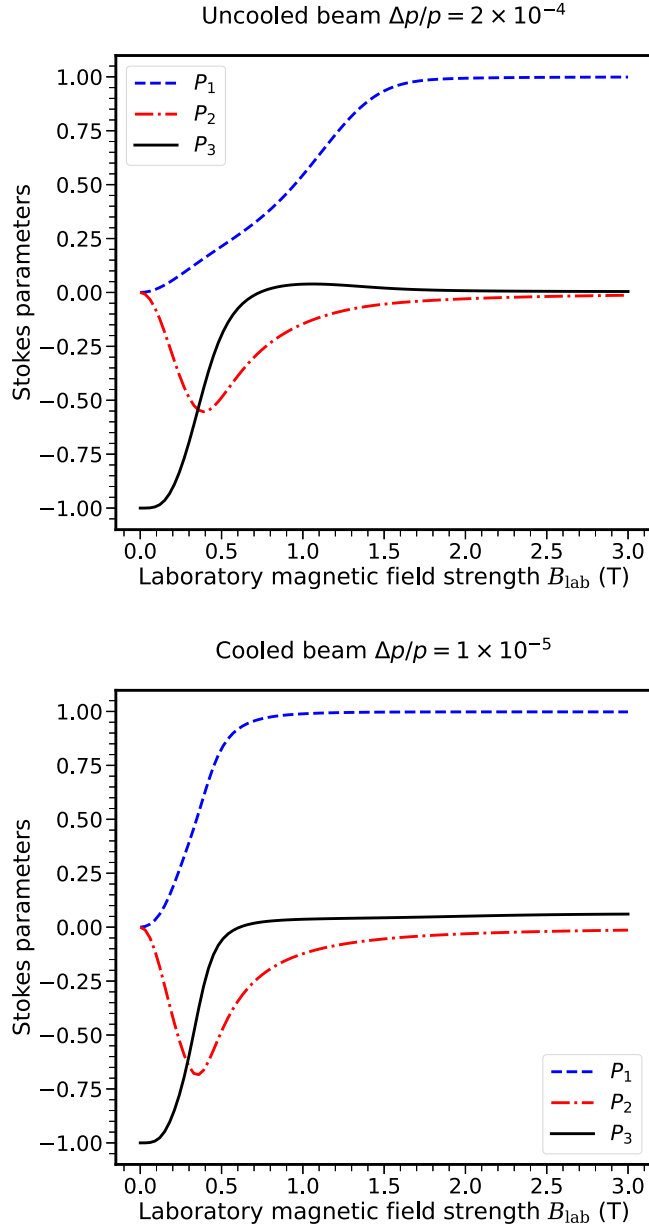


FIG. 9. The three Stokes parameters $P^{(f),(\text{lab})}$ of the emitted photons for $\theta_f^{(\text{lab})} = \phi_f^{(\text{lab})} = 0$ as a function of the laboratory magnetic field strength. Calculations were performed for the scenario of an uncooled ion beam (upper plot) as well as for a cooled beam (lower plot).

$P_1^{(f)}$, which describes linear polarization of outgoing radiation within or perpendicular to the scattering plane, almost reaches unity for $B_{\text{lab}} > 0.5$ T for both the cooled and uncooled beam scenario. Such a conversion of circular polarization into linear polarization may offer interesting applications for the Gamma Factory, which will be discussed in the next section.

VI. PHOTON SCATTERING AT GAMMA FACTORY

While the theory presented in the previous sections is general and can be applied to any storage ring experiment, below we discuss how its predictions can be used to advance the

research program of the Gamma Factory (GF) at CERN. The Gamma Factory scientific program opens new perspectives for particle, nuclear, atomic, and applied physics, see Ref. [55] for further details. Less exposed is its capacity to improve the quality of the beams accelerated and stored in the SPS and LHC rings of CERN and to reach an unprecedented precision of their control. These tasks are of vital importance for the general scientific program of CERN.

In this section we explore how the GF project can benefit from investigations of resonant photon scattering off partially stripped ions (PSIs) in the presence of an external magnetic field in the collision zone. The specific case of a He-like calcium beam discussed in this paper can be extrapolated to other PSI beams. The theoretical framework, presented in this paper, is used to assess the precision of the calibration of the SPS/LHC beam momentum as well as of the momentum spread. In addition, we discuss below the potential role of varying the strength of the magnetic field for the GF beam-cooling scheme proposed in Ref. [32] and developed further in Ref. [33]. Finally, the experimental control of the GF photon beam polarization by an external magnetic field is explored.

A. GF photon-ion collision scheme

1. Technological challenges

The GF beam test demonstrated that PSI beams can be produced, collimated, and stored in the CERN's LHC and SPS rings [56–58]. Their γ_L can span the values between 15 and 3300.

In the GF scheme, PSI bunches collide with photon pulses, which can be produced with commercially available, low-phase-noise lasers operating in ultraviolet, visible, or near-infrared regimes. These laser pulses are stored in the high-finesse Fabry-Pérot cavity to collide them with the PSI bunches at a 20-MHz repetition rate. Recent studies demonstrated that the GF laser system can be installed in the high-radiation environment of the CERN accelerator tunnels and confirmed the stable operation of the Fabry-Pérot cavity storing light pulses with an average power of 500 kW [59,60]. These results demonstrate the technological readiness for the implementation of the GF project at CERN. The final GF research and development step, the proof-of-principle experiment [61], which will be installed in the SPS tunnel, aims to deliver the final proof of the GF project feasibility, not only in its technological but also in its operational aspects.

For collisions of the laser pulses with the PSI bunches in the presence of the magnetic field, as considered in the present study, the Fabry-Pérot cavity has to be placed inside the dipole magnet. Dipole magnets, respecting the requisite aperture and length constraints, and generating the magnetic field of up to 3 Tesla, can be built using the present technology.

The trajectories of the ions circulating in the SPS/LHC rings will be perturbed by introducing the magnetic field in the collision zone. The 15-cm-long dipole magnet, which generates a 3-Tesla field, introduces an additional 0.45 Tesla-meter beam bending power. The bending power of the SPS and the LHC dipoles are 12.6 and 120 Tesla-meter, respectively. The small perturbation of the beam-particle trajectories will have

to be compensated by introducing the requisite correction magnets in the vicinity of the collision zone.

Commercially available instruments measure magnetic fields, of the strength of up to 13 Tesla, with an accuracy better than 10 ppm. The B -field-dependent effects, discussed in this paper, can thus be measured with high precision. It will be limited only by the calibration accuracy of the PSI beam parameters and by the theoretical calculation precision of the ionic energy levels and the scalar and tensor polarizabilities.

2. Configuration parameters

The specific choice of laser, delivering pulses of photons of a small energy band centered at a chosen value of ω_{lab} , the choice of the atomic transition, characterized by the energy difference between the ground state and excited state of the PSIs ω_{ion} , and the angle between the Fabry-Pérot cavity and the beam axis Θ determine the requisite β and γ_L of the ion beam which satisfy the resonance condition:

$$\omega_{\text{ion}} = (1 + \beta \cos \Theta) \gamma_L \omega_{\text{lab}}. \quad (22)$$

The Fourier-limited laser-photon bandwidth can be tuned to match, or be smaller than the γ_L spread of the PSI beam. The spectral and timing characteristics of the laser pulses and the ion beam optics will be optimized individually for each of the Gamma Factory research domains.

The length of the dipole magnet has to be larger than the longitudinal size of the PSI bunches, which for the present SPS and LHC beams are approximately 15 cm. The PSI bunch length, and the corresponding γ_L spread, can be reduced by the longitudinal cooling of the beam [32,33].

The propagation length of the excited Ca ions in the laboratory reference frame, at both the SPS and LHC energies, is below 1 cm. It is significantly smaller than the ion bunch length. In such a configuration both the ground state and excited level energy shifts have to be taken into account while calculating the energy flow of the emitted γ rays.

The Θ angle for all the considered GF applications ranges between 1° and 2.6° . The maximal Θ angle and the PSI bunch length determine the minimal aperture of the dipole magnet. One should note that in our geometry, displayed in Fig. 1, the polar angle of the incident radiation is $\theta_i = \pi - \Theta = \pi$, while in reality the angle will be slightly smaller, $\theta_i < \pi$. However, as discussed in Ref. [5], the effect of a small but nonzero angle Θ on the scattering process is negligible and hence is not considered in our calculations presented in the previous sections.

B. Gamma Factory applications

1. Resonance condition tuning

The energy differences between the excited $1s2p\ ^1P_1$ and the ground state $1s^2\ ^1S_0$ of He-like PSIs can be calculated with a precision better than 0.01% [49]. For the transition in He-like Ca ions considered in this paper, the calculated [49] and the measured [62,63] values agree within the present measurement errors, which are also below 0.01%. The γ_L of the SPS beams is presently calibrated with the precision of 0.02% [64]. However, the absolute beam-energy uncertainty increases to 0.1% [65] at the top LHC beam energy

and becomes significantly larger than the expected theoretical precision of the transition frequency. As a consequence, γ_L of the beam has to be tuned in a dedicated beam-energy scan to find its optimal value satisfying the resonance condition (22). For the canonical high-energy LHC beams, having the γ_L bandwidth of 2×10^{-4} , the scan has to be made in approximately 40 beam momentum setting steps covering $\pm 2\sigma$ beam momentum uncertainty interval, with the step size of 10^{-4} of its nominal value. Such a scan requires changing, in each step, the PSI beam optics over the whole ring circumference, which will be both time consuming and cumbersome.

The resonance-tuning procedure can be vastly simplified by replacing the γ_L scans with the magnetic field scan at a fixed γ_L value. For the LHC He-like Ca beam, the Stark shift of the excited energy level for $B_{\text{lab}} = 3$ T is of the order of 0.2% of its nominal value. It is thus two times larger than the initial beam-energy (γ_L) calibration uncertainty. By choosing the initial setting of the γ_L as the calculated mean value for its $B_{\text{lab}} = 0$ and $B_{\text{lab}} = 3$ T predictions, and by stepwise decreasing B_{lab} in the collision zone from the value of 3 Tesla down to zero, the peak in the production rate of the GF photons can be found and used for the fast and simple determination of the resonant value γ_L .

2. LHC beam-energy calibration

Once the resonance-finding step is finalized, the precision of the γ_L calibration reflects the present accuracy of calculations of the involved energy levels. As illustrated in Fig. 10 for Li-like Pb and He-like Ca, this results in an uncertainty in the order of 10^{-4} . This corresponds to a tenfold improvement in the beam-energy calibration precision of the LHC beams, becoming the most precise method of energy calibration of the high-energy hadronic beams.

Building on the results from the previous sections, we propose a complementary calibration method. For ions with scalar and tensor polarizabilities known with sufficient accuracy, the γ_L^2 dependence of the Stark shift can be exploited. The high-precision calibration procedure involves relative B -field-dependent adjustments of the laser photon energy $\omega_{\text{lab}}^{\text{res}}(B_{\text{lab}})$, maintaining the resonance condition with the $|^1P_1, M_v = \pm 1\rangle$ sublevel at each B -field strength setting. Using Eq. (15), we derive the expression

$$\gamma_{L,\text{calib}} = \frac{(1 + \cos \Theta) [\omega_{\text{lab}}^{\text{res}}(B_{\text{lab}}) - \omega_{\text{lab}}^{\text{res}}(B_{\text{lab}} = 0)]}{\left[\frac{1}{4} \alpha_2(^1P_1) - \frac{1}{2} \alpha_0(^1P_1) + \frac{1}{2} \alpha_0(^1S_0) \right] c^2 B_{\text{lab}}^2}, \quad (23)$$

where for high γ_L we set $v \approx c$ and hence $\beta \approx 1$. The calibrated $\gamma_{L,\text{calib}}$ is obtained as the average over all B -field strength settings. To eliminate the contribution of the scalar polarizabilities, one can take, for each B -field strength, the difference of the resonant laser photon energy for the $|^1P_1, M_v = 0\rangle$ and the $|^1P_1, M_v = \pm 1\rangle$ sublevels. In this case, only the tensor polarizability contributes and the expression simplifies to

$$\gamma_{L,\text{calib}} = \frac{(1 + \cos \Theta) [\omega_{\text{lab}}^{\text{res}}(M_v = \pm 1) - \omega_{\text{lab}}^{\text{res}}(M_v = 0)]}{\frac{3}{4} \alpha_2(^1P_1) c^2 B_{\text{lab}}^2}. \quad (24)$$

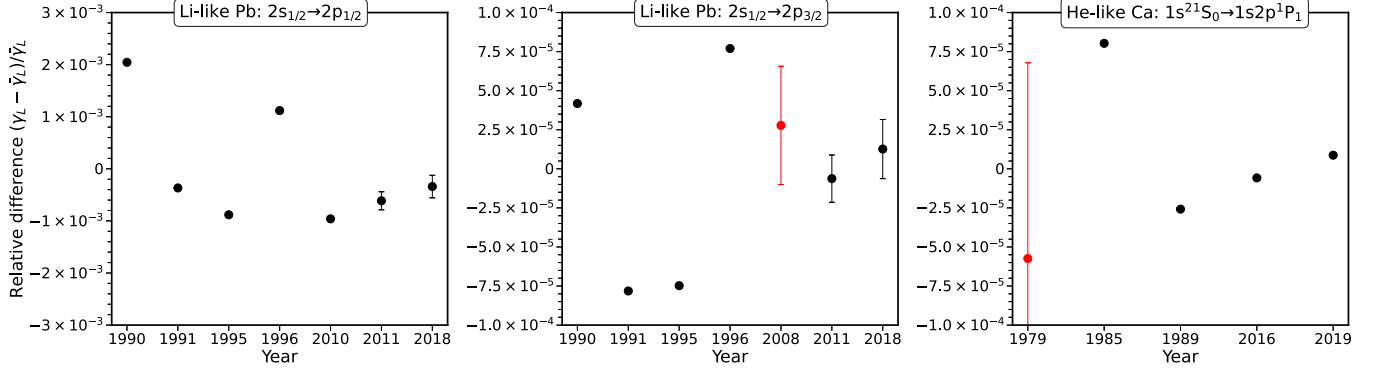


FIG. 10. The relative difference between the resonant value of the Lorentz factor γ_L , derived from calculated (black) and measured (red) transition energies, to their average value $\bar{\gamma}_L$. The transitions considered are $2s_{1/2} \rightarrow 2p_{1/2}$ and $2s_{1/2} \rightarrow 2p_{3/2}$ in Li-like Pb [66–73], and $1s^2 \ ^1S_0 \rightarrow 1s2p \ ^1P_1$ in He-like Ca [49,50,62,74,75]. The Lorentz factors are calculated for a fixed laser photon energy of 0.815 eV and an angle $\Theta = 1^\circ$.

The precision of this method is no longer constrained by the accuracy of the energy-level calculations but instead by the precision of the polarizability calculations. If calculations of transition energies or polarizabilities are sufficiently accurate, the calibration precision could be improved to 10^{-5} , achieving the same level of precision as the large electron-positron collider (LEP) electron/positron beams.

3. Measurement of polarizabilities of highly charged ions

The calibration procedure described earlier relies on the precise knowledge of the scalar and tensor polarizabilities of the involved states. However, as proposed in Ref. [20], if these polarizabilities are not known with sufficient accuracy, the method can be turned around to extract these values instead. The strong enhancement of the electric field in the ion frame and the resulting visible Stark shift opens the possibility to study polarizabilities for medium- Z highly charged ions.

4. Beam cooling and its experimental control

Cooling scheme. Laser beam cooling [31–33] is based on the selective excitation of only a fraction of ions stored in the PSI bunches. By tuning the spectral width of laser-photon pulses such that it is smaller than the PSI ion momentum smearing and detuning the resonant γ_L to lower values, only the fastest ions of the ion bunch can resonantly absorb photons. This cooling process involves a gradual increase of γ_L until the slowest ions are also excited. However, introducing a magnetic field in the ion-photon collision zone allows the replacement of the technically challenging γ_L ramp by a magnetic field strength ramp. In this scenario, γ_L can be chosen such that, for $B_{\text{lab}} = 3$ T, only the fastest ions are excited to the $|^1P_1, M_v = \pm 1\rangle$ sublevel. By decreasing the magnetic field strength, the resonant transition frequency of the $|^1P_1, M_v = \pm 1\rangle$ sublevel can be lowered to excite slower ions as well. One may note, however, that the magnetic-field-dependent cooling is by a factor of ~ 2 slower than the γ_L -ramp cooling, as only the $|^1P_1, M_v = \pm 1\rangle$ sublevels contribute to the cooling process (see Fig. 6).

Control of the momentum spread. The achieved degree of longitudinal beam cooling, observed as the reduction of the energy spread of the PSI beams, can be monitored by observ-

ing the appearance of the Zeeman splitting of the resonant γ_L values as shown in Fig. 11. In this plot the Zeeman splitting is shown for three values of the PSI beam momentum spread: for the nominal noncooled one, and for the beam with reduced Gaussian width of the beam-energy spread by a factor of 5 and 20. The characteristic double-peak structure invisible for the initial momentum spread of the LHC beams becomes visible as the cooling of the beam progresses. The measured shape of the resonant γ_L distribution for $B = 3$ T will determine the spectral distribution of the PSI beam at consecutive stages of the beam-cooling procedure.

5. GF photon beam polarization

Several domains of the GF experimental program require the precise polarization control of the generated photon

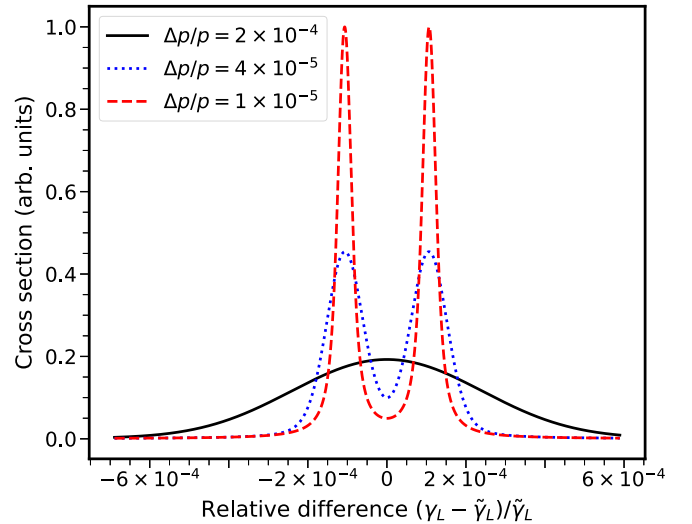


FIG. 11. The cross section of photons scattered in a finite solid angle in the y direction with an opening angle of 1 mrad in the laboratory frame as a function of the relative difference of the Lorentz factor γ_L with respect to $\bar{\gamma}_L$, corresponding to the center of the $M_v = \pm 1$ sublevel. Calculations were performed for different momentum spreads of the ion beam, showing how the Zeeman splitting of the $M_v = \pm 1$ sublevel becomes visible for a sufficiently cooled beam.

beams. Circular polarization, for instance, is crucial for the production of longitudinally polarized leptons [16] or potential atomic parity violation studies [40]. Meanwhile, linearly polarized γ rays can be used for studies of vacuum birefringence [17,23] and Delbrück scattering [20,76]. As discussed in Sec. VB 2, circularly polarized γ rays can be produced in collisions of circularly polarized laser photons with He-like ions due to perfect polarization transfer. We demonstrated that the external laboratory magnetic field has a strong impact on this transfer, see Figs. 8 and 9. For instance, the applied B_{lab} field can transform circular polarization into linear polarization, offering additional control over the polarization of scattered radiation. This opens up the possibility of tailoring the polarization of γ rays to meet specific experimental requirements on demand.

VII. CONCLUSION

We presented a theoretical study on resonant photon scattering by partially stripped ions exposed to external electric and magnetic fields. Special emphasis was placed on the geometry in which laser photons collide with counterpropagating relativistic ion beams. We assume, moreover, that a dipole magnet is placed in the collision zone, leading to strong electric and magnetic fields in the ion rest frame as a result of the Lorentz transformation. For this experimental setup, we investigated the effect of external fields both on the scattering rate and on the angular distribution and polarization of outgoing photons.

The developed theoretical approach was applied to explore in detail the resonant photon scattering by He-like Ca^{18+} ions for magnetic field strengths and collision energies typical

for the GF project at CERN. Detailed calculations indicate a strong impact of the laboratory magnetic field on the rate of detected scattered photons as well as on their polarization which arises due to Zeeman and Stark shifts of the ionic states. In particular, we showed that the rate of detected photons decreases with the growth of B_{lab} , depending on the energy spread off the ion beam. Moreover, we demonstrated that an external magnetic field can be utilized to convert initially circularly polarized photons into linearly polarized ones.

Based on the result of our calculations, we argue that the magnetic field sensitivity of the resonant scattering process can be of importance for various aspects of the GF project. For example, the B_{lab} dependence of the detection rate and the visible Stark splitting of the excited sublevels can be used to facilitate the resonance condition tuning and offers a new beam cooling scheme. Furthermore, the laboratory magnetic field provides precise and flexible polarization control of the outgoing photon beam.

ACKNOWLEDGMENTS

J.R. and A.S. acknowledge funding by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strategy – EXC-2123 QuantumFrontiers – 390837967.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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