

High-order above-threshold ionization by squeezed coherent light

D. Habibović¹ and D. B. Milošević^{1,2,*}

¹*Faculty of Science, University of Sarajevo, Zmaja od Bosne 35, 71000 Sarajevo, Bosnia and Herzegovina*

²*Academy of Sciences and Arts of Bosnia and Herzegovina, Bistrik 7, 71000 Sarajevo, Bosnia and Herzegovina*



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Squeezed coherent quantum light is described by the complex squeeze parameter $\zeta = se^{i\vartheta}$ and the coherent-state displacement parameter $\beta = |\beta|e^{i\phi_\beta}$. Our study of strong-field ionization [D. Habibović and D. B. Milošević, *Phys. Rev. A* **112**, L051103 (2025)], which focused on bright squeezed vacuum quantum light with $\beta = 0$ and $s \gg 1$, is extended to ionization by squeezed coherent light. It is shown that the amplitudes s and $|\beta|$ strongly influence the ionization process, whereas phases ϑ and ϕ_β play a less significant role. The present work provides a unified treatment of coherent, squeezed vacuum, and squeezed coherent light within a single theoretical framework. The quantum nature of the driving field is incorporated by averaging the strong-field ionization probability over the Husimi-Kano phase-space distribution. Using neon as a representative target, we analyze how the parameters characterizing squeezed coherent light modify photoelectron spectra and intensity-dependent enhancements. We show that nonclassical photon statistics can lead to a pronounced steepening of the rescattering plateau and, at low mean intensities, to an effective suppression of the nominal high-energy cutoff. These effects originate from the reduced weight of high-intensity field components inherent to squeezed states and have no analog in strong-field ionization driven by a coherent light. Our results highlight the potential for quantum-statistical control of strong-field ionization and provide a foundation for exploring laser-matter interactions driven by quantum light.

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I. INTRODUCTION

Significant advancements in quantum optics and the discovery of nonclassical states of light, such as squeezed light [1], have led to many applications [2], including quantum metrology [3] and quantum information [4,5]. In parallel, achievements in strong-field physics and attoscience have been recognized by the Nobel Prize [6–8]. Two laser-induced processes deserve particular attention: high-order harmonic generation (HHG) and high-order above-threshold ionization (HATI). In these processes, high-energy photons (in HHG) and electrons (in HATI) are emitted when a medium is exposed to intense laser fields [9–15]. With recent technological advances, quantum light sources such as bright squeezed vacuum (BSV) have become intense enough to drive such strong-field processes, stimulating an interplay of quantum optics and strong-field physics [16–18] and attoscience [19,20]. BSV-light-induced HHG in solids [21,22] and HATI from metal tips [23,24] have recently been observed.

Ionization by quantized laser field was analyzed in Refs. [25–27], while ionization by squeezed quantum light was considered, for example, in Refs. [28–30]. Ionization in the context of recently available strong quantum light was

investigated theoretically in Refs. [31–38]. In the present paper, we consider HATI driven by squeezed coherent light. Above-threshold ionization is a process in which an electron is released from its parent atom or molecule by absorbing more laser photons than required for ionization [39]. If this electron, driven by intense laser field, returns and rescatters off the parent ion, gaining additional energy, a plateau with an approximately energy-independent electron yield appears in the photoelectron spectrum [40]. This plateau terminates with an abrupt cutoff, which, for a linearly polarized laser field of intensity I_0 and angular frequency ω , occurs at the energy $10U_{p0}$, where $U_{p0} = I_0/(4\omega^2)$ is the ponderomotive energy [10]. Within this plateau, groups of enhanced narrow peaks, the energy positions of which do not shift with changes in the laser-field intensity, have been observed in experiments with inert gases [41,42]. For more details about these resonantlike enhancements in HATI, see review articles [10,43] and Refs. [44–47].

In our recent Letter [35], we formulated a theory of HATI by quantum light and applied it to investigate the intensity-dependent enhancements described above. We presented results for HATI by bright squeezed vacuum. In the present paper, we extend this analysis to squeezed coherent states, which encompass both coherent and squeezed vacuum states as special cases. We analyze the spectra of photoelectrons liberated by this squeezed coherent light and investigate how parameters characterizing the quantum state of the driving field affect different structures in the spectra and modify the intensity-dependent enhancements.

This paper is organized as follows. In Sec. II, we present the theoretical framework for high-order above-threshold

*Contact author: milo@bih.net.ba

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ionization driven by squeezed coherent light. Section III is devoted to numerical results and their discussion, and our conclusions are summarized in Sec. IV. In the Appendices we analyzed the Husimi-Kano Q function and the associated integrals, as well as their dependence on the parameters of squeezed coherent light. Atomic units are used throughout unless otherwise specified.

II. THEORY

We suppose that our atomic system is initially, at the time t_0 , in the ground state $|\psi_a(t_0)\rangle = |\psi_a\rangle e^{iI_p t_0}$, where I_p is the ionization potential and $|\psi_a\rangle$ is an eigenstate of the atomic Hamiltonian $H_a = -\nabla^2/2 + V(\mathbf{r})$, $H_a|\psi_a\rangle = -I_p|\psi_a\rangle$, with $V(\mathbf{r})$ the atomic potential. At the time t_0 the atom is exposed to the quantum field described by the squeezed coherent state [48,49]

$$|\beta, \zeta\rangle = \hat{D}(\beta)\hat{S}(\zeta)|0\rangle = e^{\beta\hat{a}^\dagger - \beta^*\hat{a}} e^{-(\zeta\hat{a}^{\dagger 2} - \zeta^*\hat{a}^2)/2}|0\rangle, \quad (1)$$

with the vacuum state $|0\rangle$ and the photon creation (annihilation) operator $\hat{a}^\dagger$ ($\hat{a}$), and where $\hat{S}(\zeta)$, with $\zeta = s e^{i\vartheta}$ and $s = |\zeta|$, is the squeeze operator and $\hat{D}(\beta)$, with $\beta = |\beta| e^{i\phi_\beta}$, is the coherent-state displacement operator (ϑ and ϕ_β are real phases). For $\beta = 0$ we have squeezed vacuum light, while for $\phi_\beta = (\vartheta + \pi)/2$ ($\phi_\beta = \vartheta/2$) our light is phase (amplitude) squeezed [49]. The density operator of the pure initial state is

$$\hat{\rho}(t_0) = \hat{\rho}_F \otimes |\psi_a(t_0)\rangle\langle\psi_a(t_0)|, \quad \hat{\rho}_F = |\beta, \zeta\rangle\langle\beta, \zeta|. \quad (2)$$

In the interaction picture the total Hamiltonian of our system of atom and single-mode quantum field is

$$\hat{H}(t) = H_a + \mathbf{r} \cdot \hat{\mathbf{E}}(t), \quad \hat{\mathbf{E}}(t) = i\epsilon\hat{\mathbf{e}}(\hat{a}e^{-i\omega t} - \hat{a}^\dagger e^{i\omega t}), \quad (3)$$

where $\hat{\mathbf{e}}$ is the unit linear polarization vector and $\epsilon = \sqrt{2\pi\omega/\mathcal{V}}$, with $\mathcal{V}$ the quantization volume. The time-dependent Schrödinger equation for the total state $|\Psi(t)\rangle$, with $i\partial|\Psi(t)\rangle/\partial t = \hat{H}(t)|\Psi(t)\rangle$, is transformed using the unitary operator $\hat{D}(\alpha)$ into equation $i\partial|\Psi_\alpha(t)\rangle/\partial t = \hat{H}_\alpha(t)|\Psi_\alpha(t)\rangle$, with $|\Psi_\alpha(t)\rangle = \hat{D}^\dagger(\alpha)|\Psi(t)\rangle$ and $\hat{H}_\alpha(t) = \hat{D}^\dagger(\alpha)\hat{H}(t)\hat{D}(\alpha)$, and

$$\hat{H}_\alpha(t) = H_a + \mathbf{r} \cdot [\mathbf{E}_\alpha(t) + \hat{\mathbf{E}}(t)], \quad \mathbf{E}_\alpha(t) = \langle\alpha|\hat{\mathbf{E}}(t)|\alpha\rangle. \quad (4)$$

Using the notation $E_{\alpha 0} = \mathcal{E}_\alpha e^{i\phi_\alpha} = 2\epsilon\alpha$, with $\mathcal{E}_\alpha = |E_{\alpha 0}|$, for the mean field we get $\mathbf{E}_\alpha(t) = (\text{Re } E_{\alpha 0} \sin \omega t - \text{Im } E_{\alpha 0} \cos \omega t)\hat{\mathbf{e}}$, and

$$\mathbf{E}_\alpha(t) = \mathcal{E}_\alpha \sin(\omega t - \phi_\alpha)\hat{\mathbf{e}}. \quad (5)$$

Important is that the dependence on the real phase ϕ_α is only in the term $\omega t - \phi_\alpha$ so that, by redefining the time t , this phase can be eliminated from the transition probability, which considerably simplifies the calculations.

Introducing the evolution operators $U_\alpha(t, t_0)$ and $\hat{U}_\alpha(t, t_0)$ which correspond to the Hamiltonians $H_\alpha(t) = H_a + \mathbf{r} \cdot \mathbf{E}_\alpha(t)$ and $\hat{H}_\alpha(t)$, respectively, we obtain [35]

$$\hat{U}_\alpha(t, t_0) = U_\alpha(t, t_0)\mathcal{T}e^{-i\int_{t_0}^t d\tau U_\alpha(t_0, \tau)\mathbf{r} \cdot \hat{\mathbf{E}}(\tau)U_\alpha(\tau, t_0)}, \quad (6)$$

with $\mathcal{T}$ the time-ordering operator. Since the fluctuation of the number of photons in the coherent state [caused by the operator $\mathbf{r} \cdot \hat{\mathbf{E}}(\tau)$] can be neglected, we approximate $\hat{U}_\alpha(t, t_0)$

with $U_\alpha(t, t_0)$. In this case the quantum light can be traced out and we can use the Husimi-Kano Q function [50,51] which is the coherent-state expectation value of the density operator, $Q(\alpha) = \langle\alpha|\hat{\rho}_F|\alpha\rangle/\pi = |\langle\alpha|\beta, \zeta\rangle|^2/\pi$ (see Appendix A). For a coherent state $|\alpha\rangle$, the probability of detection of an electron with momentum $\mathbf{p}$ and energy $E_p = p^2/2$, with $p = |\mathbf{p}|$, is given by [12,52] $|M_\alpha(\mathbf{p})|^2 d^3\mathbf{p}/(d\Omega_p dE_p) = p|M_\alpha(\mathbf{p})|^2$ where $d\Omega_p$ is the solid-angle element and $M_\alpha(\mathbf{p})$ is the ionization probability amplitude. For the statistical mixture of the coherent states, for the ionization probability with emission of an electron with the momentum $\mathbf{p}$ we obtain [35]

$$W(\mathbf{p}) = p \int d^2\alpha Q(\alpha) |M_\alpha(\mathbf{p})|^2, \quad (7)$$

where the ionization probability amplitude is [10–12,53]

$$M_\alpha(\mathbf{p}) = \lim_{t \rightarrow \infty, t_0 \rightarrow -\infty} \langle\psi_p(t)|U_\alpha(t, t_0)|\psi_a(t_0)\rangle, \quad (8)$$

with $|\psi_p(t)\rangle = |\mathbf{p}\rangle e^{-iE_p t}$ ($|\mathbf{p}\rangle$ is the plane-wave ket vector). The averaging over the Husimi-Kano Q function has previously been applied to strong-field processes. This framework was employed for analysis of HHG [16] and strong-field ionization [34,35] driven by quantum light. Our work builds upon this general perspective and extends it to HATI spectra by squeezed coherent states.

Our averaged quantum field (5) is periodic with the period $T = 2\pi/\omega$, and the ionization probability amplitude, which does not depend on the phase ϕ_α , can be written in the form [11]

$$M_\alpha(\mathbf{p}) = -2\pi i \sum_n \delta(E_p + U_{p\alpha} + I_p - n\omega) T_{p\mathcal{E}_\alpha}(n), \quad (9)$$

where the δ function expresses the energy conservation, $E_p = n\omega - U_{p\alpha} - I_p$, with n the number of absorbed photons and $U_{p\alpha} = \mathcal{E}_\alpha^2/(4\omega^2)$ the ponderomotive energy which corresponds to the field with the amplitude $\mathcal{E}_\alpha$. The T -matrix element $T_{p\mathcal{E}_\alpha}(n)$ we calculate using the improved strong-field approximation [10,11,53].

The integral over the coherent-state amplitude can be presented as $\int d^2\alpha = \int d\text{Re } \alpha d\text{Im } \alpha = \int_0^\infty d\mathcal{E}_\alpha \mathcal{E}_\alpha \int_0^{2\pi} d\phi_\alpha / (4\epsilon^2)$. The intensity of the squeezed coherent light with the averaged number of photons $\bar{n}$ in the state is [49]

$$\mathcal{E}_0^2 = 8\pi\bar{n}\omega/\mathcal{V} = 4\epsilon^2\bar{n}, \quad \bar{n} = \sinh^2 s + |\beta|^2. \quad (10)$$

It should be noticed that for large s the number $\bar{n}$ is also large. In order to hold the intensity $\mathcal{E}_0^2$ fixed the quantization volume $\mathcal{V}$ should also be large, which is true for a typical gas-laser cavity. For example, for $\mathcal{E}_0^2 = 10^{14}$ W/cm² and wavelength 800 nm we get $\mathcal{V} \approx 500\bar{n}$ a.u. Since the T -matrix element does not depend on the phase ϕ_α , the integral over $d\phi_\alpha$ in Eq. (7) gives the factor

$$\Phi(\mathcal{E}_\alpha, \beta, \zeta) = \int_0^{2\pi} d\phi_\alpha Q(\alpha). \quad (11)$$

Using this and Eq. (7) for the differential ionization rate we obtain

$$w(\mathbf{p}) = 2\pi p \sum_n \int_0^\infty d\mathcal{E}_\alpha \mathcal{E}_\alpha \delta(E_p + U_{p\alpha} + I_p - n\omega) \times |T_{p\mathcal{E}_\alpha}(n)|^2 \Phi(\mathcal{E}_\alpha, \beta, \zeta) / (4\epsilon^2), \quad (12)$$

where the integral $\Phi(\mathcal{E}_\alpha, \beta, \zeta)$ is analyzed in Appendix B. The δ function cancels the integral over $d\mathcal{E}_\alpha$, so that, analogously as in Ref. [35], we get

$$w(\mathbf{p}) = \frac{4\pi\omega^2 p}{\mathcal{E}_0^2} \bar{n} \sum_{n \geq n_0} \Phi(\mathcal{E}_\alpha, \beta, \zeta) |T_{\mathbf{p}\mathcal{E}_\alpha}(n)|_{\mathcal{E}_\alpha=\mathcal{E}_{\alpha,0}}^2, \quad (13)$$

where $n_0 = \text{int}[(E_p + I_p)/\omega] + 1$ and $\mathcal{E}_{\alpha,0} = 2\omega\sqrt{n\omega - E_p - I_p}$. Therefore, the differential ionization rate for HATI by squeezed coherent light is expressed as a sum over the number of absorbed photons of the product of the differential rate for ionization by coherent laser light with intensity $\mathcal{E}_{\alpha,0}^2$, $w_{\mathcal{E}_{\alpha,0}}(\mathbf{p}) = 2\pi p |T_{\mathbf{p}\mathcal{E}_{\alpha,0}}(n)|^2$, and the factor $\bar{n}\Phi(\mathcal{E}_\alpha, \beta, \zeta)/(2U_{p0})$, with $U_{p0} = I_0/(4\omega^2)$, $I_0 = \mathcal{E}_0^2$.

III. NUMERICAL RESULTS

We illustrate our findings using the Ne atom, which has a p ground state and an ionization potential of $I_p = 21.56$ eV. The squeezed coherent state (the so-called ideal squeezed state [48]) $|\beta, \zeta\rangle$ is characterized by the complex coherent-state amplitude $\beta = |\beta|e^{i\phi_\beta}$ and the complex squeeze parameter $\zeta = se^{i\vartheta}$, where $s > 0$ and $0 \leq \vartheta \leq 2\pi$. Our numerical results are presented for the emission in the driving-field polarization direction, i.e., in the direction defined by the unit vector $\hat{e}$.

A. Squeezed vacuum light

We begin our analysis by considering squeezed vacuum light, for which $\beta = 0$. In this case, the differential ionization rate (13) does not depend on the phase ϑ . The main goal of this subsection is to examine how the squeezing parameter s influences the HATI spectrum.

In Ref. [35], we analyzed HATI by BSV quantum light for which $s \gg 1$. There, the cutoff position was determined by the saturation intensity I_{sat} ; that is, the sharp cutoff appears at $10U_{p\alpha}^{\text{sat}}$, where $U_{p\alpha}^{\text{sat}} = I_{\text{sat}}/(4\omega^2)$ is the ponderomotive energy corresponding to the saturation intensity. In Ref. [35], the saturation intensity was determined using the barrier-suppression ionization model, which for Ne atoms yields $I_{\text{sat}} = 8.64 \times 10^{14}$ W/cm².

We first analyze how the photoelectron spectrum depends on the mean intensity of the BSV light. Since the mean intensity, parameter s , and volume $\mathcal{V}$ are connected by Eq. (10), at fixed $\mathcal{V}$, varying I_0 changes s . However, this change does not influence our results as long as we are in the BSV regime ($s \gg 1$). In Fig. 1, we present the logarithm of the differential ionization rate for Ne atoms exposed to BSV light as a function of the photoelectron energy, expressed in units of the ponderomotive energy $U_{p0} = I_0/(4\omega^2)$. The laser wavelength is 800 nm, and the mean intensities considered are $I_0 = 10^{14}$ W/cm² (black line), $I_0/2$ (red line), $I_0/5$ (green line), and $I_0/10$ (blue line). For all mean intensities, the photoelectron energy is expressed in units of U_{p0} . Consequently, the cutoff position should be close to $10U_{p\alpha}^{\text{sat}} = 86.4U_{p0}$. This expectation is well satisfied for relatively high mean intensities ($\gtrsim 5 \times 10^{13}$ W/cm²). However, as the mean intensity decreases, the plateau becomes increasingly inclined. Although a sharp cutoff still exists, the differential ionization rate before the cutoff

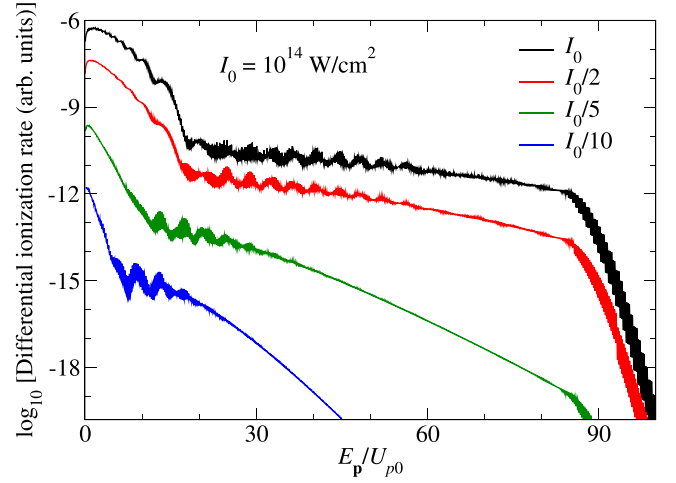


FIG. 1. Logarithm of the differential ionization rate for the Ne atom exposed to quantum BSV light as a function of the photoelectron energy, expressed in units of the ponderomotive energy U_{p0} . The laser wavelength is 800 nm, and the mean intensities considered are $I_0 = 10^{14}$ W/cm² (black line), $I_0/2$ (red line), $I_0/5$ (green line), and $I_0/10$ (blue line). The photoelectrons are emitted in the driving-field polarization direction.

can be strongly suppressed. In such cases, the cutoff may not be observable at all (see the blue line in Fig. 1).

This behavior is a clear consequence of the quantum nature of the driving light, which is encoded in the product of the averaged number of photons $\bar{n}$ and the integral $\Phi(\mathcal{E}_\alpha, \beta, \zeta)$. For BSV light, this product is given by Eq. (B8). In Fig. 2 we present the logarithm of the partial differential ionization rate $w_\alpha(\mathbf{p})$ [defined as the summand in Eq. (13)] as a function of the intensity $I_\alpha = \mathcal{E}_\alpha^2$ for the photoelectron energies indicated in the legend. The mean intensities considered are I_0 [Fig. 2(a)] and $I_0/10$ [Fig. 2(b)]. As expected, higher values of I_α correspond to higher photoelectron energies. As the mean intensity decreases, the contributions to the differential ionization rate become progressively steeper as functions of I_α , a trend that persists across the entire rescattering part of the photoelectron spectrum. This behavior can be understood by examining the product $\bar{n}\Phi$, the logarithm of which is shown by the blue solid lines in Fig. 2. Its slope is clearly negative and becomes steeper as the mean intensity decreases. In particular, the slope of the product $\bar{n}\Phi$ matches the slope of the partial rates $w_\alpha(\mathbf{p})$. Consequently, the spectrum becomes increasingly steep at lower mean intensities. Importantly, this behavior is a direct manifestation of the quantum nature of the driving light, encoded in the structure of the corresponding Husimi-Kano distribution. The quantum character of the light is therefore more pronounced at mean intensities lower than those typically used for coherent light.

We now analyze how the spectrum obtained with squeezed vacuum light depends on the value of the squeezing parameter s . Changing s for fixed I_0 implies that volume $\mathcal{V}$ also changes. In this case, in an experiment we measure the mean intensity and the experimental conditions (and therefore the volume $\mathcal{V}$) are adjusted to achieve a particular value of s . In Fig. 3, we show the results analogous to those of Fig. 1 but for the mean intensity fixed at 2×10^{13} W/cm² and for the values of the

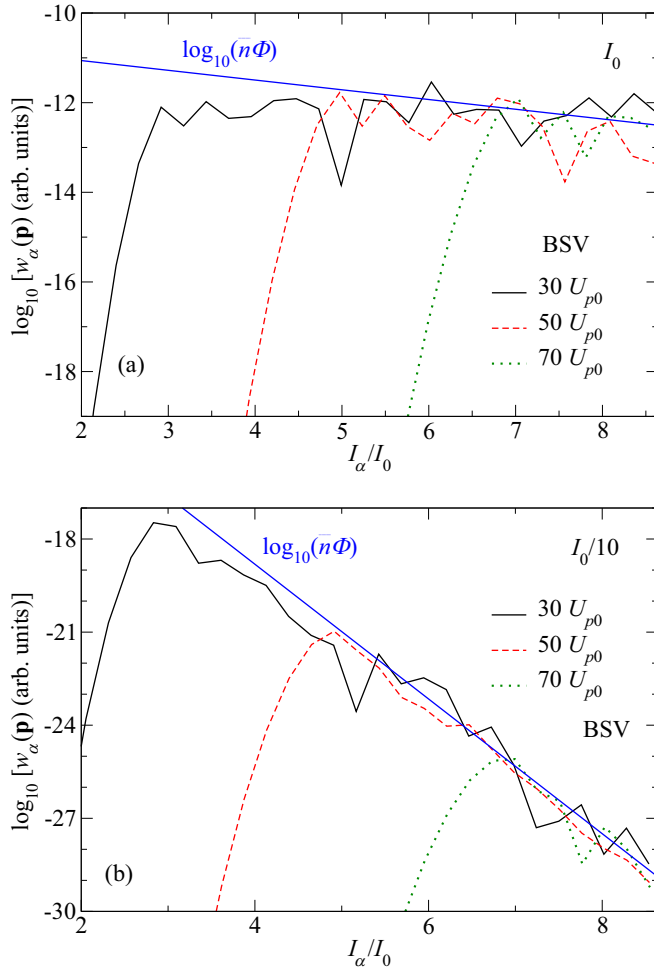


FIG. 2. Logarithm of the partial differential ionization rate $w_\alpha(\mathbf{p})$ for the Ne atom exposed to quantum BSV light as a function of the intensity $I_\alpha = \mathcal{E}_\alpha^2$, for the photoelectron energy as indicated in the legend. The mean intensities are $I_0 = 10^{14}$ W/cm² (a) and $I_0/10$ (b). The logarithm of the product $\bar{n}\Phi$ is presented by the blue solid lines. The photoelectrons are emitted in the driving-field polarization direction.

squeezing parameter indicated in the legend. The inclination of the photoelectron spectrum becomes less pronounced as the squeezing parameter decreases [cf. the cyan (BSV quantum light with $s \gg 1$), magenta ($s = 1.5$), green ($s = 1$), and red ($s = 0.5$) curves in Fig. 3]. This effect is particularly prominent for $s \leq 1$.

The dependence of the photoelectron spectrum on the squeezing parameter s can be traced to the Husimi-Kano Q function, i.e., to the dependence of the product $\bar{n}\Phi$ on the squeezing parameter s and on the intensity I_α . In Fig. 4, we show the logarithm of the partial differential ionization rate $w_\alpha(\mathbf{p})$ for Ne atoms exposed to squeezed vacuum light as a function of $I_\alpha = \mathcal{E}_\alpha^2$ for a photoelectron energy $E_p = 150U_{p0}$ and a mean intensity of $I_0 = 2 \times 10^{13}$ W/cm². The results are presented for the values of the squeezing parameter s indicated in the legend. For large values of s ($s \gtrsim 2$), the differential ionization rate exhibits behavior characteristic of BSV light, showing a steep dependence on I_α . However, as s decreases, the slope becomes less pronounced. Importantly, the change

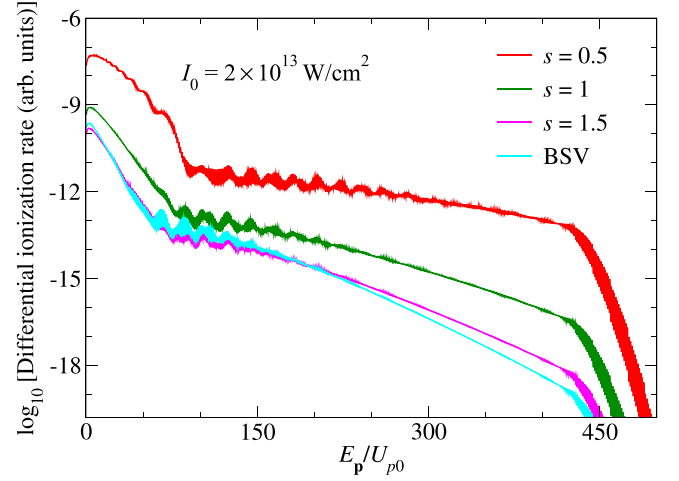


FIG. 3. Logarithm of the differential ionization rate for the Ne atom exposed to squeezed vacuum light as a function of the photoelectron energy, expressed in units of the ponderomotive energy U_{p0} . The laser wavelength is 800 nm, and the mean intensity is $I_0 = 2 \times 10^{13}$ W/cm². The values of the squeezing parameter s are indicated in the legend. The photoelectrons are emitted in the driving-field polarization direction.

in slope is not a linear function of s (cf. the slopes of the red solid and green dashed lines, and the magenta dotted and cyan dash-dotted lines in Fig. 4). This demonstrates that the quantum nature of the light is more pronounced at low mean intensities, and under these conditions the dependence of the differential ionization rate on the squeezing parameter s becomes especially significant.

Finally, we analyze how the photoelectron spectrum behaves over the full range of final photoelectron energies. In

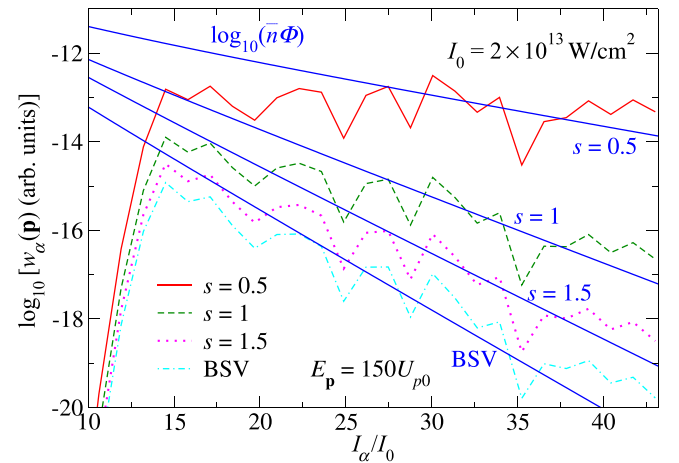


FIG. 4. Logarithm of the partial differential ionization rate $w_\alpha(\mathbf{p})$ for the Ne atom exposed to squeezed vacuum light as a function of the intensity $I_\alpha = \mathcal{E}_\alpha^2$, for the photoelectron energy $E_p = 150U_{p0}$ and a mean intensity of $I_0 = 2 \times 10^{13}$ W/cm². Results are shown for $s = 0.5$ (red solid line), $s = 1$ (green dashed line), $s = 1.5$ (magenta dotted line), and BSV quantum light $s \gg 1$ (cyan dash-dotted line). The logarithm of the product $\bar{n}\Phi$ is presented by the blue solid lines. The photoelectrons are emitted in the driving-field polarization direction.

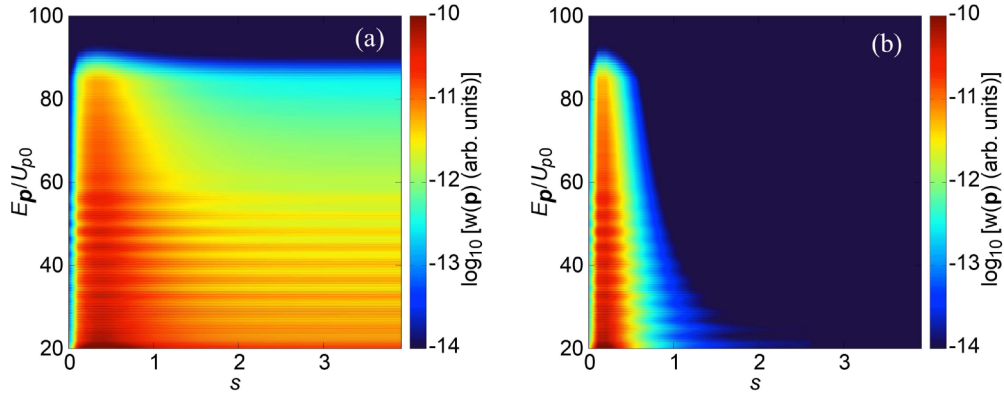


FIG. 5. Logarithm of the differential ionization rate for the Ne atom exposed to squeezed vacuum light as a function of the photoelectron energy, expressed in units of the ponderomotive energy U_{p0} , and of the squeezing parameter s . The laser wavelength is 800 nm, and the mean intensities are $I_0 = 10^{14}$ W/cm² (a) and $I_0/5$ (b). The photoelectrons are emitted in the driving-field polarization direction.

Fig. 5, we present the logarithm of the differential ionization rate for Ne atoms exposed to squeezed vacuum light as a function of the photoelectron energy and the squeezing parameter s . The laser wavelength is 800 nm, and the mean intensities are 10^{14} W/cm² [Fig. 5(a)] and 2×10^{13} W/cm² [Fig. 5(b)]. In both cases, the differential ionization rate exhibits a clear dependence on the squeezing parameter. However, this dependence becomes much more pronounced at the lower mean intensity [cf. Figs. 5(a) and 5(b)]. For a mean intensity of 10^{14} W/cm², the differential ionization rate remains substantial for all values of s , and the intensity-dependent enhancements are visible across the spectrum [see Fig. 5(a)]. In contrast, at the lower mean intensity, the differential ionization rate decreases rapidly with increasing s [see Fig. 5(b)]. In this case, intensity-dependent enhancements appear only for small values of the squeezing parameter. It is interesting to note that for $s \rightarrow 0$ the differential ionization rate becomes strongly suppressed regardless of the value of the mean intensity. This happens because in this limit $\bar{n} \rightarrow 0$, thus suppressing the differential ionization rate [see Eq. (13)].

B. Squeezed coherent light

Having examined the special case of squeezed vacuum light ($\beta = 0$), we now turn to the general situation of squeezed coherent light. In this case, in addition to the real squeezing parameters s and ϑ , two further parameters appear: the displacement amplitude $|\beta|$ and its phase ϕ_β . Our goal is to analyze how the photoelectron spectrum depends on these additional degrees of freedom. Among them, the most significant influence arises from the amplitude $|\beta|$, or equivalently from the associated intensity $I_\beta = \mathcal{E}_\beta^2$. Using Eq. (10), the averaged photon number can be expressed as $\bar{n} = \sinh^2 s I_0 / (I_0 - I_\beta)$ which immediately implies the constraint $I_\beta < I_0$. Since $\bar{n} \propto \mathcal{V} I_0$ we have $\mathcal{V} \propto \sinh^2 s / (I_0 - I_\beta)$ so that for fixed I_0 and s changing I_β implicitly changes $\mathcal{V}$. To isolate the role of I_β , we fix the squeezing strength to $s = 1$ and set the squeezing angle to $\vartheta = 0$, corresponding to $\zeta = 1$. We also choose the displacement phase $\phi_\beta = 0$. Figure 6 displays the logarithm of the differential ionization rate for Ne atoms exposed to squeezed coherent light as a function of the photoelectron energy, expressed in units of the ponderomotive

energy U_{p0} . The values of I_β are indicated in the legend, while the mean intensity is held constant at 2×10^{13} W/cm².

As I_β increases, the slope of the photoelectron spectrum becomes steeper, with the effect becoming particularly pronounced as I_β approaches I_0 . This behavior is qualitatively similar to the dependence on the squeezing parameter s : in both cases, increasing the corresponding parameter enhances the rate at which the differential ionization rate decreases with intensity.

As in the case of squeezed vacuum light, the behavior of the photoelectron spectrum can again be traced to the product $\bar{n}\Phi$. Figure 7 shows the logarithm of the partial differential ionization rate $w_\alpha(\mathbf{p})$ for Ne atoms irradiated with squeezed coherent light as a function of the intensity $I_\alpha = \mathcal{E}_\alpha^2$ evaluated at a photoelectron energy of $E_p = 250U_{p0}$ and for a mean intensity of $I_0 = 2 \times 10^{13}$ W/cm². The curves correspond to the values of I_β indicated in the legend. The differential

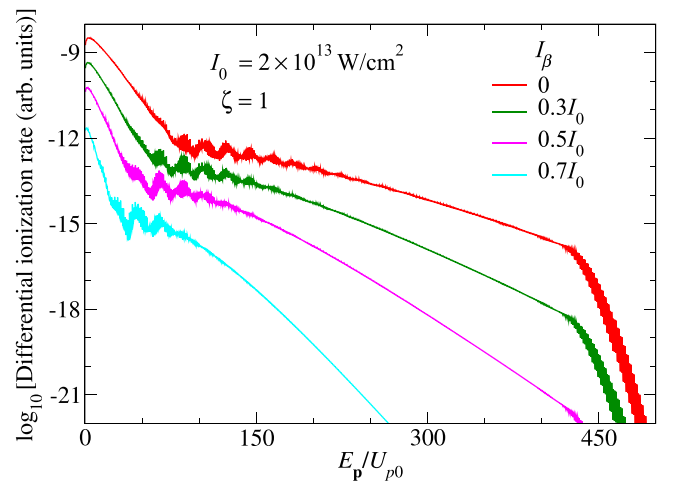


FIG. 6. Logarithm of the differential ionization rate for the Ne atom exposed to squeezed coherent light as a function of the photoelectron energy, expressed in units of the ponderomotive energy U_{p0} . The laser wavelength is 800 nm, and the mean intensity is $I_0 = 2 \times 10^{13}$ W/cm². The squeezing parameter is fixed at $\zeta = 1$, and the values of the coherent displacement are indicated in the legend. The photoelectrons are emitted in the driving-field polarization direction.

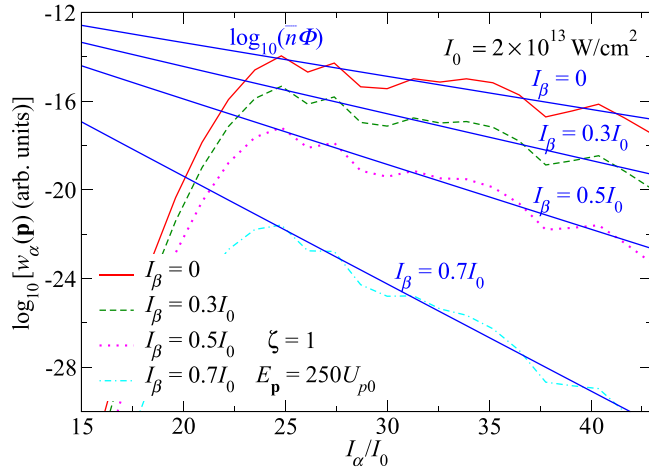


FIG. 7. Logarithm of the partial differential ionization rate $w_\alpha(\mathbf{p})$ for the Ne atom exposed to squeezed coherent light as a function of the intensity $I_\alpha = \mathcal{E}_\alpha^2$, for the photoelectron energy $E_p = 250U_{p0}$ and a mean intensity of $I_0 = 2 \times 10^{13} \text{ W/cm}^2$. Results are shown for $I_\beta = 0$ (red solid line), $I_\beta = 0.3I_0$ (green dashed line), $I_\beta = 0.5I_0$ (magenta dotted line), and $I_\beta = 0.7I_0$ (cyan dash-dotted line). The squeezing parameter is fixed at $\zeta = 1$. The logarithm of the product $\bar{n}\Phi$ is presented by the blue solid lines. The photoelectrons are emitted in the driving-field polarization direction.

ionization rate decreases steeply with increasing I_α , and the slope of this dependence becomes steeper as I_β increases. The effect is particularly pronounced as I_β approaches I_0 .

The influence of the remaining parameters is comparatively weaker. Variations of the squeezing angle ϑ and the displacement phase ϕ_β modify the slope of the spectrum, making it slightly more or less pronounced, but do not alter the overall qualitative behavior. Additional details on these dependencies are provided in Appendix C.

In conclusion, the change in the slope of the photoelectron spectrum induced by squeezed coherent state $|\beta, \zeta\rangle$, with $\beta = |\beta|e^{i\phi_\beta}$ and $\zeta = se^{i\vartheta}$, depends sensitively on the parameters characterizing this quantum state as well as on the mean intensity. The most important parameters are $|\beta|$ and s , the increase of which leads to a more rapid decrease of the differential ionization rate as a function of the intensity I_α . This effect becomes particularly pronounced at low mean intensities I_0 , where the sharp cutoff associated with saturation can be obscured by the quantum-light-induced decrease of the differential ionization rate.

IV. CONCLUSIONS

In this work, we have developed a quantum strong-field approximation framework for high-order above-threshold ionization driven by squeezed coherent light, thereby extending our earlier analysis of HATI induced by bright squeezed vacuum light. The formulation treats coherent, squeezed vacuum, and squeezed coherent states within a unified approach by averaging the strong-field ionization probability over the Husimi-Kano distribution of the driving field. This enables a systematic investigation of how nonclassical photon statistics modify strong-field ionization beyond the standard description based on classical or coherent laser fields.

Our numerical results demonstrate that the photoelectron spectra are strongly influenced by the quantum state of the driving light, particularly at low mean intensities. We have shown that both squeezing and coherent displacement lead to a pronounced steepening of the rescattering plateau and, in some cases, to an effective masking of the nominal HATI cutoff and of the intensity-dependent enhancements. This behavior originates from the suppression of high-intensity field components in the underlying phase-space distribution, which reduces the weight of the contributions responsible for generating high-energy electrons. This mechanism is distinct from saturation or rescattering dynamics in a given laser field. The dependence of the spectrum on the squeezing parameter and on the coherent contribution thus provides a clear and direct signature of the quantum nature of the driving field. Finally, we mention that a classical stochastic field with the same intensity distribution as our quantum light produces similar spectra within the strong-field approximation [37].

These findings establish squeezed coherent light as a versatile tool for controlling strong-field ionization through quantum-statistical properties rather than through classical pulse shaping, polarization control, or intensity scaling. Our results indicate that quantum-optical effects become increasingly important in regimes where classical strong-field intuition would predict well-defined plateaus and cutoffs. This opens perspectives for tailoring photoelectron spectra using nonclassical light sources and for exploring the interplay between strong-field physics and quantum optics. The present framework can be extended to other strong-field ionization scenarios and to more complex atomic or molecular targets, providing a foundation for future experimental and theoretical studies of laser-matter interactions driven by quantum light.

DATA AVAILABILITY

The data are available from the authors upon reasonable request.

APPENDIX A: HUSIMI-KANO Q FUNCTION FOR SQUEEZED COHERENT STATES

The Husimi-Kano Q function for the squeezed coherent state $|\beta, \zeta\rangle$, Eq. (1), is defined as

$$Q(\alpha) = \frac{1}{\pi} |\langle \alpha | \beta, \zeta \rangle|^2 = \frac{1}{\pi} |\langle \alpha - \beta | \hat{S}(\zeta) | 0 \rangle|^2. \quad (\text{A1})$$

The squeeze operator can be factorized as [54]

$$\hat{S}(\zeta) = \frac{e^{-\frac{1}{2}(\hat{a}^\dagger)^2 e^{i\vartheta} \tanh s - \hat{a}^\dagger \hat{a} \ln(\cosh s) + \frac{1}{2}\hat{a}^2 e^{-i\vartheta} \tanh s}}{\sqrt{\cosh s}}. \quad (\text{A2})$$

Thus $\hat{S}(\zeta)|0\rangle = \exp[-\frac{1}{2}(\hat{a}^\dagger)^2 e^{i\vartheta} \tanh s]|0\rangle/\sqrt{\cosh s}$ and

$$\langle \alpha - \beta | \hat{S}(\zeta) | 0 \rangle = \frac{1}{\sqrt{\cosh s}} e^{-\frac{1}{2}(\alpha^* - \beta^*)^2 e^{i\vartheta} \tanh s - \frac{1}{2}|\alpha - \beta|^2}. \quad (\text{A3})$$

Here the relations $\hat{a}|\alpha - \beta\rangle = (\alpha - \beta)|\alpha - \beta\rangle$ and $\langle \alpha - \beta | 0 \rangle = \exp(-|\alpha - \beta|^2/2)$ have been used. From (A1) and

(A2) we obtain

$$Q(\alpha) = \frac{\exp[-q(\zeta, \alpha - \beta)]}{\pi \cosh s}, \quad (\text{A4})$$

where $q(\zeta, \alpha - \beta) = (1 + \cos \vartheta \tanh s) \gamma_r^2 + (1 - \cos \vartheta \tanh s) \gamma_i^2 + 2\gamma_r \gamma_i \sin \vartheta \tanh s$, with $\gamma_r = \text{Re}(\alpha - \beta)$ and $\gamma_i = \text{Im}(\alpha - \beta)$. For $\vartheta = 0$ this reduces to the known result [55]: $q(s, \alpha - \beta) = 2\gamma_r^2/(1 + e^{-2s}) + 2\gamma_i^2/(1 + e^{2s})$.

APPENDIX B: INTEGRAL OVER THE PHASE ϕ_α

In this appendix we analyze the phase integral $\Phi(\mathcal{E}_\alpha, \beta, \zeta)$, Eq. (11). Introducing the notation

$$a = \frac{\bar{n}\mathcal{E}_\alpha^2}{\mathcal{E}_0^2}, \quad c_\beta = \frac{\mathcal{E}_\beta \cos \phi_\beta}{\mathcal{E}_\alpha}, \quad s_\beta = \frac{\mathcal{E}_\beta \sin \phi_\beta}{\mathcal{E}_\alpha},$$

$$a_\pm = a(1 \pm \cos \vartheta \tanh s), \quad b = 2a \sin \vartheta \tanh s, \quad (\text{B1})$$

we obtain

$$\begin{aligned} \Phi(\mathcal{E}_\alpha, \beta, \zeta) &= \frac{1}{\pi \cosh s} \int_0^{2\pi} d\phi_\alpha e^{-a_+(\cos \phi_\alpha - c_\beta)^2} \\ &\quad \times e^{-a_-(\sin \phi_\alpha - s_\beta)^2 - b(\cos \phi_\alpha - c_\beta)(\sin \phi_\alpha - s_\beta)} \\ &= \frac{J(\mathcal{E}_\alpha, \beta, \zeta)}{\pi \cosh s} e^{-a_+c_\beta^2 - a_-s_\beta^2 - bc_\beta s_\beta - a}, \end{aligned} \quad (\text{B2})$$

where

$$\begin{aligned} J(\mathcal{E}_\alpha, \beta, \zeta) &= \int_0^{2\pi} d\phi_\alpha e^{-\frac{1}{2}(a_+ - a_-) \cos 2\phi_\alpha - \frac{b}{2} \sin 2\phi_\alpha} \\ &\quad \times e^{(2a_+c_\beta + bs_\beta) \cos \phi_\alpha + (2a_-s_\beta + bc_\beta) \sin \phi_\alpha} \\ &= \int_0^{2\pi} d\phi_\alpha e^{r \cos(\phi_\alpha - \psi) - R \cos(2\phi_\alpha - \vartheta)}, \end{aligned} \quad (\text{B3})$$

$$R = a \tanh s, \quad r = \sqrt{A_+^2 + A_-^2}, \quad \tan \psi = A_-/A_+,$$

$$A_+ = 2a_+c_\beta + bs_\beta, \quad A_- = 2a_-s_\beta + bc_\beta. \quad (\text{B4})$$

Using the expansion [56] $e^{z \cos \gamma} = \sum_{n=-\infty}^{\infty} I_n(z) e^{in\gamma}$, where $I_n(z)$ is the modified Bessel function of the first kind, we obtain

$$\begin{aligned} J(\mathcal{E}_\alpha, \beta, \zeta) &= \int_0^{2\pi} d\phi_\alpha \sum_{n,m} I_n(-R) e^{in(2\phi_\alpha - \vartheta)} \\ &\quad \times I_m(r) e^{im(\phi_\alpha - \psi)}. \end{aligned} \quad (\text{B5})$$

Finally, using the relations $\int_0^{2\pi} dx e^{ix(2n+m)} = 2\pi \delta_{2n,-m}$ and $I_{-n}(x) = I_n(x) = (-1)^n I_n(-x)$, we find

$$\begin{aligned} J(\mathcal{E}_\alpha, \beta, \zeta) &= 2\pi \sum_{n=-\infty}^{\infty} (-1)^n I_n(R) I_{2n}(r) e^{in(2\psi - \vartheta)} \\ &= 4\pi \sum_{n>0} (-1)^n I_n(R) I_{2n}(r) \cos[n(2\psi - \vartheta)] \\ &\quad + 2\pi I_0(R) I_0(r). \end{aligned} \quad (\text{B6})$$

For $\beta = 0$ we have $r = 0$, so that, using the relation $I_n(0) = \delta_{n,0}$, we obtain

$$\Phi(\mathcal{E}_\alpha, 0, \zeta) = 2I_0(a \tanh s) e^{-a/\cosh s}. \quad (\text{B7})$$

Thus, for the squeezed vacuum state the integral does not depend on the phase ϑ . For the bright squeezed

vacuum state, where the squeezing parameter s is large, we have $\tanh s \approx 1 - 2e^{-2s}$, $\sinh s \approx \cosh s \approx e^s/2$, $\bar{n} = \sinh^2 s \approx e^{2s}/4$, and $a(\tanh s - 1) \approx -\mathcal{E}_\alpha^2/(2\mathcal{E}_0^2)$. For the modified Bessel function, the asymptotic formula for large z is [56] $e^{-z} I_0(z) \approx 1/\sqrt{2\pi z}$, so that $I_0(a \tanh s) e^{-a \tanh s} \approx 1/\sqrt{2\pi a \tanh s}$. Therefore, for large s we obtain

$$\sinh^2 s \Phi(\mathcal{E}_\alpha, 0, \zeta) \approx \frac{2\mathcal{E}_0 \exp[-\mathcal{E}_\alpha^2/(2\mathcal{E}_0^2)]}{\mathcal{E}_\alpha \sqrt{2\pi}}. \quad (\text{B8})$$

Substituting this into (13) reproduces the result for $w(\mathbf{p})$ from Ref. [35], originally derived for $s \rightarrow \infty$.

APPENDIX C: DEPENDENCE OF THE PRODUCT $\bar{n}\Phi(\mathcal{E}_\alpha, \beta, \zeta)$ ON THE SQUEEZED COHERENT LIGHT PARAMETERS

The dependence of the differential ionization rate, Eq. (13), on the parameters of squeezed coherent light is contained in the product of the averaged number of photons $\bar{n}$, Eq. (10), and the integral $\Phi(\mathcal{E}_\alpha, \beta, \zeta)$, Eq. (11), analyzed in Appendix B. This product depends on five real parameters: $\mathcal{E}_\alpha, \mathcal{E}_\beta, \phi_\beta, s$, and ϑ .

We first consider the squeezed vacuum state for which $\beta = 0$ and our product is independent of the phase ϑ . From Eqs. (10) and (B7), we obtain $\bar{n}\Phi(\mathcal{E}_\alpha, 0, s) = 2 \sinh s \tanh s e^{-a} I_0(a \tanh s)$, with $a = I_\alpha \sinh^2 s / I_0$, $I_\alpha = \mathcal{E}_\alpha^2$, and $I_0 = \mathcal{E}_0^2$. In the upper panel of Fig. 8 we show the logarithm of $\bar{n}\Phi(\mathcal{E}_\alpha, 0, s)$ as a function of the parameter I_α/I_0 , i.e., the ratio of the intensity I_α of the coherent-state component $|\alpha\rangle$ in the distribution of squeezed coherent state $|\beta, \zeta\rangle$, over the intensity I_0 of the state itself. We find that as s increases from 1 to 3, the exponential decrease of $\bar{n}\Phi$ becomes faster. For larger s , however, the dependence approaches that of bright squeezed vacuum light in the limit $s \rightarrow \infty$ (analyzed in Ref. [35]; for illustration, the case $s = 9$ is shown).

The inset in the upper right of Fig. 8 shows (for $s = 1$) the variances of the quadrature operators $\hat{X}$ and $\hat{Y}$, defined in Ref. [49]: $\hat{a} = \hat{X} + i\hat{Y}$, $\hat{a}^\dagger = \hat{X} - i\hat{Y}$, $\langle \Delta X \rangle^2 = \langle \hat{X}^2 \rangle - \langle \hat{X} \rangle^2$, and $\langle \Delta Y \rangle^2 = \langle \hat{Y}^2 \rangle - \langle \hat{Y} \rangle^2$. Already for $s = 1$, squeezing along the ΔX axis is substantial. The inset also shows (again for $s = 1$) the variance of the electric field (3) (in units 2ϵ) defined as $\langle \Delta E \rangle^2 = \langle \hat{\mathbf{E}}^2(t) \rangle - \langle \hat{\mathbf{E}}(t) \rangle^2$, plotted against $[\omega t - (\pi + \vartheta)/2]/\pi$. The shift by $\pi/2$ arises from the factor $i = e^{i\pi/2}$ in (3). We used the formulas [49]

$$\begin{aligned} \langle \Delta X \rangle^2 &= \frac{1}{4} [e^{2s} \sin^2(\vartheta/2) + e^{-2s} \cos^2(\vartheta/2)], \\ \langle \Delta Y \rangle^2 &= \frac{1}{4} [e^{2s} \cos^2(\vartheta/2) + e^{-2s} \sin^2(\vartheta/2)], \end{aligned} \quad (\text{C1})$$

and

$$\langle \Delta E \rangle^2 = \frac{1}{4} (e^{2s} \sin^2 \chi + e^{-2s} \cos^2 \chi), \quad \chi = \omega t - \frac{\pi + \vartheta}{2}, \quad (\text{C2})$$

for $\vartheta = 0$ and $s = 1$. For other parameter values, the elliptical noise disk rotates by $\vartheta/2$, while for $s > 1$ the squeezing is stronger. The mean electric field vanishes,

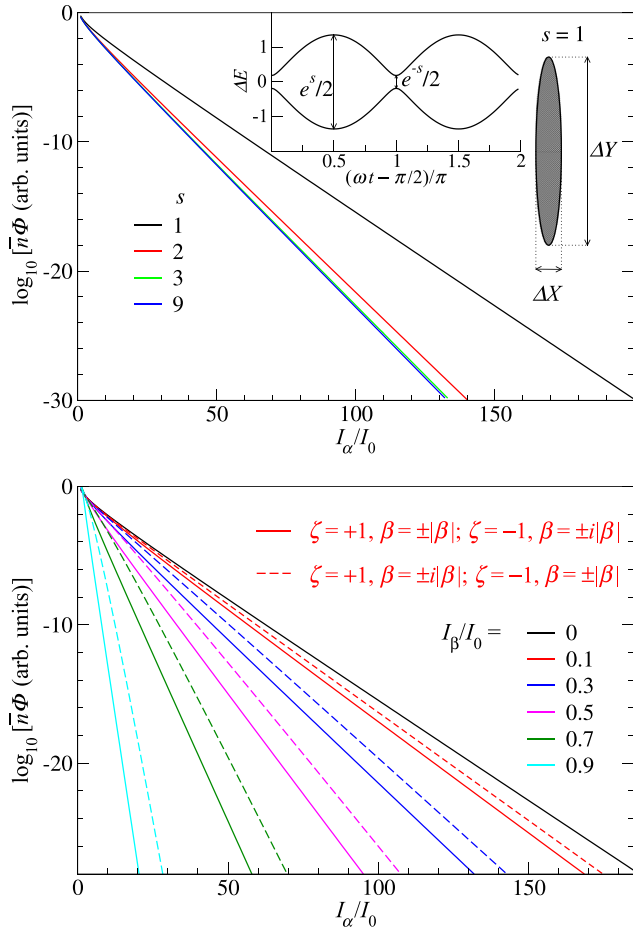


FIG. 8. Logarithm of $\bar{n}\Phi$ as function of the intensity ratio I_α/I_0 for various squeezed coherent light parameters. Upper panel: Results for $\beta = 0$ and $s = 1, 2, 3, 9$. The inset shows the variance of the quadrature operators (upper right) and the variance of the electric field as a function of $(\omega t - \pi/2)/\pi$ for $s = 1$ (upper middle). Lower panel: Results for $s = 1$ and various values of the parameter I_β/I_0 and phases ϕ_β and ϑ (i.e., of the parameters β and ζ as denoted).

$\langle \beta = 0, \zeta | \hat{\mathbf{E}}(t) | \beta = 0, \zeta \rangle = \mathbf{0}$, while the variance yields $\Delta E(\chi) = \pm \langle \Delta E(\chi) \rangle$. The separation between the curves represents the field uncertainty, with maximum $e^s/2$ and minimum $e^{-s}/2$. Results for $s = 1$ are shown. For $s = 0$ (coherent vacuum) the noisy band is bounded by constant lines at $\pm 1/4$. For arbitrary s , the product satisfies $\Delta E_{\min} \Delta E_{\max} = 1/4$.

For squeezed coherent light, the expectation values of the quadrature operators are $\langle \beta, \zeta | \hat{X} | \beta, \zeta \rangle = \text{Re } \beta$ and $\langle \beta, \zeta | \hat{Y} | \beta, \zeta \rangle = \text{Im } \beta$, so that the center of the elliptical disc in Fig. 8 is shifted to $(\text{Re } \beta, \text{Im } \beta)$. The mean field or coherent signal is $\langle \beta, \zeta | \hat{\mathbf{E}}(t) | \beta, \zeta \rangle = \mathbf{E}_\beta(t) = \mathcal{E}_\beta \cos(\omega t - \pi/2 - \phi_\beta) \hat{\mathbf{e}}$. Formulas (C1) and (C2) for variances remain valid for squeezed coherent states. The maximum signal-to-noise ratio is $4e^{2s}|\beta|^2$. For $\phi_\beta = \vartheta/2$ the light is amplitude squeezed: the electric field is cosinusoidal with amplitude $\mathcal{E}_\beta$ and phase $\omega t - (\pi + \vartheta)/2$ with phase-dependent noise—maximum $e^s/2$ at $\omega t - (\pi + \vartheta)/2 = \pi/2$, minimum $e^{-s}/2$ at $\omega t - (\pi + \vartheta)/2 = \pi$. For $\phi_\beta = (\vartheta + \pi)/2$ the light is phase squeezed [49].

In the lower panel of Fig. 8 we analyze the dependence of our product $\bar{n}\Phi$ on the values of the parameters $I_\beta/I_0 = \mathcal{E}_\beta^2/\mathcal{E}_0^2$, ϕ_β , and ϑ , for fixed $s = 1$. The case $\beta = 0$ (black line) is already shown in the upper panel. As $|\beta|$ increases ($I_\beta/I_0 = 0.1, 0.3, 0.5, 0.7, 0.9$), the exponential decrease of $\bar{n}\Phi$ with increasing I_α/I_0 becomes faster. The dependence on ϕ_β and ϑ is less pronounced. For fixed $|\beta|$, the product $\bar{n}\Phi$ lies between the solid and dashed lines of the corresponding color. The solid and dashed lines correspond to the values of phases as denoted. The decrease is fastest for $\zeta = 1$ ($\vartheta = 0$) and $\beta = \pm|\beta|$ ($\phi_\beta = 0, \pi$) (solid line). The same result holds for $\zeta = -1$ ($\vartheta = \pi$) and $\beta = \pm i|\beta|$ ($\phi_\beta = \pm\pi/2$). Conversely, the slowest decrease (dashed lines) occurs for $\vartheta = 0$ and $\phi_\beta = \pm\pi/2$ (the same result is obtained for $\vartheta = \pi$ and $\phi_\beta = \pi, 0$). In the above analysis we used the fact that the integral $\Phi(\mathcal{E}_\alpha, |\beta|e^{i\phi_\beta}, se^{i\vartheta})$ is invariant under the transformation $\phi_\beta \rightarrow \phi_\beta \pm \pi$. We also used the fact that, for fixed s and $|\beta|$, the phase integral corresponding to (ϕ_β, ϑ) is identical to that for $(\phi_\beta + \pi/2, \vartheta + \pi)$.

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