

Synchronization in a dissipative quantum many-body system

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We study synchronization in the XX qubit chain subject to local or multilocal amplitude-damping noise. We show that generic stable synchronization of the edge qubits occurs *if and only if* the decoherence-free subspace (DFS) supports exactly one single-excitation eigenstate. Analyzing the DFS structure of the model, we show that it is completely determined by a simple number-theoretic function involving the noise sites and the chain length. We further show that this same condition also guarantees constant asymptotic entanglement between the edge qubits, so that generic stable synchronization and constant asymptotic entanglement necessarily coexist. By contrast, when the DFS supports multiple single-excitation eigenstates, synchronization becomes initial state dependent and may be entirely absent, even though stable oscillatory entanglement can persist indefinitely. Our analysis can be extended to any excitation-conserving qubit chain mappable to free fermions and is directly testable on platforms with site-selective dissipation.

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Introduction. From the flashing of fireflies [1] and the rhythmic firing of cardiac pacemaker cells [2] to the coherent motion of coupled metronomes [3] and the phase locking of power grids [4], synchronization is a universal manifestation of collective dynamical order across nature and technology [5–10]. Over the last decade, it has also emerged as a central subject in quantum physics, where both forced synchronization under external driving and spontaneous synchronization arising solely from mutual coupling between systems have been explored. Quantum synchronization has been identified in systems as diverse as quantum van der Pol oscillators, harmonic oscillators, optomechanical and nanomechanical systems, trapped ions, superconducting circuits, and spin systems [11–64], with experimental realizations demonstrated across multiple platforms [65–70].

In open quantum systems, spontaneous synchronization is shaped by the interplay between coherent dynamics and environmental decay. When the decay rates of different eigenmodes are well separated, transient synchronization can appear, with one or several slowly decaying modes dominating the dynamics over a long intermediate time window before eventually fading away [15]. By contrast, stable synchronization requires the existence of a decoherence-free subspace (DFS) [49,71], i.e., a noiseless sector of the dynamics that remains decoupled from the environment and thus can sus-

tain persistent oscillations indefinitely. This also leads to nonstationary phenomena in the steady states of open quantum systems, such as dissipative time-crystalline behavior [72–74].

Quantum synchronization in an open quantum many-body setting was recently predicted by Schmolke and Lutz [40] for a chain of qubits subjected to local Gaussian white noise, using a perturbative approach in Liouville space, and was subsequently observed experimentally in superconducting transmon circuits [70]. However, emergence of stable synchronization in dissipative qubit chains under amplitude-damping (AD) noise, and more importantly its connection to the DFS structure, remains unexplored. In this Letter, we study synchronization by directly analyzing the DFS structure of the XX qubit chain subject to local or multilocal AD noise. We first demonstrate that the DFS structure is entirely fixed by a simple number-theoretic function, namely, the greatest common divisor of the noise sites and the chain length. Then, we derive a closed-form expression for the expectation values of local qubit observables restricted to the DFS and establish a necessary and sufficient condition for generic stable synchronization of the edge qubits. More precisely, we prove that generic stable synchronization occurs *if and only if* the DFS supports exactly one single-excitation eigenstate. We also show that generic stable synchronization and constant asymptotic entanglement between the edge qubits always coexist, as both are enforced by exactly the same condition on the number of DFS eigenstates. However, when synchronization loses its generic character and becomes initial state dependent, oscillatory entanglement can still persist indefinitely without synchronization ever being established.

The model and its DFS structure. We focus on the XX chain of qubits subjected to local or multilocal AD noise.

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Under certain physically motivated assumptions, the dynamics of this open many-body system is described by the Gorini-Kossakowski-Sudarshan-Lindblad equation [75–77]

$$\dot{\rho} = -i[H, \rho] + \mathcal{D}(\rho), \quad (1)$$

where $\mathcal{D}(\rho) = \sum_i \gamma_i [L_i \rho L_i^\dagger - \frac{1}{2}\{L_i^\dagger L_i, \rho\}]$ is the dissipator with γ_i being the strength of the coupling to the environment, L_i are the jump operators, and $H = H_S + H_{\text{int}}$ includes the self-Hamiltonians of the qubits making up the chain and the pairwise coupling between them, respectively. Specifically, the Hamiltonian that governs the unitary part of the evolution is given by

$$H = -\frac{\omega}{2} \sum_{i=1}^N \sigma_z^{(i)} + \frac{J}{2} \sum_{i=1}^{N-1} (\sigma_x^{(i)} \sigma_x^{(i+1)} + \sigma_y^{(i)} \sigma_y^{(i+1)}), \quad (2)$$

where σ_x , σ_y , and σ_z are the usual Pauli operators, and N denotes the number of qubits in the chain. Note that H conserves the number of excitations and does not couple excitation sectors with different numbers of excitations. Consequently, we start our DFS analysis by focusing on the single-excitation subspace of H .

We denote the basis vectors of the single-excitation subspace with $|j\rangle = |0 \cdots 1_j \cdots 0\rangle$ for $j = 1, \dots, N$. Using $\sigma_\pm = (\sigma_x \mp i\sigma_y)/2$ and setting the ground-state energy to $E_0 = 0$, it is possible to express the single-excitation sector of the full Hamiltonian H given in Eq. (2) in this basis as a nearest-neighbor hopping Hamiltonian, that is, $H^{(1)} = \omega \sum_{j=1}^N |j\rangle\langle j| + J \sum_{j=1}^{N-1} (|j\rangle\langle j+1| + |j+1\rangle\langle j|)$, which is a Toeplitz tridiagonal matrix. The eigenvalues of such matrices are given by $E_n = \omega + 2J \cos(\frac{n\pi}{N+1})$ and the corresponding eigenvectors read [78]

$$|\phi_n\rangle = \sum_{j=1}^N \phi_n(j) |j\rangle, \quad \phi_n(j) = \sqrt{\frac{2}{N+1}} \sin\left(\frac{nj\pi}{N+1}\right), \quad (3)$$

where $n = 1, \dots, N$ and the amplitudes satisfy the parity symmetry relation $\phi_n(N+1-j) = (-1)^{n+1} \phi_n(j)$. Let us now focus on the DFS structure of the XX chain under local AD noise in the single-excitation subspace. A subspace is called decoherence-free if (i) every state $|\psi\rangle$ in it is a simultaneous eigenstate of all jump operators, $L_i |\psi\rangle = c_i |\psi\rangle$, with c_i being independent of the state $|\psi\rangle$ within this subspace, and (ii) the subspace is invariant under H so that the unitary dynamics does not couple it to the decohering sector [71,79–81]. For AD noise on the site m , $L = \sigma_-^{(m)}$ acts on the single-excitation basis states as $\sigma_-^{(m)} |j\rangle = 0$, for $j \neq m$. Furthermore, $\sigma_-^{(m)} |m\rangle = |G\rangle$, where $|G\rangle \equiv |0\rangle^{\otimes N}$ denotes the ground state. Physically, this means that all single-excitation basis states with no excitation on the noise site are unaffected by the noise and thus constitute the set of dark basis states. By contrast, $|m\rangle$ is the only bright basis state, since it is coupled by dissipation to the ground state and thus leaks excitation out of the single-excitation sector.

In principle, the linear combinations of the dark basis states are decoupled from dissipation due to the AD jump operator. However, to belong to the DFS, they must also remain in the dark sector under unitary dynamics. Since the XX Hamiltonian generates nearest-neighbor hopping, a generic

superposition of dark basis states evolves into a state with nonzero amplitude on the noisy site and thus leaks out of the dark sector. The single-excitation states that remain dark for all times are thus the eigenstates of H with zero amplitude on the noise site m . This requires that $\phi_n(m) = 0$, leading to $\frac{nm}{N+1} \in \mathbb{Z}$, which in turn implies that the number of single-excitation DFS states is $r = g - 1$, where $g = \text{gcd}(m, N+1)$, and the eigenstate labels that belong to the DFS are given by $n_l = l(N+1)/g$, with $l = 1, \dots, g-1$ in Supplemental Material [82]. Here, gcd denotes the greatest common divisor function. This will allow us to fully characterize the DFS for an XX chain of arbitrary length subject to local AD noise at arbitrary sites and thus constitutes the first main result of this Letter.

More generally, let $\{|D_\alpha\rangle\}$, where $\alpha \in \{n_l\}$, denote the set of single-excitation DFS states with coefficients $\phi_\alpha(j) = \langle j | D_\alpha \rangle$. Note that the DFS also includes the zero-excitation state $|G\rangle$. Since the XX chain Hamiltonian can be mapped to a free-fermion model [78,83], the multiexcitation DFS sectors can be obtained from distinct single-excitation DFS eigenmodes. Particularly, for $\alpha \neq \beta$, two-excitation DFS states are constructed as

$$|D_{\alpha\beta}\rangle = \sum_{1 \leq i < j \leq N} \Psi_{\alpha\beta}(i, j) |i, j\rangle, \quad (4)$$

where $\Psi_{\alpha\beta}(i, j) = \phi_\alpha(i)\phi_\beta(j) - \phi_\alpha(j)\phi_\beta(i)$, with additive energies, that is, $E_{D_{\alpha\beta}} = E_{D_\alpha} + E_{D_\beta}$. All higher-excitation DFS states are obtained as $k \times k$ Slater determinants built from k distinct single-excitation DFS eigenmodes. Thus, if the single-excitation DFS contains r eigenmodes, the k -excitation DFS sector has dimension $\binom{r}{k}$, where $k = 0, 1, \dots, r$. Then, $r = \text{gcd}(m, N+1) - 1$ not only determines the number of single-excitation DFS states but also identifies all the higher-excitation DFS sectors. Hence, the full DFS structure of the XX chain under AD noise at site m is completely characterized.

The above result extends naturally to the case of multi-local AD noise. When the noise acts on sites $m_1, \dots, m_q$, a single-excitation eigenmode belongs to the DFS if it has zero amplitude on each noise site, i.e., $\phi_\alpha(m_a) = 0$ for all $a = 1, \dots, q$. This leads to

$$r = g - 1, \quad g = \text{gcd}(m_1, \dots, m_q, N+1), \quad (5)$$

so that, as in the local AD noise case, both the number of single-excitation DFS states and the full DFS structure are completely determined by a gcd condition in Supplemental Material [82].

Our DFS construction identifies invariant state vectors in Hilbert space. A more general Liouville space approach can admit additional nondecaying operators [71]. For the present model, however, the two approaches coincide, since nearest-neighbor hopping moves excitations across the chain, and any component that reaches a noise site is removed by AD noise. Thus, all nondecaying density operators have support entirely within the subspace spanned by the DFS eigenstates identified above.

Lastly, the outlined DFS structure cannot be preserved for finite-temperature AD noise, which will introduce an additional jump operator $\sigma_+^{(m)}$ alongside $\sigma_-^{(m)}$. All DFS eigenstates identified above have, by construction, vanishing amplitude at

the noise site m . The jump operator $\sigma_+^{(m)}$ creates an excitation precisely at that site, moving these states out of the DFS. Consequently, no nontrivial DFS exists under generalized local AD noise [74].

We emphasize that the construction above is not specific to the uniform XX chain. In fact, it relies only on two structural properties of the model: (i) the Hamiltonian conserves the total number of excitations, and (ii) it can be mapped to noninteracting fermions. For any qubit chain having these properties, the single-excitation DFS states are the eigenmodes with vanishing amplitude on every noise site, the multiexcitation DFS states are their Slater determinants with additive energies, and the synchronization analysis developed below carries over without modification. The gcd criterion is the manifestation of this dark-mode counting for the uniform chain, where the eigenstate amplitudes are of sine form.

Conditions for generic synchronization. In this part, we establish the conditions under which stable synchronization is achieved between the two edge qubits, and we determine when this synchronization is generic. By stable synchronization, we mean sustained nondecaying oscillations in the expectation values, as expected from a nonstationary steady state. The expectation value of interest in our treatment is $\langle \sigma_x^{(k)}(t) \rangle = \text{Tr}[\rho(t)\sigma_x^{(k)}]$. Subsequent to the transient dynamics, the asymptotic state of the system is confined to the DFS, where the dynamics is purely unitary, which yields the closed form expression in Supplemental Material [82]

$$\langle \sigma_x^{(k)}(t) \rangle_{\text{DFS}} = 2\text{Re} \sum_{\mu, \nu \in \text{DFS}} \rho_{\nu\mu}^{\infty} X_{\mu\nu}^{(k)} e^{-i(E_{\nu} - E_{\mu})t}. \quad (6)$$

In the above equation, the summation runs over pairs of DFS eigenstates $|\mu\rangle$ and $|\nu\rangle$, whose excitation numbers differ by 1, i.e. $\mathcal{N}_{\nu} = \mathcal{N}_{\mu} + 1$, since $\sigma_x = \sigma_+ + \sigma_-$, and the matrix element $\rho_{\nu\mu}^{\infty}$ is the coherence between $|\mu\rangle$ and $|\nu\rangle$ in the density operator ρ^{∞} at the onset of the unitary regime, after all bright components have decayed. $X_{\mu\nu}^{(k)} = \langle \mu | \sigma_x^{(k)} | \nu \rangle$ encodes whether flipping the qubit at site k can connect the two DFS states $|\mu\rangle$ and $|\nu\rangle$. Hence, each allowed DFS transition contributes an oscillatory component at frequency $\omega_{\nu\mu} = E_{\nu} - E_{\mu}$. If multiple distinct transitions survive, the dynamics exhibits multifrequency oscillations, whereas a single surviving transition leads to oscillations at a single frequency.

We define generic stable synchronization between the edge qubits by the condition

$$\langle \sigma_x^{(1)}(t) \rangle_{\text{DFS}} = C \langle \sigma_x^{(N)}(t) \rangle_{\text{DFS}}, \quad (7)$$

for all t , once the dynamics is confined to the DFS, where C is a real constant, and neither asymptotic edge expectation value vanishes at all times. Even though this criterion excludes arbitrary locked phases, it is well suited to the present setting, where the surviving asymptotic solutions are exactly in phase or in antiphase, consistent with the parity symmetry of the eigenstate amplitudes below Eq. (3). Since generic synchronization requires Eq. (7) to hold for arbitrary initial states [84], Eq. (6) implies $X_{\mu\nu}^{(1)} = C X_{\mu\nu}^{(N)}$ for all contributing DFS pairs μ, ν . If there exists only one single-excitation DFS state, i.e., $g = 2$, then $X_{GD_{\alpha}}^{(1)} = \phi_{\alpha}(1)$ and $X_{GD_{\alpha}}^{(N)} = \phi_{\alpha}(N)$, which yields $\phi_{\alpha}(1) = (-1)^{\alpha+1} \phi_{\alpha}(N)$, where the latter equality follows from the inversion symmetry of the eigenstate

amplitudes given in Eq. (3). The edge qubits are thus synchronized for odd α and antisynchronized for even α , where $\alpha = n_1 = (N+1)/g = (N+1)/2$. This clearly implies that N is odd and synchronization is generic. The frequency of the synchronized oscillations is dictated by the energy difference $E_{D_{\alpha}} - E_G = E_{D_{\alpha}}$.

It can also be shown that generic synchronization occurs *only if* the DFS supports one single-excitation eigenstate. In order to see this, consider the case of two single-excitation DFS states, which allows for the construction of a single two-excitation DFS state. Then, we need to account for matrix elements connecting single- and two-excitation DFS states. We focus on the matrix elements contributing to the same oscillatory term as the $X_{GD_{\alpha}}$ transition. Since $E_{D_{\alpha\beta}} - E_{D_{\beta}} = (E_{D_{\alpha}} + E_{D_{\beta}}) - E_{D_{\beta}} = E_{D_{\alpha}}$ matches the frequency of the $X_{GD_{\alpha}}$ transition, those matrix elements are $X_{D_{\beta}D_{\alpha\beta}}^{(1)}$ and $X_{D_{\beta}D_{\alpha\beta}}^{(N)}$ and must thus satisfy the same sign condition. Nonetheless, it is possible to show that $X_{D_{\beta}D_{\alpha\beta}}^{(1)} = \phi_{\alpha}(1)$ and $X_{D_{\beta}D_{\alpha\beta}}^{(N)} = -\phi_{\alpha}(N)$, and therefore $\phi_{\alpha}(1) = -(-1)^{\alpha+1} \phi_{\alpha}(N)$. In fact, this is incompatible with the previously obtained relation above, $\phi_{\alpha}(1) = (-1)^{\alpha+1} \phi_{\alpha}(N)$, making it impossible for all the terms to satisfy the synchronization condition simultaneously. Consequently, generic stable synchronization emerges *if and only if* there exists only one single-excitation DFS state in Supplemental Material [82], i.e.,

$$g = \text{gcd}(m_1, \dots, m_q, N+1) = 2, \quad (8)$$

which is the second main result of this Letter.

The same condition also fixes the synchronization behavior at every site of the chain, that is, the amplitudes $\phi_{\alpha}(j) = \sqrt{2/(N+1)} \sin(j\pi/2)$ vanish on even sites and alternate in sign on consecutive odd sites. Thus, the odd-site qubits split into two classes that are synchronized within each class and antisynchronized between them, consistent with the parity symmetry of the eigenstates.

Case study. Here, we present a concrete demonstration of our general results in a chain consisting of $N = 11$ qubits, considering both local and multilocal noise cases. To explicitly track the synchronization behavior at all times during the dynamics, we adopt the well-established figure of merit for this purpose, namely, the Pearson coefficient (PC), defined as $\text{PC} = \text{Cov}(x, y) / \sqrt{\text{Var}(x)\text{Var}(y)}$, where $x = \langle \sigma_x^{(1)}(t) \rangle$ and $y = \langle \sigma_x^{(N)}(t) \rangle$ in our case [13].

For local noise, we take the noise site to be $m = 6$, which admits $r = \text{gcd}(6, 12) - 1 = 5$ single-excitation DFS states, i.e., $\{|D_2\rangle, |D_4\rangle, |D_6\rangle, |D_8\rangle, |D_{10}\rangle\}$, with corresponding energies $E_{\alpha} = \omega + \sqrt{3}J, \omega + J, \omega, \omega - J, \text{ and } \omega - \sqrt{3}J$, respectively. By our second main result, this rules out generic synchronization. All the same, for certain classes of initial states, the system can still sustain multifrequency synchronization in Supplemental Material [82]. In particular, it is clear from Eq. (6) that the frequencies of the oscillations are set by the energy difference between the DFS states that differ by a single excitation, namely, $\omega_{\nu\mu}$. Recalling that the energies of the multiexcitation DFS states are determined additively from the single-excitation DFS energies, we see that the relevant frequencies are dictated only by the energy differences between the ground state and single-excitation DFS eigenstates,

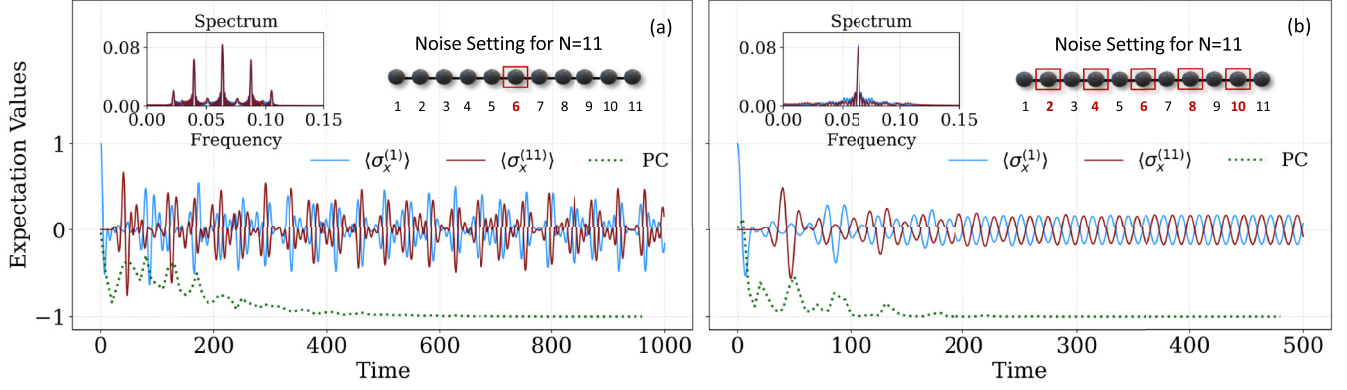


FIG. 1. Stable synchronization of expectation values $\langle \sigma_x \rangle$ for the first and the last qubits of the XX chain of $N = 11$ qubits. (a) Only the middle qubit at the site $m = 6$ is affected by the AD noise. (b) The qubits at sites $m = 2, 4, 6, 8,$ and 10 are affected by the AD noise having identical noise strengths γ . While the insets display the frequency spectrum of the $\langle \sigma_x \rangle$ oscillations, the dynamical establishment of stable antisynchronization is demonstrated by the behavior of the Pearson coefficient (PC). In both plots, the initial state is $|\psi\rangle = |+\rangle_1 \otimes \bigotimes_{k=2}^{11} |0\rangle_k$ and we fix the model parameters as $\gamma = 0.05$, $J = 0.15$, and $\omega = 0.4$. It is evident that while there are five distinct frequencies in panel (a) contributing to the antisynchronized dynamics of the edge qubits, and in panel (b) we observe single-frequency synchronization of the edge qubits. The antisynchronization observed in panel (a) is state dependent rather than generic, since the chosen initial state has support only in the zero- and single-excitation sectors.

$\omega_{D_\alpha G} = E_{D_\alpha} - E_G = E_{D_\alpha}$. In Fig. 1(a), we display the full dynamics of our model with the aforementioned noise setting for the initial state $|\psi\rangle = |+\rangle_1 \otimes \bigotimes_{k=2}^{11} |0\rangle_k$ and model parameters $\gamma = 0.05$, $J = 0.15$, and $\omega = 0.4$. Following a transient dynamics, we can clearly observe the establishment of antisynchronization between the edge qubits indicated by the settlement of PC to -1 . This behavior can be understood by recalling the parity symmetry of the eigenstates forming the DFS, together with the fact that all α values are even in the present case, i.e., $\phi_\alpha(1)/\phi_\alpha(N) = -1$. Due to the same parity symmetry argument, every pair that is reflection symmetric with respect to the center of the chain is also automatically synchronized. Moreover, it is possible to see from the behavior of the $\langle \sigma_x \rangle$ values that we do not have a simple single-frequency antisynchronization. By taking the Fourier transform of the $\langle \sigma_x \rangle$ data, we explicitly show in the upper-left inset of Fig. 1(a) that there exist in fact five distinct frequencies contributing to the antisynchronized dynamics of the edge qubits. These are exactly the predicted frequencies dictated by the single-excitation DFS energies, which, for the considered model parameters, are $\omega_{D_\alpha G}/2\pi = 0.022, 0.040, 0.064, 0.088,$ and 0.105 . This presents an explicit demonstration of a fully coherent multifrequency synchronization in a controlled setting.

For multilocal noise, we extend the noise sites to include $m_a = 2, 4, 6, 8,$ and 10 , as depicted in the upper-right inset of Fig. 1(b). Since $\gcd(2, 4, 6, 8, 10, 12) = 2$, this configuration reduces the size of the single-excitation DFS set to a single element, which is $|D_6\rangle$. Note that the current setting satisfies Eq. (8); therefore, we observe generic synchronization with its frequency, $f = \omega/2\pi$, determined by the energy difference $E_{D_6} - E_G = \omega$. Using the same initial state and model parameters as the local noise case, from Fig. 1(b) we clearly observe the establishment of antisynchronization of the edge qubits sustained by a single frequency that is extracted from the Fourier transform of the expectation values and displayed in the upper-left inset of the same figure. A direct comparison

between Figs. 1(a) and 1(b) shows that, as the number of noise sites is increased, we observe earlier establishment of synchronization. This can be understood from the fact that increased noise allows a faster decay into the DFS.

Whenever the *if and only if* condition for generic synchronization in Eq. (8) is satisfied, which requires odd N , it is necessarily accompanied by a constant finite asymptotic entanglement between the edge qubits in Supplemental Material [82]. For example, the asymptotic concurrence for the initial state considered in Fig. 1 scales as $C_{(1,N)}^\infty = 4/(N+1)^2$ [85]. Nevertheless, entanglement between the edge qubits can also survive in the absence of generic synchronization, and it is not necessarily constant. In Fig. 2, we consider the exact same scenario as in Fig. 1(a) but with a different initial state,

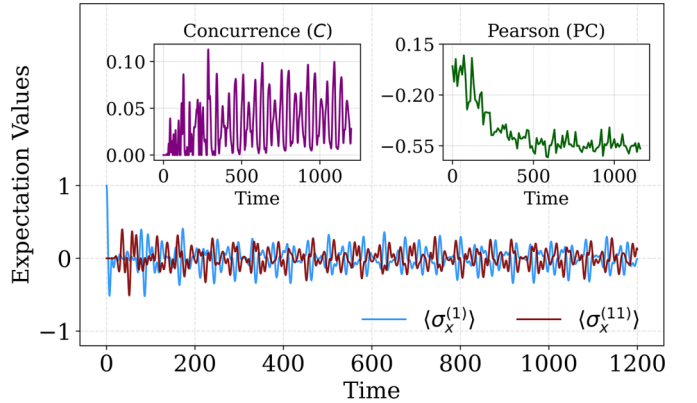


FIG. 2. Asynchronous dynamics of expectation values $\langle \sigma_x \rangle$ for the first and the last qubits of the XX chain of $N = 11$ qubits, where the relaxation noise is at the site $m = 6$. The insets show the dynamics of entanglement and Pearson coefficient for the edge qubits. Whereas the initial state is fixed to $|\psi\rangle = |+\rangle_1 \otimes |-\rangle_2 \otimes \bigotimes_{k=3}^{11} |0\rangle_k$, the model parameters are taken as $\gamma = 0.05$, $J = 0.15$, and $\omega = 0.4$.

$|\psi\rangle = |+\rangle_1 \otimes |-\rangle_2 \otimes \bigotimes_{k=3}^{11} |0\rangle_k$. The PC displayed in the upper-right inset confirms synchronization is not established, yet the upper-left inset reveals oscillatory but finite concurrence between the edge qubits. While the interplay between synchronization and quantum correlations has been examined in a number of different works [22,23,33,34,37], our results sharpen their relation in the present many-body setting; that is, constant asymptotic edge entanglement necessarily accompanies generic synchronization, whereas persistent oscillatory entanglement can survive even when synchronization is completely absent. This demonstrates that, despite their coexistence when $g = 2$, synchronization and entanglement are ultimately driven by different physical mechanisms in general.

Conclusion. Our analysis shows that a single integer, namely, the greatest common divisor of the noise sites and the chain length, fully determines the DFS structure governing the long-time coherent behavior of the dissipative XX chain under local or multilocal noise. Generic stable synchronization of the edge qubits emerges if and only if the DFS supports exactly one single-excitation eigenstate. Under the same condition, the edge qubits also develop constant asymptotic entanglement, so that both phenomena are in fact controlled by a single arithmetic condition. When the DFS supports multiple single-excitation eigenstates, synchronization loses its generic character, becomes sensitive to the initial state, and may exhibit a multifrequency structure dictated by the single-excitation DFS spectrum. Entanglement, however, can persist indefinitely even without any synchronization

being established. Our results are analytical and testable in current experimental platforms with site-selective dissipation, such as superconducting qubit arrays [70] and trapped ions [69,86]. Since the underlying mechanism requires only that the Hamiltonian conserves the total number of excitations and can be mapped to noninteracting fermions, the present framework extends directly to more general qubit chains with nonuniform couplings and nonresonant on-site energies. There, the sine wave eigenmodes no longer apply and the gcd criterion can be lost, yet the dark-mode counting that governs generic synchronization remains exactly characterizable.

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Data availability. There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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- [1] J. Buck and E. Buck, Mechanism of rhythmic synchronous flashing of fireflies, *Science* **159**, 1319 (1968).
 - [2] R. E. Mirolo and S. H. Strogatz, Synchronization of pulse-coupled biological oscillators, *SIAM J. Appl. Math.* **50**, 1645 (1990).
 - [3] J. Pantaleone, Synchronization of metronomes, *Am. J. Phys.* **70**, 992 (2002).
 - [4] G. Filatrella, A. H. Nielsen, and N. F. Pedersen, Analysis of a power grid using a Kuramoto-like model, *Eur. Phys. J. B* **61**, 485 (2008).
 - [5] I. I. Blekhman, *Synchronization in Science and Technology* (ASME, New York, 1988).
 - [6] A. Pikovsky, M. Rosenblum, and J. Kurths, *Synchronization: A Universal Concept in Nonlinear Sciences* (Cambridge University, Cambridge, England, 2001).
 - [7] A. Balanov, N. Janson, D. Postnov, and O. Sosnovtseva, *Synchronization: From Simple to Complex* (Springer, Berlin, 2009).
 - [8] S. H. Strogatz, Exploring complex networks, *Nature (London)* **410**, 268 (2001).
 - [9] S. Boccaletti, J. Kurths, G. Osipov, D. L. Valladares, and C. S. Zhou, The synchronization of chaotic systems, *Phys. Rep.* **366**, 1 (2002).
 - [10] J. A. Acebrón, L. L. Bonilla, C. J. Pérez Vicente, F. Ritort, and R. Spigler, The Kuramoto model: A simple paradigm for synchronization phenomena, *Rev. Mod. Phys.* **77**, 137 (2005).
 - [11] A. Mari, A. Farace, N. Didier, V. Giovannetti, and R. Fazio, Measures of quantum synchronization in continuous variable systems, *Phys. Rev. Lett.* **111**, 103605 (2013).
 - [12] W. Li, W. Zhang, C. Li, and H. Song, Properties and relative measure for quantifying quantum synchronization, *Phys. Rev. E* **96**, 012211 (2017).
 - [13] F. Galve, G. L. Giorgi, and R. Zambrini, Quantum correlations and synchronization measures, in *Lectures on General Quantum Correlations and Their Applications*, edited by F. F. Fanchini, D. d. O. Soares Pinto, and G. Adesso (Springer, Cham, 2017), pp. 393–420.
 - [14] A. Roulet and C. Bruder, Synchronizing the smallest possible system, *Phys. Rev. Lett.* **121**, 053601 (2018).
 - [15] G. L. Giorgi, A. Cabot, and R. Zambrini, Transient synchronization in open quantum systems, in *Advances in Open Systems and Fundamental Tests of Quantum Mechanics*, edited by B. Vacchini, H.-P. Breuer, and A. Bassi (Springer, Cham, 2019), pp. 73–89.
 - [16] O. V. Zhirov and D. L. Shepelyansky, Synchronization and bistability of a qubit coupled to a driven dissipative oscillator, *Phys. Rev. Lett.* **100**, 014101 (2008).
 - [17] O. V. Zhirov and D. L. Shepelyansky, Quantum synchronization and entanglement of two qubits coupled to a driven dissipative resonator, *Phys. Rev. B* **80**, 014519 (2009).

- [18] T. E. Lee and H. R. Sadeghpour, Quantum synchronization of quantum van der Pol oscillators with trapped ions, *Phys. Rev. Lett.* **111**, 234101 (2013).
- [19] S. Walter, A. Nunnenkamp, and C. Bruder, Quantum synchronization of a driven self-sustained oscillator, *Phys. Rev. Lett.* **112**, 094102 (2014).
- [20] S. Sonar, M. Hajdušek, M. Mukherjee, R. Fazio, V. Vedral, S. Vinjanampathy, and L.-C. Kwek, Squeezing enhances quantum synchronization, *Phys. Rev. Lett.* **120**, 163601 (2018).
- [21] I. Goychuk, J. Casado-Pascual, M. Morillo, J. Lehmann, and P. Hänggi, Quantum stochastic synchronization, *Phys. Rev. Lett.* **97**, 210601 (2006).
- [22] G. L. Giorgi, F. Galve, G. Manzano, P. Colet, and R. Zambrini, Quantum correlations and mutual synchronization, *Phys. Rev. A* **85**, 052101 (2012).
- [23] G. Manzano, F. Galve, G. Giorgi, E. Hernández-García, and R. Zambrini, Synchronization, quantum correlations and entanglement in oscillator networks, *Sci. Rep.* **3**, 1439 (2013).
- [24] G. Manzano, F. Galve, and R. Zambrini, Avoiding dissipation in a system of three quantum harmonic oscillators, *Phys. Rev. A* **87**, 032114 (2013).
- [25] C. Benedetti, F. Galve, A. Mandarino, M. G. A. Paris, and R. Zambrini, Minimal model for spontaneous quantum synchronization, *Phys. Rev. A* **94**, 052118 (2016).
- [26] T. E. Lee, C.-K. Chan, and S. Wang, Entanglement tongue and quantum synchronization of disordered oscillators, *Phys. Rev. E* **89**, 022913 (2014).
- [27] S. Walter, A. Nunnenkamp, and C. Bruder, Quantum synchronization of two van der Pol oscillators, *Ann. Phys.* **527**, 131 (2015).
- [28] G. Heinrich, M. Ludwig, J. Qian, B. Kubala, and F. Marquardt, Collective dynamics in optomechanical arrays, *Phys. Rev. Lett.* **107**, 043603 (2011).
- [29] M. Ludwig and F. Marquardt, Quantum many-body dynamics in optomechanical arrays, *Phys. Rev. Lett.* **111**, 073603 (2013).
- [30] M. R. Hush, W. Li, S. Genway, I. Lesanovsky, and A. D. Armour, Spin correlations as a probe of quantum synchronization in trapped-ion phonon lasers, *Phys. Rev. A* **91**, 061401(R) (2015).
- [31] A. Cabot, G. L. Giorgi, F. Galve, and R. Zambrini, Quantum synchronization in dimer atomic lattices, *Phys. Rev. Lett.* **123**, 023604 (2019).
- [32] P. P. Orth, D. Roosen, W. Hofstetter, and K. Le Hur, Dynamics, synchronization, and quantum phase transitions of two dissipative spins, *Phys. Rev. B* **82**, 144423 (2010).
- [33] G. L. Giorgi, F. Plastina, G. Francica, and R. Zambrini, Spontaneous synchronization and quantum correlation dynamics of open spin systems, *Phys. Rev. A* **88**, 042115 (2013).
- [34] B. Bellomo, G. L. Giorgi, G. M. Palma, and R. Zambrini, Quantum synchronization as a local signature of super- and subradiance, *Phys. Rev. A* **95**, 043807 (2017).
- [35] G. L. Giorgi, F. Galve, and R. Zambrini, Probing the spectral density of a dissipative qubit via quantum synchronization, *Phys. Rev. A* **94**, 052121 (2016).
- [36] G. Karpát, I. Yalçinkaya, and B. Çakmak, Quantum synchronization of few-body systems under collective dissipation, *Phys. Rev. A* **101**, 042121 (2020).
- [37] G. Karpát, I. Yalçinkaya, and B. Çakmak, Quantum synchronization in a collision model, *Phys. Rev. A* **100**, 012133 (2019).
- [38] G. Karpát, I. Yalçinkaya, B. Çakmak, G. L. Giorgi, and R. Zambrini, Synchronization and non-Markovianity in open quantum systems, *Phys. Rev. A* **103**, 062217 (2021).
- [39] Y. Li, X. Li, and J. Jin, Quantum nonstationary phenomena of spin systems in collision models, *Phys. Rev. A* **107**, 042205 (2023).
- [40] F. Schmolke and E. Lutz, Noise-induced quantum synchronization, *Phys. Rev. Lett.* **129**, 250601 (2022).
- [41] X. Xiao, T.-X. Lu, W.-J. Zhong, and Y.-L. Li, Classical-driving-assisted quantum synchronization in non-Markovian environments, *Phys. Rev. A* **107**, 022221 (2023).
- [42] N. Es'haqi-Sani, G. Manzano, R. Zambrini, and R. Fazio, Synchronization along quantum trajectories, *Phys. Rev. Res.* **2**, 023101 (2020).
- [43] G. J. Qiao, X. Y. Liu, H. D. Liu, C. F. Sun, and X. X. Yi, Quantum ϕ synchronization in a coupled optomechanical system with periodic modulation, *Phys. Rev. A* **101**, 053813 (2020).
- [44] X. Li, Y. Li, and J. Jin, Synchronization of persistent oscillations in spin systems with nonlocal dissipation, *Phys. Rev. A* **107**, 032219 (2023).
- [45] W. Li, Analyzing quantum synchronization through Bohmian trajectories, *Phys. Rev. A* **106**, 023512 (2022).
- [46] C. W. Wächtler and G. Platero, Topological synchronization of quantum van der Pol oscillators, *Phys. Rev. Res.* **5**, 023021 (2023).
- [47] M. Cattaneo, G. L. Giorgi, S. Maniscalco, G. S. Paroanu, and R. Zambrini, Bath-induced collective phenomena on superconducting qubits: Synchronization, subradiance, and entanglement generation, *Ann. Phys.* **533**, 2100038 (2021).
- [48] A. Cabot, G. L. Giorgi, and R. Zambrini, Synchronization and coalescence in a dissipative two-qubit system, *Proc. R. Soc. A* **477**, 20200850 (2021).
- [49] B. Buča, C. Booker, and D. Jaksch, Algebraic theory of quantum synchronization and limit cycles under dissipation, *SciPost Phys.* **12**, 097 (2022).
- [50] F. Impens and D. Guéry-Odelin, Shortcut to synchronization in classical and quantum systems, *Sci. Rep.* **13**, 453 (2023).
- [51] D. Štěrba, J. Novotný, and I. Jex, Asymptotic phase-locking and synchronization in two-qubit systems, *J. Phys. Commun.* **7**, 045003 (2023).
- [52] S. Kalita, S. Chakraborty, and A. K. Sarma, Switching of quantum synchronization in coupled optomechanical oscillators, *J. Phys. Commun.* **5**, 115006 (2021).
- [53] A. Parra-López and J. Bergli, Synchronization in two-level quantum systems, *Phys. Rev. A* **101**, 062104 (2020).
- [54] J. Tindall, C. S. Muñoz, B. Buča, and D. Jaksch, Quantum synchronisation enabled by dynamical symmetries and dissipation, *New J. Phys.* **22**, 013026 (2020).
- [55] T. Murtadho, S. Vinjanampathy, and J. Thingna, Cooperation and competition in synchronous open quantum systems, *Phys. Rev. Lett.* **131**, 030401 (2023).
- [56] T. Nadolny and C. Bruder, Macroscopic quantum synchronization effects, *Phys. Rev. Lett.* **131**, 190402 (2023).
- [57] P. Solanki, F. M. Mehdi, M. Hajdušek, and S. Vinjanampathy, Symmetries and synchronization blockade, *Phys. Rev. A* **108**, 022216 (2023).
- [58] L. Razzoli, M. Carrega, F. Cavaliere, G. Benenti, and M. Sassetti, Synchronization-induced violation of thermodynamic uncertainty relations, *Quantum Sci. Technol.* **9**, 045032 (2024).

- [59] T. Kehrer, T. Nadolny, and C. Bruder, Quantum synchronization through the interference blockade, *Phys. Rev. A* **110**, 042203 (2024).
- [60] C. W. Wächtler and J. E. Moore, Topological quantum synchronization of fractionalized spins, *Phys. Rev. Lett.* **132**, 196601 (2024).
- [61] P. Solanki, M. Krishna, M. Hajdušek, C. Bruder, and S. Vinjanampathy, Exotic synchronization in continuous time crystals outside the symmetric subspace, *Phys. Rev. Lett.* **133**, 260403 (2024).
- [62] F. Schmolke and E. Lutz, Measurement-induced quantum synchronization and multiplexing, *Phys. Rev. Lett.* **132**, 010402 (2024).
- [63] T. Nadolny, C. Bruder, and M. Brunelli, Nonreciprocal synchronization of active quantum spins, *Phys. Rev. X* **15**, 011010 (2025).
- [64] C. W. Wächtler and G. Platero, Synchronized Aharonov-Bohm motifs via engineered dissipation, *Phys. Rev. Lett.* **137**, 030405 (2026).
- [65] A. W. Laskar, P. Adhikary, S. Mondal, P. Katiyar, S. Vinjanampathy, and S. Ghosh, Observation of quantum phase synchronization in spin-1 atoms, *Phys. Rev. Lett.* **125**, 013601 (2020).
- [66] V. R. Krithika, P. Solanki, S. Vinjanampathy, and T. S. Mahesh, Observation of quantum phase synchronization in a nuclear-spin system, *Phys. Rev. A* **105**, 062206 (2022).
- [67] L. Zhang, Z. Wang, Y. Wang, J. Zhang, Z. Wu, J. Jie, and Y. Lu, Quantum synchronization of a single trapped-ion qubit, *Phys. Rev. Res.* **5**, 033209 (2023).
- [68] M. Koppenhöfer, C. Bruder, and A. Roulet, Quantum synchronization on the IBM Q system, *Phys. Rev. Res.* **2**, 023026 (2020).
- [69] Y. Li, Z. Xie, X. Yang, Y. Li, X. Zhao, X. Cheng, X. Peng, J. Li, E. Lutz, Y. Lin, and J. Du, Experimental realization and synchronization of a quantum van der Pol oscillator, *Sci. Adv.* **11**, eady5649 (2025).
- [70] Z. Tao, F. Schmolke, C.-K. Hu, W. Huang, Y. Zhou, J. Zhang, J. Chu, L. Zhang, X. Sun, Z. Guo, J. Niu, W. Weng, S. Liu, Y. Zhong, D. Tan, D. Yu, and E. Lutz, Noise-induced quantum synchronization with entangled oscillations, *Nat. Commun.* **16**, 8457 (2025).
- [71] D. A. Lidar and K. Birgitta Whaley, Decoherence-free subspaces and subsystems, in *Irreversible Quantum Dynamics*, edited by F. Benatti and R. Floreanini (Springer, Berlin, 2003), pp. 83–120.
- [72] B. Buča, J. Tindall, and D. Jaksch, Non-stationary coherent quantum many-body dynamics through dissipation, *Nat. Commun.* **10**, 1730 (2019).
- [73] G. Guarnieri, M. T. Mitchison, A. Purkayastha, D. Jaksch, B. Buča, and J. Goold, Time periodicity from randomness in quantum systems, *Phys. Rev. A* **106**, 022209 (2022).
- [74] S. Campbell, J. Eisert, and G. Guarnieri, Stability of emergent time periodicity in a few-body interacting system, *New J. Phys.* **27**, 023001 (2025).
- [75] G. Lindblad, On the generators of quantum dynamical semigroups, *Commun. Math. Phys.* **48**, 119 (1976).
- [76] V. Gorini, A. Kossakowski, and E. C. G. Sudarshan, Completely positive dynamical semigroups of N -level systems, *J. Math. Phys.* **17**, 821 (1976).
- [77] H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University, Oxford, 2002).
- [78] M. Takahashi, *Thermodynamics of One-Dimensional Solvable Models* (Cambridge University, Cambridge, England, 1999).
- [79] P. Zanardi and M. Rasetti, Noiseless quantum codes, *Phys. Rev. Lett.* **79**, 3306 (1997).
- [80] D. A. Lidar, I. L. Chuang, and K. B. Whaley, Decoherence-free subspaces for quantum computation, *Phys. Rev. Lett.* **81**, 2594 (1998).
- [81] E. Knill, R. Laflamme, and L. Viola, Theory of quantum error correction for general noise, *Phys. Rev. Lett.* **84**, 2525 (2000).
- [82] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/yrfl-y3z8> for the determination of the DFS structure, the derivation of the closed form of $\langle \sigma_x^{(k)}(t) \rangle_{\text{DFS}}$, the proof of the necessary and sufficient condition for generic edge synchronization, and the proof of the coexistence of generic synchronization and the asymptotic entanglement for the edge qubits.
- [83] E. Lieb, T. Schultz, and D. Mattis, Two soluble models of an antiferromagnetic chain, *Ann. Phys.* **16**, 407 (1961).
- [84] To be precise, the term arbitrary initial states here refers to the states that have nonzero coherences between the DFS states, which guarantees that the oscillatory terms in Eq. (6) remain finite. Initial states that do not satisfy this condition are measure zero within the set of all possible initial states.
- [85] W. K. Wootters, Entanglement of formation of an arbitrary state of two qubits, *Phys. Rev. Lett.* **80**, 2245 (1998).
- [86] J. Liu, Q. Wu, J. E. Moore, H. Haeflner, and C. W. Wächtler, Observation of synchronization between two quantum van der Pol oscillators in trapped ions, *Phys. Rev. X* **16**, 021062 (2026).