

Equal-spin and opposite-spin density-density correlations in the BCS-BEC crossover: Gauge symmetry, Pauli exclusion principle, Wick's theorem, and experiments

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We develop a general theory of *spin*-dependent density-density correlations, that is valid for any temperature, interactions, dimensions, and mass or population status of Fermi gases with two internal states. We use gauge invariance and the Pauli principle to establish constraints on the *spin*-dependent density-density correlations that are consistent with the fluctuation-dissipation and Wick's theorem. As an example, we study the *spin*-dependent density-density correlations from the BCS to the Bose regime in two dimensions at zero temperature, inspired by recent continuum quantum gas microscope experiments in ⁶Li. We show that two-particle irreducible contributions, involving collective excitations, many-particle scattering, and vertex corrections, are essential to describe the experimental data. In particular, the two-particle irreducible terms are responsible for the emergence of an experimentally observed minimum in the opposite-*spin* density-density correlations.

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I. INTRODUCTION

Ultracold Fermi gases represent a versatile platform for investigating fundamental properties of quantum many-body physics, such as correlations that have been explored in condensed matter [1–5], nuclear physics [6–10], astrophysics [11–14], and ultracold gases [15–22], where the latter serve as quantum simulators [23–26]. The measurement of correlations provides deep insight into degenerate Bose [27–30] and Fermi [31–36] systems. Due to the existence of Fano-Feshbach resonances [37–39], a prime example of the manifestation of correlations is the BCS-BEC crossover explored both theoretically [40–45] and experimentally [46–51]. Correlations were further investigated in momentum-frequency space via collective modes in the low-energy long-wavelength limit [35,36,41,52–58]. However, correlations in real space and time have been little explored because of limited experimental accessibility to date.

With the recent development of continuous quantum gas microscopes (CQGM) [59–64], it is now possible to directly measure spatially resolved equal-time density-density correlations in two-dimensional (2D) Fermi gases. These new microscopes differ from previous lattice quantum gas microscopes [24,26,65–71] that were restricted to measurements of spatial correlations at large distances only. The

CQGM has better spatial resolution than the typical inter-particle distance, allowing the detection of antibunching in real space for opposite-*spin* density-density correlations of ⁶Li atoms [63] in 2D.

In this paper, we develop a theory of *spin*-dependent density-density correlations that is valid for any temperature, interactions, dimensions, and mass or population status of Fermi gases with two internal states. We impose two general constraints, gauge symmetry and the Pauli exclusion principle, that must be satisfied by *spin*-dependent density-density correlations and that are consistent with the fluctuation-dissipation and Wick's theorem. Using these conditions, we obtain an equation of state that includes two-particle reducible and irreducible contributions. To show the power of these requirements, we investigate *spin*-dependent density-density correlations from the BCS to the Bose regime in 2D at zero temperature, similar to recent CQGM experiments in ⁶Li [63]. We demonstrate that two-particle irreducible contributions, such as collective excitations, many-particle scattering, and vertex corrections, are essential to explain experiments. In particular, we show that two-particle irreducible terms are responsible for the observed minimum in the opposite-*spin* density-density correlation function below the uncorrelated value of 1 [63]. Finally, we make quantitative predictions about anticorrelations by evaluating both position and depth of this minimum.

The remainder of the paper is organized as follows. In Sec. II, we discuss general constraints that the *spin*-resolved density-density correlation functions need to satisfy, such as gauge invariance and the Pauli exclusion principle. We also connect these correlations to the fluctuation-dissipation theorem. In Sec. III, we apply the general form of Wick's theorem to separate the *spin*-resolved correlation functions

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and the equations of state into sums of two-particle reducible and two-particle irreducible terms. In Sec. IV, we consider a translationally invariant continuum model for the BCS-BEC crossover and explicitly obtain the *spin*-resolved density-density response, as well as the *spin*-dependent equation of state in D dimensions and for any temperature or population imbalance. In Sec. V, we particularize our general results to two dimensions (2D), zero temperature, and equal-*spin* populations, as the simplest example that has connections to recent experiments [63]. In this section, we show the results of several theoretical approximations and reemphasize the importance that the correlation functions investigated need to be gauge invariant and respect the Pauli principle. We also make comparisons with experimental results and their accompanying Monte Carlo simulation [63] for a variety of experimental and numerical data. Finally, we explore essential features of the density-density correlation functions such as the Pauli-hole width in equal-*spin* correlations, the minimum location, and depth in the opposite-*spin* correlations as a function of interactions beyond what can be experimentally measured currently. In Sec. VI, we compare our approach to a very recent work on fermionic correlation functions that use the time-dependent linearized BCS approach. Finally, in Sec. VII, we state our conclusions and indicate an immediate outlook that employs the general expressions derived.

II. CORRELATION FUNCTIONS

We are interested in the normalized equal-time density-density correlation functions

$$g_{ss'}(\mathbf{r}, \mathbf{r}') = \frac{\langle \hat{n}_s(\mathbf{r}) \hat{n}_{s'}(\mathbf{r}') \rangle}{\langle \hat{n}_s(\mathbf{r}) \rangle \langle \hat{n}_{s'}(\mathbf{r}') \rangle}, \quad (1)$$

where $\hat{n}_s(\mathbf{r}) = \hat{\psi}_s^\dagger(\mathbf{r}) \hat{\psi}_s(\mathbf{r})$ is the density operator for *spin* projection s , with $\hat{\psi}_s^\dagger(\mathbf{r})$ and $\hat{\psi}_s(\mathbf{r})$ being the field operators for fermions at position $\mathbf{r}$ in continuum systems. The notation $\langle \hat{A} \rangle$ stands for the ensemble expectation value of operator $\hat{A}$ at temperature T . We use the word *spin* to represent either the internal states of identical fermions with mass $m_\uparrow = m_\downarrow = m$ or different species of fermions with $m_\uparrow \neq m_\downarrow$. For fermions with equal or different masses $m_\uparrow, m_\downarrow$, there are two *spin* projections $s = \{\uparrow, \downarrow\}$ such that the equal-*spin* (opposite-*spin*) correlation function corresponds to $s = s'$ ($s \neq s'$).

Before explicitly calculating $g_{ss'}(\mathbf{r}, \mathbf{r}')$ for a specific Lagrangian, we highlight a few of its symmetries that are independent of dimensionality, interactions, temperature, masses, and population status of *spin* projections. Any theory that aims to calculate real-space correlation functions over a wide range of spatial separations has to satisfy those constraints to be physical.

First, we emphasize the invariance of $g_{ss'}(\mathbf{r}, \mathbf{r}')$ under the local U(1) gauge transformation $\hat{\psi}_s(\mathbf{r}) \rightarrow \hat{\psi}_s(\mathbf{r}) e^{i\phi_s(\mathbf{r})}$, since the local density operator $\hat{n}_s(\mathbf{r})$ is unchanged by it. This is important, because even if the system spontaneously breaks a local U(1) gauge symmetry due to local pairing $\hat{\psi}_\uparrow(\mathbf{r}) \hat{\psi}_\downarrow(\mathbf{r}) \rightarrow \hat{\psi}_\uparrow(\mathbf{r}) \hat{\psi}_\downarrow(\mathbf{r}) e^{i[\phi_\uparrow(\mathbf{r}) + \phi_\downarrow(\mathbf{r})]}$, the physical correlation $g_{ss'}(\mathbf{r}, \mathbf{r}')$ must be gauge invariant.

Second, we investigate the Pauli principle. Using $\{\hat{\psi}_s(\mathbf{r}), \hat{\psi}_{s'}^\dagger(\mathbf{r}')\} = \delta_{ss'} \delta(\mathbf{r} - \mathbf{r}')$, we write $\langle \hat{n}_s(\mathbf{r}) \hat{n}_{s'}(\mathbf{r}') \rangle = \langle \hat{n}_s(\mathbf{r}) \hat{n}_{s'}(\mathbf{r}') \rangle + \delta_{ss'} \delta(\mathbf{r} - \mathbf{r}') \langle \hat{n}_s(\mathbf{r}) \rangle$ in terms of the

normal ordered pair correlations $\langle \hat{n}_s(\mathbf{r}) \hat{n}_{s'}(\mathbf{r}') \rangle = \langle \hat{\psi}_s^\dagger(\mathbf{r}) \hat{\psi}_{s'}^\dagger(\mathbf{r}') \hat{\psi}_{s'}(\mathbf{r}') \hat{\psi}_s(\mathbf{r}) \rangle$. Thus we obtain

$$g_{ss'}(\mathbf{r}, \mathbf{r}') = g_{ss'}^{\text{reg}}(\mathbf{r}, \mathbf{r}') + \frac{\delta_{ss'} \delta(\mathbf{r} - \mathbf{r}')}{\langle \hat{n}_{s'}(\mathbf{r}') \rangle}, \quad (2)$$

where the regular part reads

$$g_{ss'}^{\text{reg}}(\mathbf{r}, \mathbf{r}') = \frac{\langle \hat{n}_s(\mathbf{r}) \hat{n}_{s'}(\mathbf{r}') \rangle}{\langle \hat{n}_s(\mathbf{r}) \rangle \langle \hat{n}_{s'}(\mathbf{r}') \rangle}. \quad (3)$$

We emphasize that the only difference between $g_{ss'}(\mathbf{r}, \mathbf{r}')$ and $g_{ss'}^{\text{reg}}(\mathbf{r}, \mathbf{r}')$ in Eqs. (2) and (3) is the singular term that arises for $s = s'$ and $\mathbf{r} = \mathbf{r}'$. The Pauli principle manifests itself in $\langle \hat{n}_s(\mathbf{r}) \hat{n}_s(\mathbf{r}) \rangle = 0$, leading to $\lim_{\mathbf{r}' \rightarrow \mathbf{r}} g_{ss}^{\text{reg}}(\mathbf{r}, \mathbf{r}') = 0$, but $\langle \hat{n}_s(\mathbf{r}) \hat{n}_s(\mathbf{r}) \rangle = \delta(\mathbf{0}) \langle \hat{n}_s(\mathbf{r}) \rangle$ is singular, yielding $\lim_{\mathbf{r}' \rightarrow \mathbf{r}} g_{ss}(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{0}) / \langle \hat{n}_s(\mathbf{r}) \rangle$.

Using the decomposition $\hat{n}_s(\mathbf{r}) = \langle \hat{n}_s(\mathbf{r}) \rangle + \delta \hat{n}_s(\mathbf{r})$, with the ensemble average $\langle \hat{n}_s(\mathbf{r}) \rangle$ being equal to the local density $n_s(\mathbf{r}) = \langle \hat{n}_s(\mathbf{r}) \rangle$, gives

$$g_{ss'}(\mathbf{r}, \mathbf{r}') = 1 + \frac{\chi_{ss'}(\mathbf{r}, \mathbf{r}')}{n_s(\mathbf{r}) n_{s'}(\mathbf{r}')}, \quad (4)$$

where $\chi_{ss'}(\mathbf{r}, \mathbf{r}') = \langle \delta \hat{n}_s(\mathbf{r}) \delta \hat{n}_{s'}(\mathbf{r}') \rangle$ represents the connected density-density correlation tensor. In the case of $g_{ss'}(\mathbf{r}, \mathbf{r}') = 1$, the particles at $\mathbf{r}$ and $\mathbf{r}'$ are uncorrelated, while for $g_{ss'}(\mathbf{r}, \mathbf{r}') < 1$, the fermions exclude (correlation holes, anti-bunching) and for $g_{ss'}(\mathbf{r}, \mathbf{r}') > 1$, they approach each other (clustering, pairing, bunching). Comparing Eqs. (2) and (4), we write

$$\chi_{ss'}(\mathbf{r}, \mathbf{r}') = \chi_{ss'}^{\text{reg}}(\mathbf{r}, \mathbf{r}') + \delta_{ss'} \delta(\mathbf{r} - \mathbf{r}') n_s(\mathbf{r}), \quad (5)$$

where the regular part of $\chi_{ss'}(\mathbf{r}, \mathbf{r}')$ is

$$\chi_{ss'}^{\text{reg}}(\mathbf{r}, \mathbf{r}') = \langle \hat{n}_s(\mathbf{r}) \hat{n}_{s'}(\mathbf{r}') \rangle - n_s(\mathbf{r}) n_{s'}(\mathbf{r}'), \quad (6)$$

leading to the manifestation of the Pauli principle as

$$n_s^2(\mathbf{r}) = - \lim_{\mathbf{r}' \rightarrow \mathbf{r}} \chi_{ss}^{\text{reg}}(\mathbf{r}, \mathbf{r}'). \quad (7)$$

The relation in Eq. (7) corresponds to the local equation of state (EoS) that fixes the *spin*-resolved density $n_s(\mathbf{r})$. The resulting EoS is consistent with the fluctuation-dissipation theorem [72], that is,

$$\chi_{ss'}(\mathbf{r}, \mathbf{r}') = - \frac{T}{V^2} \frac{\delta^2 \Omega[j_s]}{\delta j_s(\mathbf{r}) \delta j_{s'}(\mathbf{r}')}, \quad (8)$$

where $\Omega[j_s]$ stands for the thermodynamic potential including the density-fluctuation source term $j_s(\mathbf{r})$. The tensor $\chi_{ss'}(\mathbf{r}, \mathbf{r}')$ is sometimes called the spatially dependent compressibility matrix [72].

Having established the general symmetry constraints on the fermionic correlation functions and the *spin*-resolved EoS, we turn our attention to the general form of Wick's theorem [73]. Within this framework, the *spin*-resolved density-density correlations and the *spin*-resolved EoS are expressed in terms of two-particle reducible and irreducible contributions, as discussed next.

III. WICK'S THEOREM

Using Wick's decomposition for the equal-time pair-correlation function gives [73]

$$\langle \hat{n}_s(\mathbf{r}) \hat{n}_{s'}(\mathbf{r}') \rangle = \langle \hat{n}_s(\mathbf{r}) \rangle \langle \hat{n}_{s'}(\mathbf{r}') \rangle - |G_{ss'}(\mathbf{r}, \mathbf{r}')|^2 + |F_{ss'}(\mathbf{r}, \mathbf{r}')|^2 + \langle \hat{n}_s(\mathbf{r}) \hat{n}_{s'}(\mathbf{r}') \rangle_{\text{irr}}. \quad (9)$$

Here, the first term $\langle \hat{n}_s(\mathbf{r}) \rangle \langle \hat{n}_{s'}(\mathbf{r}') \rangle$ is the product of local densities $n_s(\mathbf{r}) n_{s'}(\mathbf{r}')$. The second term represents the negative of the squared modulus of the normal (standard) Green's functions $G_{ss'}(\mathbf{r}, \mathbf{r}') = \langle \hat{\psi}_s^\dagger(\mathbf{r}) \hat{\psi}_{s'}(\mathbf{r}') \rangle$. The third term denotes the squared modulus of anomalous (pair) Green's functions $F_{ss'}(\mathbf{r}, \mathbf{r}') = \langle \hat{\psi}_s(\mathbf{r}) \hat{\psi}_{s'}(\mathbf{r}') \rangle$. The last term $\langle \hat{n}_s(\mathbf{r}) \hat{n}_{s'}(\mathbf{r}') \rangle_{\text{irr}} = \langle \delta \hat{n}_s(\mathbf{r}) \delta \hat{n}_{s'}(\mathbf{r}') \rangle_{\text{irr}}$ stands for the two-particle irreducible part of the *spin*-dependent density-density correlations.

Using Wick's theorem in Eq. (9) and the relation in Eq. (6), we write

$$\chi_{ss'}^{\text{reg}}(\mathbf{r}, \mathbf{r}') = \chi_{\text{red}, ss'}(\mathbf{r}, \mathbf{r}') + \chi_{\text{irr}, ss'}(\mathbf{r}, \mathbf{r}'), \quad (10)$$

where the two-particle reducible contribution takes the form

$$\chi_{\text{red}, ss'}(\mathbf{r}, \mathbf{r}') = -|G_{ss'}(\mathbf{r}, \mathbf{r}')|^2 + |F_{ss'}(\mathbf{r}, \mathbf{r}')|^2, \quad (11)$$

while the two-particle irreducible contribution reads

$$\chi_{\text{irr}, ss'}(\mathbf{r}, \mathbf{r}') = \langle \delta \hat{n}_s(\mathbf{r}) \delta \hat{n}_{s'}(\mathbf{r}') \rangle_{\text{irr}}. \quad (12)$$

Combining Eqs. (2) and (10), we obtain

$$g_{ss'}^{\text{reg}}(\mathbf{r}, \mathbf{r}') = 1 + \delta g_{\text{red}, ss'}(\mathbf{r}, \mathbf{r}') + \delta g_{\text{irr}, ss'}(\mathbf{r}, \mathbf{r}'), \quad (13)$$

with the correlation contributions $\delta g_{\alpha, ss'}(\mathbf{r}, \mathbf{r}') = \chi_{\alpha, ss'}(\mathbf{r}, \mathbf{r}')/n_s(\mathbf{r}) n_{s'}(\mathbf{r}')$, where $\alpha \in \{\text{red}, \text{irr}\}$ corresponds to deviations from 1, that is, the uncorrelated limit. Furthermore, the Pauli principle from Eq. (7) separates the local EoS into reducible and irreducible contributions

$$n_s^2(\mathbf{r}) = - \lim_{\mathbf{r}' \rightarrow \mathbf{r}} [\chi_{\text{red}, ss}(\mathbf{r}, \mathbf{r}') + \chi_{\text{irr}, ss}(\mathbf{r}, \mathbf{r}')]. \quad (14)$$

In translationally invariant systems, $n_s(\mathbf{r}) = n_s$ and the general relations from Eqs. (13) and (14) become

$$n_s^2 = - \lim_{\delta \mathbf{r} \rightarrow 0} [\chi_{\text{red}, ss}(\delta \mathbf{r}) + \chi_{\text{irr}, ss}(\delta \mathbf{r})] \quad (15)$$

for the EoS that obeys the Pauli principle, and

$$g_{ss'}^{\text{reg}}(\delta \mathbf{r}) = 1 + \delta g_{\text{red}, ss'}(\delta \mathbf{r}) + \delta g_{\text{irr}, ss'}(\delta \mathbf{r}) \quad (16)$$

for the normalized *spin*-resolved density-density correlation function. Here, $\delta \mathbf{r} = \mathbf{r}' - \mathbf{r}$ labels the relative position and $\delta g_{\alpha, ss'}(\delta \mathbf{r}) = \chi_{\alpha, ss'}(\delta \mathbf{r})/n_s n_{s'}$ are correlation contributions.

Next, we specialize to a translationally invariant continuum model for the BCS-BEC crossover, for which we obtain explicitly the *spin*-resolved density-density response and the *spin*-resolved EoS.

IV. BCS-BEC CROSSOVER

From now on, we consider a *spin* mixture of fermions in D dimensions with *spin* $s = \{\uparrow, \downarrow\}$ described by the Lagrangian

density

$$\mathcal{L} = \sum_s \bar{\psi}_s(x) \left[\partial_\tau - \frac{\nabla^2}{2m_s} - \mu_s \right] \psi_s(x) - g \bar{\psi}_\uparrow(x) \bar{\psi}_\downarrow(x) \psi_\downarrow(x) \psi_\uparrow(x) - \sum_s j_s(x) n_s(x), \quad (17)$$

defined in the volume L^D . Here, $\bar{\psi}_s(x)$ and $\psi_s(x)$, having dimensions $L^{-D/2}$, are the Grassmann fields of *spin* s describing field operators $\hat{\psi}_s^\dagger(x)$ and $\hat{\psi}_s(x)$ at $x = (\mathbf{r}, \tau)$, with $\mathbf{r}$ being the position and $\tau \in [0, \beta]$ being the imaginary time. Furthermore, μ_s represents the chemical potential, m_s the mass of the *spin* species, and $g \geq 0$ denotes the local SU(2)-invariant attractive interaction strength with dimensions of energy times volume. We use $\beta = 1/T$ as inverse temperature and units where $k_B = 1 = \hbar$.

The Lagrangian density (17) includes a coupling between real *spin*-dependent source fields $j_s(x)$ with dimensions of energy, and real density fields $n_s(x)$ with dimensions of inverse volume. This leads to the dimensionless action $\mathcal{S}[\bar{\psi}, \psi; j_s] = \int dx \mathcal{L}$, with $\int dx = \int_0^\beta d\tau \int_{L^D} d^D r$ used to obtain the source-dependent thermodynamic potential $\Omega[j_s] = -T \ln \mathcal{Z}[j_s]$, where the source-dependent partition function reads $\mathcal{Z}[j_s] = \oint \mathcal{D}[\bar{\psi}, \psi] e^{-\mathcal{S}[\bar{\psi}, \psi; j_s]}$.

To compute the *spin*-resolved density-density correlator in the superfluid state, we decouple the interaction in terms of the complex pair field $\Delta(x)$ via a Hubbard-Stratonovich transformation [74,75], and go to Fourier space. Integrating out both the fermions and the pair fluctuations to lowest order, see Appendix A, leads to the action contribution

$$\mathcal{S}_2[j_s] = \frac{\beta}{2} \sum_q \mathbf{j}_q^T \tilde{\chi}(q) \mathbf{j}_q, \quad (18)$$

where the *spin*-resolved density-density tensor with dimensions of inverse energy is

$$\tilde{\chi}(q) = \tilde{\chi}_0(q) + \tilde{\chi}_{\text{irr}}(q). \quad (19)$$

Here, $\mathbf{j}_q^T$ denotes the Fourier transform of $\mathbf{j}^T(x) = (j_\uparrow(x), j_\downarrow(x))$ with $q = (\mathbf{q}, iq_\ell)$, where $\mathbf{q}$ is the momentum and q_ℓ represents the bosonic Matsubara frequency. Note that $\tilde{\chi}(q)$ in Eq. (19) is an approximate, Pauli-principle preserving, and gauge-invariant solution to the generalized Bethe-Salpeter equation [76]. The matrix elements of $\tilde{\chi}(q)$ are $\tilde{\chi}_{ss'}(q) = L^D \int_0^\beta d\tau \int d^D r \chi_{ss'}(\delta \mathbf{r}, \delta \tau) e^{-i\mathbf{q} \cdot \delta \mathbf{r} + iq_\ell \delta \tau}$, describing the Fourier transform of the real-space density-density correlators $\chi_{ss'}(\delta \mathbf{r}, \delta \tau) = \langle \delta \hat{n}_s(\mathbf{0}, 0) \delta \hat{n}_{s'}(\delta \mathbf{r}, \delta \tau) \rangle$ with dimensions of L^{-2D} .

The matrix $\tilde{\chi}_0(q)$ in Eq. (19) describes the connected reducible *spin*-resolved density-density response including both $\chi_{\text{red}, ss'}(\delta \mathbf{r})$ from Eq. (11) and a constant term arising from the singular part of $\chi_{ss'}(\delta \mathbf{r})$, defined in Eq. (5), when $n_s(\mathbf{r}) = n_s$. The second term $\tilde{\chi}_{\text{irr}}(q)$ represents the two-particle irreducible contribution describing collective excitations, many-body scattering states, and vertex corrections, as discussed in Eq. (12) for its real-space counterpart.

To obtain the Gaussian fluctuation (GF) EoS, arising only from pair fluctuations, we set $j_s = 0$ and use $\Omega = -T \ln \mathcal{Z}[j_s = 0]$, yielding $n_s = n_{s, \text{sp}} + n_{s, \text{fl}}$. Here, the first term $n_{s, \text{sp}}$ comes from the saddle point and the second $n_{s, \text{fl}}$

is due to pair fluctuations, see Eq. (A14) in Appendix A. The GF EoS is globally gauge invariant in the sense that it depends only on the order-parameter modulus $|\Delta_0|$. But it violates the Pauli principle, because it does not include the two-particle irreducible contributions.

Identifying the reducible part as $\lim_{\delta\mathbf{r} \rightarrow 0} \chi_{\text{red,ss}}(\delta\mathbf{r}) = \sum_q \tilde{\chi}_{\text{red,ss}}(q)/(\beta L^{2D})$ and the irreducible component as $\lim_{\delta\mathbf{r} \rightarrow 0} \chi_{\text{irr,ss}}(\delta\mathbf{r}) = \sum_q \tilde{\chi}_{\text{irr,ss}}(q)/(\beta L^{2D})$, Eq. (15) becomes

$$n_s^2 = -\frac{1}{\beta L^{2D}} \sum_q \tilde{\chi}_{\text{red,ss}}(q) - \frac{1}{\beta L^{2D}} \sum_q \tilde{\chi}_{\text{irr,ss}}(q), \quad (20)$$

representing the Pauli-respecting (PR) EoS, which is also gauge invariant. A discussion of solutions for different EoS in 2D is relegated to Appendix B.

Thus far, we have discussed general results for the *spin*-resolved density-density response in the BCS-BEC crossover for SU(2) invariant (*s*-wave) contact interactions at any spatial dimension, temperature, interaction, mass or population imbalance. Next, we specialize $\chi_{ss'}(\delta\mathbf{r}) = \langle \delta\hat{n}_s(\mathbf{0})\delta\hat{n}_{s'}(\delta\mathbf{r}) \rangle$ and $g_{ss'}(\delta\mathbf{r})$ for the BCS-BEC crossover in two dimensions at $T = 0$ for equal masses and populations to compare with recent experiments [63], which were performed close to this limiting case.

V. CORRELATION FUNCTIONS IN 2D

Using rotational invariance at $T = 0$, we find

$$\chi_{ss'}(|\delta\mathbf{r}|) = \frac{1}{L^2} \int_{-\infty}^{\infty} \frac{dq_\ell}{2\pi} \int_0^{\infty} \frac{dQ}{2\pi} Q \tilde{\chi}_{ss'}(Q, iq_\ell) J_0(Q|\delta\mathbf{r}|), \quad (21)$$

where integration over the azimuthal angle leads to the zeroth-order Bessel function $J_0(Q|\delta\mathbf{r}|)$ with $|\delta\mathbf{r}|$ and $Q = |\mathbf{q}|$ being the modulus of $\delta\mathbf{r}$ and $\mathbf{q}$, respectively. The expression above comes from $\chi_{ss'}(|\delta\mathbf{r}|) = (T/L^4) \sum_{iq_\ell, \mathbf{q}} \tilde{\chi}_{ss'}(\mathbf{q}, iq_\ell) \exp(i\mathbf{q} \cdot \delta\mathbf{r})$, where the summations over three-momenta ($\mathbf{q}, iq_\ell$) are made explicit. At $T = 0$, we evaluate the bosonic Matsubara sum as a Riemann integral after a Wick rotation into imaginary frequency. This procedure captures the virtual contributions of phonons (longitudinal modes), vortices-antivortices (transverse modes), gapped excitations, and the particle-particle continuum all at once, without invoking any momentum or energy cutoff.

We apply the Lippmann-Schwinger relation [77] to replace the interaction g by the two-body bound-state energy $\varepsilon_B > 0$ and use the Fermi momentum (energy) k_F (ε_F) to relate ε_B to the 2D scattering length a as $\ln k_F a = \ln(8\varepsilon_F/\varepsilon_B)/2 - \gamma_E$, with $\gamma_E \approx 0.577$ being the Euler-Mascheroni constant. When comparing to experimental data, we need to consider that the parameter a used in Ref. [63] represents the size of a dimer in quasi-2D, while our parameter a denotes the scattering length in 2D. Thus, the experimental scattering parameter η in Ref. [63] and our theoretical scattering parameter $\ln k_F a$ differ by a constant numerical factor, which is given by $\ln k_F a = \eta - \gamma_E + \ln 2 \approx \eta + 0.116$.

In Fig. 1, we illustrate the behavior of $g_{ss'}(\delta\mathbf{r})$, defined in Eq. (16), and of the full density-density correlation function $g_{nn}(\delta\mathbf{r}) = \sum_{ss'} g_{ss'}(\delta\mathbf{r})/4$ versus separation $k_F|\delta\mathbf{r}|$. The interaction parameters are $\ln k_F a = 2.15$ (BCS regime) and

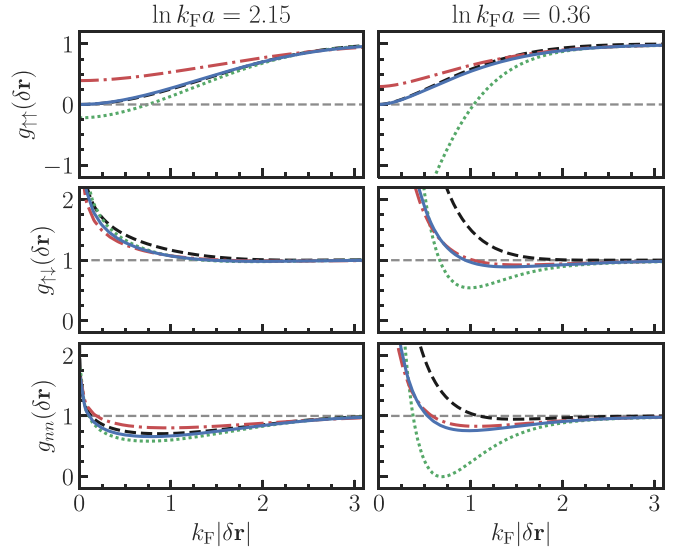


FIG. 1. Spatial behavior of correlation functions $g_{ss'}(\delta\mathbf{r})$ and $g_{nn}(\delta\mathbf{r})$ for translationally and rotationally invariant systems. The interaction parameters are $\ln k_F a = 2.15$ (BCS regime) and $\ln k_F a = 0.36$ (crossover region). Line types indicate used approximation level, see Table I.

$\ln k_F a = 0.36$ (crossover region). The different approximation levels and line types are summarized in Table I. For $g_{\uparrow\uparrow}(\delta\mathbf{r})$, the dotted green lines (case II) and dash-dotted red lines (case III) violate the Pauli exclusion principle, while the dashed black lines (case I) and the solid blue lines (case IV) satisfy it. For $g_{\uparrow\downarrow}(\delta\mathbf{r})$, the dotted green lines (case II) and the dash-dotted red lines (case III) do not differ substantially from the dashed black lines (case I) and the solid blue lines (case IV) in the BCS regime ($\ln k_F a = 2.15$). But they start to behave differently in the crossover region ($\ln k_F a = 0.36$) and beyond due to contributions from the two-particle irreducible terms. The main purpose of Fig. 1 is to show that the approximations used in cases II and III of Table I are gauge invariant but violate the Pauli principle, while the approaches in cases I and IV satisfy both gauge invariance and the Pauli principle.

In Fig. 2, we depict only cases I and IV, showing the correlation functions $g_{\uparrow\uparrow}(|\delta\mathbf{r}|)$, $g_{\uparrow\downarrow}(|\delta\mathbf{r}|)$, $g_{nn}(|\delta\mathbf{r}|)$ versus $k_F|\delta\mathbf{r}|$ for scattering parameters $\ln k_F a = \{2.24, 1.09, 0.43\}$. Furthermore, a comparison to Monte Carlo (dotted cyan lines) and experimental (solid orange circles) data from Refs. [63, 78] is

TABLE I. Different cases covering distinct approximation levels, depending on preservation (✓) or violation (×) of gauge invariance (Gauge) and Pauli principle (Pauli) for computing correlation function $\chi_{ss'}$. Gauge invariance applied to EoS ($\chi_{ss'}$) is global (local). Corresponding line styles used in subsequent figures are also indicated.

Case	Approximation	Gauge	Pauli	Line style
I	SP EoS + only $\chi_{ss'}^{(0)}$	✓	✓	Dashed black
II	SP EoS + full $\chi_{ss'}$	✓	×	Dotted green
III	GF EoS + full $\chi_{ss'}$	✓	×	Dash-dotted red
IV	PR EoS + full $\chi_{ss'}$	✓	✓	Solid blue

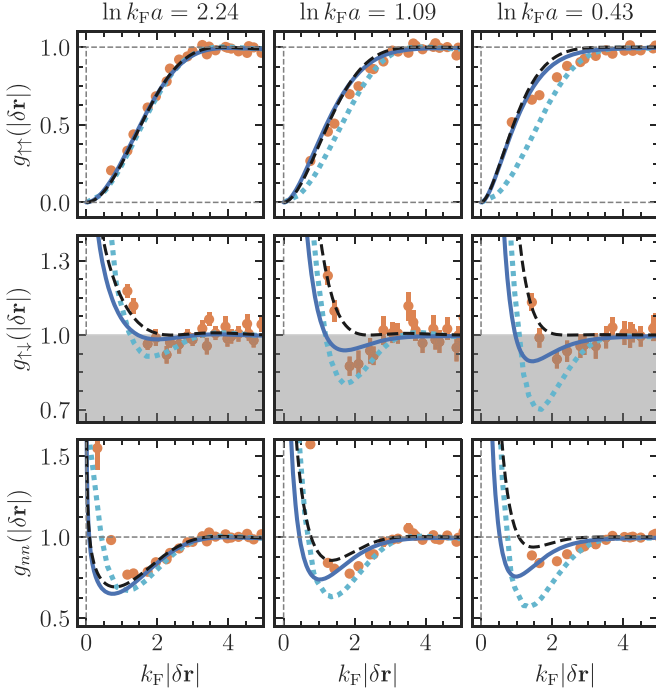


FIG. 2. Numerical results for $g_{\uparrow\uparrow}(\delta\mathbf{r})$ ranging from 0 and 1 (top panel), $g_{\uparrow\downarrow}(\delta\mathbf{r})$ (middle panel) ranging from 0.7 to 1.4, and $g_{nn}(\delta\mathbf{r})$ (bottom panel) ranging from 0.5 to 1.5 vs distance $k_F|\delta\mathbf{r}|$ ranging from 0 to 5. The dashed black (solid blue) lines illustrate case I (IV) in Table I, with the gray region in middle panel showing that $g_{\uparrow\downarrow}(\delta\mathbf{r}) \geq 1$ with no minimum for dashed black line, while $g_{\uparrow\downarrow}(\delta\mathbf{r})$ has clear minimum below 1 for solid blue line. The dotted cyan line and solid orange circles describe Monte Carlo and experimental data from Refs. [63,78] near $T/T_F = \{0.11, 0.09, 0.08\}$ for $\ln k_F a = \{2.24, 1.09, 0.43\}$.

made. An extended version of Fig. 2 is shown in Fig. 5 of Appendix B.

In Fig. 2 (top panel), we demonstrate that the difference between cases I and IV is small in $g_{\uparrow\uparrow}(\delta\mathbf{r})$ for the chosen parameters. Due to the absence of triplet pairing in our model, that is, $F_{\uparrow\uparrow}(\delta\mathbf{r}) = 0$, the only contribution to $g_{\uparrow\uparrow}(\delta\mathbf{r})$, for case I, comes from the two-particle reducible correlations $\delta g_{\text{red},\uparrow\uparrow}(\delta\mathbf{r}) = \chi_{\text{red},\uparrow\uparrow}(\delta\mathbf{r})/n_{\uparrow}^2 = -|G_{\uparrow\uparrow}(\delta\mathbf{r})|^2/n_{\uparrow}^2$. This term is always comparable to the sum of $\delta g_{\text{red},\uparrow\uparrow}(\delta\mathbf{r})$ and $\delta g_{\text{irr},\uparrow\uparrow}(\delta\mathbf{r}) = \chi_{\text{irr},\uparrow\uparrow}(\delta\mathbf{r})/n_{\uparrow}^2$ for case IV in the parameter range shown. The spatial extent $k_F r_p$ describes the size of the *Pauli hole* when $g_{\uparrow\uparrow}(\delta\mathbf{r}) = 0.5$.

In Fig. 2 (middle panel), we show that the differences between cases I and IV are more dramatic in $g_{\uparrow\downarrow}(\delta\mathbf{r})$ for the parameters shown. In case I, the only contribution to $g_{\uparrow\downarrow}(\delta\mathbf{r})$ comes from the two-particle reducible correlations $\delta g_{\text{red},\uparrow\downarrow} = \chi_{\text{red},\uparrow\downarrow}(\delta\mathbf{r})/n_{\uparrow}n_{\downarrow} = |F_{\uparrow\downarrow}(\delta\mathbf{r})|^2/n_{\uparrow}n_{\downarrow}$ being always positive. There is no negative contribution arising, since $G_{\uparrow\downarrow}(\delta\mathbf{r}) = 0$ due to *spin*-projection conservation along the *spin*-*z* axis. In case IV, however, there are two contributions: $\delta g_{\text{red},\uparrow\downarrow}(\delta\mathbf{r})$, which is always positive (bunching), and $\delta g_{\text{irr},\uparrow\downarrow}(\delta\mathbf{r})$, which is always negative (antibunching) due to the coupling to pair fluctuations. Case I (dashed black line) always gives $g_{\uparrow\downarrow}(\delta\mathbf{r}) \geq 1$ and does not produce a minimum in $g_{\uparrow\downarrow}(\delta\mathbf{r})$. The minimum in $g_{\uparrow\downarrow}(\delta\mathbf{r})$ arises from the interplay

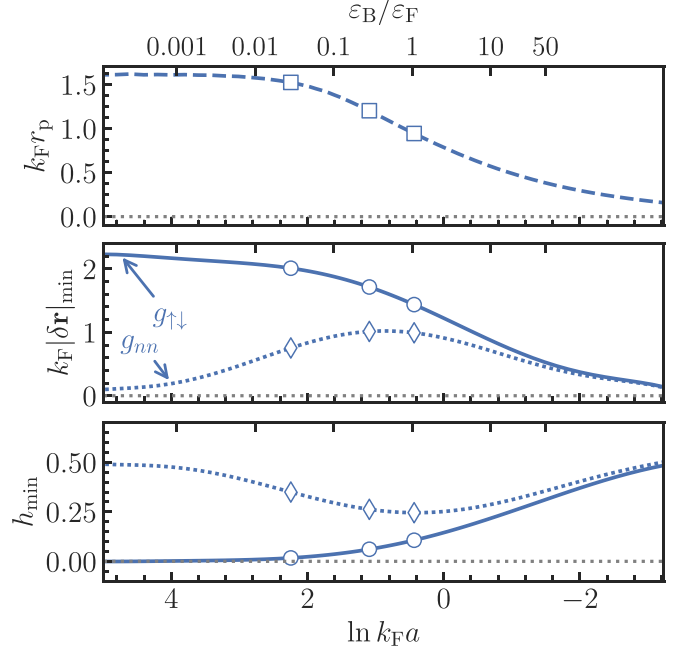


FIG. 3. Plots of $k_F r_p$ (top panel), $k_F|\delta\mathbf{r}|_{\min}$ (middle panel), and $h_{\min}$ (bottom panel) vs $\ln k_F a$. The depth of the minimum is $h_{\min} = |g_{\uparrow\downarrow}(\delta\mathbf{r}_{\min}) - 1|$ or $h_{\min} = |g_{nn}(\delta\mathbf{r}_{\min}) - 1|$. The solid blue lines refer to $g_{\uparrow\downarrow}(\delta\mathbf{r})$, the dotted blue lines represent $g_{\uparrow\downarrow}(\delta\mathbf{r})$, and the dashed blue line reflects $g_{\uparrow\uparrow}(\delta\mathbf{r})$. The blue circles, diamonds, and squares describe the scattering parameters used in Fig. 2.

between two-particle reducible and irreducible contributions, where the latter are included only in case IV (solid blue line).

In Fig. 2 (bottom panel), we establish that cases I and IV also disagree substantially for $g_{nn}(\delta\mathbf{r})$, for the parameters used, due to the differences in $g_{\uparrow\downarrow}(\delta\mathbf{r})$. In the range of parameters shown, both cases I and IV produce a minimum for $g_{nn}(\delta\mathbf{r})$ because of the increase in $g_{\uparrow\uparrow}(\delta\mathbf{r}) = g_{\downarrow\downarrow}(\delta\mathbf{r})$ and the decrease in $g_{\uparrow\downarrow}(\delta\mathbf{r}) = g_{\downarrow\uparrow}(\delta\mathbf{r})$ with growing $k_F|\delta\mathbf{r}|$ in the range from 0 to $\lesssim 1$. However, the location and depth of the minimum in $g_{nn}(\delta\mathbf{r})$ differ from that of $g_{\uparrow\downarrow}(\delta\mathbf{r})$. To clarify this behavior further, we discuss next both the location and the depth of the minima in $g_{\uparrow\downarrow}(\delta\mathbf{r})$ and $g_{nn}(\delta\mathbf{r})$, as well as $k_F r_p$ characterizing the *Pauli hole* in $g_{\uparrow\uparrow}(\delta\mathbf{r})$.

Although a comparison of our work, at $T = 0$, with current Monte Carlo (MC) and experimental results [63], at $T/T_F \approx 0.1$ is good in the BCS regime ($\ln k_F a = 2.24$), deviations occur as the crossover region ($\ln k_F a = 0.43$) is approached. These differences are attributed to finite-temperature effects and to experiments no longer being in 2D as the crossover region is entered. Furthermore, MC calculations potentially deal with a different equation of state that exaggerates the minimum in $g_{nn}(|\delta\mathbf{r}|)$. Further comparisons are available in Appendix B.

In Fig. 3, we show the size of the *Pauli-hole* $k_F r_p$ (top panel), the position of the minimum $k_F|\delta\mathbf{r}|_{\min}$ (middle panel) in $g_{\uparrow\downarrow}(\delta\mathbf{r})$ and $g_{nn}(\delta\mathbf{r})$, as well as its depth $h_{\min}$ (bottom panel) versus $\ln k_F a$, for case IV only. As seen in the top panel, $k_F r_p$ becomes smaller with growing $\ln k_F a$ approaching zero asymptotically in the extreme Bose regime for low densities. The location and depth of the minimum in $g_{\uparrow\downarrow}(\delta\mathbf{r})$ and

$g_{nn}(\delta\mathbf{r})$ differ substantially in the BCS regime, because $k_F r_p$ is large enough to produce a deeper minimum located at shorter distances in $g_{nn}(\delta\mathbf{r})$. However, as $k_F r_p$ decreases with increasing $\ln k_F a$, the location $k_F |\delta\mathbf{r}|_{\min}$ and depth $h_{\min}$ of the minimum in $g_{nn}(\delta\mathbf{r})$ approach those of $g_{\uparrow\downarrow}(\delta\mathbf{r})$, since the latter correlations become more important.

Having analyzed the correlation functions for a 2D system at $T = 0$ and discussed our numerical results, in the next section, we compare our approach to a quite recent work on fermionic correlation functions, using the time-dependent linearized BCS approach.

VI. COMPARISON WITH RECENT WORK

After the completion of our work, we became aware of a preprint in French by Y. Castin [79] that analyzed *spin*-dependent density-density correlations using the linearized time-dependent BCS (LTD-BCS). Therein, the equal-*spin* and opposite-*spin* density-density correlation functions were investigated for Fermi superfluids with equal populations both in 2D and 3D at temperatures $T \rightarrow 0^+$, where particle-hole excitations and their associated Landau damping were neglected. Furthermore, analytical asymptotic limits at short and long distances were obtained for the correlation functions by including linearized phonon excitations up to a momentum cutoff. In addition, a comparison between the LTD-BCS theory and experiments was made in 2D for a single value of $\ln(k_F a) = 1.11$. The LTD-BCS theory was also compared to Monte Carlo [17] in 3D for a single value of the scattering length $a_{3D} = \infty$ (unitarity limit).

Our present work, submitted prior to the published version of that preprint [80], contains substantial conceptual differences both in methods employed and in symmetries invoked. We derived general expressions for the *spin*-resolved density-density correlation functions using the fluctuation-dissipation theorem and the general form of Wick's theorem [73] relating time-ordered and normal ordered correlations via contractions and two-particle irreducible contributions. Then, we obtained the Pauli-respecting equation of state for each *spin* component in population balanced or imbalanced Fermi systems with arbitrary interactions. Subsequently, we analyzed a specific model for the BCS-BEC crossover in D dimensions for contact interactions, where analytic expressions for the *spin*-dependent correlation functions were derived for arbitrary population imbalance. Only after all that, we specialized our general approach to equal populations in two dimensions and zero temperature while providing an extensive comparison between our results as well as Monte Carlo and experimental data for a variety of scattering parameters from the BCS to the BEC regime.

Finally, we should emphasize that at $T = 0$, our work differs from Castin's in that we calculate our correlation functions using a different method. We perform the summation over Matsubara frequencies viewed as a Riemann sum, which can be calculated via a Wick's rotation and integration over the imaginary frequency axis, thus capturing the virtual contributions of phonons (longitudinal modes), vortices–antivortices (transverse modes), gapped excitations, and the particle-particle continuum all at once, without invoking any momentum or energy cutoff.

On a more quantitative level, at $\ln k_F a = 1.11$ and for $T = 0.09T_F$, Castin finds a chemical potential of $\mu \approx 0.56\varepsilon_F$, compared to our result of $\mu \approx 0.54\varepsilon_F$ at $T = 0$. We note that $\mu = 0.86\varepsilon_F$ at the mean-field (saddle-point) level, while $\mu \approx 0.44\varepsilon_F$ for the standard Gaussian fluctuation theory used in case III of Table I. The minimum in Castin's opposite-*spin* correlation function is located at $k_F |\delta\mathbf{r}|_{\min} \approx 1.738$ and $g_{\uparrow\downarrow}(|\delta\mathbf{r}|_{\min}) \approx 0.941$, compared to our value $k_F |\delta\mathbf{r}|_{\min} \approx 1.741$ and $g_{\uparrow\downarrow}(|\delta\mathbf{r}|_{\min}) \approx 0.938$. These results are basically the same at the numerical accuracy used in the comparison. Thus, for this particular choice of parameters, where the Fermi system is still substantially degenerate, the results are nearly indistinguishable. However, our results might no longer be similar for stronger interactions, where the chemical potential is negative, that is, $\mu/\varepsilon_F < 0$. In this case, the relative spectral weights of virtual contributions from phonons (longitudinal modes), vortices–antivortices (transverse modes), gapped excitations, and the particle-particle continuum may change.

Having completed a comparison to recent theoretical work, we are ready to state our conclusions next.

VII. CONCLUSIONS

We developed a general theory of *spin*-dependent density-density correlations that is valid for any temperature, interactions, dimensions, and mass or population status of Fermi *spin* mixtures. We used gauge invariance and the Pauli principle to establish constraints on the *spin*-dependent density-density correlations that are consistent with the fluctuation-dissipation and Wick's theorem. As an example, we studied the *spin*-dependent density-density correlations from the BCS to the Bose regime in two dimensions at zero temperature, inspired by experiments in ^6Li [63]. We showed that two-particle irreducible contributions, such as collective excitations, many-particle scattering, and vertex corrections, are responsible for the differences between saddle-point and fluctuation approaches. Two-particle irreducible terms slow down the closing of the *Pauli hole* with increasing interactions when compared to saddle-point results; they are also responsible for the minimum in the opposite-*spin* density-density correlations, which is absent for saddle-point theories, in agreement with recent experiments [63].

As an outlook, we will calculate density-density correlations at finite temperatures, which are technically more challenging, because it is necessary to either perform the full Matsubara summation or introduce a contour to handle branch cuts and poles (collective excitations) of the correlation function with the additional complication of including particle-hole excitations with Landau damping, as well as thermal vortices and antivortices in two dimensions.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: RESPONSE FUNCTIONS

Performing a Fourier transformation on the fermions,

$$c_s(\mathbf{k}, ik_n) = \int_0^\beta \frac{d\tau}{\beta} \int \frac{d^D x}{\sqrt{L^D}} \psi_s(\mathbf{x}, \tau) e^{-i\mathbf{k} \cdot \mathbf{x} + ik_n \tau}, \quad (\text{A1})$$

and integrating them through a standard procedure [40] leads to the dimensionless effective action

$$S_{\text{eff}}[j_s] = \beta L^D \sum_q \frac{|\Delta_q|^2}{g} - \sum_{k, k'} \ln \det[-\beta \mathbf{G}^{-1}]. \quad (\text{A2})$$

Here, the label $q = (\mathbf{q}, iq_\ell)$ represents momentum $\mathbf{q}$ and bosonic Matsubara frequency q_ℓ , while “det” denotes the determinant on *spin* indices. Using the momentum space relation $\Delta_q = \int_0^\beta \frac{d\tau}{\beta} \int \frac{d^D x}{L^D} \Delta(x) e^{-i\mathbf{q} \cdot \mathbf{x} + iq_\ell \tau}$, we obtain

$$\mathbf{G}^{-1}(k, k') = \begin{pmatrix} \alpha_\uparrow(k, k') - j_{\uparrow, k-k'} & \Delta_{k-k'} \\ \bar{\Delta}_{k'-k} & \alpha_\downarrow(k, k') + j_{\downarrow, k-k'} \end{pmatrix}, \quad (\text{A3})$$

with diagonal terms $\alpha_\uparrow(k, k') = (ik_n - \xi_{\uparrow, \mathbf{k}})\delta_{k, k'}$ and $\alpha_\downarrow(k, k') = (ik_n + \xi_{\downarrow, \mathbf{k}})\delta_{k, k'}$ involving the kinetic energies $\xi_{s, \mathbf{k}} = \mathbf{k}^2/2m_s - \mu_s$ with respect to chemical potentials μ_s . We use the notation $k = (\mathbf{k}, ik_n)$ for momentum $\mathbf{k}$ and fermionic Matsubara frequency k_n . The term $j_{s, q}$ is related to $j_s(x)$ via the transformation $j_{s, k-k'} = \int_0^\beta \frac{d\tau}{\beta} \int \frac{d^D x}{L^D} j_s(\mathbf{x}, \tau) e^{-i(\mathbf{k}-\mathbf{k}') \cdot \mathbf{x} + i(k_n - k'_n)\tau}$.

Since our goal is to compute the locally gauge-invariant *spin*-resolved density-density correlation functions, it is essential to include the gauge freedom of the pair field. We decompose $\Delta_q = \Delta_0 \delta_{q,0} + \lambda_q + i\theta_q$, where Δ_0 is the complex time-independent and spatially uniform order parameter, and λ_q and θ_q are pair-fluctuation real fields necessary to guarantee the local gauge invariance of the full *spin*-dependent density-density correlations.

We expand $S_{\text{eff}}[j_s]$ to quadratic order in λ_q and θ_q , followed by an expansion to Gaussian order in $j_{s, q}$. This procedure gives three contributions:

$$S_{\text{eff}}[j_s] = S_0 + S_1[j_s] + S_2[j_s, \lambda, \theta] + \mathcal{O}(j_s^3, \lambda^3, \theta^3). \quad (\text{A4})$$

The leading-order term of the effective action is

$$S_0 = \beta L^D \frac{|\Delta_0|^2}{g} - \sum_k \ln \det[-\beta \mathbf{G}_0^{-1}(k)], \quad (\text{A5})$$

where the saddle-point inverse Green's function reads

$$\mathbf{G}_0^{-1}(k) = \begin{pmatrix} \alpha_\uparrow(k, k) & \Delta_0 \\ \bar{\Delta}_0 & \alpha_\downarrow(k, k) \end{pmatrix}. \quad (\text{A6})$$

Extremizing S_0 , that is, $\partial S_0 / \partial \bar{\Delta}_0 = 0$, leads to

$$\Delta_0 = \frac{g}{\beta L^D} \sum_k \text{tr} \left\{ \mathbf{G}_0(k) \frac{\partial \mathbf{G}_0^{-1}(k)}{\partial \bar{\Delta}_0} \right\}. \quad (\text{A7})$$

The first-order correction amounts to

$$S_1[j_s] = - \sum_{k, s} G_{0, ss}(k) j_{s, 0}, \quad (\text{A8})$$

where $G_{0, ss}(k)$ is a diagonal matrix element of $\mathbf{G}_0(k)$. The linear terms in λ and θ vanish due to the saddle-point condition in Eq. (A7). The second term yields

$$S_2[j_s, \lambda, \theta] = \frac{\beta}{2} \sum_q \boldsymbol{\phi}_{-q}^T \boldsymbol{\Pi}(q) \boldsymbol{\phi}_q, \quad (\text{A9})$$

where the fields $\boldsymbol{\phi}_{-q}^T = (j_{-q}^T \quad \eta_{-q}^T)$ have dimensions of energy and include the real and imaginary fluctuations of the pair field $\eta_{-q}^T = (\lambda_{-q} \quad \theta_{-q})$. The source currents $j_{-q}^T = (j_{\uparrow, -q} \quad j_{\downarrow, -q})$ play the role of a nonuniform *fluctuation* in the chemical potentials $\mu_\uparrow$ and $\mu_\downarrow$. The elements of $\boldsymbol{\Pi}$, having dimensions of inverse energy, are

$$\Pi_{ab}(q) = v_{ab} + \frac{1}{\beta} \sum_k \text{tr} \{ \mathbf{G}_0(k) \boldsymbol{\gamma}_a \mathbf{G}_0(k+q) \boldsymbol{\gamma}_b \}, \quad (\text{A10})$$

where $v_{ab} = 2\delta_{ab}(\delta_{ax} + \delta_{ay})/(L^D g)$, with $\{a, b\} \in \{\uparrow, \downarrow, x, y\}$. The $\boldsymbol{\gamma}_a$ are 2×2 matrices defined as $\boldsymbol{\gamma}_\uparrow = -(\mathbf{I} + \boldsymbol{\sigma}_z)/2$, $\boldsymbol{\gamma}_\downarrow = (\mathbf{I} - \boldsymbol{\sigma}_z)/2$, $\boldsymbol{\gamma}_x = \boldsymbol{\sigma}_x$, $\boldsymbol{\gamma}_y = \boldsymbol{\sigma}_y$, where $\mathbf{I}$ is the identity, and $\boldsymbol{\sigma}_x$, $\boldsymbol{\sigma}_y$, and $\boldsymbol{\sigma}_z$ are the Pauli matrices.

The notation above simplifies the description of $\boldsymbol{\Pi}$ but hides its physical meaning. Thus we write

$$\begin{aligned} \boldsymbol{\Pi}(q) &= \begin{pmatrix} \Pi_{\uparrow\uparrow}(q) & \Pi_{\uparrow\downarrow}(q) & \Pi_{\uparrow x}(q) & \Pi_{\uparrow y}(q) \\ \Pi_{\downarrow\uparrow}(q) & \Pi_{\downarrow\downarrow}(q) & \Pi_{\downarrow x}(q) & \Pi_{\downarrow y}(q) \\ \Pi_{x\uparrow}(q) & \Pi_{x\downarrow}(q) & \Pi_{xx}(q) & \Pi_{xy}(q) \\ \Pi_{y\uparrow}(q) & \Pi_{y\downarrow}(q) & \Pi_{yx}(q) & \Pi_{yy}(q) \end{pmatrix} \\ &= \left(\begin{array}{c|c} \tilde{\boldsymbol{\chi}}_0(q) & \boldsymbol{\Lambda}^T(q) \\ \hline \boldsymbol{\Lambda}(q) & \boldsymbol{\Gamma}(q) \end{array} \right) \end{aligned} \quad (\text{A11})$$

in a block-diagonal form. The matrix elements $\tilde{\boldsymbol{\chi}}_0(q)$ are simply source-source (jj) correlation functions, that is, the symbols $\{\uparrow, \downarrow\}$ represent $\{j_\uparrow, j_\downarrow\}$, respectively. The matrix $\boldsymbol{\Lambda}(q)$ describes the coupling between the source terms $\{j_\uparrow, j_\downarrow\} \leftrightarrow \{\uparrow, \downarrow\}$ and the pair fluctuations $\{\lambda, \theta\} \leftrightarrow \{x, y\}$. The block matrix $\boldsymbol{\Gamma}(q)$ represents the pair fluctuations only $\{\lambda, \theta\} \leftrightarrow \{x, y\}$. Integrating out the fluctuation fields η_q leads to

$$S_{\text{eff}}[j_s] = S_{\text{GF}} + S_1[j_s] + S_2[j_s], \quad (\text{A12})$$

where the Gaussian fluctuation (GF) action is

$$S_{\text{GF}} = S_0 + \sum_q \ln \det[\beta \boldsymbol{\Gamma}(q)]. \quad (\text{A13})$$

Here, $S_1[j_s]$ and $S_2[j_s]$ are defined in Eqs. (A8) and (18), respectively.

From $S_2[j_s]$, we can read off $\tilde{\boldsymbol{\chi}}(q) = \tilde{\boldsymbol{\chi}}_0(q) + \tilde{\boldsymbol{\chi}}_{\text{irr}}(q)$, displayed in Eq. (19) of the main text. The first term $\tilde{\boldsymbol{\chi}}_0(q)$ contains the connected two-particle reducible contributions and a constant term from the singular part of $\chi_{ss'}(\delta\mathbf{r})$. The second term $\tilde{\boldsymbol{\chi}}_{\text{irr}}(q) = \boldsymbol{\Lambda}^T(-q) \boldsymbol{\Gamma}^{-1}(q) \boldsymbol{\Lambda}(q)$ represents the two-particle irreducible contribution describing collective excitations, many-body scattering states, and vertex corrections arising from the coupling between density and pair fluctuations. For instance, the integer zeros of $\boldsymbol{\Gamma}(q)$ represent the collective excitations due to pair fluctuations [58]. When relating $\tilde{\chi}_{ss'}(q)$ to the dynamical structure factor tensor $S_{ss'}(q)$,

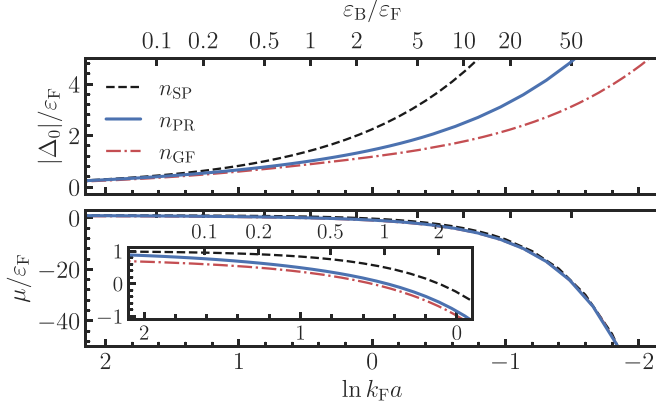


FIG. 4. Self-consistent results for $|\Delta_0|/\varepsilon_F$ (top panel) and μ/ε_F (bottom panel) vs $\ln k_F a$ (lower x axis) or binding energy $\varepsilon_B/\varepsilon_F$ (upper x axis). The dashed black line shows saddle-point (SP), the dash-dotted red line describes Gaussian fluctuations (GF), while the solid blue line represents Pauli-respecting (PR) equation of state (EoS).

the contribution from $\chi_{\text{irr,ss}}(q)$ is essential to fulfill the compressibility and the f-sum rules [20,72].

Finally, the action $\mathcal{S}_{\text{GF}}$ in Eq. (A13) gives the GF equation of state (EoS),

$$n_s = \frac{1}{L^D \beta} \sum_k G_{0,\text{ss}}(k) - \frac{1}{2L^D \beta} \sum_q \frac{\partial}{\partial \mu_s} \ln \det[\beta \Gamma(q)], \quad (\text{A14})$$

where the first term comes from $\mathcal{S}_0$ and represents the saddle-point (SP) EoS, whereas the second term is due to pair fluctuations.

APPENDIX B: CROSSOVER IN 2D

Two-dimensional ($D = 2$) systems exhibit a two-body bound state with binding energy ε_B for arbitrarily small contact interactions. In contrast, for three dimensions ($D = 3$), there is a threshold for the emergence of two-body bound states. Furthermore, in 3D, the inclusion of longitudinal fluctuations in λ, θ are sufficient to describe the BCS-BEC crossover at finite temperatures. However, in 2D, we must also include transverse fluctuations (vortices and antivortices) due to the Berezinskii-Kosterlitz-Thouless (BKT) mechanism. Thus, for now, we only investigate the simplest experimentally relevant example of a 2D Fermi system with balanced masses ($m_\uparrow = m_\downarrow = m$) and populations ($n_\uparrow = n_\downarrow = n/2$) at $T = 0$.

In Fig. 4, using the Fermi energy ε_F , we show self-consistent solutions of $|\Delta_0|/\varepsilon_F$ and μ/ε_F for saddle-point (SP) (dashed-black line), saddle-point plus Gaussian fluctuations (GFs) (dashed-dotted red line), and Pauli-respecting (PR) (solid blue). The results for $|\Delta_0|/\varepsilon_F$ (μ/ε_F) are illustrated in the top (bottom) panel versus $\ln k_F a$ ($\varepsilon_B/\varepsilon_F$), depicted as lower (upper) x axis. The values of $|\Delta_0|/\varepsilon_F$ and μ/ε_F for the PR equation of state (EoS) always lie between SP and GF results.

In Fig. 5 we present an extension of the data found in Fig. 2 of the main text. The interaction parameters range now from $\ln k_F a = 4.00$ (deeper into the BCS region) to $\ln k_F a = -0.37$ (further into the BEC regime). The plots in Fig. 5 further support the findings in the main text.

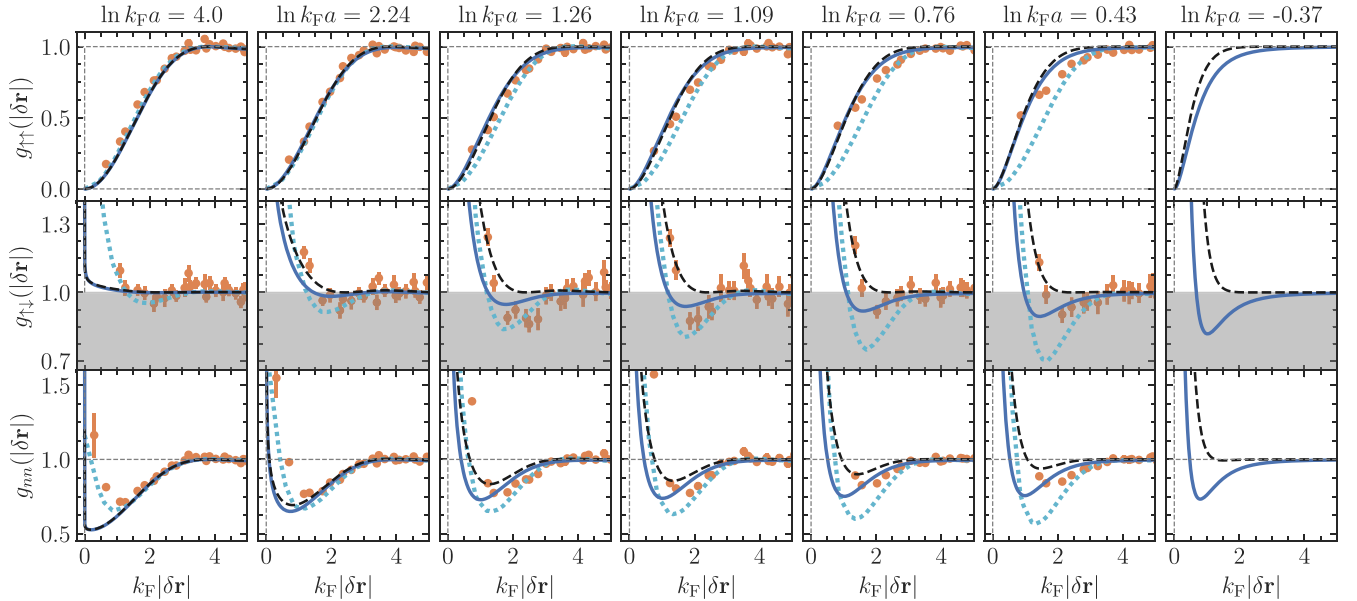


FIG. 5. Numerical results for $g_{\uparrow\uparrow}(\delta\mathbf{r})$ (top panel), $g_{\uparrow\downarrow}(\delta\mathbf{r})$ (middle panel), and $g_{nn}(\delta\mathbf{r})$ (bottom panel) versus $k_F|\delta\mathbf{r}|$ with $\ln k_F a$ ranging from 4.0 (BCS regime) to -0.37 (BEC regime). The dashed black (solid blue) lines illustrate case I (IV) in Table I. Here, the gray region in the middle panel shows that $g_{\uparrow\downarrow}(\delta\mathbf{r}) \geq 1$ with no minimum for the dashed black line, while $g_{\uparrow\downarrow}(\delta\mathbf{r})$ has a clear minimum below 1 for the solid blue line. The dotted cyan line and solid orange circles describe Monte Carlo and experimental data, respectively, from Refs. [63,78].

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