

Chiral phases and dynamics of dipoles in triangular optical ladders

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Dipoles in triangular optical ladders constitute a flexible platform for the study of the interplay between geometric frustration and long-range anisotropic interactions, and in particular for the observation of the spontaneous onset of chirality. Frustration magnifies the effect of the dipolar interactions in itinerant polarized dipolar bosons. As a result, the dipole-induced transition between a chiral superfluid and a nonchiral two-component superfluid may be observed for current state-of-the-art temperatures even for the weak intersite interaction characterizing magnetic atoms in standard optical lattices. On the other hand, pinned spin-1/2 dipoles, which we discuss in the context of polar molecules in two rotational states, realize frustrated dipolar XXZ spin models. By controlling the external electric field strength and orientation, these systems can explore a rich ground-state landscape including chiral and nematic phases, as well as intriguing chiral dynamics.

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Frustrated many-body lattice systems are crucial in quantum magnetism [1,2]. Prominent examples include antiferromagnets on triangular [3,4] or kagome lattices [5,6], and one-dimensional (1D) J_1 - J_2 models [7]. Frustration suppresses conventional magnetic order and amplifies the role of quantum fluctuations, thereby opening the way to exotic ground states such as quantum spin liquids [8], valence-bond solids [9] and chiral phases [10]. The latter exhibit a preferred handedness, breaking spatial inversion symmetry. In 1D chains, they are characterized by the vector chirality $\kappa_i = \mathbf{S}_i \times \mathbf{S}_{i+1}$, which measures the rotational sense of neighboring spins. An analogous behavior arises in itinerant bosons on frustrated lattices or under artificial gauge fields, which can host chiral superfluids (CSFs), where currents circulate spontaneously [11–25].

Ultracold particles in optical lattices and tweezer arrays are an excellent platform for the study of many-body quantum systems. They allow for the realization of models in which lattice geometry, dimensionality, and connectivity can be precisely engineered. In particular, CSFs, resulting from contact interactions and two nonequivalent minima in the Brillouin zone, have been realized in the second Bloch band of s - p and hexagonal lattices [16,19]. Chiral currents have been also observed in the presence of synthetic magnetism [13,15] and in triangular flux ladders [26]. Frustrated classical magnetism has been simulated in triangular lattices [27], but the study of

frustrated quantum magnetism with contact-interacting atoms is limited by weak superexchange intersite couplings. Lattice experiments using dipolar gases, formed by either magnetic atoms [28], polar molecules [29], or Rydberg atoms [30], open interesting possibilities for the study of frustration, since the anisotropic and long-range character of the dipolar interaction results in strong direction-dependent intersite interactions. Extended Bose-Hubbard models (EBHMs) formed by polarized itinerant bosons have been recently engineered in square optical lattices using magnetic atoms [31,32], whereas spin models, formed by pinned dipoles with internal degrees of freedom, have been realized using magnetic atoms [33,34], polar molecules [35–38], and Rydberg arrays [39,40]. Experiments on polar molecules and Rydberg atoms have explored dipolar t - J models [41,42], including the formation of kinetically induced bound states in triangular-ladder arrays [43]. Recently, it was proposed that an effective frustrated triangular ladder for magnetic atoms could be realized with two Raman-coupled Zeeman states in a 1D antimagic wavelength lattice, significantly enhancing the relatively weak nearest-neighbor interactions, which could permit the observation of a CSF-to-supersolid transition [24].

In this Letter, we study the interplay between a frustrated triangular-ladder geometry and the anisotropic long-range dipolar interactions, with a particular emphasis on how that interplay may allow for the observation of chiral phases and dynamics. Geometric frustration magnifies the effect of dipolar interactions in itinerant dipoles allowing for the observation of a dipole-induced transition between a CSF and a nonchiral two-component superfluid for state-of-the-art temperatures [32,44,45] even for the weak intersite interaction characterizing magnetic atoms in standard lattices [32]. In

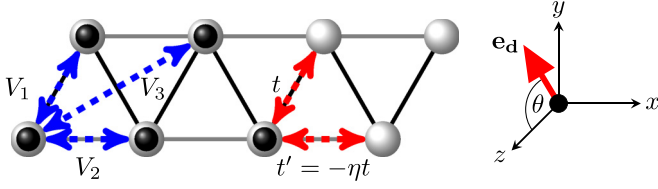


FIG. 1. Scheme of dipolar bosons on a triangular ladder.

dipolar spin models, which we discuss in the context of polar molecules, we show that by controlling the strength and orientation of an external electric field, it may be possible to explore a rich ground-state phase diagram including chiral and nematic phases, and intriguing chiral dynamics.

Itinerant dipoles. We first consider itinerant dipolar bosons in a triangular ladder on the xy plane, with legs along x (Fig. 1). An external field orients the dipoles along the direction $\mathbf{e}_d$ on the yz plane at an angle θ with the z axis. For deep-enough lattices the system is described by the EBHM,

$$\hat{H} = -t \sum_i (\hat{b}_i^\dagger \hat{b}_{i+1} + \text{H.c.}) - t' \sum_i (\hat{b}_i^\dagger \hat{b}_{i+2} + \text{H.c.}) + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) + \sum_{i,r>0} V(\mathbf{R}_{i+r} - \mathbf{R}_i) \hat{n}_i \hat{n}_{i+r}, \quad (1)$$

where $\hat{b}_i^\dagger$ creates bosons at site i , and $\hat{n}_i = \hat{b}_i^\dagger \hat{b}_i$. Sites with odd (even) index i belong to the lower (upper) leg, and are centered at $\mathbf{R}_i = \frac{i}{2} \mathbf{e}_x$ ($\mathbf{R}_i = \frac{i}{2} \mathbf{e}_x + \frac{\sqrt{3}}{2} \mathbf{e}_y$), where we set the intersite spacing to 1. The hopping rate along the rungs is $t > 0$, whereas, crucially, that along the legs is $t' = -\eta t < 0$. This may be achieved using Floquet engineering [46] or Raman-assisted hopping [47], or alternatively using negative temperature simulations [25]. On-site repulsive interactions, resulting from the interplay of contact and dipolar interactions, are characterized by $U > 0$. In contrast, intersite interactions result purely from the dipole-dipole potential $V(\mathbf{R}) = \frac{V}{|\mathbf{R}|^3} [1 - 3(\mathbf{e}_d \cdot \mathbf{R})^2 / |\mathbf{R}|^2]$, where $V > 0$ determines the dipolar strength.

The single-particle dispersion $\varepsilon_q = -2t[\cos(q) - \eta \cos(2q)]$ has a single minimum at $q = 0$ for $\eta < 1/4$, while for $\eta > 1/4$ it displays two degenerate minima at $q = \pm Q_0$, with $Q_0 = \pm \arccos(1/4\eta)$. For interacting bosons, we may expect that, as long as insulating phases do not occur, the ground-state remains a superfluid (SF) phase for $\eta < 1/4$, with a single peak at $q = 0$ in the quasimomentum distribution $n(q) = \sum_{ij} e^{iq(i-j)} \langle \hat{b}_i^\dagger \hat{b}_j \rangle$. For $\eta > 1/4$, interactions may favor one of two phases. In one of them, the bosons occupy just one of the two minima, spontaneously breaking the time-reversal symmetry ($q \rightarrow -q$) of (1), forming a superfluid for which $n(q)$ has a single peak at $q = \pm Q_0$. This so-called chiral superfluid (CSF) phase [12,17,18,20,22,24] exhibits a vortex-antivortex current distribution around the plaquettes. We characterize this phase as that with $\kappa^2 > 0$, where $\kappa^2 = \lim_{|i-j| \rightarrow \infty} \langle \hat{k}_i \hat{k}_j \rangle$ is the long-range chirality-chirality correlation, with $\hat{k}_j = \frac{1}{2i} (\hat{b}_{j+1}^\dagger \hat{b}_j - \hat{b}_j^\dagger \hat{b}_{j+1})$ the current operator. Alternatively, the bosons may occupy two equally populated peaks at $q = \pm Q$, with $Q > 0$. This two-component superfluid phase (2SF) [12,17,18,20] is a Tomonaga-Luttinger liquid (TLL) with conformal central

charge $c = 2$ formed by two mutually incoherent intertwined superfluids. Since time-reversal symmetry is not broken, the 2SF presents no chiral currents.

Purely on-site interactions ($V = 0$) favor the CSF. In contrast, as illustrated in Fig. 2(a) for $U/t = 1$, $\theta = 0$, and lattice filling $\rho = 0.2$ [results obtained using density-matrix renormalization group (DMRG), employing the TeNPy library [48], for 120 sites with open-boundary conditions], repulsive dipolar interactions induce a CSF-2SF transition at a critical V/t (see Supplemental Material Ref. [49]). This may be qualitatively understood from a mean-field approach in momentum space, $\hat{b}_q \rightarrow \alpha_q$, which results in the mean-field energy

$$E[\{\alpha_q\}] = \sum_q \varepsilon_q \alpha_q + \frac{\rho}{2} \sum_q \tilde{V}_q \left| \sum_k \alpha_{k+q}^* \alpha_k \right|^2, \quad (2)$$

where the momentum distribution $|\alpha_q|^2$ is normalized to 1. The momentum-dependent interactions $\tilde{V}_q = U + 2 \sum_{r>0} V_r \cos(qr)$ favor pairs of peaks at $\pm Q$, which minimize $\tilde{V}_{2Q}$, and hence the 2SF. It is clear that the inherent anisotropy of the dipolar interaction may play a crucial role here. Indeed, the chiral character may be controlled by the dipole orientation. Tilting the dipoles towards the y axis (increasing θ) reduces the nearest-neighbor (NN) interaction $V(1 - \frac{9}{4} \sin^2 \theta)$, while keeping the next-to-NN interactions fixed at V . This may drive a 2SF-CSF transition at fixed V/t and η , by tuning θ [see Fig. 2(b)].

Interestingly, whereas large V/t , well over 1, is typically necessary to reveal dipolar effects, such as density waves [32] or Haldane insulators [50], the critical V/t for observing the dipole-induced CSF-2SF transition may be very small, approaching zero at the vicinity of $\eta = 1/4$, as a result of the flat-band dispersion at $q = 0$ induced by frustration [17]. This is particularly interesting for experiments with magnetic atoms [31,32], for which the intersite interaction is relatively weak, e.g., $V/h \simeq 30$ Hz for erbium atoms at an intersite separation of 266 nm. Hence, dipolar effects at $V/t \sim 5$ could be observed only at the expense of severely reducing the hopping rate t/h down to few Hz, which results in problems related to lattice stability and finite temperature. Crucially, working at much lower V/t , and hence larger t , eases the constraints related to finite temperature effects. For example, working with $V/t = 0.5$ in erbium gases in standard optical lattices would demand $t/h \simeq 60$ Hz, over one order of magnitude more than in Ref. [32]. The mean-field picture of Eq. (2) reveals that the CSF and 2SF phases are separated by an energy gap $\Delta_{\text{CSF-2SF}} \sim \rho \tilde{V}_{2Q}$. For low filling factors $\rho \ll 1$, this implies $\Delta_{\text{CSF-2SF}} \ll t$, limiting the observation of the dipole-induced CSF-2SF transition to temperatures $T \ll t/k_B$, with k_B the Boltzmann constant. Fortunately, at larger ρ the critical temperature may be significantly higher, as illustrated in Fig. 2(c) for $\rho = 1$, $U/t = 1$, $\eta = 0.6$, and $\theta = 0$, for a system of $L = 30$ sites [51]. We use imaginary time evolution of purification matrix product states [52,53] to obtain a thermal density matrix which describes the system at a finite T . The measured κ^2 indicates that, as discussed above, chirality may be destroyed by both thermal and dipolar effects, but realistically reachable temperatures of $k_B T \sim 0.3t$ [44,45] are enough to reveal the dipole-induced destruction of chirality at $V/t \sim 0.5$.

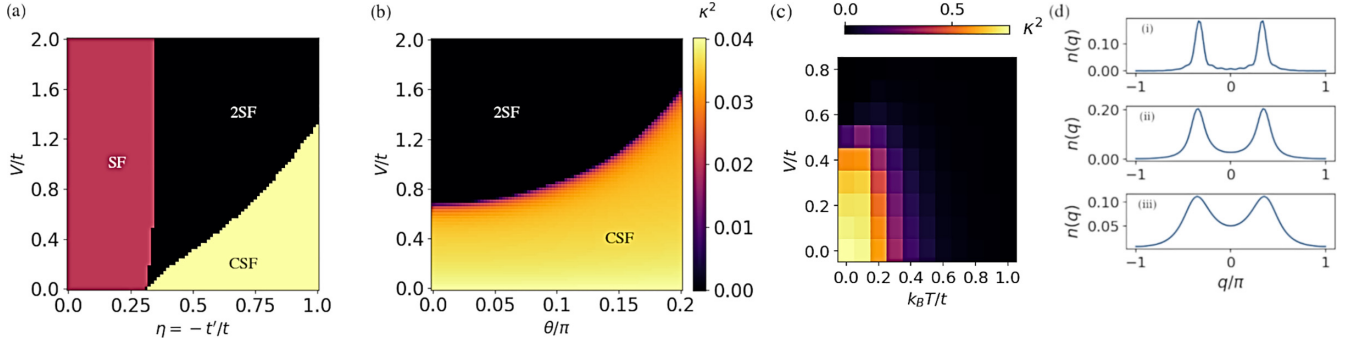


FIG. 2. (a) Phase diagram for the EBHM (1) with $U/t = 1$, $\theta = 0$, and lattice filling $\rho = 0.2$ on a ladder with 120 sites, obtained using DMRG, employing the TeNPy library [48], with a maximum bond dimension of 300, applying open-boundary conditions, and considering up to three particles per site. (b) κ^2 as a function of θ and V/t for the same system at fixed $U/t = 1$, and $\eta = 0.75$. (c) κ^2 as a function of V/t and $k_B T/t$ for $U/t = 1$, $\eta = 0.6$, and $\rho = 1$ for a ladder with 30 sites. (d) Momentum distribution $n(q)$ in the 2SF at fixed $\eta = 0.6$ (i) $k_B T = 0$, $V = 0.7t$, (ii) $k_B T = 0.6t$, $V = 0.4t$, and (iii) $k_B T = 1.8t$, $V = 0.2t$.

Finally, note that the resulting thermally induced nonchiral phase still presents two well-resolved quasimomentum peaks even at $k_B T > t$ [see Fig. 2(d)].

Pinned dipolar spins. Dipolar gases offer an alternative scenario for the onset of chirality without the need of Floquet engineering, which we discuss in the context of polar molecules. We consider a triangular ladder as that of Fig. 1 (formed by an optical lattice or a tweezer array) with each site occupied by a single pinned polar molecule. We assume an external electric field $\mathbf{E}$ along the direction $\mathbf{e}_d$ on the yz plane, forming an angle θ with the z axis ($\mathbf{e}_d$ sets the quantization axis). As in recent KRb experiments [41] we consider the pseudo-spin-1/2 system given by the two lowest dressed rotational states with zero rotational angular momentum projection along the quantization axis ($|0, 0\rangle' \equiv |\downarrow\rangle$ and $|1, 0\rangle' \equiv |\uparrow\rangle$). Dipole-dipole interactions result in spin-exchange and Ising interactions [54], well described by an extended version (beyond next-to-NNs) of the anisotropic J_1 - J_2 Hamiltonian,

$$\hat{H} = \sum_{i,r>0} J_r \left[\frac{1}{2} (\hat{S}_i^+ \hat{S}_{i+r}^- + \hat{S}_i^- \hat{S}_{i+r}^+) + \Delta \hat{S}_i^z \hat{S}_{i+r}^z \right], \quad (3)$$

where $\hat{S}_i^{\pm,z}$ are the spin-1/2 operators for the molecule at site i , $J_{2j+1} = \frac{V}{(j^2-j+1)^{3/2}} (1 - \frac{9 \sin^2 \theta}{4(j^2-j+1)})$, and $J_{2j} = V/j^3$. The exchange anisotropy Δ is determined by $|\mathbf{E}|$ [see Fig. 3(a) for the case of KRb molecules [41]]. For all θ values, the spin model is frustrated and the magnon dispersion at $\Delta = 0$, $\epsilon_q = \sum_{r>0} J_r \cos(qr)$, presents two nonzero minima in the Brillouin zone.

Figure 3(b) shows the ground-state phase diagram for a magnetization $\langle \hat{S}^z \rangle = -0.4$ [49]. Dominant exchange (small enough Δ) results in a chiral phase [55] (a magnon CSF), similar to that reported in 1D multiferroic cuprates [56]. In the spin model, the chiral phase is characterized by a nonzero chirality $\langle \hat{k}_i \rangle$, with $\hat{k}_i = (\hat{\mathbf{S}}_i \times \hat{\mathbf{S}}_{i+1})^z$. As with the EBHM, we identify the chiral phase as that with $\kappa^2 > 0$, where again $\kappa^2 = \lim_{|i-j| \rightarrow \infty} \langle \hat{k}_i \hat{k}_j \rangle$. For small enough $\theta < \arcsin(2/3)$, all Ising interactions $J_r \Delta$ are repulsive, and hence at a critical Δ (which plays a similar role as V/t in the EBHM) there is a transition from the chiral to the nonchiral two-component TLL (TLL₂ phase [57]), which may be considered a magnon

2SF. In the vicinity of $\theta \sim \arcsin(2/3)$, $J_1 \simeq 0$, causing the two legs to decouple, leading to a regime of two separate TLLs in each leg (1D-TLLs), for which the interleg correlations $\langle \hat{S}_i^+ \hat{S}_{i+2n-1}^- \rangle \simeq 0$. For larger $\theta > \arcsin(2/3)$, the chiral phase is recovered at small Δ , but it does not transition into a TLL₂ when Δ grows. In this regime, $J_1 < 0$ and $J_{r>1} > 0$, resulting at a critical Δ in the formation of NN magnon pairs which delocalize along the legs via resonant next-to-NN exchange. The chiral phase thus transitions into a quadrupolar nematic phase [10,58,59], i.e., a magnon pair superfluid, characterized by exponentially decaying $C(r) \equiv \langle \hat{S}_j^+ \hat{S}_{j+r}^- \rangle$, and polynomially decaying $D(r) = \langle \hat{S}_j^+ \hat{S}_{j+1}^+ \hat{S}_{j+r}^- \hat{S}_{j+r+1}^+ \rangle$.

The anisotropy of the dipolar interactions allows for the exploration of markedly different phase diagrams by merely changing the dipole orientation, as illustrated in Fig. 3(c) [49] for $\langle \hat{S}^z \rangle = -0.4$ and dipoles on the xz plane forming an angle θ with the z axis. In that case, $J_{2j-1} = \frac{V}{(j^2-j+1)^{3/2}} (1 - \frac{3(j-1/2)^2 \sin^2(\theta)}{(j^2-j+1)})$ and $J_{2j} = \frac{V}{j^3} (1 - 3 \sin^2 \theta)$. For $\theta < \theta_0 \simeq 0.16\pi$, the system is frustrated, experiencing a chiral-to-TLL₂ transition at a critical Δ . For $\theta > \theta_0$, the system is unfrustrated (single minimum in ϵ_q), and at low enough Δ may be either in an antiferromagnetic-XY (AFM-XY) or ferromagnetic-XY (FM-XY) phase, i.e., magnon TLLs with a quasimomentum peak at $q = \pi$ or $q = 0$. For large enough θ and Δ , the attractive $J_{2j} < 0$ results in the formation of a self-bound spin-density wave (SB-SDW), i.e., a self-bound magnon domain at one of the two legs. Finally, we note that other orientation planes different than yz or xz break the sublattice symmetry, allowing for the exploration of dimerized dipolar spin-1/2 chains [60].

Chiral dynamics. Dipoles in triangular ladders open additional interesting possibilities in what concerns nonequilibrium dynamics, since the system parameters, and even the lattice geometry, may be easily changed in real time. This may be employed to investigate the onset of out-of-equilibrium chiral dynamics from an initial nonchiral state. Let us consider in particular, the case of polar molecules in a tweezer array. The legs of the array are initially separated at enough distance to create two separate magnon TLLs, with magnetization $\langle \hat{S}^z \rangle = -0.4$. The electric field strength is chosen

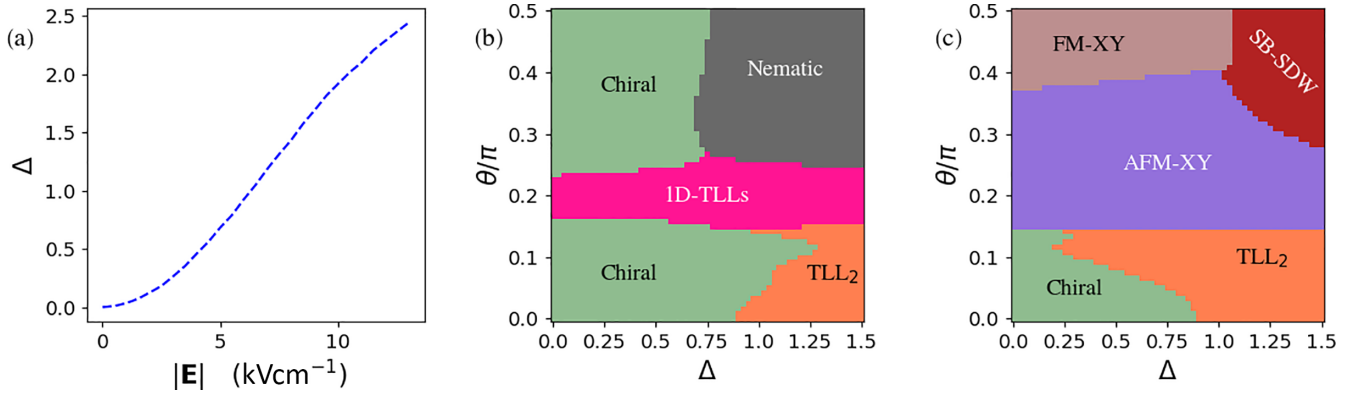


FIG. 3. (a) Anisotropy Δ as a function of the electric field for KRb molecules [41]. (b) Phase diagram of the dipolar XXZ model as a function of Δ and θ for dipoles on the yz plane. (c) Same for dipoles on the xz plane. Results obtained for a magnetization $\langle S^z \rangle = -0.4$ using DMRG, employing TeNPy [48], with a maximum bond dimension of 200, and considering interactions up to ten neighbors on a system of 120 sites with open-boundary conditions.

such that $\Delta = 0.05$, and oriented on the yz plane forming an angle $\theta = 0.15\pi$ with the z axis [within the chiral phase in Fig. 3(b)]. At time $\tau = 0$, the two legs are approached forming an equilateral triangular ladder. As shown in Fig. 4(a), the initially nonchiral system undergoes a local spontaneous breaking of the chiral symmetry that results in the formation of domains which alternate their chirality in time. By tuning the electric field such that $\Delta = 1.5$, we can instead choose a system with a ground state which preserves time-reversal symmetry. The dynamics in this case does not show a clear development of chirality [Fig. 4(b)].

Conclusions. Dipolar bosons in triangular ladders constitute an interesting platform to study the interplay of geometric frustration and long-range anisotropic interactions. For mobile particles, frustration resulting from negative in-leg hopping

magnifies the effect of dipolar interactions. The dipole-induced CSF-2SF transition may be observed even at very small ratios $V/t < 1$ for experimentally reachable temperatures. This is particularly interesting for experiments with magnetic atoms, where V is relatively small. Furthermore, in dipolar spin models, frustration is induced by the dipolar interaction itself. Focusing on the implementation with polar molecules, we have shown that chiral magnon phases, as well as other intriguing phases, including nematic (bound bimagnon superfluids) and two-component magnon superfluids (TLL₂) may be easily realized by properly setting the orientation and magnitude of the applied electric field. Finally, let us note that chirality may be experimentally revealed using the dimerization technique recently employed in Ref. [61]. In dipolar gases, due to intersite interactions, isolating a rung of the triangular ladders demands either selectively removing the particles at the neighboring rungs or physically separating the rungs (in accordion lattices or tweezer arrays). Once the rungs are isolated, currents may be determined from the measurement of the intersite imbalance of the density (Hubbard model) or the magnetization (spin model). In addition to this method, the three superfluid phases in the model of itinerant bosons can also be distinguished through their momentum distribution, which may be imaged in time-of-flight experiments.

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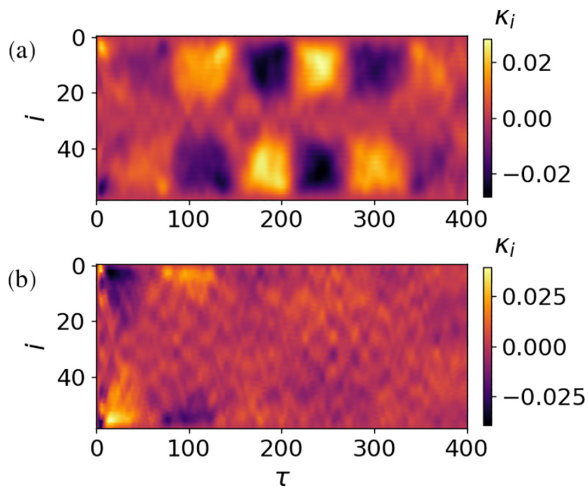


FIG. 4. Local chirality $\langle K_i \rangle$ as a function of time τ (in units of $1/V$) for dipoles on the yz plane at an angle $\theta = 0.15\pi$ with the z axis, $\langle S^z \rangle = -0.4$, and (a) $\Delta = 0.05$ and (b) $\Delta = 1.5$. The initial state is that of two independent magnon TLLs in each leg. The calculations were performed using the time-dependent variational principle algorithm (TDVP), employing TeNPy [48], for 60 sites with open-boundary conditions, keeping a maximum bond dimension of 300.

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Data availability. The data that support the findings of this article are openly available [62].

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