

Coherent control of quantum states by quadratically chirped short laser pulses

G. P. Djotyan¹, A. A. Avetisyan^{2,*} and A. P. Djotyan²

¹*HUN-REN Wigner Research Centre for Physics, Budapest, Hungary*

²*Yerevan State University, Institute of Physics, A. Manoogian 1, Yerevan - 0025, Armenia*



(Received 22 October 2025; accepted 1 April 2026; published 29 April 2026)

We present a scheme for coherent manipulation of populations and robust creation of arbitrary coherent superposition of metastable states of a quantum system with Λ configuration of operating energy levels using laser pulses with a quadratically chirped carrier frequency. The case of a single “broadband” laser pulse is considered, when the width of the laser pulse frequency spectrum (without chirp) exceeds the frequency distance between the metastable energy levels of the system. The results of the dressed state analysis and numerical simulation are presented, demonstrating reliable, robust, and rapid creation of an arbitrary coherent superposition of metastable states by changing the parameters of the laser pulses.

DOI: [10.1103/wfp1-m3m3](https://doi.org/10.1103/wfp1-m3m3)

I. INTRODUCTION

Coherent control of quantum states finds important applications in numerous fields of science and technology. Among them are atomic interferometry and atomic beam splitters [1–5], quantum chemistry [6–8], quantum information and data processing [9–14], quantum optics and manipulation of quantum states of multilevel quantum systems [15–28], generation of high harmonics and improving efficiency of nonlinear processes in resonant gases [29–35], laser cooling [36], Bose-Einstein condensation [37], writing and storage of optical phase information [38,39], and other fields; see also the review papers [40–45].

Different schemes of coherent population transfer and coherent creation of superposition states have been investigated extensively in recent years [15–29]. Among these, schemes based on stimulated Raman adiabatic passage (STIRAP) play an important role (see, for example, [42]). Although the STIRAP-based methods have proven effective, alternative methods should also be considered that may extend or complement, and in some cases prove more advantageous than, STIRAP-based schemes, contributing to the development of more effective methods in the field of coherent control.

In [23] a single laser pulse with linear frequency chirp (LFC) in the adiabatic following regime was used for population transfer between two metastable (ground) states of an atomic system with Λ configuration of operating energy levels without considerable excitation of the atom.

Note that LFC laser pulses were used for coherent control of quantum states of the graphene [46], quantum dots [47],

and of the lowest states of a shallow impurity in graphene monolayer [48]. The effect of dissipation processes on coherent population transfer and coherence creation between states of quantum systems also was considered; see [49–52]. In Ref. [53] a quadratically frequency-chirped (QFC) laser pulse was applied to demonstrate a robust and rapid population transfer between the states of a two-level quantum system with the time of the population transfer significantly shorter than in the case of a laser pulse with linear chirp.

In this work, we apply a short QFC laser pulse to a quantum system of Λ configuration of working energy levels to create a coherent superposition of metastable states of the system with a negligible and short-term excitation of the system. Based on dressed states analysis and numerical simulations, we show that the absolute value of the generated coherence can be controlled by changing the central carrier frequency and amplitude of the QFC laser pulse.

The proposed method for controlled and reliable creation of arbitrary coherence between metastable states of a quantum system under short-term and negligible excitation by a single laser pulse can find important applications in quantum information and data processing, as well as other areas of science and technology that use coherent control of quantum states.

The underlying physics of the process is as follows: due to the quadratic chirp, the laser pulse enters into resonance with the quantum system twice instead of one resonance in the case of linearly frequency-chirped pulses. In the case of a two-level atom initially in the ground state, the atom returns to its original state after adiabatic excitation following interaction with the first branch of the parabolic frequency chirp, and deexcitation after interaction with the second branch of the parabolic frequency chirp.

In the case of a Λ -structured atom with a “broadband” laser pulse interacting with both allowed transitions simultaneously, only the “bright” superposition component of the metastable states is excited and deexcited as a result of interaction with the QFC pulse, while the “dark” component of the superposition remains unaffected.

*Contact author: artakav@ysu.am

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

While the amplitude of the bright component at the end of the interaction is the same as before the interaction (similar to the case of the above-considered two-level atom), the phase of the bright component after the interaction depends on the parameters of the laser pulse.

Since the values of the probability amplitudes of metastable states are equal to a linear combination of the bright and dark components, the final probability amplitudes, and therefore the coherence between them, depend on the parameters of the laser pulse, including its central carrier frequency, the amplitude of the Rabi frequency, and the chirp speed. By changing these parameters, it is possible to obtain a given value of coherence between metastable states with a negligible and short-term excitation of the system, avoiding the effects of spontaneous relaxation.

As the analysis below shows, the resulting populations of metastable states and the generated coherence between these states depend on the laser pulse parameters, even if the QFC pulse frequency does not pass through resonance but has a turning point near resonance.

In this paper, two different cases are studied: when the time-dependent frequency of the QFC laser pulse resonates (twice) with the transition between the metastable and excited states (the case of negative central detuning), and when the time-dependent QFC laser pulse frequency has a turning point near the resonance but does not resonate with the transition (the case of positive central detuning).

Section II presents the mathematical formalism of the interaction of a QFC laser pulse with a Λ -structured quantum system. Then a dressed states analysis is performed for the probability states amplitude dynamics of the quantum system in the field of the laser pulse. The analysis is carried out on the basis of bright and dark superposition of metastable states of the system.

Section III discusses the creation of a coherent superposition of metastable states with given (arbitrary) coherence values based on numerical simulation of the Schrödinger equation. The obtained solutions are compared with the results of the dressed state analysis.

The details of the obtained results are discussed in Sec. IV, and the main conclusions of the study are summarized in Sec. V.

In the Appendix, the coherence generation time is estimated and compared with the QFC laser pulse duration for different values of the central detuning and frequency chirp speed parameter.

II. MATHEMATICAL FORMALISM

We consider interaction of a phase modulated (frequency chirped) laser pulse with a quantum system having a Λ configuration of the working levels (see Fig. 1). We assume that the laser pulse is broadband, with the width of its frequency spectrum (without chirp) exceeding the frequency spacing between two metastable states of the system. It allows the pulse to interact simultaneously with both allowed transitions of the Λ system being in Raman resonance with the system. An important feature of our consideration is the assumption of quadratic chirping of the laser pulse.

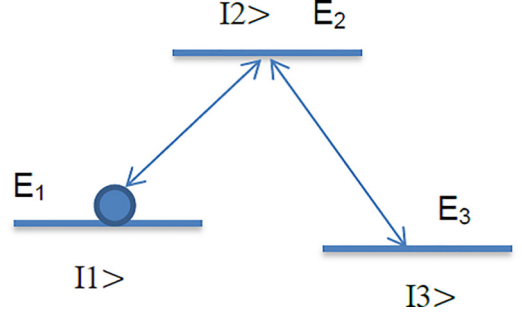


FIG. 1. The scheme of the Λ -atom energy levels.

Assuming the pulse duration to be shorter than all relaxation times of the system, we use the Schrödinger equation to describe the interaction of the laser pulse with the system. The Schrödinger equation for the column state vector $\underline{a} = (a_1, a_2, a_3)^T = a_1(1 \ 0 \ 0)^T + a_2(0 \ 1 \ 0)^T + a_3(0 \ 0 \ 1)^T$ has the following form,

$$\frac{d}{dt} \underline{a} = i \hat{H} \underline{a}, \quad (1)$$

where the column vectors $(1 \ 0 \ 0)^T$, $(0 \ 1 \ 0)^T$, and $(0 \ 0 \ 1)^T$ stand for the (bare) states $|1\rangle$, $|2\rangle$, and $|3\rangle$ respectively (see Fig. 1). The interaction Hamiltonian in the rotating-wave approximation is [23]

$$\hat{H} = \begin{bmatrix} 0 & \Omega_{12} & 0 \\ \Omega_{21} & \varepsilon_{21} + t \frac{d}{dt} \varepsilon_{21} & \Omega_{23} \\ 0 & \Omega_{32} & \omega_R \end{bmatrix}, \quad (2)$$

where $\Omega_{ij} = \Omega_{ji}^* = (1/2\hbar)d_{ij}A(t)$ ($i, j = 1, 2, 3$) is the Rabi frequency and d_{ij} is the dipole moment matrix element for laser-induced transition from state $|j\rangle$ to state $|i\rangle$.

Note that the transition between the metastable states $|1\rangle$ and $|3\rangle$ is forbidden, and only transitions between the states $|1\rangle$, $|3\rangle$, and the excited $|2\rangle$ state are allowed in the dipole approximation. $A(t)$ is the (real) envelope of the laser pulse, $A(t) = A_0 \exp(-\tau^2/2\tau_p^2)$ assumed of Gaussian shape, with τ_p being the half duration of the pulse intensity and A_0 the amplitude of the pulse. $\varepsilon_{21} = \omega_L(t) - \omega_{21}$, and $\varepsilon_{23} = \omega_L(t) - \omega_{23}$ are the detuning from the one-photon resonances, where $\omega_L(t)$ is the time-dependent carrier frequency of the laser pulse; ω_{21} and ω_{23} are the resonant transition frequencies between the corresponding states. Note that in the case of a single laser pulse interacting with the quantum system under consideration, the Raman detuning $\omega_R = \varepsilon_{21}(t) - \varepsilon_{23}(t) = \omega_{13}$ is a time-independent parameter equal to $\omega_{13} = (E_1 - E_3)/\hbar$ being the angular frequency distance between the two ground states of the quantum system with energies E_1 and E_3 (see Fig. 1). By assuming that the width $\Delta\omega_p (\sim 1/\tau_p)$ of the frequency spectrum of the broadband laser pulse exceeds the frequency distance ω_{13} between the two metastable states, $\Delta\omega_p > \omega_{13}$, we can assume the Raman resonance for interaction of the broadband laser pulse with the quantum system, and put the Raman detuning $\omega_R = 0$ in the Hamiltonian in Eq. (2). Note that the situation in this case is analogous to the interaction of the quantum system with a bichromatic frequency-chirped laser pulse, consisting of two laser pulses

with identical envelopes and identical frequency chirps and the carrier frequencies shifted by the frequency distance between the two ground states (see also Refs. [22,38,39]).

In what follows we assume a quadratic chirp in time for the laser carrier frequency: $\omega_L(t) = \omega_{L0} + \beta t^2$, where ω_{L0} is the central frequency and β is the chirp speed parameter. At this step it is convenient to introduce the bright g_b and dark (or “trapped”) g_d superposition of the (bare) probability amplitudes of the ground states of the Λ system:

$$\begin{aligned} g_b(t) &= [a_1(t)d_{21} + a_3(t)d_{32}]/d, \\ g_d(t) &= [a_1(t)d_{12} - a_3(t)d_{23}]/d, \end{aligned} \quad (3)$$

where

$$d = \sqrt{|d_{12}|^2 + |d_{23}|^2}.$$

From the Schrödinger equation (1), we obtain the following equations for the amplitudes of the excited a_2 , as well as the bright g_b and dark g_d superposition states:

$$\begin{aligned} \frac{d}{dt}a_2 - i\varepsilon(t)a_2 &= iF(t)g_b, \\ \frac{d}{dt}g_b &= iF(t)a_2, \\ \frac{d}{dt}g_d &= 0, \end{aligned} \quad (4)$$

where the effective detuning $\varepsilon(t) = \varepsilon_{21} + \frac{d}{dt}\varepsilon_{21}$ and $F(t) = \frac{1}{2\hbar}A(t)d$.

As it follows from Eq. (4), the amplitude of the dark superposition state remains equal to its initial value before interaction with the laser pulse: $g_d(t) = g_d(-\infty)$. On the other hand, the equations for the amplitudes of the bright component and the excited state correspond to an equivalent two-level quantum system with probability amplitudes of the ground and excited states equal to $g_b(t)$ and $a_2(t)$, respectively.

A. Dressed states picture

In this section, we proceed to the analysis of the problem of coherent control of the states of the equivalent two-level quantum system under consideration [see Eq. (4)] using the dressed state approach. The solution of Eq. (4) can be represented on the basis of the adiabatic dressed states $\underline{b}^{(k)}$ as follows:

$$\underline{c}(t) = \sum_{k=1}^2 r_k(t) \underline{b}_k(t) \exp\left[-i \int_{-\infty}^t w_k(t') dt'\right], \quad (5)$$

with the initial condition at $t \rightarrow -\infty$,

$$\underline{c}(t \rightarrow -\infty) = \sum_{k=1}^2 r_k(-\infty) \underline{b}_k(-\infty), \quad (6)$$

where the column state vector $\underline{c}(t) = [a_2(t) g_b(t)]^T$ is the solution of the Schrödinger equation (4); $\underline{b}_k(t)$ is the adiabatic dressed state eigenvector, corresponding to the eigenvalue w_k of the Hamiltonian $\hat{H}_b$:

$$\hat{H}_b \underline{b}_k(t) = w_k(t) \underline{b}_k(t), \quad (7)$$

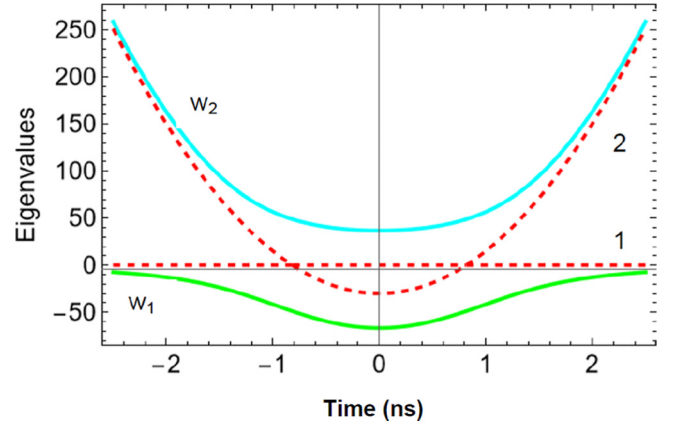


FIG. 2. Time dependence of eigenvalues w_1 (green) and w_2 (blue), normalized by the Planck's constant $\hbar$, as well as the eigenvalues in the absence of the laser field (dashed lines). A quadratic chirp of the laser carrier frequency is assumed. Parameters applied are as follows: half pulse duration $\tau_p = 5$ ns, central detuning $e_{0p} = -30$ GHz, $\beta = 15$ GHz³, and $w_p = 35$ GHz.

where

$$\hat{H}_b = \begin{pmatrix} \varepsilon(t) & F(t) \\ F(t) & 0 \end{pmatrix} \quad (8)$$

from Eq. (4).

The function $r_k(t)$ in Eq. (5) is the statistical weight of the adiabatic dressed state vector $\underline{b}_k(t)$ in the bare state vector $\underline{c}(t)$. According to the adiabatic theorem [54,55], $r_k(t) \equiv r_k(-\infty)$ if the change in time-dependent parameters of the system is adiabatically slow (see the relevant conditions below).

We obtain the following expressions for the eigenvalues $w_k(t)$ and for the dressed state eigenvectors $\underline{b}_k(t)$ as solutions of Eq. (7):

$$w_{1,2} = \frac{\varepsilon(t)}{2} \mp \sqrt{\frac{\varepsilon(t)^2}{4} + F(t)^2}, \quad (9)$$

$$\underline{b}_k(t) = b_1^{(k)}(0 \ 1)^T + b_2^{(k)}(1 \ 0)^T, \quad (10)$$

$$\begin{aligned} b_1^{(k)}(t) &= \frac{w_k(t) - \varepsilon(t)}{\sqrt{[w_k(t) - \varepsilon(t)]^2 + F(t)^2}} g_b(-\infty), \\ b_2^{(k)}(t) &= \frac{F(t)}{\sqrt{[w_k(t) - \varepsilon(t)]^2 + F(t)^2}} g_b(-\infty), \end{aligned} \quad (11)$$

where for the quadratic frequency chirp under consideration the effective detuning $\varepsilon(t) = e_{0p} + 3\beta t^2$ [see Eq. (4)], $g_b(-\infty)$ is the amplitude of the bright superposition component before interaction with the laser pulse and $e_{0p} = \omega_{L0} - \omega_{21}$ is the central detuning. In the following, for simplicity, we assume equal transition dipole elements for the dipole allowed transitions of the system $d_{12} = d_{32}$.

$F(t) = \frac{1}{2\hbar}A(t)d = w_p\sqrt{2}\exp[-t^2/\tau_p^2]$ with $w_p = |d_{12}|A_0/2\hbar$. The dependence of the eigenvalues $w_1(t)$ and $w_2(t)$ on time is shown in Fig. 2.

Let us assume at this point that the system is in one of the ground states initially ($t \rightarrow -\infty$). This means that the state vector $\underline{c}(t \rightarrow -\infty) = (0 \ g_b)^T$, and in the dressed states

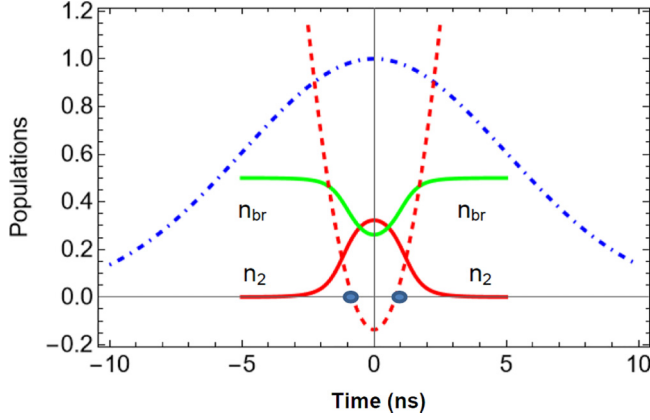


FIG. 3. Time dependence of populations of the bright superposition (green) and of the excited state (red) in the field of a QFC laser pulse. For reference, the temporal shape of the laser pulse and the frequency detuning with two resonance intersection points are shown by dashed-dotted and dashed lines, respectively. Parameters applied are the same as in Fig. 2. Note that, with equal dipole moments for transitions between the metastable and excited states, the initial population of the bright state is 0.5.

approach,

$$\begin{aligned} \underline{c}(-\infty) &= \sum_{k=1}^2 r_k(-\infty) \underline{b}_k(t) \\ &= \sum_{k=1}^2 r_k(-\infty) [\underline{b}_1^{(k)}(-\infty) (0 \ 1)^T]. \end{aligned}$$

As it follows from Eqs. (9)–(11), only for $w = w_1 = \frac{\varepsilon(t)}{2} - \sqrt{\frac{\varepsilon(t)^2}{4} + F(t)^2}$, $\underline{b}_1^{(1)} \rightarrow \underline{g}_b(-\infty)$, and $\underline{b}_2^{(1)} \rightarrow 0$ at $t \rightarrow -\infty$. Therefore, the dressed state $\underline{b}_1(t) = \underline{b}_1^{(1)}(1 \ 0)^T + \underline{b}_2^{(1)}(0 \ 1)^T$ is the dressed state vector, which must be identified with the one that is adiabatically transformed into the initial state vector of the system before interaction with the laser pulse (at $t \rightarrow -\infty$) [see Eq. (6)]: $\underline{c}(t \rightarrow -\infty) = \underline{b}_1^{(1)}(t \rightarrow -\infty)$. According to the adiabatic theorem, the system remains in this state if the conditions of adiabaticity are met (see [54,55]).

The general condition for realization of the adiabaticity condition in the case of the two-level system is (see also [44])

$$|d\theta/dt| \ll \sqrt{\varepsilon(t)^2 + F(t)^2}, \quad (12)$$

where the mixing angle $\theta = \tan^{-1}[\varepsilon(t)/F(t)]$.

Assuming that $F(t)^2 \gg \varepsilon(t)^2$ during the states populations transition time in the system, we arrive at the following condition from Eq. (12) (see also [44]):

$$F_0^2 \gg d\varepsilon(t)/dt,$$

where F_0 is the maximum Rabi frequency (coupling strength): $F_0 = |d_{12}|A_0/\sqrt{2}\hbar$. For our case of a quadratically chirped laser pulse, we will have the following adiabaticity condition:

$$F_0^2 \gg 6\beta t_{tr}, \quad (13)$$

where t_{tr} is the population transition time in the system that is a function of the chirp speed parameter β and other parameters of the laser field (see the Appendix). In Fig. 3 the dynamics

of the population of states in the field of a QFC laser pulse is presented as a result of numerical simulation of Eq. (4).

As can be seen in Fig. 3, resonance with the allowed transitions of the system is reached twice in the case of a quadratic frequency chirp (instead of a single resonance crossing in the case of a linearly chirped pulse), which leads to a return to the ground state of the population of the bright component after a temporary population of the excited state.

This effect of the QFC laser pulse is used to create a coherent superposition of metastable states of the considered Λ -structured quantum system. As mentioned above, the superposition of two metastable states is created by the interference of the complex bright component, the phase of which depends on the QFC pulse parameters, with the dark component [see Eqs. (3) and (4)], which does not interact with the laser pulse.

Using Eqs. (5) and (9)–(11) with eigenvalue $w = w_1$, we obtain for the amplitudes of the bright $g_b(t)$ and of the excited state $a_2(t)$

$$\begin{aligned} g_b(t) &= \frac{[w_1 - \varepsilon(t)]}{V(t)} g_b(-\infty) \exp[-i\varphi(t)], \\ a_2(t) &= \frac{F(t)}{V(t)} g_b(-\infty) \exp[-i\varphi(t)], \\ V(t) &= \sqrt{[w_1 - \varepsilon(t)]^2 + F(t)^2}, \end{aligned} \quad (14)$$

where $\varphi(t) = \int_{-\infty}^t w_1(t') dt'$.

For the probability amplitudes $a_{1,3}(t)$ of the metastable states of our quantum system, we obtain, using Eqs. (3) and (14),

$$\begin{aligned} a_1(t) &= \frac{d_{12}}{d} \frac{w_1 - \varepsilon(t)}{V(t)} g_b(-\infty) \exp[-i\varphi(t)] + C_1, \\ a_3(t) &= \frac{d_{23}}{d} \frac{w_1 - \varepsilon(t)}{V(t)} g_b(-\infty) \exp[-i\varphi(t)] - C_2, \end{aligned} \quad (15)$$

where

$$\begin{aligned} C_1 &= [a_1(-\infty)|d_{23}|^2 - a_3(-\infty)d_{12}d_{23}]/d^2, \\ C_2 &= [a_1(-\infty)d_{21}d_{23} - a_3(-\infty)d_{12}^2]/d^2. \end{aligned}$$

At this step, we assume that initially the population of the quantum system under consideration is in one of the metastable states, for example, in the state $|1\rangle$: $a_1(-\infty) = 1$, $a_3(-\infty) = 0$. Also, for simplicity, we assume that the transition dipole moments are equal: $d_{12} = d_{23}$. These assumptions simplify the expressions (15) significantly: $g_b(-\infty) = g_d(-\infty) = 1/\sqrt{2}$, and

$$\begin{aligned} a_1(t) &= \frac{1}{2} \left[\frac{w_1 - \varepsilon(t)}{V(t)} \exp[-i\varphi(t)] + 1 \right], \\ a_3(t) &= \frac{1}{2} \left[\frac{w_1 - \varepsilon(t)}{V(t)} \exp[-i\varphi(t)] - 1 \right]. \end{aligned} \quad (16)$$

Next, we study the dependence of the coherence $\rho_{13}(t) = a_1(t)a_3^*(t)$ between the two metastable states of the quantum system established after action of a QFC laser pulse on the laser pulse parameters, using the results of the dressed states

analysis [Eq. (16)], and direct numerical simulation of the Schrödinger equation [Eq. (1)].

Assuming a positive chirp speed parameter $\beta > 0$ for a quadratic frequency chirp (see Fig. 3), two different cases are considered below: (a) the case of negative central detuning, when the central frequency of the laser is less than the resonant frequency of the transition between the metastable and excited states (the detuning is negative at the central frequency of the laser), and (b), the case of positive central detuning, when the central frequency of the pulse is greater than the resonant frequency (the detuning is positive at the central frequency of the laser and has a turning point at the resonance).

These two cases demonstrate two different physical mechanisms of interaction. While in case (a) the laser carrier frequency passes through resonance with the transition twice, in case (b) the laser frequency has a minimal (positive) detuning at the center of the pulse, without reaching resonance with the transition.

The population value of the bright component, temporarily transferred to the excited state, depends differently on the central detuning of the laser pulse in these two cases. In case (a), the greater the central (negative) detuning, the greater the part of the bright population that temporarily is transferred into the excited state, with the maximum value equal to the entire initial population of the bright component. In case (b), the greater the central (positive) detuning, the smaller the part that temporarily goes into the excited state. Note that changing the sign of the chirp speed parameter from $\beta > 0$ to $\beta < 0$ leads to replacement of the situations corresponding to the cases (a) and (b) considered above.

III. CREATION OF A COHERENT SUPERPOSITION OF THE METASTABLE STATES BY A QFC LASER PULSE

A. Case of negative central detuning

We proceed with numerical simulation of the Schrödinger equation (1) for the case of a negative central detuning, as discussed above. The dynamics of the populations of quantum states, as well as the absolute value of the coherence between the metastable states is presented in Fig. 4.

The color (density) graph of the absolute value of coherence between two metastable states of the system depending on the detuning of the central frequency and the amplitude of the Rabi frequency is shown in Fig. 5.

As follows from Fig. 5, there is a region of QFC pulse parameters where the maximum (near 0.5) coherence can be achieved with extremely high robustness. In addition, from Fig. 5, one can observe a quasiperiodic dependence of the coherence on the central detuning and the Rabi frequency amplitude.

This quasiperiodic dependence can also be seen in Fig. 6, where the dependence of the absolute value of the created coherence between metastable states is presented as a function of the (negative) central detuning for a given value of the Rabi frequency amplitude, and also in Fig. 7—depending on the amplitude of the Rabi frequency for a given (negative) value of the central frequency detuning.

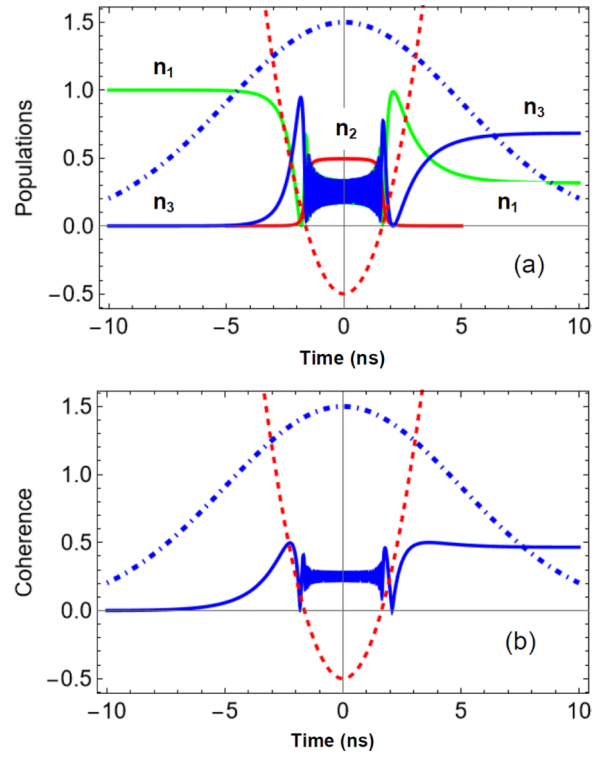


FIG. 4. Time dependence of the populations of the metastable states $|1\rangle$, n_1 (green), $|3\rangle$, n_3 (blue), and the excited state $|2\rangle$, n_2 (red) (a), and (b) the absolute value of the coherence between the metastable states $|1\rangle$ and $|3\rangle$ in the field of a QFC laser pulse. The system is supposed to be in the state $|1\rangle$ initially. For reference, the frequency detuning $\varepsilon(t)$ is represented by the dashed line, and the laser pulse shape is represented by the dashed-dotted line. The parameters applied are as follows: the half pulse duration $\tau_p = 5$ ns, amplitude of the Rabi frequency $w_p = 15$ GHz, and the chirp speed parameter $\beta = 25$ GHz³.

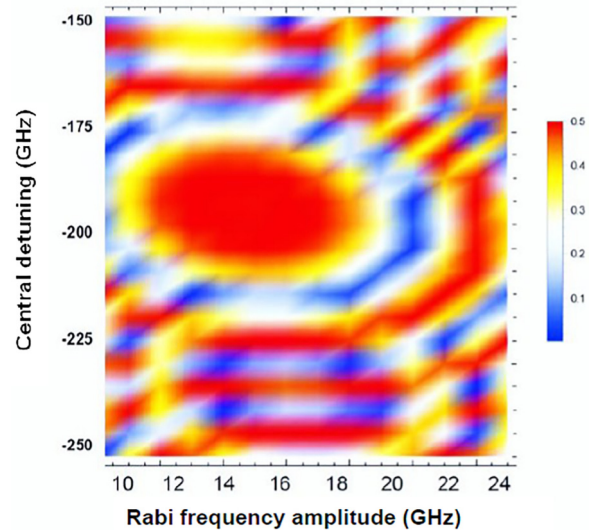


FIG. 5. Color (density) plot of the absolute value of coherence established after action of a QFC laser pulse as a function of *negative* central detuning and the amplitude of the Rabi frequency. In the simulations along the X and Y axes, 1 and 10 GHz computational steps were used, respectively. The QFC pulse duration and the chirp speed parameter are the same as in Fig. 4.

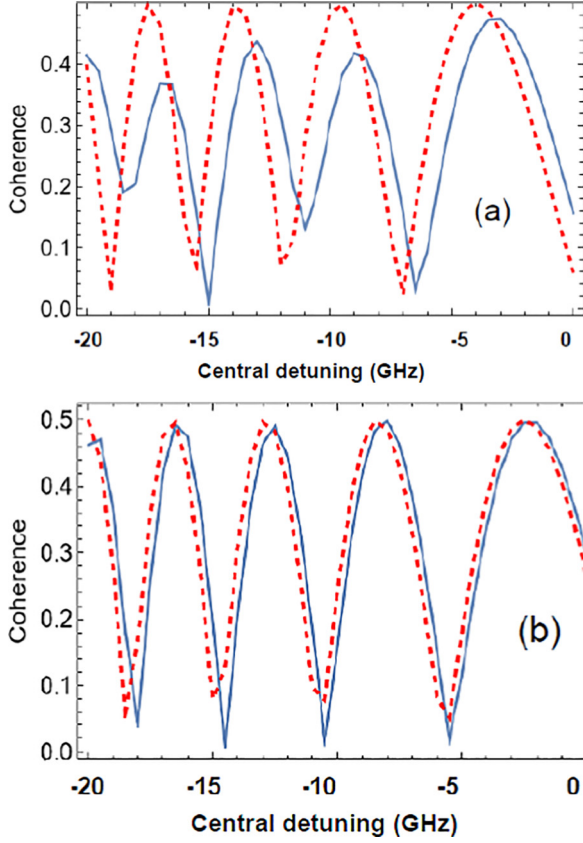


FIG. 6. Dependence of the absolute value of coherence between metastable states after action of a QFC laser pulse on the central (negative) detuning e_{0p} for different values of the amplitude $\hbar w_p$ of the Rabi frequency of the laser pulse as a result of numerical simulation of the Schrödinger equation [Eq. (1)] (solid lines) and based on the dressed states analysis [Eq. (16)] (dashed lines): (a) $w_p = 1$ GHz; (b) $w_p = 5$ GHz. The QFC pulse duration and the chirp speed parameter are the same as in Fig. 5.

As can be seen from Figs. 6 and 7, the results based on the numerical solution of the Schrödinger equation are close to the results obtained based on the dressed states approximation, for sufficiently large values of the maximum Rabi frequency amplitudes.

B. Case of positive central detuning

In this section, we study the effect of a QFC laser pulse on a quantum system with Λ structure of operating energy levels under the condition of a positive value of the central frequency detuning (see Fig. 8). We perform an analysis similar to the one above for a negative central frequency detuning. While in the previous case with a negative central frequency detuning the QFC pulse frequency passes through resonance with the system twice, in the case of a positive central detuning, the resonance with the system is not reached at all. This has a number of interesting and useful consequences. One of them is the negligible short-term excitation of the system, as can be seen in Fig. 8(a) [compare with the case of negative central detuning, Fig. 4(a)].

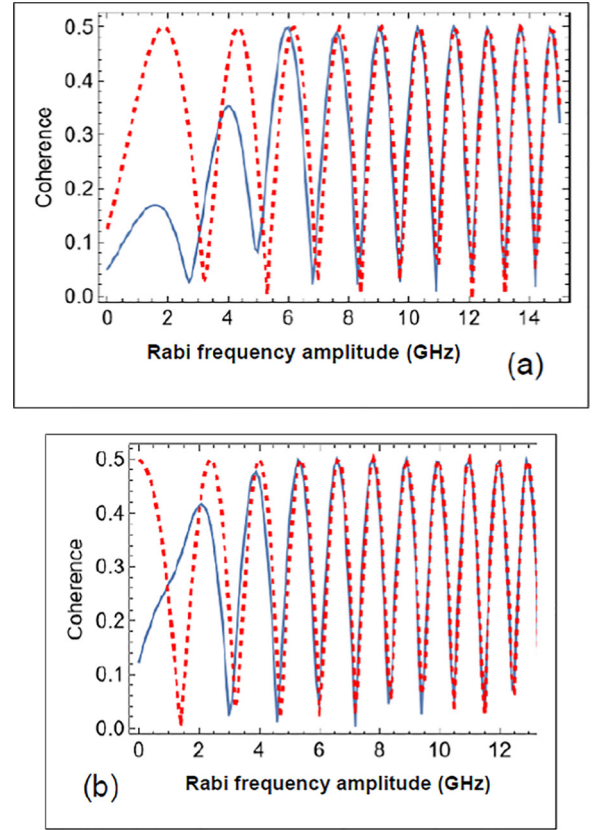


FIG. 7. Dependence of the absolute value of coherence between the metastable states after action of a QFC laser pulse as a function of the pulse Rabi frequency amplitude for different values of the central (negative) detuning e_{0p} as a result of numerical simulation of the Schrödinger equation [Eq. (1)] (solid lines), and based on the dressed states analysis [Eq. (16)] (dashed lines): (a) $e_{0p} = -100$ GHz; (b) $e_{0p} = -20$ GHz. The QFC pulse duration and the chirp speed parameter are the same as in Fig. 6.

Below, color (density) graphs are presented for dependence of the coherence established at the end of the QFC pulse on the (positive) central detuning and the amplitude of the Raman frequency.

As it follows from Fig. 9, there are parameter regions in which the creation of a given coherence may be realized with high robustness. It should be noted that the dependence of coherence on the central frequency detuning and the Rabi frequency amplitude has quasiperiodic character, similar to the case of negative detuning considered above.

In Fig. 10 the dependence of the absolute value of the created coherence between metastable states is presented as a function of the (positive) central detuning for a given value of the Rabi frequency amplitude, and in Fig. 11 as a function of the Rabi frequency amplitude for a given (positive) value of the central frequency detuning.

As can be seen from Figs. 10 and 11, the solutions based on numerical simulation of the Schrödinger equation [Eq. (1)] and those obtained on the basis of the dressed states approximation [Eq. (16)] coincide with each other in the considered parameters range.

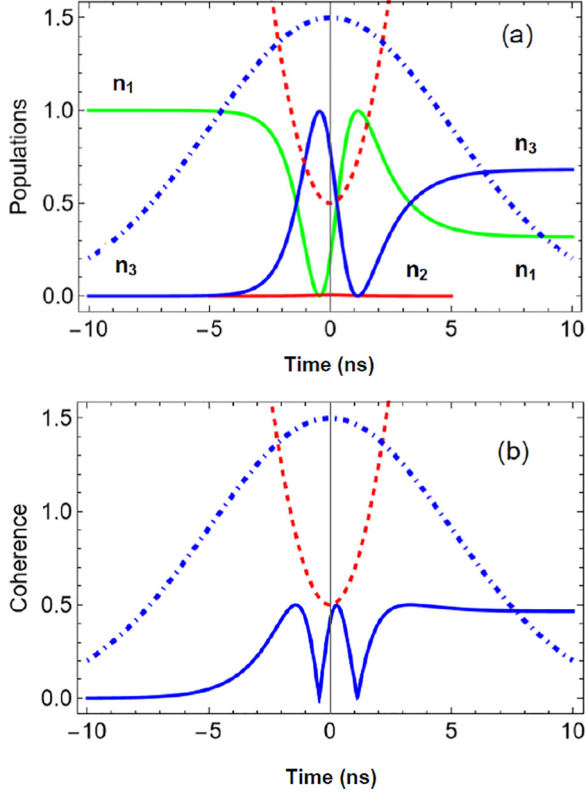


FIG. 8. Time dependence of the populations of metastable states $|1\rangle$, n_1 (green), and $|3\rangle$, n_3 (blue) and the excited state $|2\rangle$, n_2 (red) (a), and (b) of the absolute value of the coherence between the metastable states $|1\rangle$ and $|3\rangle$ in the field of a QFC laser pulse with positive central frequency detuning. The system is supposed to be in the state $|1\rangle$ initially. The frequency detuning is represented by a dashed line and the laser pulse shape by a dot-dashed line for reference. The parameters applied are as follows: half duration of the pulse $\tau_p = 5$ ns, the amplitude of the Rabi frequency $w_p = 15$ GHz, and the chirp speed parameter $\beta = 25$ GHz³.

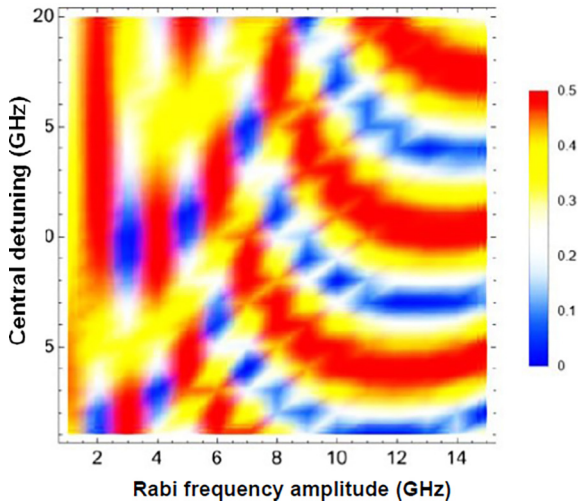


FIG. 9. Color (density) plot of the absolute value of the coherence as a function of *positive* central detuning and Rabi frequency amplitude. A computational step corresponding to 1 GHz was used in the simulations along both the X and Y axes. The QFC pulse duration and the chirp speed parameter are the same as in Fig. 8.

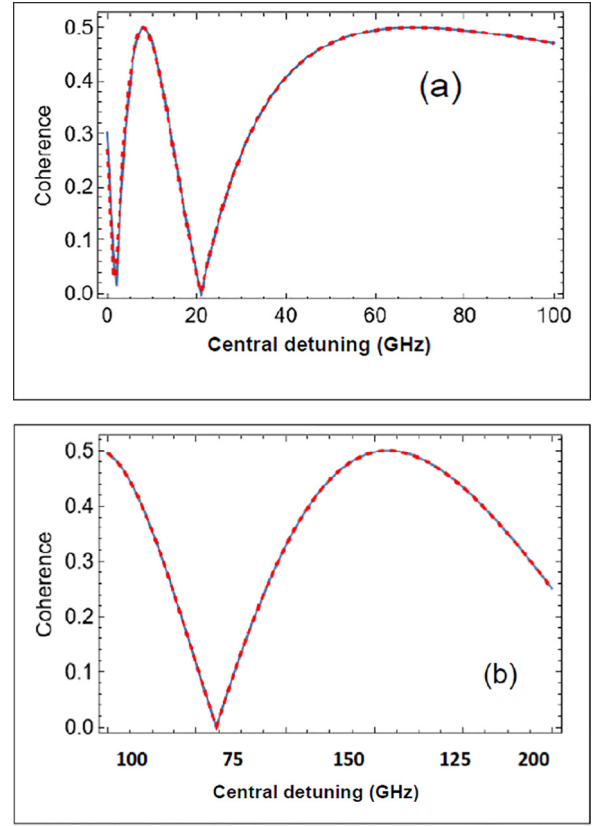


FIG. 10. Dependence of the absolute value of the coherence between the metastable states after action of a QFC laser pulse on the central (positive) detuning e_{0p} for different values of the Rabi frequency amplitude w_{p0} as a result of numerical simulation of the Schrödinger equation [Eq. (1)] (solid lines) and based on the dressed states analysis [Eq. (16)] (dashed lines): (a) $w_p = 5$ GHz; (b) $w_p = 15$ GHz. The QFC pulse duration and the chirp speed parameter are the same as in Fig. 9.

IV. DISCUSSION

Our results show that a given (arbitrary) value of coherence between metastable states of a Λ -structured quantum system can be created by exposure to a *single* QFC laser pulse with variable amplitude and central detuning.

An important condition is that the width of the laser pulse frequency spectrum (without chirp) exceeds the frequency distance between the metastable states.

Coherence can be created by negligible and short-term excitation of the system. The population of the excited state depends strongly on the central detuning of the laser carrier frequency.

The maximum value of the short-term population of the excited state, which is equal to the initial population of the bright component, is reached in the case of negative central detuning (for a positive chirp parameter β) when the frequency of the laser pulse crosses the resonance twice.

In contrast, the short-term population of the excited state is negligible if there is no resonance crossing at positive values of the central detuning.

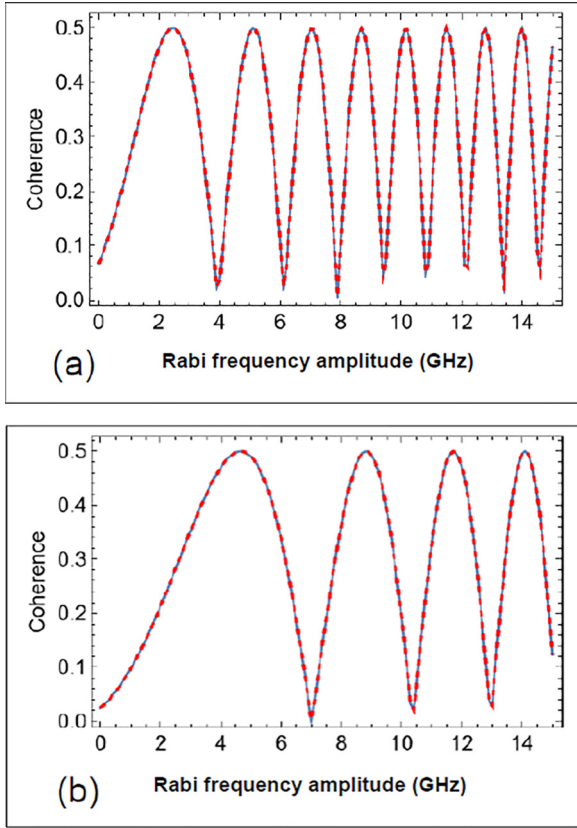


FIG. 11. Dependence of the absolute value of the coherence between the metastable states after action of a QFC laser pulse on the Rabi frequency amplitude for different values of the central (positive) detuning e_{0p} as a result of numerical simulation of the Schrödinger equation [Eq. (1)] (solid lines), and based on the dressed states analysis [Eq. (16)] (dashed lines): (a) $e_{0p} = 20$ GHz; (b) $e_{0p} = 100$ GHz. The QFC pulse duration and the chirp speed parameter are the same as in Fig. 10.

In both cases, there are ranges of laser pulse parameters where coherence generation is robust to small and medium changes in the parameters.

And in both cases, the absolute value of the created coherence is a quasiperiodic function of the central frequency and the (Rabi frequency) amplitude of the laser pulse.

However, the case of positive central detuning seems more preferable due to the insignificant temporary excitation of the system, as well as being due to the larger values of the (quasi-) periods of coherence change when varying the parameters of the laser pulse, making its creation more reliable and predictable.

As follows from Figs. 6 and 7, and 10 and 11, the results obtained during numerical simulation at sufficiently high amplitudes of laser pulses are close to the results obtained based on the consideration of dressed states.

It should be noted that the coherent control scheme presented in this paper uses one laser pulse instead of two pulses of a given shape and delay, as in the case of STIRAP-based schemes [42].

Another advantage of the presented scheme is its high speed, which allows the creation of coherence in the adiabatic

following regime in a much shorter period of time than the duration of the laser pulse (see the Appendix).

It is also important to note that the fidelity of population transfer is not inferior to STIRAP-based methods. The fidelity of population transfer using QFC laser pulses in a two-level system was investigated in detail in ref. [53] and shown to be high. Since the physics of our method is based on the population transfer of the bright component of the superposition to the excited state and back in an effective two-level system, all the conclusions from [53] regarding the high fidelity of population transfer using a QFC laser pulse are applicable to our case.

V. CONCLUSIONS

In conclusion, interaction of a QFC laser pulse with a quantum system having a Λ configuration of working energy levels has been studied with the aim of creating arbitrary values of coherence between metastable states of the system with negligibly small and short-term excitation of the system. Two cases of central frequency detuning have been considered: negative, when the laser frequency passes through resonance with the system twice, and positive, when the laser pulse frequency approaches resonance without crossing it.

It is shown that in both cases it is possible to create an arbitrary coherence value by changing the parameters of the laser pulse in the variation ranges in which the creation of coherence is robust to small and medium changes in these parameters.

The case of positive central detuning seems more preferable due to the insignificant short-term excitation of the system, as well as being due to the larger values of the (quasi-) periods of coherence change when varying the parameters of the laser pulse, making its creation more reliable and predictable.

As a suitable quantum system for the experimental implementation of the scheme described above, one can consider the magnetic sublevels of the transition $J = 1 \leftrightarrow J' = 0$, for example, in the Sm atom, where the total momentum numbers $J = 1$ and $J' = 0$ correspond to the ground and excited states, respectively. The role of the metastable states of the lambda-linkage atom will be played by the magnetic sublevels with quantum numbers $m = -1$ and $m = 1$. Transitions from these states to the excited state with magnetic quantum number $m = 0$ will be provided by the σ^+ and σ^- circular components of the linearly polarized laser pulse (see also [25] and references therein). Another candidate for experimental realization of the scheme may be metastable states of impurities in graphene systems (see, for example, [48]).

A high speed of the coherence generation has been demonstrated, when the coherence creation time can be much shorter than the QFC laser pulse duration in the adiabatic following regime.

As our simulation results (which will be presented in our upcoming paper) show, even shorter coherence creation times can be achieved in the nonadiabatic interaction regime using QFC laser pulses with higher-frequency chirp speed parameters.

An important application of the presented scheme can be the generation and processing of qubits, when only

metastable states of the quantum system are involved, which allows eliminating the negative effects of spontaneous relaxation.

It is worth noting that the quadratic frequency chirp of the laser pulse considered in this paper is merely a simplification of a nonmonotonic temporal behavior of a bell-shaped (or inverse bell-shaped) frequency chirp. Thus all the effects of creating coherence between metastable states also apply to laser pulses with a more general bell-shaped frequency chirp.

ACKNOWLEDGMENT

The work is performed with the financial support of the Higher Education and Science Committee of the Republic of Armenia within the framework of Project No. 21AG-1C048.

The authors declare no conflict of interests.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX

In the Appendix, we analyze the dependence of the coherence creation time between metastable states of the quantum system on the value of the central detuning and the frequency chirp speed parameter.

Since the results of numerical simulations and dressed states analysis are close at sufficiently high Rabi frequency amplitudes (see Figs. 6 and 7, and 10 and 11), we estimate the coherence creation time using the dressed states analysis results.

As the results of the analysis below show, the duration of the coherence creation process can be significantly reduced making it much shorter than the laser pulse duration, by increasing the value of the frequency chirp speed parameter.

Below, we consider variation in time of the population of the excited state of the quantum system [see Eq. (14)].

We claim that the coherence generation process is practically complete at the moment when the population of the excited state approaches its zero value, assuming identical values of the transition dipole moments ($d_{12} = d_{23}$), for simplicity.

We analyze the time dependence of the function

$$f(t) = |a_2|^2 = \frac{1}{2} - |g_b(t)|^2 = \frac{1}{2} \frac{F(t)^2}{[w_1 - \varepsilon(t)]^2 + F(t)^2},$$

where $|a_2|^2$ and $|g_b(t)|^2$ are populations of the excited and bright components [see Eq. (14)], the detuning $\varepsilon(t) = e_{0p} + 3\beta t^2$, the Rabi frequency $F(t) = F_0 \exp[-t^2/2\tau_p^2]$, and w_1 is the eigenvalue from Eq. (9).

When using dimensionless parameters, the function $f(t)$ has the following form:

$$f(\tau) = \frac{1}{2} \frac{\exp(-\tau^2)}{\exp(-\tau^2) + [\frac{1}{2}(e_0 + k\tau^2) + \sqrt{\frac{1}{4}(e_0 + k\tau^2)^2 + \exp(-\tau^2)}]^2},$$

where $\tau = t/\tau_p$, $F(\tau) = F_0 \exp(-\tau^2/2)$, $e_0 = e_{0p}/F_0$, and $k = 3\beta\tau_p^2/F_0$.

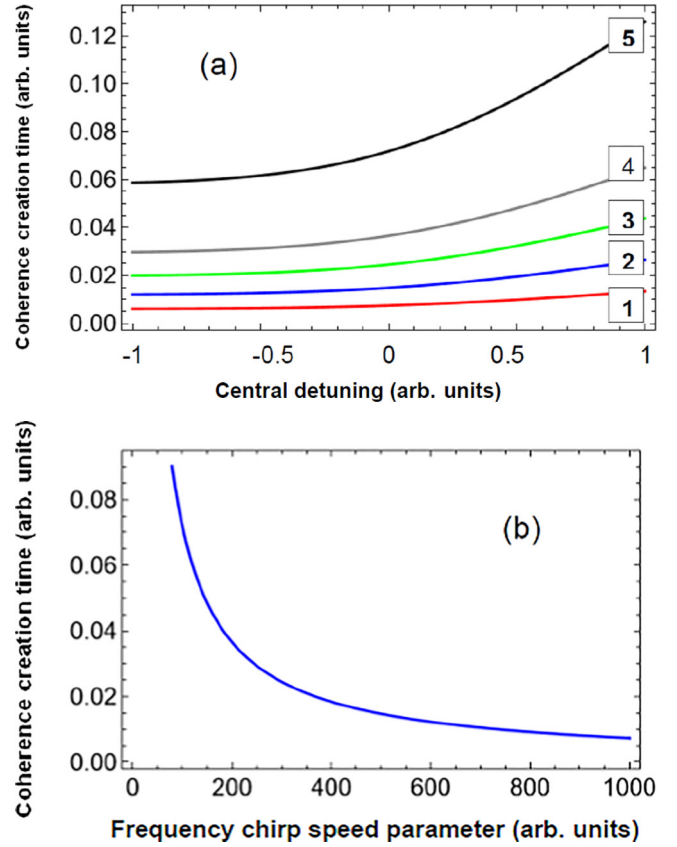


FIG. 12. Dependence of the coherence creation time on (a) normalized central detuning for different values of the dimensionless frequency chirp speed parameter k : 1 – $k = 1000$ (red), 2 – $k = 500$ (blue), 3 – $k = 300$ (green), 4 – $k = 200$ (gray), and 5 – $k = 100$ (black), and on the frequency chirp speed parameter k at zero central detuning (b). The small parameter $\alpha = 10^{-3}$.

Our goal is to determine the time instant $\tau = \tau^*$, at which

$$f(\tau = \tau^*) = \alpha f_{\max}, \quad (\text{A1})$$

where α is a small parameter, and $f_{\max}$ is the maximum value of the function $f(\tau)$ that is achieved at $\tau = 0$.

Equation (A1) is solved numerically to find τ^* as a function of the normalized frequency chirp speed parameter k , and the central detuning e_0 . We take as a small parameter $\alpha = 10^{-3}$, assuming that the coherence creation process is completed when the population of the excited state is approximately 0.1% of its maximum value. It should be noted that the maximum value of the excited state population that can be achieved is 0.5.

The adiabaticity condition (13) takes the following form in dimensionless parameters:

$$F_0\tau_p \gg 2k\tau_{\text{tr}}, \quad (\text{A2})$$

where the dimensionless time of coherence creation $\tau_{\text{tr}} = 2\tau^*$.

The results of evaluation of the coherence creation time by numerical solution of Eq. (A1) are presented in Fig. 12.

As can be seen from Fig. 12, the duration of the coherence creation can be significantly shorter than the duration of the laser pulse at a sufficiently large frequency chirp speed parameter k .

It should also be noted that the adiabaticity condition (A2) still holds for the simulation results shown in Fig. 12.

-
- [1] M. Kasevich and S. Chu, Atomic interferometry using stimulated Raman transitions, *Phys. Rev. Lett.* **67**, 181 (1991).
 - [2] M. Weitz, B. C. Young, and S. Chu, Atomic interferometer based on adiabatic population transfer, *Phys. Rev. Lett.* **73**, 2563 (1994).
 - [3] J. Lawall and M. Prentiss, Demonstration of a novel atomic beam splitter, *Phys. Rev. Lett.* **72**, 993 (1994).
 - [4] D. L. Butts, J. M. Kinast, K. Kotru, A. M. Radojevic, B. P. Timmons, and R. E. Stoner, Coherent population trapping in Raman-pulse atom interferometry, *Phys. Rev. A* **84**, 043613 (2011).
 - [5] H. Theuer, R. Unanyan, C. Habscheid, K. Klein, and K. Bergmann, Novel laser controlled variable matter wave beam-splitter, *Opt. Express* **4**, 77 (1999).
 - [6] P. Brumer and M. Shapiro, Laser control of molecular processes, *Annu. Rev. Phys. Chem.* **43**, 257 (1992).
 - [7] X. Li and G. A. Parker, Theory of laser enhancement of ultracold reactions, *J. Chem. Phys.* **128**, 184113 (2008).
 - [8] S. Chatterjee and S. S. Bhattacharyya, Selective excitation of LI2 by chirped laser pulses with all possible interstate radiative couplings, *J. Chem. Phys.* **133**, 164313 (2010).
 - [9] C. P. Williams and S. H. Clearwater, *Explorations in Quantum Computing* (Springer-Verlag, Berlin, 1997).
 - [10] D. Bouwmeester, A. Ekert, and A. Zeilinger, *The Physics of Quantum Information: Quantum Cryptography, Quantum Teleportation, Quantum Computation* (Springer-Verlag, Berlin, 2000).
 - [11] P. Kaufmann, T. F. Gloger, D. Kaufmann, M. Johanning, and C. Wunderlich, High-fidelity preservation of quantum information during trapped-ion transport, *Phys. Rev. Lett.* **120**, 010501 (2018).
 - [12] M. G. Bason, M. Viteau, N. Malossi, P. Huillery, E. Arimondo, D. Ciampini, R. Fazio, V. Giovannetti, R. Mannella, and O. Morsch, High-fidelity quantum driving, *Nat. Phys.* **8**, 147 (2012).
 - [13] M. Saffman, T. G. Walker, and K. Mølmer, Quantum information with Rydberg atoms, *Rev. Mod. Phys.* **82**, 2313 (2010).
 - [14] C. H. Bennett and D. P. DiVincenzo, Quantum information and computation, *Nature (London)* **404**, 247 (2000).
 - [15] A. S. Parkins, P. Marte, P. Zoller, and H. J. Kimble, Synthesis of arbitrary quantum states via adiabatic transfer of Zeeman coherence, *Phys. Rev. Lett.* **71**, 3095 (1993).
 - [16] G. P. Djotyan, J. S. Bakos, G. Demeter, P. N. Ignácz, M. A. Kedves, Zs. Sörlei, J. Szigeti, and Z. L. Tóth, Coherent population transfer in Rb atoms by frequency-chirped laser pulses, *Phys. Rev. A* **68**, 053409 (2003).
 - [17] G. Liu, V. Zakharov, T. Collins, P. Gould, and S. A. Malinovskaya, Population inversion in hyperfine states of Rb, with a single nanosecond chirped pulse in the framework of a four-level system, *Phys. Rev. A* **89**, 041803(R) (2014).
 - [18] K. Varga-Umbrich, J. S. Bakos, G. P. Djotyan, Zs. Sörlei, G. Demeter, P. N. Ignácz, B. Ráczkevi, J. Szigeti, and M. A. Kedves, Coherent manipulation of trapped Rb atoms by overlapping frequency-chirped laser pulses: Theory and experiment, *Eur. Phys. J. D* **76**, 70 (2022).
 - [19] F. Vewinger, M. Heinz, R. Garcia Fernandez, N. V. Vitanov, and K. Bergmann, Creation and measurement of a coherent superposition of quantum states, *Phys. Rev. Lett.* **91**, 213001 (2003).
 - [20] N. Sangouard, S. Guérin, L. P. Yatsenko, and T. Halfmann, Preparation of coherent superposition in a three-state system by adiabatic passage, *Phys. Rev. A* **70**, 013415 (2004).
 - [21] L. Yatsenko, N. V. Vitanov, B. W. Shore, T. Riekens, and K. Bergmann, Creation of coherent superpositions using Stark-chirped rapid adiabatic passage, *Opt. Commun.* **204**, 413 (2002).
 - [22] H. K. Avetissian, B. R. Avchyan, and G. F. Mkrtchian, Two-color multiphoton resonant excitation of three-level atoms, *Phys. Rev. A* **74**, 063413 (2006).
 - [23] G. P. Djotyan, J. S. Bakos, Zs. Sörlei, and J. Szigeti, Coherent control of atomic quantum states by single frequency-chirped laser pulses, *Phys. Rev. A* **70**, 063406 (2004).
 - [24] Z. Zhang, X. Yang, and X. Yan, Population transfer and generation of arbitrary superpositions of quantum states in a four-level system using a single-chirped laser pulse, *J. Opt. Soc. Am. B* **30**, 1017 (2013).
 - [25] G. P. Djotyan, J. S. Bakos, G. Demeter, Zs. Sörlei, J. Szigeti, and D. Dzsotjan, Creation of a coherent superposition of quantum states by a single frequency-chirped short laser pulse, *J. Opt. Soc. Am. B* **25**, 166 (2008).
 - [26] H. K. Avetissian, A. Brown, and G. F. Mkrtchian, Induced multiphoton adiabatic passage and coherent control of population transfer in quantum systems with high-energy transitions, *Phys. Rev. A* **80**, 033413 (2009).
 - [27] G. P. Djotyan, N. Sandor, J. S. Bakos, and Zs. Sörlei, An extremely robust strong-field control of atomic coherence, *Opt. Express* **19**, 17493 (2011).
 - [28] M. Ndong, G. P. Djotyan, A. Ruschhaupt, and S. Guérin, Robust coherent superposition of states by single-shot shaped pulse, *J. Phys. B: At., Mol. Opt. Phys.* **48**, 174007 (2015).
 - [29] J. B. Watson, A. Sanpera, X. Chen, and K. Burnett, Harmonic generation from a coherent superposition of states, *Phys. Rev. A* **53**, R1962 (1996).
 - [30] M. Jain, Hui Xia, G. Y. Yin, A. J. Merriam, and S. E. Harris, Efficient nonlinear frequency conversion with maximal atomic coherence, *Phys. Rev. Lett.* **77**, 4326 (1996).
 - [31] A. V. Sokolov, D. R. Walker, D. D. Yavuz, G. Y. Yin, and S. E. Harris, Raman generation by phased and antiphased molecular states, *Phys. Rev. Lett.* **85**, 562 (2000).
 - [32] M. D. Lukin, P. R. Hemmer, M. Löffler, and M. O. Scully, Resonant enhancement of parametric processes via radiative interference and induced coherence, *Phys. Rev. Lett.* **81**, 2675 (1998).
 - [33] E. Korsunsky, T. Halfmann, J. P. Marangos, M. Fleischhauer, and K. Bergmann, Analytical study of fourwave mixing with large atomic coherence, *Eur. Phys. J. D* **23**, 167 (2003).

- [34] T. Rickes, J. P. Marangos, and T. Halfmann, Enhancement of third-harmonic generation by Stark-chirped rapid adiabatic passage, *Opt. Commun.* **227**, 133 (2003).
- [35] H. K. Avetissian, A. G. Markossian, and G. F. Mkrtchian, High-order harmonic generation on atoms and ions with laser fields of relativistic intensities, *Phys. Rev. A* **84**, 013418 (2011).
- [36] J. T. Bahns, W. C. Stwalley, and P. L. Gould, Laser cooling of molecules: A sequential scheme for rotation, translation, and vibration, *J. Chem. Phys.* **104**, 9689 (1996).
- [37] M. Dupont-Nivet, M. Casiulis, T. Laudat, C. I. Westbrook, and S. Schwartz, Microwave-stimulated Raman adiabatic passage in a Bose–Einstein condensate on an atom chip, *Phys. Rev. A* **91**, 053420 (2015).
- [38] G. P. Djotyan, J. S. Bakos, and Zs. Sörlei, Coherent writing and reading of information using frequency-chirped short bichromatic laser pulses, *Opt. Express* **4**, 113 (1999).
- [39] G. P. Djotyan, J. S. Bakos, and Zs. Sörlei, Three-level Λ atom in the field of frequency-chirped bichromatic laser pulses: Writing and storage of optical phase information, *Phys. Rev. A* **64**, 013408 (2001).
- [40] O. V. Ivakhnenko, S. N. Shevchenko, and F. Nori, Nonadiabatic Landau–Zener Stückelberg–Majorana transitions, dynamics, and interference, *Phys. Rep.* **995**, 1 (2023).
- [41] K. Bergmann, H. Theuer, and B. W. Shore, Coherent population transfer among quantum states of atoms and molecules, *Rev. Mod. Phys.* **70**, 1003 (1998).
- [42] N. V. Vitanov, A. A. Rangelov, B. W. Shore, and K. Bergmann, Stimulated Raman adiabatic passage in physics, chemistry, and beyond, *Rev. Mod. Phys.* **89**, 015006 (2017).
- [43] N. V. Vitanov, T. Halfmann, B. W. Shore, and K. Bergmann, Laser-induced population transfer by adiabatic passage techniques, *Annu. Rev. Phys. Chem.* **52**, 763 (2001).
- [44] V. S. Malinovsky and J. L. Krause, General theory of population transfer by adiabatic rapid passage with intense, chirped laser pulses, *Eur. Phys. J. D* **14**, 147 (2001).
- [45] P. Král, I. Thanopoulos, and M. Shapiro, Colloquium: Coherently controlled adiabatic passage, *Rev. Mod. Phys.* **79**, 53 (2007).
- [46] C. Ding, R. Yu, X. Hao, and D. Zhang, Controllable population dynamics in Landau-quantized graphene, *Sci. Rep.* **8**, 1530 (2018).
- [47] F. Kappe, Y. Karli, G. Wilbur, R. G. Krämer, S. Ghosh, R. Schwarz, M. Kaiser, T. K. Bracht, D. E. Reiter, S. Nolte, K. C. Hall, G. Weihs, and V. Remesh, Chirped pulses meet quantum dots: Innovations, challenges, and future perspectives, *Adv. Quantum Technol.* **8**, 2300352 (2025).
- [48] A. P. Djotyan, G. P. Djotyan, and A. A. Avetisyan, Coherent control of lowest states of a shallow impurity in graphene monolayer by phase modulated laser pulses, *J. Contemp. Phys.* **59**, 272 (2024).
- [49] J. S. Bakos N.Sandor, Zs. Sörlei, and G. P. Djotyan, Creation of coherent superposition states in inhomogeneously broadened media with relaxation, *J. Opt. Soc. Am. B* **28**, 2785 (2011).
- [50] C. Xu, A. Poudel, and M. G. Vavilov, Nonadiabatic dynamics of a slowly driven dissipative two-level system, *Phys. Rev. A* **89**, 052102 (2014).
- [51] A. Levy, E. Torrontegui, and R. Kosloff, Action-noise-assisted quantum control, *Phys. Rev. A* **96**, 033417 (2017).
- [52] A. A. Avetisyan, A. P. Djotyan, G. P. Djotyan, A. L. Vartanian, and A. L. Asatryan, Coherent control of a shallow impurity quantum states in graphene monolayer by bichromatic phase modulated laser pulses: Influence of relaxation processes, *J. Contemp. Phys.* **60**, 42 (2025).
- [53] F. Ahmadinouri, M. Hosseini, and F. Sarreshtedari, Investigation of robust population transfer using quadratically chirped laser interacting with a two-level system, *Phys. Scr.* **94**, 105404 (2019).
- [54] P. Meystre and M. Sargent III, *Elements of Quantum Optics* (Springer-Verlag, New York, 1991).
- [55] L. Allen and J. H. Eberly, *Optical Resonance and Two-Level Atoms* (Dover, New York, 1987).