

Measuring deviations from a perfectly circular cross section of an optical nanofiber at the angstrom scale

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Tapered optical fibers (TOFs) with sub-wavelength-diameter waists, known as optical nanofibers, are powerful tools for interfacing quantum emitters and nanophotonics. These applications demand stable polarization of the fiber-guided light field. However, the linear birefringence resulting from Ångström-scale (Å-scale) deviations in the nanofiber's ideally circular cross section can lead to significant polarization changes within mm of light propagation. Here, we experimentally investigate such deviations using two *in situ* approaches. First, we measure the resonance frequencies of hundreds of flexural modes along the nanofiber, which exhibit splitting due to the noncircular cross section. By analyzing the mean resonance frequencies of each pair and the corresponding frequency splitting, we conclude that the nanofiber can be well described as having an elliptical cross section with a mean radius of 255.2(8) nm, where the semiaxes differ by only about 2 Å. Second, we monitor the polarization of the guided light field by imaging the light scattered out of the nanofiber and observe a periodic polarization change along it. From the linear birefringence due to the elliptical cross section, we infer a comparable difference in the semiaxes as the first method, and determine the orientation of the polarization eigenaxes. Our work is crucial for any fundamental or applied study that requires a well-controlled interaction between guided light and matter, in particular for quantum memories, frequency conversion, or lasing that require a large interaction length.

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I. INTRODUCTION

Optical nanofibers can be realized as the waist of tapered optical fibers (TOFs), and are powerful experimental tools [1], e.g., to interface guided light with emitters such as NV centers, cryogenically cooled molecules, or laser-cooled atoms [2–4]. The strong transverse confinement of guided light in nanofibers leads to high intensities [5], rendering this system well suited also for nonlinear optics experiments [6–9]. Other areas where TOFs find applications are in fiber-based beam splitters [10], active components such as fiber null couplers [11,12], and particle trapping [13].

The manufacture of TOFs has matured over the last decades, enabling, e.g., power transmissions in excess of 99% [14,15] and the controlled excitations of higher-order modes guided in the nanofiber part [16,17]. Moreover, a high accuracy of the radius profile of the produced TOF [8,18–21], e.g., thanks to real-time feedback [22,23], has been achieved, and excellent mechanical properties, e.g., sustaining GPa-scale tensile strains [24], have been observed. In most works thus far, the cross section of TOFs is assumed to be circular, which appears plausible as a result of the surface tension aiming to minimize the nanofiber's circumference during the heat-and-pull production [25,26]. Remarkably, if the nanofiber had a

slightly elliptical cross section with one semiaxis being only a few Å larger than the other, the polarization of the guided light exhibits a periodic modulation after only a few mm of propagation [27], due to the resulting linear birefringence of the nanofiber wave guide. A characterization of TOFs and their cross section with Å-scale resolution has, however, not been available until very recently [21,28].

Here, we experimentally determine Å-scale deviations from the ideally circular cross section of optical nanofibers using two methods. The first one relies on measuring mechanical resonances of the flexural modes, which appear as frequency-split pairs. From the average frequencies of each mode pair, we extract the mean radius of the nanofiber, while the splitting reveals the difference between the ellipse's semiaxes. Moreover, we introduce a general asymmetry parameter for TOFs with elliptical cross sections and analytically link it to the frequencies of the two split mechanical modes, valid also if the mean radius of the TOF changes along the fiber. This well describes our experimental observations across hundreds of modes. Our second method relies on the characterization of light scattered out of the nanofiber, which reveals the local polarization of the guided light. We provide an analytic formula relating the nanofiber's asymmetry parameter and its linear birefringence, yielding comparable deviations from the ideally circular nanofiber cross section as the first method. Moreover, the optical method allows determining the orientation of the two polarization eigenaxes of the linearly birefringent nanofiber with respect to the laboratory frame. Our first method is related to Ref. [28], in which flexural modes of TOFs and their frequency splitting are detected via an amplitude modulation of the transmitted light. While Ref. [28] requires a microfiber tip to be brought into overlap

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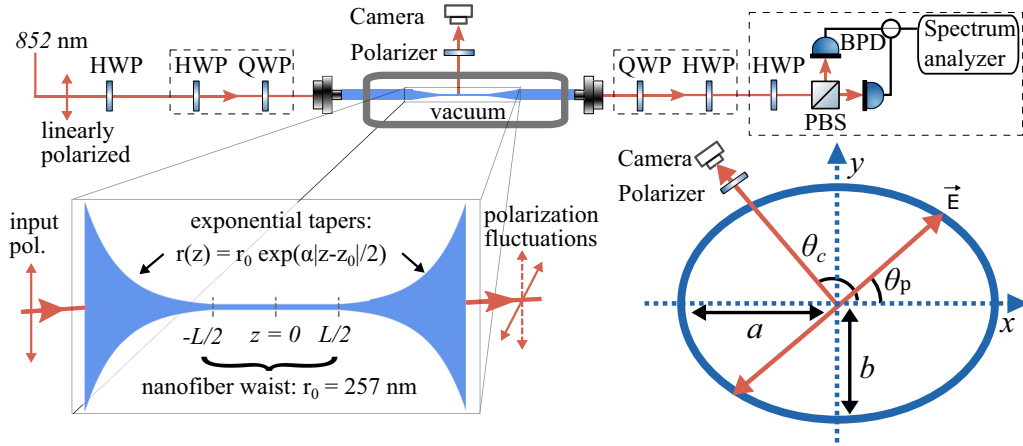


FIG. 1. Schematic of the experimental setup. We launch linearly polarized laser light into the TOF, which is placed inside a vacuum chamber. The radius profile of the TOF, including the exponential tapers and the nanofiber waist with nominal radius $r_0 = 257$ nm, is illustrated in the bottom left. The transmitted light is directed into a balanced detection setup to measure polarization fluctuations induced by the mechanical motion of the TOF. We monitor the scattering from the nanofiber waist using a camera outside the chamber while controlling the polarization along the nanofiber by the wave plates in front. In the bottom right, we sketch the elliptical cross section of the nanofiber with two semiaxes, a and b . The camera is aligned at an angle θ_c with respect to the semimajor axis, while the polarization angle is θ_p . BPD: Balanced photodiode, HWP: half-wave plate, QWP: quarter-wave plate, PBS: polarizing beam splitter.

with the evanescent field of the guided mode, both of our methods do not need a tip, simplifying the experimental implementation.

II. EXPERIMENTAL SETUP

Our experimental setup is depicted in Fig. 1. We mount a TOF with a nanofiber waist in a vacuum chamber with a typical pressure of 2×10^{-8} mbar. As illustrated in the inset of Fig. 1, the nanofiber waist has a nominal radius of $r_0 = 257$ nm over a length of $L = 10$ mm. The two ends of the nanofiber waist are connected to exponential tapers that extend from the waist radius r_0 up to approximately $9 \mu\text{m}$, following the profile $r(z) = r_0 \exp(\alpha|z - z_0|/2)$, with $\alpha/2 \approx 0.38 \text{ mm}^{-1}$ and $z_0 = \mp L/2$ for the left and right taper, respectively. We launch linearly polarized laser light with a wavelength of $\lambda = 852$ nm into the TOF. The TOF is manufactured from a commercial single-mode fiber (SM800-5.6-125) in a heat-and-pull process [25,26]. The birefringence between the input and output fiber ends and the nanofiber waist is prior unknown. To compensate, we place a half-wave plate (HWP) and a quarter-wave plate (QWP) before and after the fiber. A camera (CS165MU1/M, Thorlabs) outside the vacuum chamber images the light scattered from the nanofiber waist. We define the fiber axis as the z axis and place a polarizer before the camera, aligned to block the longitudinal component of the electrical field E_z [29]. More details will be discussed later. In transmission, a polarization analysis setup, consisting of a HWP, a polarizing beamsplitter (PBS), and a balanced photodiode (PDB210A, Thorlabs), measures the polarization fluctuations.

III. FLEXURAL MODES

In Fig. 2(a), we present the power spectral density (PSD), $S_{VV}(f)$, of the signal measured with the balanced photodiode

in the polarization analysis setup recorded with a spectrum analyzer (“Spectrum” module of HF2LI, Zurich Instruments), while the system is in equilibrium with its environment. The PSD covers a frequency range from 0–220 kHz with a frequency resolution of 0.4 Hz. The dominant peak at 166.3 kHz corresponds to the fundamental torsional mode of the nanofiber [30–32]. Upon fabrication, the TOF is glued across two separated fiber-holder halves on a translation stage and initially stretched, yielding only torsional modes in the PSD. While monitoring the frequency shift of the torsional mode, we gradually relax the TOF to reduce the strain until it is close to zero, at which point the flexural modes appear in the spectrum as numerous peaks [33]. As shown in the inset (a1) of Fig. 2(a), each flexural mode exhibits two resonant peaks. A simple extension beyond a circular cross section is to treat our nanofiber as a cylinder with a slightly elliptical cross section, defined by semimajor axis a and semiminor axis b . Since the flexural modes penetrate into the tapered regions, we model the entire structure as a beam with slightly elliptical cross section, connected to two exponential tapers. Remarkably, the resulting resonance frequencies follow the familiar formula for a simple beam with fixed-fixed boundary conditions [34]

$$\Omega_{a,b} = \left(n + \frac{1}{2}\right)^2 \frac{\pi^2}{L_{\text{eff}}^2} \sqrt{\frac{EI_{a,b}}{\rho A}} \quad (1a)$$

$$\stackrel{\text{circular}}{=} \left(n + \frac{1}{2}\right)^2 \frac{\pi^2}{2L_{\text{eff}}^2} \sqrt{\frac{E}{\rho}} r_0. \quad (1b)$$

In these expressions, we absorb all effects of the taper’s varying radius into a single effective length,

$$L_{\text{eff}} \approx L + 8/\alpha, \quad (2)$$

which in our case yields $L_{\text{eff}} \approx 20.52$ mm. In Eq. (1), $\Omega_{a,b}$ are the angular resonance frequencies for the oscillations bending about the respective semiaxis, n is the mode

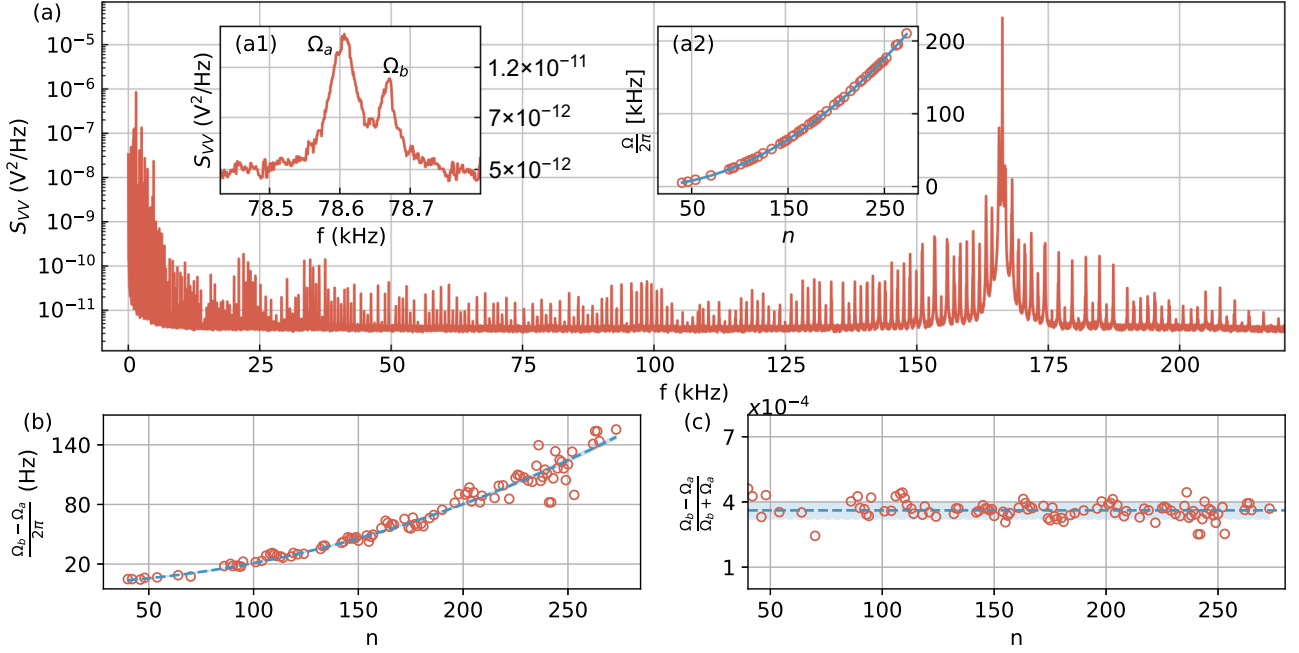


FIG. 2. Measurement and analysis of flexural modes, focusing on mode splitting. (a) Power spectral density (PSD), $S_{VV}(f)$, at thermal equilibrium. The fundamental torsional mode at 166.3 kHz appears as the most prominent peak, while hundreds of flexural modes manifest themselves as smaller peaks in the PSD. Inset (a1): A zoomed-in view of the PSD reveals a clear splitting of a single flexural mode. Inset (a2): the average resonance frequency, $\Omega = \frac{\Omega_a + \Omega_b}{2}$, as a function of the mode number n . To these data points (red dots), we fit with Eq. (1b) (blue line) to determine the mode number of the first captured peak, and the mean radius r_0 . (b) Mode splitting, $\frac{\Omega_b - \Omega_a}{2}$, as a function of the mode number n . Red dots are the data points while the blue line is a fit according to Eq. (3), yielding $a - b = 0.180(2)$ nm. (c) The normalized mode splitting, $\frac{\Omega_b - \Omega_a}{\Omega_a + \Omega_b}$, as a function of the mode number, n , remains constant. The blue dashed line shows the average value of the data points (red dots), corresponding to an asymmetry factor of $3.62(5) \times 10^{-4}$. The light blue area represents the standard deviation of the data points.

number, E is Young's modulus, ρ is the mass density, A is the cross-sectional area, and $I_{a,b}$ are the second moments of area, namely $I_a = A b^2/4$ and $I_b = A a^2/4$. Flexural modes bending about the semimajor axis have a different second moment of area than those about the semiminor axis, leading to the frequency splitting for each mode pair

$$\Omega_b - \Omega_a = \left(n + \frac{1}{2}\right)^2 \frac{\pi^2}{2L_{\text{eff}}^2} \sqrt{\frac{E}{\rho}} (a - b). \quad (3)$$

Based on the data shown in Fig. 2(a), we identify the resonance frequencies from approximately 3 kHz onward and record the relative mode number, n_{rel} , with respect to the first observed peak. We take the mean value of each double peak and fit the resonance frequencies using Eq. (1b) [inset (a2) of Fig. 2(a)], assuming a circular cross section with mean radius r_0 and $I_a = I_b = \frac{\pi}{4} r_0^4$. From the fit, we determine the absolute mode number as $n = n_{\text{rel}} + n_0$, where $n_0 = 27.3(2) \approx 27$ is the fit parameter corresponding to the mode number of the first observed peak. We also extract the mean radius of the nanofiber as $r_0 = 255.2(8)$ nm, closely matching the nominal value of 257 nm. Second, we calculate the mode splitting, $\Omega_b - \Omega_a$, and fit it using Eq. (3), as shown in Fig. 2(b). The fitting yields $a - b \approx 0.180(2)$ nm, which is much smaller than the mean radius. To confirm this small deviation, we introduce the asymmetry parameter $\zeta = \frac{a-b}{a+b} \ll 1$, assume it remains constant even in the tapers, and relate it to the

frequency splitting according to (see Appendix A)

$$\zeta = \frac{a - b}{a + b} \approx \frac{\Omega_b - \Omega_a}{\Omega_a + \Omega_b}. \quad (4)$$

Remarkably, this expression is independent of other fit parameters (e.g., L_{eff} and n) as well as material properties (e.g., ρ and E). In Fig. 2(c), we plot $\frac{\Omega_b - \Omega_a}{\Omega_a + \Omega_b}$ as a function of the mode number n . By taking the mean value of the data points, we obtain $\zeta = 3.62(5) \times 10^{-4}$, which is in good agreement with the asymmetry parameter calculated from the previously estimated values of $a - b$ and r_0 .

The asymmetry of the fiber's cross section not only causes mode splitting in the mechanical modes but also gives rise to linear birefringence with two principal axes for the guided optical mode. The effective refractive index is larger when the guided light is polarized along the semimajor axis, whereas light polarized along the semiminor axis experiences a smaller effective index. Thereby, the nanofiber section features linear birefringence, characterized by a difference in the propagation constants of the two light fields as (see Appendix B)

$$\Delta\beta = \beta_b - \beta_a = \frac{2\pi}{\lambda} \times 0.176 \zeta = 0.442(9) \text{ mm}^{-1}. \quad (5)$$

Here, the factor 0.176 is specific to the parameters, $\lambda = 852$ nm, $r_0 = 257$ nm, and a refractive index of the fiber material of $n = 1.452$, while β_a and β_b denote the propagation constants for light polarized along the eigenaxes a and b , respectively. In this case, an initially linearly polarized input light field whose polarization is not aligned to

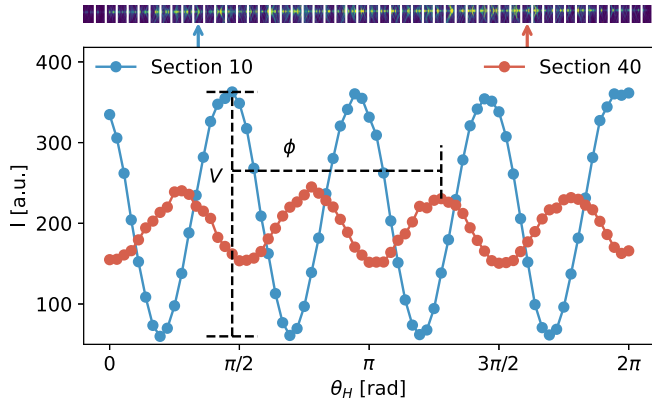


FIG. 3. Polarization-dependent scattering pattern of the nanofiber. The top panel shows an image of the nanofiber averaged over θ_H , see main text. The horizontal bright line corresponds to the scattering from the nanofiber while the white vertical lines divide the nanofiber into multiple sections. The bottom panel presents the intensities of sections 10 and 40, showing periodic modulation as a function of θ_H . From this modulation, we extract the visibility and phase of the scattered light. Note that TOFs produced with the flame-brushing technique exhibit a small but measurable surface roughness, typically with an rms amplitude of a few Å. The leading effect of surface roughness for our experiment is that a larger roughness gives rise to a stronger scattering of light out of the guided mode, thereby yielding a larger total signal on the camera.

one of the polarization eigenaxes undergoes periodic changes in its polarization state. During the propagation through the fiber, the light also assumes elliptical and circular polarization states. After a so-called beat length of $L_{\text{beat}} = 2\pi/\Delta\beta = 14.2(3)$ mm, the light field goes back to the initial linear polarization state.

IV. POLARIZATION IMAGING

The idea of the polarization imaging is based on Refs. [29,35]. We analyze the light scattered out of the nanofiber by surface roughness and impurities, and density inhomogeneities in the bulk silica. Assuming Rayleigh scatterers, they are expected to preserve the local polarization of the guided light. Following the dipole radiation pattern, maximum intensity is expected when the optical axis of the imaging system is perpendicular to the local linear polarization of the guided light, while minimum intensity is expected when they are parallel. The resulting image recorded with the camera is shown in Fig. 3 (top), where the horizontal bright line corresponds to the light scattered by the nanofiber. We then divide this region of interest (size: $14.88(9)$ mm $\times$ $0.42(1)$ mm) into 51 sections, as indicated by the white vertical lines, and record the integrated intensity I . We adjust the first HWP at the input to rotate the polarization of the guided field and measure the visibility of the integrated intensity, which is defined as $V = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$. We then tune a second HWP and a QWP before the fiber to maximize the visibility at a specific section, where we set $z = 0$ and assume the light field is linearly polarized [29,35]. At that position the polarization angle follows $\theta_p = \theta_0 + 2\theta_H$ [35], where θ_0 is the initial polarization angle and θ_H is the rotation angle of the

first HWP. In Fig. 3 (bottom), we display the intensities, I , of sections 10 and 40 as a function of θ_H ; see the Supplemental Material [36] for two videos (Movies S1 and S2) showing the images recorded by cameras 1 and 2 during rotation of the first HWP. The intensity in each section exhibits a periodic modulation in response to the HWP rotation, allowing us to extract the visibility and relative phase (with respect to θ_H) at every section. In Appendix C, we attribute this behavior to the linear birefringence of the nanofiber as a result of an elliptical cross section, where the principal axes for the guided optical modes align with the two semiaxes. As sketched in Fig. 1 (bottom right), we define the fiber frame, such that the x axis is oriented along the semimajor axis and the y axis along the semiminor axis. The predicted intensity is

$$I(\theta_c, \theta_p, z) \propto 1 - \text{Re}\{e^{i2\theta_p}W\}, \quad (6)$$

where θ_c is the camera angle, z is the position along the fiber axis, and $W = \cos 2\theta_c - i \sin 2\theta_c \cos \Delta\beta z$. The visibility and phase are then given as

$$V_W = |W|, \quad \phi_W = \text{Arg}(W), \quad (7)$$

where Arg denotes the argument of a complex number. At each position z , the visibility and phase depend only on two factors: the camera angle θ_c and the linear birefringence $\Delta\beta$. Initially, prior to the measurement, θ_c is unknown. If the camera happens to be aligned along one of the two principal axes such that $\theta_c = 0$, the measured signal will become independent of $\Delta\beta$. To avoid this scenario, we mount two cameras at different angles to do the intensity measurement, with an angle separation of $\Delta\theta_c = 72^\circ \neq 90^\circ$. In Fig. 4, we plot the visibility and phase recorded by camera 1 and camera 2, respectively. By simultaneously fitting the visibility data from both cameras using Eq. (7), we find that $\Delta\beta = 0.222(9)$ mm $^{-1}$, $L_{\text{beat}} = 28.2(12)$ mm, $a - b \approx 0.085(4)$ nm, and the camera angles, $\theta_{c,1} = 129.9(4)^\circ$ and $\theta_{c,2} = 57.9(4)^\circ$. We infer that the angle between the semimajor axis and the horizontal plane is about 36° . The phase data is then fitted while fixing $\Delta\beta$ to the above value.

We briefly compare the findings obtained with the flexural mode and the polarization imaging method. In both cases, we find Å-scale deviations from the commonly assumed circular cross section of the nanofiber. All experimental data can be well described assuming a TOF with an elliptical cross section for which the relative deviation of the semimajor and the semiminor axis is at the 10^{-4} level. Although it is remarkable that such small deviations can be detected, the actual values found with the two methods differ by about a factor of 2.

We speculate that this deviation arises from adsorption and desorption processes, which, depending on the conditions (pressure in the vacuum chamber, power and polarization of guided light, etc.) experienced by the nanofiber prior to the measurement, may add and remove adsorbates on the nanofiber surface. This hypothesis is compatible with the observation that the mode splitting observed in the flexural mode measurement always stays approximately constant and agrees within better than 10% when measured for different pressures in the vacuum chamber or different probe laser powers: it is easy to show that the normalized frequency splitting, $(\Omega_b - \Omega_a)/(\Omega_a + \Omega_b)$, does not change when adsorbates build up on the nanofiber surface, i.e., when material is deposited that

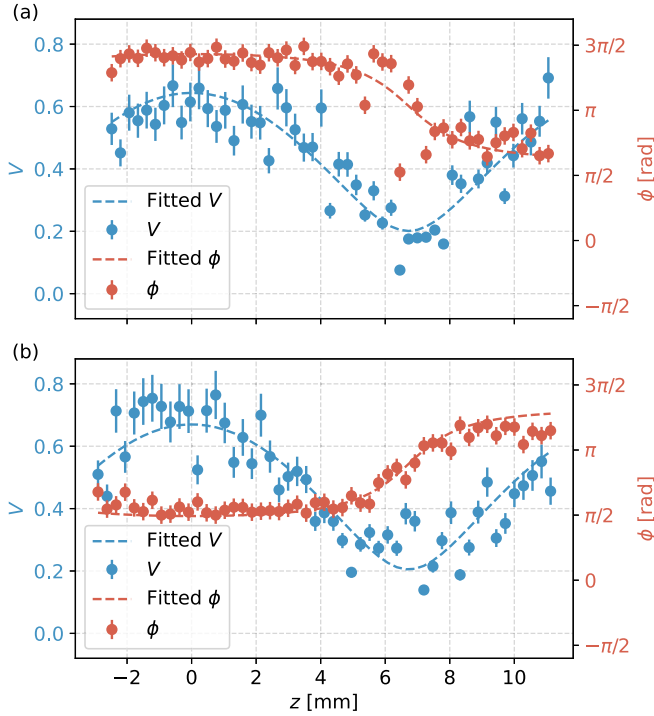


FIG. 4. Visibility and phase measurement for (a) camera 1, and (b) camera 2. From a joint fit of the visibility data for both cameras using Eq. (7), we extract the linear birefringence, $\Delta\beta = 0.222(9)\text{ mm}^{-1}$, and camera angles, $\theta_{c,1} = 129.9(4)^\circ$ and $\theta_{c,2} = 53.9(9)^\circ$. The phase is then fitted using the obtained value for $\Delta\beta$.

adds to the nanofibers inertia but can be assumed not to alter the restoring force that underlies the flexural mode vibrations. The beat length found in the polarization imaging and, thus, the inferred asymmetry of the nanofiber, however, scatters by about 10% when carried out at the same pressure and by about 50% when markedly varying pressures or probe laser powers (and thus the equilibrium temperature of the nanofiber). Surface coverage of the nanofiber may change under these different conditions, thereby modifying its birefringence in a way that is compatible with the observed variations in beat length [37,38].

V. CONCLUSIONS

In conclusion, we demonstrated two experimental methods that allow us to characterize the elliptical cross section of the nanofiber waist of a TOF, with immediate relevance for defining and maintaining a well-defined polarization of the guided light in optical nanofibers. Both methods, one based on analyzing the frequency splitting of flexural mode mechanical resonances and the other based on analyzing the polarization of the light scattered out of the fiber, yield Å-scale deviations from an ideally circular fiber cross section. Moreover, both methods are simple to implement, work with any tapered optical fiber, and are compatible with in-vacuum applications. Specifically, we recommend method 1 (flexural modes) for a precise measurement of the nanofiber's cross section ($a - b$), whereas method 2 (polarization imaging) is better suited for

setting a well-defined polarization of the nanofiber-guided light.

Similar deviations from a perfectly round cross section are observed for most tapered fibers produced with our fiber pulling rig, and similarly seem to be present for tapered fibers made by other research groups [28]. In future work, the origin of those imperfections will be investigated. We speculate that gravity or the direction of the gas flow in the fabrication process might play a role as they, evidently, break the radial symmetry of the problem. Once the mechanism is understood, we aim at controlling the exact nanofiber shape during the fabrication. For example, more strongly elliptical fiber cross sections appear as a promising approach in order to create polarization-maintaining optical nanofibers [39,40]. Our work will enable an improvement in tapered-fiber coupling, and in light-matter interaction, e.g., via Brillouin scattering [41,42]. It will also allow a higher level of control in interfacing, e.g., plasmonic particles [43] and quantum emitters such as laser-cooled atoms [27,29,44–49]. From another perspective, shape control during fabrication may also allow optimization of the optomechanical coupling strength for both flexural and torsional modes [31,32,50,51]. This could offer the possibility to cool the mechanical motions to the quantum regime, making TOFs a promising platform for hybrid quantum systems [52].

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: LATERAL VIBRATIONS OF SLIGHTLY ELLIPTICAL BEAMS

The equation of motion for the free lateral vibration of a homogeneous beam oriented along the z axis with nonuniform cross section is [34]

$$\partial_z^2 [EI(z)\partial_z^2 w(z, t)] + \rho A(z)\partial_t^2 w(z, t) = 0, \quad (\text{A1})$$

where E is Young's modulus, $I(z)$ is the second moment of area, $w(z, t)$ is the deflection, ρ is the mass density, and $A(z)$ is the area of the cross section. We assume that the beam has an elliptical cross section with semiaxes $a(z)$ and $b(z)$, and define the asymmetry as $\zeta = \frac{a-b}{a+b}$, such that

$$a(z) = r_0 r(z)[1 + \zeta(z)], \quad (\text{A2a})$$

$$b(z) = r_0 r(z)[1 - \zeta(z)], \quad (\text{A2b})$$

where r_0 is the characteristic radius of the structure and $r(z)$ is the (unitless) radius profile. For $\zeta \ll 1$,

we find

$$\begin{aligned} A(z) &= \pi a(z)b(z) = \pi r_0^2 r(z)^2 + O(\zeta^2) \\ &= A_0 r(z)^2 + O(\zeta^2), \end{aligned} \quad (\text{A3a})$$

$$\begin{aligned} I_a(z) &= \frac{A(z)}{4} b(z)^2 = \frac{\pi r_0^4}{4} r(z)^4 [1 - 2\zeta(z)] + O(\zeta^2) \\ &= I_0 r(z)^4 [1 - 2\zeta(z)] + O(\zeta^2), \end{aligned} \quad (\text{A3b})$$

$$\begin{aligned} I_b(z) &= \frac{A(z)}{4} a(z)^2 = \frac{\pi r_0^4}{4} r(z)^4 [1 + 2\zeta(z)] + O(\zeta^2) \\ &= I_0 r(z)^4 [1 + 2\zeta(z)] + O(\zeta^2), \end{aligned} \quad (\text{A3c})$$

where $A_0 = \pi r_0^2$ and $I_0 = \frac{\pi r_0^4}{4}$. Substituting Eqs. (A3) into Eq. (A1), we obtain

$$\frac{c_0^2}{r(z)^2} \partial_z^2 [r(z)^4 [1 \mp 2\zeta(z)] \partial_z^2 w(z, t)] = -\partial_t^2 w(z, t), \quad (\text{A4})$$

where $c_0^2 = \frac{E I_0}{\rho A_0}$. The solution can be found using the method of separation of variables,

$$w(z, t) = W(z)T(t). \quad (\text{A5})$$

This leads to

$$\begin{aligned} \frac{1}{W(z)} \frac{c_0^2}{r(z)^2} \partial_z^2 [r(z)^4 [1 \mp 2\zeta(z)] \partial_z^2 W(z)] \\ = -\frac{\partial_t^2 T(t)}{T(t)} = \Omega^2, \end{aligned} \quad (\text{A6})$$

where any solution Ω is an angular resonance frequency. Eq. (A6) can be written as two equations,

$$\partial_t^2 T(t) + \Omega^2 T(t) = 0, \quad (\text{A7a})$$

$$\frac{1}{r(z)^2} \partial_z^2 [r(z)^4 [1 \mp 2\zeta(z)] \partial_z^2 W(z)] - \frac{\Omega^2}{c_0^2} W(z) = 0. \quad (\text{A7b})$$

For constant asymmetry, $\zeta(z) = \zeta$, we will find

$$\frac{1}{r(z)^2} \partial_z^2 [r(z)^4 \partial_z^2 W(z)] - s^4 W(z) = 0, \quad (\text{A8a})$$

$$\text{with } s = \sqrt{\frac{\Omega^2}{c_0^2(1 \mp 2\zeta)}} \approx \sqrt{\frac{\Omega}{c_0(1 \pm \zeta)}}. \quad (\text{A8b})$$

For any beam, there exists an infinite set of normal modes, each associated with a distinct resonance frequency [34]. Accordingly, we assume the solutions to Eq. (A8a) are given by $s = s_n$, where $n = 1, 2, 3, \dots$ is the mode number, which yields

$$\Omega_n = s_n^2 c_0 (1 \pm \zeta) \Rightarrow \frac{\Omega_b - \Omega_a}{\Omega_a + \Omega_b} = \zeta. \quad (\text{A9})$$

Despite the inhomogeneous radius profile of the beam, as long as ζ stays constant along the beam, the normalized mode splitting will always follow Eq. (A9). This is important for our analysis, as the flexural modes propagate not only through the nanofiber waist but also into the exponential tapers, where the radius varies continuously.

APPENDIX B: LINEAR BIREFRINGENCE FOR A DIELECTRIC WAVE GUIDE WITH A SLIGHTLY ELLIPTICAL CROSS SECTION

The analysis of this section follows Ref. [53]. We define the elliptical coordinates (ξ, η, z) as

$$x = q \cosh \xi \cos \eta, \quad (\text{B1a})$$

$$y = q \sinh \xi \sin \eta, \quad (\text{B1b})$$

$$z = z, \quad (\text{B1c})$$

where q is half the distance between the foci (i.e., the focal length in elliptical coordinates), $\xi \in [0, \infty)$, and $\eta \in [0, 2\pi)$. In these coordinates, $\xi = \xi_0 = \text{constant}$ defines an ellipse with the semiaxes

$$a = q \cosh \xi_0 = \frac{q}{2} (e^{\xi_0} + e^{-\xi_0}), \quad (\text{B2a})$$

$$b = q \sinh \xi_0 = \frac{q}{2} (e^{\xi_0} - e^{-\xi_0}). \quad (\text{B2b})$$

We then find the asymmetry as

$$\zeta = \frac{a - b}{a + b} = e^{-2\xi_0}. \quad (\text{B3})$$

For $\xi_0 \rightarrow \infty$, the shape approaches a circle, but the area,

$$A = \pi ab = \pi \frac{q^2}{4} \left(\frac{1}{\zeta} + \zeta \right), \quad (\text{B4})$$

diverges in this limit. For a finite circle of area $A = \pi r_0^2$, we thus additionally require that

$$\frac{q^2}{4} = r_0^2 \frac{\zeta}{1 + \zeta^2} = r_0^2 \zeta [1 + O(\zeta^2)]. \quad (\text{B5})$$

For an elliptical waveguide, whose elliptical cross section is parameterized by q and ξ_0 , with relative permittivity ϵ_0 (outside the ellipse) and ϵ_1 (inside the ellipse), the approximate characteristic equations [53] for the ordinary and extraordinary HE_{nm} modes are

$$\begin{aligned} \left[\frac{1}{\gamma_1^2} \frac{Ce'_n(\xi_0)}{Ce_n(\xi_0)} + \frac{1}{\gamma_0^2} \frac{Fek'_n(\xi_0)}{Fek_n(\xi_0)} \right] \left[\frac{1}{\gamma_1^2} \frac{Se'_n(\xi_0)}{Se_n(\xi_0)} + \frac{\epsilon_0}{\epsilon_1} \frac{1}{\gamma_0^2} \frac{Gek'_n(\xi_0)}{Gek_n(\xi_0)} \right] \\ - n^2 \frac{(\gamma_0^2 + \gamma_1^2)(\gamma_0^2 + \frac{\epsilon_0}{\epsilon_1} \gamma_1^2)}{\gamma_0^4 \gamma_1^4} = 0, \end{aligned} \quad (\text{B6a})$$

$$\begin{aligned} \left[\frac{1}{\gamma_1^2} \frac{Se'_n(\xi_0)}{Se_n(\xi_0)} + \frac{1}{\gamma_0^2} \frac{Gek'_n(\xi_0)}{Gek_n(\xi_0)} \right] \left[\frac{1}{\gamma_1^2} \frac{Ce'_n(\xi_0)}{Ce_n(\xi_0)} + \frac{\epsilon_0}{\epsilon_1} \frac{1}{\gamma_0^2} \frac{Fek'_n(\xi_0)}{Fek_n(\xi_0)} \right] \\ - n^2 \frac{(\gamma_0^2 + \gamma_1^2)(\gamma_0^2 + \frac{\epsilon_0}{\epsilon_1} \gamma_1^2)}{\gamma_0^4 \gamma_1^4} = 0. \end{aligned} \quad (\text{B6b})$$

The functions Se_n , Ce_n , Fek_n , and Gek_n are modified (or radial) Mathieu functions [54]. The parameters γ_0 and γ_1 are defined as

$$\gamma_0^2 = \frac{q^2}{4} (\beta^2 - k_0^2), \quad (\text{B7a})$$

$$\gamma_1^2 = \frac{q^2}{4} (k_1^2 - \beta^2). \quad (\text{B7b})$$

Here, $k_0 = (\omega/c)\sqrt{\epsilon_0}$ is the wave number in the surrounding medium, $k_1 = (\omega/c)\sqrt{\epsilon_1}$ is the wave number inside the bulk material, and β is the effective wave number of the guided mode. Similar to the case of a circular cross section,

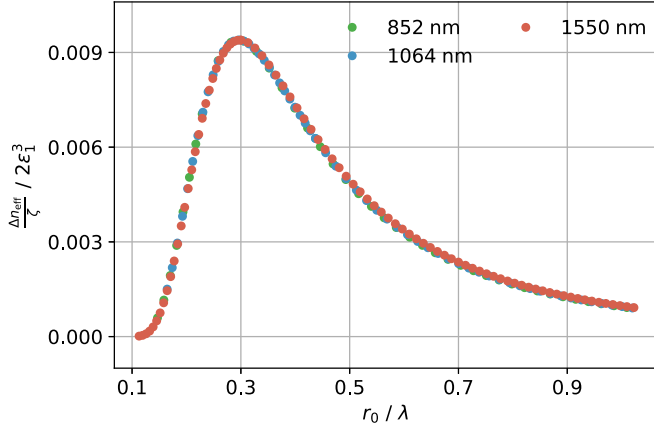


FIG. 5. Differences in the effective refractive indices as a function of r_0/λ . The normalized quantity, $\Delta n_{\text{eff}}/2\epsilon_1^3$, allows to relate the geometry of the elliptical cross section, ζ , to the birefringence of the nanofiber in a generally applicable way, i.e., valid for different wavelengths. Dots in different colors correspond to three wavelengths, 852 nm, 1064 nm, and 1550 nm, respectively. All calculations are for the fundamental guided mode.

the only modes without any cutoff frequency are the HE_{11} modes, where $n = m = 1$.

In the following, we Taylor expand Eqs. (B6) to first order in ζ . Since these equations scale with $1/\zeta^2$ for vanishing ζ , we first multiply both sides with ζ^2 to define

$$f(\zeta; \beta) = \zeta^2 \times [\text{left-hand side of Eqs. (B6)}]. \quad (\text{B8})$$

With this, we need to solve $f(\zeta; \beta) = 0$ by finding the zeros as a function of β . As we are interested in the solution to first order in ζ , we use the ansatz

$$\beta(\zeta) = \beta_0 + \zeta \Delta\beta + O(\zeta^2), \quad (\text{B9})$$

which leads to

$$\begin{aligned} f[\zeta; \beta(\zeta)] &= f[\zeta; \beta_0 + \zeta \Delta\beta + O(\zeta^2)] \\ &= f(0; \beta_0) + \zeta [\partial_\zeta f(0; \beta_0) + \Delta\beta \partial_\beta f(0; \beta_0)] + O(\zeta^2) \\ &= 0. \end{aligned} \quad (\text{B10})$$

The solution $(\beta_0, \Delta\beta)$ is thus found by first finding the solution to $f(0; \beta_0) = 0$. This equation will actually reduce to the characteristic equation for a spherical wave guide. Once β_0 is known, we find $\Delta\beta$ by

$$\Delta\beta = -\frac{\partial_\zeta f(0; \beta_0)}{\partial_\beta f(0; \beta_0)}. \quad (\text{B11})$$

We notice that $\Delta\beta$ of the ordinary mode equals $-\Delta\beta$ of the extraordinary mode. This follows from the fact that $\partial_\zeta f(0; \beta_0)$ changes sign from Eq. (B6a) to Eq. (B6b), while $\partial_\beta f(0; \beta_0)$ stays the same. We find β_0 by numerically solving $f(0; \beta_0) = 0$, which yields a single solution if r_0/λ

is sufficiently small, such that the cylinder operates as a single-mode wave guide. We then numerically compute $\Delta\beta = -\partial_\zeta f(0; \beta_0)/\partial_\beta f(0; \beta_0)$. For the parameters $\{\lambda, r_0, \epsilon_0, \epsilon_1\} = \{852 \text{ nm}, 257 \text{ nm}, 1.00^2, 1.45^2\}$ (the value of ϵ_1 follows from the Sellmeier's equation), we find

$$n_{\text{eff}} = \frac{\beta_0}{2\pi/\lambda} = 1.143, \quad (\text{B12a})$$

$$\Delta n_{\text{eff}} = \frac{\beta_b - \beta_a}{2\pi/\lambda} = \frac{-2\Delta\beta}{2\pi/\lambda} \zeta = 0.176 \zeta. \quad (\text{B12b})$$

Figure 5 shows the normalized prefactor from Eq. (B12b), defined as $\Delta n_{\text{eff}}/2\epsilon_1^3$, plotted against r_0/λ . The normalization factor $1/(2\epsilon_1^3)$ is an empirical choice that ensures good overlap between the curves at different wavelengths. Using the data points from Fig. 5, one can estimate the birefringence of an optical nanofiber for a given radius, laser wavelength, and asymmetry.

APPENDIX C: VISIBILITY AND PHASE FROM THE POLARIZATION IMAGING

Compared to a linearly polarized light field in the free space, the fundamental HE_{11} mode in the nanofiber is only quasilinearly polarized [5]. The guided light has a nontrivial longitudinal component along the fiber axis (z axis), which we filter out by placing a polarizer before the camera. For the polarization imaging, we approximate the transverse electric field component in the plane orthogonal to the fiber axis as a linear polarization at an angle θ_p at $z = 0$, as sketched in Fig. 1 (bottom right). This allows us to define the normalized electric field vector as $\vec{u}(z=0) = \frac{\vec{E}(z=0)}{|\vec{E}(z=0)|} = \begin{pmatrix} \cos \theta_p \\ \sin \theta_p \end{pmatrix}$, which is unitless and has unit amplitude. Due to the linear birefringence of the nanofiber, the electric field at position z is then

$$\vec{u}(z) = \begin{pmatrix} e^{i\beta_a z} \cos \theta_p \\ e^{i\beta_b z} \sin \theta_p \end{pmatrix} = e^{i\beta_a z} \begin{pmatrix} \cos \theta_p \\ e^{i\Delta\beta z} \sin \theta_p \end{pmatrix}. \quad (\text{C1})$$

The camera is at an angle of θ_c , such that the camera axis is $\begin{pmatrix} \cos \theta_c \\ \sin \theta_c \end{pmatrix}$. The projection of the electrical field onto the direction orthogonal to the camera axis is

$$|\vec{u}_p(z)| = |\cos \theta_p \sin \theta_c - e^{i\Delta\beta z} \sin \theta_p \cos \theta_c|. \quad (\text{C2})$$

Thereby, the predicted intensity captured by the camera is

$$I_1(\theta_p, z) \propto |\vec{u}_p(z)|^2 = 1 - \text{Re}\{e^{i2\theta_p} W\}, \quad (\text{C3})$$

where we define $W = \cos 2\theta_c - i \sin 2\theta_c \cos \Delta\beta z$. We observe that at every position z along the fiber and for a fixed camera position θ_c , the intensity is a sinusoidal function of the polarization angle. In the experiment, we tune this angle by a half-wave plate in order to extract the interference visibility

$$V_W = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} = |W|. \quad (\text{C4})$$

- [1] L. Tong, F. Zi, X. Guo, and J. Lou, *Opt. Commun.* **285**, 4641 (2012).
- [2] M. J. Morrissey, K. Deasy, M. Frawley, R. Kumar, E. Prel, L. Russell, V. G. Truong, and S. Nic Chormaic, *Sensors* **13**, 10449 (2013).

- [3] P. Solano, J. A. Grover, J. E. Hoffman, S. Ravets, F. K. Fatemi, L. A. Orozco, and S. L. Rolston, *Advances in Atomic, Molecular, and Optical Physics* (Elsevier, Amsterdam, 2017), Vol. 66, pp. 439–505

- [4] K. P. Nayak, M. Sadgrove, R. Yalla, F. Le Kien, and K. Hakuta, *J. Opt.* **20**, 073001 (2018).
- [5] F. Le Kien, J. Liang, K. Hakuta, and V. Balykin, *Opt. Commun.* **242**, 445 (2004).
- [6] T. A. Birks, W. J. Wadsworth, and P. S. J. Russell, *Opt. Lett.* **25**, 1415 (2000).
- [7] S. M. Spillane, G. S. Pati, K. Salit, M. Hall, P. Kumar, R. G. Beausoleil, and M. S. Shahriar, *Phys. Rev. Lett.* **100**, 233602 (2008).
- [8] U. Wiedemann, K. Karapetyan, C. Dan, D. Pritzkau, W. Alt, S. Irsen, and D. Meschede, *Opt. Express* **18**, 7693 (2010).
- [9] J. Zhang, Y. Kang, X. Guo, Y. Li, K. Liu, Y. Xie, H. Wu, D. Cai, J. Gong, Z. Shi *et al.*, *Light Sci. Appl.* **12**, 89 (2023).
- [10] G. Keiser, *Fiber Optic Communications* (Springer, Berlin, 2021).
- [11] T. A. Birks, P. S. J. Russell, and D. Culverhouse, *J. Lightwave Technol.* **14**, 2519 (1996).
- [12] M. Blaha, A. Rauschenbeutel, and R. Pennetta, *Phys. Rev. Appl.* **22**, 054008 (2024).
- [13] P. Praveen Kamath, S. Sil, V. G. Truong, and S. Nic Chormaic, *Biomed. Opt. Express* **14**, 6172 (2023).
- [14] R. Nagai and T. Aoki, *Opt. Express* **22**, 28427 (2014).
- [15] J. Hoffman, S. Ravets, J. Grover, P. Solano, P. Kordell, J. Wong-Campos, L. Orozco, and S. Rolston, *AIP Adv.* **4**, 067124 (2014).
- [16] M. C. Frawley, A. Petcu-Colan, V. G. Truong, and S. Nic Chormaic, *Opt. Commun.* **285**, 4648 (2012).
- [17] S. Ravets, J. Hoffman, L. Orozco, S. Rolston, G. Beadie, and F. Fatemi, *Opt. Express* **21**, 18325 (2013).
- [18] F. Warken and H. Giessen, *Opt. Lett.* **29**, 1727 (2004).
- [19] M. Sumetsky, Y. Dulashko, J. Fini, A. Hale, and J. Nicholson, *Opt. Lett.* **31**, 2393 (2006).
- [20] L. S. Madsen, C. Baker, H. Rubinsztein-Dunlop, and W. P. Bowen, *Nano Lett.* **16**, 7333 (2016).
- [21] F. K. Fatemi, J. E. Hoffman, P. Solano, E. F. Fenton, G. Beadie, S. L. Rolston, and L. A. Orozco, *Optica* **4**, 157 (2017).
- [22] Y. Yu, X. Zhang, Z. Song, J. Wang, and Z. Meng, *Appl. Opt.* **53**, 8222 (2014).
- [23] Y. Kang, J. Gong, Y. Xu, N. Yao, W. Fang, X. Guo, and L. Tong, *IEEE Photon. Technol. Lett.* **32**, 219 (2020).
- [24] S. Holleis, T. Hoinkes, C. Wuttke, P. Schneeweiss, and A. Rauschenbeutel, *Appl. Phys. Lett.* **104**, 163109 (2014).
- [25] T. Birks and Y. Li, *J. Lightwave Technol.* **10**, 432 (1992).
- [26] J. Ward, A. Maimaiti, V. H. Le, and S. Chormaic, *Rev. Sci. Instrum.* **85**, 111501 (2014).
- [27] A. Goban, K. S. Choi, D. J. Alton, D. Ding, C. Lacroûte, M. Pototschnig, T. Thiele, N. Stern, and H. J. Kimble, *Phys. Rev. Lett.* **109**, 033603 (2012).
- [28] C. Wang, L. Song, J. Wang, J. Zhou, K. Feng, Q. Zhang, C. L. Zou, G. Li, P. Zhang, and T. Zhang, *Opt. Express* **33**, 14964 (2025).
- [29] E. Vetsch, S. T. Dawkins, R. Mitsch, D. Reitz, P. Schneeweiss, and A. Rauschenbeutel, *IEEE J. Sel. Top. Quantum Electron.* **18**, 1763 (2012).
- [30] C. Wuttke, G. D. Cole, and A. Rauschenbeutel, *Phys. Rev. A* **88**, 061801(R) (2013).
- [31] F. Tebbenjohanns, J. Jia, M. Antesberger, A. S. Prasad, S. Pucher, A. Rauschenbeutel, J. Volz, and P. Schneeweiss, *Phys. Rev. A* **108**, L031101 (2023).
- [32] D. Su, Y. Jiang, P. Solano, L. A. Orozco, J. Lawall, and Y. Zhao, *Photon. Res.* **11**, 2179 (2023).
- [33] L. Song, C. Wang, Y. Hu, J. Zhou, Q. Zhang, C.-L. Zou, G. Li, P. Zhang, and T. Zhang, *Phys. Rev. Lett.* **132**, 033801 (2024).
- [34] S. Rao, *Mechanical Vibrations in SI Units*, 6th ed. (Addison-Wesley, New York, 2017).
- [35] M. Joos, A. Bramati, and Q. Glorieux, *Opt. Express* **27**, 18818 (2019).
- [36] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/w1vv-4ys7> for two videos (Movies S1 and S2) showing images recorded by cameras 1 and 2 during rotation of the first half-wave plate.
- [37] U. Delic, Cavity cooling by coherent scattering of a levitated nanosphere in vacuum, Ph.D. thesis, University of Vienna, 2019, <https://theses.univie.ac.at/detail/50464>.
- [38] F. Ricci, M. T. Cuairan, A. W. Schell, E. Hebestreit, R. A. Rica, N. Meyer, and R. Quidant, *ACS Nano* **16**, 8677 (2022).
- [39] Y. Jung, G. Brambilla, K. Oh, and D. J. Richardson, *Opt. Lett.* **35**, 378 (2010).
- [40] H. Xuan, J. Ju, and W. Jin, *Opt. Express* **18**, 3828 (2010).
- [41] J. C. Beugnot, S. Lebrun, G. Pauliat, H. Maillotte, V. Laude, and T. Sylvestre, *Nat. Commun.* **5**, 5242 (2014).
- [42] O. Florez, P. F. Jarschel, Y. A. Espinel, C. Cordeiro, T. Mayer Alegre, G. S. Wiederhecker, and P. Dainese, *Nat. Commun.* **7**, 11759 (2016).
- [43] J. Petersen, J. Volz, and A. Rauschenbeutel, *Science* **346**, 67 (2014).
- [44] J. Lee, J. Grover, J. Hoffman, L. Orozco, and S. Rolston, *J. Phys. B* **48**, 165004 (2015).
- [45] S. Ruddell, K. Webb, I. Herrera, A. Parkins, and M. Hoogerland, *Optica* **4**, 576 (2017).
- [46] C. Østfeldt, J.-B. S. Béguin, F. T. Pedersen, E. S. Polzik, J. H. Müller, and J. Appel, *Opt. Lett.* **42**, 4315 (2017).
- [47] N. V. Corzo, J. Raskop, A. Chandra, A. S. Sheremet, B. Gouraud, and J. Laurat, *Nature (London)* **566**, 359 (2019).
- [48] R. K. Gupta, J. L. Everett, A. D. Tranter, R. Henke, V. Gokhroo, P. K. Lam, and S. Nic Chormaic, *AVS Quantum Sci.* **4**, 026801 (2022).
- [49] G. Kestler, K. Ton, D. Filin, C. Cheung, P. Schneeweiss, T. Hoinkes, J. Volz, M. S. Safronova, A. Rauschenbeutel, and J. T. Barreiro, *PRX Quantum* **4**, 040308 (2023).
- [50] R. Pennetta, S. Xie, R. Zeltner, J. Hammer, and P. S. J. Russell, *Photon. Res.* **8**, 844 (2020).
- [51] M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, *Rev. Mod. Phys.* **86**, 1391 (2014).
- [52] P. Treutlein, C. Genes, K. Hammerer, M. Poggio, and P. Rabl, Hybrid mechanical systems, in *Cavity Optomechanics: Nano- and Micromechanical Resonators Interacting with Light*, edited by M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt (Springer, Berlin, 2014), pp. 327–351.
- [53] C. Yeh and F. I. Shimabukuro, *The Essence of Dielectric Waveguides* (Springer, Berlin, 2008).
- [54] N. W. McLachlan, *Theory and Application of Mathieu Functions* (Clarendon Press, Oxford, 1947).