

Entangling two exciton modes placed in two separate exciton-optomechanical cavities

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Distributing robust and long-distance entanglement between repeater nodes is a critical task for both foundational tests of quantum physics and the development of scalable quantum networks. In this paper, we propose a scheme for entangling two spatially separated exciton modes located in two exciton-optomechanical cavities. Each cavity, consisting of a semiconductor optomechanical cavity integrated with a quantum well, is simultaneously driven by a red-detuned laser and a squeezed vacuum light. Under realistic experimental conditions, we show that steady-state entanglement can be established between the two exciton modes by transferring quantum correlations from the squeezed light to them. Maximum entanglement is achieved when the exciton modes are tuned to resonance with the anti-Stokes sidebands of the driving lasers. With a high mechanical quality factor, the resulting entanglement exhibits robust behavior against thermal noise and remains significantly nonzero even at room temperature. Furthermore, we discuss a method to access and detect the excitonic entanglement. Our scheme provides a promising platform for practical long-distance entanglement distribution, which is of fundamental importance for quantum information processing and network architectures.

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I. INTRODUCTION

Entanglement, a cornerstone of quantum mechanics [1], has attracted renewed interest with the rise of quantum information science [2]. It is now widely recognized as a vital resource underpinning various quantum information processing tasks, including quantum teleportation [3], quantum dense coding [4], quantum computation [5], and quantum cryptography [6]. One of the major challenges in this field is the distribution of robust and long-distance entanglement in noisy and dissipative environments [2]. Achieving this goal is crucial for the implementation of quantum communication protocols across large-scale quantum networks via repeater nodes [7]. In this context, numerous schemes have been proposed to generate entanglement between spatially separated systems using platforms such as cavity optomechanics [8], cavity magnonics [9], two-qubit states [10], coupled quantum dots [11], atomic ensembles [12], and Bose-Einstein condensates [13].

In recent years, the exploration of quantum phenomena in exciton optomechanics, i.e., semiconductor optomechanical cavities integrated with quantum wells (QWs), has attracted growing interest [14]. Notable studies have focused on topics such as nonlinear optics mediated by exciton-polaritons [15], squeezed light generation via exciton-phonon interaction [16], giant optomechanical coupling and dephasing protection [17], the enhancement of optomechanical interactions

using quantum well exciton-polaritons [18], polariton light-emitting diodes [19], unconventional optical bistability [20], and the generation of superhigh-frequency mechanical oscillations [14]. Despite these advances, quantum entanglement in exciton optomechanics remains relatively unexplored.

More recently, a systemic theoretical framework for demonstrating genuine tripartite exciton-photon-phonon entanglement in a single exciton-optomechanical cavity was developed in [21,22]. In contrast, Zuo *et al.* [23] proposed a scheme for entangling two exciton modes, while genuine tripartite entanglement among three exciton modes was reported in [24]. However, these proposals are limited to entanglement within a single cavity and thus cannot facilitate long-distance entanglement between spatially separated exciton modes. To the best of our knowledge, no scheme has yet been reported for entangling two exciton modes located in spatially separated exciton-optomechanical cavities.

Distributing entanglement over remote nodes in a quantum network is one of the extremely challenging and rewarding tasks in quantum technologies [2]. It allows extending some quantum information processing tasks over long distances with the help of quantum repeaters [25,26]. For instance, atoms or ions, coupled via electromagnetic fields, have been shown to be meaningful for transferring entanglement from site to site or for quantum memory [27]. In particular, optomechanical cavities could be profitably employed for implementing quantum communication networks, where mechanical modes can be considered local nodes, in which information can be memorized, while optical modes carry this information between remote nodes [28]. Unfortunately, in these schemes, the distributed entanglement is found to be more fragile to thermal noise, having a tendency to even

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disappear completely for temperatures far lower than room temperature (300 K). Therefore, distributing long-distance entanglement using systems operating at room temperature is of great interest and may open up new perspectives for future quantum technologies.

With this in mind and motivated by the fact that exciton modes in optically driven systems stand out as favorable candidates for implementing a variety of quantum information-processing tasks in temperatures up to room temperature [29–33], the purpose of the present paper is to extend the work in Refs. [21–24] by proposing a scheme for distributing entanglement between two exciton modes positioned in spatially separated exciton-optomechanical cavities. The two cavities are driven by red-detuned lasers and coupled via two-mode squeezed light. Each cavity consists of a QW embedded between two distributed Bragg reflectors (DBRs) [34], where the QW supports an exciton mode that interacts with the intracavity field through a beam-splitter-type interaction. Due to the radiation pressure force, the DBRs undergo mechanical vibrations, leading to an optomechanical coupling between the cavity field and a mechanical degree of freedom. Since our objective is to entangle two exciton modes, we consider a weak exciton-photon coupling regime to avoid the formation of polariton modes [20].

Finally, we note that an exciton is a quasiparticle formed by the Coulombic binding of an electron and a hole [35]. Excitons primarily exist in semiconductor nanostructures such as quantum wells and quantum dots [36], and they can couple to a quantized electromagnetic field via exciton-photon dipole interaction [36]. The mutual interaction among excitons, photons, and phonons in a cavity forms the basis of a versatile platform for exploring various quantum phenomena across a wide temperature range, i.e., from 10 to 300 K [14]. Notably, a strong exciton-photon coupling regime gives rise to a hybrid light-matter quasiparticle known as a polariton [37].

The remainder of this paper is organized as follows. In Sec. II, we introduce the system at hand; thereafter, we present its Hamiltonian, and we derive the corresponding linearized quantum Langevin equations. We then compute, in the unresolved sideband regime, the stationary covariance matrix of the full system including two exciton modes, two optical modes, and two mechanical modes. In Sec. III, we analyze the entanglement between two exciton modes, examining its dependence on various physical and environmental parameters. Also, we discuss how the generated excitonic entanglement can be experimentally measured. Finally, Sec. IV concludes the paper with a summary of our findings.

II. THE MODEL AND QUANTUM LANGEVIN EQUATIONS

In Fig. 1, we consider two spatially separated semiconductor cavities, where each cavity is formed by a combination of movable DBRs and a QW. The j th cavity, having frequency ω_{c_j} , is driven by a laser of frequency ω_{l_j} and power $\mathcal{P}_j$. An optical cavity mode is simultaneously coupled to a mechanical mode, resulting from the displacement of the DBRs under the radiation pressure effect [34], and to an exciton mode in the QW. In addition, the two cavities are jointly pumped by two-mode squeezed light, where the first (second) squeezed mode is sent to the first (second) cavity. The Hamiltonian of

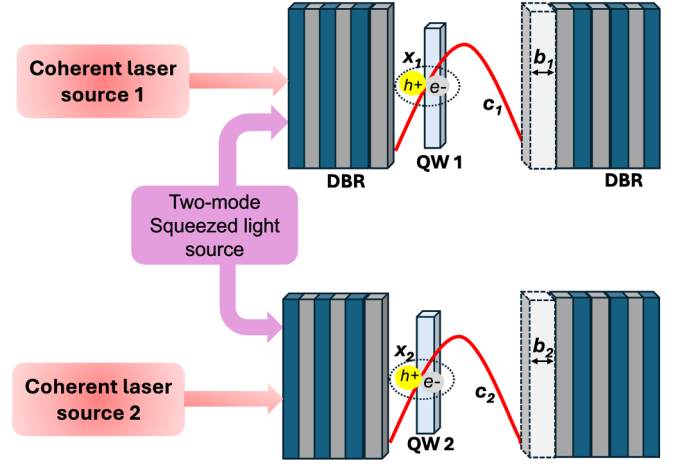


FIG. 1. Sketch of the whole system: Two semiconductor cavities are driven by red-detuned lasers and coupled by two-mode squeezed light. Each cavity, formed by two DBRs, confines an optical mode c_j and a mechanical mode b_j . A quantum well, supporting an exciton mode x_j , should be positioned near, but not at, the antinodes of the cavity field. This setup is for vanishing phonon strain; then the exciton-phonon coupling can be safely neglected. Furthermore, this setup reduces the exciton-photon coupling strength, avoiding the formation of exciton polaritons. Note that two DBRs, supplemented with a QW, were realized experimentally in [34].

the two cavities reads [21]

$$\begin{aligned} \mathcal{H} = \sum_{j=1}^2 & [\hbar\omega_{x_j}x_j^\dagger x_j + \hbar\omega_{c_j}c_j^\dagger c_j + \hbar\omega_{b_j}b_j^\dagger b_j \\ & + \hbar\chi_j(x_j^\dagger c_j + x_j c_j^\dagger) + \hbar g_{0j}c_j^\dagger c_j(b_j^\dagger + b_j) \\ & + i\hbar\varepsilon_j(c_j^\dagger e^{-i\omega_{l_j}t} - c_j e^{i\omega_{l_j}t})], \end{aligned} \quad (1)$$

where x_j is the annihilation operator for the exciton mode in the j th QW with frequency ω_{x_j} and decay rate κ_{x_j} , c_j is the annihilation operator for the optical mode in the j th cavity with frequency ω_{c_j} and decay rate κ_{c_j} , and b_j is the annihilation operator of the mechanical mode with frequency ω_{b_j} and damping rate κ_{b_j} . Moreover, χ_j denotes the coupling strength of the beam-splitter-like interaction between the j th cavity mode and its corresponding exciton mode, while g_{0j} denotes the optomechanical coupling strength between the j th cavity mode and its corresponding mechanical mode. Finally, $\varepsilon_j = \sqrt{2\kappa_{c_j}\mathcal{P}_j/\hbar\omega_{l_j}}$ is the coupling strength between the j th input drive laser and the j th cavity.

A proper analysis of the system at hand should take into account the loss process as well as the Brownian noises acting on the considered modes. This can be performed, in a frame rotating at the drive frequency ω_{l_j} , via the following nonlinear quantum Langevin equations:

$$\dot{x}_j = -(\kappa_{x_j} + i\Delta_{x_j})x_j - i\chi_j c_j + \sqrt{2\kappa_{x_j}}x_j^{\text{in}}, \quad (2)$$

$$\begin{aligned} \dot{c}_j = & -(\kappa_{c_j} + i\Delta_{c_j})c_j - i\chi_j x_j - ig_{0j}c_j(b_j^\dagger + b_j) \\ & + \varepsilon_j + \sqrt{2\kappa_{c_j}}c_j^{\text{in}}, \end{aligned} \quad (3)$$

$$\dot{b}_j = -(\kappa_{b_j} + i\omega_{b_j})b_j - ig_{0j}c_j^\dagger c_j + \sqrt{2\kappa_{b_j}}b_j^{\text{in}}, \quad (4)$$

where $\Delta_{x_j} = \omega_{x_j} - \omega_{l_j}$ and $\Delta_{c_j} = \omega_{c_j} - \omega_{l_j}$. In Eq. (2), x_j^{in} denotes the vacuum radiation input noise acting on the j th exciton mode, with the correlation function $\langle x_j^{\text{in}}(t)x_j^{\text{in}\dagger}(t') \rangle = \delta(t - t')$ [38], while in Eq. (3), c_j^{in} denotes the input squeezed noise operator affecting the j th cavity mode, with the correlation functions

$$\langle c_k^{\text{in}\dagger}(t)c_k^{\text{in}}(t') \rangle = N\delta(t - t'), k = 1, 2, \quad (5)$$

$$\langle c_k^{\text{in}}(t)c_k^{\text{in}\dagger}(t') \rangle = (N + 1)\delta(t - t'), \quad (6)$$

$$\langle c_k^{\text{in}}(t)c_{k'}^{\text{in}}(t') \rangle = M\delta(t - t'), k \neq k' = 1, 2, \quad (7)$$

$$\langle c_k^{\text{in}\dagger}(t)c_{k'}^{\text{in}\dagger}(t') \rangle = M\delta(t - t'), \quad (8)$$

where $N = \sinh^2 r$ and $M = \sinh r \cosh r$, with r being the squeezing parameter [39]. In Eq. (4), b_j^{in} denotes the Brownian noise operator affecting the j th mechanical mode. This operator is, in general, not δ correlated, and it exhibits a non-Markovian correlation function between two instants t and t' [40]. However, using a high mechanical quality factor $\mathcal{Q}_j = \omega_{b_j}/\kappa_{b_j} \gg 1$ allows us to recover the Markovian process. Therefore, we have [41]

$$\langle b_j^{\text{in}\dagger}(t)b_j^{\text{in}}(t') \rangle = n_{\text{th},j}\delta(t - t'), j = 1, 2, \quad (9)$$

$$\langle b_j^{\text{in}}(t)b_j^{\text{in}\dagger}(t') \rangle = (n_{\text{th},j} + 1)\delta(t - t'), \quad (10)$$

where $n_{\text{th},j} = [\exp(\hbar\omega_{b_j}/k_B T_j) - 1]^{-1}$ is the j th mean thermal phonon number, with T_j and k_B being the temperature of the j th mechanical thermal bath and the Boltzmann constant, respectively.

Due to the quadratic terms $c_j^\dagger c_j$, $c_j b_j^\dagger$, and $c_j b_j$, Eqs. (3) and (4) cannot be solved exactly [40]; however, assuming weak coupling between the j th cavity mode and the corresponding mechanical mode, the terms δc_j and δb_j will be much smaller than the steady-state mean values $\langle c_j \rangle$ and $\langle b_j \rangle$. Therefore, these equations can be linearized around the steady state. To do this, we write each operator $\mathcal{O}_j \equiv \{x_j, c_j, b_j\}$ as $\mathcal{O}_j = \langle \mathcal{O}_j \rangle + \delta \mathcal{O}_j$ [42], i.e., a sum of its steady-state mean value $\langle \mathcal{O}_j \rangle$ and a small fluctuation $\delta \mathcal{O}_j$ with zero mean value, where the mean values can be obtained by solving the expectation values of Eqs. (2), (3), and (4) in the steady state. Thus, we get $\langle x_j \rangle = \frac{-i\varepsilon_j \chi_j}{\chi_j^2 + (\kappa_{c_j} + i\tilde{\Delta}_{c_j})(\kappa_{x_j} + i\Delta_{x_j})}$, $\langle c_j \rangle = \frac{\varepsilon_j(\kappa_{x_j} + i\Delta_{x_j})}{\chi_j^2 + (\kappa_{c_j} + i\tilde{\Delta}_{c_j})(\kappa_{x_j} + i\Delta_{x_j})}$, and $\langle b_j \rangle = \frac{-i g_{0j} |\langle c_j \rangle|^2}{\kappa_{b_j} + i\omega_{b_j}}$, with $\tilde{\Delta}_{c_j} = \Delta_{c_j} + g_{0j}(\langle b_j \rangle + \langle b_j \rangle^*)$ being the j th effective cavity-drive detuning including the frequency shift due to the radiation pressure force [28]. We assume that the two cavities are strongly driven, which leads to a large amplitude, i.e., $|\langle c_j \rangle| \gg 1$, and due to the exciton-photon beam-splitter interaction, the exciton modes also have a large amplitude, i.e., $|\langle x_j \rangle| \gg 1$; hence, the second-order fluctuation terms $\delta c_j^\dagger \delta c_j$, $\delta c_j \delta b_j$, and $\delta c_j \delta b_j^\dagger$ can be safely neglected. Consequently, we get the linearized Langevin equations for the quantum fluctuations:

$$\dot{\delta x_j} = -(\kappa_{x_j} + i\Delta_{x_j})\delta x_j - i\chi_j \delta c_j + \sqrt{2\kappa_{x_j}}x_j^{\text{in}}, \quad (11)$$

$$\begin{aligned} \dot{\delta c_j} = & -(\kappa_{c_j} + i\tilde{\Delta}_{c_j})\delta c_j - i\chi_j \delta x_j - g_j(\delta b_j^\dagger + \delta b_j) \\ & + \sqrt{2\kappa_{c_j}}c_j^{\text{in}}, \end{aligned} \quad (12)$$

$$\dot{\delta b_j} = -(\kappa_{b_j} + i\omega_{b_j})\delta b_j - (g_j \delta c_j^\dagger - g_j^* \delta c_j) + \sqrt{2\kappa_{b_j}}b_j^{\text{in}}, \quad (13)$$

where $g_j = i g_{0j} \langle c_j \rangle$ is the j th effective optomechanical coupling strength [28].

We note that the linearized equations [(11)–(13)] can be described by the effective Hamiltonian

$$\begin{aligned} \mathcal{H}_{\text{eff}} = & \sum_{j=1}^2 [\hbar \Delta_{x_j} \delta x_j^\dagger \delta x_j + \hbar \tilde{\Delta}_{c_j} \delta c_j^\dagger \delta c_j + \hbar \omega_{b_j} \delta b_j^\dagger \delta b_j \\ & + \hbar \chi_j (\delta x_j^\dagger \delta c_j + \delta x_j \delta c_j^\dagger) \\ & + \hbar (g_j^* \delta c_j - g_j \delta c_j^\dagger) (\delta b_j^\dagger + \delta b_j)]. \end{aligned} \quad (14)$$

If we assume $\omega_{b_j} \gg \kappa_{b_j}, \kappa_{c_j}$ and then apply the rotating-wave approximation for $\tilde{\Delta}_{c_j} = \Delta_{x_j} + \omega_{b_j}$ [43], the interaction Hamiltonian obtained from Eq. (14) reads

$$\begin{aligned} \mathcal{H}_{\text{int}} = & \sum_{j=1}^2 [\hbar \chi_j (\delta x_j^\dagger \delta c_j + \delta x_j \delta c_j^\dagger) \\ & + \hbar (g_j^* \delta c_j \delta b_j^\dagger - g_j \delta c_j^\dagger \delta b_j)], \end{aligned} \quad (15)$$

which is relevant for quantum state transfer and cooling [44].

Using Eqs. (11), (12), and (13) together with the quadrature position $\delta q_{\mathcal{O}_j} = (\delta \mathcal{O}_j^\dagger + \delta \mathcal{O}_j)/\sqrt{2}$ and momentum $\delta p_{\mathcal{O}_j} = i(\delta \mathcal{O}_j^\dagger - \delta \mathcal{O}_j)/\sqrt{2}$ and the corresponding input noise operators $q_{\mathcal{O}_j}^{\text{in}} = (\mathcal{O}_j^{\text{in}\dagger} + \mathcal{O}_j^{\text{in}})/\sqrt{2}$ and $p_{\mathcal{O}_j}^{\text{in}} = i(\mathcal{O}_j^{\text{in}\dagger} - \mathcal{O}_j^{\text{in}})/\sqrt{2}$ for $\mathcal{O}_j \equiv \{x_j, c_j, b_j\}$, we obtain

$$\partial_t \delta q_{x_j} = -\kappa_{x_j} \delta q_{x_j} + \Delta_{x_j} \delta p_{x_j} + \chi_j \delta p_{c_j} + \sqrt{2\kappa_{x_j}} q_{x_j}^{\text{in}}, \quad (16)$$

$$\partial_t \delta p_{x_j} = -\Delta_{x_j} \delta q_{x_j} - \kappa_{x_j} \delta p_{x_j} - \chi_j \delta q_{c_j} + \sqrt{2\kappa_{x_j}} p_{x_j}^{\text{in}}, \quad (17)$$

$$\partial_t \delta q_{b_j} = -\kappa_{b_j} \delta q_{b_j} + \omega_{b_j} \delta p_{b_j} + \sqrt{2\kappa_{b_j}} q_{b_j}^{\text{in}}, \quad (18)$$

$$\begin{aligned} \partial_t \delta p_{b_j} = & -\omega_{b_j} \delta q_{b_j} - \kappa_{b_j} \delta p_{b_j} - 2\text{Im}(g_j) \delta q_{c_j} \\ & + 2\text{Re}(g_j) \delta p_{c_j} + \sqrt{2\kappa_{b_j}} p_{b_j}^{\text{in}}, \end{aligned} \quad (19)$$

$$\begin{aligned} \partial_t \delta q_{c_j} = & \chi_j \delta p_{x_j} - 2\text{Re}(g_j) \delta q_{b_j} - \kappa_{c_j} \delta q_{c_j} \\ & + \tilde{\Delta}_{c_j} \delta p_{c_j} + \sqrt{2\kappa_{c_j}} q_{c_j}^{\text{in}}, \end{aligned} \quad (20)$$

$$\begin{aligned} \partial_t \delta p_{c_j} = & -\chi_j \delta q_{x_j} - 2\text{Im}(g_j) \delta q_{b_j} - \tilde{\Delta}_{c_j} \delta q_{c_j} - \kappa_{c_j} \delta p_{c_j} \\ & + \sqrt{2\kappa_{c_j}} p_{c_j}^{\text{in}}, \end{aligned} \quad (21)$$

which can be written in matrix form as

$$\partial_t \vartheta = \mathcal{K} \vartheta + \xi, \quad (22)$$

with $\vartheta^T = (\delta q_{x_1}, \delta p_{x_1}, \delta q_{x_2}, \delta p_{x_2}, \delta q_{b_1}, \delta p_{b_1}, \delta q_{b_2}, \delta p_{b_2}, \delta q_{c_1}, \delta p_{c_1}, \delta q_{c_2}, \delta p_{c_2})$, $\xi^T = (\sqrt{2\kappa_{x_1}} q_{x_1}^{\text{in}}, \sqrt{2\kappa_{x_1}} p_{x_1}^{\text{in}}, \sqrt{2\kappa_{x_2}} q_{x_2}^{\text{in}}, \sqrt{2\kappa_{x_2}} p_{x_2}^{\text{in}}, \sqrt{2\kappa_{b_1}} q_{b_1}^{\text{in}}, \sqrt{2\kappa_{b_1}} p_{b_1}^{\text{in}}, \sqrt{2\kappa_{b_2}} q_{b_2}^{\text{in}}, \sqrt{2\kappa_{b_2}} p_{b_2}^{\text{in}}, \sqrt{2\kappa_{c_1}} q_{c_1}^{\text{in}}, \sqrt{2\kappa_{c_1}} p_{c_1}^{\text{in}}, \sqrt{2\kappa_{c_2}} q_{c_2}^{\text{in}}, \sqrt{2\kappa_{c_2}} p_{c_2}^{\text{in}})$.

$\sqrt{2\kappa_{c1}}p_{c1}^{\text{in}}, \sqrt{2\gamma_{c2}}q_{c2}^{\text{in}}, \sqrt{2\kappa_{c2}}p_{c2}^{\text{in}}$, and

$$\mathcal{K} = \begin{pmatrix} -\kappa_{x1} & \Delta_{x1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \chi_1 & 0 & 0 \\ -\Delta_{x1} & -\kappa_{x1} & 0 & 0 & 0 & 0 & 0 & 0 & -\chi_1 & 0 & 0 & 0 \\ 0 & 0 & -\kappa_{x2} & \Delta_{x2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \chi_2 \\ 0 & 0 & -\Delta_{x2} & -\kappa_{x2} & 0 & 0 & 0 & 0 & 0 & 0 & -\chi_2 & 0 \\ 0 & 0 & 0 & 0 & -\kappa_{b1} & \omega_{b1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\omega_{b1} & -\kappa_{b1} & 0 & 0 & -2g_{1I} & 2g_{1R} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\kappa_{b2} & \omega_{b2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\omega_{b2} & -\kappa_{b2} & 0 & 0 & -2g_{2I} & 2g_{2R} \\ 0 & \chi_1 & 0 & 0 & -2g_{1R} & 0 & 0 & 0 & -\kappa_{c1} & \tilde{\Delta}_{c1} & 0 & 0 \\ -\chi_1 & 0 & 0 & 0 & -2g_{1I} & 0 & 0 & 0 & -\tilde{\Delta}_{c1} & -\kappa_{c1} & 0 & 0 \\ 0 & 0 & 0 & \chi_2 & 0 & 0 & -2g_{2R} & 0 & 0 & 0 & -\kappa_{c2} & \tilde{\Delta}_{c2} \\ 0 & 0 & -\chi_2 & 0 & 0 & 0 & -2g_{2I} & 0 & 0 & 0 & -\tilde{\Delta}_{c2} & -\kappa_{c2} \end{pmatrix}, \quad (23)$$

where we used the notations $g_{jR} = \text{Re}(g_j)$ and $g_{jI} = \text{Im}(g_j)$.

The solution of Eq. (22) reads [38]

$$\vartheta(t) = \mathcal{F}(t)\vartheta(0) + \int_0^t ds \mathcal{F}(s)\xi(t-s), \quad (24)$$

with $\mathcal{F}(t) = \exp\{\mathcal{K}t\}$. The system under consideration is stable and reaches its steady state if and only if all the eigenvalues of the drift matrix $\mathcal{K}$ have negative real parts, which can be analyzed by applying the Routh-Hurwitz criteria [45]. Because of the 12×12 dimension of the drift matrix $\mathcal{K}$, the analytic expressions of the stability conditions are too cumbersome and cannot be reported here.

The dynamics is linearized around the steady state; moreover, $x_j^{\text{in}}, c_j^{\text{in}}$, and b_j^{in} are zero-mean Gaussian noises; because of these facts, the steady state of the system is a zero-mean Gaussian state which can be completely specified by the following 12×12 covariance matrix $\mathcal{V}$ [39]:

$$\mathcal{V}_{ii'} = \langle \vartheta_i(\infty)\vartheta_{i'}(\infty) + \vartheta_{i'}(\infty)\vartheta_i(\infty) \rangle / 2, \quad (25)$$

where $\vartheta_i(\infty)$ is the vector of quadrature fluctuations in the steady state ($t \rightarrow \infty$). When the system is stable, with the use of Eq. (24), Eq. (25) is [38]

$$\mathcal{V}_{ii'} = \sum_{k,k'} \int_0^\infty ds \int_0^\infty ds' \mathcal{F}_{ik}(s) \mathcal{F}_{i'k'}(s') \Xi_{kk'}(s-s'), \quad (26)$$

where $\Xi_{kk'}(s-s') = \langle \xi_k(s)\xi_{k'}(s') + \xi_{k'}(s')\xi_k(s) \rangle / 2 = \mathcal{N}_{kk'}\delta(s-s')$ is the matrix of the steady-state noise correlations, in which $\mathcal{N} = \mathcal{N}_{\kappa_x} \oplus \mathcal{N}_{\kappa_b} \oplus \mathcal{N}_{\kappa_c}$, with $\mathcal{N}_{\kappa_x} = \kappa_{x1}\mathbb{1}_2 \oplus \kappa_{x2}\mathbb{1}_2$, $\mathcal{N}_{\kappa_b} = \kappa_{b1}(2n_{\text{th}1} + 1)\mathbb{1}_2 \oplus \kappa_{b2}(2n_{\text{th}2} + 1)\mathbb{1}_2$, and

$$\mathcal{N}_{\kappa_c} = \begin{pmatrix} \kappa_{c1} \cosh(2r) & 0 & \sqrt{\kappa_{c1}\kappa_{c2}} \sinh(2r) & 0 \\ 0 & \kappa_{c1} \cosh(2r) & 0 & -\sqrt{\kappa_{c1}\kappa_{c2}} \sinh(2r) \\ \sqrt{\kappa_{c1}\kappa_{c2}} \sinh(2r) & 0 & \kappa_{c2} \cosh(2r) & 0 \\ 0 & -\sqrt{\kappa_{c1}\kappa_{c2}} \sinh(2r) & 0 & \kappa_{c2} \cosh(2r) \end{pmatrix}. \quad (27)$$

When the stability conditions are fulfilled, we have $\mathcal{F}(\infty) = 0$; therefore, Eq. (26) becomes $\mathcal{V} = \int_0^\infty ds \mathcal{F}(s) \mathcal{N} \mathcal{F}(s)^T$, which is equivalent to the Lyapunov equation [39]

$$\mathcal{K}\mathcal{V} + \mathcal{V}\mathcal{K}^T = -\mathcal{N}. \quad (28)$$

Equation (28) is linear for $\mathcal{V}$ and can be straightforwardly solved. However, the general expression of the matrix $\mathcal{V}$ is too cumbersome and cannot be reported here. The covariance matrix $\mathcal{V}_{x_1x_2}$ describing exciton modes x_1 and x_2 can be obtained from the matrix $\mathcal{V}$ by taking the first four rows and columns. Then, in a block form, $\mathcal{V}_{x_1x_2}$ can be written as

$$\mathcal{V}_{x_1x_2} = \begin{pmatrix} \mathcal{V}_{x_1} & \mathcal{V}_{x_1/x_2} \\ \mathcal{V}_{x_1/x_2}^T & \mathcal{V}_{x_2} \end{pmatrix}, \quad (29)$$

where the two 2×2 matrices $\mathcal{V}_{x_1}$ and $\mathcal{V}_{x_2}$ represent the first and second exciton modes, respectively, while $\mathcal{V}_{x_1/x_2}$ describes the correlation between them.

III. ENTANGLEMENT BETWEEN TWO-EXCITON MODES

Now, we are in a position to assess the quantum entanglement that settled between exciton modes x_1 and x_2 . To establish the conditions under which modes x_1 and x_2 are entangled, we employ the logarithmic negativity, defined as $\mathcal{E}_{\mathcal{N}} = \max[0, -\ln(2\tilde{\Lambda}_-)]$ [46,47], where $\tilde{\Lambda}_- = \sqrt{[\Theta - \sqrt{\Theta^2 - 4 \det \mathcal{V}_{x_1x_2}}]/2}$ is the lowest symplectic eigenvalue of the partially transposed covariance matrix (29), with $\Theta = \det \mathcal{V}_{x_1} + \det \mathcal{V}_{x_2} - 2 \det \mathcal{V}_{x_1/x_2}$ [46,47]. Then, modes x_1 and x_2 are entangled if and only if $\tilde{\Lambda}_- < 1/2$, which is equivalent to Simon's necessary and sufficient entanglement

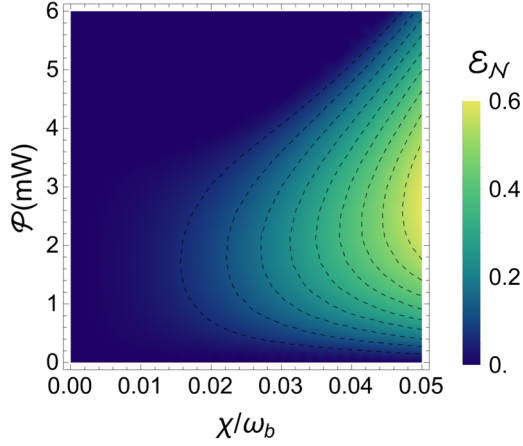


FIG. 2. Density plot of the entanglement $\mathcal{E}_N$ of the two exciton modes, x_1 and x_2 , against the driving laser power $\mathcal{P}$ and the normalized exciton-photon coupling strength χ/ω_b for $T \approx 8.5$ K, $r = 0.35$, and $\tilde{\Delta}_c = \Delta_x = +\omega_b$. The other parameters are provided in the text. Small values of the drive power $\mathcal{P}$ are required to ensure that the system maintains a stable steady state.

nonpositive partial transpose criterion for Gaussian states [48].

It is well-known that the mechanical frequency in exciton-optomechanical cavities is within the gigahertz range [18]. As a result, at low mechanical bath temperatures, the mean thermal phonon number will be very small; nevertheless, it becomes enormous at high temperatures, which prevents the generation of entanglement near room temperature, i.e., 300 K. To overcome the destructive effect of thermal noise, we drive the two cavities by red-detuned lasers, i.e., $\tilde{\Delta}_c > 0$, which is relevant for both quantum state transfer and cooling [44]. Furthermore, we set the two exciton modes to be in resonance with the cavities, i.e., $\Delta_x = \tilde{\Delta}_c = +\omega_b$, which resonantly activates the anti-Stokes scattering. This will therefore induce the cooling of the phonon modes, and optimal entanglement between the two exciton modes can be achieved by transferring quantum correlations from the squeezed light to them.

To have a fairly good idea of entanglement between exciton modes x_1 and x_2 , we take the set of experimentally feasible parameters from [20,34,49], which makes our proposal closer to experimental reality. In particular, we take identical cavities and identical driving lasers, i.e., $\omega_b = 2\pi \times 20$ GHz, $\kappa_b = 2\pi \times 1$ MHz, $\kappa_c = 2\pi \times 1$ GHz, $\kappa_x = 2\pi \times 100$ MHz, $\chi = 2\pi \times 0.92$ GHz, $\varepsilon = 2\pi \times 6 \times 10^{12}$ Hz, $g_0 = 2\pi \times 10$ MHz, and $\omega_l = 2\pi \times 3 \times 10^{14}$ Hz.

In Fig. 2, we plot the entanglement $\mathcal{E}_N$ of the two exciton modes, x_1 and x_2 , versus the driving power $\mathcal{P}$ and the normalized exciton-photon coupling χ/ω_b for mechanical bath temperature $T \approx 8.5$ K and squeezing $r = 0.35$. Even though exciton modes x_1 and x_2 are spatially separated, entanglement can be established between them via an appropriate choice of the laser power $\mathcal{P}$ and the exciton-photon coupling strength χ . This feature can be understood as follows: Ref. [50] showed that two spatially disjoint optical modes can be entangled by transferring quantum correlations from squeezed light to them. Moreover, driving the two cavities

with strong red-detuned lasers—provided that $\omega_b \gg \kappa_x, \kappa_b$ —places the two cavities within the resolved sideband regime [43]. Therefore, the rotating-wave terms $\propto (\delta c_j \delta b_j^\dagger - \delta c_j^\dagger \delta b_j)$ in Eq. (15) induce the cooling of the mechanical modes with effective state-swap interaction, which is a precondition for preparing quantum states in the system. The cooling of the mechanical modes in the system can be understood as a red-detuned laser entering the cavity, then exciting an intracavity mode by absorbing—through the beam-splitter interaction $(\delta c_j \delta b_j^\dagger - \delta c_j^\dagger \delta b_j)$ —a phonon mode from the vibrating DBRs, and finally decaying out of the cavity [51]. On the other hand, through the beam-splitter interaction $(\delta x_j^\dagger \delta c_j + \delta x_j \delta c_j^\dagger)$ between the j th exciton mode and the corresponding cavity mode, entanglement can be transferred from the two-mode squeezed light to the state of the two exciton modes by the mediation of the two optical modes. Hence, our proposal shows that by virtue of the entanglement transfer, it is possible to prepare entangled exciton modes without direct interaction between them.

Quite remarkably, Fig. 2 indicates that for $\chi = 0$, $\mathcal{E}_N$ remains zero regardless of the values of the laser power $\mathcal{P}$, which reflects the primordial role of the beam-splitter interaction $(\delta x_j^\dagger \delta c_j + \delta x_j \delta c_j^\dagger)$ in the creation of entanglement between exciton modes x_1 and x_2 . Also, it shows that without the driving laser, i.e., $\mathcal{P} = 0$, no entanglement can be created between exciton modes x_1 and x_2 , and the entanglement $\mathcal{E}_N$ undergoes a resonancelike behavior under the effect of the driving power $\mathcal{P}$. Indeed, with the gradual increase of the power $\mathcal{P}$, the entanglement $\mathcal{E}_N$ arises and grows monotonically until it reaches a maximum value and then diminishes to zero. This is because, as the power $\mathcal{P}$ increases, the photon number in the cavities increases, which enhances the entanglement between the two optical modes and consequently enhances entanglement transfer from the optical modes to the two exciton modes. However, with larger values of the driving laser $\mathcal{P}$, thermal noise entering each cavity becomes more aggressive, which destroys the optical entanglement and then the excitonic one.

It is interesting to note that even though the entanglement $\mathcal{E}_N$ grows monotonically with the coupling strength χ in Fig. 2, $\mathcal{E}_N$ must remain finite. This is because the stability conditions, which impose a strong upper bound on the maximum allowable exciton-photon coupling strength χ , limit the amount of the attained entanglement at the instability threshold.

In Fig. 3, we plot the entanglement $\mathcal{E}_N$ as a function of the normalized detuning Δ_x/ω_b using $\tilde{\Delta}_c = +\omega_b$, $r = 0.35$, $\mathcal{P} = 3.5$ mW, $T = 8.5$ K, and various values of the normalized exciton dissipation κ_x/ω_b . As can be seen, $\mathcal{E}_N$ increases with the increase of Δ_x/ω_b until it reaches a maximum value for $\Delta_x/\omega_b = +1$, and then it starts to diminish. This therefore confirms the fact that optimal entanglement between the two exciton modes can be achieved when both of them are tuned to resonance with the anti-Stokes sidebands, i.e., $\Delta_x = +\omega_b$, scattered by the mechanical motions. Strikingly, Fig. 3 shows that increasing the dissipation κ_x not only reduces the degree of entanglement $\mathcal{E}_N$ but also significantly enlarges the existent region of the excitonic entanglement.

The impact of the mechanical bath temperature T and the squeezing parameter r on the entanglement $\mathcal{E}_N$ is plotted

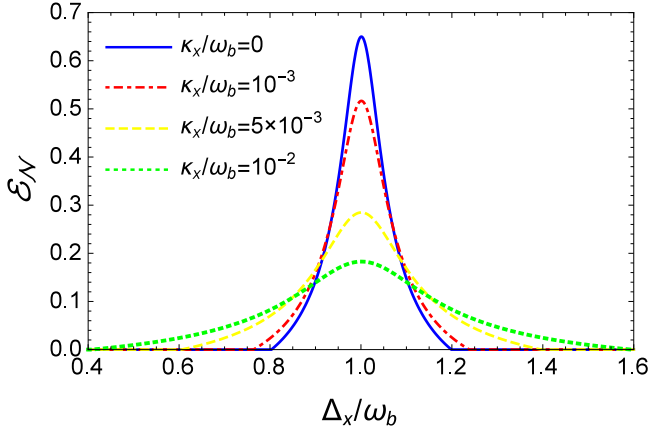


FIG. 3. Plot of the entanglement $\mathcal{E}_N$ of the two exciton modes against the normalized detuning Δ_x/ω_b using various values of the normalized exciton decay rate κ_x/ω_b . We take $\tilde{\Delta}_c = +\omega_b$, $r = 0.35$, $\mathcal{P} = 3.5$ mW, and $T = 8.5$ K. Remarkably, the two exciton modes are maximally entangled when both of them are tuned to resonance with the anti-Stokes sidebands, i.e., $\Delta_x = +\omega_b$. It is clear that the increase in the exciton decay rate κ_x not only diminishes the maximum of entanglement $\mathcal{E}_N$ but also significantly enlarges the region where $\mathcal{E}_N > 0$.

in Fig. 4. We have chosen $\tilde{\Delta}_c = \Delta_x = +\omega_b$ such that the cooling of the phonon modes achieves optimal efficiency; then significant excitonic entanglement can be attained. For the mechanical quality factor, we used $Q \approx 1.2 \times 10^5$. As can be seen, the exciton-exciton entanglement is quite robust against the mechanical bath temperature T , where maximum entanglement is reached close to $T = 0$. In addition, as the temperature T increases, the thermal noise entering the cavities increases too; therefore, $\mathcal{E}_N$ monotonically decreases, and the region of values of the squeezing r where $\mathcal{E}_N \neq 0$ narrows. Manifestly, Fig. 4 reveals that with the use of the high mechanical quality factor $Q \approx 1.2 \times 10^5$, which is currently accessible [34,49,52], the two exciton modes will be

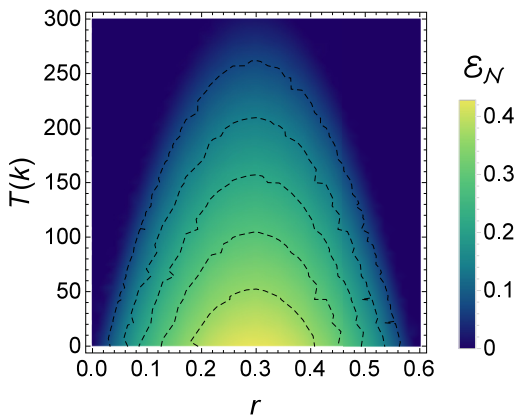


FIG. 4. Density plot of the entanglement $\mathcal{E}_N$ versus the squeezing parameter r and the temperature of the mechanical bath T using $\tilde{\Delta}_c = \Delta_x = +\omega_b$ and $\mathcal{P} = 3.5$ mW. For the mechanical quality factor, we used $Q = 1.2 \times 10^5$ [34,49,52]. Manifestly, the entanglement $\mathcal{E}_N$ survives up to room temperature $T \approx 300$ K.

entangled even at room temperature, i.e., $T \approx 300$ K. We emphasize that several theoretical investigations have shown that some classes of semiconductors, such as lead-halide perovskites [30], semiconductor heterostructures [31], gallium nitride [32], and ternary semiconductor alloy microstructures [33], can exhibit excitons at room temperature due to their larger exciton-binding energies. In addition, miscellaneous experiments have been accomplished with excitons at room temperature [53–55].

On the other hand, Fig. 4 shows that without squeezed light, i.e., $r = 0$, $\mathcal{E}_N$ always remains zero, meaning that there is no entanglement between the exciton modes. However, with the injection of squeezed light, i.e., $r \neq 0$, entanglement between the two exciton modes can be established even though they are separated in space. Also, Fig. 4 indicates that the entanglement $\mathcal{E}_N$ displays a resonancelike behavior under the squeezing effect, which is similar to the behavior of $\mathcal{E}_N$ under the effect of the laser power $\mathcal{P}$ in Fig. 2. This can be understood based on the fact that the reduced state of two-mode squeezed light is a thermal state with a mean photon number proportional to the squeezing parameter r [8].

Last, the generated entanglement between exciton modes x_1 and x_2 can be read out experimentally by measuring their covariance matrix (29). This can be accomplished by sending, into each cavity, a much weaker probe field that realizes a state-swap interaction with the corresponding exciton mode. Moreover, by homodyning the cavity output of the j th probe field, the quadratures of the two exciton modes can be measured; consequently, the covariance matrix (29) can be numerically reconstructed. We should mention that this strategy requires the exciton decay rate κ_x to be much smaller than the optical decay rate κ_c , such that when the drives are switched off and all intracavity photons decay, the exciton states remain almost unchanged, at which time the probe field is sent [56]. In addition, it is interesting to note that the excitonic entanglement can be measured without need of homodyne detection by virtue of the approach developed in [57]. Indeed, due to the beam-splitter-like interaction between the j th exciton mode x_j and optical mode c_j , the state of mode x_j can be mapped onto the output of the j th probe field. Then, using only beam splitters and single-photon detectors, the excitonic entanglement can be detected from coincidences between photodetectors placed on the two output probe fields [57].

The feasibility of our proposal is verified under realistic conditions using state-of-the-art experimental parameters. This makes our proposal a foreseeable possibility toward the distribution of long-distance entanglement across repeater nodes operating at room temperature, which is of great importance for real-world quantum networks [7].

IV. CONCLUSIONS

We studied a scheme for entangling two exciton modes placed in two spatially separated exciton-optomechanical cavities. The two cavities are driven by red-detuned lasers and coupled via two-mode squeezed vacuum light. Under realistic experimental conditions, we showed that steady-state entanglement between two exciton modes can be established and controlled by tuning key physical and environmental parameters, such as the drive laser power, exciton-photon coupling

strength, mechanical bath temperature, and the squeezing degree. Optimal entanglement is achieved when the exciton modes are tuned in resonance with the anti-Stokes sidebands scattered by the mechanical motion. The resulting entanglement exhibits strong resilience to thermal noise and remains significantly nonzero up to room temperature using a high mechanical quality factor. We also discussed practical methods for detecting the generated entanglement. Although various quantum phenomena have been explored in exciton optomechanics, entanglement has received relatively little attention. This work represents a proposal to investigate entanglement between two spatially separated exciton modes within the framework of exciton optomechanics. The demonstrated potential of our scheme to distribute robust entanglement between noninteracting excitons, up to room temperature, offers

a promising route toward building quantum networks in which processors and memories are connected via shared quantum correlations [7].

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The authors declare no conflict of interest.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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