

Transduction-enabled superconducting quantum repeater: Toward deterministic entanglement distribution with high-fidelity gates

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(Received 13 March 2026; accepted 14 September 2026; published 5 October 2026)

Long-distance entanglement distribution is hindered by photon loss in optical fibers and the no-cloning theorem. Optical quantum repeater (QR) protocols rely on Bell-state measurements (BSMs); they are intrinsically limited to probabilistic photon operations and fail 50% of the time. We propose a hybrid approach to building quantum repeaters that combines the high transmission speed of photonic qubits in optical fiber with the high-fidelity quantum processing capabilities enabled by superconducting circuits. The transduction-enabled superconducting QR (TESQR) architecture eliminates the need for probabilistic BSMs and allows deterministic processing operations. The TESQR framework always yields a final state at the remote nodes rather than aborting on photon loss, manifesting deterministic entanglement distribution within certain parameter regimes. We evaluate the performance by assessing output-state fidelities and success probabilities of entanglement distribution using realistic noise models. Additionally, we integrate an entanglement purification scheme and evaluate the performance through numerical simulations in the QUTIP environment. Our results show that, for entanglement swapping, the proposed scheme improves the entanglement distribution rate by an average of 63% and by up to 159% compared with photonic-only architectures. Moreover, after purification, the end-to-end fidelities exceed 0.8 over distances up to 20 km.

DOI: [10.1103/r577-33dc](https://doi.org/10.1103/r577-33dc)

I. INTRODUCTION

Entanglement distribution is a fundamental component of future quantum internet networks. Realizing long-distance quantum communication remains a major challenge due to the exponential attenuation of photons in optical fibers and operational errors in intermediate processing nodes. Moreover, qubits cannot be amplified or cloned like classical signals without destroying their quantum information, in accordance with the no-cloning theorem. For these reasons, quantum repeaters (QRs) [1] are needed, as they split the end-to-end channel into multiple shorter segments and employ various error-suppression techniques to securely extend the communication range.

Over the past two decades, numerous QR architectures have been proposed. The Duan-Lukin-Cirac-Zoller (DLCZ) protocol [2] employs atomic-ensemble memories and heralded Raman emissions, together with optical Bell-state measurements (BSMs) on the *readout* photons for entanglement swapping. Although it paved the way for the first

experimental demonstrations, the DLCZ protocol exhibits a low per-trial success probability (typically spanning the 10^{-6} – 10^{-4} range) and requires phase stabilization and high detection efficiencies, limiting the achievable entanglement distribution rate [3].

Other protocols adopt purely photonic strategies, avoiding stationary memories and instead relying on large photonic cluster states with adaptive single-qubit measurements [1]. Examples include multiplexed photonic cluster approaches [4] and all-photonic repeater schemes based on Calderbank-Shor-Steane (CSS) codes trees and homodyne detection [5]. While these approaches eliminate the need for matter qubit storage, they do so at the cost of very large photon overheads and high sensitivity to channel and component losses [6].

Hybrid approaches attempt to combine the advantages of both matter and photonic systems. Van Loock *et al.* [7] proposed using dispersive interactions between matter qubits (e.g., atoms in cavities) and bright coherent pulses, where a single optical pulse sequentially traverses two cavities and acquires phase rotations conditional on the state of each qubit. A homodyne measurement on the outgoing field then heralds entanglement, with success probabilities exceeding those of purely photonic schemes. Subsequent work showed that, while such schemes enable faster entanglement generation, the initial fidelity is sensitive to channel loss and cavity decay, typically requiring many purification rounds that increase system complexity and latency [8,9]. Trapped-ion-based nodes provide another hybrid route: photons entangled

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with the ions' internal states are interfered to herald remote entanglement, while deterministic local gates (e.g., Mølmer-Sørensen gates) enable high-fidelity swapping and readout. Variants employing co-trapped species address wavelength matching to telecom bands and have demonstrated promising simulated ranges [10,11].

Despite these advances, a recurring bottleneck across most implementations is the optical BSM used for link-level entanglement generation or swapping. Its success probability in linear-optical implementations is fundamentally capped at 50%, since only two of the four Bell states can be distinguished [12,13]. In practice, the entanglement rates are further reduced by photonic losses and by the need for precise synchronization to enable optical interference [6,14]. Alternative strategies do exist to exceed the 50% linear-optical bound, such as those based on hyperentanglement, nonlinear interactions, or ancillary resources [15,16]. However, these solutions introduce additional experimental overheads and trade-offs.

Recent years have witnessed significant progress in bidirectional microwave-optical quantum transduction, realized on several platforms such as electro-optic crystalline cavities [17,18], electro- and piezo-optomechanics [19], rare-earth ions [20], atomic assembly, and others [21]. They have provided a useful platform to combine the high-fidelity quantum operations of superconducting circuits with room-temperature communication capabilities via optical flying photons. Here, we propose a transduction-enabled superconducting quantum repeater (TESQR), a hybrid architecture that extends the outlook of previous works [22] and draws inspiration from the general setup described in Ref. [23]. This scheme differs from existing solutions by eliminating probabilistic optical BSMs during both entanglement generation and swapping, thereby enabling deterministic processing operations. With TESQR, we combine the advantages of superconducting quantum computing devices and optical fibers, i.e., integrating fast gate operations with low-loss communication. It also overcomes the intrinsic limitation of 50% Bell-state discrimination and synchronization challenges, while improving the final output fidelity.

In this work, we analyze the processing performance of the hybrid architecture by evaluating (i) the output fidelity and (ii) the protocol's success probability for entanglement distribution. This analysis is conducted by evaluating various noise models. To this end, we characterize the processing operations using several figures of merit for the different setup components. Since our primary objective is to assess the feasibility and potential advantages of the proposed hybrid scheme at the level of quantum processing operations, we do not model imperfections in quantum memories and instead assume ideal memory performance, which is consistent with previous studies [9,24]. Specifically, we assume that all operations are completed within a time $t < t_{\text{coh}}$, where t_{coh} denotes the memory coherence time. Furthermore, we highlight that our analysis focuses on quantum-processing performance metrics rather than on resource costs. A systematic resource-cost investigation would require defining nonstandard, cross-platform metrics to account for the fundamentally different physical constituents involved (superconducting qubits, transducers, photodetectors, etc.), and is therefore left for dedicated future work.

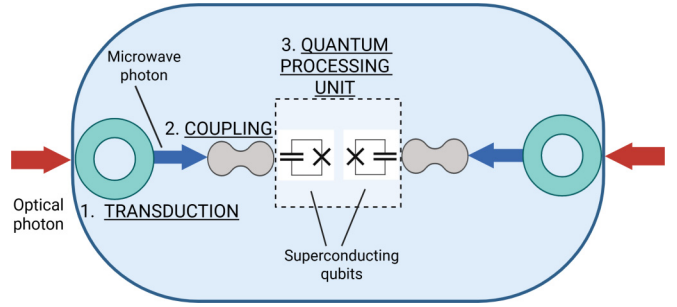


FIG. 1. Illustration of TESQR node architecture performing joint measurement on two optical photons via transduction, coupling, and superconducting QPU.

Finally, we propose entanglement purification in a superconducting quantum processing unit (QPU) before the swapping step. For this purpose, we develop a QUTIP-based simulation framework tailored to the considered network scenario. Numerical results demonstrate the effectiveness of the proposed architecture, achieving end-to-end fidelity values exceeding 0.8 for link distances up to 20 km between the two parties under realistic processing imperfections.

To summarize, the main contributions of this work are as follows:

- (1) We propose a hybrid quantum repeater architecture (TESQR) that leverages quantum transduction to enable deterministic entanglement distribution within certain parameter regimes.
- (2) We develop a comprehensive analytical model of the network subsystems, including a detailed noise characterization, and evaluate the TESQR performance in terms of fidelity and success probability.
- (3) We introduce a performance enhancement strategy based on an additional dedicated purification stage and validate the resulting improvement through a publicly available software implementation [25].

The paper is structured as follows. In Sec. II, we describe the TESQR architecture; in Sec. III, we present the analytical model of the various noisy operations in the network scenario. Section IV evaluates the performance of the TESQR architecture for entanglement swapping in terms of output fidelity and success probability. In Sec. V, we introduce the purification scheme and report the corresponding numerical results obtained with the QUTIP simulation framework. Finally, Sec. VI concludes the paper and outlines future development directions.

II. TESQR ARCHITECTURE

The protocol considers entangled pairs between a photonic qubit (Fock-state, single-rail encoding) and superconducting circuits. The TESQR node is built on a functional layer comprising three main components (as illustrated in Fig. 1):

- (1) *Transduction*. After propagation via optical fibers, the photons are converted into microwave photons via a quantum transducer.
- (2) *Coupling*. After the conversion, the microwave photon couples directly to a superconducting QPU.

TABLE I. Network and TESQR node stages and corresponding processes to perform entanglement swapping.

Stage	Process	Description
Communication, stage 1	Source	Alice and Bob locally generate entangled Bell-like states.
Communication, stage 2	Transmission	The photonic qubits propagate through an optical fiber of length L , characterized by an attenuation length l_{att} .
TESQR, stage 1	Transduction	Optical-to-microwave frequency conversion is characterized by thermal noise with mean photon occupancy $\bar{n}$ and transduction efficiency η_t .
TESQR, stage 2	Coupling	The converted microwave photons couple to the superconducting qubits.
TESQR, stage 3	Swapping	Local controlled-NOT (CNOT) and Hadamard gates and measurements are applied to the superconducting qubits to swap the entanglement between Alice and Bob.

(3) *Swapping*. Entanglement swapping is performed locally on the QPU, using microwave gates followed by measurements.

Key advantages of this approach are (1) leveraging the high-fidelity gate operations achievable with superconducting circuits (single-qubit error rates below 10^{-4}) [26–29]; (2) combining the benefits of using low-loss optical photons at standard telecom frequencies for transmission with the robust microwave-domain processing capabilities of stationary superconducting qubits, made possible by recent advances in quantum transduction and signal processing [30]; (3) overcoming the intrinsic 50% theoretical limitation of linear-optical BSMs [31] by employing a deterministic Bell-measurement scheme for the entanglement distribution at the repeater node; and (4) guaranteeing a final end-to-end entangled state, which could be noisy and suitable for further processing, such as purification. Heralded photonic schemes, instead, inherently discard failures and thus prevent performance improvement [24].

Table I provides a summary of the main sequential steps required to implement entanglement swapping within the network scenario under consideration.

III. MODEL ANALYSIS

The network model under consideration involves the two communication parties, Alice and Bob, each equipped with entanglement sources, aiming to perform entanglement distribution. The TESQR architecture is the intermediate repeater node located halfway along the optical fiber connecting Alice

and Bob. It executes entanglement swapping and entanglement purification.

Here, we analytically examine the effect of each processing step on the source states, modeling the various operations in a cascade with appropriate noise models. Figure 2 illustrates the network architecture under consideration. The model parameters are discussed in the next sections and summarized in Table II. The network scenario depicted in Fig. 2 highlights the symmetry of the setup in the two segments Alice-TESQR and Bob-TESQR. Accordingly, to avoid overburdening the mathematical notation, in the following we consider only one of the two equivalent links and omit, where possible, the numerical subscripts. A numerical treatment of the asymmetric scenario, in which the parameters of the two segments are independently varied, is provided and discussed in Appendix E.

A. Source states

The sources generate Bell-type entangled states composed of transmitted photonic subsystems (T_A for Alice and T_B for Bob) and a remote superconducting subsystem (R_A for Alice and R_B for Bob). For subsystem T , we adopt a single-rail Fock state encoding ($|0\rangle, |1\rangle$), defined by the absence ($|0\rangle$) or presence ($|1\rangle$) of a single photon in a bosonic mode [24]. This choice of photonic encoding is motivated by the subsequent physical feasibility of the coupling process, which would otherwise be hindered by alternative encoding methods (such as time-bin encoding) [32]. Moreover, in standard implementations with photonic BSM, this encoding requires single-photon interference at the repeater node, thereby allowing a higher entangling probability compared to double-photon interference [24,33,34].

For the remote stationary qubit subsystem R , let $|g\rangle$ and $|e\rangle$ denote the ground and excited states, respectively. Accordingly, the source state (for Alice or Bob, equivalently) is defined as

$$|\psi\rangle_{TR} = \frac{1}{\sqrt{2}}(|0\rangle_T \otimes |g\rangle_R + |1\rangle_T \otimes |e\rangle_R). \quad (1)$$

The generation of entangled states of the form in Eq. (1) is experimentally achievable, as documented in Ref. [35]. Imperfections are introduced from the quantum channel onward, while the modeling of imperfect sources is intentionally left outside the scope of this work. This choice reflects the aim of the paper, which is to evaluate the performance of the proposed hybrid repeater node independently of the specific physical implementation of the other protocol components.

To simplify the derivation below, it is convenient to introduce the displacement operator acting in the photonic mode, as in Ref. [36]: $D_T(\xi) = \exp(\xi a^\dagger - \xi^* a)$, where $a^\dagger$ and a are the creation and annihilation operators, and ξ is a complex phase-space variable. Its matrix elements are defined as $X_{mn}(\xi) = \langle m|D_T(\xi)|n\rangle$, $m, n \in \{0, 1\}$.

Similarly to the quantum characteristic function, we now define the reduced characteristic operator by applying the displacement operator to the subsystem T and then tracing the photonic mode:

$$\rho_R^{\text{source}}(\xi) = \text{Tr}_T\{\rho_{TR}^{\text{source}} D_T(\xi)\}, \quad (2)$$

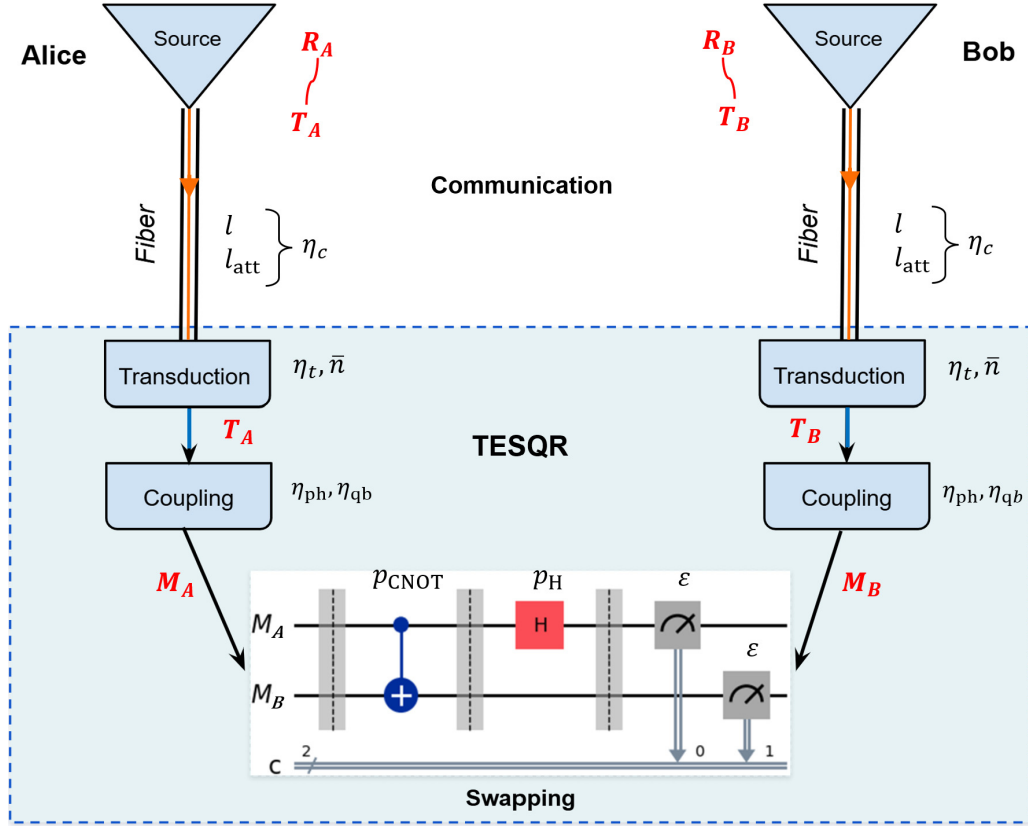


FIG. 2. Schematic of the network architecture. The setup features Alice and Bob at the end points, each producing an entangled pair: a transmitted photonic qubit (T_A or T_B) in single-rail Fock-state encoding and a remote superconducting qubit (R_A or R_B). Photonic qubits travel through optical fibers of length l with transmissivity $\eta_c = e^{-l/l_{\text{att}}}$ ($l_{\text{att}} = 22$ km, equivalent to 0.2 dB/km at 1550 nm). At the TESQR node, optical-to-microwave conversion occurs via electro-optic transduction with efficiency η_t and thermal noise photon number $\bar{n}$. Microwave photons couple to superconducting qubits M_A and M_B with photon availability η_{ph} and transition efficiency η_{qb} . Gate-based entanglement swapping uses a CNOT gate (error p_{CNOT}), Hadamard gate (error p_H), and positive operator-valued measures (POVMs) (error ε). Optical and microwave photon paths are highlighted in orange and blue, respectively.

TABLE II. Network model parameter definitions.

Process	Parameter	Definition
Transmission	l	Fiber length (km) between Alice/Bob–TESQR
Transmission	l_{att}	Fiber attenuation length (km)
Transmission	η_c	Fiber transmissivity
Transduction	η_t	Transduction efficiency
Transduction	$\bar{n}$	Mean noise photon number in the transduction process
Transduction	N_t	Transduction output added noise
Transduction	T_{sys}	Transduction system temperature (K)
Transduction	ζ_m	Microwave extraction efficiency
Transduction	ζ_o	Optical extraction efficiency
Coupling	η_{ph}	Probability of microwave photon availability
Coupling	η_{qb}	Probability of superconducting qubit excitation upon photon absorption
Swapping	p_H	Hadamard gate error rate
Swapping	p_{CNOT}	CNOT gate error rate
Swapping	ε	POVM measurement error rate

where $\rho_{TR}^{\text{source}} = |\psi\rangle_{TR}\langle\psi|_{TR}$. We may also recover the state from the reduced operator by the Glauber equation [36]:

$$\rho_{TR}^{\text{source}} = \frac{1}{\pi} \int \rho_R^{\text{source}}(\xi) D_T^\dagger(\xi) d^2\xi. \quad (3)$$

B. Fiber transmission

The optical fiber channel between the source and TESQR, of length l , is modeled as a Gaussian pure-loss bosonic channel [1,37], with associated transmissivity $\eta_c \in [0, 1]$, which physically corresponds to the probability of photon survival. For simplicity, we include other sources of attenuation (e.g., connector losses) in the channel transmissivity. This parameter is related to the fiber attenuation length l_{att} through the exponential relation

$$\eta_c = e^{-l/l_{\text{att}}}. \quad (4)$$

We assume a standard telecommunication wavelength of 1550 nm, with $l_{\text{att}} = 22$ km, corresponding to an attenuation coefficient of 0.2 dB/km. The attenuation coefficient chosen for this analysis is aligned with realistic standard for fiber-based deployment [1].

The Gaussian pure-loss channel with transmissivity η_c transforms the displacement operator as $D_T(\xi) \rightarrow D_T$

$(\sqrt{\eta_c} \xi) e^{-\frac{1-\eta_c}{2} |\xi|^2}$. The state after transmission through the fiber is expressed as

$$\rho_R^{\text{ch}}(\xi) = \rho_R^{\text{source}}(\sqrt{\eta_c} \xi) e^{-\frac{1-\eta_c}{2} |\xi|^2}. \quad (5)$$

The explicit analytical expressions, obtained both here and in the subsequent steps, are reported in Appendix A.

C. Transduction

For the optical-to-microwave quantum transduction process, we refer to the electro-optic implementation approach [17,18]. This process can be modeled as a single-mode Gaussian quantum channel with transmissivity $\eta_t \in [0, 1]$ and added noise:

$$N_t = (1 - \eta_t)(\bar{n} + \frac{1}{2}), \quad (6)$$

where $\bar{n}$ denotes the mean noise photon number in the beam-splitter model [18]. The latter also determines the efficiency-noise trade-off of the transducer and depends on physical parameters such as system temperature (T_{sys}) as well as microwave (ζ_m) and optical (ζ_o) extraction efficiencies. Accordingly, the transduction process is characterized by the pair of parameters $[\eta_t, \bar{n}]$, as noted in Fig. 2. In evaluating the thermal noise contribution of the transducer, we assume a representative microwave frequency of 8 GHz; the corresponding thermal occupation at optical frequencies is negligible. Realistic values of $\bar{n}$ used in the subsequent performance evaluation are discussed in Appendix B.

Using the displacement-operator approach, the state after the transduction channel is given by [38]

$$\rho_R^{\text{tr}}(\xi) = \rho_R^{\text{ch}}(\sqrt{\eta_t} \xi) e^{-N_t |\xi|^2}. \quad (7)$$

D. Coupling mechanism

The next step involves evaluating the coupling mechanism between the microwave photon output from the transducer and a two-level superconducting matter qubit of subsystem M initialized in the ground state (M_A for Alice's segment and M_B for Bob's segment). The eigenstates in the bare basis are $\{|g\rangle, |e\rangle\}$. This process ideally performs a swap of the flying qubit state to the matter qubit, according to the transformations $|g0\rangle \rightarrow |g0\rangle$, $|g1\rangle \rightarrow |e0\rangle$. This dynamics is described by circuit quantum electrodynamics (CQED) notation, in which a resonator is capacitively coupled to a superconducting qubit (e.g., a transmon qubit) [39]. On resonance, i.e., when the qubit frequency matches the resonator frequency (zero detuning) [40], the energy exchange between the two systems gives rise to Rabi oscillations whereby a single photon is transferred between the matter qubit and resonator with unit probability. This process has been experimentally realized in several works using an external tunable coupler based on a superconducting quantum interference device (SQUID) [41,42]. The dynamics of this interaction can be approximated within the rotating-wave approximation (RWA) by the Jaynes-Cummings interaction Hamiltonian [43],

$$H_{\text{int}} = \hbar g_r (a^\dagger \sigma^- + a \sigma^+), \quad (8)$$

where $a^\dagger$ and a denote the microwave photon creation and annihilation operators, respectively; $\sigma^+ = |e\rangle\langle g|$ and $\sigma^- = |g\rangle\langle e|$

$\langle e|$ are the qubit raising and lowering operators, respectively; g_r is the vacuum Rabi coupling strength (in rad/s); and $\hbar$ is the reduced Planck constant. After a coupling time $t = \frac{\pi}{2g_r}$, the input state $|g1\rangle$ evolves to $-i|e0\rangle$, while $|g0\rangle$ remains unchanged [40].

We propose the following joint Kraus operators to analytically model the imperfect coupling process,

$$\begin{aligned} K_0 &= |g0\rangle_{MT} \langle g0|_{MT} - i\sqrt{\eta_{\text{qb}}\eta_{\text{ph}}} |e0\rangle_{MT} \langle g1|_{MT}, \\ K_1 &= \sqrt{1 - \eta_{\text{qb}}\eta_{\text{ph}}} |g0\rangle_{MT} \langle g1|_{MT}, \end{aligned} \quad (9)$$

where the efficiency parameters are defined as follows: (1) η_{ph} is the probability that the microwave photon is not lost during the coupling interaction with the qubit, and (2) η_{qb} is the probability that the qubit makes the correct transition to the excited state upon absorbing the photon, conditional on the photon being available.

Starting from the reduced state in Eq. (7), the full density matrix is obtained using the Glauber formula [36] $\rho_{TR}^{\text{tr}} = \frac{1}{\pi} \int \rho_R^{\text{tr}}(\xi) D_T^\dagger(\xi) d^2\xi$. Finally, including the superconducting subsystem M into the post-transduction global state $\rho_{MTR}^{\text{tr}} = |g\rangle_M \langle g|_M \otimes \rho_{TR}^{\text{tr}}$, we obtain the full state description:

$$\rho_{MTR}^{\text{coupl}} = \sum_{i=0,1} (K_i \otimes \mathbb{I}_R) \rho_{MTR}^{\text{tr}} (K_i^\dagger \otimes \mathbb{I}_R), \quad (10)$$

where $\mathbb{I}_R$ is the identity operator that acts on subsystem R . As shown in Appendix A 5, the photonic subsystem T in $\rho_{MTR}^{\text{coupl}}$ is disentangled from subsystems M and R , and can therefore be traced out and neglected in the subsequent analysis.

E. Swapping circuit

Recalling the symmetry of the two links Alice-TESQR and Bob-TESQR, we can write $\rho_{MR}^{\text{coupl}} = \rho_{MA RA}^{\text{coupl}} = \rho_{MB RB}^{\text{coupl}}$ and obtain the full system density matrix as

$$\rho_0 = \rho_{MA RA MB RB}^{\text{coupl}} = \rho_{MA RA}^{\text{coupl}} \otimes \rho_{MB RB}^{\text{coupl}}. \quad (11)$$

The TESQR's QPU applies the gate-based version of the BSM [44] on its two superconducting qubits, M_A and M_B . We model noisy gate operations through depolarizing noise [45,46]. Consequently, the transformations (for application of the CNOT unitary U_C on qubits i and j , and of the Hadamard unitary U_H on qubit i) are expressed in our model as

$$\begin{aligned} U_C^{(ij)} \rho_0 U_C^{(ij)\dagger} &\longrightarrow \rho_1 = (1 - p_{\text{CNOT}}) U_C^{(ij)} \rho_0 U_C^{(ij)\dagger} \\ &\quad + \frac{p_{\text{CNOT}}}{4} \text{Tr}_{ij}[\rho_0] \otimes \mathbb{I}_{ij}, \end{aligned} \quad (12)$$

$$\begin{aligned} U_H^{(i)} \rho_1 U_H^{(i)\dagger} &\longrightarrow \rho_2 = (1 - p_H) U_H^{(i)} \rho_1 U_H^{(i)\dagger} \\ &\quad + \frac{p_H}{2} \text{Tr}_i[\rho_1] \otimes \mathbb{I}_i, \end{aligned} \quad (13)$$

where Tr_{ij} and Tr_i denote the partial trace over, respectively, subsystems i, j and subsystem i , while $\mathbb{I}_{ij}$ and $\mathbb{I}_i$ are the identity operators on the corresponding subsystems. p_{CNOT} and p_H represent the gate error probabilities for the CNOT and Hadamard gates, respectively, as indicated in Fig. 2. In reference to Eq. (11), $i = M_A$ and $j = M_B$.

Finally, we describe the imperfect single-qubit measurements on both M_A and M_B in the computational basis by

TABLE III. Ideal entangled states of remote qubits R_A and R_B , conditioned on the measurement outcome of $M_A M_B$.

Outcome ($M_A M_B$)	Ideal postmeasurement state $ \psi_{R_A R_B}^{\text{ideal}}\rangle$
$ gg\rangle_{M_A M_B}$	$\frac{1}{\sqrt{2}}(gg\rangle_{R_A R_B} - ee\rangle_{R_A R_B})$
$ eg\rangle_{M_A M_B}$	$\frac{1}{\sqrt{2}}(gg\rangle_{R_A R_B} + ee\rangle_{R_A R_B})$
$ ge\rangle_{M_A M_B}$	$-i \frac{1}{\sqrt{2}}(ge\rangle_{R_A R_B} + eg\rangle_{R_A R_B})$
$ ee\rangle_{M_A M_B}$	$i \frac{1}{\sqrt{2}}(- ge\rangle_{R_A R_B} + eg\rangle_{R_A R_B})$

a positive operator-valued measure (POVM) [47] with the corresponding imperfect measurement operators for outcomes $|g\rangle$ and $|e\rangle$ defined, respectively, as

$$\begin{aligned} P_g &= (1 - \varepsilon)|g\rangle_M \langle g|_M + \varepsilon|e\rangle_M \langle e|_M, \\ P_e &= \varepsilon|g\rangle_M \langle g|_M + (1 - \varepsilon)|e\rangle_M \langle e|_M, \end{aligned} \quad (14)$$

where ε quantifies the quality of the measurement on the computational basis.

IV. PERFORMANCE EVALUATION

To evaluate the performance of the proposed scheme, it is necessary to first define the ideal entangled state of the remote matter qubits R_A and R_B . Assuming the absence of noise and losses during channel transmission, transduction, and coupling processes (i.e., $\eta_c = 1$, $\eta_t = 1$, $\bar{n} = 0$, $\eta_{\text{ph}} = 1$, $\eta_{\text{qb}} = 1$), the ideal state after coupling is given by

$$\begin{aligned} |\psi\rangle_{R_A M_A R_B M_B} &= \left[\frac{1}{\sqrt{2}}(|gg\rangle_{R_A M_A} - i|ee\rangle_{R_A M_A}) \right] \\ &\otimes \left[\frac{1}{\sqrt{2}}(|gg\rangle_{R_B M_B} - i|ee\rangle_{R_B M_B}) \right]. \end{aligned} \quad (15)$$

During the subsequent swapping phase, the ideal CNOT ($p_{\text{CNOT}} = 0$) and Hadamard ($p_H = 0$) gates, together with the ideal POVMs ($\varepsilon = 0$) on qubits M_A and M_B , project the remote qubits R_A and R_B into an entangled state $|\psi_{R_A R_B}^{\text{ideal}}\rangle$. Its explicit form follows from Eqs. (12)–(14) and is summarized in Table III.

A. Output fidelity

For a given measurement result of $M_A M_B$, let $\rho_{R_A R_B}^{\text{final}}$ denote the $R_A R_B$ density matrix and let $|\psi_{R_A R_B}^{\text{ideal}}\rangle$ be the associated ideal target state. Accordingly, $\rho_{R_A R_B}^{\text{final}}$ denotes the general scenario with nonzero network noise parameters and is analytically obtained by applying the noise operation model introduced in Sec. III. The quality of the entangled state generated between qubits R_A and R_B is quantified by computing its fidelity with respect to the corresponding ideal state reported in Table III. The final fidelity is defined as

$$F_{\text{final}} = \langle \psi_{R_A R_B}^{\text{ideal}} | \rho_{R_A R_B}^{\text{final}} | \psi_{R_A R_B}^{\text{ideal}} \rangle, \quad (16)$$

which measures the overlap between the experimentally obtained state and the ideal entangled state corresponding to the specific measurement outcome. Indeed, fidelity is a straightforward yet crucial performance metric for assessing quantum-state quality, as maintaining high entanglement

TABLE IV. Numerical values of the network model parameters used for performance evaluation.

Parameter	Value
l_{att}	22 km
T_{sys}	0.1 K
ζ_m	0.9
ζ_o	0.9
η_{ph}	0.99
η_{qb}	0.99
p_H	2×10^{-4}
p_{CNOT}	2×10^{-3}
ε	5×10^{-3}

fidelity directly impacts the performance and security of quantum applications such as key distribution and teleportation [48].

The typical values of the relevant network architecture parameters are reported in Table IV [26,28,49–51]. In Fig. 3, we show the behavior of F_{final} as a function of the length of a single source-TESQR segment; hence, the total distance between Alice and Bob corresponds to twice the value reported. Concerning the values of $\bar{n}$, we consider the setup analyzed in Appendix B, with a temperature $T_{\text{sys}} = 0.1$ K. The curves are computed for different values of the transduction efficiency, and they are computed analytically according to the model developed in Sec. III. Similarly, Fig. 4 illustrates the corresponding fidelity trends as a function of the transduction efficiency for several fiber length values.

Several features can be observed from Figs. 3 and 4. The fidelity shows a symmetric trend between $|gg\rangle_{M_A M_B}$ and $|eg\rangle_{M_A M_B}$, and $|ge\rangle_{M_A M_B}$ and $|ee\rangle_{M_A M_B}$. This is consistent with the corresponding symmetry of the ideal postmeasurement states, reported in Table III. As a result of fiber photon loss, transduction thermal noise, and imperfect coupling, qubits M_A and M_B enter the swapping procedure in the ground state. This, when followed by the CNOT and the Hadamard gates, effectively leads to the Bell-measurement output combinations $|gg\rangle_{M_A M_B}$ and $|eg\rangle_{M_A M_B}$.

In addition, all cases exhibit a common exponential decay of the fidelity with increasing source-TESQR segment length, due to the exponential attenuation of the optical fiber. However, the outcomes $|ge\rangle_{M_A M_B}$ and $|ee\rangle_{M_A M_B}$ display a more favorable behavior compared to the other two cases ($|gg\rangle_{M_A M_B}$ and $|eg\rangle_{M_A M_B}$). In particular, Fig. 4(b) shows that a transduction efficiency above 0.4 is sufficient to reach a fidelity greater than 0.5 even for $l = 20$ km (corresponding to a total Alice-Bob separation of 40 km). Indeed, the value $F_{\text{final}} = 0.5$ represents the minimum threshold required to certify the presence of entanglement [52], which is indicated in Figs. 3 and 4 by a horizontal gray dashed line.

The other two outcomes ($|gg\rangle_{M_A M_B}$ and $|eg\rangle_{M_A M_B}$) correspond to cases in which noise contributions—primarily due to photon loss and transduction noise—are more pronounced. This is confirmed by the fact that, with transduction efficiency approaching unity and short distances, i.e., negligible photon losses from the communication link, the fidelities associated with the four measurement outcomes approach the same

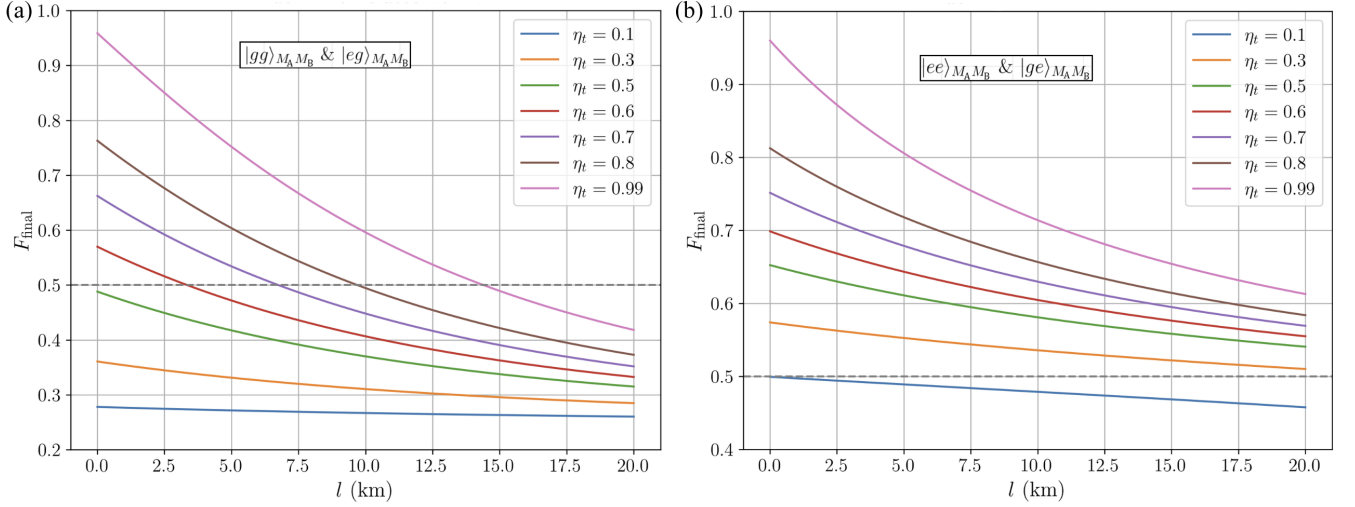


FIG. 3. Final output fidelity F_{final} as a function of fiber length l (km), for various transduction efficiency values and for $M_A M_B$ measurement outcomes: (a) $|gg\rangle_{M_A M_B}$ and $|eg\rangle_{M_A M_B}$, (b) $|ee\rangle_{M_A M_B}$ and $|ge\rangle_{M_A M_B}$. The other network parameters are set according to Table IV.

values, as shown in Fig. 4 in the upper-right region. Conversely, in the low-efficiency and long-distance regime, the fidelity trend corresponding to $|gg\rangle_{M_A M_B}$ and $|eg\rangle_{M_A M_B}$ outcomes diverge maximally from those associated with $|ge\rangle_{M_A M_B}$ and $|ee\rangle_{M_A M_B}$. For a link distance of 10 km, the fidelity trend shown in Fig. 4(a) surpasses the 0.5 threshold only for $\eta_t \geq 0.83$, reaching a maximum of $F_{\text{final}} = 0.6$ at ideal unit transduction efficiency. By contrast, in Fig. 4(b), the fidelity already exceeds the 0.5 threshold for $\eta_t \geq 0.25$, eventually attaining a maximum value of 0.71.

B. Probability of success

Figure 5 shows the outcome measurement probabilities of $M_A M_B$ qubits as a function of the transduction efficiency, for a fiber length $l = 10$ km and the parameters listed in Table IV. Figure 5 confirms the symmetry of the possible output states and, in the considered scenario, the higher

occurrence probability of the states $|gg\rangle_{M_A M_B}$ and $|eg\rangle_{M_A M_B}$. Moreover, as $\eta_t \rightarrow 1$, the probabilities redistribute and approach the equiprobable value of 0.25, which would ideally be reached in a noiseless environment.

To assess the protocol's probability of success, we note that the hybrid TESQR scheme always produces a final state shared by qubits R_A and R_B . As shown in Fig. 5, noise redistributes the outcome probabilities while preserving their total sum. Hence, a measurement result is always obtained, although the fidelity of the corresponding state may be reduced by losses and imperfections. In contrast, optical BSM schemes [24,53] are intrinsically probabilistic and therefore do not always yield an outcome—for instance, this happens due to photon loss, imperfect interference, or indistinguishability [6,53,54]. Such events must therefore be discarded. The TESQR approach, instead, always provides a measurement outcome. As a result, even noisy states remain available for further processing, including entanglement purification as

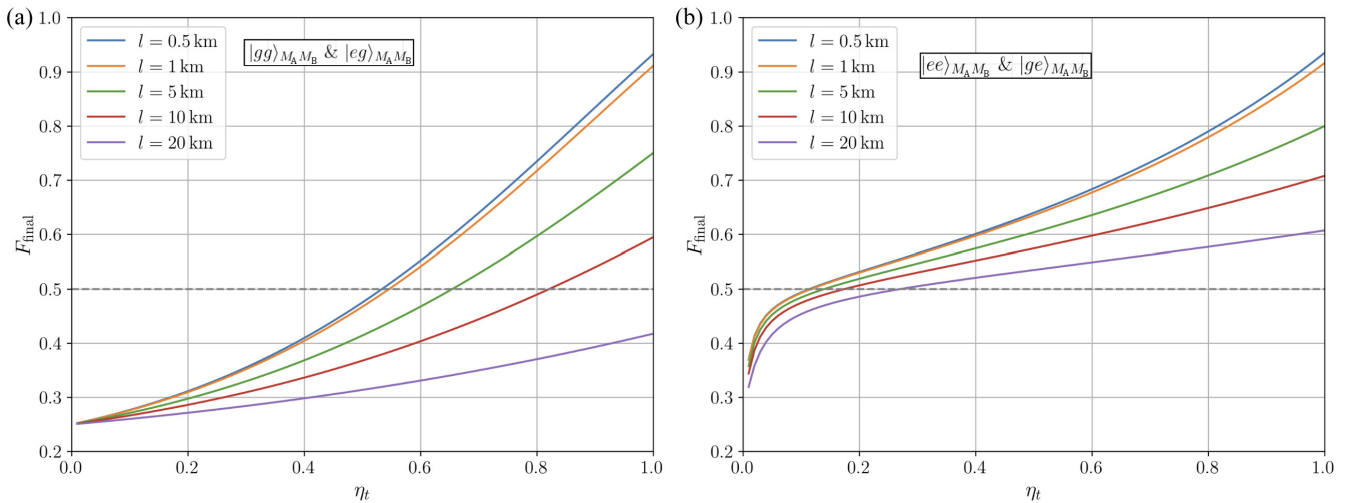


FIG. 4. Final output fidelity F_{final} as a function of transduction efficiency η_t , for various fiber length values and for $M_A M_B$ measurement outcomes: (a) $|gg\rangle_{M_A M_B}$ and $|eg\rangle_{M_A M_B}$, (b) $|ee\rangle_{M_A M_B}$ and $|ge\rangle_{M_A M_B}$. The other network parameters are set according to Table IV.

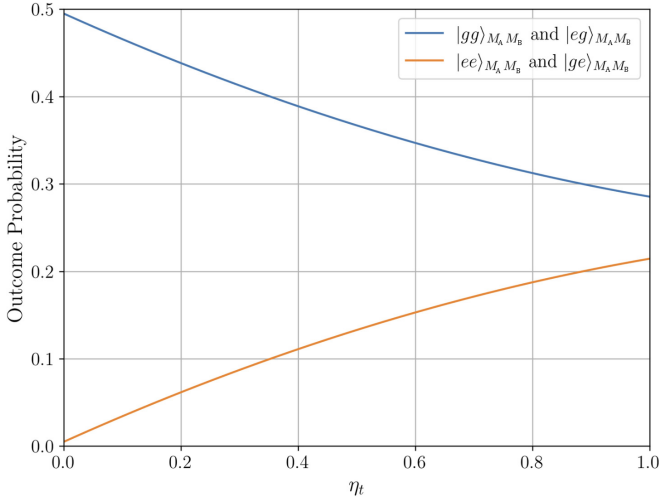


FIG. 5. Measurement outcome probabilities for M_A and M_B superconducting qubits as a function of transduction efficiency η_t , with $l = 10$ km and other network parameters set according to Table IV.

discussed below. In this sense, the protocol is operationally deterministic, as it eliminates the probabilistic nature of optical BSMs.

Building on this distinction, it is convenient to introduce a quantitative measure of success that captures the usefulness of the outcomes produced by the TESQR scheme, hence allowing for a comparison with the fully photonic swapping protocol. In particular, we consider an output state $R_A R_B$ to be entangled and thus useful for further processing only if its fidelity exceeds the threshold of 0.5 [24,52]. Accordingly, in our network scenario, where we deal with four possible measurement outcomes on M_A and M_B , we define the probability of success as

$$P_{\text{success}} = \sum_{i,j \in \{|g\rangle, |e\rangle\}} p_{i,j} \Theta(F_{\text{final}}^{i,j} - 0.5), \quad (17)$$

where $p_{i,j}$ denotes the probability of obtaining the $M_A M_B$ measurement outcome corresponding to $|ij\rangle_{M_A M_B}$, $F_{\text{final}}^{i,j}$ represents the corresponding output fidelity with respect to the ideal Bell state $|\psi_{R_A R_B}^{\text{ideal}}\rangle$ [as defined in Eq. (16)], and $\Theta(\cdot)$ is the Heaviside step function [$\Theta(x) = 0$ if $x < 0$; $\Theta(x) = 1$ if $x \geq 0$]. Under this definition, an outcome counts as successful only if its individual fidelity exceeds the 0.5 threshold.

As a benchmark for our approach, we consider the equivalent optical BSM in a single-rail encoding scheme, consisting of a balanced beam splitter and two number-resolving photodetectors [24]. In this setup, a successful heralded swapping event corresponds to the detection patterns [0,1] and [1,0], where exactly one of the photodetectors is triggered by a single photon. As the only source of noise in this scheme, we account for the detection efficiency η_D of the photodetectors. Accordingly, in Fig. 6 we compare the swapping success probability of the proposed hybrid TESQR scheme, P_{success} [Eq. (17)] (solid curves), with that of the photonic counterpart (dashed curves) [24], as a function of the fiber length l , and for different values of η_t and η_D , respectively. Unlike the purely photonic case, and following a worst-case analysis approach,

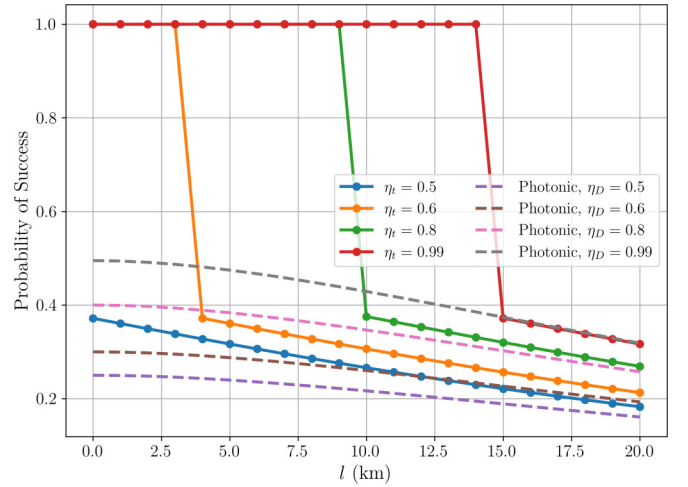


FIG. 6. Swapping success probability versus fiber length l (km) for the hybrid TESQR scheme (solid curves) and the photonic counterpart (dashed curves), plotted for different transduction efficiencies η_t and photodetection efficiencies η_D .

all other noise parameters listed in Table IV are taken into account in the TESQR scheme.

Figure 6 shows that, for equal values of transduction efficiency and detection efficiency, the TESQR scheme consistently achieves a higher probability of success, and thus a larger expected entangling rate, compared to the purely photonic counterpart. In particular, for $\eta_t \gtrsim 0.6$, there exists an initial fiber range where P_{success} is equal to unity for the proposed scheme. In this regime, all possible measurement outcomes of M_A and M_B yield to entangled states of R_A and R_B with fidelity above 0.5. Within this parameter range, entanglement distribution can therefore be regarded as deterministic. The extent of this deterministic region increases with η_t , reaching a limit distance of approximately 14 km for $\eta_t = 0.99$. Beyond this flat region, P_{success} exhibits a drop, which arises because, above this distance threshold, only two of the four possible measurement outcomes yield fidelities above 0.5. Nevertheless, even after this drop, the TESQR curves remain consistently above those of the photonic scheme. The performance gap further widens as the corresponding transduction or detection efficiency decreases. We highlight that the success probability of the photonic scheme is fundamentally bounded above by 0.5, in agreement with the ideal theoretical limit [13]. Finally, the TESQR approach exhibits an average and maximum improvement of 63% and 159%, respectively, in the entanglement distribution success probability compared to the photonic-only counterpart across the tested scenarios. We emphasize that the reported percentage gains quantify the rate of generating an entangled resource suitable for further processing, rather than the quality of the resulting entangled states; therefore, the comparison with the photonic scheme should be interpreted in terms of the rate of usable entanglement generation, not output-state fidelity.

V. PURIFICATION ENHANCEMENT

Entanglement purification is a well-established method to enhance the fidelity of output states, at the expense of

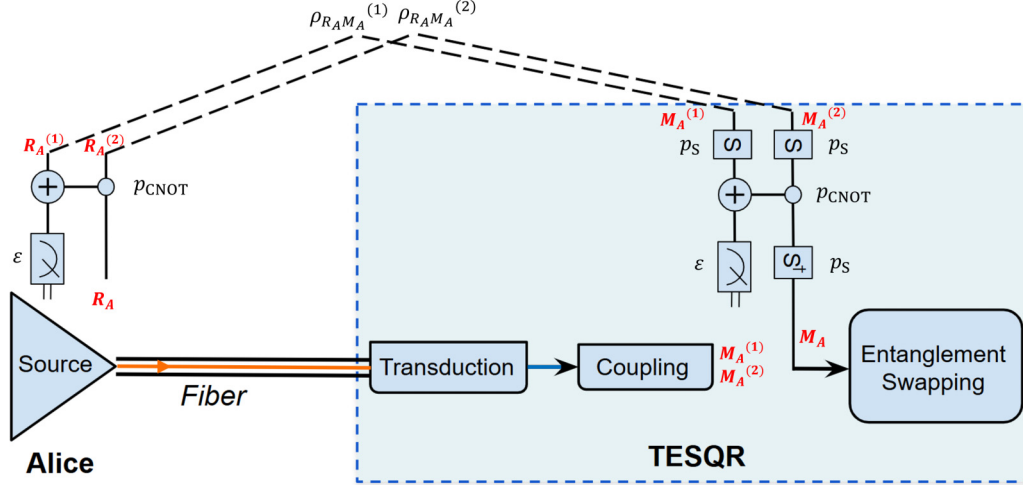


FIG. 7. Alice-TESQR segment incorporating single-round BBPSSW entanglement purification prior to swapping. Two instances of the source state undergo noisy fiber transmission, transduction, and coupling, producing the noisy states $\rho_{R_A M_A}^{(1)}$ and $\rho_{R_A M_A}^{(2)}$. TESQR applies local S gates to $M_A^{(1,2)}$ to obtain Werner form. Bilateral CNOT gates (at both Alice and TESQR) and measurements on the target qubits follow. Success is heralded if the outcomes match. In that case, an $S^\dagger$ gate is applied to the surviving TESQR qubit before the swapping procedure (Fig. 2). The scheme is symmetric for the Bob-TESQR segment. Optical and microwave photon paths are highlighted in orange and blue, respectively.

additional quantum state processing at the network nodes. In this regard, our hybrid solution with TESQR mitigates this issue by operating with superconducting qubits, thereby enabling a direct circuit-based implementation. By contrast, in photonic schemes entanglement purification can be realized, for instance, through recurrence protocols [55], cross-Kerr nonlinearities, hyperentanglement [56], or multipartite entangled states [57]. However, such approaches are generally highly resource demanding [58].

In this scenario, we are primarily interested in assessing the performance contribution of the TESQR node, and given that the swapping scheme introduces additional noise, it is advantageous to apply entanglement purification immediately after the coupling process. This is performed independently on the two segments Alice-TESQR and Bob-TESQR, acting on the pairs $\rho_{M_A R_A}$ and $\rho_{M_B R_B}$. In each segment, the sequence of operations is TESQR $\rightarrow$ entanglement purification $\rightarrow$ entanglement swapping. For the purification scheme, we adopt the classical BBPSSW (Bennett-Brassard-Popescu-Schumacher-Smolín-Wootters) protocol [59]. Figure 7 illustrates the single-round protocol applied to our network model with reference to the Alice-TESQR segment. Alice (Bob) generates two instances of the source state $\rho_{TR}^{\text{source}}$, which are subsequently processed in the photonic domain as described earlier, eventually yielding, after coupling, two instances $\rho_{R_A M_A}^{(1)}$ and $\rho_{R_A M_A}^{(2)}$. We first transform the states into a Werner-like form, as required by the original protocol [59]. In our scenario, the TESQR node applies a local S gate to both superconducting matter qubits $M_A^{(1)}$ and $M_A^{(2)}$ (see Appendix C). Subsequently, both parties perform a bilateral CNOT gate followed by computational-basis POVMs on the target qubits. The purification step is deemed successful when the measurement outcomes coincide. In that case, the TESQR node applies an adjoint S gate ($S^\dagger$) to the remaining superconducting qubit, thereby restoring it to its original form. This purified

qubit is then used in the entanglement swapping procedure (Fig. 2).

Consistent with our previous noise modeling, the S gates are characterized by a depolarizing channel noise acting with probability p_S . To ensure a realistic performance evaluation, the other purification operations (CNOT gates and measurements) are also modeled with the noise parameters described in Sec. III E.

A. Simulation framework

To numerically assess the fidelity after purification, we implement a Monte Carlo simulator based on the QUTIP library [60]. In this framework, we implement the building blocks of the proposed hybrid network architecture. To ensure reproducibility, the software code is available at Ref. [25]. Fiber propagation is described as an amplitude-damping (pure-loss) channel acting on the photonic Fock space. Kraus operators for the lossy photonic mode are applied probabilistically to state vectors in order to generate stochastic trajectories (implemented via sampling over Kraus outcomes). In the simulation environment, the photonic Hilbert space is truncated to a finite Fock basis in order to balance modeling accuracy and computational complexity. This approximation does not significantly affect the subsequent modeling of the transduction process, as further discussed in Appendix D.

Electro-optic quantum transduction is modeled as a beam-splitter-type unitary interaction between the system mode and a thermal environment, $U(\theta) = \exp\{\theta(a^\dagger b - ab^\dagger)\}$, where a and $a^\dagger$ are, respectively, the lowering and raising operator for the input signal state (and b and $b^\dagger$ are the equivalent versions for the environment state), and $\theta = \arccos(\sqrt{\eta_t})$. The environment is initialized in a thermal bath state $\rho_{\text{th}}(\bar{n})$ [61]. Importantly, the photon conversion is implemented at the density-matrix level: the joint system-environment state evolves under the unitary, after which a

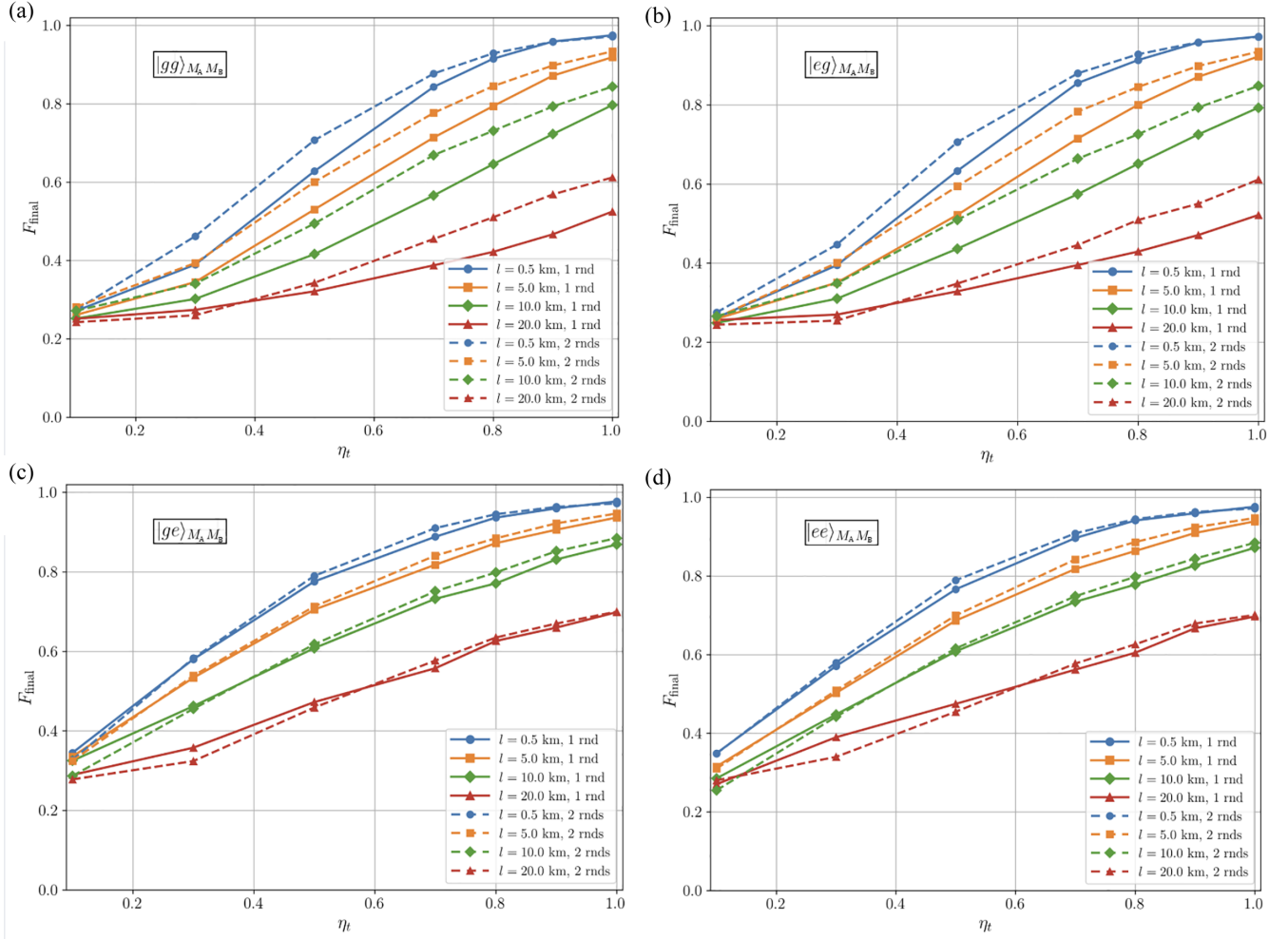


FIG. 8. Fidelity from simulation results with single (1 rnd) and double (2 rnds) round of purification, shown as a function of transduction efficiency η_t for different fiber lengths. Each plot corresponds to a specific measurement outcome of $M_A M_B$: (a) $|gg\rangle_{M_A M_B}$, (b) $|eg\rangle_{M_A M_B}$, (c) $|ge\rangle_{M_A M_B}$, and (d) $|ee\rangle_{M_A M_B}$. p_S is set to 0.0002, and the remaining network parameters are as in Table IV.

partial trace over the environment is performed to obtain the reduced system state. This realizes the Gaussian-loss model, i.e., the beam splitter with added noise [18]. From the resulting reduced density matrix, we then sample a pure state via eigendecomposition, which enables straightforward state-vector propagation in the subsequent simulation steps.

Photonic-matter qubit coupling is modeled with the appropriate set of previously introduced Kraus elements, which are applied probabilistically to the input state vector. Gate imperfections (in both purification and swapping) are described as local depolarizing channels, with Kraus set $\{\sqrt{1-p}I, \sqrt{p/3}X, \sqrt{p/3}Y, \sqrt{p/3}Z\}$ applied to the affected qubit(s) [61]. Imperfect POVMs are incorporated by probabilistic Kraus sampling, parametrized by the measurement error rate ε . Finally, the postswap reduced state of the remote qubits is retained in density-matrix form and used for fidelity evaluation.

B. Results and discussion

In Fig. 8, we report the simulation results for the final fidelity of $R_A R_B$ qubits, for each $M_A M_B$ measurement outcome

case. Results are plotted as a function of the transduction efficiency and for different fiber lengths. We have applied both a single round and two rounds of purification using the previously introduced protocol. In the latter case, this procedure requires four initial instances of ρ_{RM} on each source-TESQR link. Additional purification rounds would cause a corresponding exponential scaling of initial resource cost. To limit statistical fluctuations, we have performed 10^4 simulation runs for each purification round and have computed the average of the resulting fidelities. For a fair comparison with the earlier performance analysis, we employ the same noise parameters (see Table IV) and, analogously to the Hadamard-gate case, we fix the S gate error probability at $p_S = 0.0002$ [28].

Figure 8 demonstrates the efficacy of single-round purification. Across all four measurement outcomes, we observe a marked fidelity increase relative to the unpurified baseline (Fig. 4). The symmetries between outcomes $|gg\rangle_{M_A M_B}$ and $|eg\rangle_{M_A M_B}$ (and similarly $|ge\rangle_{M_A M_B}$ and $|ee\rangle_{M_A M_B}$) are preserved, and in the low-noise limit ($\eta_t \rightarrow 1$, $l \rightarrow 0$ km), the performance across all the outcome scenarios becomes nearly identical. Quantitatively, the purification improvement is significant: for $l = 0.5$ km and $\eta_t = 0.7$ [Figs. 8(a) and 8(b)],

TABLE V. Simulation measurement outcome probabilities of $M_A M_B$ qubits, for single- and double-round purification at $l = 10$ km and for different transduction efficiencies. Each entry shows the probability pairs $[p(\cdot), p(\cdot)]$ for the corresponding grouped outcome $M_A M_B$. The other network parameters are set according to Table IV.

η_t	Single round		Double round	
	$[gg\rangle_{M_A M_B}, eg\rangle_{M_A M_B}]$	$[ge\rangle_{M_A M_B}, ee\rangle_{M_A M_B}]$	$[gg\rangle_{M_A M_B}, eg\rangle_{M_A M_B}]$	$[ge\rangle_{M_A M_B}, ee\rangle_{M_A M_B}]$
0.10	[0.0979, 0.1044]	[0.0118, 0.0101]	[0.0074, 0.0073]	[0.0007, 0.0009]
0.30	[0.1047, 0.1064]	[0.0273, 0.0283]	[0.0089, 0.0086]	[0.0028, 0.0028]
0.50	[0.1116, 0.1107]	[0.0466, 0.0459]	[0.0123, 0.0124]	[0.0069, 0.0069]
0.80	[0.1150, 0.1141]	[0.0815, 0.0826]	[0.0265, 0.0264]	[0.0216, 0.0214]
1.00	[0.1272, 0.1248]	[0.1097, 0.1092]	[0.0464, 0.0459]	[0.0431, 0.0404]

fidelity rises from about 0.65 to over 0.85. Likewise, for $l = 5$ and 10 km, maximum fidelities reach about 0.92 and 0.80, respectively (compared to prepurification values of 0.75 and 0.60). At long distances, such as $l = 20$ km, the critical fidelity value of 0.5 is achievable, assuming unit transduction efficiency.

Similar improvements are shown in Figs. 8(c) and 8(d). For $l = 0.5$ km, a fidelity of 0.9 is reached at $\eta_t = 0.7$, whereas the unpurified case required $\eta_t > 0.9$. Similarly, for $l = 10$ km, the maximum fidelity increases to 0.88 (from 0.7), and even the $l = 20$ km case improves from 0.6 to 0.7.

Notably, the transduction efficiency threshold for effective purification shifts to higher η_t as fiber length increases. This occurs because purification requires an input fidelity above 0.5; as l grows, the postcoupling fidelity drops, eventually falling below this critical value. We emphasize that this threshold cannot be inferred directly from Fig. 4, which depicts the final fidelity after swapping rather than the intermediate state immediately after coupling.

Regarding the double-round purification, the results in Fig. 8 show that it generally improves performance in all scenarios, with a more pronounced advantage when the process is dominated by noise, such as in the measurement outcome cases $|gg\rangle_{M_A M_B}$ and $|eg\rangle_{M_A M_B}$, as previously discussed. However, there exist parameter regimes where double purification fails to outperform single-round purification and may even degrade performance. This effect occurs because the purification protocol enhances fidelity only for states with an initial fidelity above 0.5. If the output of the first round remains below this value, applying a second round—subject to realistic gate noise and decoherence—necessarily deteriorates the state.

Finally, since the purification protocol is probabilistic, it is interesting to examine the outcome probabilities associated with the four measurement outcomes of $M_A M_B$ qubits, both with one-round and two-round purification. As an illustrative example, Table V reports these values as a function of the transduction efficiency for the case of fiber length $l = 10$ km.

The numerical results reported in Table V first confirm, under nonideal transduction efficiency conditions, the expected imbalance in the outcome probabilities of the measurement results $|gg\rangle_{M_A M_B}$, $|eg\rangle_{M_A M_B}$ compared to the other two. Moreover, as η_t increases, the probabilities of both classes of outcomes tend to rise. In this regard, a more pronounced improvement is observed for the $|ge\rangle_{M_A M_B}$, $|ee\rangle_{M_A M_B}$ cases, where the absolute increase between the minimum and maximum values of η_t amounts to nearly one order of magnitude with a single purification round. A similar behavior is

observed for the double-purification case. Here, the gain in fidelity is obtained at the cost of a global reduction of approximately one order of magnitude in the outcome probabilities. Nevertheless, with an ideal unit transduction efficiency, the four outcomes tend to become nearly equiprobable in both schemes. Even in such noisy scenarios, the overall outcome probability reaches maxima of 47% and 18% for single- and double-round purification, respectively. This confirms the feasibility of the proposed enhancement approach.

VI. CONCLUSION

In this work, we introduce TESQR, a hybrid quantum repeater architecture that integrates optical-to-microwave transduction with quantum processing based on superconducting qubits. A key innovation of the proposed approach is the complete elimination of optical BSs. Instead, entanglement swapping operations are directly performed in superconducting circuits. This architecture combines the excellent transmission capabilities of optical photons in classical communication channels with the high-fidelity quantum processing provided by superconducting matter qubits.

To evaluate the feasibility of this approach, we have analytically developed a comprehensive model that describes fiber optic transmission, transduction, coupling with superconducting qubits, and quantum computation. Realistic noise models are used for each component, leading to final fidelity expressions dependent on various nonideal network parameters. Analytical results of the entanglement-swapping process demonstrate that, within suitable operating regimes, the proposed scheme can generate final entangled pairs with fidelities exceeding 0.5 over a total end-to-end distance of 20 km. Furthermore, it achieves average and maximum improvements of, respectively, 63% and 159% in entanglement distribution success probability compared to the corresponding photonic interference-based swapping scheme. Unlike the latter, the TESQR approach always produces a final output state at the remote nodes, rather than aborting on failed photon-interference events. This operationally deterministic feature is advantageous because it preserves a usable state for subsequent processing, even when the output is not yet entangled.

We further investigate the enhancement of performance by adding an entanglement purification stage applied at the repeater node. To assess the effectiveness of this method, we have implemented a full network simulation using the QUTIP framework. Numerical tests with realistic device parameters

resulted in end-to-end fidelity gains in excess of 0.8 for distances up to 20 km, both with single and double purification rounds.

Although current experimental demonstrations of microwave-to-optical transduction have not yet reached the efficiency levels required for immediate large-scale deployment, rapid progress and increasing interest in the field suggest that substantial improvements are achievable in the near term [62,63]. Therefore, the TESQR architecture provides a paradigm and a realistic pathway toward high-rate hybrid quantum repeaters, enabling efficient entanglement distribution in future quantum internet implementations. Future work will explore optimized purification strategies and potential performance enhancements based on quantum error correction techniques.

ACKNOWLEDGMENTS

This manuscript has been authored by employees of the Fermi Forward Discovery Group, LLC, under Contract No. 89243024CSC000002 with the U.S. Department of Energy, Office of Science, Office of High Energy Physics. The U.S. Department of Energy, Office of Science, Early Career Research Program supports S.Z., C.W., and F.F. (partially). J.W. and A.C. are supported by the Fermi Forward Discovery Group, LLC, under Contract No. FWP-23-24 with the U.S. Department of Energy, Office of Science, Advanced Scientific Computing Research (ASCR) Program. D.M.K. is supported by the U.S. Department of Energy, Office of Science, National Quantum Information Science Research Centers, Superconducting Quantum Materials and Systems (SQMS) Center. The SQMS Center supports theory for quantum state preparation in superconducting quantum computing. This work has been partially supported by the Italian Ministry of University and Research (MUR) in the framework of the FoReLab project (Departments of Excellence), that supported F.F., R.G.G., and M.P.

DATA AVAILABILITY

The data that support the findings of this article are openly available [64].

APPENDIX A: DENSITY-MATRIX EXPRESSIONS

The source joint state of the photonic mode T and the remote matter qubit R is

$$|\psi\rangle_{TR} = \frac{1}{\sqrt{2}}(|0\rangle_T \otimes |g\rangle_R + |1\rangle_T \otimes |e\rangle_R). \quad (\text{A1})$$

Accordingly, the source state density matrix is

$$\begin{aligned} \rho_{TR}^{\text{source}} = & \frac{1}{2}(|0\rangle_T \langle 0|_T \otimes |g\rangle_R \langle g|_R + |1\rangle_T \langle 1|_T \otimes |e\rangle_R \langle e|_R \\ & + |0\rangle_T \langle 1|_T \otimes |g\rangle_R \langle e|_R + |1\rangle_T \langle 0|_T \otimes |e\rangle_R \langle g|_R). \end{aligned} \quad (\text{A2})$$

1. Displaced-mode (reduced characteristic) operator

The displacement operator on the photonic mode (of subsystem T) is [36]

$$D_T(\xi) = \exp(\xi a^\dagger - \xi^* a),$$

and we use the matrix elements (in the $|0\rangle, |1\rangle$ basis)

$$X_{00}(\xi) = \langle 0|D_T(\xi)|0\rangle = e^{-|\xi|^2/2}, \quad (\text{A3})$$

$$X_{10}(\xi) = \langle 1|D_T(\xi)|0\rangle = e^{-|\xi|^2/2} \xi, \quad (\text{A4})$$

$$X_{01}(\xi) = \langle 0|D_T(\xi)|1\rangle = -e^{-|\xi|^2/2} \xi^*, \quad (\text{A5})$$

$$X_{11}(\xi) = \langle 1|D_T(\xi)|1\rangle = e^{-|\xi|^2/2} (1 - |\xi|^2). \quad (\text{A6})$$

We define the reduced characteristic operator on the qubit R by

$$\rho_R^{\text{source}}(\xi) = \text{Tr}_T \{ \rho_{TR}^{\text{source}} D_T(\xi) \}. \quad (\text{A7})$$

Computing the contributions term by term [using the source-state form (A2)], we get

$$\begin{aligned} \rho_R^{\text{source}}(\xi) = & \frac{1}{2} e^{-|\xi|^2/2} (|g\rangle_R \langle g|_R + (1 - |\xi|^2) |e\rangle_R \langle e|_R \\ & + \xi |g\rangle_R \langle e|_R - \xi^* |e\rangle_R \langle g|_R). \end{aligned} \quad (\text{A8})$$

2. Fiber channel

A pure-loss channel with transmissivity η_c acts on the displacement as

$$D_T(\xi) \rightarrow D_T(\sqrt{\eta_c} \xi) e^{-\frac{1-\eta_c}{2} |\xi|^2},$$

so that

$$X_{mn}(\xi) \mapsto X_{mn}(\sqrt{\eta_c} \xi) e^{-\frac{1-\eta_c}{2} |\xi|^2}.$$

Applying this to Eq. (A8) gives

$$\begin{aligned} \rho_R^{\text{ch}}(\xi) = & \frac{1}{2} e^{-|\xi|^2/2} (|g\rangle_R \langle g|_R + (1 - \eta_c |\xi|^2) |e\rangle_R \langle e|_R \\ & + \sqrt{\eta_c} \xi |g\rangle_R \langle e|_R - \sqrt{\eta_c} \xi^* |e\rangle_R \langle g|_R). \end{aligned} \quad (\text{A9})$$

3. Transduction Gaussian channel

The transduction channel is modeled by transmissivity η_t and added noise,

$$N_t = (1 - \eta_t)(\bar{n} + \frac{1}{2}),$$

with action

$$D_T(\xi) \mapsto D_T(\sqrt{\eta_t} \xi) e^{-N_t |\xi|^2}.$$

Substituting $\xi \rightarrow \sqrt{\eta_t} \xi$ in Eq. (A9) and adding the extra damping factor yields

$$\begin{aligned} \rho_R^{\text{tr}}(\xi) = & \frac{1}{2} e^{-(\frac{\eta_t}{2} + N_t) |\xi|^2} (|g\rangle_R \langle g|_R + (1 - \eta_c \eta_t |\xi|^2) |e\rangle_R \langle e|_R \\ & + \sqrt{\eta_c \eta_t} \xi |g\rangle_R \langle e|_R - \sqrt{\eta_c \eta_t} \xi^* |e\rangle_R \langle g|_R). \end{aligned} \quad (\text{A10})$$

4. Full system density matrix

First of all, we derive the full system density matrix by using the Glauber formula [36],

$$\rho_{TR}^{\text{tr}} = \frac{1}{\pi} \int \rho_R^{\text{tr}}(\xi) D_T^\dagger(\xi) d^2 \xi, \quad (\text{A11})$$

which explicitly produces

$$\begin{aligned}\rho_{TR}^{\text{tr}} = & \frac{1}{2A} |0\rangle_T \langle 0|_T \otimes |g\rangle_R \langle g|_R + \frac{1}{2} \left(\frac{1}{A} - \frac{1}{A^2} \right) |1\rangle_T \langle 1|_T \otimes |g\rangle_R \langle g|_R \\ & + \frac{1}{2} \left(\frac{1}{A} - \frac{\eta_c \eta_t}{A^2} \right) |0\rangle_T \langle 0|_T \otimes |e\rangle_R \langle e|_R + \frac{1}{2} \left(\frac{1}{A} - \frac{1 + \eta_c \eta_t}{A^2} + \frac{2\eta_c \eta_t}{A^3} \right) |1\rangle_T \langle 1|_T \otimes |e\rangle_R \langle e|_R \\ & + \frac{1}{2} \sqrt{\eta_c \eta_t} \frac{1}{A^2} [|0\rangle_T \langle 1|_T \otimes |g\rangle_R \langle e|_R + |1\rangle_T \langle 0|_T \otimes |e\rangle_R \langle g|_R],\end{aligned}\quad (\text{A12})$$

where $A = \frac{1+\eta_t}{2} + N_t$.

A new superconducting qubit M is initialized in its ground state:

$$\rho_{MTR}^{\text{tr}} = |g\rangle_M \langle g|_M \otimes \rho_{TR}^{\text{tr}}.$$

5. Coupling

The joint Kraus operators incorporating photon and qubit efficiencies η_{ph} and η_{qb} are defined as

$$K_0 = |g0\rangle_{MT} \langle g0|_{MT} - i\sqrt{\eta_{\text{qb}}\eta_{\text{ph}}} |e0\rangle_{MT} \langle g1|_{MT}, \quad (\text{A13})$$

$$K_1 = \sqrt{1 - \eta_{\text{qb}}\eta_{\text{ph}}} |g0\rangle_{MT} \langle g1|_{MT}. \quad (\text{A14})$$

The trace-preserving property can be readily verified as

$$K_0^\dagger K_0 + K_1^\dagger K_1 = \mathbb{I}_{\text{sub}}, \quad (\text{A15})$$

where $\mathbb{I}_{\text{sub}}$ denotes the identity operator on the input subspace $\{|g0\rangle, |g1\rangle\}$ (i.e., assuming that the superconducting qubit M is prepared in the ground state).

The final state after applying these operators is given by

$$\rho_{MTR}^{\text{coupl}} = \sum_{i=0,1} (K_i \otimes \mathbb{I}_R) \rho_{MTR}^{\text{tr}} (K_i^\dagger \otimes \mathbb{I}_R), \quad (\text{A16})$$

where $\mathbb{I}_R$ is the identity operator acting on subsystem R .

Collecting all transformed terms, the coupled output state is

$$\begin{aligned}\rho_{MTR}^{\text{coupl}} = & \frac{1}{2A} |g0\rangle_{MT} \langle g0|_{MT} \otimes |g\rangle_R \langle g|_R + \frac{1}{2} \left(\frac{1}{A} - \frac{1}{A^2} \right) [\eta_{\text{qb}}\eta_{\text{ph}} |e0\rangle_{MT} \langle e0|_{MT} \otimes |g\rangle_R \langle g|_R \\ & + (1 - \eta_{\text{qb}}\eta_{\text{ph}}) |g0\rangle_{MT} \langle g0|_{MT} \otimes |g\rangle_R \langle g|_R] + \frac{1}{2} \left(\frac{1}{A} - \frac{\eta_c \eta_t}{A^2} \right) |g0\rangle_{MT} \langle g0|_{MT} \otimes |e\rangle_R \langle e|_R \\ & + \frac{1}{2} \left(\frac{1}{A} - \frac{1 + \eta_c \eta_t}{A^2} + \frac{2\eta_c \eta_t}{A^3} \right) [\eta_{\text{qb}}\eta_{\text{ph}} |e0\rangle_{MT} \langle e0|_{MT} \otimes |e\rangle_R \langle e|_R + (1 - \eta_{\text{qb}}\eta_{\text{ph}}) |g0\rangle_{MT} \langle g0|_{MT} \otimes |e\rangle_R \langle e|_R] \\ & + i \frac{1}{2} \sqrt{\eta_c \eta_t} \frac{\sqrt{\eta_{\text{qb}}\eta_{\text{ph}}}}{A^2} [|g0\rangle_{MT} \langle e0|_{MT} \otimes |g\rangle_R \langle e|_R - |e0\rangle_{MT} \langle g0|_{MT} \otimes |e\rangle_R \langle g|_R].\end{aligned}\quad (\text{A17})$$

Rewriting Eq. (A17) as $\rho_{MTR}^{\text{coupl}} = \rho_{MR}^{\text{coupl}} \otimes |0\rangle_T \langle 0|_T$, one observes that the photonic subsystem T is disentangled from the remaining subsystems M and R . It can therefore be traced out from the global state and neglected in the subsequent analysis.

6. Swapping and fidelity evaluation

Recalling the symmetry of the two segments Alice-TESQR and Bob-TESQR (Sec. III), we can assert that $\rho_{MR}^{\text{coupl}} = \rho_{MAR_A}^{\text{coupl}} = \rho_{MBR_B}^{\text{coupl}}$ and obtain the full system density matrix as

$$\rho_o = \rho_{MAR_A MBR_B}^{\text{coupl}} = \rho_{MAR_A}^{\text{coupl}} \otimes \rho_{MBR_B}^{\text{coupl}}. \quad (\text{A18})$$

The subsequent calculations involving the application of noisy CNOT and Hadamard gates, followed by the noisy POVM operation (as described in Sec. III E), as well as the

fidelity evaluation with respect to the corresponding ideal states, are not analytically tractable by hand. Instead, they are symbolically solved with the aid of the PYTHON Sympy library [65,66]. These derivations are omitted here so as not to compromise readability.

APPENDIX B: THERMAL NOISE IN ELECTRO-OPTIC QUANTUM TRANSDUCTION

To assess realistic values of the thermal noise photon number $\bar{n}$ used throughout this work, we assume a fixed microwave extraction efficiency $\zeta_m = 0.9$ and optical extraction efficiency $\zeta_o = 0.9$, consistent with experimentally realistic parameters [49]. Under these assumptions, Fig. 9 shows the dependence of $\bar{n}$ on the transduction efficiency η_t for different

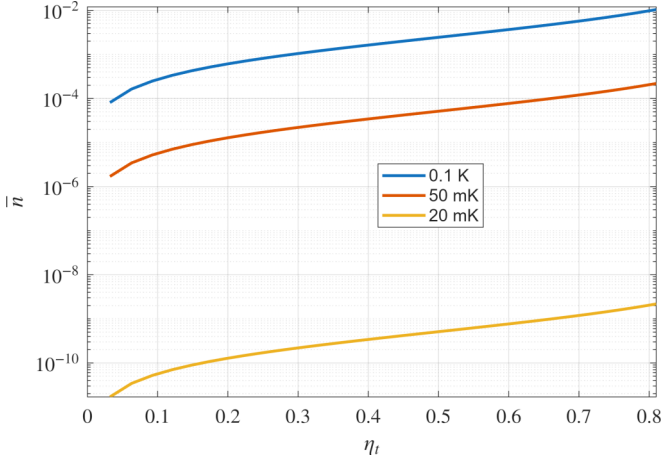


FIG. 9. Transducer thermal noise photon number $\bar{n}$ (logarithmic scale) as a function of the transduction efficiency η_t for different system temperatures T_{sys} . The microwave and optical extraction efficiencies are fixed to $\zeta_m = \zeta_o = 0.9$, and a microwave frequency of 8 GHz is assumed in the transducer.

system temperatures T_{sys} , considering the electro-optic direct transduction scheme without squeezing [30].

We assume a representative microwave frequency of 8 GHz when evaluating the Bose-Einstein thermal photon contribution in the transducer (with mean microwave occupation number n_{in}). The corresponding thermal occupation at optical frequencies is negligible [21]. Moreover, in the absence of squeezing, the added noise parameter N_t is invariant under the conversion direction, i.e., optical to microwave or microwave to optical [18]. The overall transduction efficiency and output added noise parameter in such a transduction scheme are expressed, respectively, by [18]

$$\eta_t = \zeta_m \zeta_o \frac{4C}{(1+C)^2}, \quad (\text{B1a})$$

$$N_t = \frac{1}{2} + \frac{2C\zeta_o[2(1-\zeta_m)n_{\text{in}} - \zeta_m]}{(1+C)^2}, \quad (\text{B1b})$$

where C indicates the interaction cooperativity ($\in [0, 1)$). Recalling the beam-splitter model expression $N_t = (1 - \eta_t)(\bar{n} + \frac{1}{2})$, it becomes evident from Eqs. (B1a) and (B1b) that the transduction efficiency and the mean thermal noise photon number are intrinsically linked.

The absolute magnitude of $\bar{n}$ displayed in Fig. 9 is strongly temperature dependent. At $T_{\text{sys}} = 0.1$ K, the thermal occupation reaches maximum values on the order of 10^{-2} at high η_t , whereas reducing the temperature to 50 mK suppresses $\bar{n}$ by approximately two orders of magnitude over the entire efficiency range. At $T_{\text{sys}} = 20$ mK, the thermal contribution becomes negligible, with $\bar{n}$ remaining below 10^{-8} even at the largest transduction efficiencies considered.

APPENDIX C: LOCAL S CORRECTION FOR ENTANGLEMENT PURIFICATION

The BBPSSW entanglement purification protocol [59] assumes as input a pair of two-qubit states of the Werner form.

For two qubits the Werner state can be written as

$$\rho_W(F) = F|\Phi^+\rangle\langle\Phi^+| + \frac{1-F}{3} \sum_{\beta \in \{\Phi^-, \Psi^+, \Psi^-\}} |\beta\rangle\langle\beta|, \quad (\text{C1})$$

where $0 \leq F \leq 1$ denotes the fidelity with respect to the Bell state $|\Phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$, and $\{|\Phi^-\rangle, |\Psi^\pm\rangle\}$ are the remaining Bell basis states.

In our notation the (ideal, noiseless) $R_A M_A$ state produced by the coupling process is

$$|\psi\rangle_{R_A M_A} = \frac{1}{\sqrt{2}}(|gg\rangle_{R_A M_A} - i|ee\rangle_{R_A M_A}), \quad (\text{C2})$$

where we identify $|g\rangle \equiv |0\rangle$ and $|e\rangle \equiv |1\rangle$ for the computational basis.

The single-qubit phase (“ S ”) gate is defined by the matrix

$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}, \quad S^\dagger = \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}. \quad (\text{C3})$$

Applying S on the second qubit (here, M_A), we obtain

$$\begin{aligned} (\mathbb{I} \otimes S)|\psi\rangle_{R_A M_A} &= \frac{1}{\sqrt{2}}(|gg\rangle_{R_A M_A} - i \cdot i|ee\rangle_{R_A M_A}) \\ &= \frac{1}{\sqrt{2}}(|gg\rangle_{R_A M_A} + |ee\rangle_{R_A M_A}) \\ &= |\Phi^+\rangle. \end{aligned} \quad (\text{C4})$$

Thus a local application of S transforms the state in Eq. (C2) into the $|\Phi^+\rangle$ Bell state, which is the canonical target for BBPSSW-style purification.

After performing purification, the local phase introduced by the S operation can be undone by applying $S^\dagger$ on the same qubit. Indeed,

$$\begin{aligned} (\mathbb{I} \otimes S^\dagger)|\Phi^+\rangle_{R_A M_A} &= \frac{1}{\sqrt{2}}(|gg\rangle_{R_A M_A} + |e\rangle(-i)|e\rangle_{R_A M_A}) \\ &= \frac{1}{\sqrt{2}}(|gg\rangle_{R_A M_A} - i|ee\rangle_{R_A M_A}), \end{aligned} \quad (\text{C5})$$

which recovers the original state in Eq. (C2).

APPENDIX D: ROLE OF THE PHOTONIC TRUNCATION DIMENSION IN THE SIMULATOR

In our simulation framework, the photonic Hilbert space is truncated to a finite Fock basis of dimension `dim_photonic`. This parameter determines the maximum number of photonic excitations that can be explicitly represented in each photonic mode. The truncation is a practical necessity, as it reduces both memory usage and CPU requirements.

To quantify the effect of increasing the photonic truncation dimension, we have performed a dedicated test to verify variations in the photon number (in one of the two equivalent segments containing T_A or T_B) after the transduction process. In Fig. 10 we report the mean photon number $\langle n \rangle$ as a function of `dim_photonic`, for different values of η_t , with $l = 10$ km and all other simulation parameters set according to Table IV. To obtain averaged quantities, we have performed 10^5 independent Monte Carlo simulation runs. The numerical results in Fig. 10 show that increasing `dim_photonic` beyond two produces only a negligible change in the mean photon number

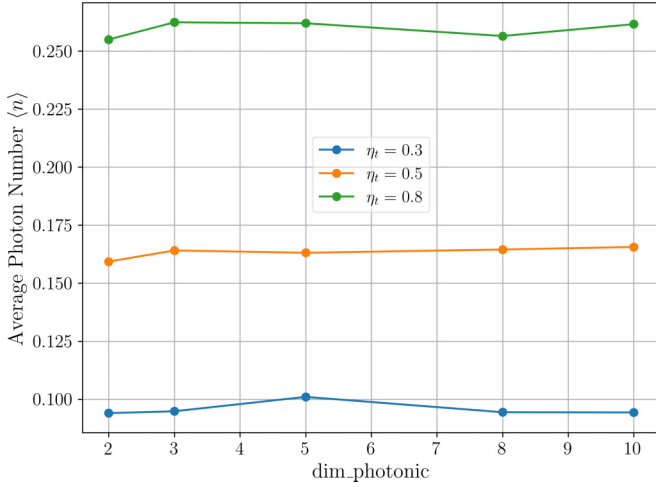


FIG. 10. Average photon number $\langle n \rangle$ versus dim_photonic after the transduction process, for various transduction efficiency values, with $l = 10$ km and all other simulation parameters according to Table IV.

after transduction: the increase remains smaller than 0.007, 0.008, and 0.01 for, respectively, $\eta_t = 0.3, 0.5$, and 0.8 . Moreover, the trend is essentially stable, with maxima of 0.101, 0.164, and 0.262 for the three respective transduction efficiency values. Additional simulations further confirm that the empirical probability $p_{>1}$ (i.e., the probability that the photon number after transduction exceeds 1 in the simulation trials) remains approximately constant at 0.1%, 0.4%, and 1% for $\eta_t = 0.3, 0.5$, and 0.8 , respectively, when dim_photonic is increased beyond 2. Taken together, these results indicate that photon-addition events arising from the transduction model occur infrequently and are already well described within the $\{|0\rangle, |1\rangle\}$ truncated basis for the considered operating regime. Moreover, this effect would be further enhanced by reducing the transduction system temperature T_{sys} . From a practical standpoint, this justifies the use of $\text{dim_photonic} = 2$ as an efficient and reliable approximation, valid on average more than 99% of the time in our network configuration.

APPENDIX E: ASYMMETRIC LINK SCENARIO

Throughout the main text, we have assumed a symmetric network configuration, in which the two segments Alice-TESQR and Bob-TESQR share identical physical parameters. This assumption was adopted to keep the analytical model tractable and to allow a clean assessment of the fundamental performance and feasibility of the TESQR scheme under standard operating conditions. It is also physically motivated: since all components of the TESQR architecture reside within the same integrated system, they are ideally engineered and controlled to exhibit matching properties, and deviations from symmetry can in principle be suppressed through careful calibration.

For completeness, we investigate here TESQR's entanglement-swapping fidelity performances when the two segments exhibit asymmetries in the following key physical parameters:

(1) *Fiber length asymmetry.* Distinct lengths l_A and l_B yield different channel transmissivities $\eta_{c,k} = e^{-l_k/l_{\text{att}}}$, $k \in \{A, B\}$, where k denotes Alice's or Bob's side.

(2) *Transduction efficiency asymmetry.* Distinct efficiencies $\eta_{t,A}$ and $\eta_{t,B}$ arise, e.g., from fabrication tolerances or different cavity operating points.

(3) *Thermal noise asymmetry.* Distinct mean thermal photon numbers $\bar{n}_A$ and $\bar{n}_B$ reflect different cryogenic bath temperatures $T_{\text{sys},A}$ and $T_{\text{sys},B}$.

All other parameters (η_{qb} , η_{ph} , p_{CNOT} , p_H , and ε) remain identical for the two segments, as they are performed locally on the same QPU.

1. Model extension

The derivation of each single-link density matrix proceeds as in Appendix A, independently for each segment, with link-specific parameters ($\eta_{c,k}$, $\eta_{t,k}$, $\bar{n}_k$). In particular, the transduction added-noise and normalization factor become

$$N_{t,k} = (1 - \eta_{t,k}) \left(\bar{n}_k + \frac{1}{2} \right), \quad A_k = \frac{1 + \eta_{t,k}}{2} + N_{t,k}, \quad (\text{E1})$$

for $k \in \{A, B\}$. All model steps up to and including the individual coupled density matrices remain formally identical to the symmetric case. The sole modification is in the assembly of the full four-partite state, where Eq. (A18) is generalized to

$$\rho_o^{\text{asym}} = \rho_{M_A R_A}^{\text{coupl}} \otimes \rho_{M_B R_B}^{\text{coupl}}, \quad (\text{E2})$$

with the two factors now in general distinct. The subsequent swapping circuit and fidelity evaluation proceed identically to the symmetric case, using SymPy [65,66] for symbolic computation.

2. Numerical results and discussion

We evaluate F_{final} after entanglement swapping for three scenarios, each isolating one asymmetry parameter. The shared fixed parameters follow Table IV, with central operating point $\eta_{t,0} = 0.5$, $l_0 = 10$ km, and $T_{\text{sys}} = 0.1$ K. Each asymmetry is introduced symmetrically as $x_A = x_0 - \Delta/2$, $x_B = x_0 + \Delta/2$, recovering the symmetric case at $\Delta = 0$.

Transduction efficiency asymmetry. Figure 11 shows F_{final} versus $\Delta\eta_t = \eta_{t,B} - \eta_{t,A}$, with $\bar{n}_k$ derived self-consistently from $\eta_{t,k}$ via the transducer model (Appendix B).

All four outcomes degrade monotonically and symmetrically as $|\Delta\eta_t|$ grows. To understand this, we recall from Eq. (A17) that the coupled density matrix of each link contains two structurally distinct contributions: diagonal population terms (proportional to $1/A_k$, $1/A_k^2$, $1/A_k^3$), which depend on $\eta_{t,k}$ through A_k ; and an off-diagonal coherence term with coefficient

$$a_{5,k} = \frac{i}{2} \frac{\sqrt{\eta_{c,k} \eta_{t,k}} \sqrt{\eta_{\text{qb}} \eta_{\text{ph}}}}{A_k^2}, \quad (\text{E3})$$

which carries the quantum correlations between the matter qubit M_k and the remote qubit R_k . Since $\eta_{t,k}$ appears in *both* the diagonal and off-diagonal elements, any change in $\eta_{t,k}$ simultaneously affects all density-matrix entries for that link. The joint coherence contributing to the final entangled state after swapping is therefore proportional to $\sqrt{\eta_{t,A} \cdot \eta_{t,B}}$. Since

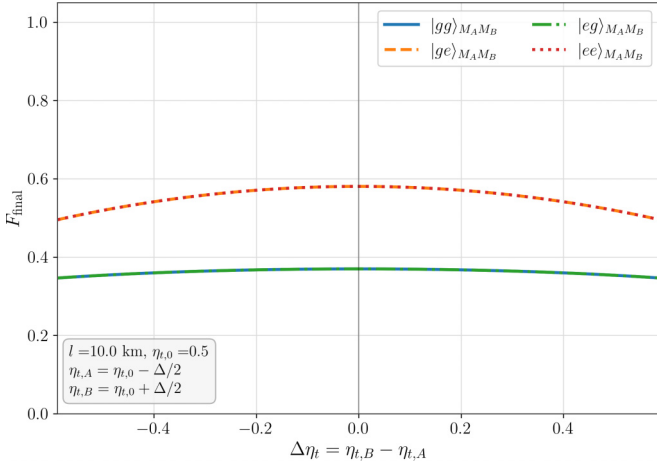


FIG. 11. Entanglement-swapping output fidelity F_{final} vs transduction efficiency imbalance $\Delta\eta_t = \eta_{t,B} - \eta_{t,A}$, with $\eta_{t,A} = \eta_{t,0} - \Delta\eta_t/2$, $\eta_{t,B} = \eta_{t,0} + \Delta\eta_t/2$. Fixed: $l_0 = 10$ km, $\eta_{t,0} = 0.5$; other parameters as in Table IV.

$\eta_{t,A} + \eta_{t,B} = 2\eta_{t,0}$ is fixed, the AM-GM inequality gives

$$\sqrt{\eta_{t,A} \cdot \eta_{t,B}} \leq \eta_{t,0}, \quad (\text{E4})$$

with equality only when $\eta_{t,A} = \eta_{t,B}$. Because η_t affects all density-matrix elements, this suppression acts uniformly on all four measurement outcomes, explaining the identical qualitative behavior seen in Fig. 11.

Fiber length asymmetry. Figure 12 shows F_{final} versus $\Delta l = l_B - l_A$, with $\eta_{t,A} = \eta_{t,B} = \eta_{t,0}$ fixed.

Unlike the $\Delta\eta_t$ case, this scenario reveals a *split* behavior: outcomes $|ge\rangle_{M_A M_B}$ and $|ee\rangle_{M_A M_B}$ degrade with increasing $|\Delta l|$, while $|gg\rangle_{M_A M_B}$ and $|eg\rangle_{M_A M_B}$ remain nearly flat. To understand this, recall that the target states for the latter two outcomes are $|gg\rangle \pm |ee\rangle$, whose fidelity depends on the probability of both remote qubits R_A and R_B being found in the

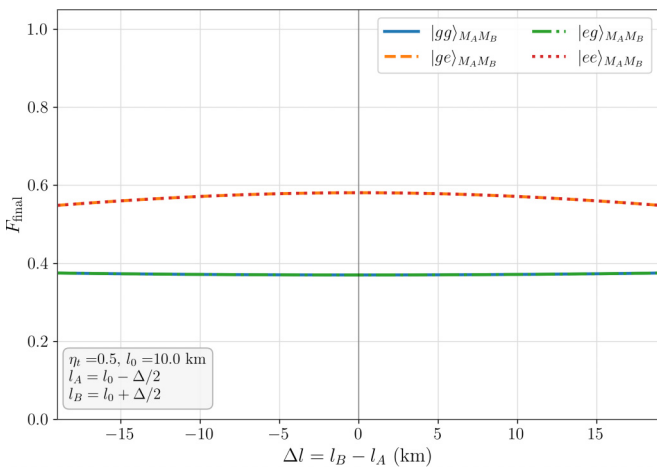


FIG. 12. Entanglement-swapping output fidelity F_{final} vs fiber length imbalance $\Delta l = l_B - l_A$, with $l_A = l_0 - \Delta l/2$, $l_B = l_0 + \Delta l/2$. Fixed: $\eta_{t,0} = 0.5$, $l_0 = 10$ km; other parameters as in Table IV.

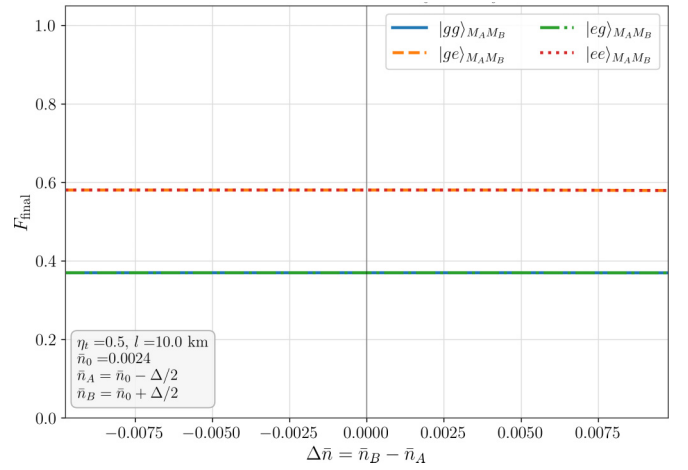


FIG. 13. Entanglement-swapping output fidelity F_{final} vs thermal noise imbalance $\Delta\bar{n} = \bar{n}_B - \bar{n}_A$, with $\bar{n}_A = \bar{n}_0 - \Delta\bar{n}/2$, $\bar{n}_B = \bar{n}_0 + \Delta\bar{n}/2$. Fixed: $\eta_{t,0} = 0.5$, $l_0 = 10$ km; other parameters as in Table IV.

same state ($|g\rangle$ or $|e\rangle$). This is governed by the diagonal coefficients $a_{1,k}$ and $a_{2,k}$ of Eq. (A17), which are independent of $\eta_{c,k}$ since the normalization factor A_k depends only on $\eta_{t,k}$ and $\bar{n}_k$. These terms correspond to events in which the photon is lost before reaching R_k , so their magnitude is set entirely by the transducer and is unaffected by fiber attenuation. As a consequence, varying Δl —which only modifies $\eta_{c,k}$ while leaving A_k unchanged—does not alter $a_{1,k}$ or $a_{2,k}$, and the fidelity of $|gg\rangle$ and $|eg\rangle$ remains flat. The target states $|ge\rangle \pm |eg\rangle$ for the other two outcomes instead require quantum coherence between $|g\rangle_R$ and $|e\rangle_R$ on each link, carried by the off-diagonal term $a_{5,k} \propto \sqrt{\eta_{c,k}\eta_{t,k}}/A_k^2$ and by the $\eta_{c,k}$ -dependent diagonal terms $a_{3,k}$ and $a_{4,k}$. These coefficients are directly affected by fiber asymmetry, and their joint degradation across the two links causes the fidelity of $|ge\rangle$ and $|ee\rangle$ to decrease with $|\Delta l|$. This behavior contrasts sharply with the $\Delta\eta_t$ case: there, $\eta_{t,k}$ enters A_k directly, so any transducer imbalance simultaneously modifies all coefficients $a_{1,k}$ through $a_{5,k}$, degrading all four outcomes uniformly.

Thermal noise asymmetry. Figure 13 shows F_{final} versus $\Delta\bar{n} = \bar{n}_B - \bar{n}_A$, with η_t and l fixed at their nominal values.

All four curves are essentially flat. At $T_{\text{sys}} = 0.1$ K and $f = 8$ GHz, the thermal occupation $\bar{n}_0 \approx 2.4 \times 10^{-3}$ is negligible compared to the vacuum term in $N_t = (1 - \eta_t)(\bar{n} + 1/2)$. The absolute perturbation on A_k induced by the full sweep of $\Delta\bar{n}$ is of order $(1 - \eta_t)\bar{n}_0 \sim 10^{-3}$, entirely masked by the contributions from η_t , η_c , and the gate errors. This confirms that, at the cryogenic temperatures of interest, thermal noise is not a limiting factor of the TESQR scheme, and moderate intersegment temperature differences have no measurable impact on fidelity. The dominant sources of degradation remain η_t , η_c , and gate imperfections, consistently with the main text analysis. These results jointly support the physical validity and robustness of the symmetric scenario adopted as the primary framework throughout this work.

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