

Light-induced localized vortices in multicomponent Bose-Einstein condensates

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We study continuous interaction of a trapped two-component Bose-Einstein condensate with light fields in a Λ -type configuration. Using light beams with orbital angular momentum, we theoretically show how to create a stable, pinned vortex configuration, where the rotating component is confined to the region surrounded by the second, nonrotating component. The atoms constituting this vortex can be localized in volumes much smaller than the volume occupied by the second component. We also show that the vortex position can be changed dynamically by moving the laser beams, provided the beams' movement speed remains below the speed of sound. This allows us to use the localized vortex to stir the second component, and to determine the superfluid flow's critical velocity.

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I. INTRODUCTION

Bose-Einstein condensates (BECs) provide a unique platform for studying quantum fluids, which can support macroscopic quantum phenomena such as superfluidity and quantized vortices. Vortices in BECs are characterized by phase singularities in the condensate wave function and are fundamental excitations that reveal insights into angular momentum quantization and topological defects in quantum systems [1]. The study of vortices has broad implications, ranging from quantum turbulence [2,3] to connections with superconductivity [4] and the structure of neutron stars [5].

A key challenge in the study of BEC vortices is their controlled creation and manipulation. Typically, the vortices are produced by transferring angular momentum to the atoms using optical means: through phase imprinting [6], stirring [7,8], or using beams carrying orbital angular momentum, such as Laguerre-Gaussian (LG) beams [9–17]. More recently, interest has also increased in coupling light to condensates using the so-called Λ -type configurations, which create an internal dark state. The properties of this state can be used to realize atom control at subwavelength resolution [18,19], create optical lattices featuring barriers of subwavelength width [20,21], and make narrow structures in the BEC [22]. The Λ systems have also been used to create vortices in BEC via time-dependent transfer of population between the atomic internal states by applying Raman-type schemes [11,12,23,24].

In this work, we consider a trapped two-component BEC continuously interacting with two light fields in a Λ -type configuration. One of the two laser fields used in the setup is an LG beam, which can transfer angular momentum to the BEC atoms. We show in the following that the driven system possesses stationary states such that either one or both components of the BEC are in a vortex state. These stationary

states belong to the manifold of dark states and are therefore immune to decay via spontaneous emission. Compared to the familiar coreless-vortex profiles (whereby the core of the rotating vortex is filled by the nonrotating component), the stationary states can also have inverted profiles, with the rotating component being surrounded by the nonrotating one. In this case, the rotating atoms of one component are tightly localized in a region of the order of the healing length of the other component. The degree of localization can be controlled by tuning the relative strength of the two laser beams. Remarkably, when this relative strength exceeds a certain threshold, the localized-vortex state becomes the lowest-energy state of the dark-state manifold. To further demonstrate the stability of these states, we show that the resulting vortices can be robustly moved around the trap by moving the laser beams. As long as the beam movement speed is below the speed of sound, the vortex follows the beams without notably disturbing the nonrotating component. To quantify this effect, we move the vortex in circular paths, thereby stirring the other component with a localized impurity. By examining the resulting heating, we determine the critical velocity of the superfluid flow below which the vortex moves without dissipation.

II. THEORY

A. Single-particle Hamiltonian of the system

We consider a general three-level atomic system arranged in a Λ -type configuration of the atom-light coupling shown in Fig. 1 [18–20,25–29]. Two copropagating light beams provide the position-dependent couplings between the atomic internal states. The beams are characterized by the Rabi frequencies:

$$\begin{aligned}\Omega_1(\rho, \varphi) &= \Omega_0 \left(\frac{\rho}{a} \right)^v e^{-\rho^2/w_0^2} e^{ik_z z + i\nu\varphi}, \\ \Omega_2(\rho, \varphi) &= \Omega_0 e^{-\rho^2/w_0^2} e^{ik_z z},\end{aligned}\quad (1)$$

where (ρ, φ) are the polar coordinates, w_0 is the beam waist (the same for both beams), Ω_0 is the amplitude of the Rabi frequencies, and k_z is the wave number for the paraxial propagation of the light beams in the z direction. The parameter

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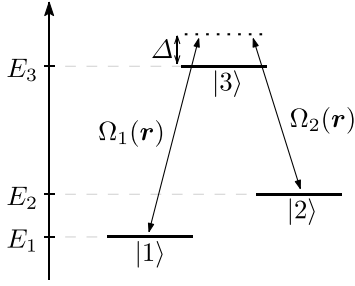


FIG. 1. Schematic representation of the relevant energy levels of the Λ scheme of atom-light coupling.

a , having dimension of length, controls the subwavelength nature of the setup. The beam characterized by the Rabi frequency Ω_1 couples the atomic internal states $|1\rangle$ and $|3\rangle$ and represents a Laguerre-Gaussian mode LG_0^ν with a vorticity (winding number) $\nu > 0$. On the other hand, the atomic states $|2\rangle$ and $|3\rangle$ are coupled by a Gaussian beam characterized by the Rabi frequency Ω_2 . Both beams copropagate along the z axis [26–28], oscillating at frequencies ω_1 and ω_2 , respectively.

On a single-particle level, the dynamics of an atom is governed by the Hamiltonian

$$\hat{H}(\mathbf{r}) = -\frac{\hbar^2}{2m}\Delta + V(\mathbf{r}) + \hat{H}_0(\mathbf{r}). \quad (2)$$

The kinetic energy term and the trapping potential $V(\mathbf{r})$ (assumed to be the same for the atomic internal states $|1\rangle$, $|2\rangle$, and $|3\rangle$) act as identity operators in the space of atom's internal states. Transitions between the latter states are described by the internal-space Hamiltonian

$$\begin{aligned} \hat{H}_0(\mathbf{r}) = & \epsilon|2\rangle\langle 2| + \left(-\Delta - i\frac{\Gamma}{2}\right)|3\rangle\langle 3| \\ & + \hbar[\Omega_1(\mathbf{r})|3\rangle\langle 1| + \Omega_2(\mathbf{r})|3\rangle\langle 2| + \text{H.c.}], \end{aligned} \quad (3)$$

The temporal dependence of this Hamiltonian has been eliminated by transitioning to the rotating frame and adopting the rotating wave approximation [30]. Two parameters emerge as a result, including the single-photon detuning $-\Delta = E_3 - E_1 - \hbar\omega_1$ and the two-photon detuning $\epsilon = E_2 - E_1 + \hbar\omega_2 - \hbar\omega_1$; here E_i denotes the energy of the i th energy level (see Fig. 1). Furthermore, we have introduced the spontaneous decay rate Γ for the excited state $|3\rangle$ represented through the imaginary part of the excited-state energy. In the following, we consider the two-photon resonance, for which $\epsilon = 0$.

It is instructive to consider the states $|\alpha(\mathbf{r})\rangle$ representing the eigenstates of the internal-space Hamiltonian (3), such that $\hat{H}_0(\mathbf{r})|\alpha(\mathbf{r})\rangle = \varepsilon_\alpha(\mathbf{r})|\alpha(\mathbf{r})\rangle$, with $\mathbf{r}$ treated parametrically. Of special interest is the dark eigenstate of $\hat{H}_0$, given by:

$$|D(\mathbf{r})\rangle = \frac{1}{\sqrt{1+|\zeta|^2}}(|1\rangle - \zeta|2\rangle), \quad (4)$$

where

$$\zeta(\mathbf{r}) = \frac{\Omega_1(\mathbf{r})}{\Omega_2(\mathbf{r})}. \quad (5)$$

The dark state is characterized by zero eigenenergy. Having no contribution from the lossy excited state $|3\rangle$, the dark state has

an infinite lifetime as far as evolution under $\hat{H}_0$ is concerned. For $\Delta = \Gamma = 0$, the remaining two eigenstates of the internal-space Hamiltonian are given by

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|B\rangle \pm |3\rangle), \quad \text{where } |B\rangle = \frac{\zeta^*|1\rangle + |2\rangle}{\sqrt{1+|\zeta|^2}}, \quad (6)$$

with

$$\hat{H}_0|\pm\rangle = \pm\hbar\Omega|\pm\rangle \quad \text{and} \quad \Omega = \sqrt{|\Omega_1|^2 + |\Omega_2|^2}. \quad (7)$$

In Eq. (6), $|B\rangle$ is the so-called bright state, representing a superposition of atomic ground states orthogonal to the dark state $|D\rangle$. Later we will consider the adiabatic motion of atoms in the dark-state manifold, which is relevant if the total Rabi frequency Ω is much larger than the characteristic kinetic energy of the atomic center-of-mass motion.

B. Dynamics of the BEC

Now let us consider the equations for an atomic BEC interacting with the laser fields. We start with the Schrödinger equation for the full state vector $|\Phi(\mathbf{r})\rangle$ of a single atom

$$i\hbar\frac{\partial}{\partial t}|\Phi(\mathbf{r})\rangle = \hat{H}(\mathbf{r})|\Phi(\mathbf{r})\rangle \quad (8)$$

and use the expansion

$$|\Phi(\mathbf{r})\rangle = \sum_{i=1}^3 \Phi_i(\mathbf{r})|i\rangle. \quad (9)$$

Here $|i\rangle$ are the bare atomic states featured in the definition of $\hat{H}_0$ in Eq. (3). To account for the interaction between the atoms, we use the Gross-Pitaevskii approach for the multicomponent BECs [31]. In practice, this amounts to supplementing the Schrödinger equations for $\Phi_i(\mathbf{r})$ with the nonlinear terms, thereby promoting the single-particle wave functions to the wave functions (order parameters) of the components of the condensate [27]. This way we arrive at

$$\begin{aligned} i\hbar\dot{\Phi}_1 = & \left(-\frac{\hbar^2}{2m}\Delta + V + g_{11}|\Phi_1|^2 + g_{12}|\Phi_2|^2\right)\Phi_1 + \hbar\Omega_1^*\Phi_3, \\ i\hbar\dot{\Phi}_2 = & \left(-\frac{\hbar^2}{2m}\Delta + V + g_{12}|\Phi_1|^2 + g_{22}|\Phi_2|^2\right)\Phi_2 + \hbar\Omega_2^*\Phi_3, \\ i\hbar\dot{\Phi}_3 = & \left(-\frac{\hbar^2}{2m}\Delta + V - \Delta - i\frac{\Gamma}{2}\right)\Phi_3 + \hbar\Omega_1\Phi_1 + \hbar\Omega_2\Phi_2. \end{aligned} \quad (10)$$

For atoms adiabatically following the dark state, the population of the excited state $|3\rangle$ described by the wave function Φ_3 is small at all times, so atom collisions are not taken into account in the equation for Φ_3 . The coefficients g_{ij} describing interaction between the atoms in the corresponding internal states are related to the s -wave scattering lengths a_{ij} as $g_{ij} = 4\pi\hbar^2 a_{ij}/m$. The multicomponent wave function of the BEC is normalized to the total number of atoms, $\sum_{i=1}^3 \int d\mathbf{r} |\Phi_i|^2 = N$. We will also use parameters η_i to quantify the relative population of each component:

$$\eta_i = \frac{1}{N} \int |\Phi_i|^2 d\mathbf{r}. \quad (11)$$

To obtain an equation for the evolution of atoms adiabatically following the dark state, we introduce the following superpositions of Φ_1 and Φ_2 describing the wave functions for atoms in the dark and bright states introduced in the previous Sec. II A:

$$\begin{aligned}\Phi_D &= \frac{1}{\sqrt{1+|\zeta|^2}}(\Phi_1 - \zeta^* \Phi_2), \\ \Phi_B &= \frac{1}{\sqrt{1+|\zeta|^2}}(\zeta \Phi_1 + \Phi_2).\end{aligned}\quad (12)$$

Inserting these definitions into Eq. (10), we obtain a set of equations for Φ_D , Φ_B , and Φ_3 , in which the dark-state and bright-state wave functions Φ_D and Φ_B are coupled via nonadiabatic terms. Under the adiabatic assumption of slow center-of-mass motion for atoms in the dark internal states, justified for the systems to be considered, and the assumption of equal nonlinear couplings ($g_{11} = g_{12} = g_{22} = \beta$), we reach the dark-state GPE [27]

$$i\hbar\dot{\Phi}_D = \frac{1}{2m}(-i\hbar\nabla - \mathbf{A})^2\Phi_D + (U + V)\Phi_D + \beta|\Phi_D|^2\Phi_D.\quad (13)$$

It is worth noting that, although Φ_D is the solution to a nonlinear mean-field equation for a BEC, the term “dark-state wave function” remains appropriate. This state contains no atoms in the internal state $|3\rangle$, and, provided the adiabatic approximation holds, this remains true at all times, since the equation for Φ_D is fully decoupled from those for Φ_B and Φ_3 . In contrast, the equation for the “bright-state BEC” Φ_B is coupled to Φ_3 , allowing atoms in Φ_B to transition into $|3\rangle$ and decay.

It is apparent from Eq. (13) that reduction to the dark-state manifold results in the appearance of geometric vector and scalar potentials $\mathbf{A}(\mathbf{r})$ and $U(\mathbf{r})$ emerging due to the position dependence of the atom-light coupling and given by [27,28]

$$\begin{aligned}\mathbf{A} &= i\hbar \frac{\zeta^* \nabla \zeta - \zeta \nabla \zeta^*}{2(1+|\zeta|^2)}, \\ U &= \frac{\hbar^2}{2m} \frac{\nabla \zeta^* \cdot \nabla \zeta}{(1+|\zeta|^2)^2}.\end{aligned}\quad (14)$$

For the beams given by Eq. (1) the geometric gauge potentials do not depend on z and read

$$\begin{aligned}\mathbf{A} &= -\frac{\hbar\nu}{a} \frac{\left(\frac{\rho}{a}\right)^{2\nu-1}}{1+\left(\frac{\rho}{a}\right)^{2\nu}} \mathbf{e}_\varphi, \\ U &= \frac{1}{m} \left(\frac{\hbar\nu}{a}\right)^2 \frac{\left(\frac{\rho}{a}\right)^{2\nu-2}}{\left[1+\left(\frac{\rho}{a}\right)^{2\nu}\right]^2}.\end{aligned}\quad (15)$$

Here $\mathbf{e}_\varphi$ denotes a unit vector in the azimuthal direction. The artificial magnetic field resulting from the geometric potential $\mathbf{A}$ of Eq. (15) is

$$\mathbf{B} = \nabla \times \mathbf{A} = -\frac{2m}{\hbar} U \mathbf{e}_z,\quad (16)$$

where $\mathbf{e}_z$ is a unit vector directed along the z -axis. For the light beams given by Eq. (1), the effective magnetic field is thus proportional to the scalar potential.

Once the solution of Eq. (13) is found, one can return to the wave functions Φ_1 and Φ_2 of the internal-state basis according to

$$\begin{aligned}\Phi_1 &= \frac{1}{\sqrt{1+|\zeta|^2}}\Phi_D, \\ \Phi_2 &= -\frac{\zeta}{\sqrt{1+|\zeta|^2}}\Phi_D.\end{aligned}\quad (17)$$

This follows from Eq. (12) with $\Phi_B = 0$, since for adiabatic atomic motion in the dark state described by Eq. (13) for the wave-function Φ_D , the system is almost completely in the dark state with a negligible population of the bright and excited states.

In this way, when the atoms forming the BEC are in the dark state, the components Φ_1 and Φ_2 are related to each other via Eq. (17), in which the component Φ_2 has an extra factor $\zeta = \Omega_1/\Omega_2$. This factor is proportional to $e^{i\nu\varphi}$, as the Rabi frequency Ω_1 has a vortex with a winding number ν . As a result, at least one of the components Φ_1 or Φ_2 should have a vortex when the BEC atoms are in the dark state.

Our analysis will be mostly based on numerical calculations of the stationary states described by the full three-component wave function, governed by Eqs. (10). We will concentrate on the solutions belonging to the dark-state manifold, referring to the center-of-mass state for the dark state atoms with the lowest energy as the ground state. Moreover, we will be interested in stationary vortex solutions of the form $\Phi_i(\mathbf{r}, t) \propto f_i(\rho) e^{il_i\varphi} e^{-i\mu t/\hbar}$, where $i = 1, 2, 3$ and μ is the chemical potential of the system, and we will refer to the integers l_i as vorticities for each internal state. On the other hand, Eq. (13) for the dark state, valid under the assumption of sufficiently slow atomic motion and equal intracomponent scattering lengths, will be used to perform an analytic consideration in Sec. III A.

We remark that the dark state potentials arising from coupling BECs with LG beams have previously been considered (see, in particular, Ref. [27]). However, the stationary states emerging under continuous interaction with the laser beams have not been analyzed before.

C. Trap geometry and parameter values

As the geometric vector and scalar potentials $\mathbf{A}$ and U of Eq. (15), corresponding to the light beams (1), do not depend on z , we can consider the two-dimensional (2D) motion of dark-state atoms in the xy plane for fixed z . Specifically, for a cylindrically symmetric setup of interest, in the dark-state GPE (13) we can separate the variables as $\Phi_D(\rho, \varphi, z, t) = \Phi_D^{(2)}(\rho, \varphi, t)Z(z)$ and take $Z(z) = 1/\sqrt{d}$, thereby using the Thomas-Fermi approximation for the atomic motion in the z direction extended over the distance d . The resulting equation for $\Phi_D^{(2)}(\rho, \varphi, t)$ has the same form as Eq. (13) but the interaction strength changes as $\beta \rightarrow \beta/d$. A similar procedure can be performed for the exact system (10), yielding accurate results in the adiabatic regime.

In our numerical simulations we consider a BEC cloud of $N = 5 \times 10^4$ atoms confined in a cylindrical trap [32] of the radius R and the extent d along the z axis with $R = d = 15.0 \mu\text{m}$. The states $|1\rangle$ and $|2\rangle$ are taken to be two

hyperfine levels of ^{87}Rb : $|1\rangle = |F=1, m_F=-1\rangle$ and $|2\rangle = |F=2, m_F=1\rangle$, while state $|3\rangle$ is the $5^2\text{P}_{3/2}$ state, whose decay rate is $\Gamma = 38.1\text{ MHz}$ [33]. The scattering lengths are [34]: $a_{11} = 100a_0$, $a_{12} = 98.0a_0$, $a_{22} = 95.4a_0$, where a_0 is the Bohr radius. The assumption of equal scattering lengths, used in deriving the dark-state GPE, is thus justified for this system.

The typical values of the Rabi frequencies used in Λ -scheme experiments range from $\sim 10\text{ MHz}$ to $\sim 1\text{ GHz}$. Our calculations show that the dark-state GPE yields very accurate results [compared to the solution of the full system of GPEs (10)] already at $\Omega_0 = 1\text{ MHz}$, meaning that the lowest states of the dark-state manifold indeed become effectively decoupled from the states of the $|\pm\rangle$ manifolds. Generally, the nonadiabatic coupling may be disregarded provided $U/\hbar \ll \Omega(\mathbf{r})$ for all $\mathbf{r}$ [20,27,28], where $\Omega \equiv \Omega(\mathbf{r})$ is the total Rabi frequency given by Eq. (7). For $v=1$, the maximum of U is located at the origin, while Ω attains its minimal value there (we neglect the beam-waist term $e^{-\rho^2/w_0^2}$ in the present reasoning), equal to $\Omega(\mathbf{r}=0) = \Omega_0$. However, even for the lowest value of a considered in this work ($a = 0.3\text{ }\mu\text{m}$), one finds $U/\hbar = 8\text{ kHz}$, which is orders of magnitude smaller than the typical value of Ω_0 . Furthermore, although generally including the decay rate of $|3\rangle$ improves the validity of the dark-state description [20,21,29], in the regime in question we obtained equally accurate results even for $\Gamma = 0$ —again because Ω_0 is very large compared to U . Finally, we mention the effect of the detuning Δ . Nonzero positive values of Δ shift the manifold of the $|-\rangle$ states down in energy, further reducing the coupling with the states of the $|D\rangle$ manifold. On the other hand, the states of the $|+\rangle$ manifold are then also shifted to lower energies, and this enhances the coupling with the $|D\rangle$ -manifold states. Thus, the dark-state GPE is expected to be valid for some intermediate values of Δ . For example, we have found that for $\Omega_0 \sim 10\text{ MHz}$, values of Δ from 0 to up to $\sim 10\Omega_0/a^v$ lead to good agreement between the dark-state analysis and the full treatment. Meanwhile, negative values of Δ shift to higher energies the manifold of $|-\rangle$ states. This facilitates coupling between the $|-\rangle$ and $|D\rangle$ states, reducing the accuracy of the dark-state description.

In all our calculations we used $\Omega_0 = 20.0\text{ MHz}$ and $\Delta = 10\Omega_0/a^v$, where the parameter a characterizes the Rabi frequency of the vortex light beam in Eq. (1). The beam-waist term $e^{-\rho^2/w_0^2}$ can be set to unity assuming the waist is much larger than the radial extent of the cloud. Moreover, the term $e^{-\rho^2/w_0^2}$ plays no role in the adiabatic dynamics of atoms in the dark state, since it drops out of the relative Rabi frequency ζ given by Eq. (5). However, we did include this term in the exact numerical calculations by setting $w_0 = 2R$ —while the results were the same, this provided a slight speed up for the calculations.

Henceforth we adopt the dimensionless units, measuring length in units of R , energy in units of kinetic energy $E_R = \hbar^2/2mR^2$ corresponding to the momentum $k_R = 1/R$, and time in units of $\hbar/E_R$. In these units the radial coordinate ρ varies between 0 and 1. We refer the reader to the Appendix for additional details on the units and the dimensionless form of the equations used for numerical calculations.

III. STATIONARY VORTEX SOLUTIONS

A. Approximate analysis of motion of dark-state atoms

To gain physical insight into the expected structure of the possible stationary states of the system undergoing the 2D motion, it is instructive to consider the dark-state GPE (13) first. To find stationary solutions, we put $\Phi_D(\mathbf{r}, t) = e^{-i\mu t} \psi(\mathbf{r})$ and further separate the variables due to the cylindrical symmetry of the system:

$$\psi(\rho, \varphi) = f(\rho)e^{il\varphi} \quad (18)$$

with $f(\rho)$ being real and l being integer. In the specific case of the geometric potentials given in Eq. (15), this leads to the equation of the Bessel type (with an additional nonlinear term) for $f \equiv f(\rho)$:

$$\rho^2 \partial_\rho^2 f + \rho \partial_\rho f + [(\mu - U - \beta f^2)\rho^2 - (l - \rho A_\varphi)^2]f = 0. \quad (19)$$

Here the external trapping potential term is not written explicitly, but it controls the boundary conditions: $f(1) = 0$.

To allow for an analytical treatment, in the following Secs. III A 1 and III A 2 we will neglect the interaction term by setting $\beta = 0$. Our numerical calculations confirm that the conclusions obtained regarding the value of l of the ground-state wave function remain valid in the presence of interactions. We will analyze two limiting regimes for the solutions of Eq. (19) with $\beta = 0$. The regimes are defined by the distance a describing the extent of the geometric scalar potential U and the effective magnetic field $\mathbf{B}$ given by Eqs. (15) and (16).

1. Large a

First, we consider the case when the distance a is large compared to the system radius R , so that $\rho \ll a$. Then, according to Eq. (15), for $l \neq 0$ one can neglect the term $\rho A_\varphi \approx -(\rho/a)^{2v}v$ in comparison with l in Eq. (19), whereas for $l = 0$ this term may be disregarded because $A_\varphi^2 \ll \mu$. In a similar way, the geometric scalar potential U is small in comparison with μ and thus can be also neglected. In such a situation, the solutions satisfying the boundary condition are the properly scaled Bessel functions of the first kind J_l . From the sequence of zeros of J_l we find that the ground state corresponds to $l = 0$, and the degenerate pair of the lowest excited states corresponds to $l = \pm 1$. Explicitly, the chemical potential $\mu^{(l)}$ (the eigenenergy) of the ground state is given by $\mu^{(0)} = (j_{0,1})^2$, where $j_{l,n}$ is the n th root of the Bessel function $J_l(x)$. The corresponding wave function $\psi^{(l)}$ is given by $\psi^{(0)}(\rho, \varphi) = f^{(0)}(\rho) = C J_0(\rho j_{0,1})$, with C being a normalization constant.

These findings are illustrated in Figs. 2(a1)–2(a5), corresponding to $a = 10$, $v = 1$. Figures 2(a1) and 2(a2) display the curves of $U(\rho)$ and $\rho A_\varphi(\rho)$. It is apparent that the two functions are low in magnitude compared to the chemical potential and can be disregarded as noted above. Indeed, numerically solving Eq. (19) (with U and ρA_φ included) we find that the chemical potential of the ground state is $\mu^{(0)} = 5.805$, close to the limiting value $(j_{0,1})^2 \approx 5.783$. The solution $f^{(0)}(\rho)$ also matches the limiting solution $J_0(\rho j_{0,1})$, as shown in Fig. 2(a3). The blue curve additionally shows the

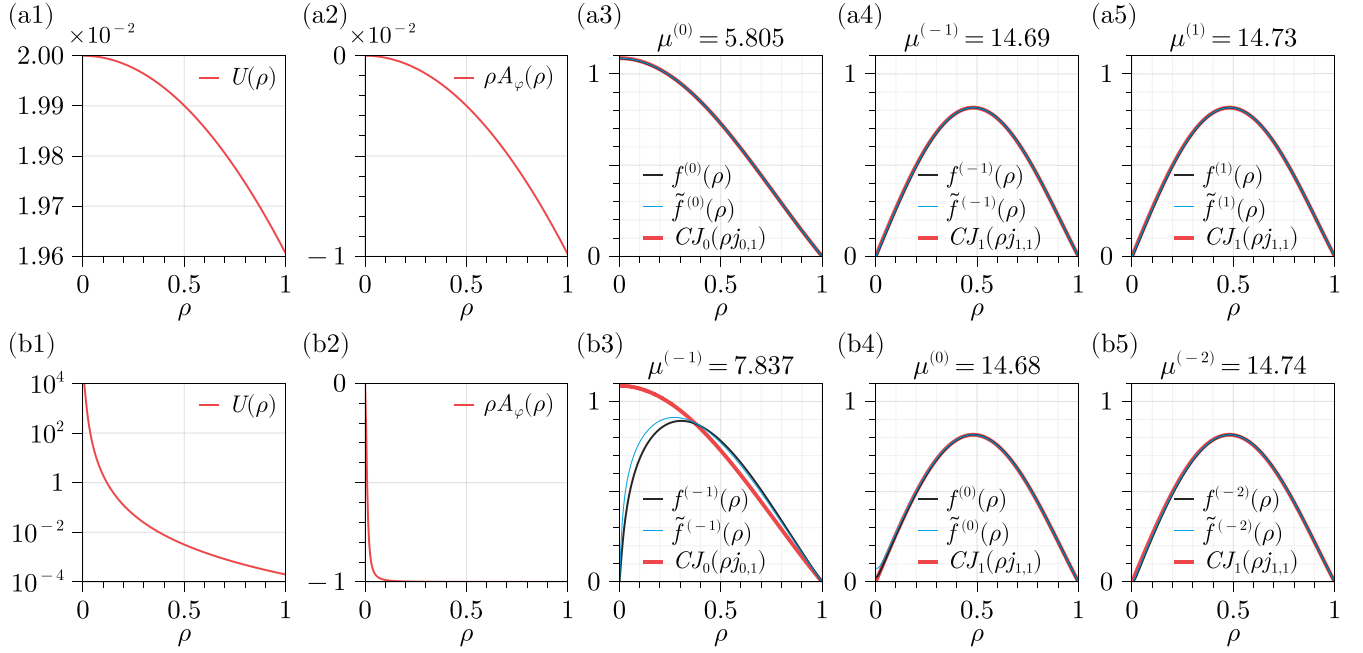


FIG. 2. The potentials U , ρA_φ and the solutions of Eq. (19). (a1)–(a5) show, respectively, the curves U , ρA_φ , and three solutions of Eq. (19) (with $\beta = 0$) for $\nu = 1$ and $a = 10$. In (a3)–(a5), curves $\tilde{f}^{(l)}(\rho)$ depict the solution obtained under an additional assumption $U = 0$. (b1)–(b5) show the same as (a1)–(a5) for $\nu = 1$ and $a = 0.01$. The depicted wave functions are normalized such that $\int_S |\psi^{(l)}(\rho, \varphi)|^2 dS = 2\pi \int_0^1 [f^{(l)}(\rho)]^2 \rho d\rho = 1$.

solution $\tilde{f}^{(0)}(\rho)$ obtained from Eq. (19) with U disregarded, confirming that the scalar potential has almost no impact on the solution. Finally, Figs. 2(a4) and 2(a5) display the excited states corresponding to $l = \pm 1$; the degeneracy is slightly lifted by the ρA_φ term.

To draw conclusions regarding the motion of atoms, we have to return to the wave functions of the two components given in Eq. (17). Substituting the ground-state solution $\Phi_D^{(0)} = e^{-i\mu t} \psi^{(0)}(\mathbf{r})$, we find that in the ground state, $\Phi_1^{(0)}$ is vortex free and is localized near the origin (with a maximum at $\rho = 0$), while $\Phi_2^{(0)}$ describes a vortex having zero density at the origin and rotating around the first component with vorticity equal to $l_2 = l + \nu = +1$. It should be noted that, according to Eq. (17), the vorticities of the components always differ by ν units. Therefore, at least one of the two components will necessarily be in a vortex state as long as the spatial part of Φ_D is of the form (18). For example, even if $l = 0$, then Φ_1 will be vortex free, while Φ_2 will have vorticity equal to ν .

2. Small a

Now let us consider the case when the distance a is small compared to the system radius, $a \ll R$. In that case, one has $\rho \gg a$ for most of the area occupied by the condensate, except for a small area close to the center, $\rho \lesssim a$ in which the geometric scalar and vector potentials are concentrated. For $\rho \gg a$, it follows from Eq. (15) that $\rho A_\varphi(\rho) \rightarrow -\nu$. The ground-state solution can be found by choosing l such that $l + \nu = 0$; the solution will again be given by J_0 . Thus, for $\rho \gg a$ the ground-state wave function is given by $\psi^{(-\nu)}(\rho, \varphi) = C J_0(\rho j_{0,1}) e^{-i\nu\varphi}$. However, close to the origin, where $\rho \lesssim a$, the radial part of the solution will have a different form. Indeed, l is now fixed to $-\nu$, and we can neglect the term ρA_φ .

The solution in the region $\rho \ll a$ is then given by J_ν , having a zero at the origin as a vortex solution should. Meanwhile, in the region $\rho \gg a$, the two lowest excited states will come in a degenerate pair having the radial dependence J_1 and corresponding to $l + \nu = \pm 1$. Close to the origin ($\rho \ll a$), they will have the radial dependencies $J_{\pm 1 - \nu}$. These results are illustrated in Figs. 2(b1)–2(b5) for the case where $a = 0.01$, $\nu = 1$. In Fig. 2(b1), it is apparent that the maximum value of U greatly exceeds that of the chemical potential. However, the presence of the potential does not appreciably influence the resulting wave functions due to the potential being localized at the origin. Figure 2(b2) displays the curve ρA_φ , which has a value close to $\nu = -1$ in most of space occupied by the condensate. In agreement with the preceding arguments, the state with the lowest chemical potential is obtained for $l = -1$ [see Fig. 2(b3)]. Away from the origin, the wave function tends to J_0 , while at the origin it attains a zero. Further reducing a increases the steepness of the wave function near the origin and enhances the match with J_0 (not shown). The degenerate pair of the lowest excited states is shown in Figs. 2(b4) and 2(b5). These wave functions mostly follow the shape of J_1 except for the small portion of space near the origin where the condition $\rho \gg a$ no longer holds. There, the state with $l = +1 - \nu = 0$ has the form of a J_0 function, while the $l = -1 - \nu = -2$ state has the shape of J_2 .

Thus, if parameter a is small enough so that the condition $\rho \gg a$ holds in most of space occupied by the condensate, the dark-state solution of the lowest energy will be the one corresponding to $l = -\nu$. In this limit the gradient of the phase ϕ of wave function equals $-\mathbf{e}_\varphi \nu / \rho$, thereby canceling the contribution of vector potential $\mathbf{A}$ to the velocity field: $\mathbf{v} \propto \nabla \phi - \mathbf{A} \rightarrow 0$. As a result, Φ_1 will be in a vortex state, while Φ_2 will be vortex free.

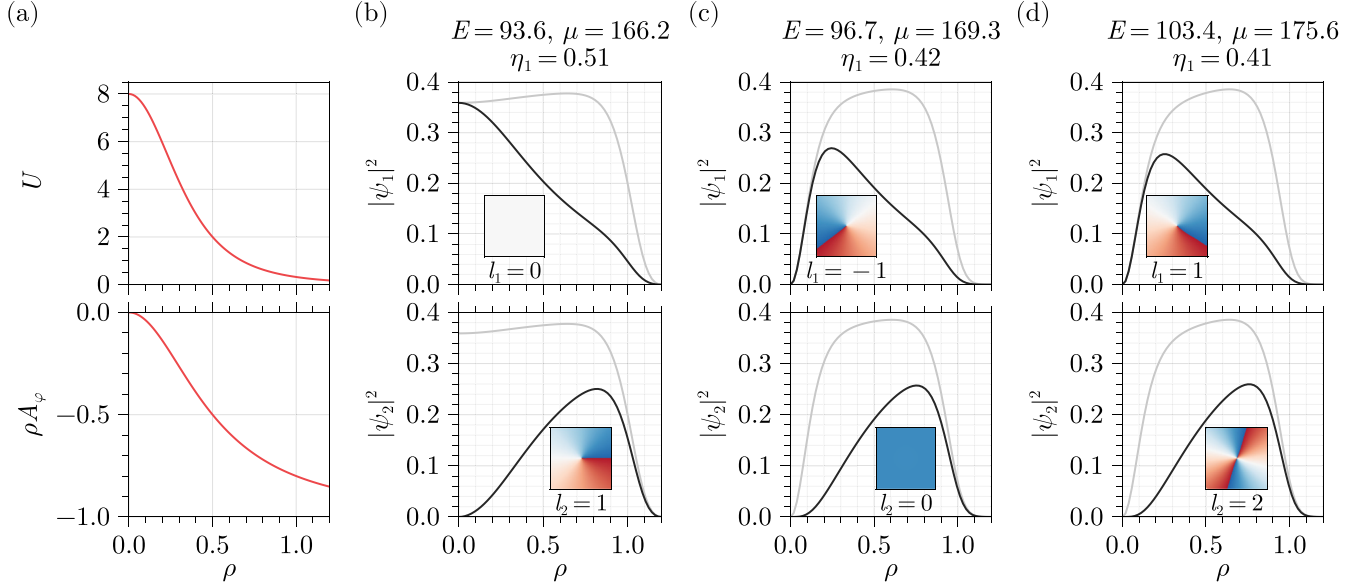


FIG. 3. The potentials U and ρA_φ , as well as the solutions of Eq. (10) for $\nu = 1$ and $a = 0.5$. (a) Potentials U and ρA_φ . (b)–(d) show, respectively, three lowest-energy solutions of Eq. (10); wave-function normalization is $\sum_{i=1}^3 \int_S |\psi_i|^2 dS = 1$. The black lines in the top and bottom panels show, respectively, the radial cuts of the densities $|\psi_1|^2$ and $|\psi_2|^2$ of the first and the second component. The gray lines show the total density. The values of the density of the third component are vanishingly small in all cases (calculation yields values no larger than 10^{-7}) and are therefore not shown. For the same reason, the relative occupation of the second component can be taken to be $\eta_2 = 1 - \eta_1$. The vorticities l_i of the two components are displayed together with the insets showing the two-dimensional phase profiles in the xy plane. The value of the phase is color-coded as follows: dark blue = $-\pi$, white = 0 , dark red = π .

B. Full treatment

We will now turn to the solutions of the full coupled GPEs (10), including the atom-atom interactions, and study the structure of the lowest-energy states of the dark-state manifold. To obtain the numerical solutions, we employed the method of imaginary-time evolution: one makes a change $t \rightarrow -i\tau$ and propagates the equations in time τ until convergence is reached, starting from a certain trial wave function. The algorithm will then converge to the lowest-energy state provided it has nonzero overlap with the trial wave function. To enable convergence to the states of the dark-state manifold, we used the solution of Eq. (13) as the trial function for solving coupled GPEs (10). In turn, the trial wave functions required to solve Eq. (13) were constructed by solving this equation using the Thomas-Fermi approximation and multiplying the solution by a phase factor $e^{il\varphi}$ with a chosen value of l . Since states of the form (18) with different l are orthogonal, the trial wave function of this form converges to a state with the same value of l . This allowed us to obtain stationary solutions of different vorticities. The ground state can then be found by ordering the states based on the value of the energy per particle E . The population of each component was calculated from the obtained solutions rather than being enforced. We refer the reader to the Appendix for additional numerical details.

In a stationary state, the wave functions of the components have the form $\Phi_i(\mathbf{r}, t) = e^{-i\mu t} \psi_i(\mathbf{r})$. Below, we show the results obtained using full three-level calculation so as to take into account the different scattering lengths. If they are taken to be equal, then the results match with the dark-state calculations based on solving Eq. (13) or Eq. (19) (the resulting total chemical potentials agree with at least three-digit accuracy).

We start by setting $\nu = 1$ and $a = 0.5$. Such a value of a corresponds to an intermediate regime whereby the term ρA_φ is not small (compared to unity) and cannot be approximated by $-\nu$, as one can see in the bottom panel of Fig. 3(a). Numerical calculations show that ground state is the one corresponding to an $l = 0$ solution of the dark-state GPE (13). As noted above, this corresponds to Φ_1 being vortex free and Φ_2 being in a vortex state. Such a solution is shown in Fig. 3(b). It is apparent that the first component fills the core of the second component, a situation known to stabilize vortices with vorticities higher than $l = 1$ [35,36]. Notably, which of the components acquires vorticity is determined by the inequivalent coupling fields Ω_1 and Ω_2 rather than the intrinsic properties of the two states (the scattering lengths). The total density of the BEC, shown in gray in Fig. 3(b), indicates that the condensate as a whole does not have a pronounced density dip at the origin, meaning that the core of the vortex is completely filled.

Two lowest excited states are shown in Figs. 3(c) and 3(d). Their energies per particle are 96.7 and 103.4 units, respectively, showing that the term ρA_φ has lifted the degeneracy. Notably, the interaction potentials $g_{ij}|\Phi_k|^2$ do not depend on the vorticities of the components and, therefore, do not lift the degeneracy. In the first excited state, the first component is in a vortex state, rotating in the direction opposite to the direction of the vector field $\mathbf{A}$, while the second component is vortex free. The vortex on the first component mostly occupies the space at the center of the second component. Such an approximate phase separation is in line with the criterion $g_{12}^2 > g_{11}g_{22}$ [37,38], which, however, does not take into account the kinetic energy [39], which is especially important in the present setting. In the second excited state, vorticities

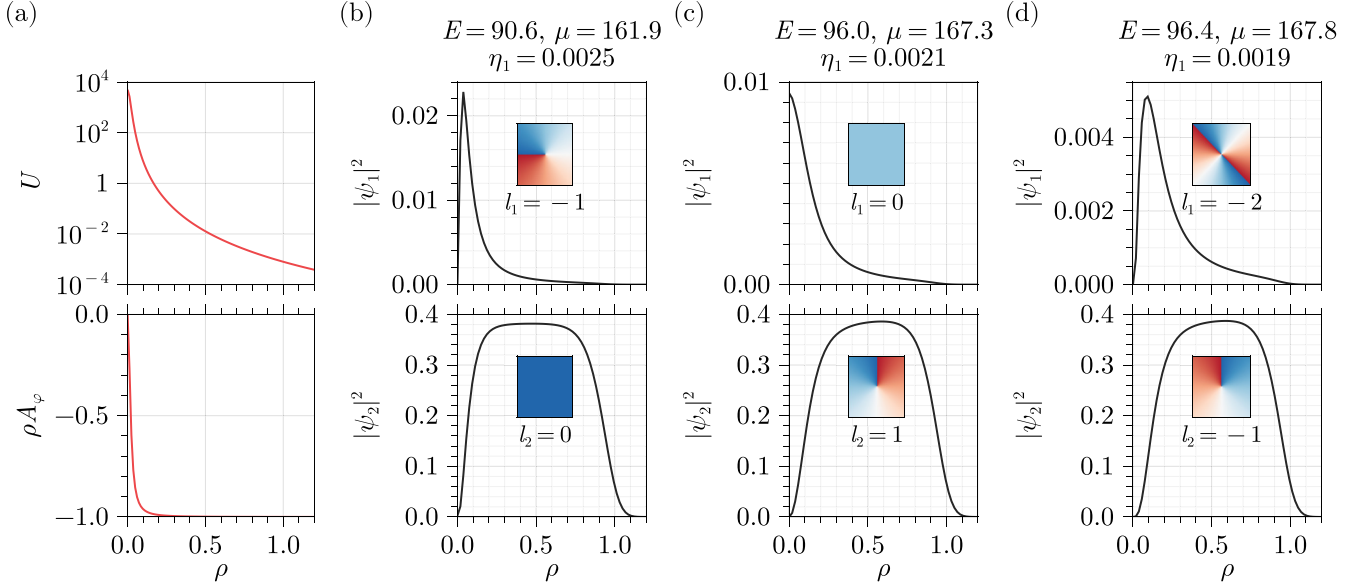


FIG. 4. Same as Fig. 3 for $\nu = 1$ and $a = 0.02$. The total densities are not explicitly shown since they would appear to largely coincide with $|\psi_2|^2$ due to the highly imbalanced occupation of the two modes.

of the two components are given by $l_1 = 1$ and $l_2 = 2$, while the spatial profiles are almost the same as those of the respective components of the first excited state. In both cases, the components contribute to a nearly uniform total density of the cloud in the bulk.

Let us now study the regime where the distance a is much smaller than the system radius R , specifically $a = 0.02$. In this case the condition $\rho \gg a$ holds in most of the space, and ρA_φ tends to $-\nu$ already in the vicinity of the origin [see Fig. 4(a), bottom panel]. The ground state now corresponds to the $l = -\nu = -1$ solution of the dark-state GPE (13). This means that Φ_1 is a vortex state, and it occupies a small region of space near the origin [see Fig. 4(b)]. Specifically, the density of the first component is concentrated in a region of the order of the healing length ξ_2 of the second component. Assuming all atoms are in the second component, we have (in dimensionless units) $\xi_2 = \sqrt{S/g_{22}} = 0.086$, where we approximately took $S = \pi$ for the occupied area. The localized nature of the vortex results from the $\frac{1}{\sqrt{1+|\epsilon|^2}}$ term in the expression for Φ_1 (17), which effectively cuts the vortex off when $\rho \gg a$. Another consequence is the small resulting population of the first component, which in the present case is $\eta_1 = 0.25\%$. Thus, decreasing the a parameter localizes the vortex more tightly, but also reduces the population of the first component. The second component occupies the space surrounding the first component and has zero vorticity.

The first excited state, on the other hand, has the typical structure whereby the nonrotating component fills the core of the rotating one [see Fig. 4(c)]. The almost-degenerate partner of this state is shown in Fig. 4(d). These two states correspond, respectively, to the $l = 0$ and $l = -2$ solutions of the dark-state GPE (13). The degeneracy results from the fact that the kinetic energy is the same, whether the cloud does not rotate ($l = 0$) or rotates in the direction opposite to the $\mathbf{A}$ field but with twice the speed ($l = -2$).

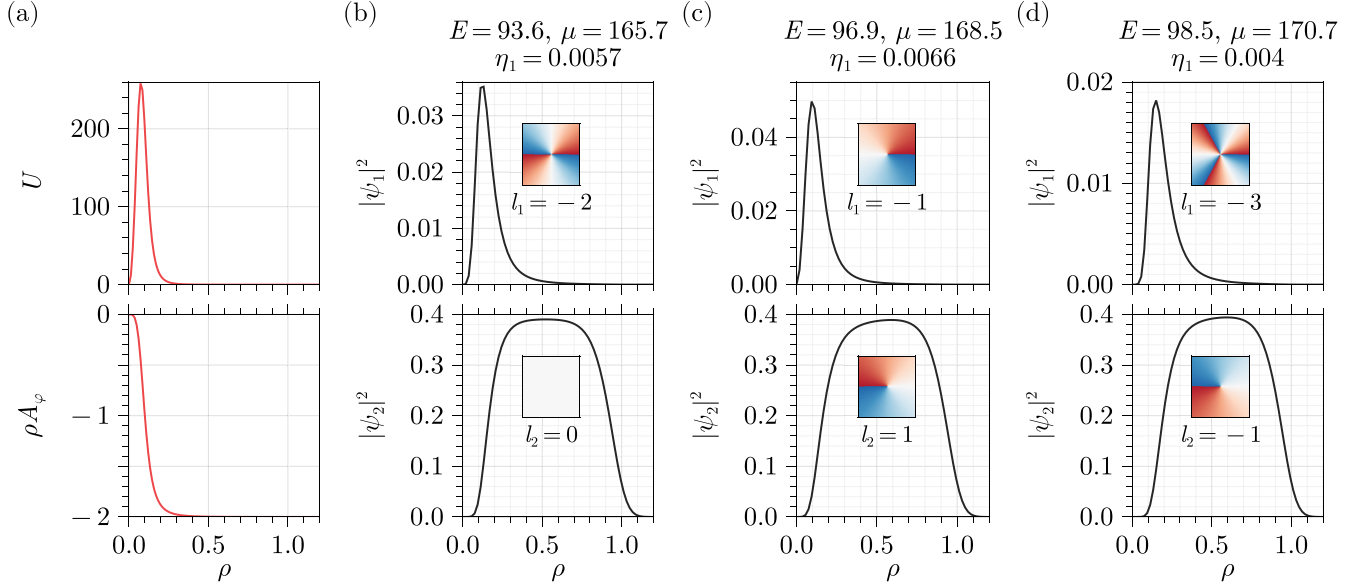
Localized vortices with vorticities higher than one can be analogously created by using beams with $\nu > 1$. The results for the case $\nu = 2$ are presented in Fig. 5. Choosing $a = 0.1$ already leads to the potential ρA_φ tending to $-\nu$ in most of space [see Fig. 5(a)]. This leads to a ground state with the first component having vorticity $l_1 = -2$ and the second being vortex free: $l_2 = l_1 + \nu = 0$. The vortex has a density maximum farther from origin compared to the $l_1 = -1$ vortex in Fig. 4(b) due, in part, to the former having the form of J_2 and thus rising less sharply compared to J_1 . The degree of localization can be increased by reducing a , but the number of atoms in the vortex component will then decrease. The lowest excited states [see Figs. 5(c) and 5(d)] also contain localized vortices in the first component.

It is worth noting that the density profiles of both components remain essentially unchanged as g_{12} is reduced into the miscible regime, $g_{12}^2 < g_{11}g_{22}$, provided the coupling constants g_{ij} are of similar magnitude. In particular, the densities shown in Figs. 4(b) and 5(b) are virtually unaffected. This invariance is expected, as the localized-vortex configuration is sustained by the external laser beams rather than interparticle interactions, as demonstrated in Sec. III A, where interactions are entirely neglected.

IV. MOVING THE VORTEX

To demonstrate the stability of the localized lowest-energy vortex states and show the high level of available control, we consider changing the position of the vortex by moving the laser beams. A similar protocol has been considered in Ref. [40], however, that work considered moving a vortex created by a phase-imprinting technique. Our approach is more robust because the laser beams create a pinned vortex, which cannot break free.

We focus on the regime $\nu = 1$, $a = 0.02$ studied in Fig. 4 but this time consider an off-axis vortex created by centering

FIG. 5. Same as Fig. 4 for $\nu = 2$ and $a = 0.1$.

the laser beams at $(x, y) = (0.5, 0)$. Once the state is prepared, the transverse profile of the laser beams Ω_1 and Ω_2 start moving counterclockwise on a circular path around the origin at a constant tangential speed v . Figure 6 shows the wave functions obtained after completing one full circle. In Fig. 6(a) corresponding to $v_1 = 150 \mu\text{m/s}$ (time to complete the circle is $t = 0.314 \text{ s}$), it is apparent that the vortex of the first component has retained its structure, and the density of the second component has not been strongly distorted. The value of E has increased by only 3% of the energy (per particle) gap between the ground state and the first excited one. Here the velocity is chosen to be lower than the speed of sound in the second

component, which equals to $v_s = \hbar/(m\xi_2\sqrt{2}) = 400 \mu\text{m/s}$. Moving the vortex at twice the speed, $v_2 = 2v_1$, results in setting the whole of the second component in circular motion. The distortion of the density is apparent in Fig. 6(b).

Finally, the case of supersonic movement is studied in Fig. 6(c) corresponding to the movement speed $v_3 = 2v_s$. The density profiles of both components are strongly distorted and the energy is increased 1.5 times. Nevertheless, the first component retains its signature phase profile, and no additional vortices are created in the second component in the process. Let us stress that in all three cases, the third component remained essentially unpopulated, and the total norm of the

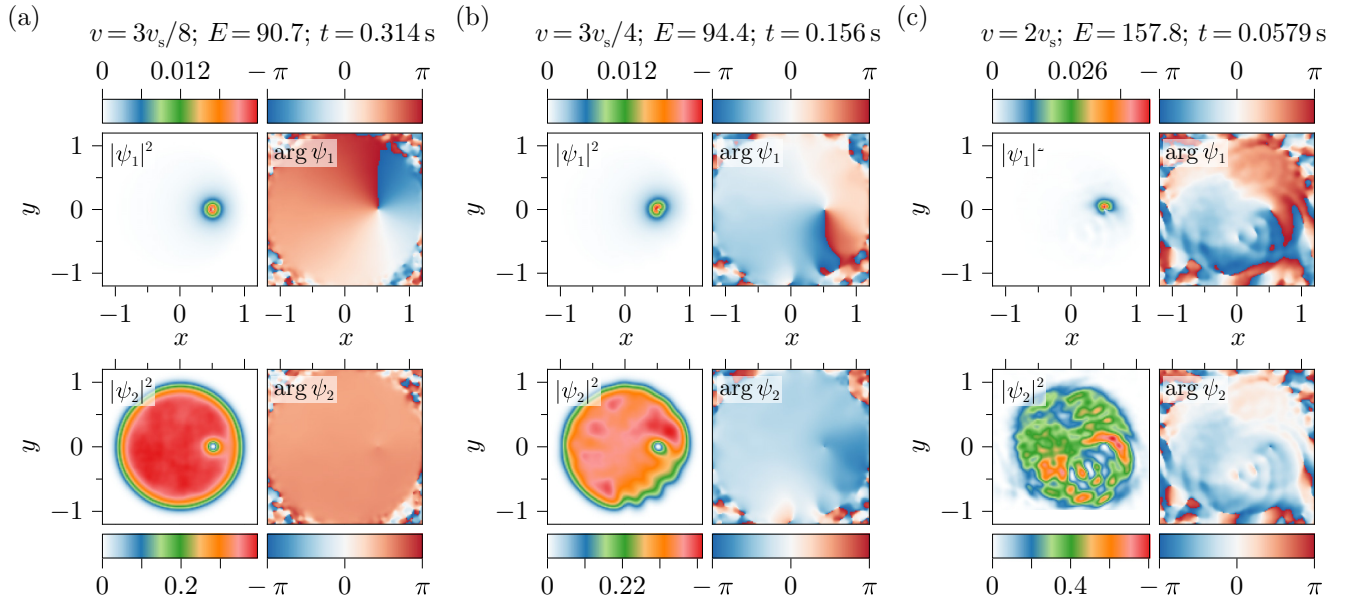


FIG. 6. Wave functions of components 1 and 2 after completing one circular sweep of the laser beams around the origin. The tangential beam movement speed is (a) $v_1 = 3v_s/8 = 150 \mu\text{m/s}$, (b) $v_2 = 3v_s/4 = 300 \mu\text{m/s}$, and (c) $v_3 = 2v_s = 800 \mu\text{m/s}$. In all three cases the relative population of the first component is $\eta_1 \approx 0.0025$, while the third component is essentially unpopulated.

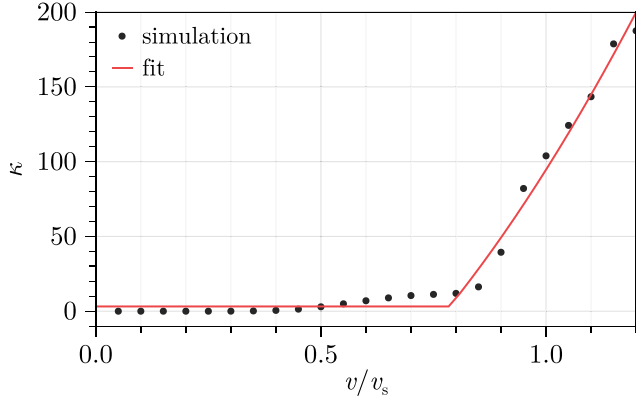


FIG. 7. Condensate heating rate as a function of the movement speed of the beams.

wave functions was conserved on the level of the numerical accuracy used, meaning that, in practice, no loss of atoms took place.

It is also instructive to use the localized vortex in the first component as a probe for the critical superfluid velocity v_c of the second component. In this case, the vortex acts as an impurity that can move without resistance provided its speed remains below v_c . According to Landau's criterion, the phononic excitation spectrum of the condensate sets the critical velocity equal to the sound velocity [41]. However, both experimental results [42–45] and detailed theoretical studies [46–48] indicate that v_c is typically lower, depending on factors such as impurity size [44], the nature of the impurity potential (attractive or repulsive) [46], and the impurity trajectory (linear or circular) [46,48]. To measure v_c in our system, we repeat the circular trajectory simulations, moving the beams for a fixed duration of 0.1 s. We monitor the total energy per particle, $E(t)$, and fit it with a linear function to obtain the heating rate $\kappa = dE/dt$. Since the first component's population is much smaller than that of the second, $E(t)$ primarily reflects the energy of the atoms in the second component. The results for velocities between $0.1v_s$ and $1.2v_s$ are shown in Fig. 7, revealing a clear threshold behavior where κ increases rapidly once the velocity exceeds a certain value.

We quantified this threshold by fitting the data to

$$\kappa(v) = \kappa_0 + b \max(v^2 - v_c^2, 0), \quad (20)$$

where κ_0 , b , and v_c are fit parameters. This form is suitable for 2D systems [49] and has proven effective in experimental analyses [44,47]. The least-squares fit shown in Fig. 7 yields $v_c = 0.8v_s$, which is lower than the sound velocity and consistent with the above observations. The precise dissipation mechanisms warrant further study and will be addressed in a future work.

V. CONCLUSIONS

In summary, we have studied the interaction of a two-component BEC mixture with the light fields in a Λ -type configuration and investigated the stationary states of the dark-state manifold. The angular momentum of $v\hbar$ per photon carried by one of the two fields leads to either one or

both components being in a vortex state, with their vorticities differing by v units. In the regime $a \ll R$, the structure of the stationary states is dictated by the vector potential term. In this limit, $\mathbf{A}$ tends to $-\frac{v}{\rho}\mathbf{e}_\varphi$ and its contribution to the velocity field (and hence to the kinetic energy) can be canceled if the dark state is described by a wave function $f(\rho)e^{-iv\varphi}$. We have demonstrated that the lowest-energy state of the dark-state manifold indeed has this form. In this case, the first component (the one interacting with the LG beam) contains a vortex of vorticity v , while the second component is vortex free. The core of the former vortex coincides with the center of the beams, while the density profile of the vortex demonstrates a strong degree of localization as the density falls off as $[1 + (\rho/a)^2]^{-1/2}$ away from the vortex core. Such a vortex can be moved around by moving the laser beams. Provided the movement speed is less than approximately half the speed of sound in the second component, the shape of the vortex retains its structure during the movement, and the density of the second component does not get distorted. More specifically, moving the vortex at a speed below the critical value of about $0.8v_s$ causes almost no heating of the condensate, indicating its superfluid nature.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX: DIMENSIONLESS UNITS AND NUMERICAL DETAILS

For our numerical calculations we used dimensionless units, measuring length in units of cylindrical trap radius R and energy in units of $E_R = \hbar^2/2mR^2$, and time in units of $\hbar/E_R$. In these units, the system of equations (10) becomes

$$\begin{aligned} i\dot{\Phi}_1 &= (-\Delta + V + g_{11}|\Phi_1|^2 + g_{12}|\Phi_2|^2)\Phi_1 + \Omega_1^*\Phi_3, \\ i\dot{\Phi}_2 &= (-\Delta + V + g_{12}|\Phi_1|^2 + g_{22}|\Phi_2|^2)\Phi_2 + \Omega_2^*\Phi_3, \\ i\dot{\Phi}_3 &= \left(-\Delta + V - \Delta - i\frac{\Gamma}{2}\right)\Phi_3 + \Omega_1\Phi_1 + \Omega_2\Phi_2. \end{aligned} \quad (\text{A1})$$

Our calculations were carried out in the dark-state regime, where the amplitudes of Φ_3 were vanishingly small. Consequently, the non-Hermitian nature of the dynamics arising

from the imaginary decay term Γ was effectively negligible, allowing us to adopt the conventional normalization used for Hermitian systems, $\sum_{i=1}^3 \int d\mathbf{r} |\Phi_i|^2 = 1$ (the sum can equally be taken over Φ_1 and Φ_2 only). Furthermore, the energies and chemical potentials of the stationary states had vanishingly small imaginary parts, implying that the wave-function norm remained essentially constant over time. We note that in our numerical calculations, the wave functions were normalized to unity rather than to the total particle number.

We take $R = 15.0 \mu\text{m}$ and the mass of an ^{87}Rb atom $m = 1.443 \times 10^{-25} \text{ kg}$. The amplitudes of the Rabi frequencies Ω_1 and Ω_2 are expressed in terms of Ω_0 [see Eq. (1)] whose value is $\Omega_0 = \frac{20.0 \text{ MHz}}{E_R/\hbar} = 1.23 \times 10^7$; the decay rate is $\Gamma = \frac{38.1 \text{ MHz}}{E_R/\hbar} = 2.35 \times 10^7$, and the detuning is $\Delta = 10\Omega_0/a^v$. The interaction strengths are characterized by $g_{ij} = 8\pi a_{ij}N/d$, where division by the cylindrical trap height d appears as a result of reduction to the 2D equations (see main text). We take $d = R$, $N = 5 \times 10^4$, $a_{11} = 100a_0$, $a_{12} = 98.0a_0$, and $a_{22} = 95.4a_0$, where a_0 is the Bohr radius. This results in $g_{11} = 443$, $g_{12} = 434$, $g_{22} = 423$.

The radial profile of the cylindrical trap was modeled by a logistic function [35]

$$V(\rho) = \frac{V_0}{1 + V_0 e^{-b(\rho - \rho_0)}} \quad (\text{A2})$$

with $V_0 = 1000$, $b = 17$, $\rho_0 = 0.7$, ensuring a steep rise at $\rho \approx 1$. The computational grid contained 129 points in each dimension, spanning the interval $\rho \in [-1.2, 1.2]$.

The dark-state GPE in dimensionless units becomes

$$i\dot{\Phi}_D = (-i\nabla - \mathbf{A})^2 \Phi_D + (U + V)\Phi_D + \beta|\Phi_D|^2 \Phi_D \quad (\text{A3})$$

while the components of the artificial gauge field are given by

$$\mathbf{A} = -\frac{v}{a} \frac{(\frac{\rho}{a})^{2v-1}}{1 + (\frac{\rho}{a})^{2v}} \mathbf{e}_\varphi, \quad U = 2\left(\frac{v}{a}\right)^2 \frac{(\frac{\rho}{a})^{2v-2}}{[1 + (\frac{\rho}{a})^{2v}]^2}. \quad (\text{A4})$$

We remind that the dark-state GPE is obtained under the assumption of equal inter- and intracomponent scattering lengths; in our calculations we used $\beta = g_{11} = 443$.

The solutions Φ_D of the dark-state GPE (A3) were obtained using the imaginary-time evolution algorithm. The trial wave functions were constructed by solving the equation using the Thomas-Fermi approximation and multiplying the solution by a phase factor $e^{il\varphi}$ with a chosen value of l . Since states (18) with different l 's are orthogonal, the trial wave function of this form converges to a state with the same value of l . To solve the GPE system (A1), we used the following trial functions: Φ_3 was simply set to zero, while Φ_1 and Φ_2 were obtained from the corresponding dark-state solution Φ_D using Eq. (17).

The chemical potential was calculated from the stationary solution of the system (A1) using

$$\mu = \sum_{i,j} \int \Phi_i^* (D_{ij} + F_{ij}) \Phi_j d\mathbf{r}, \quad (\text{A5})$$

where

$$\hat{D} = \begin{pmatrix} -\Delta + V & 0 & \Omega_1^* \\ 0 & -\Delta + V & \Omega_2^* \\ \Omega_1 & \Omega_2 & -\Delta + V - \Delta - i\frac{\Gamma}{2} \end{pmatrix} \quad (\text{A6})$$

and

$$\hat{F} = \begin{pmatrix} g_{11}|\Phi_1|^2 + g_{12}|\Phi_2|^2 & 0 & 0 \\ 0 & g_{12}|\Phi_1|^2 + g_{22}|\Phi_2|^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (\text{A7})$$

Meanwhile, the energy per particle E is given by Eq. (A5) with $F_{ij} \rightarrow \frac{1}{2}F_{ij}$.

Calculations have been performed using the GPELAB software package [50,51]. The figures in the text have been produced using the Makie.jl software package [52].

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