

Numerical evaluation of Casimir forces using the discontinuous Galerkin time-domain method

Carles Martí Farràs^{1,*}, Bettina Beverungen¹, Philip Trøst Kristensen², Francesco Intravaia¹, and Kurt Busch^{1,3}

¹*Institut für Physik, Humboldt-Universität zu Berlin, Newtonstraße 15, 12489 Berlin, Germany*

²*DTU Electro, Technical University of Denmark, Ørstedes Plads 343, 2800 Kgs. Lyngby, Denmark*

³*Max-Born-Institut, Max-Born-Straße 2a, 12489 Berlin, Germany*



(Received 4 March 2026; revised 10 August 2026; accepted 14 August 2026; published 22 October 2026)

Casimir forces dominate the interaction between electrically neutral macroscopic objects at nanoscale separations, making their quantitative description highly relevant to modern technology and fundamental physics. In this work, we present a time-domain scheme for computing Casimir forces between vacuum-separated bodies based on the Maxwell stress tensor formalism. In particular, we provide a concrete implementation using the finite-element-based discontinuous Galerkin time-domain method. The approach enables accurate evaluation of finite-temperature Casimir–Lifshitz interactions in arbitrary geometries and for general material models. At the core of the method, the electromagnetic Green tensor is expressed as the system’s response to dipolar excitations, thereby recasting the Maxwell stress tensor into a set of classical scattering problems driven by electric and magnetic dipoles. We validate the approach against reference calculations of the Casimir interaction between parallel half-spaces at both zero and finite temperature. We further demonstrate its applicability to the interaction between a finite cylindrical object and a half-space, for which semi-analytical expressions are not available, showing excellent agreement with both asymptotic predictions based on physical considerations and alternative numerical approaches. These findings illustrate the method’s potential for studying Casimir interactions in micro- and nanoscale structures of relevance to nanodevice design and experimental settings.

DOI: [10.1103/pllp-htg3](https://doi.org/10.1103/pllp-htg3)

I. INTRODUCTION

Fluctuation-induced phenomena, arising from quantum and thermal fluctuations of the electromagnetic field, lead to fascinating effects that extend beyond the domain of classical electrodynamics. A prominent example is the Casimir effect, a typically attractive force between electrically neutral macroscopic objects. First predicted by H. B. G. Casimir for perfectly reflecting mirrors in vacuum and attributed to zero-point fluctuations of the quantum electromagnetic field [1], it was later extended by Lifshitz to realistic materials, incorporating properties such as dispersion and dissipation at finite temperature [2]. The Casimir force can be regarded as a macroscopic manifestation of the entire system’s quantum fluctuations, including those within the materials. In this sense, it encompasses van der Waals interactions, originally introduced by van der Waals and later described by London as an attractive force between closely spaced particles with fluctuating dipole moments [3]. Casimir and Polder extended London’s theory to include retardation effects, describing the interaction between an atom and a macroscopic object [4],

with the resulting fluctuation-induced phenomena commonly referred to as the Casimir–Polder interaction.

While negligible at macroscopic scales, Casimir forces become significant at submicrometer distances, rapidly increasing as the separation decreases. Beyond its theoretical relevance, the Casimir effect has been consistently confirmed through experimental observations [6]. Modern measurements rely on precision instruments such as torsion pendulums [7], atomic force microscopes [8–10], and microelectromechanical torsional oscillators [11–15]. Continuing advances in experimental sensitivity have brought the detection of finite-temperature corrections within reach [16–20], yet some related controversies remain unresolved [13,21,22]. Although successful plane-plane measurements have been reported [23], experimental setups frequently employ curved surfaces in close proximity to a substrate in order to minimize alignment errors. In such configurations, the interaction is commonly estimated using the proximity-force approximation (PFA) [24], which describes the interacting surfaces as locally parallel and integrates the corresponding infinitesimal parallel-plate contributions. From a technological standpoint, the ongoing miniaturization of nanostructured devices has made the Casimir effect increasingly relevant in the design of nano- and micro-electromechanical systems, sparking interest within both the experimental and theoretical research communities [25,26]. One consequence is that, due to its typically attractive nature, the Casimir effect can lead to stiction—undesired adhesion of movable components—thereby disrupting device performance [11,27,28].

*Contact author: marticar@physik.hu-berlin.de

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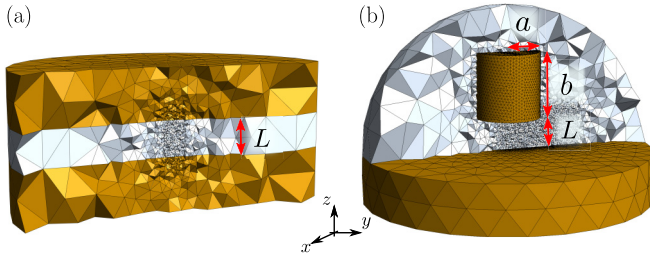


FIG. 1. Representative tetrahedral meshes generated with GMSH [5] for (a) plane-plane and (b) cylinder-plane geometries. L is the minimum surface-to-surface separation, while a and b denote the cylinder radius and height, respectively.

In this context, theoretical descriptions of Casimir interactions are essential for understanding geometry- and material-dependent effects, designing nanostructured devices, and optimizing experimental setups. Over time, semi-analytical approaches based on scattering methods have been developed to describe practical systems, including complex geometries, realistic material models, and finite-temperature corrections [29–35]. However, these methods remain strongly geometry-dependent. As an alternative, full numerical techniques provide a more flexible framework for treating arbitrary three-dimensional configurations without increasing the mathematical complexity. Common approaches include boundary-element methods (BEMs) [36–39], as well as finite-difference techniques in both the frequency domain (FDFD) [40] and the time domain (FDTD) [41,42]. These approaches incorporate temperature effects [43], a broad range of material models, and arbitrary geometries, revealing interesting phenomena such as nonmonotonic force behavior [44,45], geometry-induced repulsion for cylindrical objects above a perforated plate [39,46,47], or material-induced suspension in fluids [48,49]. In this context, achieving numerically accurate Casimir-force calculations for fully three-dimensional systems with arbitrary geometries and material properties, while consistently incorporating finite-temperature effects, remains technically challenging due to the presence of multiple relevant length scales.

In this work, we extend the numerical framework introduced by Kristensen *et al.* in Ref. [50] for high-accuracy Casimir-Polder calculations to include both Casimir interactions and finite-temperature corrections within the Maxwell stress tensor formalism. Our approach employs the discontinuous Galerkin time-domain (DGTD) method [51], a finite-element-based technique for numerically solving Maxwell’s equations in the time domain. As representative examples, we consider the geometries shown in Fig. 1. First, we examine the Casimir interaction between vacuum-separated metallic half-spaces and compare our results with established semi-analytical solutions based on Lifshitz theory [2]. We further consider the interaction between a finite cylinder and a half-space separated by vacuum. Although this configuration has previously been addressed semi-analytically for a perfectly conducting, infinitely thin disk with a radius smaller than the separation distance [52], the finite cylinder lacks the symmetry required for an efficient semi-analytical treatment. However, BEM approaches [38] and rapidly

converging multiple-scattering expansion methods [53,54] have recently been reported for Casimir-force calculations in arbitrary geometries involving dispersive and dissipative materials. Consequently, we validate our results by confirming the expected asymptotic behavior at separations both large and small compared with the cylinder radius, as well as through comparison with alternative established numerical methods [55].

The present framework is based on an earlier approach by Rodríguez *et al.* and McCauley *et al.* in Refs. [41,42], which employed the FDTD method to compute the fields excited by a harmonic expansion of extended sources in an artificial dissipative system. This approach reformulates the dielectric permittivity to effectively deform the complex-frequency integration contour, thereby reducing the required computational time. Instead, our approach retains the original electromagnetic problem and employs tailored broadband dipole sources positioned at the nodes of a Gaussian quadrature scheme. This ensures accurate and efficient evaluation of the temporal and spatial integrals in a postprocessing step. Compared with standard FDTD methods, which rely on regular staggered grids in both space and time [56], the DGTD method enables adaptive meshing and achieves higher-order convergence through local polynomial basis functions and efficient time-stepping schemes [51].

This paper is organized as follows: In Sec. II, we outline the theoretical framework for evaluating Casimir interactions within the Maxwell stress tensor formalism. Section III describes the numerical implementation and its relation to classical scattering problems in the time domain. In Sec. IV, we present two case studies: (i) the Casimir force between parallel half-spaces, illustrating the temporal integration scheme; and (ii) the interaction between a finite metallic cylindrical object and a half-space, introducing the surface integration scheme necessary for the treatment of systems lacking translational symmetry. Finally, Sec. V summarizes the results and outlines potential future directions.

II. MAXWELL STRESS TENSOR APPROACH

We compute fluctuation-induced forces in systems at thermal equilibrium using the Maxwell stress tensor formalism. Specifically, we consider a collection of arbitrarily shaped, nonoverlapping bodies embedded in vacuum at a temperature T . The net force on any given object, resulting from quantum and thermal fluctuations of the electromagnetic field, is given by the surface integral of the expectation value of the Maxwell stress tensor operator evaluated in the system’s thermal state

$$\mathbf{F} = \oint_S dS \langle \hat{\mathbf{T}}(\mathbf{r}, t) \rangle_T \cdot \mathbf{n}(\mathbf{r}). \quad (1)$$

Here, S denotes any closed surface surrounding the body on which the force is evaluated, and $\mathbf{n}(\mathbf{r})$ is the outward-pointing unit vector normal to the surface at position $\mathbf{r}$. In empty space, the expectation value of the stress tensor is expressed in terms of the quantum correlation functions of the fluctuating macroscopic electric- and magnetic-field operators $\hat{\mathbf{E}}(\mathbf{r}, t)$ and $\hat{\mathbf{H}}(\mathbf{r}, t)$ evaluated at the same position $\mathbf{r}$ and time t ,

as follows [57]:

$$\begin{aligned} \langle \hat{T}_{ij}(\mathbf{r}, t) \rangle_T &= \epsilon_0 \left[\langle \hat{E}_i(\mathbf{r}, t) \hat{E}_j(\mathbf{r}, t) \rangle_T - \frac{\delta_{ij}}{2} \langle \hat{\mathbf{E}}^2(\mathbf{r}, t) \rangle_T \right] \\ &+ \mu_0 \left[\langle \hat{H}_i(\mathbf{r}, t) \hat{H}_j(\mathbf{r}, t) \rangle_T - \frac{\delta_{ij}}{2} \langle \hat{\mathbf{H}}^2(\mathbf{r}, t) \rangle_T \right], \end{aligned} \quad (2)$$

where δ_{ij} is the Kronecker delta with $i, j \in \{x, y, z\}$, while ϵ_0 and μ_0 are the vacuum permittivity and permeability, respectively. In thermal equilibrium, the field correlation functions can be evaluated using the fluctuation-dissipation theorem (FDT) as [58,59]

$$\begin{aligned} \langle \hat{E}_i(\mathbf{r}, t) \hat{E}_j(\mathbf{r}', t) \rangle_T &= \frac{\hbar \mu_0}{\pi} \int_0^\infty d\omega \omega^2 \coth \left[\frac{\hbar \omega}{2k_B T} \right] \\ &\times \text{Im} \{ G_{ij}^E(\mathbf{r}, \mathbf{r}'; \omega) \}, \quad (3a) \\ \langle \hat{H}_i(\mathbf{r}, t) \hat{H}_j(\mathbf{r}', t) \rangle_T &= \frac{\hbar \epsilon_0}{\pi} \int_0^\infty d\omega \omega^2 \coth \left[\frac{\hbar \omega}{2k_B T} \right] \\ &\times \text{Im} \{ G_{ij}^H(\mathbf{r}, \mathbf{r}'; \omega) \}. \quad (3b) \end{aligned}$$

Although the field operators fluctuate in time, the stationarity of the thermal state implies that their equal-time correlations are time-independent; therefore, the right-hand side contains no explicit time dependence. Accordingly, the expectation value of the stress tensor operator and the Casimir force in Eq. (1) are time-independent, and, for simplicity, their explicit time dependence is omitted throughout the remainder of the paper. These expressions relate the quantum and thermal field fluctuations to the imaginary parts of the electromagnetic Green tensors $G_{ij}^\sigma(\mathbf{r}, \mathbf{r}'; \omega)$ at frequency ω , where $\sigma \in \{E, H\}$ distinguishes between the electric and magnetic contributions.

The Green tensors encode the geometry and material properties of the system and depend on both the observation point $\mathbf{r}$, where the fields are evaluated, and the source position $\mathbf{r}'$, which gives rise to scattering in the system. For linear and local objects embedded in vacuum, they satisfy the following inhomogeneous vector Helmholtz equations [57]:

$$\left[\nabla \times \frac{1}{\mu(\mathbf{r}, \omega)} \nabla \times - \frac{\omega^2}{c^2} \epsilon(\mathbf{r}, \omega) \right] \underline{G}^E(\mathbf{r}, \mathbf{r}'; \omega) = \delta(\mathbf{r} - \mathbf{r}') \underline{\mathbb{1}}, \quad (4a)$$

$$\left[\nabla \times \frac{1}{\epsilon(\mathbf{r}, \omega)} \nabla \times - \frac{\omega^2}{c^2} \mu(\mathbf{r}, \omega) \right] \underline{G}^H(\mathbf{r}, \mathbf{r}'; \omega) = \delta(\mathbf{r} - \mathbf{r}') \underline{\mathbb{1}}, \quad (4b)$$

subject to outgoing radiation boundary conditions [60]. Here, $\epsilon(\mathbf{r}, \omega)$ and $\mu(\mathbf{r}, \omega)$ are the piecewise-homogeneous permittivity and permeability functions, and $\underline{\mathbb{1}}$ represents the unit dyadic. The total Green tensor decomposes as $\underline{G}^\sigma(\mathbf{r}, \mathbf{r}'; \omega) = \underline{G}_0^\sigma(|\mathbf{r} - \mathbf{r}'|; \omega) + \underline{G}_{SC}^\sigma(\mathbf{r}, \mathbf{r}'; \omega)$, where $\underline{G}_0^\sigma$ is the vacuum contribution, which solves Eqs. (4) in the isotropic and homogeneous vacuum. Although $\underline{G}_0^\sigma$ diverges in the coincidence limit $\mathbf{r}' \rightarrow \mathbf{r}$, its contribution to Eq. (1) vanishes as it becomes independent of $\mathbf{r}$ and integrates to zero over any closed surface. On the other hand, the scattered Green tensor $\underline{G}_{SC}^\sigma$ represents the response to a point-like source located at $\mathbf{r}'$ in the presence of material inhomogeneities, such as geometric

boundaries and interface reflections. Since $\underline{G}_0^\sigma$ does not contribute to the net Casimir force, we focus only on $\underline{G}_{SC}^\sigma$ and, for simplicity, omit the subscript SC in the subsequent sections.

III. NUMERICAL IMPLEMENTATION

The FDT in Eqs. (3) connects the expectation value of the Maxwell stress tensor in Eq. (2) to the imaginary parts of the Green tensors. This relation recasts a quantum electrodynamic effect into a classical electrodynamics problem, governed by Eqs. (4). However, as mentioned above, semi-analytical solutions for these equations exist only for a limited set of highly symmetric geometries. The challenge, therefore, is to develop efficient numerical methods that can evaluate the scattered Green tensor accurately for arbitrary geometries.

A practical strategy involves interpreting each column of the dyadic Green tensor as the field response of the system to a point-like dipole source. Specifically, the ij component of the electric Green tensor, $G_{ij}^E(\mathbf{r}, \mathbf{r}'; \omega)$, is proportional to the i th component of the electric field at position $\mathbf{r}$ produced by an infinitesimal oscillating electric dipole aligned along the j axis and located at $\mathbf{r}'$ [61]. The corresponding source current density is expressed by $\mathbf{j}^E(\mathbf{r}, \omega) = j^E(\omega) \delta(\mathbf{r} - \mathbf{r}') \mathbf{e}_j$, where $j^E(\omega)$ is the frequency-domain amplitude. Analogously, the magnetic Green tensor, $G_{ij}^H(\mathbf{r}, \mathbf{r}'; \omega)$, is obtained by introducing a magnetic dipole. At coincidence, the Green tensor components follow from the fields as

$$G_{ij}^E(\mathbf{r}, \mathbf{r}; \omega) = \frac{E_i^{(j)}(\mathbf{r}, \omega)}{i \omega \mu_0 j^E(\omega)}, \quad G_{ij}^H(\mathbf{r}, \mathbf{r}; \omega) = \frac{H_i^{(j)}(\mathbf{r}, \omega)}{i \omega \epsilon_0 j^H(\omega)}, \quad (5)$$

where $E_i^{(j)}$ and $H_i^{(j)}$ are the i th components of the scattered electric and magnetic fields induced by dipoles oriented along the j axis [50].

By substituting Eqs. (3), together with Eqs. (5), into Eq. (2), the expectation value of the Maxwell stress tensor appearing in Eq. (1) can be expressed in terms of the scattered electromagnetic fields as

$$\langle \hat{T}_{ij}(\mathbf{r}) \rangle_T = \frac{\hbar}{\pi} \sum_\sigma \text{Im} \left\{ \int_{-\infty}^\infty d\omega g_T^\sigma(\omega) \Delta_{ij}^\sigma(\mathbf{r}, \omega) \right\}, \quad (6)$$

where the geometry-dependent scattered-field contribution is defined as

$$\Delta_{ij}^\sigma(\mathbf{r}, \omega) = \Lambda_{ij}^\sigma(\mathbf{r}, \omega) - \frac{\delta_{ij}}{2} \sum_k \Lambda_{kk}^\sigma(\mathbf{r}, \omega). \quad (7)$$

Here, the sum runs over the spatial components $k \in \{x, y, z\}$. The geometry-dependent scattered-field quantities are given by

$$\Lambda_{ij}^E(\mathbf{r}, \omega) = \epsilon_0 E_i^{(j)}(\mathbf{r}, \omega) \quad \text{and} \quad \Lambda_{ij}^H(\mathbf{r}, \omega) = \mu_0 H_i^{(j)}(\mathbf{r}, \omega). \quad (8)$$

In Eq. (6), we introduced the temperature-dependent function

$$g_T^\sigma(\omega) = -i \frac{\omega}{j^\sigma(\omega)} \coth \left[\frac{\hbar \omega}{2k_B T} \right] \Theta(\omega), \quad (9)$$

where $\Theta(\omega)$ is the Heaviside step function. In the limit $\omega \rightarrow \infty$, g_T^σ diverges as ω^{l+2} , where l is the order of the first non-vanishing derivative at time $\tau = 0$ of the time-domain current

response $j^\sigma(\tau)$, whose Fourier transform is $j^\sigma(\omega)$ [50]. Despite its divergent behavior, the imaginary-part operator “Im” in Eq. (6) commutes with the integration, leading to a finite result. This can be justified by interpreting g_T^σ as a tempered distribution, since it appears only under integration against the smooth, decaying field functions Δ_{ij}^σ .

A. From frequency to time domain

Direct numerical evaluation of the Maxwell stress tensor through frequency-domain integration along the real axis is hindered by highly oscillatory integrands [62]. These oscillations degrade both the accuracy and efficiency of the broadband integral in Eq. (6), which accounts for contributions from all fluctuating frequency modes. A standard alternative is contour deformation via Wick rotation, which involves evaluating the Green tensor along the imaginary frequency axis. This transformation smooths the integrand, turning it into an exponentially decaying function [63]. While frequency-domain methods were among the first numerical approaches for Casimir-type calculations [40,64,65], solving Maxwell’s equations at many discrete frequencies renders this method computationally expensive. As an alternative, time-domain methods extract the full spectral response from a single pulsed excitation [41,42]. This efficiency motivates our choice to evaluate Casimir interactions by solving Maxwell’s equations in the time domain.

Following Refs. [41,42], the expectation value of the stress tensor in Eq. (6) can be expressed as a temporal convolution,

$$\langle \hat{T}_{ij}(\mathbf{r}) \rangle_T = 2\hbar \sum_{\sigma} \int_0^{\infty} d\tau \operatorname{Im} \{ g_T^\sigma(-\tau) \} \Delta_{ij}^\sigma(\mathbf{r}, \tau), \quad (10)$$

where τ denotes the time elapsed since the dipolar excitation, and the lower integration limit follows from causality, which requires the system response to vanish for $\tau < 0$, with $\tau = 0$ marking the excitation onset. Within the proposed framework, the evaluation of the averaged Maxwell stress tensor reduces to calculating the temporal kernel $\operatorname{Im}\{g_T^\sigma(-\tau)\}$ and computing the scattered electromagnetic response $\Delta_{ij}^\sigma(\mathbf{r}, \tau)$. The former admits an analytic treatment, while the latter is obtained by numerically solving the time-dependent Maxwell’s equations using our own implementation of the DGTD method, cf. Fig. 2. It is worth noting that the numerical scheme is general and can be applied to other numerical electromagnetic field solvers in the time domain [66]. Moreover, the scattered response Δ_{ij}^σ is temperature-independent, allowing different temperatures to be explored in a postprocessing step by modifying only the kernel, without additional computational cost.

1. Source current density and temporal kernel

The behavior of both the kernel and the scattered response is tied to the temporal profile of the source current density. This is formally arbitrary within the above-described approach, provided it is causal, continuous, and produces divergence-free fields. Causality and physical realizability require the current density to be real and vanish for $\tau < 0$. To ensure consistency with Maxwell’s curl equations and a divergence-free evolution, the current density and its first two derivatives must vanish at $\tau = 0$ [50,56].

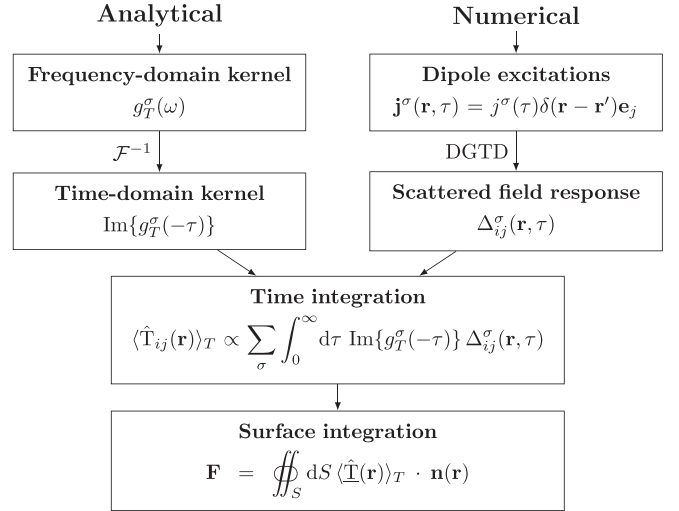


FIG. 2. Schematic overview of the computational workflow. The analytically calculated time-domain kernel $\operatorname{Im}\{g_T^\sigma(-\tau)\}$ and the numerically computed scattered response $\Delta_{ij}^\sigma(\mathbf{r}, \tau)$ are combined in a temporal convolution and integrated over a closed surface to yield the Casimir force.

In practice, a specific form for the current profile is chosen to accelerate the convergence of the temporal integral in Eq. (10). Building on and extending the approach in Ref. [50], we have found an exponentially damped polynomial current of the form

$$j^\sigma(\tau) = j_0^\sigma (\gamma \tau)^{s-1} [s - \chi^\sigma \gamma \tau] e^{-\gamma \tau} \Theta(\tau), \quad (11)$$

where the integer $s \geq 4$ ensures that both the current and its first two time derivatives vanish at $\tau = 0$. The amplitude is set by j_0^σ , and causality is enforced by the Heaviside step function $\Theta(\tau)$. The parameter $\gamma > 0$ controls the decay rate and thereby the spectral bandwidth of the excitation, while the discrete parameter

$$\chi^\sigma = \begin{cases} 1 & \text{if } \sigma = E \\ 0 & \text{if } \sigma = H, \end{cases} \quad (12)$$

distinguishes electric ($\sigma = E$) from magnetic ($\sigma = H$) dipole excitations, modifying the pulse shape. In practice, the choice of $s = 4$ and $\gamma \approx c/2L$ in Eq. (11), where L is the minimum surface-to-surface separation between bodies, cf. Fig. 1, ensures proper temporal convergence for both electric and magnetic dipole excitations at zero and finite temperature, cf. Appendix B. In contrast with Ref. [50], where only the electric field is relevant, the slow decay of the scattered magnetic field, due to the existence of long-lived nonoscillatory modes [67–69], affects the convergence of the time-domain integration since it contributes nonnegligibly at long times.

The kernel $g_T^\sigma(\omega)$, as defined in Eq. (9), depends on the Fourier-transformed current density, which reads

$$j^\sigma(\omega) = j_0^\sigma s! \gamma^{s-1} \frac{[(1 - \chi^\sigma) \gamma - i\omega]}{(\gamma - i\omega)^{s+1}}. \quad (13)$$

The full time dependence of $\operatorname{Im}\{g_T^\sigma(-\tau)\}$ is obtained by Fourier transforming the kernel in Eq. (9) by deforming the integration contour into the complex-frequency plane, with $\omega \rightarrow \zeta = \omega + i\xi$ [50]. Due to causality, $j^\sigma(\zeta)$ is analytic

in the upper half-plane and satisfies the Schwarz reflection principle $[j^\sigma(\zeta)]^* = j^\sigma(-\zeta^*)$, as required for the Fourier transform of any real-valued time-domain function. Therefore, the only term contributing with single poles to Eq. (9) when closing the contour in the upper half-plane is the thermal factor $\coth[\hbar\omega/2k_B T]$. The corresponding poles are located at the Matsubara frequencies $\zeta_m = i\xi_m = i m \omega_{\text{th}}$, with $\omega_{\text{th}} =$

$2\pi k_B T / \hbar$. Applying the residue theorem yields

$$\text{Im} \{g_T^\sigma(-\tau)\} = \frac{\omega_{\text{th}}}{2\pi} \sum_{m=0}^{\infty} \frac{\xi_m}{j^\sigma(i\xi_m)} e^{-\xi_m \tau}, \quad (14)$$

where the prime indicates that the $m = 0$ term is weighted by a factor $1/2$. For the current density in Eq. (13), the Matsubara sum in Eq. (14) admits a closed-form representation as

$$\text{Im} \{g_T^\sigma(-\tau)\} = \tilde{g}_0^\sigma \begin{cases} \frac{\omega_{\text{th}}}{2\gamma} + \sum_{l=0}^{s+1} \binom{s+1}{l} \left(\frac{\omega_{\text{th}}}{\gamma}\right)^{l+1} \text{Li}_{-l}(e^{-\omega_{\text{th}} \tau}), & \sigma = E, \\ \sum_{l=0}^s \binom{s}{l} \left(\frac{\omega_{\text{th}}}{\gamma}\right)^{l+2} \text{Li}_{-(l+1)}(e^{-\omega_{\text{th}} \tau}), & \sigma = H, \end{cases} \quad (15)$$

where $\tilde{g}_0^\sigma = \gamma^3 / 2\pi j_0^\sigma s!$ and $\text{Li}_l(z)$ is the polylogarithm of order l [70].

2. The DGTD method

To solve Maxwell's equations numerically for arbitrary scattering configurations, we employ the nodal DGTD method in the form introduced by Hesthaven and Warburton [71]. This finite-element-based approach is designed for time-domain integration of partial differential equations in flux-conservative form [72]. In this method, the computational domain is discretized using an unstructured mesh composed of finite elements that fully cover the simulation region. Within each element, Maxwell's curl equations are solved locally by expanding the electromagnetic fields in a set of local basis functions given by Lagrange polynomials of order p , with the expansion coefficients related to the field amplitudes at the corresponding nodal points [51]. This local expansion improves memory efficiency and parallelization [51,71,72], thereby reducing the computational resources required for large-scale three-dimensional simulations. A global solution is then constructed by introducing the so-called upwind numerical flux at element interfaces, ensuring interelement coupling and enforcement of boundary conditions [71]. Through spatial discretization, Maxwell's equations are cast into a semidiscrete form, i.e., a set of first-order coupled ordinary differential equations for the time-dependent expansion coefficients. These are integrated in time using an explicit fourth-order low-storage Runge–Kutta scheme with 14 stages [73], yielding a flexible, high-order time-stepping technique.

In numerical time-domain approaches, frequency-dependent permittivities cannot be efficiently incorporated into Maxwell's equations through pointwise multiplication, as in the frequency domain; such an implementation would require a computationally expensive convolution in time. Instead, an auxiliary current density is introduced to represent the material polarization response, satisfying an auxiliary differential equation (ADE) derived from the frequency-domain constitutive relation. This ADE is solved simultaneously with Maxwell's equations within the same time-stepping scheme [51,56].

Within the same framework, the total-field–scattered-field (TF-SF) technique is employed to inject the field distribution produced by dipolar excitations and decouple the scattered response from the total field. This approach exploits the

linearity of Maxwell's equations to decompose the electromagnetic fields into incident and scattered components by splitting the computational domain into a total-field (TF) region, where both the incident and scattered fields are evolved in time, and a scattered-field (SF) region, where only the latter is retained. The incident field is injected into the simulation through the numerical flux at the TF-SF interface by adding or subtracting the analytically evaluated source-induced field distribution [51,56]. The DGTD method is particularly well suited to this formulation, as field values are stored independently on either side of the element interfaces. Notably, singularities from localized point sources are absent when the source is placed in the SF region, thereby aligning with the scattered Green tensor formalism introduced earlier.

Having established the numerical formulation, we now examine its computational efficiency and convergence properties. The performance of a numerical method is commonly characterized by three main factors: (i) the computational time to obtain a solution, (ii) the associated memory requirement, and (iii) the accuracy of the resulting solution. For explicit DGTD schemes, the computational complexity scales as $O(NN_t)$, where N is the number of mesh elements and N_t the number of time steps, which is constrained by the inradius of the smallest element [51], analogous to the Courant–Friedrichs–Lewy stability condition in FDTD methods [56]. For a three-dimensional discretization with uniform mesh refinement, the characteristic element size scales as $h \sim N^{-1/3}$, implying $\Delta t \sim N^{-1/3}$. Therefore, for a fixed physical simulation time, this leads to $N_t \sim N^{1/3}$ and, consequently, to an overall computational time scaling as $O(N^{4/3})$. For locally refined meshes, such as those typically employed in Casimir-force computations, DGTD methods can support efficient local time-stepping algorithms that allow larger time steps in coarser regions of the mesh [74]. This can lead to significant reductions in computational time. In contrast, the memory requirement is primarily determined by the storage of field unknowns within each element and a fixed number of local flux interactions with neighboring elements, resulting in an asymptotic scaling of $O(N)$, consistent with other time-domain methods [56].

The spatial convergence of the scheme depends on both the characteristic edge length, h , and the polynomial order, p . Increasing p enhances spatial resolution by adding more interpolation nodes within each element. In this work, we use straight-sided tetrahedral elements, which are particularly

convenient for resolving multiscale features and handling curved geometries more naturally than regular grids, reducing staircasing artifacts. When the geometry is well-represented by the mesh, i.e., it is discretization-conforming, the method achieves high-order convergence of $O(h^{p+1})$ [51]. However, for nonconforming meshes with unresolved curvature, convergence may be limited to $O(h^2)$ [75]. For smoothly varying boundaries, curvilinear elements can be used to restore high-order convergence [51,75].

IV. APPLICATIONS

In this section, we present the numerical results of the proposed scheme, focusing on the Casimir force between two vacuum-separated metallic objects. A key step in the procedure is the integration over a closed surface in Eq. (1). Unlike boundary-element methods, which obtain the Casimir force without requiring a numerical stress-tensor integration [39], the present approach evaluates the force through a discretized surface integral. In principle, this requires evaluating the stress tensor at multiple points of the closed surface S , leading to a large number of time-domain simulations—up to six per evaluation point, corresponding to three orthogonal dipole orientations and two source types. Although these simulations can be easily parallelized, high spatial resolutions can significantly increase the required computational resources. While the geometry of S is theoretically arbitrary, certain choices can be numerically more efficient than others. For example, if one body is enclosed by a cube, the numerical evaluation of the integral simplifies considerably. In the limit of an infinitely extended cube, the only contributing surface is the face between the two bodies, S_0 , since the amplitudes of the scattered field on the remaining faces decay rapidly with distance and their contributions vanish at infinity. The essential physics is then fully captured by integrating only over the infinitely extended surface S_0 . Further simplifications can rely on the symmetries of specific systems.

As a benchmark, we first consider the Casimir interaction between metallic half-spaces separated by a vacuum gap of width L . This setup provides insight into the temporal integration in Eq. (10). In addition, this geometry offers computational simplicity and admits a semi-analytical treatment based on Lifshitz theory [2], which provides an independent reference calculation for assessing the accuracy and convergence of the numerical approach. As a second example, we consider a finite-size metallic cylindrical object positioned above a half-space, with its axis perpendicular to the surface. This less symmetric configuration underscores the surface-integration scheme that links the stress tensor to the force in Eq. (1).

A. Casimir force between parallel half-spaces

We consider two identical, metallic half-spaces separated by a distance L along the z axis, cf. Fig. 1(a). The optical response of the metal is modeled as that of silver, described by a standard Drude dielectric function

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\Gamma)}, \quad (16)$$

with plasma frequency $\omega_p = 1.39 \times 10^{16} \text{ s}^{-1}$ ($\hbar\omega_p = 9 \text{ eV}$) and damping rate $\Gamma = 3.23 \times 10^{13} \text{ s}^{-1}$ ($\hbar\Gamma = 21 \text{ meV}$) [76]. Due to translational invariance in the xy plane, we choose the integration surface S_0 to lie at the midplane at $z = L/2$, equidistant from both interfaces. This surface extends infinitely in the x and y directions and has a unit normal vector $\mathbf{n} = -\mathbf{e}_z$. The symmetry of the system ensures that the net Casimir force is directed along the z axis, i.e., $\mathbf{F} = F_z \mathbf{e}_z$. Accordingly, the relevant component of the stress tensor is $\langle \hat{T}_{zz}(\mathbf{r}) \rangle_T$, which needs to be integrated over the in-plane coordinates $\mathbf{r}_\parallel = (x, y)$ lying in the plane $z = L/2$. However, due to lateral invariance, $\langle \hat{T}_{zz}(\mathbf{r}) \rangle_T$ is spatially uniform over $z = L/2$. The Casimir force per unit area S_0 can therefore be computed at a single spatial position, say $\mathbf{r}_0 = (0, 0, L/2)$, as

$$\frac{F_z}{S_0} = -\langle \hat{T}_{zz}(\mathbf{r}_0) \rangle_T, \quad (17)$$

thus reducing the numerical problem to a single time integration.

As mentioned above, the symmetry of the system enables a semi-analytical solution, and Eq. (17) can be recast in terms of the Lifshitz formula [2,77]

$$\frac{F_z}{S_0} = -k_B T \sum_{m=0}^{\infty} \int \frac{d^2 \mathbf{q}}{(2\pi)^2} \sum_u 2\kappa \frac{[r_u(i\xi_m, q)e^{-\kappa L}]^2}{1 - [r_u(i\xi_m, q)e^{-\kappa L}]^2}, \quad (18)$$

which describes the pressure on one of the half-spaces in terms of the component of the electromagnetic wave-vector parallel to the planes, $\mathbf{q}$, with modulus $q = |\mathbf{q}|$, and the orthogonal wave-vector component in vacuum, $\kappa = (q^2 + \xi_m^2/c^2)^{1/2}$, evaluated at the imaginary Matsubara frequencies. The reflection coefficients r_u , with $u \in \{\text{TE}, \text{TM}\}$, describe the scattering of the fields at the half-space. Within the present context, r_u are given by the Fresnel formulas [57,78]. As above, the prime in the sum indicates that the $m = 0$ term carries a weight $1/2$.

In terms of the Lifshitz formula, the interaction can be characterized by the relative size of L with respect to two characteristic length scales: the plasma wavelength $\lambda_p = 2\pi c/\omega_p$, which is approximately 136 nm for silver, and the thermal wavelength $\lambda_{\text{th}} = 2\pi c/\omega_{\text{th}} = \hbar c/k_B T$, which roughly corresponds to the characteristic length scale in Wien's displacement law and is on the order of 7.6 μm at room temperature. For $L \ll \lambda_p$, the interaction is in the nonretarded regime, where propagation delays can be neglected. In this limit, Eq. (18) becomes [78,79]

$$\frac{F_z}{S_0} \sim -\frac{\hbar}{8\pi^2 L^3} \int_0^\infty d\xi \text{Li}_3 \left(\left[\frac{\epsilon(i\xi) - 1}{\epsilon(i\xi) + 1} \right]^2 \right) \\ \stackrel{\Gamma/\omega_p \ll 1}{\sim} -\frac{\hbar c \pi^2}{240 L^4} \left[\eta \frac{L}{\lambda_p} - \frac{15\zeta(3)}{\pi^4} \frac{\Gamma L}{c} \right], \quad (19)$$

where $\eta \approx 1.193$ and $\zeta(x)$ is the Riemann zeta function for which $\zeta(3) \approx 1.202$. Note that the previous expression does not depend on the temperature of the system, indicating that, in this regime, the quantum fluctuations dominate over the thermal fluctuations. For separations $L \gg \lambda_p$, retardation effects become significant, and in the zero-temperature limit, the

Lifshitz formula recovers Casimir's original result [1], given by

$$\frac{F_z}{S_0} \sim -\frac{\hbar c \pi^2}{240 L^4} \quad (20)$$

for the attraction between two perfectly reflecting plates. In this limit, the force loses all dependence on the material parameters, which remain relevant only through the condition $L \gg \lambda_p$. Corrections due to finite temperature become appreciable when $L \gtrsim \lambda_{th}$. In the limit $L \gg \lambda_{th}$, the classical thermal fluctuations dominate the interaction and the Lifshitz formula tends to

$$\frac{F_z}{S_0} \sim -\frac{\zeta(3)}{8\pi} \frac{k_B T}{L^3}. \quad (21)$$

The above expressions and associated quantities serve as a benchmark for our numerical computations.

1. Time integration

Due to the symmetry of the system, we have carried out the computations within a cylindrical computational domain centered at $\mathbf{r}_0$, with radius R and height $2H + L$, terminated with Silver–Müller boundary conditions [51]. The metallic half-spaces are approximated as finite disks of radius R and thickness H , forming the TF region. Due to the dissipative nature of metals, the thickness H is chosen such that contributions from the opposite interfaces can be neglected, effectively recovering the configuration of two facing half-spaces originally considered by Lifshitz [2]. The radius R is chosen to minimize spurious back-reflections from the edge of the computational domain reaching $\mathbf{r}_0$ [50]. Within the metallic disks, Maxwell's equations are solved in conjunction with the ADE for the Drude model, cf. Sec. III A 2. The SF region corresponds to the vacuum gap between the disks, and the TF-SF boundaries coincide with the metal-vacuum interfaces. The tetrahedral mesh, generated by GMSH [5], is locally refined near the evaluation position $\mathbf{r}_0$ with an average edge length of $h = L/10$ within a cylindrical region of radius L . The mesh is linearly coarsened toward the boundaries of the computational domain, reaching $h = L$ at the outermost limits. We use a global polynomial order $p = 3$, ensuring a resolution of at least two interpolation nodes per smallest relevant wavelength [51,56]. For an effective numerical treatment of Maxwell's equations, it is also advantageous to use dimensionless units. This approach entails scaling all fields and lengths with a reference field strength E_0 and a length scale ℓ_0 . For the application considered here, we use $E_0 = 1$ V/m and $\ell_0 = 1$ nm.

The excitation sources are located at $\mathbf{r} = \mathbf{r}_0$ and introduced into the simulation scheme through the TF-SF technique, cf. Sec. III A 2. Azimuthal invariance implies that at $\mathbf{r}_0$, we need only distinguish between $j = \parallel$ for the direction parallel to the planes and $j = \perp$ for the direction perpendicular to them. In Figs. 3(a), we illustrate the temporal behavior of the current density for a parallel-oriented electric dipole (left column) and a perpendicular-oriented magnetic dipole (right column). The pulse profiles correspond to the definition in Eq. (11), where for both the electric and magnetic dipoles we use $s = 4$ [50]. The pulse decays at a rate γ , a tunable parameter typically set to $\gamma \approx c/2L$, which controls the spectral bandwidth, as discussed in Appendix B.

In Figs. 3(b), we illustrate in the left and right columns the time evolution of the corresponding scattered fields recorded directly at the nodal point $\mathbf{r} = \mathbf{r}_0$, without spatial interpolation. Due to the symmetry of the configuration, the scattered fields $E_i^{(j)}(\mathbf{r}_0, \tau)$ and $H_i^{(j)}(\mathbf{r}_0, \tau)$ vanish for $i \neq j$. This property is inherited by the related quantities defined below. Note that in Figs. 3(b), the field amplitudes remain zero for $\tau \leq L/c$, reflecting the roundtrip time for the excitation to reach the interface and return to the source position. For $\tau \geq L/c$, transient reflections appear, followed by a decay regime once the initial pulse has sufficiently faded. In the source-free region, the electric-field component in the left panel of Figs. 3(b) exhibits exponentially damped oscillations at a frequency close to the surface-plasmon resonance $\omega_{sp} \approx \omega_p/\sqrt{2}$, corresponding to the dominant pole of the nonretarded electric Green tensor [77]. On the other hand, the amplitude of the magnetic field in the right panel of Figs. 3(b) decays exponentially without oscillatory behavior.

To assess the temporal convergence of Eq. (10), we define the component-wise contributions to the local Casimir force per unit area as

$$f_{ij,T}^\sigma(\mathbf{r}) = 2\hbar \int_0^{t_{\max} \rightarrow \infty} d\tau \operatorname{Im} \{g_T^\sigma(-\tau)\} \Lambda_{ij}^\sigma(\mathbf{r}, \tau). \quad (22)$$

Numerical evaluation of Eq. (22) requires truncating the upper limit at sufficiently large $t_{\max}$, introducing a trade-off between computational efficiency and numerical accuracy. In Ref. [41], $t_{\max}$ is chosen such that the field amplitude decays below a specified error threshold in an artificially dissipative medium with tunable dissipation. However, extending this criterion to physical systems can be computationally demanding and may result in long simulation times, particularly for cavities and resonators [80]. To reduce computation time while maintaining accuracy, we instead choose $t_{\max}$ to capture only the relevant field information based on the system's physical characteristics [80,81]. The long-time response is then extrapolated by fitting the finite-duration signal over the interval $[t_0, t_{\max}]$ to a linear superposition of exponentially damped oscillatory modes, where t_0 is chosen such that the initial pulse has sufficiently decayed, e.g., $\gamma t_0 = 30$ in Figs. 3(b). For this purpose, we use a time-domain harmonic inversion technique based on the filter-diagonalization method [82,83], which has the advantage of providing a semi-analytical expression for the long-time response field. The modal parameters are extracted using the HARMINV implementation within a finite frequency window [84]. In the left and right panels of Figs. 3(b), we illustrate the excellent agreement between reconstructed and simulated fields for $\gamma\tau \geq 30$. Therefore, long-time corrections are obtained semi-analytically by effectively integrating Eq. (22) from $t_{\max}$ to infinity using the harmonic inversion parameters extracted from the simulated data, i.e.,

$$f_{ij,T}^\sigma(\mathbf{r}) = 2\hbar \left[\int_0^{t_{\max}} d\tau \operatorname{Im} \{g_T^\sigma(-\tau)\} \Lambda_{ij}^\sigma(\mathbf{r}, \tau) + \sum_n \operatorname{Re} \left\{ d_n \int_{t_{\max}}^\infty d\tau \operatorname{Im} \{g_T^\sigma(-\tau)\} e^{-ia_n\tau} \right\} \right]. \quad (23)$$

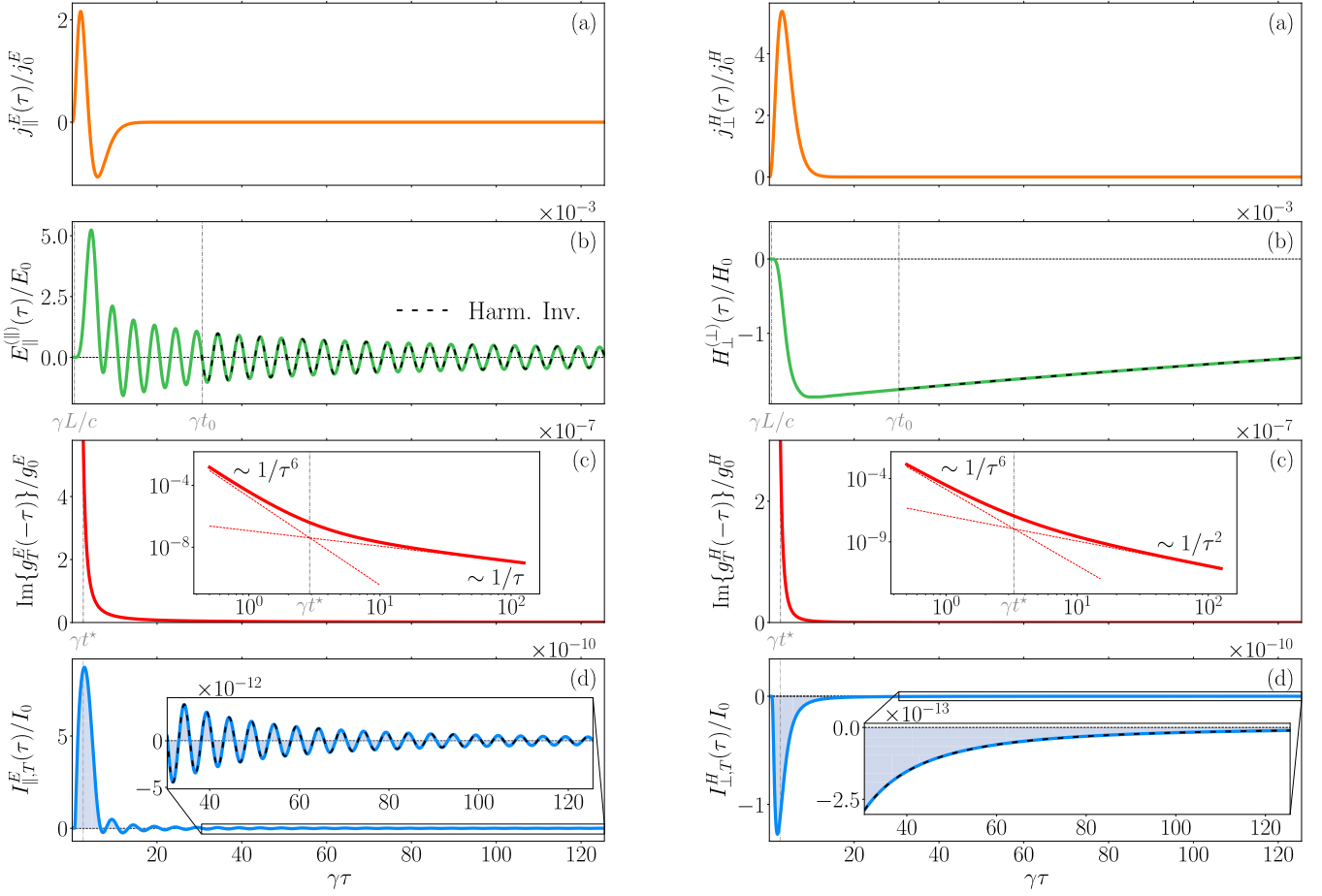


FIG. 3. (left column) Relevant quantities characterizing the time-integration scheme for two silver half-spaces separated by $L = 20$ nm and excited by an electric dipole oriented parallel to the planes. All quantities are scaled with a reference field strength $E_0 = 1$ V/m and a length scale $\ell_0 = 1$ nm. Each panel depicts a different component of the scheme. (a) Temporal profile of the current density, as defined in Eq. (11) for $s = 4$ and $\gamma \approx c/2L$, expressed in units of $j_0^E = \epsilon_0 E_0 \ell_0^2 c$. (b) Scattered electric field $E_{||}^{(||)}(\tau)$ at $\mathbf{r}_0 = (0, 0, L/2)$ in units of E_0 . (c) Kernel function in units of $g_0^E = c^3/\ell_0^3 j_0^E$, with a logarithmic scale inset showing the asymptotic behavior at zero temperature, cf. Appendix A. (d) Time integrand $I_{||,T}^E(\tau) = \text{Im}\{g_T^E(-\tau)\} \epsilon_0 E_{||}^{(||)}(\mathbf{r}_0, \tau)$ in units of $I_0 = g_0^E \epsilon_0 E_0 = c^2/\ell_0^5$, with an inset focusing on the long-time behavior. (right column) Same as the left column, but for a perpendicular magnetic dipole. All quantities are expressed in the following units: (a) $j_0^H = E_0 \ell_0^2$; (b) $H_0 = E_0 \sqrt{\epsilon_0/\mu_0}$; (c) $g_0^H = c^3/\ell_0^3 j_0^H$; (d) $I_0 = c^2/\ell_0^5$.

The first integral on the right-hand side of Eq. (23) is evaluated using standard quadrature techniques, with Λ_{ij}^σ the simulated scattered fields, while the second is performed analytically, where $d_n = A_n e^{i\phi_n}$ and $a_n = \omega_n - i\alpha_n$ are the harmonic inversion parameters [84]. Here, A_n is the mode amplitude, ϕ_n the phase shift, ω_n the resonant frequency, and $\alpha_n > 0$ the decay constant of the n th mode. The combination of direct numerical integration and the semi-analytical long-time correction accelerates convergence compared with an approach based only on the first term in Eq. (23). Although the harmonic inversion provides a semi-analytical description of the long-time dynamics, it remains a numerical technique that introduces approximation errors. Consequently, the accuracy of Eq. (23) still depends on the choice of $t_{\max}$; therefore, a sufficiently rapid decay of the integrands remains essential to minimize errors. Besides field dissipation, convergence is also governed by the behavior of the kernel $\text{Im}\{g_T^E(-\tau)\}$, defined in Eq. (15) and illustrated in the left and right panels of Figs. 3(c). As shown in Figs. 3(d), the kernel amplifies short-time field contributions while suppressing those at

longer times. Notably, the divergence at $\tau = 0$ is physically regularized by causality, since the scattered field vanishes for $\tau < L/c$.

Inserting the numerical results from Eq. (23) into the zz component of the Maxwell stress tensor in Eq. (10) yields the Casimir force per unit area along the z direction. Figure 4 shows the numerical results for the force as a function of the separation L , normalized to the asymptotic value of the Lifshitz formula in Eq. (19). The outcome of our numerical approach (symbols) agrees very well with the semi-analytical results in Eq. (18) (solid lines) across a wide range of separations and temperatures. When retardation is negligible, $L \ll \lambda_p$, the numerical data approach unity, become insensitive to temperature, and reproduce the expected asymptotic scaling $F_z \sim -1/L^3$ predicted by Eq. (19). At larger separations, two different trends are observed. First, when the finite speed of light becomes relevant [78,85], the force tends toward Casimir's original expression in Eq. (20), approaching the scaling $F_z \sim -1/L^4$. In this region, the contribution from thermal fluctuations is still sufficiently small, although

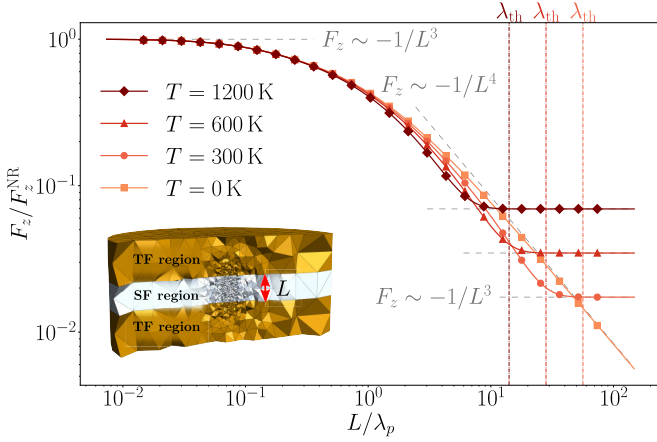


FIG. 4. Casimir force between parallel half-spaces as a function of their separation L . The force is normalized to the nonretarded asymptotic value, F_z^{NR} , cf. Eq. (19). Symbols denote numerical results, while solid lines represent semi-analytical reference calculations based on the Lifshitz formula in Eq. (18). At short separations, the numerical results recover the $F_z \sim -1/L^3$ behavior predicted by Eq. (19) and are independent of temperature, whereas at large separations they reproduce the $F_z \sim -1/L^4$ and $F_z \sim -1/L^3$ dependencies expected from Eqs. (20) and (21) at zero and finite temperature, respectively. The inset shows a representative mesh of the plane-plane geometry generated with GMSH [5].

deviations are visible. Second, when $L \gg \lambda_{\text{th}}$, thermal effects become significant and, in accordance with Eq. (21), the data progressively recover the expected scaling of $F_z \sim -1/L^3$. The temperature dependence of the prefactor in Eq. (21) is distinctly visible within the normalization scheme used in Fig. 4.

2. Convergence studies

To quantify the accuracy and convergence properties of the numerical scheme, we evaluate the relative error of the Casimir force per unit area, computed independently for each field component as

$$\left| \frac{\Delta f_{ij,T}^\sigma}{\bar{f}_{ij,T}^\sigma} \right| = \left| \frac{f_{ij,T}^\sigma - \bar{f}_{ij,T}^\sigma}{\bar{f}_{ij,T}^\sigma} \right|. \quad (24)$$

Here, $f_{ij,T}^\sigma$ is the result from the numerical time-integration scheme, as described in Sec. IV A 1, while $\bar{f}_{ij,T}^\sigma$ denotes the reference value obtained from complex-frequency-domain integration of the corresponding Green tensor component, cf. Appendix C.

The accuracy of the method is limited by different error sources, most notably the finite spatial discretization inherent to any mesh-based approach. To assess its effect, we perform a convergence study with respect to both mesh refinement, by uniformly reducing the average edge length h through subdivision of each tetrahedral element, and polynomial refinement, by increasing the global polynomial order p of the local basis functions [86].

The study is conducted for the same configuration as in the inset of Fig. 4, and the results are averaged over 22 nominally identical simulations with different tetrahedral distributions obtained by displacing $\mathbf{r}_0$ within the plane $z = L/2$ [50]. This

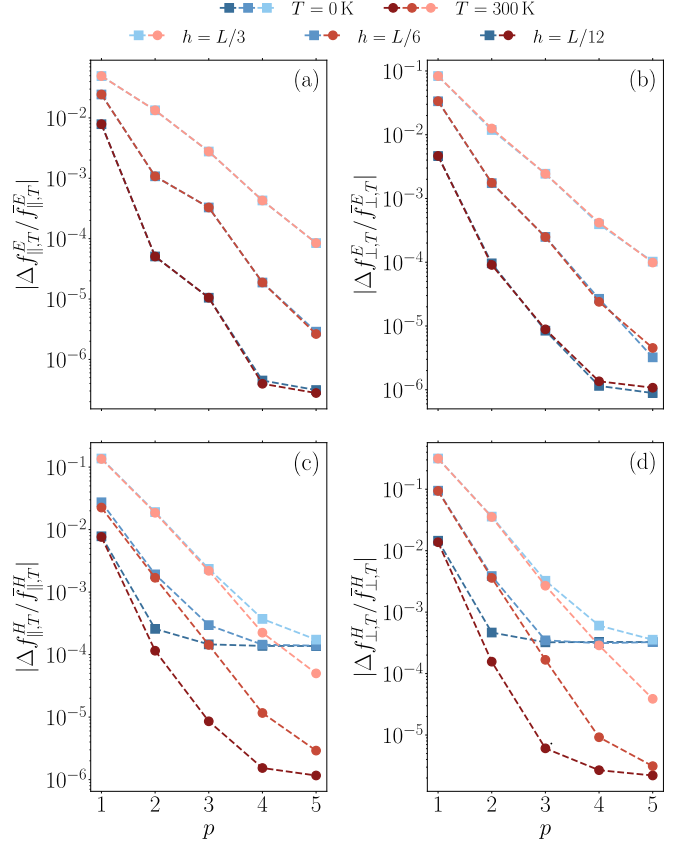


FIG. 5. Relative error of the component-wise Casimir force per unit area at $L = 100$ nm, averaged over 22 nominally identical simulations with total duration $\gamma t_{\text{max}} = 30$. The error is shown as a function of the polynomial order p for different mesh resolutions. The mesh size h is defined as the average edge length in the central region of the computational domain. Results are presented for both zero (blue) and room temperature (red). Individual contributions from the electromagnetic field components are shown separately: panels (a) and (b) correspond to the parallel ($\parallel$) and perpendicular ($\perp$) electric-field components, respectively, while panels (c) and (d) show the corresponding magnetic-field components.

averaging reduces sensitivity to a particular mesh placement and yields a more robust estimate of the spatial discretization error. Figure 5 presents the averaged relative error as a function of the polynomial order for different mesh resolutions. Electric-field contributions in Figs. 5(a) and 5(b) exhibit the expected convergence behavior $O(h^{p+1})$, consistent with theoretical predictions for conformal meshes, cf. Sec. III A 2. In contrast, the relative error for magnetic-field contributions at $T = 0$ K in Figs. 5(c) and 5(d) plateaus beyond a certain polynomial order and mesh resolution, suggesting a transition to a regime dominated by temporal truncation errors rather than spatial discretization, as discussed below. This behavior differs at finite temperature, e.g., $T = 300$ K, where the faster decay of the temporal kernel reduces truncation errors, cf. Appendix A for details.

To quantify the impact of t_{max} on the temporal integration in Eq. (23), Fig. 6 shows the relative error as a function of t_{max} at both zero and finite temperature for a fixed mesh size. Direct quadrature integration over $[0, t_{\text{max}}]$ is compared with

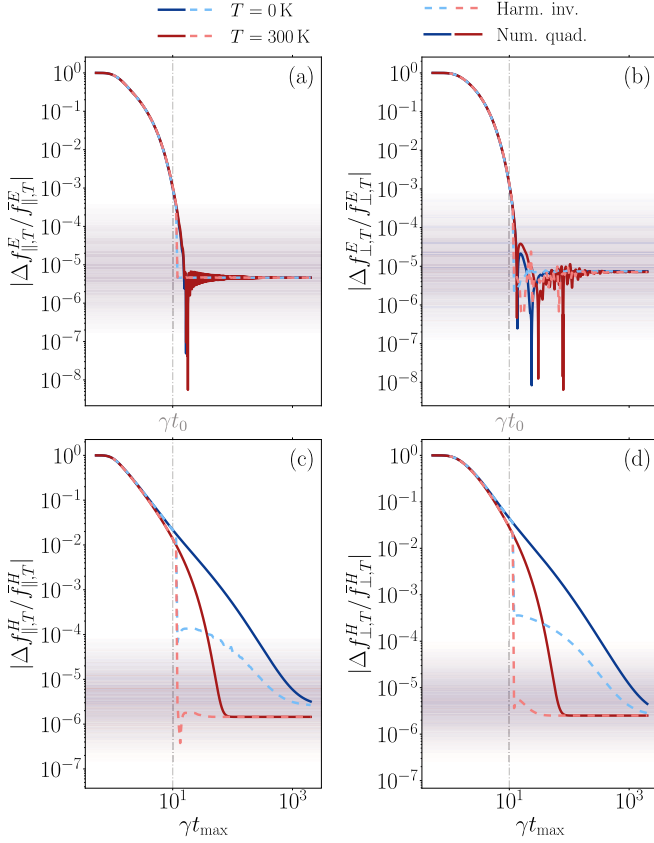


FIG. 6. Relative error of the component-wise Casimir force per unit area as a function of the simulation time $t_{\max}$. The error is shown for $L = 100$ nm, polynomial order $p = 5$, and mesh size $h = L/6$. Solid lines represent direct quadrature of the time-domain response over $[0, t_{\max}]$, while dashed lines include analytically integrated long-time contributions from $t_{\max}$ to $\tau \rightarrow \infty$ using the modal parameters obtained via harmonic inversion over $[t_0, t_{\max}]$, according to Eq. (23). Results are shown for both zero (blue) and room temperature (red). The shaded region represents the standard deviation of the relative error from 22 nominally identical simulations with computational domains sufficiently large to prevent spurious back-reflections within the simulation window. Panels (a) and (b) show the parallel ($\parallel$) and perpendicular ($\perp$) electric-field components, respectively, while panels (c) and (d) show the corresponding magnetic-field components.

results obtained by adding harmonic-inversion-based long-time contributions according to Eq. (23). For the electric-field components, Figs. 6(a) and 6(b), increasing $t_{\max}$ leads to rapid convergence toward a residual error set by spatial discretization, as discussed above. The dependence on long-time contributions is weak as the net dipole moment vanishes at large τ , causing the second integral in Eq. (23) to oscillate around its asymptotic value. In contrast, the magnetic-field components, Figs. 6(c) and 6(d), are more sensitive to temporal truncation, with errors being significantly reduced by the harmonic inversion approach. Increasing the time window further improves the accuracy of the harmonic inversion by better capturing the slower temporal decay characteristic of magnetic-field contributions [84]. As $t_{\max}$ increases, both direct quadrature and harmonic inversion results tend to the same residual error. The local minima in $|\Delta f_{ij,T}^\sigma / \bar{f}_{ij,T}^\sigma|$

correspond to sign changes in the error and carry no special significance.

As in Ref. [50], we find that the accuracy may additionally be limited by reflections from the boundaries. To confirm that the residual error in Figs. 5 and 6 is not dominated by such spurious back-reflections, we perform control simulations using larger domains in which the roundtrip time from the outer boundary to $\mathbf{r}_0$ exceeds $t_{\max}$, preventing boundary reflections from reaching $\mathbf{r}_0$ within the integration window. The standard deviation of the relative errors across the 22 simulations, represented by the shaded regions in Fig. 6, confirms that the residual error is dominated by spatial discretization rather than boundary reflections.

3. Computational efficiency

Having identified the main sources of numerical error, we now assess the computational efficiency of the proposed DGT framework. To put this in perspective, we compare its scaling with that of established numerical methods for Casimir-force calculations in arbitrarily shaped geometries, such as the BEM implemented in the open-source software SCUFF-EM [39,55]. Using a sequence of uniformly refined meshes, we examine the computational time, maximum memory requirement, and relative error of both approaches for the parallel half-space configuration considered in the previous section.

Figure 7 compares volumetric DGT discretizations based on tetrahedral elements with polynomial orders $p = 1$ and 3 against the surface-only triangular discretization employed by BEMs. To account for the symmetries of the parallel half-spaces, two BEM formulations are considered. In general, SCUFF-EM models arbitrary geometries as compact objects with closed surfaces. For the present geometry, this corresponds to approximating the half-spaces as finite disks of sufficiently large extent, consistent with the finite computational domain used in Sec. IV A 1. Alternatively, the translational invariance of the system can be exploited through periodic boundary conditions, yielding an exact numerical representation of the infinitely extended half-space configuration. While the extended-object formulation is the more natural choice for this geometry, the compact-object formulation is included to illustrate the computational scaling of a general-purpose BEM implementation for finite objects. In both BEM formulations, the Casimir force is obtained by evaluating the electromagnetic response at multiple imaginary frequencies to perform the contour integration. In contrast, the symmetry of the present configuration allows the DGT approach to compute the force per unit area from only four independent time-domain simulations, corresponding to two dipole orientations (parallel and perpendicular) and two source types (electric and magnetic), cf. Sec. IV A 1. The independence of these simulations makes them readily parallelizable; consequently, the elapsed time and memory requirement reported in Fig. 7 correspond to those of the most demanding individual simulation [87].

The computational behavior of the two numerical methods can be understood from their asymptotic scaling properties. As discussed in Sec. III A 2, the computational cost of an explicit DGT scheme scales as $O(N^{4/3})$, where N denotes the

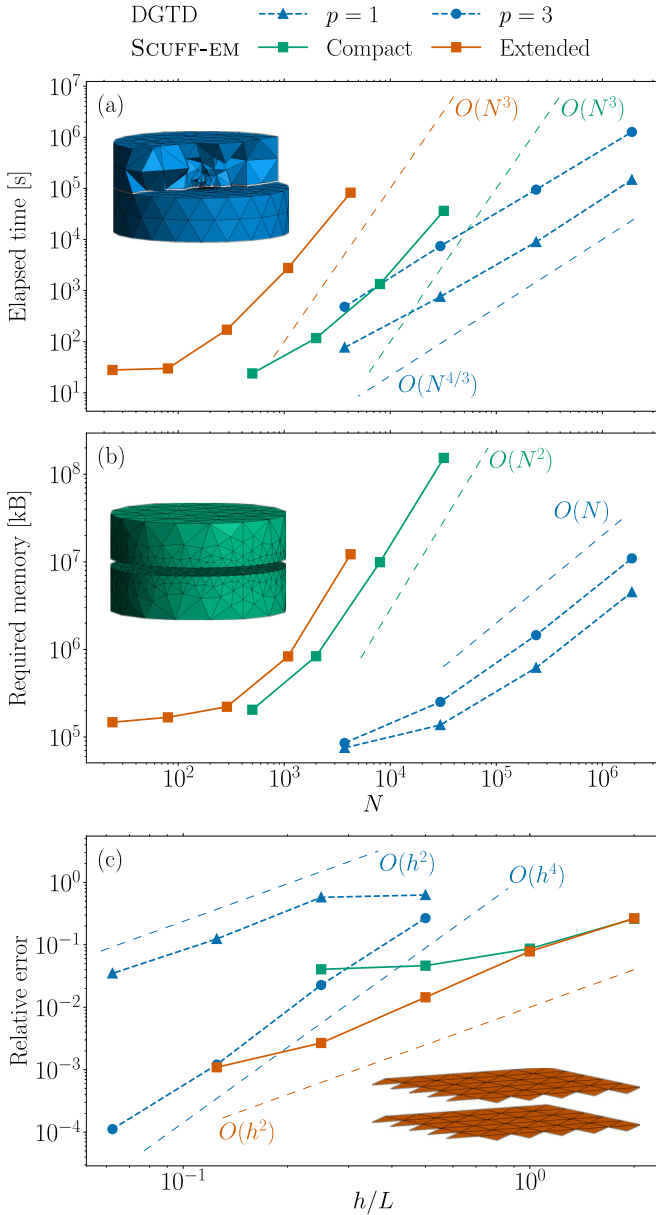


FIG. 7. Comparison of the computational performance of the DGTD method with polynomial orders $p = 1$ and 3 , and the BEM implemented in SCUFF-EM for compact and extended objects [55]. Results are shown for the Casimir force per unit area for parallel half-spaces separated by a vacuum gap of $L = 100$ nm. Panels (a) and (b) show the runtime and memory requirement for a single time- or frequency-domain simulation as functions of the number of mesh elements N . Panel (c) shows the relative error as a function of the average edge length h .

number of mesh elements. In contrast, the cost of SCUFF-EM is dominated by operations involving dense interaction matrices, leading to an asymptotic scaling of $O(N^3)$ for sufficiently large N when dense direct solvers are employed [39]. This behavior is reflected in Fig. 7(a), where the computational time of SCUFF-EM increases more rapidly under uniform mesh refinement than that of the DGTD. A similar behavior is observed in the memory requirements shown in Fig. 7(b). For the DGTD method, the memory requirement scales as $O(N)$, cf. Sec. III A 2. In contrast, the BEM requires storing

interactions between all pairs of surface elements, resulting in a dense system matrix and a memory scaling of $O(N^2)$.

Together with the runtime and memory requirements shown in Figs. 7(a) and 7(b), respectively, the relative error in Fig. 7(c) provides a direct estimate of the computational resources required to achieve a given accuracy for this particular problem. In addition, it highlights the convergence behavior of the BEMs for a fixed choice of basis functions [88], as well as the higher-order convergence of the DGTD, cf. Sec. III A 2. While the extended-object formulation exhibits the expected convergence behavior, the error for compact objects saturates under mesh refinement as finite computational-domain effects become dominant.

DGTD and BEM approaches present complementary strengths and limitations; therefore, the approach presented here should be regarded not as a replacement for established numerical methods, but rather as an additional complementary framework. BEM implementations, such as SCUFF-EM, have become a standard and efficient tool for Casimir-force calculations, as they require discretization only of material interfaces, resulting in a substantially smaller number of unknowns. This advantage is particularly pronounced when the relevant physical length scales are comparable to the characteristic geometric features of the system, which can then be accurately represented by relatively coarse surface meshes. In contrast, the DGTD method requires a volumetric discretization of the entire computational domain, resulting in a larger number of degrees of freedom for a given geometry. However, its more favorable scaling under mesh refinement makes it attractive when fine spatial resolution is required, since its computational time and memory requirements increase less rapidly than those of dense-matrix BEM.

B. Cylindrically symmetric object above a plane

We now consider a finite, cylindrically symmetric object positioned above a half-space, separated by a vacuum gap of width L . The coordinate system is chosen such that the z axis coincides with the system's axis of symmetry and is oriented normal to the substrate, which occupies $z \leq 0$. Owing to the cylindrical symmetry around the z axis, the lateral components of the Casimir force cancel, resulting in a net force directed along $\mathbf{e}_z$.

Once again, we choose a closed surface whose bottom face lies at $z = L/2$, midway between the object and the substrate, while the remaining faces extend to infinity, contributing negligibly due to the decay of the scattered fields, cf. Fig. 8. The integral in Eq. (1) therefore reduces to an effective open-surface integral over S_0 at $z = L/2$, with unit normal vector $\mathbf{n} = -\mathbf{e}_z$. Exploiting the system's axial symmetry, we work with cylindrical coordinates $\mathbf{r} \equiv (\rho, \varphi, z)$. The azimuthal dependence φ can be integrated analytically, reducing the surface integration of the stress tensor to a one-dimensional integral over the radial coordinate $\rho \in [0, \infty)$, which can be evaluated using a Gaussian quadrature scheme [89]. Although Gauss-Laguerre quadrature can, in principle, handle semi-infinite intervals, in practice, it introduces nodes far from the object that contribute negligibly to the final value, while expanding the computational domain to impractical sizes. Truncating the integral to a finite interval and using

a Gauss–Legendre quadrature scheme leads to an efficient computation [90].

We exploit the adaptive-meshing capabilities of finite-element methods and evaluate the smoothly varying stress tensor at nonuniform radial positions corresponding to the nodes of a Gauss–Legendre quadrature of order N_q [36,77], bounded by a maximum cutoff radius $\rho_{\max}$, beyond which the stress tensor is assumed to be negligible. This formulation enables efficient surface integration using standard Gaussian quadrature routines under a suitable change of variables, while requiring only a small number of evaluation points. The resulting force is then approximated as

$$F_z \approx -\frac{\pi \rho_{\max}^2}{2} \sum_{i=1}^{N_q} w_i \langle \hat{T}_{zz}(\rho_i, z = L/2) \rangle_T, \quad (25)$$

where $\rho_i = \rho_{\max} \sqrt{(x_i + 1)/2}$ is the discretized radial coordinate, and w_i and $x_i \in [-1, 1]$ are the corresponding Gauss–Legendre weights and nodes, respectively.

1. Surface integration

As an illustrative example, we consider a finite metallic cylinder with radius a and height $b = 2a$, positioned above a half-space. The material response is described by the Drude form given in Eq. (16) for silver. The computational domain is shown in Fig. 1(b), and the mesh is generated according to the procedure outlined in Sec. IV A 1. To resolve the cylinder's curvature, the mesh size is set to $h = a/10$ near the lateral surface.

The local stress-tensor contributions, $f_{ij,T}^\sigma(\rho, z = L/2)$, are evaluated at radial positions determined by a 16-point Gauss–Legendre quadrature within the domain $\rho \leq \rho_{\max} = a + 2L$. At each quadrature node, a point dipole is placed, and the corresponding scattered fields are evolved in time. The field amplitudes are recorded at the dipole positions and integrated over time using the scheme described in Sec. IV A 1. The total Maxwell stress tensor, shown in Fig. 8, is obtained by summing the individual field contributions, as given by Eq. (10). We note that near the cylinder center, at $\rho \ll a$, the configuration resembles the translationally invariant plane-plane geometry discussed in the previous section, and thus the stress tensor remains nearly uniform. For $\rho \sim a$, a peak appears, which can be attributed to the increasing contribution of scattered fields coming from the lateral surface of the cylinder. For $\rho \gg a$, the stress tensor decays exponentially at a rate scaling as the inverse of L , thereby justifying a posteriori the choice of $\rho_{\max}$. The spatial integration of these contributions yields the resulting Casimir force. Given the rapid convergence of Gaussian quadrature techniques [89], and for sufficiently large $\rho_{\max}$, we expect the dominant numerical error to arise from the spatial discretization of the mesh. For nonconformal meshes, such as in the present case, the convergence rate may be limited to $O(h^2)$, cf. Sec. III A 2.

Figure 9 shows the force as a function of the separation L , normalized by the pressure given in Eq. (19) multiplied by the area of the cylinder's base, $A = \pi a^2$. This normalization corresponds to the nonretarded limit of the PFA. Numerical results (symbols) are compared with asymptotic predictions (dashed lines) in Fig. 9, where different temperatures are considered for a cylindrical object with a fixed radius of $a =$

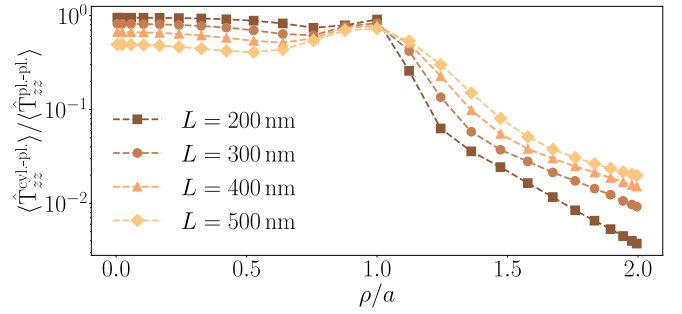


FIG. 8. Spatial distribution of the Maxwell stress tensor for a cylinder of radius $a = 500$ nm and height $b = 2a$, positioned at different separations above a half-space. Values are normalized to the plane-plane Maxwell stress tensor.

500 nm. Although no semi-analytical solution exists for this configuration, the limiting behaviors for both short- and long-distance regimes can be reliably predicted based on physical considerations. At separations $L \ll a$, when the distance is much smaller than the radius of the cylindrical structure, the interaction should approach that of the parallel half-spaces. This behavior is consistent with that predicted by the PFA [91,92]. For our configuration, this means that the impact of the scattering arising from the cylinder's boundaries can be neglected, leading to a force given by the Lifshitz pressure on the effective area of the system, i.e., the area of the cylinder's base A . Consequently, when $L \ll \lambda_p$ and $L \ll a$, the numerically evaluated Casimir force becomes temperature independent and scales as $F_z \sim -1/L^3$, thereby justifying the choice of normalization used in Fig. 9.

When the distance becomes larger than the radius of the compact object, the accuracy of the PFA degrades and eventually loses its validity, and the numerics deviate from the semi-analytical estimate, cf. Fig. 10. In this regime, the interaction is increasingly governed by the cylinder's multipole scattering contributions [93]. For $L \gg a$, when the cylinder dimensions are small compared with its distance from the surface, higher-order multipoles can be neglected, and the dipolar response dominates the scattered electromagnetic radiation. In this regime, the system can be effectively described in terms of a polarizable object interacting with a surface, and the Casimir force gradually transitions into the Casimir–Polder limit [4]. Unlike the Lifshitz expression in Eq. (18), the Casimir–Polder interaction is directly connected to the polarizability of the object. While no closed-form expression exists for the polarizability of a small cylinder [94], a reliable approximation is provided by relating it to that of a volume-equivalent spheroid [49]. Within this approach and for this specific case, a cylindrical object with volume $V = \pi a^2 b$ can be roughly approximated by a sphere of equivalent volume. Under these conditions, the frequency-dependent polarizability follows the Clausius–Mossotti relation [57,95]

$$\alpha(\omega) \approx 3\epsilon_0 V \frac{\epsilon(\omega) - 1}{\epsilon(\omega) + 2}, \quad (26)$$

where $\epsilon(\omega)$ is the dielectric function given in Eq. (16), and thus the polarizability features a resonance at $\omega_r \approx \omega_p/\sqrt{3}$, for $\Gamma \ll \omega_p$.

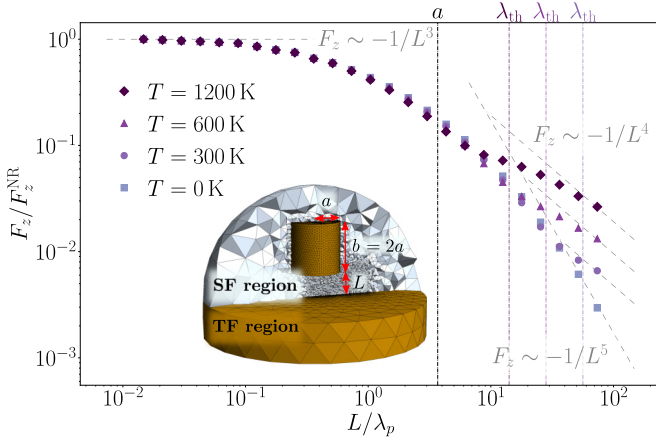


FIG. 9. Casimir force between a cylindrical object and a half-space as a function of their separation L . The cylinder has radius $a = 500$ nm and height $b = 2a$. The force is normalized to the non-retarded limit of the Casimir pressure in Eq. (19), multiplied by the area of the cylinder's base. Symbols denote numerical results, and dashed lines represent semi-analytical asymptotic predictions. At short separations, the numerical results recover the $F_z \sim -1/L^3$ behavior predicted by Eq. (19) and are independent of temperature, whereas at large separations they reproduce the $F_z \sim -1/L^5$ and $F_z \sim -1/L^4$ dependencies expected from the asymptotic expressions for the Casimir–Polder interaction in Eqs. (28) and (27) at zero and finite temperature, respectively. The inset shows a representative mesh of the cylinder-plane geometry generated with GMSH [5].

The preceding considerations enable us to estimate the behavior of the force at large separations. At finite temperature, in the thermal regime, i.e., $L \gg \lambda_{\text{th}}$, the Casimir–Polder interaction is dominated, as above, by thermal fluctuations, yielding the thermal limit [95]

$$F_z \sim -\frac{3}{16\pi} \frac{k_B T}{\epsilon_0} \frac{\alpha(0)}{L^4}. \quad (27)$$

The finite lateral extent of the object gives rise to a power law that decays one order faster than the Casimir force in the corresponding limit. This is visible in the trend of the numerical data in Fig. 9, which bends downward compared with what is observed in Fig. 4. With increasing separation, the data approach the asymptotic limit in Eq. (27), which is set by the system's temperature, validating both the numerics and the underlying physical reasoning. At zero temperature, the interaction features a different asymptotic behavior at large separations. The retarded regime of the Casimir–Polder force is defined by $L \gg \lambda_r = 2\pi c/\omega_r \approx \sqrt{3}\lambda_p$. Therefore, for $L \gg \lambda_p$ and $L \gg a$, the interaction behaves according to [85,95,96]

$$F_z \sim -\frac{3}{8\pi^2} \frac{\hbar c}{\epsilon_0} \frac{\alpha(0)}{L^5}, \quad (28)$$

which again features a characteristic power law, decaying one order faster than the Casimir force at zero temperature in the corresponding limit, cf. Eq. (20). We observe this behavior in Fig. 10, where we considered cylinders with different values of a at $T = 0$ K. To highlight the deviation from the PFA prediction (solid line), the normalization is chosen with respect to the Casimir pressure in Eq. (20) multiplied by the area of the cylinder's base. For each cylinder size, the numerical

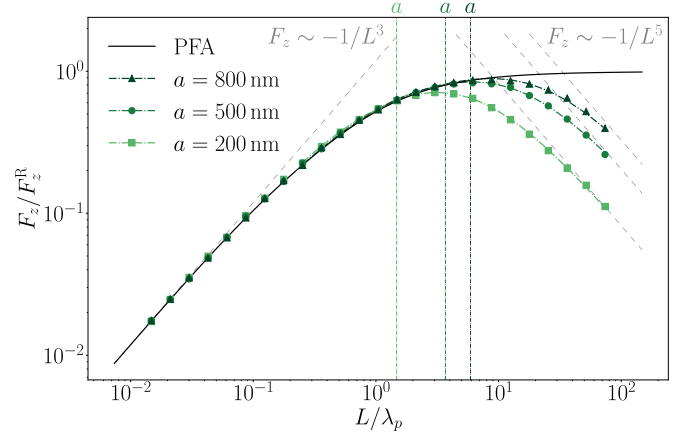


FIG. 10. Casimir force between a cylindrical object and a half-space as a function of their separation L . The temperature is fixed at $T = 0$ K, while different cylinder radii a and corresponding heights $b = 2a$ are considered. The force is normalized to the retarded limit of the Casimir pressure in Eq. (20), multiplied by the area of the cylinder's base. As in Fig. 9, symbols denote DGTD calculations, and dashed lines represent asymptotic predictions. In addition, the solid line describes semi-analytical approximations based on the PFA, while dash-dotted lines correspond to SCUFF-EM calculations. At short separations, the numerical results recover the $F_z \sim -1/L^3$ behavior predicted by Eq. (19), whereas at large separations they reproduce the $F_z \sim -1/L^5$ dependence from the Casimir–Polder interaction in Eq. (28), with $\alpha(0)$ varying according to a .

results obtained with the DGTD approach (symbols) and the BEM implemented in SCUFF-EM [39,55] (dash-dotted lines) progressively approach the asymptotic behavior described by Eq. (28). As expected, the smaller the value of a , the shorter the separation L at which the numerics start to deviate from the PFA. The dependence of the asymptotic expression on a follows directly from the relation $\alpha(0) = 6\epsilon_0\pi a^3$.

Finally, it is worth stressing that the crossover region, $L \sim a$, falls outside the reach of the asymptotic expressions above. Consequently, accurate predictions rely exclusively on numerical methods. As shown in Fig. 10, the DGTD results remain in close agreement with the SCUFF-EM calculations, thereby providing an independent validation.

V. CONCLUSIONS

In this work, we have presented a high-accuracy, time-domain finite-element-based framework for computing Casimir forces between vacuum-separated bodies within the Maxwell stress tensor formalism. The numerical scheme employs the discontinuous Galerkin time-domain method to evaluate finite-temperature Casimir interactions between arbitrarily shaped objects described by general material models. By expressing the electromagnetic Green tensor in terms of the system's response to dipolar excitations, the stress tensor is recast into a set of classical scattering problems driven by electric- and magnetic-dipole sources in the time domain. Benchmark calculations for the Casimir interaction between vacuum-separated half-spaces show excellent agreement with Lifshitz theory across multiple length scales. Relevant steps

in achieving this outcome have been the design of a specific temporal profile for the electric and magnetic sources and the development of a harmonic-inversion-based procedure to extrapolate the long-time behavior of the electromagnetic fields. Regarding convergence, our analysis has revealed that the accuracy is primarily limited by temporal truncation and spatial discretization, while the computational cost and memory requirement scale as $O(N^{4/3})$ and $O(N)$, respectively, under uniform mesh refinement.

We have further applied the framework to the Casimir interaction between a metallic cylindrical object and a half-space embedded in vacuum, a geometry lacking a semi-analytical expression. The numerical results have recovered the expected asymptotic behavior in both short- and long-distance limits, while showing good agreement with well-established numerical methods in the intermediate regime, where geometric, dispersive, and thermal effects interact in nontrivial ways. One of the primary technical challenges of our approach has been the efficient evaluation of the Maxwell stress tensor flux over a closed integration surface. For the specific configuration in this work, we have used a Gaussian quadrature scheme to reach the desired level of accuracy with a relatively small number of quadrature points. While the present computation exploits cylindrical symmetry, the extension to more general geometries is directly achievable via optimized two-dimensional Gaussian quadrature or cubature schemes [97–99]. Although such an extension would increase the number of required simulations linearly with the number of quadrature points, the independence of each run makes the method ideally suited for parallelization on high-performance computing clusters. Consequently, with sufficient computational resources, the wall-clock time is determined by the cost of the most demanding individual DGTD simulation rather than by the total number of quadrature points. This highlights the flexibility of the DGTD approach for investigating nontrivial Casimir phenomena, including geometry-induced repulsion and nonmonotonic distance dependence [39,40,44–46], in configurations that lie beyond the scope of commonly used approximations, such as the proximity-force approximation and more rigorous multipole-scattering methods. Although the present implementation is restricted to vacuum-separated bodies, the underlying framework effectively remains applicable to dispersionless-medium-assisted configurations, for which alternative electromagnetic stress tensor formulations have been proposed [100,101]. For dispersive media and Abraham-based formulations [102], the kernel in Eq. (9) acquires an additional frequency dependence through the material response functions $\epsilon(\omega)$ and $\mu(\omega)$. This dependence can be treated either numerically, following the approach of Ref. [41], or analytically, in analogy with the treatment of the polarizability in Ref. [50]. In addition, the numerical implementation would require reconstructing the incident dipole field at the TF-SF boundary in the presence of a dispersive background medium [56].

In summary, the DGTD method provides an accurate and flexible alternative for computing Casimir-type forces in the time domain. Its ability to accommodate arbitrary geometries, dispersive material models, and finite temperatures makes it particularly well suited for studying fluctuation-induced

forces in micro- and nanoscale systems. Future developments may include the implementation of curvilinear elements to preserve high-order convergence [75], which would be especially beneficial for spherical geometries commonly encountered in experimental setups. On the material side, the approach can be extended to incorporate spatially dispersive models in nanostructured geometries, which are becoming increasingly relevant in Casimir physics [103,104]. The DGTD method can accommodate spatially dispersive models through ADE formulations [50,105], offering important advantages for multiscale problems [106]. In contrast, boundary-element methods require additional formulations to incorporate nonlocal bulk response [107], while finite-difference time-domain methods may require extremely fine structured grids to simultaneously resolve subnanometer nonlocal length scales and larger geometric features [108].

ACKNOWLEDGMENTS

C.M.F. and K.B. acknowledge funding by the Deutsche Forschungsgemeinschaft (DFG) in the framework of the Collaborative Research Center (CRC) 1375 (Project ID 398816777 - Project No. A06). F.I. acknowledges BERLIN QUANTUM, an initiative endowed by the Innovation Promotion Fund of the city of Berlin, for financial support.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX A: ASYMPTOTIC BEHAVIOR OF $\text{Im}\{g_T^\sigma(-\tau)\}$

The kernel $\text{Im}\{g_T^\sigma(-\tau)\}$ in Eq. (15) is a central quantity of the approach. In this Appendix, we examine its asymptotic behavior in the short-time and long-time limits, as well as finite- and zero-temperature regimes, for electric ($\sigma = E$) and magnetic ($\sigma = H$) dipolar sources.

In the short-time limit, i.e., for $\tau \ll 1/\gamma$ and $\tau \ll 1/\omega_{\text{th}}$, with $\omega_{\text{th}} = 2\pi k_B T/\hbar$, the polylogarithm can be expanded asymptotically as $\text{Li}_{-l}(e^{-\omega_{\text{th}}\tau}) \sim l!/(\omega_{\text{th}}\tau)^{l+1}$ [70]. In this regime, the leading-order contribution to the expression in Eq. (15) is provided by $l = s + 1$ for $\sigma = E$ and $l = s$ for $\sigma = H$. Inserting it in Eq. (15) then yields the short-time behavior,

$$\lim_{\tau \rightarrow 0} \text{Im}\{g_T^\sigma(-\tau)\} \sim \tilde{g}_0^\sigma \frac{(s+1)!}{(\gamma\tau)^{s+2}}, \quad (\text{A1})$$

with $\tilde{g}_0^\sigma = \gamma^3/2\pi j_0^\sigma s!$. Note that Eq. (A1) does not depend on the temperature and is the same for both the magnetic and electric sources, revealing a source-type-independent power-law divergence at short times [50]. For our choice $s = 4$, this coincides with the $1/\tau^6$ behavior visible in the insets of Figs. 3(c).

In the opposite limit, i.e., for $\tau \gg 1/\gamma$ and $\tau \gg 1/\omega_{\text{th}}$, the condition $e^{-\omega_{\text{th}}\tau} \ll 1$ holds, leading to an exponential decay of

the polylogarithm. As a result, the Matsubara series in Eq. (14) is dominated by its first term, which leads to the asymptotic behavior,

$$\lim_{\tau \rightarrow \infty} \text{Im} \{g_T^\sigma(-\tau)\} \sim \tilde{g}_0^\sigma \begin{cases} \frac{\omega_{\text{th}}}{2\gamma}, & \sigma = E \\ e^{-\omega_{\text{th}}\tau} \sum_{l=0}^s \binom{s}{l} \left(\frac{\omega_{\text{th}}}{\gamma}\right)^{l+2}, & \sigma = H, \end{cases} \quad (\text{A2})$$

so that the electric kernel approaches a constant, whereas the magnetic kernel decays exponentially.

As the temperature tends to zero, the spacing between adjacent Matsubara frequencies becomes infinitesimal, and the discrete sum in Eq. (14) approaches an integral,

$$\omega_{\text{th}} \sum_{m=0}^{\infty} \rightarrow \int_0^{\infty} d\xi. \quad (\text{A3})$$

In this regime, the kernel in Eq. (15) reduces to a sum of pure power-law terms, expressed as

$$\lim_{T \rightarrow 0} \text{Im} \{g_T^\sigma(-\tau)\} = \tilde{g}_0^\sigma \begin{cases} \sum_{l=0}^{s+1} \binom{s+1}{l} \frac{l!}{(\gamma\tau)^{l+1}}, & \sigma = E \\ \sum_{l=0}^s \binom{s}{l} \frac{(l+1)!}{(\gamma\tau)^{l+2}}, & \sigma = H. \end{cases} \quad (\text{A4})$$

At short times, $\tau \ll 1/\gamma$, these reproduce the divergent behavior described in Eq. (A1). In the long-time limit, $\tau \gg 1/\gamma$, only the lowest-order terms contribute, yielding

$$\lim_{\substack{T \rightarrow 0 \\ \tau \rightarrow \infty}} \text{Im} \{g_T^\sigma(-\tau)\} \sim \tilde{g}_0^\sigma \begin{cases} \frac{1}{\gamma\tau}, & \sigma = E \\ \frac{1}{(\gamma\tau)^2}, & \sigma = H, \end{cases} \quad (\text{A5})$$

revealing a power-law decay, in contrast with the constant or decaying exponential behavior observed at finite temperature. As above, the long-time asymptotic trends are confirmed by the numerical evaluation in the insets of Figs. 3(c).

APPENDIX B: THE ROLE OF γ AND χ^σ

The time-dependent current density in Eq. (11) depends on two pulse-shaping parameters: γ , which determines the temporal and spectral characteristics of the pulse, and χ^σ , a discrete source-specific parameter that modulates the temporal profile of the pulse and influences the decay behavior of the integration kernel, as discussed in Appendix A. Although both parameters are formally arbitrary within the theoretical description, a comprehensive understanding of their influence is relevant for ensuring numerical convergence, computational efficiency, and accuracy.

In the time domain, in Eq. (11), γ acts as an exponential envelope controlling the pulse width, while in the frequency domain, in Eq. (13), it determines the spectral bandwidth. Increasing γ produces temporally narrower but spectrally broader pulses. A priori, a large value of γ is desirable for broadband excitations [41]. However, in addition to shaping the temporal and spectral properties of the pulse, γ also influences the convergence behavior of the numerical scheme through the kernel $\text{Im}\{g_T^\sigma(-\tau)\}$. As discussed in Appendix A, this diverges as $1/\tau^{s+2}$ for $\tau \ll 1/\gamma$ and decays

as $1/\tau^{2-\chi^\sigma}$ for $1/\gamma \ll \tau \ll 1/\omega_{\text{th}}$, where χ^σ can be either 0 or 1. These regimes intersect at a characteristic timescale $t^* = [(s+1)!]^{1/(s+\chi^\sigma)}/\gamma$, which delineates the transition from the rising flank of the integrand, $I_{ij,T}^\sigma$, at short times, $\tau < t^*$, to the trailing edge at long times, $\tau > t^*$, cf. Figs. 3(d). To prevent premature suppression of relevant field contributions, t^* is of the order of the physical arrival time L/c . Consequently, the optimal choice of γ must balance a broadband excitation with a proper alignment of the kernel and the scattered-field arrival time. In practice, we find that setting $\gamma \lesssim c/2L$ has proven effective across a wide range of length scales.

At finite temperature, the timescale $1/\omega_{\text{th}}$ modifies the long-time behavior of the kernel. For $\tau \gg 1/\omega_{\text{th}}$, thermal effects dominate, and, depending on χ^σ , the kernel either approaches a constant or decays exponentially, as seen in Eq. (A2). For electric sources with $\chi^E = 1$ [50], the kernel becomes time-independent in the long-time limit. Therefore, the convergence of the time integration in Eq. (22) in this case is primarily dominated by the decay behavior of the scattered electric field. Note that, for $\chi^E = 1$, the current profile in Eq. (11) has zero integral over positive times as the net dipole moment vanishes at long times. As a result, the time integral in Eq. (22) oscillates around its asymptotic value, becoming less sensitive to the choice of truncation time, as seen in Fig. 6.

A different situation arises for magnetic-dipole sources. If $\chi^H = 1$ is retained, the kernel again approaches a constant at long times $\tau \gg 1/\omega_{\text{th}}$. In combination with the slow decay of the scattered magnetic field, due to the slowly decaying nonoscillatory modes in the system [67–69], as seen in the right panel of Figs. 3(b); this compromises the numerical convergence. To improve performance in the time domain at finite temperature, we therefore set $\chi^H = 0$. This forces the term $\xi_m/j^\sigma(i\xi_m)$ in Eq. (14) to vanish at $\xi_m = 0$, resulting in an exponentially decaying kernel at long times, cf. Eq. (A2).

APPENDIX C: BENCHMARK QUANTITIES

The reference quantities $\tilde{f}_{ij,T}^\sigma$ are obtained through a complex-frequency-domain evaluation of the corresponding Green tensor components and are employed to assess the accuracy of the numerical time-integration scheme. In analogy with Eqs. (6) and (7), the Maxwell stress tensor can be written in terms of the electromagnetic Green tensor in the frequency domain as

$$\langle \hat{T}_{ij}(\mathbf{r}) \rangle_T = -2k_B T \sum_{\sigma} \sum_{m=0}^{\infty} \frac{\xi_m^2}{c^2} \mathcal{G}_{ij}^\sigma(\mathbf{r}, i\xi_m), \quad (\text{C1})$$

where

$$\mathcal{G}_{ij}^\sigma(\mathbf{r}, i\xi_m) = G_{ij}^\sigma(\mathbf{r}, i\xi_m) - \frac{\delta_{ij}}{2} \sum_k G_{kk}^\sigma(\mathbf{r}, i\xi_m). \quad (\text{C2})$$

The summation over imaginary frequencies results from the Wick rotation of the real-frequency stress tensor. The

component-wise contributions to the local Casimir force per unit area are therefore

$$\tilde{f}_{ij,T}^\sigma(\mathbf{r}) = -2k_B T \sum_{m=0}^{\infty} \frac{\xi_m^2}{c^2} G_{ij}^\sigma(\mathbf{r}, i\xi_m). \quad (\text{C3})$$

However, analytical expressions for the Green tensor $\underline{G}^\sigma$ exist only for a few highly symmetric geometries. One such case is

the parallel half-space configuration discussed in Sec. IV A. Due to the translational symmetry, the scattered Green tensor depends only on the perpendicular coordinate z , and azimuthal symmetry ensures that the only nonvanishing components are $G_{xx}^\sigma = G_{yy}^\sigma \equiv G_{\parallel}^\sigma$ and $G_{zz}^\sigma \equiv G_{\perp}^\sigma$. For identical half-spaces separated by a vacuum gap of width L , the electric Green tensor reads [109]

$$G_{\parallel}^E(z, i\xi_m) = \frac{1}{8\pi} \int_0^\infty dq \frac{q}{\kappa} \left[\left(\frac{\kappa c}{\xi_m} \right)^2 \frac{2r_{\text{TM}}^2(i\xi_m, q)e^{-2\kappa L} - r_{\text{TM}}(i\xi_m, q)e^{-2\kappa z} - r_{\text{TM}}(i\xi_m, q)e^{-2\kappa(L-z)}}{1 - r_{\text{TM}}^2(i\xi_m, q)e^{-2\kappa L}} + \frac{2r_{\text{TE}}^2(i\xi_m, q)e^{-2\kappa L} + r_{\text{TE}}(i\xi_m, q)e^{-2\kappa z} + r_{\text{TE}}(i\xi_m, q)e^{-2\kappa(L-z)}}{1 - r_{\text{TE}}^2(i\xi_m, q)e^{-2\kappa L}} \right], \quad (\text{C4a})$$

$$G_{\perp}^E(z, i\xi_m) = -\frac{1}{4\pi} \int_0^\infty dq \frac{q}{\kappa} \left(\frac{qc}{\xi_m} \right)^2 \frac{2r_{\text{TM}}^2(i\xi_m, q)e^{-2\kappa L} + r_{\text{TM}}(i\xi_m, q)e^{-2\kappa z} + r_{\text{TM}}(i\xi_m, q)e^{-2\kappa(L-z)}}{1 - r_{\text{TM}}^2(i\xi_m, q)e^{-2\kappa L}}, \quad (\text{C4b})$$

for $0 \leq z \leq L$. Here, $q = |\mathbf{q}|$ is the in-plane wave-vector, and $\kappa = (q^2 + \xi_m^2/c^2)^{1/2}$ is the perpendicular component in vacuum. For planar geometries and due to the symmetry of Maxwell's equations, the magnetic Green tensor is obtained from the electric one by exchanging the TE and TM reflection

coefficients ($r_{\text{TE}} \leftrightarrow r_{\text{TM}}$) in Eqs. (C4) [95]. In practice, the reference values $\tilde{f}_{ij,T}^\sigma$, used in Sec. IV A 2, are computed by numerically evaluating the Matsubara sum in Eq. (C3) with Eqs. (C4).

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