Letter

Fast and direct preparation of a genuine lattice Bose-Einstein condensate via the quantum Mpemba effect

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(Received 3 June 2025; revised 8 October 2025; accepted 24 November 2025; published 22 December 2025)

We demonstrate that dissipative state preparation protocols in many-body systems can be substantially accelerated via the quantum Mpemba effect. Our approach exploits weak symmetries to analytically identify a class of simple, experimentally realizable states that converge exponentially faster to the steady state than typical random initializations. In particular, we study the preparation of a lattice Bose-Einstein condensate (BEC), where the depletion can be controlled via the dissipation strength. We also show how to tune the momentum of the created high-fidelity BEC by combining superfluid immersion with lattice shaking. Our theoretical predictions are confirmed by numerical simulations of the dissipative dynamics. This protocol paves the way to unlock the enormous potential of the dissipative preparation of highly entangled states in analog quantum simulators.

DOI: 10.1103/nm4b-9w5c

Preparing and controlling highly entangled states is a central goal of analog quantum simulators based on ultracold atoms in optical lattices. This includes, for instance, the realization of bosonic [1] and fermionic [2] Mott insulators, topological states [3–5], antiferromagnets [6], and many-body localized states [7]. While traditional schemes are based on coherent control, it has recently been shown that adding controlled dissipation can be computationally [8,9] and experimentally [10] advantageous in terms of robustness to noise and efficiency. Despite significant progress however, so-called dissipative state preparation (DSP) protocols still suffer from long preparation times [11] and the problem of finding optimized, fast-converging initializations has remained completely unexplored.

In this letter, we show that the Mpemba effect, which originally referred to the classical nonequilibrium phenomenon of hot systems cooling faster than warm ones, can be exploited to substantially speed up DSP protocols. As a concrete many-body example, we study the preparation of a Bose-Einstein condensate (BEC) [12,13] [see Fig. 1(a)], and exploit a discrete symmetry of the system to identify simple, experimentally realizable initial configurations that converge exponentially faster to the steady state than random ones [see Fig. 1(b)]. Note that the preparation of BECs is of particular importance, since it represents the first step for preparing *any* highly entangled state in ultracold atom platforms. Our analysis even carries over to finite-momenta BECs [14], which require the modulation of the environmentally mediated hopping by an additional complex phase via lattice shaking techniques [15–17].

The quantum Mpemba effect. The dynamics of a quantum system weakly coupled to a Markovian (i.e., memoryless) environment obeys the Lindblad master equation [18]

$$\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}t} = \mathcal{L}\hat{\rho} = -\mathrm{i}[\hat{H}, \hat{\rho}] + \sum_l \hat{L}_l \hat{\rho} \hat{L}_l^\dagger - \frac{1}{2}\{\hat{L}_l^\dagger \hat{L}_l, \hat{\rho}\}, \tag{1}$$

where $[\cdot,\cdot]$ and $\{\cdot,\cdot\}$ indicate the commutator and anticommutator, respectively, $\hat{\rho}$ denotes the system's density matrix, $\hat{H}$ the Hamiltonian, and the influence of the environment is captured by so-called jump operators $\hat{L}_l$. In the eigenbasis of the Lindbladian superoperator $\mathcal{L}$, the time-evolved density matrix can be written as [19]

$$\hat{\rho}(t) = \mathrm{e}^{\mathcal{L}t}\hat{\rho}_0 = \hat{\rho}_{\mathrm{ss}} + \sum_{p=2}^{D^2} \mathrm{e}^{\lambda_p t}\,\mathrm{Tr}(\hat{\rho}_0 \hat{l}_p)\hat{r}_p. \tag{2}$$

Here, $\hat{\rho}_0$ is the initial state, λ_p indicates an eigenvalue of $\mathcal{L}$, $\hat{l}_p$ and $\hat{r}_p$ are its corresponding left and right eigenmodes, and D is the dimension of $\hat{H}$. Since all λ_p have negative real parts, every term in Eq. (2) decays exponentially in time except for the steady state $\hat{\rho}_{\mathrm{ss}}$, which, up to normalization, is given by the right eigenmode corresponding to the eigenvalue λ_1. Let us sort the eigenvalues ascendingly according to the absolute value of their real part as $\lambda_1 = 0 < |\mathrm{Re}(\lambda_2)| \leqslant |\mathrm{Re}(\lambda_3)| \leqslant \cdots$. At late times, a typical random state will approach the steady state with an equilibration speed $\propto \exp[\mathrm{Re}(\lambda_2)t]$. Instead, special states that have zero overlap with the so-called

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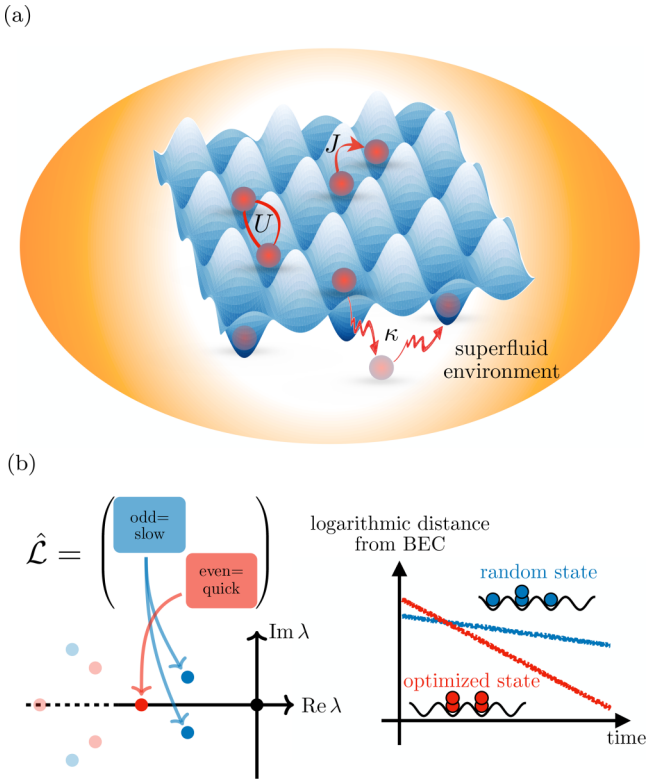


FIG. 1. Mpemba-effect-assisted preparation of a Bose-Einstein condensate (BEC) in an optical lattice. (a) A condensate of interacting bosonic particles can be prepared by combining coherent hopping with dissipation-mediated tunneling via a superfluid environment in which the optical lattice is immersed. (b) left: we consider a Lindbladian $\hat{\mathcal{L}}$ featuring a weak $\mathbb{Z}_2$ symmetry corresponding to reflections about the center of the lattice. This endows the Lindbladian with a block-diagonal structure, with one block corresponding to evenly transforming states and one to oddly transforming states. Spectral analysis shows that the slowest-decaying mode belongs to the oddly transforming block. (b) right: Based on this symmetry argument, we identify a class of (evenly transforming) product states that converge exponentially faster to the target BEC.

slowest decaying mode $\hat{l}_2$ will equilibrate exponentially faster, namely as $\exp[\mathrm{Re}(\lambda_3)t]$. Given some distance function $\mathfrak{D}$ (for instance the L_2-norm) and assuming that initially the special, fast-equilibrating states have a larger distance to the steady state $\hat{\rho}_{\mathrm{ss}}$ than typical, random ones, their distance curves with respect to $\hat{\rho}_{\mathrm{ss}}$ will cross as a function of time, which is called a Mpemba effect [20]. Besides classical Markovian systems [20–25], recently the Mpemba effect has been thoroughly investigated in isolated [26–32] and open [19,33–41] quantum systems. Yet, open many-body systems remained elusive so far, because in general the decomposition Eq. (2) can be computed only for small-scale systems amenable to exact diagonalization (ED).

Preparing a lattice BEC. We consider bosonic particles in a one-dimensional (1D) lattice described by the Bose-Hubbard Hamiltonian

$$\hat{H}_{k_0} = -J\sigma_{k_0}\sum_{j=1}^{L-1}\left(\mathrm{e}^{\mathrm{i}k_0}\hat{b}_{j+1}^{\dagger}\hat{b}_j + \mathrm{H.c.}\right) + \frac{U}{2}\sum_{j=1}^{L}[\hat{b}_j^{\dagger}]^2[\hat{b}_j]^2. \tag{3}$$

Here, $\hat{b}_j^{\dagger}$ ($\hat{b}_j$) creates (annihilates) a boson on site j, L is the number of sites, J and U represent the hopping amplitude and the on-site interaction strength, and we consider an open boundary conditions (OBCs). The prefactor σ_{k_0} is 1 for $|k_0| < \pi/2$ and -1 otherwise, ensuring that the real part of the hopping amplitude is always negative for $J > 0$ (we set the lattice spacing to one). We study the Markovian dissipative dynamics obeying Eq. (1) and choose

$$\hat{L}_j^{k_0} = \sqrt{\kappa}\left(\hat{b}_{j+1}^{\dagger} + \mathrm{e}^{-\mathrm{i}k_0}\hat{b}_j^{\dagger}\right)\left(\hat{b}_{j+1} - \mathrm{e}^{\mathrm{i}k_0}\hat{b}_j\right) \tag{4}$$

as jump operators with dissipation strength κ. Up to the phase factor $\mathrm{e}^{\mathrm{i}k_0}$ and boundary conditions, Eqs. (3) and (4) are consistent with the model proposed in Refs. [12,13]. These jump operators drive any initial state to a BEC and can be experimentally realized by immersing the system in a superfluid [42]. Using Bogoliubov theory [14,43], we derive the steady states in each k sector, which are given by $\hat{\rho}_{\mathrm{ss}}^{k} = \mathrm{e}^{-\hat{h}_{\mathrm{eff}}^{k}/T_{\mathrm{eff}}}/Z_k$ (see the Supplemental Material [44]). Here, $\hat{h}_{\mathrm{eff}}^{k} = E_{\mathrm{eff}}^{k}\hat{a}_k^{\dagger}\hat{a}_k$ is an effective single-particle Hamiltonian, T_{eff} can be interpreted as the effective temperature, and $Z_k = \mathrm{Tr}(e^{-\hat{h}_{\mathrm{eff}}^{k}/T_{\mathrm{eff}}})$. The system's steady state can be written as $\hat{\rho}_{\mathrm{ss}} = \prod_k \hat{\rho}_{\mathrm{ss}}^{k}$. Expanding E_{eff}^{k} in the limit $k \ll \sqrt{JU}/\kappa$, the effective temperature acquires a particularly simple form, which to the first nontrivial order in the dissipation strength (which had not been considered in Ref. [12]) is given by

$$T_{\mathrm{eff}} = \frac{|U|n}{2\sqrt{1+(2n\kappa/J)^2}}, \tag{5}$$

with the density $n = N/L$. Interestingly, in this limit, the effective k Hamiltonian also simplifies and reduces to Eq. (3), when replacing the interaction $U \to U_{\mathrm{eff}} = U/(1+a^2)$, where $a = 2n\kappa/J$. We stress that these results include the case $U > \kappa$, while in the limit $\kappa \gg U$ and fixed L we get $U_{\mathrm{eff}} \sim O(U/\kappa)$ and $T_{\mathrm{eff}} = 0$, as we discuss in the Supplemental Material [44]. Note how dissipation suppresses the effective repulsive interaction, eventually generating a maximal condensation in the limit $\kappa/U \gg 1$. This starkly contrasts the equilibrium case, where a true condensation is forbidden and underlines the nonequilibrium character of the steady state. In fact, our results suggest a picture in which lattice bosons scatter off the immersing BEC, yielding an effective pumping protocol. As a consequence, the effective temperatures can be significantly lower than the one of the immersing BEC, and hence, the coherence of the prepared lattice BEC much higher. To illustrate this point, we perturbatively evaluated the fraction of noncondensed particles, known as condensate depletion, finding $\delta = (N - N_0)/N \sim O[(U/\kappa)^2]$, where N_0 denotes the occupation of the condensate. Similarly, for the two-point correlation functions we obtain $\langle \hat{b}_i^{\dagger}\hat{b}_j\rangle \xrightarrow{i-j=L} n - O[(U/\kappa)^2]$, which we denote as a lattice analog of off-diagonal long-range order [45–47].

Note that both quantities can be controlled by tuning U/κ. Strikingly, the condensate depletion vanishes as $(U/\kappa)^2$, which suggests that BECs with an extremely large condensate fraction can be realized. Finally, using the phase modulation $\mathrm{e}^{\mathrm{i}k_0}$, which can be implemented via lattice shaking [16,17], i.e., by adding a fast-oscillating laser field, the mode k_0 in which the particles condense can be tuned, allowing for the

fast realization of high-quality finite-momentum BECs. We note that the case $k_0 = \pi$ is particularly simple to realize experimentally, since $e^{i\pi} = -1$ implies that both the coherent and the incoherent hopping are real and thus no lattice shaking is required.

Symmetry-adapted initial states. A set of fast-converging initial states can be found without explicitly diagonalizing $\mathcal{L}$. For that purpose, we exploit the fact that the Lindbladian and its unique steady state are invariant under a (discrete) symmetry. We emphasize that here and in the following, we refer to symmetry transformations on the vectorized, i.e., doubled, Hilbert space, which should not be confused with transformations on the physical Hilbert space only. Specifically, for $k_0 = 0$ the Lindbladian constructed from Eqs. (3) and (4) possesses the same inversion symmetry as the Hamiltonian Eq. (3), i.e., it is invariant under reflections about the center of the lattice described by the unitary transformation

$$\hat{U}_{\text{inv}} \hat{b}_j^\dagger \hat{U}_{\text{inv}}^\dagger = \hat{b}_{L+1-j}^\dagger. \tag{6}$$

The more general case is covered in the Supplemental Material [44]. This weak symmetry [48,49] decomposes $\mathcal{L}$ into two blocks, corresponding to eigenmodes that transform evenly or oddly under $\hat{U}_{\text{inv}}$. We investigate the unitary part of the Lindbladian $\mathcal{H}$ [see Eq. (1)], which in its vectorized form is given by [50]

$$\hat{\mathcal{H}} = -i\hat{H} \otimes \hat{\mathbb{1}} + \hat{\mathbb{1}} \otimes i\hat{H}^T. \tag{7}$$

In the Supplemental Material [44], we show that its vectorized eigenstates are adiabatically connected to those of $\mathcal{L}$ using perturbation theory in the limit $\kappa \to 0^+$, which is also confirmed using large-scale numerics [51]. We furthermore find that the slowest-decaying mode $\hat{I}_2$ transforms oddly under inversion for almost all parameters (U, κ), as we discuss below. An important consequence of the previous considerations is that any physically realizable state that is symmetric under reflections about the center of the lattice has zero overlap with $\hat{I}_2$ and equilibrates exponentially faster to the BEC than random initial states. This also includes the product states

$$|\psi\rangle = |n_1, n_2, \ldots, n_{L/2}, n_{L/2}, \ldots n_2, n_1\rangle, \tag{8}$$

where the j-th entry indicates the number of particles on site j. Common examples of states of the type Eq. (8) include so-called wedding-cake states $|\psi\rangle = |1, 2, \ldots, L/2-1, L/2, L/2, L/2-1, \ldots, 2, 1\rangle$, which can be readily prepared in harmonic traps. Among such symmetric, fast-converging product states, the fastest-converging is the one where all particles are initially located on the central site(s) [see Fig. 1(b)]. This can be qualitatively understood from the fact that such a state is connected to the BEC by a minimal number of hoppings and we call it the symmetrically localized (SL) state. Crucially, the SL state can also be realized experimentally in the novel hybrid setups combining optical lattices with optical tweezers [52,53]. However, we want to point out that all states of the form Eq. (8) exhibit exponential speedups. Note that the same symmetry arguments can be applied to higher-dimensional systems, and preliminary numerical evidence shows that the SL state is the fastest-converging one also in 2D.

The arguments outlined above directly apply to the zero momentum case $k_0 = 0$, only. However, Lindbladians with different characteristic momenta k_0 [see Eq. (4)] and their eigenmodes are unitarily connected to one another, as we discuss in the the Supplemental Material [44].

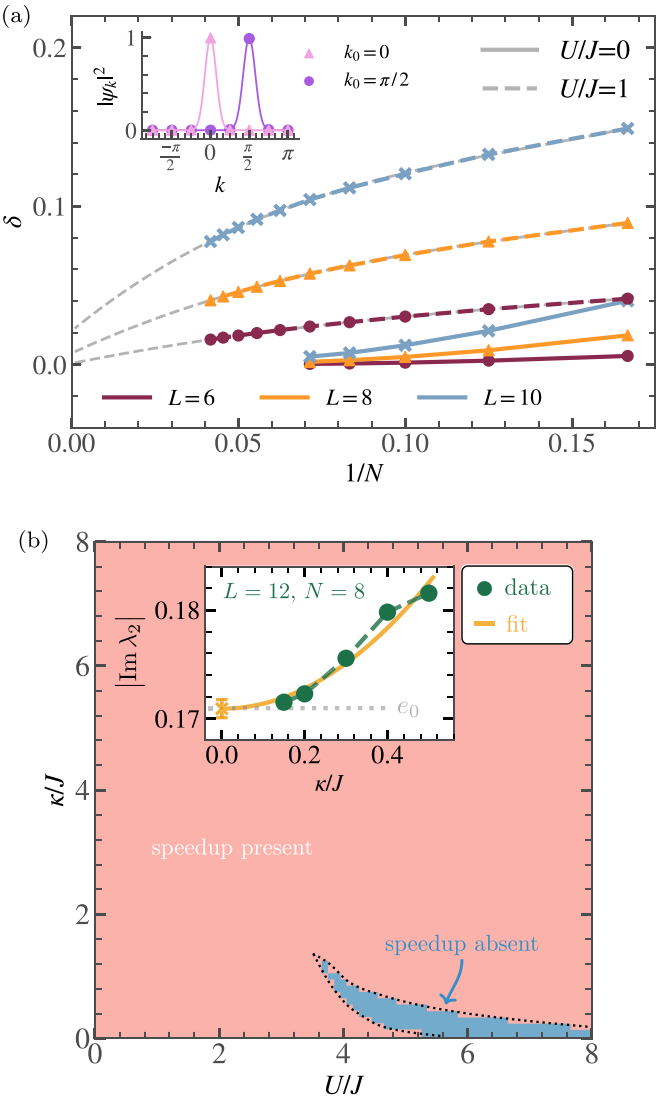


FIG. 2. (a) Simulating the dissipative preparation of a finite-momentum BEC in a 1D lattice. We show the condensate depletion as a function of the inverse total particle number N for $|k_0| < \pi/2$. The extrapolations to $1/N \to 0$ (gray lines) were performed with a third-order polynomial in $1/N$. Inset: Momentum space representation of the eigenvector ψ corresponding to the eigenvalue N_0 of $\boldsymbol{\gamma}$ for characteristic Lindbladian momenta $k_0 = 0$ and $k_0 = \pi/2$ [see Eq. (4)]. (b) Parameter regions in which symmetric initial states Eq. (8) yield a Mpemba speedup (red) and in which they do not (blue). These two regions correspond to the slowest-decaying mode of the Lindbladian transforming oddly and evenly under the inversion symmetry Eq. (6), respectively, which originate from a level crossing [64–66]. We consider $L = 6$ and $N = 3$ at $k_0 < \pi/2$ and employ ED. Inset: Criterion to numerically validate the occurrence of the Mpemba effect. In the limit $\kappa \to 0$, the imaginary part of λ_2 (green dots) converges to the eigenvalue e_0 of the first excited eigenspace of $\hat{\mathcal{H}}$ in perturbation theory [see Eq. (7)]. For the extrapolation we employ a quadratic fit (orange line) and consider $U = 0$.

Simulating the dissipative dynamics. To quantitatively assess the speed and the accuracy of our protocol, we employ matrix-product state (MPS) techniques to numerically com-

pute the dissipative evolution generated by Eqs. (3) and (4) for 1D many-body systems. We represent vectorized density matrices as MPSs [54,55] and vectorized Lindbladians as matrix-product operators (MPOs) [56–59]. The time evolution is computed with a variant of the time-dependent variational principle (TDVP) [59–63] tailored for bosonic systems (see the Supplemental Material [44] for the details of the numerical implementation).

First, in Fig. 2(a) we show the accuracy of the obtained BECs at different system sizes. For this purpose, we compute the condensate depletion δ as the deviation of the leading eigenvalue N_0 of the one-body density matrix $\boldsymbol{\gamma}$, componentwise defined via $\boldsymbol{\gamma}_{ij} = \langle \hat{b}_j^\dagger \hat{b}_i \rangle$, from the maximal possible condensate population N. The condensate depletion decreases upon increasing the total particle number N and we find an extensive scaling $\delta \sim O[1/N]$, in agreement with the Bogoliubov theory. Strikingly, extrapolating the condensate depletion as a function of $1/N$ to the limit $N \to \infty$, we observe $\delta \to 0$ also at finite interaction strengths $U/J = 1$ (dashed lines). Moreover, the inset indicates that the eigenvector corresponding to the leading eigenvalue of $\boldsymbol{\gamma}$ has almost unit weight at the characteristic momentum of the Lindbladian k_0, demonstrating the formation of a finite-momentum BEC, controlled by k_0.

Then, in Fig. 2(b) we show that in the largest part of the parameter space, the symmetric initial states Eq. (8) yield a Mpemba speedup. These ED results are supported by large-scale MPS results in the Supplemental Material [44], which demonstrate that the small area in which the Mpemba speedup is absent does not grow upon increasing the system size. In the inset we plot $|\mathrm{Im}\,\lambda_2|$ as a function of κ, which shows the slowest decaying mode of the Lindbladian stems from the first excited eigenspace of $\hat{\mathcal{H}}$ [see Eq. (7)], whose eigenvalue we denote as $\mathrm{i}e_0$, validating the perturbation theory approach that we also analyze in detail in the Supplemental Material [44].

Next, we quantify the Mpemba speedups yielded by our protocol, simulating a lattice loaded with either the SL state or random product states and different values of U/J. In Fig. 3(a), we compare the equilibration dynamics towards the BEC. As a distance measure, we consider the L_2-norm $\mathfrak{D}(t) = \|\hat{\rho}(t) - \hat{\rho}_{\mathrm{ss}}\|_2$, which is the easiest to compute in the vectorized framework. We find that the SL state converges exponentially faster than random product states for all considered bosonic repulsion strengths U/J. Taking a more practical perspective, the actual speedup to achieve a certain condensate fidelity is the relevant quantity. For that purpose, Fig. 3(b) shows the corresponding speedups $S(\epsilon)$ relative to a desired L_2 distance ϵ from the steady state $\hat{\rho}_{\mathrm{ss}}$. Notably, the speedups do not change significantly when varying the system size or the number of particles. This indicates that similar speedups can also be achieved for larger system sizes. Note that due to unitary equivalence, the speedups are the same for all $|k_0| < \pi/2$ and for all $|k_0| \geqslant \pi/2$ [44].

Discussion and conclusions. Dissipative state preparation (DSP) is finally becoming a viable experimental alternative to coherent control-based preparation schemes in terms of robustness and efficiency [8–10,67], but still suffers from long preparation times. In this letter, we show that the Mpemba effect can be exploited to find simple, experimentally realizable initializations that substantially reduce the protocol times. In particular, we study the important example of a BEC [12,13], which represents the starting point for preparing any highly entangled state in ultracold atoms. Crucially, we exploit a weak symmetry of the system to find optimized initializations without diagonalizing the Lindbladian, which allows us to study a genuine many-body system. In the limit of strong dissipation, we analytically show that the steady state exhibits true long-range order on the length scales of the lattice. Moreover, we use MPS to simulate the protocol numerically and quantitatively estimate the obtained speedups. The numerical

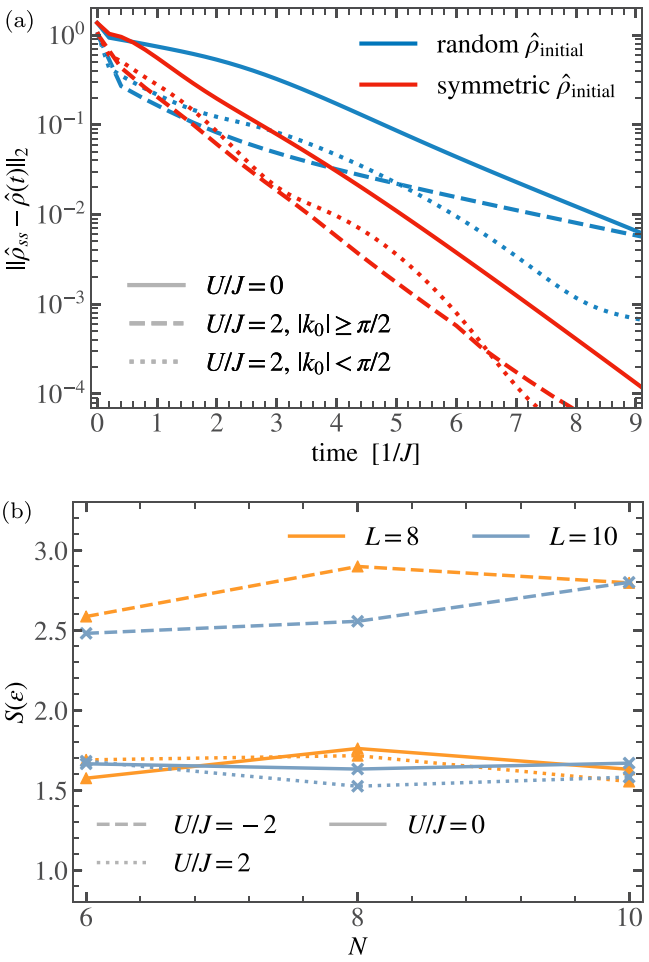


FIG. 3. Mpemba speedups in the preparation of BECs. (a) The symmetrically localized (SL) state (red lines), where all particles are initially located on the central site(s), converges exponentially faster to the steady state $\hat{\rho}_{\mathrm{ss}}$ than random initial product states (blue lines). Random states are generated by distributing N particles on the lattice, sampling the positions from a uniform distribution over the sites. Line styles indicate different bosonic interaction strengths and characteristic momenta k_0. All calculations were performed with system parameters $L = 10, N = 10$, local dimension $d = N + 1$, and $\kappa = 2J$, and we averaged over five product state realizations. (b) The corresponding speedups $S(\epsilon) = t_{\mathrm{random}}/t_{\mathrm{symmetric}}$ as a function of the total particle number. Note that the speedup is dictated by the spectrum of the Lindbladian, namely $S(\epsilon) \to \mathrm{Re}\,\lambda_3/\mathrm{Re}\,\lambda_2$ as $\epsilon \to 0$. t_{random} and $t_{\mathrm{symmetric}}$ are the times at which the L_2 distance from the steady state has dropped below a precision threshold $\epsilon = 10^{-4}$ for the random and the SL states, respectively.

results also confirm the predicted scaling for the depletion $\delta \propto (U/\kappa)^2$ already for moderate dissipation strength.

Our symmetry-based approach to find fast-converging initial states can be applied to a wide class of many-body open quantum systems. This significantly reduces the preparation times and the associated errors of DSP protocols, paving the way to the realization of complex states, such as η-paired superconducting states in ultracold atoms [47], which have thus far remained elusive.

Acknowledgments. We are grateful to Monika Aidelsburger for enlightening discussions and Henning Schlömer, Timothy Harris, and Federico Carollo for useful comments on a previous version of the manuscript. P.W. and S.P. acknowledge support by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strategy-426 EXC-2111-390814868. This work was supported by Grant No. INST 86/1885-1 FUGG of the German Research Foundation (DFG). M.M. acknowledges funding from the Royal Society and Research Ireland. All calculations were performed using the SyTen toolkit [68,69].

Data availability. The data that support the findings of this article are openly available [70].

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