

Fundamental and second-subharmonic Autler-Townes splitting in classical systems

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The dynamic Stark effect and Autler-Townes splitting (ATS) are hallmarks of driven two-level systems. We establish a direct correspondence between these quantum phenomena and the parametric normal mode splitting in coupled classical oscillators. This identifies a second-subharmonic ATS under a two-tone parametric drive. We find excellent agreement between the theory and the vibrations of a nanomechanical two-mode system, capturing both the fundamental and second-subharmonic ATS, and allowing quantitative extraction of the modal coupling irrespective of the degree of modal hybridization.

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I. INTRODUCTION

The dynamic (ac) Stark effect, first identified by Autler and Townes [1], describes the field-induced modification of energy levels in coherently driven quantum systems. In atomic and optical physics, this light shift forms the basis of optical lattices and lattice clocks [2,3] and governs coherent optical control in semiconductors [4,5]. Controlled Stark shifts are likewise essential for qubit manipulation and field sensing in Rydberg and superconducting architectures [6,7], for modulation of excitons [8–10], and for inducing dressed states in cavity and circuit quantum electrodynamics and cavity optomechanics [11,12]. Although originally described semi-classically, its quantized description arises naturally within the Jaynes-Cummings model [13].

Autler-Townes splitting (ATS) is an immediate consequence of the dynamic Stark effect when the field is tuned at the atom transition frequency [1]. An analogous phenomenology is found in classical systems, where a parametrically induced modal coupling produces the so-called parametric normal mode splitting (PNMS) [14]. This is a consequence of the similarity of the dynamical structure due to the underlying time-varying potential; however, a rigorous mathematical mapping beyond the rotating-wave approximation is yet to be established.

In the classical regime, two linearly (but not necessarily strongly) coupled mechanical modes produce PNMS upon the additional application of a parametric drive [15]. The approximate relation for PNMS in Ref. [15] has been applied to achieve tunable and controllable coupling in various

nano- and micromechanical systems [15–23]. However, the relation between the PNMS splitting width $2g$ and the modal coupling strength λ has not been appreciated to date. In the case of dispersively coupled modes, the parametric nature of the coupling induces the same phenomenon [24,25]. This is, e.g., apparent in strongly coupled cavity optomechanical systems [12,26]. Note that the term avoided crossing is often used in the literature interchangeably with normal mode splitting [26,27]. Here we will adhere to the following convention: The term “avoided crossing” will be solely employed for the case of lifting the degeneracy of the natural eigenfrequencies when tuned on resonance via an external parameter. This phenomenon is also known in mechanics as “mode veering” [28,29]. The term “parametric normal mode splitting” will refer to the splitting of *each* of the two linearly coupled normal modes into two Floquet-dressed spectral branches under the application of an additional parametric drive tuned to their frequency difference.

In this work we show that the dynamics of a driven two-level system are in one-to-one correspondence with those of parametrically coupled classical harmonic oscillators under the slow-envelope approximation. Both systems exhibit a fundamental splitting at the transition (or difference) frequency in addition to higher-order splittings at its odd fractions [1]. Moreover, we show that applying a two-tone parametric drive with a frequency ratio of 1:2 introduces an additional splitting at half the modal difference frequency—an effect that has been observed before in parametrically driven classical systems [15,18] but was not considered in the original ATS framework [1]. This *second-subharmonic* ATS can be linked to signatures of bichromatically driven atoms [30].

In addition, we present a thorough theoretical model to describe the PNMS from a basic classical model using the multiple scales perturbation method [31,32], relating it to the coupling strength and the associated Bloch-Siegert effect [33]. This facilitates the quantification of the coupling strength even in the weak limit, where the avoided crossing effect is no longer resolved [34]. The results are experimentally validated

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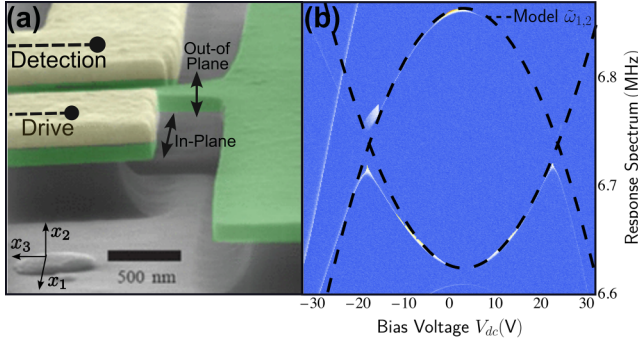


FIG. 1. (a) Scanning electron micrograph of a SiN nanostring (green) flanked by two golden electrodes (yellow), each connected to either the drive or the detection circuitry. (b) Spectrogram of the IP and OOP frequencies of the nanostring as a function of the bias voltage V_{dc} varied from -32 V to $+32$ V. The data are overlaid with a model of the uncoupled, tunable eigenfrequencies $\tilde{\omega}_i$ (dashed black lines), revealing two avoided crossing points at $V_{dc} \simeq \pm 20$ V.

by exploring the two fundamental flexural eigenmodes of a strongly prestressed nanomechanical string resonator, denoted as out-of-plane (OOP) and in-plane (IP) modes, which are dielectrically coupled by a dc voltage applied between two adjacent electrodes [see Fig. 1(a)] [35]. The modes of the string are driven by a white-noise force, while the parametric coupling is induced by modulating both modes at their difference frequency with an additional tone.

II. TWO-MODE SYSTEM MODELING

The equations of motion of the IP (x_1) and OOP (x_2) fundamental flexural eigenmode of the nanostring surrounded by the inhomogeneous electric field between the electrodes [see Fig. 1(a)] are

$$\begin{aligned} \ddot{x}_1 + 2\beta_1\dot{x}_1 + [\tilde{\omega}_1^2 + \eta_1(t)]x_1 + [\lambda^2 + \eta_c(t)]x_2 &= f_1(t), \\ \ddot{x}_2 + 2\beta_2\dot{x}_2 + [\tilde{\omega}_2^2 + \eta_2(t)]x_2 + [\lambda^2 + \eta_c(t)]x_1 &= f_2(t), \end{aligned} \quad (1)$$

with $\tilde{\omega}_{1,2}$, λ representing the dielectrically tunable eigenfrequencies and coupling strength, respectively, and $\beta_{1,2}$ are the amplitude damping rates [36–38]. A driving force $f_i(t)$ as well as inherently induced parametric actuation terms $\eta_{1,2}(t)$, $\eta_c(t)$ are applied. Under weak driving, nonlinearities and higher-order coupling terms [39] do not influence the system dynamics and are omitted for the sake of simplicity. As a result of the dielectric coupling, x_1 and x_2 hybridize, and the observable modes are found after diagonalization of Eq. (1), yielding the equations of motion in the hybridized coordinates q_i ($i = 1, 2$)

$$\begin{aligned} \ddot{q}_1 + 2\gamma\dot{q}_1 + [\Omega_1^2 + \Sigma_1(t)]q_1 + \zeta\Lambda(t)q_2 &= F_1(t), \\ \ddot{q}_2 + 2\gamma\dot{q}_2 + [\Omega_2^2 + \Sigma_2(t)]q_2 + \Lambda(t)q_1 &= F_2(t), \end{aligned} \quad (2)$$

with damping γ , modal eigenfrequencies Ω_i , drive $F_i(t)$, parametric actuation $\Sigma_i(t)$ and time-dependent parametric coupling $\Lambda(t)$ oscillating at frequency Ω_p , and a scaling factor ζ [40]. The relation between the modal eigenfrequencies of the hybridized modes Ω_i and the original modes is

$$\Omega_{1,2}^2 = \frac{1}{2}(\tilde{\omega}_1^2 + \tilde{\omega}_2^2) \mp \frac{1}{2}\sqrt{(\tilde{\omega}_2^2 - \tilde{\omega}_1^2)^2 + 4\lambda^4}. \quad (3)$$

Similarly, the coordinate transformation establishes a relation between the amplitude of the parametric coupling Λ and the original coupling strength λ [40]

$$\Lambda_\zeta = \frac{\sqrt{\zeta}V_{ac}(t)\delta\omega^2}{V_{dc}} \times \left(\frac{\delta\omega^2 + (c_{11} - c_{22})V_{dc}^2}{\sqrt{4\lambda^4 + ((c_{11} - c_{22})V_{dc}^2 + \delta\omega^2)^2}} - 1 \right), \quad (4)$$

where $\Lambda_\zeta = \sqrt{\zeta}\Lambda$, $\delta\omega^2 = (\tilde{\omega}_2^2 - \tilde{\omega}_1^2)|_{V_{dc} \approx 0}$, and c_{ii} are the tuning coefficients for $\tilde{\omega}_i$ with V_{dc} [see Eq. (S18)] [40].

At given $\Omega_i(V_{dc})$, the multiple-scale method is applied up to the second order for $\Omega_p \simeq \delta\Omega = |\Omega_2 - \Omega_1| \ll \Omega_1, \Omega_2$, as discussed in [40]. The splitting width is deduced to be

$$2g = \tilde{\Lambda} \sqrt{1 + \left(\frac{\tilde{\Lambda}}{4\delta\Omega} \right)^2}, \quad (5)$$

where $\tilde{\Lambda} = \Lambda_\zeta / (2\sqrt{\Omega_1\Omega_2})$ is the Rabi frequency. The last equation reflects the ATS expression [1], where the second term under the root is found to represent the Bloch-Siegert shift [33,40,41]. This resemblance emerges from a correspondence between the dynamical structures that induce both ATS and PNMS. A canonical change of variables $(q_i, \dot{q}_i) \mapsto (C_i, C_i^*)$ under a slow-envelope approximation converts Eq. (2) into the semiclassical ATS form [1]

$$\begin{aligned} i\dot{C}_1(t) &= \Omega_1 C_1(t) + \tilde{\Lambda} \cos(\Omega_p t) C_2(t), \\ i\dot{C}_2(t) &= \Omega_2 C_2(t) + \tilde{\Lambda} \cos(\Omega_p t) C_1(t), \end{aligned} \quad (6)$$

where $\Sigma_i = 0$, $\gamma = 0$, $\zeta = 1$ are assumed for simplicity.

III. FLOQUET SPECTRUM

The quasi-energies (-frequencies) of either the parametrically excited classical system or the driven two-level system can be calculated equivalently using Floquet analysis. We begin with the classical system (2) for $\Sigma_i = 0$, $\gamma = 0$, $\zeta = 1$, and $\Lambda(t) = \Lambda \cos(\Omega_p t)$. We introduce the ansatz

$$q_i(t) = e^{i\mu t} \sum_n q_i^{(n)} e^{in\Omega_p t}, \quad (7)$$

and by spectral decomposition, this yields

$$(\Omega_i^2 + (\mu + in\Omega_p)^2)q_i^{(n)} + \frac{\Lambda}{2}(q_j^{(n-1)} + q_j^{(n+1)}) = 0. \quad (8)$$

This corresponds equivalently to the quantum system (6), using the ansatz $C_i = e^{i\sigma t} \sum_n c_i^{(n)} e^{-in\Omega_p t}$, which yields [1]

$$(\sigma + \Omega_i - n\Omega_p)c_i^{(n)} + \frac{\tilde{\Lambda}}{2}(c_j^{(n-1)} + c_j^{(n+1)}) = 0. \quad (9)$$

In each of the classical or the quantum versions of the problem, this eigenvalue problem is represented by a series in (n) Fourier orders. However, considering the order n of a degree of freedom i , the series shows only possible couplings

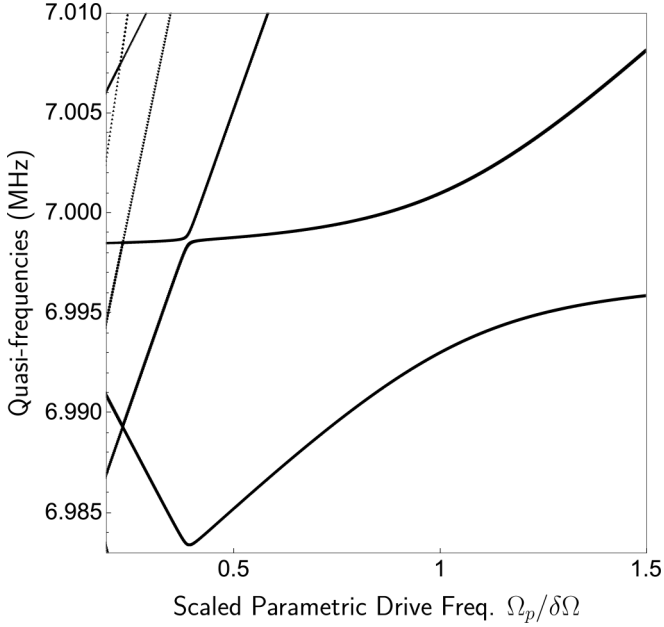


FIG. 2. Quasi-energies calculated either for the semiclassical quantum eigenvalues (9) or the classical ones (8), thus showing identical ATS/PNMS structures to the original plot of Autler and Townes [1].

with the directly neighboring $n \pm 1$ of the other degree of freedom j . This means that the odd series $n = 1, 3, 5, \dots$ of i is coupled to the even series $n = 2, 4, 6, \dots$ of j and vice versa. Nevertheless, the odd and even series of i (and also of j) are separable, creating two independent subsystems [42]. They combine to form a *Floquet Hamiltonian* H_F [43], in which these subsystems are not coupled by relevant off-diagonal terms. The eigenvalue problem $H_F \mathbf{c} = \sigma \mathbf{c}$ of Eq. (9) can be represented in matrix form as

$$\sum_{j=1}^2 \sum_{m \in \mathbb{Z}} H_{F(i,n),(j,m)} c_j^{(m)} = \sigma c_i^{(n)}, \quad i \in \{1, 2\}. \quad (10)$$

The quasi-energies (σ or $\text{Im}[\mu]$) of both (semiclassical) quantum and classical systems are depicted in Fig. 2. The ATS/PNMS at $\Omega_p = |\Omega_2 - \Omega_1|/n$, $n = 1, 3, 5, \dots$ show identical structures under the transformation $\tilde{\Lambda} = \Lambda/(2\sqrt{\Omega_1\Omega_2})$ (cf. Sec. S1 [40]). However, no splittings can take place at even fractions $\Omega_p = |\Omega_2 - \Omega_1|/n$, $n = 2, 4, \dots$ —a property well known in theory [1]. This is based on the separation of the even and odd series, as explained earlier. This condition is, however, violated if H_F includes coupling terms between both subsystems. This means that $c_i^{(n)}$ (or $q_i^{(n)}$) are not only related to $c_j^{(n\pm 1)}$ but also to $c_j^{(n\pm 2)}$, and only then can *second-subharmonic ATS/PNMS* take place. In this case the eigenvalue problems for both cases read

$$(\sigma + \Omega_i - n\Omega_p)c_i^{(n)} + \frac{\tilde{\Lambda}}{2}(c_j^{(n-1)} + c_j^{(n+1)}) + \frac{\tilde{\Lambda}_{II}}{2}(c_j^{(n-2)} + c_j^{(n+2)}) = 0, \quad (11a)$$

$$[\Omega_i^2 + (\mu + in\Omega_p)^2]q_i^{(n)} + \frac{\Lambda}{2}(q_j^{(n-1)} + q_j^{(n+1)}) + \frac{\Lambda_{II}}{2}(q_j^{(n-2)} + q_j^{(n+2)}) = 0. \quad (11b)$$

An experimental realization of Fig. 2 is shown in Fig. 5 for one nanomechanical string resonator, and a plot for another string is found in Sec. S8 [40], where in the latter the splittings are more pronounced but also include nonlinear effects. The physical ground for the appearance of second-subharmonic ATS in our experiment is related to the presence of $|V_{ac}^2|$ as discussed in a following section.

IV. MEASUREMENT

The PNMS model is applied to the two orthogonally polarized fundamental flexural modes, the OOP and IP modes, of a strongly prestressed nanomechanical string resonator. The string is made of stoichiometric silicon nitride and has a length, width, and thickness of 65 μm , 250 nm, and 100 nm, respectively. It is placed between two adjacent gold electrodes for dielectric transduction and control, and read out by means of microwave cavity-assisted heterodyne displacement detection using a coaxial $\lambda/4$ microwave cavity as described elsewhere [35–37,40,44]. Throughout this work, a microwave cavity pump power of 13 dBm is applied on resonance with the cavity mode for displacement detection.

The nanostrip is characterized by probing its resonant response to a coherent resonant drive V_{ac} with a constant amplitude and a swept frequency as a function of the applied dc bias voltage V_{dc} . Figure 1(b) shows the resulting eigenfrequency tuning of the OOP and the IP mode. Clearly,

the characteristic approximately quadratic tuning behavior leading to a stiffening (softening) of the OOP (IP) mode, respectively, is observed [37]. The avoided crossings near $V_{dc} \approx \pm 20$ V are reminiscent of the strong coupling between OOP and IP [38]. The response of the system changes strongly with the system parameters. While parts of the spectrum remain below the noise level, other parts exhibit a strongly nonlinear response. Notice that the spurious mode on the left side of the figure corresponds to the IP mode of a shorter string on the sample. The response of the OOP and IP mode is modeled using Eq. (3) for $\lambda = 0$ and fitted to the tuned eigenfrequencies $\tilde{\omega}_{1,2}$ in the vicinity of $V_{dc} = 0$ to extract the tuning coefficients c_{ii} as shown as black dashed lines in Fig. 1(b). The magnitude of the avoided crossing is indicative of the coupling λ when both modes are tuned on resonance as discussed in detail in [40].

To observe the PNMS, the string is exposed to a parametric excitation V_{pump} along with a white-noise driving force V_{noise} and the dc bias voltage V_{dc} . The voltages, V_{dc} and $V_{ac} = V_{\text{noise}} + V_{\text{pump}}$, are combined with a bias-T and applied between the two electrodes. That said, the parametric excitation results from the inherent dielectric interaction, as any oscillating voltage V_{ac} will both act as a direct drive and modulate the modes' eigenfrequencies; see [40] for details. The predominant interaction is set by the choice of frequency: Voltages close to $\tilde{\omega}_i$ drive mode i on resonance, while their

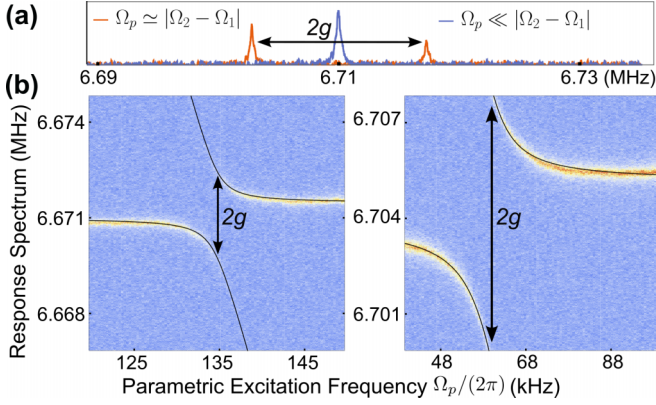


FIG. 3. (a) Unperturbed resonance of Ω_1 at $V_{dc} = -22$ V (blue) and split PNMS peaks (orange) under additional parametric perturbation at $\Omega_p \approx \delta\Omega$. (b) Sweeping Ω_p around $\delta\Omega$ reveals full PNMS branches, at $V_{dc} = -14$ V (left) and -20 V (right), encoding response amplitudes in color (blue to red). Solid black lines represent the fitted model [see Eq. (S42) [40]] showing an excellent agreement with the experiment.

parametric contribution represents a second-order effect and is thus negligible. On the other hand, voltages fulfilling a parametric resonance condition $\Omega_p = |\Omega_i \pm \Omega_j|/n$ with positive integer n produce parametric effects. In our case we focus on the difference combination frequency condition $\Omega_p = |\Omega_2 - \Omega_1|$, which yields the PNMS. This situation is also known as the beam-splitter interaction [12].

The response of the system near Ω_1 to a parametric modulation with a constant $V_{\text{pump}} = 2$ V is depicted in Figs. 3(a) and 3(b). The blue trace in Fig. 3(a) shows a single peak representing one of the two hybridized modes, Ω_1 , excited using the noise drive. The orange trace shows the response when the parametric resonance condition is additionally fulfilled, giving rise to PNMS. The apparent normal mode splitting reveals $2g$.

Figure 3(b) shows PNMS measurements for a swept parametric excitation frequency Ω_p at two different dc bias voltages, representing different coupling strengths and leading to splittings of significantly different widths. Away from the parametric resonance condition, the system converges to Ω_1 , while the response branches off in its vicinity. The response of the system is fitted using the PNMS model [see Eq. (S42) [40]]. The black solid lines in Fig. 3(b) show excellent agreement with the experimental data. The parametric coupling Λ obtained from the fit is converted into $2g$ using Eq. (5). Additionally, the vertical arrows indicate the width of the normal mode splitting $2g$ at the point of minimal separation of the two branches, $\Omega_p = |\Omega_2 - \Omega_1|$, yielding 2 and 7 kHz for Fig. 3(b) (left) and (right), respectively.

V. COUPLING STRENGTH ESTIMATION

The PNMS measurement is repeated for a large number of dc bias voltages between -32 to $+32$ V. The resulting splittings' widths and corresponding parametric coupling values are plotted as a function of V_{dc} as black dots in Fig. 4. In this figure the solid blue line depicts a fit of Eq. (4) to the data, assuming a linear relation [38], $\lambda = \sqrt{c_\lambda} V_{dc}$, between λ and

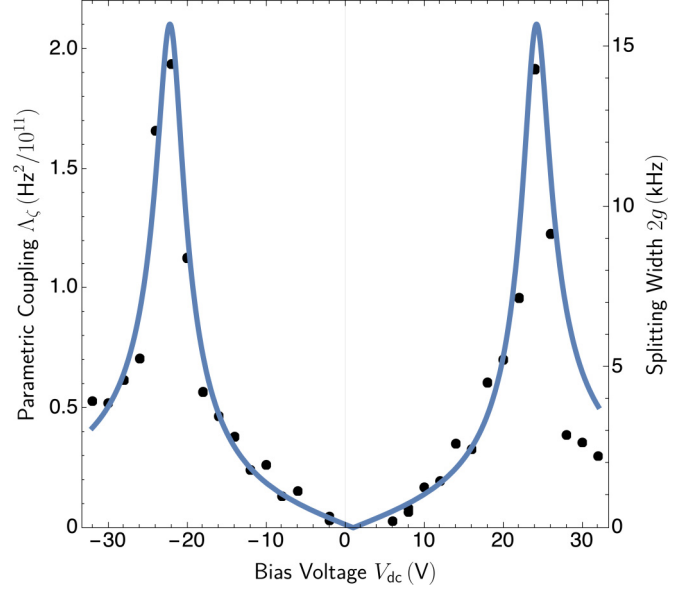


FIG. 4. Splitting width $2g$ and the corresponding scaled parametric coupling $\Lambda_\zeta = \sqrt{\zeta}\Lambda$ values across the bias voltage range (black points), fitted with Eq. (4) based on the ansatz $\lambda = \sqrt{c_\lambda} V_{dc}$ to obtain c_λ ; see Sec. S8 [40].

V_{dc} [40]. We find $c_\lambda \approx 4.41 \times 10^8 (\text{Hz/V})^2$. Figure 4 shows a very good agreement between theory and experiment. The axis on the right expresses the result in terms of the width of the normal mode splitting $2g$, taking advantage of $2g \propto \Lambda_\zeta$ according to Eq. (5), and exploiting that the second term under the square root is negligible compared to the leading one in our experimental parameters (cf. Fig. S3 [40]). Note that λ can also be calculated directly without the *a priori* assumption $\lambda = \sqrt{c_\lambda} V_{dc}$ using Eqs. (4) and (5), highlighting the $\lambda - 2g$ relation.

Both $2g$ and Λ_ζ exhibit a symmetric behavior around $V_{dc} \simeq 0$ V, as expected from the dielectric origin of the coupling. Despite the ever-increasing coupling strength λ with $|V_{dc}|$, the splitting width reaches a maximum value at the avoided crossing [cf. Fig. 1(b)] before declining further. This can be explained by the hybridization of IP and OOP modes that are coupled by λ , resulting in Eq. (4). It is instructive to compare the magnitude of the coupling strength λ obtained from the PNMS with the value of λ deduced from the width of the avoided crossing. The avoided crossing produces a splitting $\Gamma/(2\pi) \approx \lambda^2/(2\pi\tilde{\omega}) \approx 47$ kHz (see Sec. S3 [40]) at $|V_{dc}| \approx 20$ V (only the splitting at -20 V is experimentally resolved); see Fig. 1(b). With $\tilde{\omega} = \tilde{\omega}_1 = \tilde{\omega}_2 \approx 2\pi \times 6.735$ MHz on resonance, this yields $\lambda/(2\pi) \approx 0.56$ MHz. As detailed above, the PNMS allows for determining λ at any bias voltage for known c_λ . Near resonance, a coupling strength of $\lambda/(2\pi) \approx \sqrt{4.41 \times 10^8} \times 20 = 0.42$ MHz is found, in good agreement with the value determined from the avoided crossing.

VI. SECOND-SUBHARMONIC ATS

The ATS-PNMS correspondence, discussed above, suggests that the PNMS experiment should map the dynamic Stark quasi-energies as discussed earlier. As depicted in

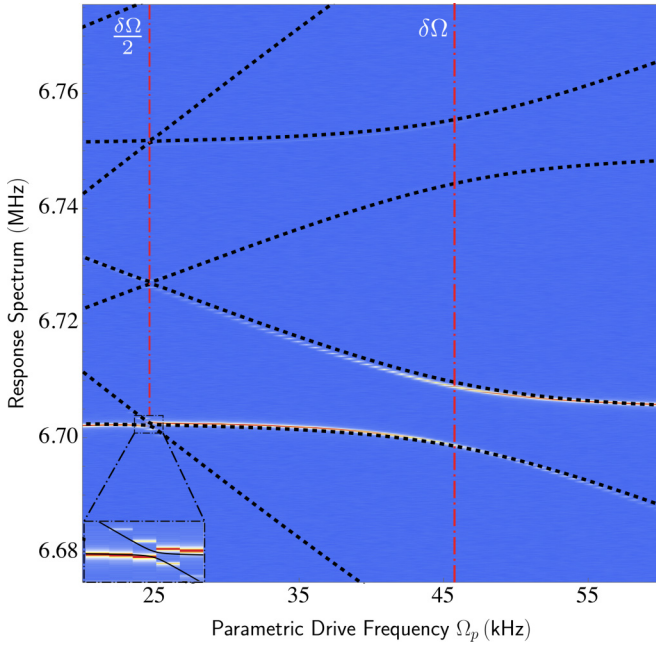


FIG. 5. Experimental map of the quasi-energies at the avoided crossing point $V_{dc} \simeq -20$ V by sweeping Ω_p , showing the fundamental ATS at $\Omega_p = \delta\Omega$ and the second-subharmonic ATS at $\delta\Omega/2$ (magnified in the inset). The splittings of Ω_1 are clearly resolved, while those of Ω_2 lie mostly under the noise level. The dashed black lines show the Floquet model for a two-tone drive.

Fig. 5, a PNMS experiment is conducted at nearly the avoided crossing resonance condition $V_{dc} \simeq -20$ V, thus bringing both modal frequencies Ω_i to the nearest possible value. The figure shows the fundamental PNMS/ATS of both hybrid modes at $\Omega_p = \delta\Omega$. In addition, we observe a splitting at $\delta\Omega/2$ that can not be explained by the original ATS model [1].

This observation suggests the presence of a second parametric coupling tone with the double frequency $2\Omega_p$, which produces the term $(\Lambda_{II}/2)(q_j^{(n-2)} + q_j^{(n+2)})$ in Eq. (11). Under the linear assumption, the plausible explanation in the current case is the invalidity of the approximation $V_{ac}^2 \propto \cos(2\Omega_p t) \approx 0$ in the expansion of the forcing term $(V_{ac} + V_{dc})^2$ (see Sec. S6 [40]). Although V_{ac}^2 remains by at least one order of magnitude smaller than $2V_{dc}V_{ac}$, its small contribution opens up a splitting at $\Omega_p = \delta\Omega/2$ as seen in the inset of Fig. 5. Constructing the Floquet Hamiltonian with two-tone drive $\Omega_p, 2\Omega_p$ captures both fundamental and second-subharmonic splittings. As shown in Fig. 5, the theory shows a very good agreement at both the fundamental and the second-subharmonic ATS. Since the model quantitatively captures the second-subharmonic ATS in the linear case, this can provide a useful tool to differentiate between two-tone parametric excitations, which are explained by the linear model, and nonlinear second-harmonic generation, e.g., at higher amplitudes, in similar systems.

VII. CONCLUSIONS

There have been several attempts to establish dynamical correspondences between quantum and classical systems

with time-dependent Hamiltonians [45–49]. A particularly relevant subclass consists of driven two-level systems with a single-tone parametric coupling, which exhibit the dynamic Stark effect and the associated Autler-Townes splitting at resonance [1]. We show that the dynamics of such systems can be mapped onto those of parametrically coupled classical harmonic oscillators under the slow-envelope approximation. As a result, under the ATS resonance condition, a splitting of the normal mode takes place in classical systems, which has been studied extensively in nanoelectromechanical and cavity optomechanical systems under the name parametric normal mode splitting (PNMS). One manifestation of this correspondence is the generation of PNMS at the difference frequency $\delta\Omega$ and its odd fractions. Splittings at $\delta\Omega/2$ are theoretically not permitted for a single-tone parametric drive. We show via Floquet analysis that using a two-tone parametric drive $\Omega_p, 2\Omega_p$ in the ATS/PNMS model, however, induces a second-subharmonic ATS at exactly $\delta\Omega/2$. This establishes a connection between second-subharmonic splittings previously observed in parametrically driven nanomechanical systems [15,18] and the generalized ATS framework for bichromatic driving fields or two-photon processes [30,50].

In the near-resonant domain $\Omega_p \sim \delta\Omega$, we develop a rigorous model for a system of two parametrically excited coupled harmonic oscillators that realizes PNMS for each of the two oscillators. Our model establishes a direct relation between the width of the normal mode splitting $2g$ and the coupling strength λ , an aspect which has not been appreciated to date. This provides a tool to quantify the coupling between two modes irrespective of its strength. For the case of weak coupling, this quantity is inaccessible to standard dynamical characterization methods due to the hybridization of the laboratory coordinates. For strong coupling, an avoided crossing directly reveals the coupling strength if the eigenfrequencies are widely tunable. The presented method overcomes this hurdle. It can be applied not only in strain-coupled nanomechanical systems [51–56], but also in optical or microwave cavity arrays [57,58], and in cavity quantum electrodynamics [59,60], just to name a few. In addition, it can be applied to hybrid systems combining several degrees of freedom, such as spin-mechanical systems [61,62]. Here the presented method supplies a means to detect the coupling to inaccessible dark states, since PNMS can be measured at only one of the two excited modes.

The theoretical model is complemented by an experimental study using a nanoelectromechanical two-mode system. We study the two orthogonal fundamental flexural modes of a dielectrically controlled SiN nanostring resonator. Dielectric driving is mediated by a dc bias voltage that couples and tunes the eigenmodes. We apply an additional parametric modulation for two different experiments. In the first, we tune the parametric frequency at the difference frequency to induce a PNMS from which the coupling strength is extracted quantitatively in a range of dc bias voltage. In the second experiment, we adjust the bias voltage to the value at which the avoided crossing takes place, then we sweep the parametric drive from 20 to 60 kHz to capture both fundamental and second-subharmonic ATS/PNMS for both hybridized modes. In both cases we compare the observed splittings with the

respective theoretical model and find an excellent traceability of the PNMS.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [63]; embargo periods may apply.

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