

Time-reversal interferometry using Schrödinger-cat states with scalable entangling resources

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We propose a method for generating Schrödinger-cat states—defined as equal superpositions of arbitrary coherent spin states—using a concise sequence of rapid, alternating one-axis twisting and rotation pulses. We demonstrate that the required shearing strength for the protocol, which scales linearly with time, decreases with increasing number of atoms (N) in proportion to $1/\sqrt{N}$. The resulting states achieve optimal quantum Fisher information among superpositions of two coherent spin states with a specified angular separation. This makes them well suited to exceed the classical phase sensitivity limit in quantum metrology applications. Notably, our protocol is compatible with a time-reversal strategy for quantum metrology, ensuring its practical viability. We also demonstrate that the Heisenberg-limit scaling remains intact when reducing the twisting alongside the number of atoms, mitigating losses from photon scattering. Although the states are then prepared only approximately, they retain enough catlike structure to sustain this scaling.

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I. INTRODUCTION

High-precision measurements are crucial in various research areas, including testing fundamental laws [1,2], general relativity [3,4], quantum computing [5], and searching for new physics [6]. Achieving higher measurement precision can impact diverse fields such as gravitational wave detection [7], timekeeping [8], and geodesy [9–11]. However, the discrete nature of light and matter limits measurement error to the standard quantum limit (SQL), which scales as $1/\sqrt{N}$ with the number of probes N . Quantum effects, such as entanglement, can surpass this limit, enabling quantum metrology to achieve the Heisenberg limit (HL) scaling, where measurement errors scale as $1/N$ [12]. This allows for ultra-high-precision sensing with relatively few atoms by exploiting quantum properties.

The capability to attain HL scaling depends on quantum correlations in the generated multiatomic states used as a probe. Noncorrelated states, such as coherent spin states (CSS), can only achieve SQL scaling, while maximally entangled states, such as Greenberger-Horne-Zeilinger (GHZ) states, enable HL scaling. However, generating and

manipulating GHZ states is challenging due to decoherence, dissipation, and atom losses [13–15].

Fortunately, other states can also achieve sub-SQL scaling [16–18], such as Schrödinger-cat states [19], which have higher immunity to decoherence [20–23]. In this paper, we adopt the convention that Schrödinger-cat states are superpositions of two CSSs. In contrast to the GHZ state, the CSSs forming the Schrödinger-cat states are not necessarily orthogonal. Thus, Schrödinger-cat states can be considered as a generalization of GHZ states. Other states that can also attain sub-SQL scaling include spin-squeezed states [24–29]. These states exhibit reduced variance in one measured quadrature, while increasing variance in an orthogonal measurement. Both the Schrödinger-cat states and spin-squeezed states can be generated using pulse sequences of one-axis twisting (OAT) [24,27] and rotations [28,30–34].

In this work, we propose to use a pulse sequence of OAT and rotations to generate Schrödinger-cat states using limited entangling resources. The central advance concerns the scaling of those resources. The total shearing strength required to generate these states favorably decreases as $1/\sqrt{N}$, unlike the time evolution under a pure OAT Hamiltonian, which also can generate Schrödinger-cat states in N -independent time [31,35]. This advantage enables the implementation of a time-reversal interferometric protocol [18,36–46] based on these sequences, in which the many-atom quantum system undergoes a pulse sequence that creates the Schrödinger-cat state, free evolution, and the inverse pulse sequence. This sequence amplifies the phase acquired during free evolution, yielding

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Heisenberg-limited scaling. Unlike standard spin squeezing for Ramsey interferometry, our approach does not reduce variance to generate enhancement, which substantially relaxes detection-noise requirements compared to squeezing-based protocols [38]. Our method is versatile and applicable to any OAT setup, using only engineered atom-atom interactions and standard rotation pulses. We focus on implementing this scheme in an optical-lattice clock experiment, where a time-reversal interferometric protocol has been demonstrated [41,42]. In this setup, photons incoherently scattered during the generation of the OAT Hamiltonian project subensembles of atoms into ground and excited states, causing coherence loss and reducing the contributing atom number at the final measurement [41,47]. Within a simplified, experimentally motivated model [41], our analysis accounts for photon-scattering-induced contrast loss that diminishes measurement precision. Using the proposed scheme, we demonstrate generating various Schrödinger-cat-like states and show that HL scaling is achieved even when target states are prepared approximately. This will enable ultra-high-precision sensing in various research areas.

The outline of our paper is as follows. In Sec. II, we characterize Schrödinger-cat states using the quantum Fisher information (QFI) and identify which states maximize the QFI for a given angular separation between the constituent coherent spin states. In Sec. III, we prove which are the necessary controls to create Schrödinger-cat states and find pulse sequences to generate them. In Sec. IV, we present a time-reversal interferometric protocol capable of achieving HL scaling by creating and dismantling the Schrödinger-cat states to amplify interferometric signals. A final discussion is in Sec. V.

II. CHARACTERIZING SCHRÖDINGER-CAT STATES

We consider an ensemble of N spin-1/2 particles, or equivalently, N identical two-level systems. The collective spin operator $\vec{S} = [\hat{S}_x, \hat{S}_y, \hat{S}_z]$ is defined by its components $\hat{S}_i = \frac{1}{2} \sum_{n=1}^N \hat{\sigma}_{i,n}$, where $i = x, y, z$ and $\hat{\sigma}_{i,n}$ is the Pauli matrix acting on the particle n . We define a CSS as a state where all individual spins are in the same state; consequently, the collective spin state is

$$|\theta, \phi\rangle = \bigotimes_{\ell=1}^N \left(\cos \frac{\theta}{2} |\uparrow\rangle_{\ell} + e^{i\phi} \sin \frac{\theta}{2} |\downarrow\rangle_{\ell} \right), \quad (1)$$

where θ and ϕ define the direction in spherical coordinates where the individual spins are oriented.

Since a Schrödinger-cat state is a superposition of two CSSs $|\theta_1, \phi_1\rangle$ and $|\theta_2, \phi_2\rangle$, its wave function can be represented as

$$|\Psi_{\text{cat}}\rangle = \frac{1}{C} (|\theta_1, \phi_1\rangle + |\theta_2, \phi_2\rangle), \quad (2)$$

where $C^2 = 2\text{Re}(1 + C_N)$ is the normalization factor, $C_N = (C_1 C_2 + e^{i\Delta\phi} S_1 S_2)^N$, $\Delta\phi = \phi_2 - \phi_1$, $C_i = \cos(\theta_i/2)$, $S_i = \sin(\theta_i/2)$.

To quantify the potential of Schrödinger-cat states for quantum metrology, we evaluate the QFI, which for pure

states is defined by

$$\mathcal{F}_Q = 4\Delta\hat{S}_z^2, \quad (3)$$

with $\Delta\hat{O}^2 = \langle\hat{O}^2\rangle - \langle\hat{O}\rangle^2$. The quantum Cramér-Rao bound (qCRB), $\Delta\varphi^2 = 1/\mathcal{F}_Q$, defines the minimum uncertainty that can be achieved for an estimation of φ using a given collective spin state $|\Psi_{\text{cat}}\rangle$. The phase $\varphi = \omega\tau$ is the one acquired while the system evolves under the Hamiltonian $\hat{H}_0 = \omega\hat{S}_z$ for a time τ , for instance, during the free evolution in a Ramsey interferometry procedure.

For an arbitrary Schrödinger-cat state, we obtain

$$\langle\hat{S}_z\rangle = \frac{N}{C^2} \text{Re}[C_+ C_- + C_{N-1} (C_1 C_2 - e^{i\Delta\phi} S_1 S_2)] \quad (4)$$

and

$$\begin{aligned} \langle\hat{S}_z^2\rangle = & \frac{N}{4} + \frac{N(N-1)}{4C^2} \text{Re}[\cos^2\theta_1 + \cos^2\theta_2 \\ & + 2C_{N-2} (C_1 C_2 - e^{i\Delta\phi} S_1 S_2)^2], \end{aligned} \quad (5)$$

where $C_{\pm} = \cos[(\theta_1 \pm \theta_2)/2]$, $C_k = (C_1 C_2 + e^{i\Delta\phi} S_1 S_2)^k$.

Given a great-circle angle θ between each CSS, the Schrödinger-cat state that is symmetric with respect to the xy and xz planes maximizes the QFI as defined by Eq. (3). Under these assumptions, we have

$$|\Psi_{\text{cat}}(\theta)\rangle = \frac{|\pi/2 - \theta/2, 0\rangle + |\pi/2 + \theta/2, 0\rangle}{\sqrt{2 + 2\cos^N(\theta/2)}}. \quad (6)$$

We show that this state is a local maximum of the QFI in Appendix A.

When considering the state in Eq. (6), we obtain

$$\mathcal{F}_Q = N + N(N-1) \frac{\sin^2(\theta/2)}{1 + \cos^N(\theta/2)}. \quad (7)$$

Similar expressions for general Schrödinger-cat states can be found for the QFI in arbitrary directions using the complementary expressions that we write in Appendix B. If $\theta = 0$, we have a CSS and the minimum uncertainty possible is $\Delta\varphi = N^{-1/2}$, corresponding to the SQL. If $\theta \neq 0$, we can take the limit $N \gg 1$, to obtain

$$\Delta\varphi^2 = \frac{1}{\mathcal{F}_Q} \approx \frac{1}{N^2 \sin^2(\theta/2)}, \quad (8)$$

which exhibits HL scaling, even for an arbitrarily small value of θ , when the overlap between CSSs, $\langle\pi/2 - \theta/2, 0|\pi/2 + \theta/2, 0\rangle$, is very large. Note that $\theta = \pi$ gives exactly the HL, as expected from a GHZ state. One can understand this from pure geometrical considerations; in the limit $N \gg 1$, the average projection onto the z axis for the CSS in the northern hemisphere is $(N/2) \sin(\theta/2)$. Similarly, the average projection onto the z axis for the CSS in the southern hemisphere is $-(N/2) \sin(\theta/2)$. Thus, the variance for the Schrödinger-cat state is $N^2 \sin^2(\theta/2)/4$, and consequently the QFI $\mathcal{F}_Q = N^2 \sin^2(\theta/2)$, as we found in Eq. (8).

III. CREATING SCHRÖDINGER-CAT STATES

We propose that alternating OAT applications with collective rotations is sufficient to drive a CSS to an optimal

Schrödinger-cat state. To prove this claim, we introduce the Dicke basis $|S, m\rangle$, with $S = N/2$ and $m = -S, -S + 1, \dots, S - 1, S$, corresponding to the eigenfunctions of $\hat{S}_z$, namely, $\hat{S}_z |S, m\rangle = m |S, m\rangle$.

We note that Schrödinger-cat states are symmetric on this basis, thus $|\Psi_{\text{cat}}(\theta)\rangle = \sum_{m=-S}^S a_m |S, m\rangle$ with $a_m = a_{-m}$. Thus, it is convenient to describe the dynamics to define the subspace of symmetrized Dicke states,

$$|S, m\rangle_s = \begin{cases} \frac{1}{\sqrt{2}}(|S, m\rangle + |S, -m\rangle) & \text{if } m \neq 0, \\ |S, 0\rangle & \text{if } m = 0. \end{cases}$$

On this basis,

$$\hat{S}_z^2 |S, m\rangle_s = m^2 |S, m\rangle_s,$$

and

$$\begin{aligned} \hat{S}_x |S, m\rangle_s &= \frac{1}{2} \sqrt{S(S+1) - m(m+1)} |S, m+1\rangle_s \\ &\quad + \frac{1}{2} \sqrt{S(S+1) - m(m-1)} |S, m-1\rangle_s, \end{aligned}$$

which shows that these operators preserve $a_m = a_{-m}$ symmetry and time evolution under them remains in the symmetrized subspace.

Now we turn to controllability. Let us consider $\hat{H}_0 = \hat{S}_z^2$ to be the drift Hamiltonian and $\hat{H}_1 = \hat{S}_x$ to be the control Hamiltonian. We can note three things: (i) $\hat{H}_1$ couples the state $|S, m\rangle_s$ with $|S, m+1\rangle_s$ and $|S, m-1\rangle_s$. Consequently, all nearest-neighbor states are coupled, and all states are accessible. (ii) All transitions are nondegenerate, as the transition frequencies are $\omega_{m,m-1} = \langle S, m | \hat{S}_z^2 | S, m \rangle_s - \langle S, m-1 | \hat{S}_z^2 | S, m-1 \rangle_s = 2m-1$, and are different for all values of m . (iii) All relevant (nearest-neighbor) fractions of transition frequencies $\omega_{m,m-1}/\omega_{m',m'-1}$ are rational numbers. According to Refs. [48–50], these three conditions are sufficient for pure state controllability. Perhaps the key point is that the $\hat{H}_0 = \hat{S}_z^2$ nonlinearity removes the degeneracy and enables control.

Summing up, OAT and rotations around the x axis are sufficient to create any state in this subspace. In particular, one can start from a CSS oriented to the x axis $|\pi/2, 0\rangle$ (which also belongs to the symmetric subspace) and create a Schrödinger-cat state. Note that these controls are available in the experimental setup in which we are focusing on our approach.

We propose a sequence of n OAT applications alternating with n collective rotations along the x direction for the creation of Schrödinger-cat states starting from a CSS oriented to the x direction, $|\pi/2, 0\rangle$. Figure 1 illustrates the proposed pulse sequence with the state distribution over the generalized Bloch sphere for an example of $N = 20$ atoms. Intuitively, OAT rotates the distribution on the generalized Bloch sphere with magnitude proportional to z . Thus, the northern and southern hemispheres rotate in opposite directions, with the magnitude being greatest at the poles. The first squeezing pulse creates a squeezed distribution, but it can break it into a two-peaked distribution upon reorienting the distribution appropriately (see Fig. 1). Finally, by repeating this process appropriately, one can generate a target Schrödinger-cat state.

To find the pulse sequence that generates the objective Schrödinger-cat state, we propagate the initial CSS given

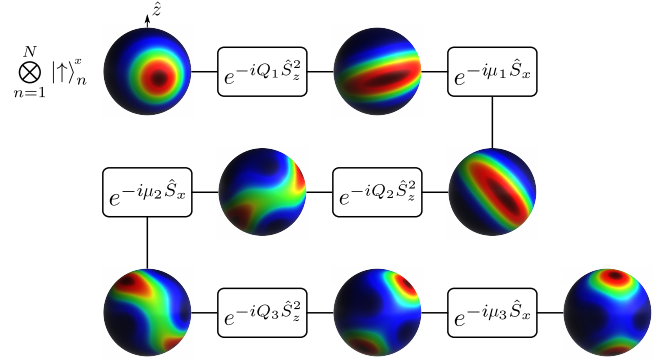


FIG. 1. Sequence example where $n = 3$ pulses are used to create a Schrödinger-cat state with $\theta = \pi/2$ in $N = 20$ atoms. The infidelity with respect to the target is $\epsilon \approx 0.02$. The spheres correspond to a Husimi Q representation.

values of Q_k and μ_k

$$\begin{aligned} |\Psi_{\text{guess}}\rangle &= \exp(-i\mu_n \hat{S}_x) \exp(-iQ_n \hat{S}_z^2) \cdots \\ &\quad \times \exp(-i\mu_1 \hat{S}_x) \exp(-iQ_1 \hat{S}_z^2) |\pi/2, 0\rangle, \end{aligned} \quad (9)$$

where $Q_k = \chi t_k$ is the shearing strength of the k th OAT pulse and μ_k is the rotation angle produced by the k th $\hat{S}_x$ pulse. Next, we calculate the gradient of the infidelity $\epsilon = 1 - |\langle \Psi_{\text{cat}}(\theta) | \Psi_{\text{guess}} \rangle|^2$ with respect to (Q_k, μ_k) as outlined in Appendix C. In this way, we iteratively find the parameters (Q_k, μ_k) that minimize ϵ . For instance, the sequence in Fig. 1 creates a Schrödinger-cat state with $\theta = \pi/2$ and infidelity $\epsilon \approx 0.02$. In general, the parameters we obtain via optimization are not trivial, in the sense that they don't correspond to a Trotterization of a known Hamiltonian, nor are they all equal.

IV. INTERFEROMETRIC PROTOCOL

In general, an interferometric protocol has three stages. The first is probe-state preparation, the second is free evolution, in which the system evolves under a free Hamiltonian, and the third is detection. An example of this is the Ramsey interferometry illustrated over the generalized Bloch sphere in Fig. 2(a). In this protocol, the first stage is to prepare a CSS at the equator using a $\pi/2$ pulse. In the free evolution, the system evolves under the Hamiltonian $\hat{H} = \omega \hat{S}_z$ for a time $\tau = \varphi/\omega$. At last, in the detection stage, a $\pi/2$ pulse maps the phase, or the rotation angle, into the projection onto the z axis. Note that this pulse is the inverse of the first $\pi/2$ pulse, exhibiting a time-reversal quality. Finally, $\langle \hat{S}_z \rangle$ is measured and the result allows the estimation of φ with SQL uncertainty.

In Fig. 2(b), we illustrate the interferometric protocol that we propose. The state preparation consists of the Schrödinger-cat state creation, and free evolution is under $\hat{H} = \omega \hat{S}_z$. However, we must determine appropriate detection, as simply applying a $\pi/2$ pulse before readout will not yield any population difference due to the probe state's symmetry. The approach we will consider here is time reversal. Consequently, we propose to apply the inverse of the pulse sequence that creates the Schrödinger-cat state after free evolution, in analogy to the standard Ramsey interferometry. In this case, the last stage maps the phase to a variation of the expected value of

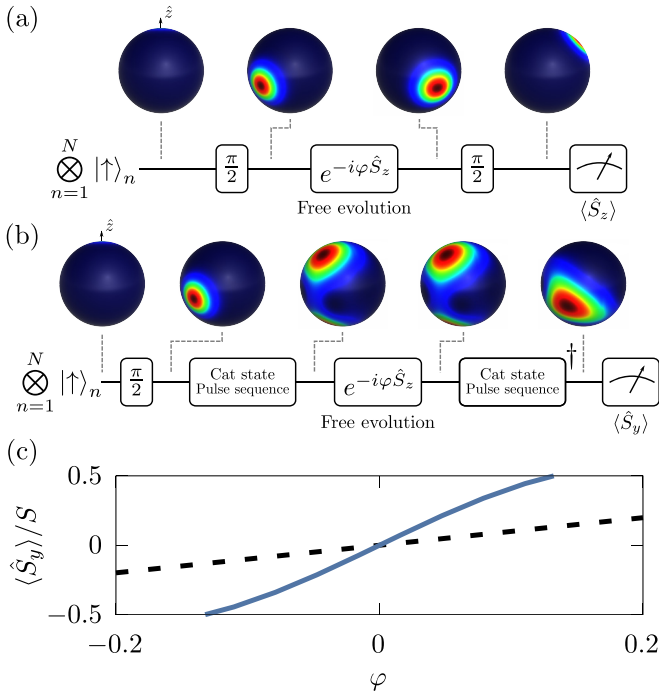


FIG. 2. (a) Schematic of the standard Ramsey interferometry scheme using a CSS. (b) Schematic of the modified Ramsey interferometry scheme using a Schrödinger-cat state. (c) The mean value $\langle \hat{S}_y \rangle / S$ as a function of the angle φ at the end of our modified Ramsey interferometry scheme. The dashed black line represents the maximum signal that can be reached with a CSS (SQL). The sequence uses $n = 3$ twisting and rotation pulses, with a total normalized shearing strength $\tilde{Q} = 1$, to generate the Schrödinger-cat state with $\theta = \pi/2$ for $N = 100$ atoms. The spheres correspond to a Husimi Q representation.

$\hat{S}_y$ (which can be mapped to a population difference by a $\pi/2$ pulse). In the absence of an accumulated phase, the protocol returns the system to its initial state. Otherwise, it provides a state close to the starting CSS but with the displacement proportional to the accumulated phase.

The minimal resolvable phase of the protocol $\Delta\varphi$ can be estimated from $\Delta\hat{S}_y$ at the end of the protocol, namely,

$$\Delta\varphi = \left| \frac{\Delta\hat{S}_y}{\partial \langle \hat{S}_y \rangle / \partial \varphi} \right|. \quad (10)$$

Thus the metrological gain with respect to the SQL is given by

$$\mathcal{G} = \frac{(\Delta\varphi)_{\text{SQL}}^2}{(\Delta\varphi)^2} = \frac{1}{N(\Delta\varphi)^2}. \quad (11)$$

In this expression, HL scaling corresponds to an exponent $b = 1$ in $\mathcal{G} = aN^b$. As in previous time-reversal implementations [41], since the resulting state is close to being a CSS, metrological gain comes from a displacement surpassing the one from Ramsey interferometry. Therefore, $\langle \hat{S}_y \rangle = \mathcal{M}\varphi S$, where the amplification factor $\mathcal{M}$ exceeds unity, which is the maximum permitted by the standard quantum limit (SQL). This relationship is illustrated by plotting $\langle \hat{S}_y \rangle$ as a function of φ in Fig. 2(c) for the proposed interferometric sequence. The

plot demonstrates an increased slope near $\varphi = 0$ compared to the dashed line, which represents the SQL, corresponding to standard Ramsey interferometry.

In an optical-lattice clock experiment [41], collective entanglement generation relies on the interaction of the atomic ensemble with a light cavity field. Cavity feedback squeezing [47,51] is a deterministic technique to generate an effective OAT Hamiltonian $\chi \hat{S}_z^2$. The spin quantum noise tunes the atom-cavity resonance such that the intracavity light intensity is proportional to $\hat{S}_z$. However, any information contained in the light field results in the nonunitary evolution of the atomic system [52]. In particular, a photon scattered in free space projects one atom into either spin-up or spin-down, thereby reducing spin coherence and the length of the spin vector. However, it has been recently demonstrated [41,47], and modeled [52,53], that decoherence can be minimized to reach a near-unitary evolution by tuning the entangling-light frequency. Finally, by switching the sign of the detuning between the atom-cavity resonance and the light, the sign of χ can be reversed [47,52].

As in Ref. [41], we parametrize the spin vector's length as $C_{\text{sc}} = \exp(-\gamma \tilde{Q})$, where

$$\tilde{Q} = \sqrt{N} \sum_k Q_k \quad (12)$$

is the total normalized shearing strength and γ is the number of free-space scattered photons per atom, per unit of shearing strength, which we choose to be equal to 0.36 addressing the experimental conditions reported in Ref. [41]. Consequently, larger values of $\tilde{Q}$ in the pulse sequence that creates Schrödinger-cat states result in a lower signal amplitude $\langle \hat{S}_y \rangle$ at the measurement. To minimize this effect in the metrological gain scaling, we fix $\tilde{Q}$ during the optimization of the pulse sequence. As a result, the contrast loss becomes a proportionality factor and does not affect the scaling. Furthermore, it ensures a time scaling proportional to $1/\sqrt{N}$. However, this could reduce the fidelity of the prepared state.

Figure 3 provides an analysis of the scaling behavior of the proposed approach with respect to the number of atoms N . In Fig. 3(a), the prepared states for $N = 50$ are shown on the generalized Bloch sphere using the Husimi Q representation. Figure 3(b) presents the infidelity of the generated Schrödinger-cat states as a function of N , illustrating convergence as N increases. Furthermore, Figure 3(b) shows that increasing θ leads to higher infidelities, which makes the states more difficult to generate using this technique. Figure 3(c) displays the achieved metrological gain for different values of θ , defined as the angle between the coherent spin states forming the target Schrödinger-cat state, and $\tilde{Q}$, the total normalized shearing strength. The scaling exponent for the metrological gains approaches unity and runs parallel to the gray zone boundary, which is indicative of Heisenberg limit (HL) scaling. This scaling is attributed to the convergence of the infidelity. Even when fidelities are imperfect, a portion of the state remains in a Schrödinger-cat configuration, which is sufficient to generate quantum Fisher information $\mathcal{F}_Q$ proportional to N^2 and thereby achieve HL scaling. This illustrates the distinction between the high-fidelity preparation of the target Schrödinger-cat state and the generation of a state that retains enough catlike structure to provide Heisenberg-like

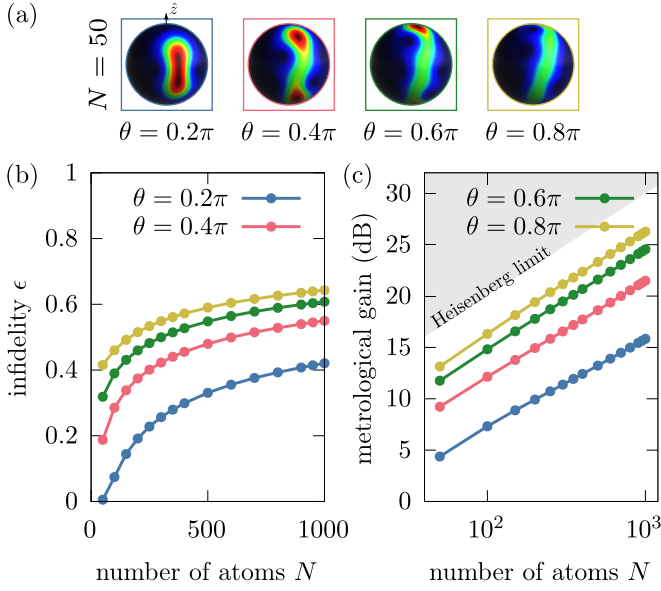


FIG. 3. Scaling of our approach when using a fixed total normalized shearing strength $\tilde{Q} = 1$ and $n = 3$ OAT and rotation pulses. (a) Examples of Schrödinger-cat states generated by pulse sequence for $N = 50$ (Husimi Q representation). (b) Infidelity of the generated Schrödinger-cat states as a function of N . (c) Metrological gain with respect to the SQL as a function of N for different angles, θ , between the coherent spin states that form the target Schrödinger-cat states. The exact HL is depicted as the boundary between accessible and inaccessible metrological gains, and is highlighted in a darker color for reference.

scaling of the metrological gain: the infidelities in Fig. 3(b) preclude the former in several parameter regimes, while the gains in Fig. 3(c) demonstrate the latter. When photon scattering losses are considered, $\tilde{Q} = 1$ corresponds to a loss of 6.4 dB. As a result, the states analyzed in this work maintain a net positive metrological gain. Comparing Fig. 3(b) and Fig. 3(c) reveals that, although larger θ values increase infidelity, they also result in higher metrological gain.

The infidelity analysis is further detailed in Appendix D. The results indicate that fidelity converges to lower values as either the number of pulses n or $\tilde{Q}$ increases. Consequently, any loss of fidelity, such as that associated with increasing θ , can be mitigated by employing longer pulse sequences or additional entangling resources. However, increasing $\tilde{Q}$ may also lead to greater contrast loss, resulting in the existence of an optimal value. A brief analysis of this tradeoff is provided in Appendix E.

A key consideration for experimental implementation is the robustness of the metrological gain to variations in pulse amplitude. To evaluate this, a systematic multiplicative error in the OAT strengths and rotation angles is introduced, and the metrological gain is recalculated. The error is drawn per realization from a Gaussian distribution of 1% width, the calibration accuracy demonstrated in the intended platform [41]. Figure 4 shows the resulting metrological gain scaling for total normalized shearing strengths $\tilde{Q} = 0.5$ and $\tilde{Q} = 1.0$ in Figs. 4(a) and 4(b), respectively. The metrological gain is essentially unaffected at all atom numbers considered. The spread is largest at the largest atom numbers, where it is smaller

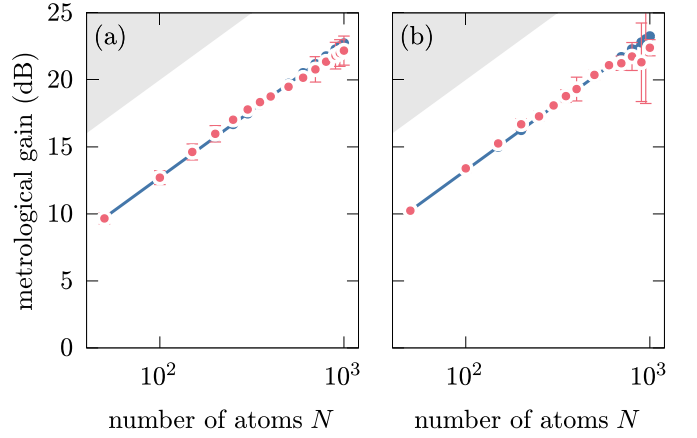


FIG. 4. Metrological gain scaling under systematic variations of the pulse parameters. (a) shows results with its standard deviations for a total normalized shearing strength $\tilde{Q} = 0.5$, a target angle $\theta = \pi/2$ between the coherent spin states, and $n = 4$ twisting and rotation pulses with a common systematic calibration error of the implemented pulse areas, drawn per realization from a Gaussian distribution of 1% width, the accuracy demonstrated in the intended platform [41]. (b) shows results for the same parameters but with $\tilde{Q} = 1.0$. The solid line in both plots represents the results for ideal pulses. For reference, we include the exact HL as the boundary between accessible and inaccessible metrological gains (in darker color).

in 4(a) than in 4(b), showing that an appropriate choice of $\tilde{Q}$ can mitigate the effect of control errors. These findings indicate that the proposed scheme exhibits robustness to pulse amplitude variations. In Appendix F we extend this analysis further into a control-error budget for the intended platform.

V. DISCUSSION AND CONCLUSION

We have demonstrated that the appropriate OAT pulses and rotations can generate any symmetric state in the Dicke basis, including symmetric Schrödinger-cat states with respect to the equatorial plane of the generalized Bloch sphere. These states have attracted some attention because of their squeezing, planar squeezing [54], and their potential in the framework of quantum computing [55]. Moreover, Schrödinger-cat states are also useful for ultra-high-sensitivity sensing and metrology under the appropriate protocol because of their HL scaling Fisher information [16,31].

We developed a time-reversal interferometric protocol and analyzed its performance, focusing on potential implementation in an optical-lattice clock experiment. The protocol employs a short pulse sequence of alternating OAT and rotation pulses to create Schrödinger-cat states, and then uses the opposite pulse sequence after free evolution, thus creating a protocol in which any accumulated phase breaks time-reversal symmetry. Our results show HL scaling even when considering contrast loss due to the same light-mediated interactions that create the squeezing, i.e., limiting the shearing strength of the OAT applications. The protocol uses the identical elements as existing OAT implementations and provides a time scaling proportional to $1/\sqrt{N}$, which is certainly favorable.

Thus, it does not require major modification to an existing practical setup. The required control accuracy is quantified in Appendix F. At the atom numbers of current experiments, the tolerated mismatch between the preparation and the time-reversal readout coincides with the demonstrated calibration accuracy.

Our analysis demonstrates that a small number of pulses, specifically three OAT and rotation pulses, is sufficient to observe Heisenberg limit (HL) scaling. Fidelities can be further improved by employing longer sequences, targeting superpositions of CSSs separated by smaller angles, or allocating additional squeezing resources. However, there is likely an optimal amount of squeezing resources, or total shearing strength, due to the detrimental effects of excessive entanglement on system coherence.

Finally, our approach is related to the kicked-top model. In this model, repeating alternating one-axis twisting and rotation pulses generates chaotic dynamics with Heisenberg-scaling quantum Fisher information [56,57]. In contrast, we investigate sequences with few pulses and optimize them for a target state. This avoids the random states of the chaotic regime, which have quantum Fisher information $\mathcal{F}_Q = N^2/3$, about 4.77 dB away from the Heisenberg limit. Our scheme is also related to the twist-and-turn model, $\hat{H} = \chi \hat{S}_z^2 + \Omega \hat{S}_x$. This model can generate near-GHZ states [42,58,59]. Our approach also allows the angle between the coherent spin states to be freely chosen.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: MAXIMUM QFI OF SYMMETRIC SCHRÖDINGER-CAT STATES

To show that the state in Eq. (6) locally maximizes the QFI, we compute how it changes when rotating the state using the unitary $\hat{R} = e^{-i\alpha \hat{S}_x - i\beta \hat{S}_y}$. Thus, the QFI after rotation becomes $\mathcal{F}_Q = 4\langle \hat{R} \hat{S}_z^2 \hat{R}^\dagger \rangle - 4\langle \hat{R} \hat{S}_z \hat{R}^\dagger \rangle^2$. Expanding up to quadratic order assuming $\alpha, \beta \ll 1$, we obtain

$$\begin{aligned} \langle \hat{R} \hat{S}_z \hat{R}^\dagger \rangle &\approx \left(1 - \frac{\alpha^2 + \beta^2}{2}\right) \langle \hat{S}_z \rangle - \alpha \langle \hat{S}_y \rangle + \beta \langle \hat{S}_x \rangle \\ &\approx \beta \langle \hat{S}_x \rangle, \end{aligned} \quad (\text{A1})$$

since $\langle \hat{S}_z \rangle = \langle \hat{S}_y \rangle = 0$ due to the symmetry of the state. Similarly,

$$\begin{aligned} \langle \hat{R} \hat{S}_z^2 \hat{R}^\dagger \rangle &\approx (1 - \alpha^2 - \beta^2) \langle \hat{S}_z^2 \rangle + \alpha^2 \langle \hat{S}_y^2 \rangle + \beta^2 \langle \hat{S}_x^2 \rangle \\ &\quad - \alpha \langle \{\hat{S}_y, \hat{S}_z\} \rangle + \beta \langle \{\hat{S}_x, \hat{S}_z\} \rangle - \alpha\beta \langle \{\hat{S}_x, \hat{S}_y\} \rangle. \end{aligned} \quad (\text{A2})$$

This reduces to

$$\langle \hat{R} \hat{S}_z^2 \hat{R}^\dagger \rangle \approx (1 - \alpha^2 - \beta^2) \langle \hat{S}_z^2 \rangle + \alpha^2 \langle \hat{S}_y^2 \rangle + \beta^2 \langle \hat{S}_x^2 \rangle. \quad (\text{A3})$$

Given the symmetry of the state, for any pair of axes u and v , we have $\langle (\hat{S}_u + \hat{S}_v)^2 \rangle = \langle (\hat{S}_u - \hat{S}_v)^2 \rangle$. This implies that

$$\langle \{\hat{S}_u, \hat{S}_v\} \rangle = \langle (\hat{S}_u + \hat{S}_v)^2 \rangle - \langle (\hat{S}_u - \hat{S}_v)^2 \rangle = 0. \quad (\text{A4})$$

Consequently, we can approximate up to second order

$$\begin{aligned} \mathcal{F}_Q &= 4(1 - \alpha^2 - \beta^2) \langle \hat{S}_z^2 \rangle + 4\alpha^2 \langle \hat{S}_y^2 \rangle \\ &\quad + 4\beta^2 (\langle \hat{S}_x^2 \rangle - \langle \hat{S}_x \rangle^2). \end{aligned} \quad (\text{A5})$$

We conclude that corrections to the QFI are quadratic and with negative concavity (since $\Delta \hat{S}_z^2 > \Delta \hat{S}_{x,y}^2$). Consequently, symmetric Schrödinger-cat states locally maximize the QFI under the stated symmetry assumptions.

APPENDIX B: EXPECTATION VALUES FOR ARBITRARY SCHRÖDINGER-CAT STATES

For an arbitrary Schrödinger-cat state, we obtain the following expressions that complement Eqs. (4) and (5),

$$\begin{aligned} \langle \hat{S}_x \rangle &= \frac{N}{2C^2} \text{Re}[\sin \theta_1 \cos \phi_1 + \sin \theta_2 \cos \phi_2 \\ &\quad + 2C_{N-1}(e^{-i\phi_1} \mathcal{S}_1 \mathcal{C}_2 + e^{i\phi_2} \mathcal{C}_1 \mathcal{S}_2)], \end{aligned} \quad (\text{B1})$$

$$\begin{aligned} \langle \hat{S}_y \rangle &= \frac{N}{2C^2} \text{Re}[\sin \theta_1 \sin \phi_1 + \sin \theta_2 \sin \phi_2 \\ &\quad + 2iC_{N-1}(e^{-i\phi_1} \mathcal{S}_1 \mathcal{C}_2 - e^{i\phi_2} \mathcal{C}_1 \mathcal{S}_2)], \end{aligned} \quad (\text{B2})$$

$$\begin{aligned} \langle \hat{S}_x^2 \rangle &= \frac{N}{4} + \frac{N(N-1)}{4C^2} \text{Re}[\sin^2 \theta_1 \cos^2 \phi_1 + \sin^2 \theta_2 \cos^2 \phi_2 \\ &\quad + 2C_{N-2}(e^{-i\phi_1} \mathcal{S}_1 \mathcal{C}_2 + e^{i\phi_2} \mathcal{C}_1 \mathcal{S}_2)^2], \end{aligned} \quad (\text{B3})$$

and

$$\begin{aligned} \langle \hat{S}_y^2 \rangle &= \frac{N}{4} + \frac{N(N-1)}{4C^2} \text{Re}[\sin^2 \theta_1 \sin^2 \phi_1 + \sin^2 \theta_2 \sin^2 \phi_2 \\ &\quad - 2C_{N-2}(e^{-i\phi_1} \mathcal{S}_1 \mathcal{C}_2 + e^{i\phi_2} \mathcal{C}_1 \mathcal{S}_2)^2]. \end{aligned} \quad (\text{B4})$$

APPENDIX C: EFFICIENT GRADIENT EVALUATION

To optimize the pulse sequence, we use a gradient descent scheme using the infidelity

$$\epsilon = 1 - |\mathcal{A}|^2, \quad (\text{C1})$$

with $\mathcal{A} = \langle \Psi_{\text{cat}}(\theta) | \Psi_{\text{guess}} \rangle$, as a figure of merit. To evaluate the gradient, we propagate the initial state, $|\Psi_0\rangle = |\pi/2, 0\rangle$, forward,

$$|\Psi_\ell\rangle = G_\ell |\Psi_{\ell-1}\rangle \quad (\text{C2})$$

with G_ℓ the time-evolution operator corresponding to the ℓ th pulse, which can be either a twisting or a rotation. Then, we

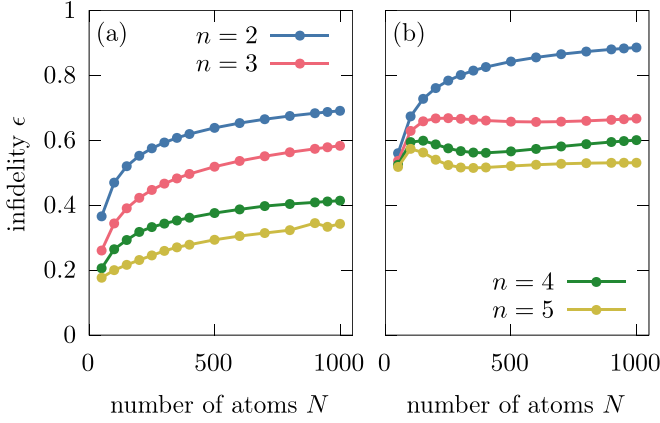


FIG. 5. Infidelity of the generated Schrödinger-cat states as a function of the number of atoms N for different OAT and rotations applications, n . (a) shows the results for a total normalized shearing strength $\tilde{Q} = 1.0$. The angle between the coherent spin states that form the target state is $\theta = \pi/2$. (b) shows the same results as (a) but for $\tilde{Q} = 0.5$.

propagate backward $|\lambda_{2n}\rangle = \mathcal{A}|\Psi_{\text{cat}}(\theta)\rangle$, using

$$|\lambda_{\ell-1}\rangle = G_{\ell}^{\dagger} |\lambda_{\ell}\rangle \quad (\text{C3})$$

to evaluate

$$\frac{\partial \epsilon}{\partial a_{\ell}} = -2 \text{Re} \left[\langle \lambda_{\ell} | \frac{\partial G_{\ell}}{\partial a_{\ell}} | \Psi_{\ell-1} \rangle \right], \quad (\text{C4})$$

where a_{ℓ} are the pulse parameters. This can be evaluated efficiently as outlined in Ref. [60].

To enforce that $\tilde{Q} = \sqrt{N} \sum Q_k$ is a constant, we parametrize Q_k as a function of x_k

$$Q_k = \frac{\tilde{Q}}{\sqrt{N}} \frac{e^{x_k}}{\sum_j e^{x_j}}, \quad (\text{C5})$$

where the x_k are unconstrained real optimization variables. This is a softmax-type mapping. Finally, $\partial \epsilon / \partial x_j$ can be evaluated using Eq. (C4) and the chain rule.

APPENDIX D: INFIDELITY SCALING FOR DIFFERENT NUMBER OF PULSES

Here, we provide a systematic numerical characterization of the infidelities achieved by the optimized pulse sequences for different numbers of pulses.

Figure 5 shows the infidelity, ϵ , as a function of the atom number N for different values of the number of pulses n , at fixed total normalized shearing strength $\tilde{Q} = 1.0$, in Fig. 5(a), and $\tilde{Q} = 0.5$, in Fig. 5(b). In general, the infidelities vary slowly with N and eventually approach a weakly varying regime. Increasing the number of pulses systematically reduces infidelity, indicating that pulse depth is the relevant control resource for improving convergence. These results show how the loss of fidelity with increasing N can be compensated by employing longer pulse sequences. We observe a similar behavior for different values of θ , as Fig. 3(b) exemplifies.

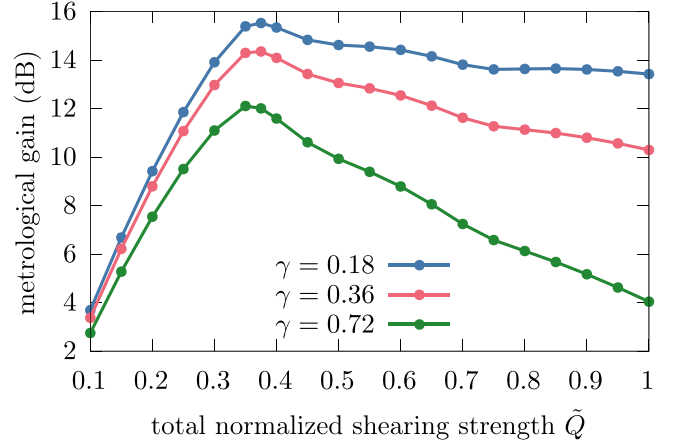


FIG. 6. Metrological gain as a function of $\tilde{Q}$ for $N = 200$ atoms, and $n = 3$ OAT and rotations. The angle between the coherent spin states that form the target state is $\theta = \pi/2$.

APPENDIX E: BALANCING SQUEEZING AND CONTRAST LOSS

Here we analyze two competing effects. From one side, increasing the total normalized shearing strength $\tilde{Q}$ leads to better fidelities. However, it also increases detrimental effects such as the contrast loss detailed in the main text. In Fig. 6, we plot the metrological gain (taking into account the contrast loss) for different values of γ , the number of free-space scattered photons per atom, per unit of shearing strength, including the one reported in recent experiments, 0.36, and the half and double of that value. We observe that the metrological gain is maximum around $\tilde{Q} = 0.4$ for all these cases. Naturally, higher values of γ give worse metrological gains as it implies more contrast loss. Yet, the dependence on $\tilde{Q}$ becomes much weaker as γ decreases.

APPENDIX F: ROBUSTNESS TO INDEPENDENT CONTROL ERRORS AND CONTROL-ERROR BUDGET

Here we quantify the protocol's sensitivity to control imperfections: systematic and random errors in pulse areas Q_k and μ_k , calibration errors in time-reversal readout, rotation axis phase errors $\delta\phi_k$, and finite detection noise. Errors are applied multiplicatively to pulse areas, so all pulse-area errors are relative (for even N , both gates are exactly 2π periodic in their parameters, and a pulse-timing error is a first-order area error). In the intended implementation, the sign change $\chi \rightarrow -\chi$ is produced by changing the detunings of the two-color drive [42,47], so calibration errors of preparation and readout are largely correlated. We model the readout as having the same common calibration error as preparation plus a residual systematic mismatch, and quantify tolerance to each separately.

The two components behave qualitatively differently. Errors common to preparation and readout are self-corrected by the time-reversal echo, with the readout traversing the same imperfect pulses as preparation in reverse (see Fig. 4). At the 3×10^{-3} level, they cost less than 0.1 dB at every atom number studied. The protocol's sensitivity is set by the

TABLE I. Control-error budget for the time-reversal protocol on the ^{171}Yb platform of Ref. [41]. The required columns correspond to the errors at which the mean metrological gain drops by 3 dB in our simulations (i.e., each value is a threshold: smaller errors cost less than 3 dB), computed at $\tilde{Q} = 0.5$ ($N = 350$ is simulated directly, being the scale of the demonstrated experiments). Entries $>3 \times 10^{-2}$ indicate that the gain stays within 3 dB of its ideal value over the entire simulated error range. Entries are relative errors of the pulse areas, except the rotation-axis phase (radians) and the detection noise (in units of the SQL variance $N/4$). The mismatch rows are residual errors of the time-reversal readout relative to the preparation, i.e., of the $\chi \rightarrow -\chi$ branch. The rotation-angle and axis-phase mismatches are applied per pulse. Demonstrated values marked † are quoted in Ref. [41] and values marked $*$ are estimates for standard techniques rather than published measurements. The projections indicate plausible upgrades.

Imperfection	Required		Demonstrated	Projected / Comment
	$N = 350$	$N = 10^3$		
Common calibration errors				
Shearing strength $\delta Q_k/Q_k$	$>3 \times 10^{-2}$	$>3 \times 10^{-2}$	$\sim 10^{-2} \star$	Comfortable at any N
Rotation angle $\delta\mu_k/\mu_k$	$>3 \times 10^{-2}$	$>3 \times 10^{-2}$	$\sim 10^{-2} \dagger$	Comfortable
Preparation-readout mismatch				
Shearing strength $\delta Q_k/Q_k$	8×10^{-3}	3×10^{-3}	$\sim 10^{-2} \star$	10^{-3} by echo-nulling calibration
Rotation angle $\delta\mu_k/\mu_k$	4×10^{-3}	9×10^{-4}	$\sim 10^{-3} \star$	10^{-4} with amplitude-stabilized rf pulses
Rotation-axis phase $\delta\phi_k$	8×10^{-3}	2×10^{-3}	$\lesssim 10^{-4} \star$	Comfortable (direct digital synthesis)
Detection noise σ_{meas}^2	1 SQL		$<0.15 \text{ SQL} \dagger$	Costs 0.6 dB at any N

residual mismatch between preparation and reversal. The relative pulse-area error at 3 dB gain loss ranges from 4×10^{-3} to 8×10^{-3} at $N = 350$, the scale of current experiments [41], and from 9×10^{-4} to 3×10^{-3} at $N = 10^3$, depending on the channel (for $\tilde{Q} = 0.5$; at $\tilde{Q} = 1.0$ tolerances tighten by a factor of 1.2–3). The corresponding tolerance on the rotation-axis phase, an additive error, is similar in radians. The two classes also affect the state differently: common errors slightly displace the prepared cat state, which the echo corrects, while mismatch errors leave the prepared state untouched and degrade only the readout. Finite detection noise is benign for the same reason. The echo returns the state to a coherent spin state, so noise at the working point is $\sqrt{N}/2$. Detection noise with variance σ_{meas}^2 , in units of the SQL variance $N/4$, costs

a fixed $10 \log_{10}(1 + \sigma_{\text{meas}}^2)$ dB of metrological gain at any N (0.6 dB at the demonstrated $\sigma_{\text{meas}}^2 = 0.15$). Table I compares these requirements with accuracies demonstrated in the intended platform [41,42,47]. The common calibrations meet their requirements comfortably. The critical one is the residual calibration mismatch of the $\chi \rightarrow -\chi$ reversal branch. The demonstrated $\sim 1\%$ sits at the $N \approx 350$ threshold, consistent with the echo-based gain measured at $N = 340$ [41], while $N = 10^3$ requires an improvement by a factor of ≈ 4 . The mismatch is directly measurable at the protocol output (as any residual signal $\langle \hat{S}_y \rangle$ or excess noise $\Delta \hat{S}_y$ when the protocol runs without an interrogation phase), so the reversed branch can be calibrated against the echo itself, beyond its open-loop accuracy.

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