

Gravitationally induced entanglement in atom interferometry

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A promising route to testing quantum gravity in the laboratory is to look for gravitationally induced entanglement (GIE) between two or more quantum matter systems. To date, proposed tests have primarily focused on microsolid platforms prepared in highly nonclassical states, such as NOON states or strongly squeezed states. Here, we instead consider measuring GIE in atom interferometry as evidence of quantum gravity. We propose placing two atom interferometers next to each other in parallel and looking for correlations in the atom-number statistics at their output ports. Importantly, GIE is possible without challenging macroscopic superposition states, such as NOON or Schrödinger-cat states. Instead, GIE can arise using coherent, near-classical atomic states, substantially reducing the experimental complexity. Observing the effect in this regime would require the total mass of the atom interferometers to be near the Planck mass scale. However, we show that state-of-the-art quantum squeezing techniques in cold-atom systems can significantly lower this mass requirement, and outline realistic pathways by which the necessary mass scales could be achieved in the near future, placing laboratory tests of GIE with atom interferometers within reach.

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I. INTRODUCTION

The last few years have witnessed an explosion of interest in testing quantum gravity with small-scale experiments [1]. The general idea is to, rather than bringing gravity to the traditional microscopic scales of quantum theory, such as using particle accelerators operating at energies that appear far out of reach [2], to instead bring quantum theory to the macroscopic realm of gravity. This approach dates back to a thought experiment of Feynman [3], where a massive quantum object is placed near another object that has been put into superposition branches of two locations by a Stern-Gerlach apparatus. In the modern-day interpretation of the experiment [4,5], if gravity obeys quantum theory, then we would expect the gravitational field or force of the superposed object to also go into superposition branches, with the other mass responding to this superposition and resulting in entanglement. However, if gravity obeys classical theory, then gravity cannot go into a quantum superposition, and we would therefore not naturally expect entanglement between the objects. The experiment thus provides a test of whether gravity is quantized.

Most proposals for table-top tests of quantum gravity have followed this line of reasoning and consider gravitationally induced entanglement (GIE) between two massive, solid objects [4–11]. However, these proposals require substantial

advancements over current state of the art [12]. For example, in the quintessential Bose *et al.* proposal [4] solid objects of masses 10^{-15} kg must be sent into a quantum superposition using a Stern-Gerlach experiment, yet so far only single atoms have been sent through Stern-Gerlach experiments [13], requiring a new technological leap where masses nine to ten orders of magnitude greater than single atoms are used. Similarly, proposals based on trapped oscillating microsols require substantial levels of quantum squeezing [12,14], with quantum squeezing not yet achieved in such experiments.

Given the challenge of these experiments, it is instructive to consider whether there are other physical systems that could be well suited to GIE tests since, as pointed out in the pioneering work of Marletto and Vedral [5], GIE need not be limited to quantum solids. Of note is atom interferometry since this experimental field has already found significant success in testing classical effects of gravity, such as tests of the equivalence principle [15] and Newton's constant [16]. Atom interferometry has also reached industrial gravitational applications [17] and seen remarkable quantum developments in recent years, such as high levels of entanglement, quantum superpositions over a distance of 54 cm, and levels of quantum squeezing that surpass any other quantum system [18–25].

Although successful in tests of classical effects of gravity, cold-atom interferometry is generally considered unsuitable for tests of GIE. This is because the states usually considered for testing quantum gravity are highly nonclassical [26], such as NOON or Schrödinger-cat states, which appear harder to achieve in cold atoms than microsols, leading to considerations that only pairs of atoms become entangled such that the signal scales as m^2 , where m is the mass of the atoms, and is independent of the number of atoms. Furthermore, it has been argued that the mass density required to test quantum

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gravity has to be that of microsolids and thus unattainable to cold-atom experiments [12].

Previous attempts to use atom interferometry for tests of quantum gravity have, therefore, considered alternative methods to measuring GIE. For example, a collapse-and-revival measurement was considered for an atom interferometer interacting with a massive solid object in a superposition of locations [27]. However, collapse and revival does not provide as general a test of quantum vs classical gravity as entanglement [28–31]. In another attempt [32], a method based on quantum Fisher information was chosen to test quantum gravity in an atom interferometer, but was considered unfeasible due to the requirement of a highly quantum initial state and strong electromagnetic interactions that provide the same signal [32,33].

It might thus appear that atom interferometry is unsuitable for tests of quantum gravity. However, here we demonstrate that atom interferometry *can* be used for GIE by implementing a scheme where two atom interferometers are placed next to each other in parallel and GIE is created between the two interferometers. Sizeable signals can be generated without the need for N00N or Schrödinger-cat-like states or mass densities of solids: with easily created coherentlike states of atoms, we show that a GIE signal scales as $G\sqrt{N_1 N_2} m^2 t / (\hbar d)$, where N_1 and N_2 are the number of atoms in the two interferometers and d is the distance between them, which is not just $Gm^2 t / (\hbar d)$ or $G\sqrt{N_1 + N_2} m^2 t / (\hbar d)$ as might be naively expected [34–36]. The enhanced scaling of $\sqrt{N_1 N_2}$ can be understood from a quantum metrology perspective, as discussed further in Sec. II D, and is due to each atom of one interferometer seeing all the atoms of the other interferometer in a superposition, such that all the atoms entangle together rather than in independent pairs. Density arguments against using cold atoms used figures of merit that were derived in the assumption of solid objects [12], and presupposed that the distances between the interacting objects d had to be at the micrometer scale, which need not be the case: distances at the 100- μm scale (which is in fact likely required to avoid Casimir interactions [4]) already allow for gas densities since the density of the objects scale like R^3 , with R the radius of a spherical object, whereas GIE can just scale as $1/d$ (with $d \geq 2R$ required physically) [4].

With state-of-the-art quantum squeezing in cold atoms, the above scaling of $\sqrt{N_1 N_2}$ is further improved, seemingly bringing a test of GIE closer to current atom interferometer experiments than GIE proposals are to current microsolid experiments. For example, there is no need for major technical leaps such as placing microsolids in quantum squeezed or Schrödinger-cat states for the first time, and instead the main challenge is to increase the number of atoms in interferometers.

II. PROPOSAL

The proposed experiment consists of two atomic interferometers A and B in a parallel Mach-Zehnder configuration separated by a distance d (see Fig. 1). The paths of interferometer A are crossed at a second beam splitter, while the paths of interferometer B are directly used for homodyne detection [37–39]. The gravitational force between the atoms

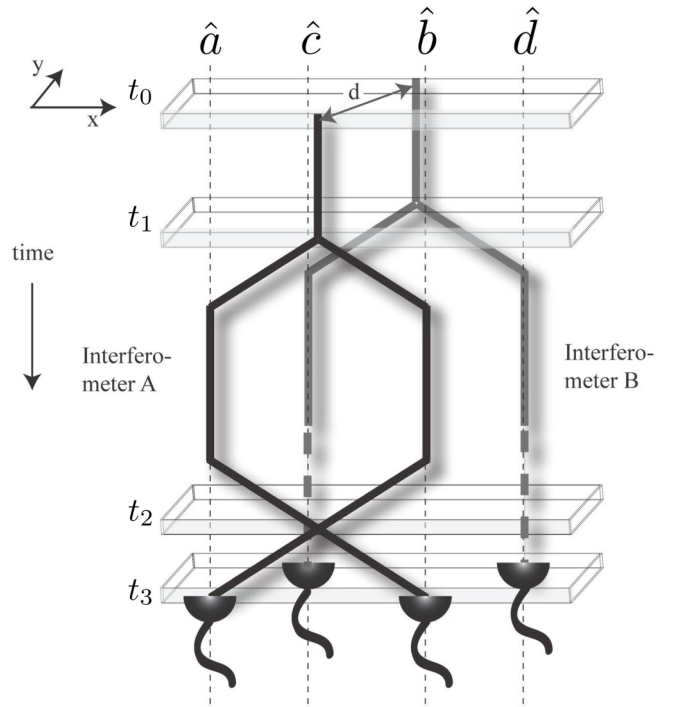


FIG. 1. Two atom interferometers (interferometer A with modes $\hat{a}$ and $\hat{b}$, and B with $\hat{c}$ and $\hat{d}$) are placed adjacent to each other and in parallel with spacing d . The interferometers are far enough apart, or a thin conducting sheet is placed between them, such that they only interact with each other gravitationally. At time t_1 the atoms of both interferometers are beam split into two arms, after which they interact with the atoms of the other interferometer. At time t_2 the arms of interferometer A are brought back together through another beam splitter, whereas the arms of interferometer B are left open, which allows for a simple differential measurement scheme to evidence entanglement, as discussed further in Sec. II C. Finally, at t_3 the number of atoms in each mode are measured, for example, with single-atom detectors [40–47]. Correlations in the differences in the number of atoms in each output port are sought as evidence of the quantum nature of gravity [see (9)]. Note that the same laser or magnetic field could be used for both interferometers to eliminate classical noise in the differential signal [48–57].

of one interferometer and the other induces entanglement between the interferometers, which is revealed by correlations in the number of atoms in the output ports of the interferometers. Physical implementations of the interferometers could be atomic fountains with free-falling atomic clouds [58–62] or trapped interferometers, such as cold atoms in double-well potentials [63–67] or optical lattices [68,69] (for reviews see, e.g., [16,70,71]). We leave the physical implementation of the atom interferometer open, demonstrating how the parameter space is, in principle, wide enough for the implementation of any of the above schemes. However, owing to their expected longer interaction times, we pay extra attention to a double-well setup (Fig. 2), which, as with atom fountains, can be used with thermal or condensed atoms [72–74].

As well as the gravitational interaction, the interferometers will also interact through electromagnetism. However, those interactions can be neglected by making sure that the interferometers are sufficiently far apart such that the

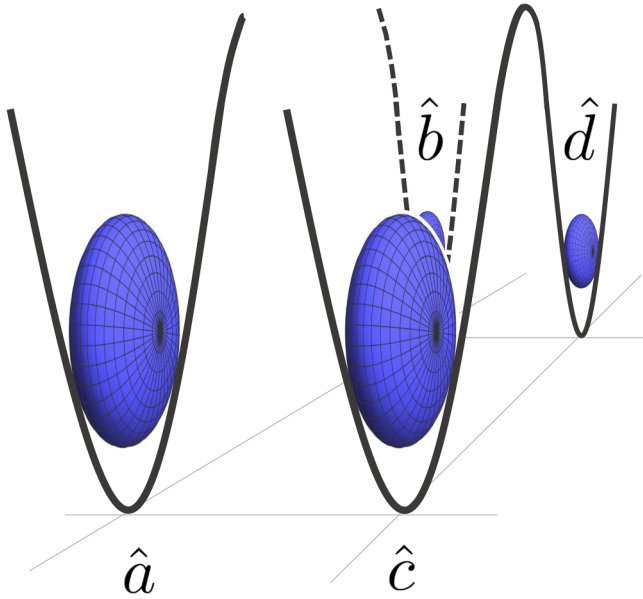


FIG. 2. An example of a physical realization of the interferometry scheme of Fig. 1. Two adjacent double-well harmonic potentials store atomic clouds with oblate spheroid (pancake) geometries. The atoms can be beam splitted by lowering the double-well potential barrier, which creates a Josephson junction. One double well is labeled by modes $\hat{a}$ and $\hat{b}$, the other by $\hat{c}$ and $\hat{d}$. The distance between the two interferometers is closer than the distance between each arm of the interferometer to enhance the correlations between the interferometers due to their gravitational interaction.

gravitational interaction dominates over the electromagnetic interactions between neutral atoms [4] or by placing a conducting membrane between the interferometers [75–77]. This is the principal reason why we consider two atomic interferometers for a test of quantum gravity rather than one [32]. Additionally, there is the issue of whether, from a theory point of view [4,5,78,79], local classical gravity could still create entanglement in a single interferometer through direct interactions as the atoms are brought together in the beam splitters [40], and the fact that, for the initial state to be nonclassical, it must already be entangled in the particle basis, while, in the mode basis, entanglement is not generated by quantum gravity when the initial state is separable (see below). These issues are not present when using two interferometers.

In contrast to typical proposals for microsolid experiments [4], in Fig. 1, the atoms in each interferometer are in independent superpositions, such that there is no N00N state of N atoms and instead a far more classical-like state, often referred to as a “coherent state.” This is a much simpler state and routinely produced in atom interferometer experiments [65]. The difference between the coherent states that are routinely produced in atom interferometers and N00N states has been the source of some confusion in the literature [24,80,81].

We will find (Sec. IID) that the basic figure of merit for entanglement from quantum gravity can be proportional to $GMmt/(\hbar d) \times 10^{r_{dB}/10}$, where d is the closest distance between the interferometers, t is the interaction time, r_{dB} is the degree of quantum squeezing in dBs, m is the mass of a single

atom, and M is the total mass of all the atoms passing through each interferometer (assuming that the total mass of atoms in each interferometer is the same). This is to be compared with the figure of merit for the Bose *et al.* proposal [4], the relative phase, which is $GM^2t/(\hbar d)$. There is thus a loss of a factor of N , with N the number of atoms, compared to the Bose *et al.* scheme. However, we will see how this can be compensated for by a larger number of atoms than in the Bose *et al.* scheme, and quantum squeezing the initial state. Ignoring the squeezing, we can understand the $Mm = \sqrt{N^2}m^2$ rather than M^2 scaling from a physical perspective: in the Bose *et al.* configuration, the atoms of the solid are in a N00N state, whereas here the atoms are in a coherent state, where each atom is *independently* in a superposition of two locations (see Sec. IID for further discussion).

A. Initial state

The first stage of the interferometer, occurring at time t_1 in Fig. 1, beam splits the atoms. Physical implementations of this include Bragg diffraction [82], modifying the potential barrier of a double-well potential [65,83,84]. We take the beam splitter to be a 50-50 beam splitter, and assume that each interferometer is in a coherent state with the full system a product of coherent states $|\psi\rangle = |\phi\rangle_{ab} \otimes |\phi'\rangle_{cd}$, where [65]

$$|\phi\rangle_{\alpha\beta} := \frac{1}{\sqrt{2^N}} \sum_{k=0}^N \sqrt{\binom{N}{k}} e^{ik\phi} |N-k\rangle_{\alpha} |k\rangle_{\beta}, \quad (1)$$

with $\alpha, \beta = \{a, b, c, d\}$ the modes of the interferometers as labeled in Figs. 1 and 2; N the number of atoms in each interferometer, which we assume to be the same just for mathematical convenience [85]; and ϕ and ϕ' the phases induced by the beam splitters: The state (1) is created by the beam-splitter unitary operator $\hat{U}_{\alpha\beta}(\theta, \phi) := \exp[\theta(e^{i\phi}\hat{\alpha}\hat{\beta}^\dagger - e^{-i\phi}\hat{\alpha}^\dagger\hat{\beta})]$ acting initially on N atoms in one mode, with $\theta = \pi/4$ [39].

Here we are describing the interferometers using second quantization (the mode basis). In first quantization (the particle basis), the coherent state is

$$|\phi\rangle_{\alpha\beta} = \frac{1}{\sqrt{2^N}} \bigotimes_{i=1}^N (|\alpha\rangle_i + e^{i\phi}|\beta\rangle_i), \quad (2)$$

where i labels the particles and α and β the modes. This emphasizes that the coherent state is just a product of independent atoms each in a superposition.

B. Evolved state

The atoms are now left to interact via gravity. Normally, in atom interferometer experiments, this involves the atoms of the interferometer interacting with a very large object, such as the Earth or a 500-kg tungsten mass [16,86], which imparts a strong gravitational force on the atoms, $F = GMm/r^2$, where m is the mass of the atom used in the interferometer, M is the mass of the large object, and r is the typical distance between the atoms and the large object. Often, as for example in atom fountain experiments, this strong gravitational interaction results in the atoms moving toward the object such that their paths in space are noticeably curved [87]. However, in the

setup considered here, the gravitational force is between the atoms of two interferometers and is thus significantly weaker, with essentially M in the force above now being replaced by $N \times m$. Such a force does not displace the atoms much from their initial positions and, as in proposals based on microsolids, such as [4], this can be ignored: taking an ^{87}Rb interferometry with $d \approx 0.1$ m and $N \approx 10^{13}$, the acceleration imparted on each atom will typically be 10^{-20} ms^{-2} , compared to 9.81 ms^{-2} when M and r are, respectively, the mass and radius of the Earth. Such a small gravitational force might at first appear to suggest that a quantum gravitational effect cannot be observed. However, the experiment here is unlike standard atom interferometer experiments: here we are looking for entanglement between the interferometers due to their superposed paths, which, as we show below, is a measurable effect, similar to how the entanglement effect in [4] can be large even though the gravitational force may be extremely weak.

In the nonrelativistic (Newtonian) limit of quantum gravity, which is a perfectly valid limit of quantum gravity [88–90], and assuming fixed positions for the atoms following the above analysis, the gravitational interaction between the atoms of the interferometers is described by a Hamiltonian of the form [40]

$$\hat{H} = -\frac{1}{2}G \int d^3x d^3x' \sum_{\alpha, \beta = \{a, b, c, d\}} \frac{:\hat{\rho}_\alpha(\mathbf{x})\hat{\rho}_\beta(\mathbf{x}') :}{|\mathbf{x} - \mathbf{x}'|} \quad (3)$$

$$=: \sum_{\alpha, \beta = \{a, b, c, d\}} \lambda'_{\alpha\beta} \hat{\alpha}^\dagger \hat{\beta}^\dagger \hat{\alpha} \hat{\beta}, \quad (4)$$

where $:\cdot:$ represents normal ordering, $\hat{\rho}_\alpha(\mathbf{x}) := m\hat{\Psi}_\alpha^\dagger(\mathbf{x})\hat{\Psi}_\alpha(\mathbf{x})$ is the mass-density operator for the atoms, $\hat{\Psi}_\alpha(\mathbf{x})$ is the nonrelativistic atomic field, and we have approximated the wave functions of each mode to be nonoverlapping and used a single-mode approximation [65]: $\hat{\Psi}_\alpha(\mathbf{x}) \approx \psi_\alpha(\mathbf{x})\hat{\alpha}$, with $\hat{\alpha}$ the annihilation operator for the corresponding mode (satisfying $[\hat{\alpha}, \hat{\alpha}^\dagger] = 1$) and $\psi_\alpha(\mathbf{x})$ the “wave function” of the mode. For $\lambda'_{\alpha\beta}$, we have

$$\lambda'_{\alpha\beta} := -\frac{1}{2}Gm^2 \int d^3x d^3x' \frac{|\psi_\alpha(\mathbf{x})|^2 |\psi_\beta(\mathbf{x}')|^2}{|\mathbf{x} - \mathbf{x}'|}, \quad (5)$$

where m is the mass of the atoms and G Newton’s constant. To obtain a concrete expression for $\lambda'_{\alpha\beta}$, we assume a trapped double-well system with a standard harmonic trapping potential for each well. Then, to maximize the strength of the gravitational interaction between the interferometers, we assume harmonic potentials that force the atomic clouds to have oblate spheroid wave functions (see Fig. 2). This is found to provide a small enhancement of $\approx \frac{3}{2}$ over using spheres of the same volume (see Appendix A). In a nontrapped scheme, $\lambda'_{\alpha\beta}$ would need to be calculated for the relevant wave functions $\psi_\alpha(\mathbf{x})$ of the particular setup in that case.

Given the configuration of Fig. 1, we ignore the interaction between the two modes within the interferometer (a with b and c with d) and the diagonal modes between the two interferometers (a with d and b with c) [91], such that the only sizable gravitational interactions are the self-interactions of each mode, and between the adjacent modes a and c and b and d of the two interferometers. We also assume for

simplicity that all modes are identical, such that $\lambda'_{aa} = \lambda'_{bb} = \lambda'_{cc} = \lambda'_{dd} =: \lambda'_s$ and $\lambda'_{ac} = \lambda'_{bd} =: \lambda'$. The state of the system after the atoms have interacted gravitationally for time t is then [92]

$$|\psi_t\rangle = \frac{1}{2^N} \sum_{k, k'=0}^N \sqrt{\binom{N}{k} \binom{N}{k'}} e^{i(k\Phi + k'\Phi')} e^{i\lambda_s(k^2 + k'^2)} \times e^{2i\lambda k k'} |N-k\rangle_a |k\rangle_b |N-k'\rangle_c |k'\rangle_d, \quad (6)$$

where $\Phi := \phi - N(\lambda_s + \lambda)$ and $\Phi' := \phi' - N(\lambda_s + \lambda)$, with $\lambda_s := 2\lambda'_s t/\hbar$ and $\lambda := 2\lambda' t/\hbar$. In (6), we have ignored the electromagnetic interactions between the atoms within each interferometer, and will introduce these later. For convenience, we have also ignored the free evolution, which will just contribute to the phases Φ and Φ' .

C. Signal

From (6), the (quantum) gravitational interaction in general entangles the two interferometers. In the neglect of decoherence, we can characterize this using the concurrence, an entanglement monotone that has recently been extended to multiparticle and continuous-variable states as [93–95]

$$C(\rho) = \sqrt{2(1 - \mu_R)}, \quad (7)$$

where μ_R is the purity of the reduced density matrix across the appropriate bipartition of the pure state. In Appendix F, using (6), we find the purity of the state of one of the interferometers to be

$$\mu = \frac{1}{2^{2N}} \sum_k \binom{2N}{k} \cos^{2N}[\lambda(N-k)]. \quad (8)$$

In the assumption that $\lambda N \ll 1$, this approximates $\mu \approx 1 - \frac{1}{2}\lambda^2 N^2$, and thus the concurrence is $C(\rho) \approx \lambda N$, such that entanglement scales as $GMmt/(\hbar d)$ as described in the Introduction.

In practice, however, it would be hard to measure the concurrence, and (7) is also only valid for pure states [96]. We therefore consider an alternative, simple measurement scheme to infer entanglement that can be readily achieved in current experiments. This measurement scheme is to simply look for correlations in the number of atoms in each mode of the interferometers after performing another beam splitter on the ab interferometer (see Fig. 1) [97,98].

We can write this measurement as

$$S := \langle \psi_f | \hat{J}_z^{ab} \hat{J}_z^{cd} | \psi_f \rangle - \langle \psi_f | \hat{J}_z^{ab} | \psi_f \rangle \langle \psi_f | \hat{J}_z^{cd} | \psi_f \rangle, \quad (9)$$

where $\hat{J}_z^{\alpha\beta} := \frac{1}{2}(\hat{N}_\alpha - \hat{N}_\beta)$ is the number difference operator, $\hat{N}_\alpha := \hat{\alpha}^\dagger \hat{\alpha}$ is the number operator of the respective mode, and $|\psi_f\rangle$ is the final state of the system (the state at time t_3 in Fig. 1). The above signal S involves measuring the difference in the number of atoms of each interferometer. This would use single-atom detectors, which have been realized for cold atoms using various different techniques, such as introducing an optical lattice exposing the atoms to near-resonant light [41–43], using a CCD camera to measure the transit of single atoms through a light sheet [44], or a high-finesse cavity where the atoms traveling through the cavity cause detectable shifts in the cavity resonance [45,101], which has been used

to demonstrate many-body entanglement [46,47]. Note that since we are interested in a differential measurement, similar to other studies using two interferometers [48–57], the same laser or magnetic field could be used for both interferometers to eliminate classical noise in the differential signal [48–57].

With the assumption that the full state remains pure, it is clear that an $S \neq 0$ implies entanglement since a product state results in $S = 0$. However, S is a more general measure of entanglement than just for the assumption of products of coherent states and can also apply, for example, when there is a symmetrical decoherence mechanism acting on the interferometers, which is natural given the symmetry of the experimental setup (Fig. 1). This is made possible by not closing the second interferometer since then measuring $|\psi_f\rangle$ with $\hat{J}_z^{cd}$ is equivalent to just measuring the difference in the number of atoms in $\hat{c}$ and $\hat{d}$ for the earlier state $|\psi_i\rangle$ [see (10) below], which is not affected by a decoherence mechanism that acts the same on both arms of the interferometer (see Appendix C for more detail). This is the principal reason why we choose to only close one of the interferometers: atomic interferometers do not always close the paths (see, for example, [37]). Therefore, looking for a signal $S > 0$ provides strong evidence that the interferometers have become entangled.

Given that $\hat{J}_\varphi^{ab} \equiv \hat{U}_{ab}^\dagger(-\pi/4, \varphi) \hat{J}_z^{ab} \hat{U}_{ab}(-\pi/4, \varphi)$, where $\hat{J}_\varphi^{ab} = (\hat{a}\hat{b}^\dagger e^{i\varphi} + \hat{a}^\dagger \hat{b} e^{-i\varphi})/2$, and $\hat{U}_{ab}(-\pi/4, \varphi)$ is the action of the final beam splitter for the ab interferometer, the signal (9) is equivalent to

$$S = \langle \psi_i | \hat{J}_\varphi^{ab} \hat{J}_z^{cd} | \psi_i \rangle - \langle \psi_i | \hat{J}_\varphi^{ab} | \psi_i \rangle \langle \psi_i | \hat{J}_z^{cd} | \psi_i \rangle, \quad (10)$$

where $|\psi_i\rangle$ is the evolved state calculated in Sec. II B. Here, $\hat{J}_\varphi^{\alpha\beta}$ is the generalized “spin” operator, with $\hat{J}_x^{\alpha\beta}$ and $\hat{J}_y^{\alpha\beta}$ defined for $\varphi = 0$ and $\pi/2$, respectively. These spin operators obey the $\mathfrak{su}(2)$ algebra $[\hat{J}_p^{\alpha\beta}, \hat{J}_q^{\alpha\beta}] = i\epsilon_{pqr} \hat{J}_r^{\alpha\beta}$, where $p, q, r \in \{x, y, z\}$ such that the scheme in Fig. 1 is closely related to the Bose *et al.* proposal [4] for microsolids.

Plugging in (6) for $|\psi_i\rangle$ into (10), we find

$$S = \frac{1}{4} N^2 \sin(\phi - \varphi) \cos^{N-1} \lambda_s \cos^{N-1} \lambda \sin \lambda. \quad (11)$$

Assuming $\lambda_s \approx \lambda$ and $\lambda^2 N \ll 1$, then $S \approx \lambda N^2 \sin(\phi - \varphi)/4$.

D. Signal-to-quantum noise

In order to determine the feasibility of the scheme, we must compare the signal to the expected level of noise. We can then formulate the signal-to-noise ratio (SNR), which must be at least greater than one for the experiment to be feasible.

A noise that is impossible to circumvent as it is fundamental to the state of the system is the quantum noise. Therefore, as a first step we should consider the quantum noise associated with the state of the system and the corresponding SNR. An SNR < 1 in this case, for a given state of the system, would then rule out the experiment for that state no matter what experimental techniques are used. The derived SNR should therefore be seen as the most fundamental SNR since the noise is the inescapable quantum noise associated with the state of the system. Experimental noise, which depends on the particular experimental setup, will be considered in Sec. II H.

The quantum noise associated with the signal S is $\sqrt{\text{Var}(S)}$ where $\text{Var}(S)$ is the variance associated with S , which is given by [102]

$$\text{Var}(\kappa_{11}) = \frac{1}{\mathbb{M}} \kappa_{22} + \frac{1}{\mathbb{M} - 1} \kappa_{20} \kappa_{02} + \frac{1}{\mathbb{M} - 1} S^2 \quad (12)$$

$$\approx \frac{1}{\mathbb{M}} (\kappa_{22} + \kappa_{20} \kappa_{02} + S^2), \quad (13)$$

where $\mathbb{M} \gg 1$ is the number of repetitions of the experiment, κ_{20} is the variance of $\hat{J}_\varphi^{ab}$ given an initial state of the system, κ_{02} is the variance of $\hat{J}_z^{cd}$, and κ_{22} is the bivariate cumulant defined in (D1).

The full expression for the SNR is nontrivial and given in (D3). However, in the assumption that $\lambda_s \approx \lambda$ and $\lambda N \ll 1$, we find the following simple expression for the optimum SNR under this approximation: $\text{SNR} \approx \sqrt{\mathbb{M}} \lambda N$, which is achieved when $\varphi = \pi/2$ so that $\hat{J}_\varphi^{ab} \equiv \hat{J}_y^{ab}$. Note that this λN scaling matches that of the concurrence entanglement monotone considered above when $\lambda N \ll 1$. This scaling is proportional to $Gm^2 N t / \hbar d$, where d is the distance between the center of the interferometers, which contrasts, for example, with the $Gm^2 N^2 t / \hbar d$ scaling found in Bose *et al.* for the size of the entangling-inducing phase. As discussed in Sec. II, this poorer scaling in N is to be expected, as here the state of the atoms is far more “classical-like” than the effective N00N state of [4]. However, although a factor of N is lost compared to a N00N state scheme, a coherent state of N atoms is significantly easier to create and maintain than a N00N state of N atoms since the decoherence effect will be far less severe (this a major current issue with quantum metrology in general).

The different scalings of N^2 in the microsolid experiment [4] and N here can also be understood from a quantum metrology perspective. In the microsolid experiment, the quartic interaction Hamiltonian (4) leads to a Heisenberg scaling of $N_1 N_2$ for an initial N00N state (compared to an N scaling for a quadratic Hamiltonian [34–36]), where N_1 and N_2 are the total number of atoms in each interferometer. In contrast, in our proposal the quartic interaction Hamiltonian leads to a shot-noise scaling of $\sqrt{N_1 N_2}$ for an initial coherent state. This is to be compared to a $\sqrt{N}$ scaling for a quadratic Hamiltonian [34–36] where $\sqrt{N}$ comes from the reduction in noise due to the N atoms in one shot being equivalent to N repetitions of the experiment (a classical result). The enhanced scalings obtained for many-body systems when moving from a quadratic to a quartic, or any higher-order Hamiltonian, is referred to as super-Heisenberg in the optimum cases [103–105].

Note that a $\sqrt{N_1 + N_2}$ scaling for the SNR would be obtained if the gravitational interaction resulted in $(N_1 + N_2)$ -independent pairs of atoms becoming entangled, as this is equivalent to repeating an experiment with two atoms $N_1 + N_2$ times. However, this is not what occurs in the experiment: each atom in one interferometer sees at once N_1 or N_2 atoms in a quantum superposition and becomes entangled with all of them, such that, in the end, all the atoms of the interferometers become entangled. The experiment can thus not be made equivalent to repeating an experiment with just two atoms $N_1 + N_2$ times, and hence the overall

scaling for the SNR is not just $\sqrt{N_1 + N_2}$ as might be at first expected.

E. Required experimental parameters

For an observable effect, an SNR at least of order one is needed. We now discuss the state of the art in cold-atom interferometer experiments and what advancements are required to achieve an SNR greater than one. For the coherence time, we have seen $t \approx 10^2$ s for an atom interferometer with optical lattices [68], $t \approx 10^4$ s predicted for rubidium atoms in optical-tweezer arrays [106], and we estimate three-body damping to provide an upper bound of $t \approx 10^3$ – 10^4 s for our proposed setup (see Sec. II H 2). These coherence times are long compared to many other GIE proposals. For example, the Bose *et al.* proposal uses 1 s [4]. However, that proposal assumes initial NOON states, which decohere much faster than coherent states that are considered here.

For the distance between the interferometers, this must be at least an order of magnitude less than the superposition size of each interferometer to justify our assumption that the cross interactions ($\hat{a}$ with $\hat{d}$ and $\hat{b}$ with $\hat{c}$) can be neglected. The state of the art here is that atoms have been superposed over 54 cm [24,25]. For the number of atoms, it is difficult to assess the state of the art in atom interferometry as there has not been a technological requirement to push these to large values, however, at least 10^6 atoms per second were achieved in an atom fountain in 1999 [59]. Interferometers are often fed from MOTs or Bose-Einstein condensates (BECs), and over 10^{11} have been reached in magneto-optical traps (MOTs) [107], and 10^8 – 10^9 in BECs [108,109]. Taking the above state-of-the-art values for the coherence time and number of atoms, with a distance d of a few centimeters, an equatorial and polar radii also of centimeters (giving a number density of order 10^{12} cm $^{-3}$), $M \approx 10^3$ repetitions, and assuming that there are of order 10 such interferometers operating independently, we obtain an SNR of $\approx 10^{-7}$, illustrating that these state-of-the-art numbers must be pushed further. A clear place to push is the number of atoms. For example, with 10^{16} atoms of a heavy element such as caesium such that the mass of the interferometers is on the Planck scale (and taking $t \approx 10^4$ s), would achieve an SNR greater than one with the above other values.

However, such a high number of atoms would be very difficult to achieve. Alternatively, to reduce the number of atoms required or reduce the challenge of the other experimental parameters, the initial state could be quantum squeezed, which decreases the noise by $10^{-r_{\text{dB}}/10}$ where r_{dB} is the squeezing in decibels. For example, a state-of-the-art squeezing of 20 dB [20] in both interferometers would provide an SNR of $\approx 10^{-5}$ for the above state-of-the-art values and thus reduce the required atom number to 10^{14} (and number density 10^{11} cm $^{-3}$) to achieve an SNR greater than 1 [110]. Going two orders of magnitude beyond the state of the art to 40 dB of squeezing (above 33 dB has been suggested for squeezing of atoms via atom-light interactions [111]), then the numbers could be further lowered to, for example, 10^{12} atoms (with number density 10^{12} cm $^{-3}$), a distance d of mm's and interrogation times of order 10^3 s

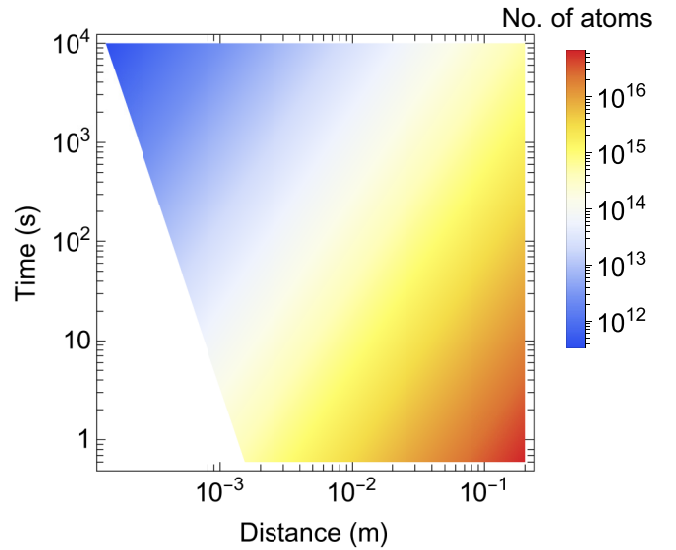


FIG. 3. Parameters required to achieve an SNR of order 1 for the interferometry scheme of Fig. 1. Distance is the separation d between the interferometers, time is the interrogation time t , and No. of atoms is the total number of atoms N . The atoms considered are caesium, squeezed at 35 dB, five independent setups are assumed, the number of repetitions are such that at $t = 10^4$ s we have $M \approx 10^3$, and only the parameters for which the number density $n < 10^{16}$ cm $^{-3}$ are shown, allowing for densities varying from MOTs to BECs (the other parameters can still be used if the interferometry scheme is slightly modified, as discussed in the main text).

or 10^{13} atoms with times of order 10^2 s. Although 10^{12} – 10^{13} atoms are still one or two orders of magnitude larger than what has currently been achieved (e.g., 10^{12} in MOTs), we provide a method for attaining this in the very near future (see Sec. II G). Compared to other GIE schemes, which use different systems, these numbers look much closer to current experiments, with a detailed comparison provided in Sec. II F.

The above parameters are just an example of what can be used to get an SNR of order 1, but the parameter space available to an atom interferometry scheme is very large, due in part to the high tunability of such setups. The main experimental parameters involved are the interrogation time; the number of repetitions and independent interferometry setups; the number of atoms; the initial squeezing; and the distance between the interferometers, which is related to the superposition size and the geometry of the clouds and interferometry setup. Figure 3 illustrates the large parameter space available to an atom interferometry scheme, considering the distances and interrogation times available given an initial squeezing of 35 dB and at least 10^3 repetitions. Only parameters for which the number density is less than 10^{16} cm $^{-3}$ are used, and the density varies between those of Bose-Einstein condensation to noncondensed atoms, with condensation not a necessary requirement for atom interferometry [16,112].

F. Comparison to other schemes

Although there is great flexibility in the numbers required, due to the high tunability of cold-atom experiments, to see

GIE, all possibilities are challenging. This is to be expected for an experiment testing the quantum nature of gravity, and should be compared to current proposals on detecting GIE. For example, there is the revolutionary Bose *et al.* proposal [4], where Stern-Gerlach devices are used to place diamonds with NV centers into N00N-like states and then entanglement is sought in their internal spin states. Other schemes include using highly squeezed microsolids in traps and looking for entanglement between the external position and momentum variables [7,14], as well as looking for entanglement between massive mirrors in optical interferometers [9–11]. Despite the differences in the various schemes, it has been argued that they all require the same level of advancements to test quantum gravity [12].

We now compare the advancements required in microsolid experiments to test quantum gravity to the advancements required for our proposal. Since, as stated above, it has been argued that the same level of improvements is required for the different microsolid schemes, we just consider the Bose *et al.* setup for simplicity. However, as microsolids have not yet been placed in macroscopic superposition states or sent through Stern-Gerlach devices, it is hard to know what is best to compare this proposal against. One option is to compare it to the experiment that has achieved the most macroscopic N00N-like states, while another is to compare it to the Stern-Gerlach experiment involving the largest mass. Since only very small molecules have been sent through Stern-Gerlach experiments so far, we choose the former comparison since it will provide the more favorable numbers for a rough guide on the level of advancements required. In this case, the most macroscopic N00N-like states created so far were in an experiment involving large molecules sent through a diffraction grating [113]. Here, the molecules had mass 10^{-23} kg, a superposition size 300 nm, and were held for 10 ms. In comparison, the Bose *et al.* proposal requires a mass 10^{-15} – 10^{-14} kg, superposition size 0.25 mm, and coherence time of 2 s. Therefore, the mass must be increased by eight or nine orders of magnitude (and still maintained in a Schrödinger-cat state), the coherence time by two orders of magnitude, and the superposition distance of the objects must go up by three orders of magnitude.

For the scheme considered here, since all the state-of-art values considered in the previous section were achieved in different experiments, it is also not straightforward to determine the exact level of advancement that is required. However, taking the different atom experiments considered above as a starting point, then for the scheme where 40 dB of squeezing is required, a distance d of mm's, interrogation times of 10^2 s, and atom numbers of 10^{13} , then the squeezing must be increased by two orders of magnitude, and the number of atoms by seven or two depending on whether we take the current values from atom fountain interferometers or MOTs, with the coherence time already matching the state of the art. Here, there is no need to create or maintain a highly challenging Schrödinger-cat state. The Bose *et al.* proposal, as with all current microsolid proposals, also requires a new technological paradigm, Stern-Gerlach experiments with microsolids, whereas the proposal considered here is based on a current experimental platform: quantum squeezed atom interferometry. Other leaps required in alternative microsolid proposals

include quantum squeezing microsolids for the first time and at very large values, e.g., squeezing of around 110 dB is required for a mass of $M = 10^{-7}$ kg (an order above the Planck mass) held in an $\omega = 100$ kHz trap for a time of seconds [12,14].

G. Achieving the required atom numbers

We now discuss a possible mechanism for getting to the required atom numbers, which, in all the above-considered examples of the parameters needed, notably must be increased over the state of the art. However, the density can be either that of thermal atoms in a magnetic optical trap (MOT) or a BEC. Since it is harder to achieve larger atom numbers in BECs, here we highlight a possible implementation with a MOT that is used to source an atom fountain or double-well interferometer, both of which can be used with noncondensed atoms at the expense of less coherence [24,80,81,112,114]. The largest number of atoms in a MOT to date was over 10^{11} [107], however, there has been no technological drive to increase the number of atoms. We propose using a two-dimensional (2D) MOT to load a three-dimensional (3D) MOT. The important innovation is then the implementation of a train of slightly offset cooling frequencies, an expansion of a two-color MOT [115,116]. A two-color MOT enables a larger range of capture velocities and therefore leads to an increase in the trappable atom number, with the loading rate scaling as the fourth power of the capture velocity. Combining this with a range of novel and existing techniques, dark spot MOTs [117], very large trapping beams, asymmetric optical frequency spreads produced by acousto-optic modulation [118], using cooling beams resonant with the D1 line of the relevant species, etc., it will be possible to overcome some of the historic pitfalls of this approach [115,119] and maximize the number of captured atoms. Initial work suggests that captured atom numbers of 10^{14} are possible [114], bringing the required parameter space for testing quantum gravity within reach.

H. Experimental noise and decoherence

Above, we have considered the signal and the inherent quantum noise of the atoms. In addition to this, there will also be experimental sources of noise and decoherence. The most prominent sources of noise and decoherence are expected to be the electromagnetic interactions of the atoms, collisions between the atoms and background gas atoms due to an imperfect vacuum, fluctuations in any trapping potentials, and any imperfections of the beam splitters and measuring devices.

1. Electromagnetic interactions

First, we consider the electromagnetic interactions. At low temperatures, the electromagnetic interactions between the atoms will be predominantly two-body interactions [65]:

$$\hat{H} = \frac{1}{2} \sum_{\alpha, \beta=\{a,b,c,d\}} \int d^3x d^3x' V_{\alpha\beta}(\mathbf{x} - \mathbf{x}') \times \hat{\Psi}_{\alpha}^{\dagger}(\mathbf{x}') \hat{\Psi}_{\beta}^{\dagger}(\mathbf{x}) \hat{\Psi}_{\alpha}(\mathbf{x}') \hat{\Psi}_{\beta}(\mathbf{x}) \quad (14)$$

$$=: \sum_{\alpha, \beta=\{a,b,c,d\}} \kappa'_{\alpha\beta} \hat{\alpha}^{\dagger} \hat{\beta}^{\dagger} \hat{\alpha} \hat{\beta}, \quad (15)$$

where

$$\kappa'_{\alpha\beta} := \frac{1}{2} \int d^3x V_{\alpha\beta}(\mathbf{x} - \mathbf{x}') |\psi_\alpha(\mathbf{x}')|^2 |\psi_\beta(\mathbf{x})|^2, \quad (16)$$

and we have assumed a one-mode approximation as we did when considering the gravitational interactions. The Hamiltonian (15) has the same form as that of the gravitational interactions (4). However, we take $\kappa'_{ac} = \kappa'_{bd} \approx 0$ assuming that these can be neglected given the distance between the interferometers or that there is a conducting membrane between them, but we keep $\kappa'_{ab} \neq 0$ and $\kappa'_{cd} \neq 0$ in general, defining $\kappa'_d := \kappa'_{ab} = \kappa'_{cd}$. Ignoring the effect of the interactions before the beam splitter, the state of the system at time t is then the same as (6) except we now add to λ_s the term $\kappa_s - \kappa_d$, where $\kappa_s := 2\kappa'_{\alpha\alpha}t/\hbar$ and $\kappa_d := 2\kappa'_d t/\hbar$, with $\alpha\beta = \{a, b, c, d\}$. Therefore, the signal and SNR are the same as (11) and (D3), respectively, except that we replace λ_s with $\lambda_s + \kappa_s - \kappa_d$.

Although the electromagnetic interactions cannot create entanglement between the interferometers given the above assumptions, they can still lead to a loss of coherence [120] and affect the signal and SNR for the gravitational correlations. In principle, this effect can be turned off or minimized using the Feshbach resonances of the atoms [40,65,121]: Due to the Feshbach resonances, a magnetic field of a certain strength can be applied to the gas such that the two-body electromagnetic interactions are switched off or significantly reduced [122], a remarkable tool not available to microsolids. Although the magnetic field strengths required to achieve this are not challenging, in order to maintain a constant zero interaction, the magnetic field must have sufficiently low noise. However, the electromagnetic interactions can in fact enhance the GIE effect in certain cases. For example, in the measurement scheme in Appendix B, where both arms of the interferometer are closed, as we increase the electromagnetic interactions between the atoms (assuming the example experimental parameters above), the signal and SNR start increasing once the strength surpasses the strength of the gravitational interactions. This increase is maintained until the electromagnetic interactions are around three orders greater than the gravitational ones, after which, although the signal carries on increasing, the SNR starts reducing. In contrast, for the scheme in Fig. 1 where only one arm is closed, the signal and the SNR slightly decrease once the electromagnetic interactions become greater than the gravitational ones, and significantly decrease once the electromagnetic interactions are around four orders of magnitude greater than the gravitational ones. Ignoring the magnetic dipole-dipole interactions, this would mean reducing the effective s -wave scattering by around five orders of magnitude below the Bohr radius (see Appendix E). However, the different dependence on the strength of the electromagnetic interactions suggests that another measurement scheme could be developed where the effect of the electromagnetic interactions is minimized further or extinguished. For example, as shown in Appendix F, the purity of each interferometer and thus the concurrence entanglement monotone considered above is only dependent on λ and not on the electromagnetic interactions, although this, and its mixed-state extensions [95], would be more challenging to measure. Alternatively, the

dependence on the electromagnetic interactions could be extracted given that the dependence of the SNR on γ exactly matches that of measuring $\langle \hat{J}_z^{ab} \rangle$ in a single, isolated interferometer (see Appendix G).

2. Three-body collisions

Above, we have assumed a two-body approximation for the electromagnetic interactions. However, three-body interactions, although suppressed, will still be present and are often the limiting factor in the lifetime of trapped atomic clouds since the collisions result in atoms being ejected out of the traps.

The master equation for this process in a two-mode interferometer is [124]

$$\begin{aligned} \frac{d\hat{\rho}(t)}{dt} = & -\frac{i}{\hbar}[\hat{H}, \hat{\rho}(t)] + \gamma_3(t) \left(\hat{a}^3 \hat{\rho}(t) \hat{a}^{\dagger 3} - \frac{1}{2} \{ \hat{a}^{\dagger 3} \hat{a}^3, \hat{\rho}(t) \} \right. \\ & \left. + \hat{b}^3 \hat{\rho}(t) \hat{b}^{\dagger 3} - \frac{1}{2} \{ \hat{b}^{\dagger 3} \hat{b}^3, \hat{\rho}(t) \} \right), \end{aligned} \quad (17)$$

where $\hat{H}$ is the Hamiltonian (4), γ_3 is the three-body decay rate, and $\hat{a}$ and $\hat{b}$ are the annihilation operators for the two modes of the interferometer. The three-body decay rate is $\gamma_3(t) = Ln^2(t)$ [65,124–126], where L is the rate coefficient and $n(t)$ is the number density of the gas at time t . For the coherent state (2), the decoherence rate matches the damping rate γ_3 [124,127]. We, therefore, estimate the decoherence time of the experiment for an initial coherent state by the half-life from three-body decay, which is $\tau = 3/[2\gamma_3(0)]$. This time will depend on the species of atoms used and the density. Taking a number density of 10^{14} cm^{-3} and an erbium gas as an example, which has $L \lesssim 3 \times 10^{-30} \text{ cm}^6 \text{ s}^{-1}$ [128], we find $\tau \gtrsim 2 \times 10^3 \text{ s}$, which matches the lifetimes times assumed for estimating the SNR in Sec. IID. This estimate is based on a 3D gas, whereas for 2D and 1D gases, which can be approximated by oblate and prolate spheroidal geometries, three-body losses can be much lower [129–131].

3. Background gas atom collisions

Another source of decoherence will come from collisions with background gas atom collisions, which are thought to be the limiting factor in GIE schemes such as Bose *et al.* [4] since, given the initial N00N state of atoms, a single loss of atom would be enough to completely decohere the state.

Assuming that all collisions cause atoms to leave the interferometer, the master equation for this process in the Born-Markov approximation is [125]

$$\begin{aligned} \frac{d\hat{\rho}(t)}{dt} = & -\frac{i}{\hbar}[\hat{H}, \hat{\rho}(t)] + \gamma_f(t) \left(\hat{a} \hat{\rho}(t) \hat{a}^\dagger - \frac{1}{2} \{ \hat{a}^\dagger \hat{a}, \hat{\rho}(t) \} \right. \\ & \left. + \hat{b} \hat{\rho}(t) \hat{b}^\dagger - \frac{1}{2} \{ \hat{b}^\dagger \hat{b}, \hat{\rho}(t) \} \right), \end{aligned} \quad (18)$$

where $\gamma_f \approx 1/\sqrt{6}\sigma(u_f)n_f u_f$ when assuming only s -wave scattering, with n_f the number density of the background gas atoms, u_f their average velocity, and $\sigma(u_f)$ is the cross section for the process. Using kinetic theory, we can approximate the above quantities as $u_f = \sqrt{2k_B T/m_f}$, $n_f = P/(k_B T)$, and $\sigma_f = 7.57(1.033)^2 [C_6/(\hbar u_f)]^{2/5}$, where P is the

pressure of the vacuum chamber and C_6 is the van der Waals constants from the van der Waals potential $V(r) = -C_6/r^6$. Again, for a coherent state (2), the decoherence rate matches the damping rate γ_f [125,132]. Estimating the decoherence time of the experiment by the half-life due to γ_f , to achieve decoherence times of order 10^3 s, we require pressures of around 10^{-9} Pa assuming background gas atoms to be at room temperature, taking interaction cross sections from [133]. The reason far more reasonable pressures are required compared to microsolid experiments, such as [4] for example which requires around 10^{-15} Pa, is because the initial state here is just a collection of independent atom superpositions, which is relatively robust to foreign atom collisions in comparison to N00N states where a single collision will completely decohere the state rather than just reducing the number of atoms in the state by one as in the coherent state.

4. Additional sources of noise and decoherence

Above we have discussed the most prominent sources of noise and decoherence in the experiment. However, there will be many other sources of noise and decoherence, such as fluctuations of the trapping potentials and the effect of any conducting membrane used to shield the electromagnetic interactions (for example, the membrane must be at low enough temperatures to not heat the atoms too much, and it may also attract the atoms). The temperature of the gas will also affect how it is split in the 50-50 beam splitter: in (1) we assumed that the gas starts off in a single-mode Fock state, whereas in reality there will be many modes and, unless there is condensation, all the modes will be in a thermal state [24]. In addition to this, it is also possible that there could be decoherence due to not closing the cd interferometer: away from the Newtonian regime of quantum gravity, it is possible that the gravitational field remains entangled with matter, which would also be the case in certain other proposed tests of quantum gravity [7,14]. This will be considered in future work, however, the measurement scheme considered in Appendix B would not suffer from any such decoherence.

Another option available to cold-atom systems is to operate the experiment in space, dramatically reducing noise and the challenge of achieving high interrogation times together with high atom numbers. Cold-atomic gas experiments, including atomic interferometers, have already operated in space [134,135], and there are many plans to carry out further gravitational tests with cold atoms in space [16,136,137].

III. SUMMARY

We have described a cold-atom experiment that tests quantum gravity through GIE. It is simple in that it does not require highly quantum states, such as N00N states or Heisenberg-limited squeezed states. However, as with any experiment to test the quantum nature of gravity, it requires advancements over the state of the art and, in this case, particularly the number of atoms. The number of atoms required, without quantum squeezing the state, means that the total mass is close to the Planck mass scale, a scale that can be used as a rough figure of merit for the ability of an experiment to test quantum gravity [125,138,139]. However, by slightly quantum

squeezing the state, for example, to current state-of-the-art levels, the mass and thus the number of atoms required can be reduced by orders of magnitude to approachable levels, as detailed in Sec. II G.

The advancements required here over the state of the art do not need technological leaps of faith, as with generating highly nonclassical states, such as N00N states or Heisenberg-limit quantum squeezing in microsolid experiments. Extending the scheme in a similar vein to extensions to the Bose *et al.* proposal [4,140–142] might also provide small improvements, such as using several atom interferometers and perhaps looking for multipartite correlations rather than bipartite correlations and entanglement. A key feature of cold-atom gases is that it is easy to manipulate them into different schemes and setups. Here, we have concentrated on the scheme in Fig. 1 where two interferometers are parallel to each other and the gases are of oblate spheroidal shape. However, this can be tailored to the requirements of the experiment. For example, it could be necessary to have a smaller superposition size, in which case extreme oblate spheroids might be preferable with the interferometers adjacent to each other but not in parallel planes, similar to the Bose *et al.* setup [4].

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APPENDIX A: GRAVITATIONAL INTERACTION OF SPHEROIDS

The coupling $\lambda'_{\alpha\beta}$ in (5) can be written as

$$\lambda'_{\alpha\beta} = \frac{1}{2} \int d^3\mathbf{x} \rho_\alpha(\mathbf{x}) \Phi_\beta(\mathbf{x}), \quad (\text{A1})$$

where $\rho_\alpha(\mathbf{x}) := m|\psi_\alpha(\mathbf{x})|^2$ is the mass density of the cold-atomic gas in mode α , and

$$\Phi_\beta(\mathbf{x}) := \int d^3\mathbf{x}' \frac{\rho_\beta(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} \quad (\text{A2})$$

is the gravitational potential of the gas in mode β .

From [125], the gravitational potential outside an oblate spheroid in cylindrical coordinates is

$$\Phi_o = \frac{3Gm}{4l^3} \left[(2l^2 - D) \sin^{-1} \left(\frac{\sqrt{2}l}{B} \right) + \frac{\sqrt{2}l(AD - l^2C)}{EB^2} \right], \quad (\text{A3})$$

where

$$A := r^2 + z^2 + \sqrt{z^4 + 2z^2(r^2 + l^2) + (l^2 - r^2)^2}, \quad (\text{A4})$$

$$B := \sqrt{A + l^2}, \quad (\text{A5})$$

$$C := r^2 + 2z^2, \quad (\text{A6})$$

$$D := r^2 - 2z^2, \quad (\text{A7})$$

$$E := \sqrt{A - l^2}, \quad (\text{A8})$$

with $l = \sqrt{a^2 - c^2}$ the focal distance, and a and c , respectively, the equatorial and polar radii.

With the gases of the two interferometers two oblate spheroids a distance d apart, and assuming no overlap of the wave functions, $\lambda'_{\alpha\beta\neq\alpha}$ can be calculated as [125]

$$\int d^3x \rho_\alpha(\mathbf{x}) \Phi_\beta(\mathbf{x}) = \rho_0 \quad (\text{A9})$$

$$\times \int_0^{2\pi} \int_{d-c}^{d+c} \int_0^a \sqrt{1-(z-d)^2/c^2} dr dz d\vartheta r \Phi_o, \quad (\text{A10})$$

where $\rho_0 := m/[(4/3)\pi a^2 c]$ is the density of a uniform spheroid. If the two gases where spherical with uniform density, then $\lambda'_{\alpha\beta}$ is simply $Gm^2/(2d)$. We find that, assuming an oblate spheroid with same volume as a sphere, this can be improved upon by a factor of $\approx \frac{3}{2}$ for oblate spheroids, assuming $d \approx 2c$. The ellipticity $e := l/a$ that achieves this is $e \approx \sqrt{26/3}/3 \approx 0.98$.

The self-interaction couplings $\lambda'_{\alpha\alpha}$ can be computed in a similar way, resulting in [125]

$$\lambda'_{\alpha\alpha} = \frac{3Gm^2}{5l} \sin^{-1} e, \quad (\text{A11})$$

which for $e \approx 0.98$ gives $\lambda'_{\alpha\alpha}/\lambda'_{\alpha\beta\neq\alpha} \approx 0.16$.

Here, we have assumed that the density of the atomic gas is uniform. In practice, the density will depend on the harmonic potential and the strength of the electromagnetic interactions, ranging from a Gaussian profile to a paraboliclike profile [65, 125].

APPENDIX B: CLOSING BOTH INTERFEROMETERS

With both interferometers closed, the covariance (9) is equivalent to $\langle \psi_t | \hat{J}_\varphi^{ab} \hat{J}_{\varphi'}^{cd} | \psi_t \rangle - \langle \psi_t | \hat{J}_\varphi^{ab} | \psi_t \rangle \langle \psi_t | \hat{J}_{\varphi'}^{cd} | \psi_t \rangle$ before the final beam splitters, where φ and φ' are the phases induced by two final beam splitters. This is found to be

$$\begin{aligned} S &= \frac{1}{8} N^2 [\cos(\mu + \nu) \cos^{2(N-1)}(\lambda + \lambda_s) \\ &\quad + \cos(\mu - \nu) \cos^{2(N-1)}(\lambda - \lambda_s) \\ &\quad - 2 \cos \mu \cos \nu \cos^{2(N-1)}(\lambda_s) \cos^{2N}(\lambda)], \end{aligned} \quad (\text{B1})$$

where $\mu := \varphi - \phi$, $\nu := \varphi' - \phi'$, and we have ignored the electromagnetic interactions between the atoms; to introduce these we just switch λ_s with γ as defined in the main text.

The variance of this signal is

$$\begin{aligned} \text{Var}(S) &\approx \frac{1}{\mathbb{M}} [(\langle \hat{J}_\varphi^{ab} \hat{J}_{\varphi'}^{cd} \rangle)^2 - 2(\langle \hat{J}_\varphi^{ab} \rangle^2 \langle \hat{J}_{\varphi'}^{cd} \rangle \langle \hat{J}_{\varphi'}^{cd} \rangle \\ &\quad - 2(\langle \hat{J}_{\varphi'}^{cd} \rangle^2 \langle \hat{J}_\varphi^{ab} \rangle \langle \hat{J}_\varphi^{ab} \rangle + \langle \hat{J}_\varphi^{ab} \rangle^2 \langle \hat{J}_{\varphi'}^{cd} \rangle^2 \\ &\quad + \langle \hat{J}_{\varphi'}^{cd} \rangle^2 \langle \hat{J}_\varphi^{ab} \rangle^2 + 6\langle \hat{J}_\varphi^{ab} \hat{J}_{\varphi'}^{cd} \rangle \langle \hat{J}_\varphi^{ab} \rangle \langle \hat{J}_{\varphi'}^{cd} \rangle \\ &\quad - 4\langle \hat{J}_\varphi^{ab} \rangle^2 \langle \hat{J}_{\varphi'}^{cd} \rangle^2], \end{aligned} \quad (\text{B2})$$

where

$$\begin{aligned} \langle (\hat{J}_\varphi^{ab})^2 \rangle &= \frac{1}{8} N(1 + N + (N-1) \cos[2(\varphi - \phi)]) \\ &\quad \times \cos^{N-2}(2\lambda_s) \cos^N(2\lambda), \end{aligned} \quad (\text{B3})$$

$$\begin{aligned} \langle (\hat{J}_{\varphi'}^{cd})^2 \rangle &= \frac{1}{8} N(1 + N + (N-1) \cos[2(\varphi' - \phi')]) \\ &\quad \times \cos^{N-2}(2\lambda_s) \cos^N(2\lambda), \end{aligned} \quad (\text{B4})$$

$$\begin{aligned} \langle (\hat{J}_\varphi^{ab})^2 \hat{J}_{\varphi'}^{cd} \rangle &= \frac{1}{32} N^2 [2 \cos(\varphi' - \phi') (\cos^{N-1} \lambda_s \cos^{N-2} \lambda [N + \cos(2\lambda)]) \\ &\quad + (N-1) (\cos(2\varphi + \varphi' - 2\phi - \phi') \cos^{N-2}(2\lambda_s + \lambda) \\ &\quad \times \cos^{N-1}(\lambda_s + 2\lambda) + \cos(2\varphi - \varphi' - 2\phi + \phi') \\ &\quad \times \cos^{N-2}(2\lambda_s - \lambda) \cos^{N-1}(\lambda_s - 2\lambda))], \end{aligned} \quad (\text{B5})$$

$$\begin{aligned} \langle (\hat{J}_\varphi^{ab})^2 (\hat{J}_{\varphi'}^{cd})^2 \rangle &= \frac{1}{128} N^2 [2(N+1)^2 + (N-1)(2 \cos[2(\varphi - \phi)] \\ &\quad + \cos[2(\varphi' - \phi')]) \cos^{N-2}(2\lambda_s) \cos^{N-2}(2\lambda) \\ &\quad \times [N + \cos(4\lambda)] + (N-1) \cos[2(\varphi + \varphi' - \phi - \phi')] \\ &\quad \times \cos^{2(N-2)}[2(\lambda_s + \lambda)] + (N-1) \\ &\quad \times \cos[2(\varphi - \varphi' - \phi + \phi')] \cos^{2(N-2)}[2(\lambda_s - \lambda)]]. \end{aligned} \quad (\text{B6})$$

In the assumption that $\lambda N \ll 1$ and $\lambda_s \approx \lambda$, we find that $\text{SNR} \approx \frac{1}{2} \lambda N$, which is a factor 2 smaller than the SNR of the scheme considered in the main text.

APPENDIX C: ENTANGLEMENT MEASURE

Clearly S in (9) is zero when the state of the atoms is a pure product state. However, classical gravity could be a nonunitary process that causes the two interferometers to couple to each other such that the full state of the system before the final beam splitter becomes mixed:

$$\rho_t = \sum_i p_i \rho_i^{ab} \otimes \rho_i^{cd}, \quad (\text{C1})$$

where $0 \leq p_i \leq 1$ and $\sum_i p_i = 1$. In the case that $\rho_i^{ab} = |\phi_i\rangle_{ab} \langle \phi_i|$ and $\rho_i^{cd} = |\phi'_i\rangle_{cd} \langle \phi'_i|$, with p_i and potentially also ϕ_i and ϕ'_i dependent on the classical-gravity interaction, S is still zero since

$$S = \sum_i p_i {}_{ab} \langle \phi_i | \hat{J}_y^{ab} | \phi_i \rangle {}_{cd} \langle \phi'_i | \hat{J}_z^{cd} | \phi'_i \rangle \quad (\text{C2})$$

$$- \sum_{i,j} p_i p_j {}_{ab} \langle \phi_i | \hat{J}_y^{ab} | \phi_j \rangle {}_{cd} \langle \phi'_j | \hat{J}_z^{cd} | \phi'_i \rangle \quad (\text{C3})$$

$$= 0 \quad (\text{C4})$$

as ${}_{cd} \langle \phi'_i | \hat{J}_z^{cd} | \phi'_i \rangle = 0$.

More generally, given the symmetry of the setup in Fig. 1, it would be expected that a classical-gravity interaction will not affect the 50-50 split of the atoms in each interferometer on average (before the final beam splitter). In this case, even if classical gravity shifted the state of the interferometers ρ_{ab}^i and ρ_{cd}^i away from coherent or squeezed coherent states, we will still obtain $\langle \hat{J}_z^{cd} \rangle = 0$ and thus $S = 0$. In the assumption of

perfect symmetry, we would also not expect nongravitational noise to introduce asymmetry in the 50-50 split. However, even if it did, if it forces the state of the interferometers to be in the number mixed states

$$\rho_{\alpha\beta}^i = \sum_k c_{\alpha\beta k}^i |N-k\rangle_{\alpha\alpha} \langle N-k| \otimes |k\rangle_{\beta\beta} \langle k|, \quad (\text{C5})$$

then also $S = 0$ for classical gravity since now $\text{Tr}(\rho_{\alpha\beta}^i \hat{f}_y^{ab}) = 0$.

APPENDIX D: SNR

From (12), the quantum noise associated with the estimation of the covariance signal S involves the bivariate cumulant κ_{22} , which is [102]

$$\begin{aligned} \kappa_{22} := & \langle (\hat{f}_\varphi^{ab} \hat{f}_z^{cd})^2 \rangle - 2 \langle (\hat{f}_\varphi^{ab})^2 \hat{f}_z^{cd} \rangle \langle \hat{f}_z^{cd} \rangle \\ & - 2 \langle \hat{f}_\varphi^{ab} (\hat{f}_z^{cd})^2 \rangle \langle \hat{f}_\varphi^{ab} \rangle - \langle (\hat{f}_\varphi^{ab})^2 \rangle \langle (\hat{f}_z^{cd})^2 \rangle \\ & + 2 \langle (\hat{f}_\varphi^{ab})^2 \rangle \langle \hat{f}_z^{cd} \rangle^2 + 2 \langle \hat{f}_\varphi^{ab} \rangle^2 \langle (\hat{f}_z^{cd})^2 \rangle \\ & + 8 \langle \hat{f}_\varphi^{ab} \hat{f}_z^{cd} \rangle \langle \hat{f}_\varphi^{ab} \rangle \langle \hat{f}_z^{cd} \rangle - 6 \langle \hat{f}_\varphi^{ab} \rangle^2 \langle \hat{f}_z^{cd} \rangle^2 \\ & - 2 \langle \hat{f}_\varphi^{ab} \hat{f}_z^{cd} \rangle^2. \end{aligned} \quad (\text{D1})$$

The noise $\text{Var}(S)$ is then

$$\begin{aligned} \text{Var}(S) = & \frac{1}{64} (2(N+1) + (N-1) \cos(2\nu) \cos^{N-2} 2\lambda_s \\ & \times \cos^{N-2} 2\lambda [2 + N(\cos 4\lambda - 1)] \\ & + 4N \cos^2 \nu \cos^{2N-2} \lambda_s \cos^{2N-2} \lambda \\ & \times [2N \sin^2 \lambda + \cos^2 \lambda - 2]), \end{aligned} \quad (\text{D2})$$

where $\nu := \varphi - \phi$. The SNR is then found through

$$\text{SNR} = |S| / \sqrt{\text{Var}(S)}, \quad (\text{D3})$$

with S given by (11).

APPENDIX E: USING FESHBACH RESONANCES

For the example parameter values in the main text where no squeezing is used, requiring of order 10^{16} atoms, the gravitational interaction coupling $2\lambda'$ is of order $10^{-58} \text{ kg m}^2 \text{ s}^{-2}$. In contrast, ignoring the much weaker magnetic dipole-dipole interactions (MDDIs), the self-interacting electromagnetic coupling κ' from (16) for an oblate spheroid gas is $\kappa' = \frac{1}{2} g / [(4/3) a^2 c] = 2\pi \hbar^2 a_s / [(4/3) a^2 c m]$ where we have used $V_{\alpha\alpha}(\mathbf{x} - \mathbf{x}') = g\delta(\mathbf{x} - \mathbf{x}')$, with $g = 4\pi \hbar^2 a_s / m$ the s -wave coupling constant, and a_s the s -wave scattering length. Setting $a_s = a_0$ the Bohr radius, κ' is of order $10^{-50} \text{ kg m}^2 \text{ s}^{-2}$. From the main text, the electromagnetic start to significantly affect the noise of the signal S once κ' becomes four orders of magnitude larger than λ' . Therefore, if Feshbach resonances are used to remove this noise, a_s must be lowered below a_0 by four or five orders of magnitude.

As stated in the main text, due to the presence of MDDIs, it would likely be preferable to set the s -wave interactions so that they cancel with the MDDIs, unless ^{88}Sr is used which has no MDDIs. However, here the same level of noise of the magnetic field would be required to cancel these effects as with setting a_s to $10^{-4} a_0$ as above.

APPENDIX F: DEPENDENCE OF PURITY ON GRAVITATIONAL INTERACTION

The density matrix after the interferometers have interacted with each other is $\rho = |\psi_t\rangle\langle\psi_t|$ where $|\psi_t\rangle$ is given by (6). Tracing over one of the interferometers (modes c and d), we are then left with

$$\begin{aligned} \rho_{ab} = & \frac{1}{2^{2N}} \sum_{k,l=0}^N \sqrt{\binom{N}{k} \binom{N}{l}} e^{i(k-l)\Phi} e^{i(k^2-l^2)\gamma} \\ & \times |N-k\rangle_a \langle N-l| \otimes |k\rangle_b \langle l| \sum_{k'} \sqrt{\binom{N}{k'}} e^{2i\lambda k'(k-l)} \\ = & \frac{1}{2^N} \sum_{k,l=0}^N \sqrt{\binom{N}{k} \binom{N}{l}} e^{i(k-l)\Phi} e^{i(k^2-l^2)\gamma} \\ & \times \cos^N[\lambda(k-l)] |N-k\rangle_a \langle N-l| \otimes |k\rangle_b \langle l|, \end{aligned} \quad (\text{F1})$$

where we have replaced λ_s in (6) with $\gamma := \lambda_s + \kappa_s - \kappa_d$ to include the electromagnetic interactions within each interferometer (see main text).

Squaring and tracing again, we find the purity of the ab interferometer $\mu := \text{Tr}(\rho_{ab}^2)$ to be

$$\mu = \frac{1}{2^{2N}} \sum_{k,l=0}^N \binom{N}{k} \binom{N}{l} \cos^{2N}[\lambda(k-l)] \quad (\text{F2})$$

$$= \frac{1}{2^{2N}} \sum_{k=0}^{2N} \binom{2N}{k} \cos^{2N}[\lambda(N-k)]. \quad (\text{F3})$$

Note that this is independent of γ . Therefore, the purity of the state is not affected by the gravitational or electromagnetic self-interactions of each interferometer, and instead only the gravitational interactions *between* the interferometers, which cause entanglement.

In the assumption that $\lambda N \ll 1$, this approximates

$$\mu \approx 1 - \frac{1}{2} \lambda^2 N^2. \quad (\text{F4})$$

APPENDIX G: NOISE OF ISOLATED INTERFEROMETER

Here we consider a single-atom interferometer that is not interacting with another one, and measuring $\langle \hat{J}_z \rangle$. The interferometer is as the ab interferometer in Fig. 1 in that the two arms are recombined at the end before $\langle \hat{J}_z \rangle$ is measured. Taking into account the self-interactions of the interferometer, the SNR for this measurement is

$$\begin{aligned} \text{SNR} = & \sqrt{2N} \cos \delta \cos^{(N-1)} \gamma / ((1+N) \\ & - 2N \cos^2 \delta \cos^{2(N-1)} \gamma \\ & + (N-1) \cos 2\nu \cos^{N-2} 2\gamma)^{\frac{1}{2}}, \end{aligned} \quad (\text{G1})$$

where δ is the difference of the phases of the two beam splitters and $\gamma := \lambda_s + \kappa_s - \kappa_d$.

Given $\lambda N \ll 1$, the SNR of S is

$$\text{SNR} = \sqrt{2N\lambda} \sin \nu \cos^{(N-1)} \gamma / ((1+N) - 2N \cos^2 \nu \cos^{2(N-1)} \gamma + (N-1) \cos 2\nu \cos^{N-2} 2\gamma)^{\frac{1}{2}},$$

which has the same dependence on γ as (G1), providing another method to isolate the electromagnetic self-interactions of the interferometers.

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