

Rabi oscillations with close-range quantum vortex states

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(Received 17 July 2025; accepted 12 May 2026; published 4 June 2026)

Quantum vortices separated by distances much larger than their core size interact via their long-range velocity field. At smaller separations, however, the influence of the core's compressibility strongly influences the vortex dynamics. Using the example of a compact necklace of five vortices, it is shown that close-range effects lead to a new (slower) state of orbital motion, which is not predicted by the usual long-range theory. It is demonstrated that control can be exerted over the vortex dynamics by modulating the interaction strength to induce Rabi oscillations between the fast and slow orbital states. A better understanding of these close-range states may have consequences for physical scenarios involving compact vortex clusters, such as quantum fluids with dissipative losses.

DOI: [10.1103/PhysRevA.113.063309](#)

I. INTRODUCTION

Vortices are key components of a wide variety of systems, examples of which include the ocean and atmosphere, optical devices, superconductors, and neutron stars [1–5]. In quantum fluids (such as atomic Bose-Einstein condensates (BECs) [6], superfluid ^4He [7], polariton condensates [8]) vortices are topological defects of the system's order parameter Ψ with quantised circulation $\oint d\mathbf{l} \cdot \mathbf{v} = 2\pi\hbar\ell/M$, where $\mathbf{v} = \frac{\hbar}{M}\nabla\arg(\Psi)$ is the fluid velocity, M is the mass of the condensate particles, and $\ell \in \mathbb{Z}$ is the number of times the phase of Ψ winds around the defect. The existence of quantum vortices (QVs) in these systems gives rise to important phenomena such as mutual friction [7], topological phase transitions [9], and quantum turbulence in 2D [10] and 3D [11].

For a collection of corotating vortices at low energy, the separation between QVs is much larger than the size of the vortex core. Hence, the core structure is neglected, and QVs are treated as infinitesimal filaments (in 3D) or points (in 2D) interacting only through their long-range velocity field. In 2D, the corresponding model is known as the point-vortex (PV) model, which takes the form

$$\dot{\mathbf{x}}_i = \sum_{j \neq i} \mathbf{v}_j(\mathbf{x}_i), \quad (1)$$

where $\mathbf{v}_j$ is the velocity field of the j^{th} vortex whose center is at the position $\mathbf{x}_j(t)$, overdot denotes time differentiation, and a physical boundary can be accounted for by including image vortices. Equation (1) states that each vortex simply

moves with a velocity given by summing up the velocity fields induced by all the others. Although the PV model captures the salient features of the vortex dynamics, it can fail quantitatively in simple scenarios where the fluid compressibility (density variations) cannot be neglected [12]. Furthermore, the model does not account for important phenomena arising for vortices at close-range, i.e., when the inter-vortex distance becomes small. Indeed, Eq. (1) has to be supplemented with ad hoc rules to describe elementary processes such as vortex mergers, annihilations, and reconnections [7,13].

QV splitting instabilities are another crucial effect in this category, since they cause QVs with $|\ell| > 1$ to decay into clusters of $|\ell| = 1$ vortices. There is a rich literature on QV splitting patterns, especially in the context of atomic BECs. The simplest case involves the splitting of $\ell = 2$, which was observed in [14]. At higher ℓ , the dynamics becomes much richer due to the competition of multiple unstable m -modes, where m is the azimuthal index of a wave in the linear theory. The $\ell = 4$ QV (which is predicted to be unstable for $m = 2, 3, 4$ [15]) can be created in ^{87}Rb by reversing the magnetic field of the trap [16]. Experiments on $\ell = 4$ have measured the linear ($m = 2$) and triangular ($m = 3$) splitting patterns [17–20], but not others due to narrow instability windows and small growth rates in finite size set-ups. The splitting dynamics is strongly influenced by the BEC interaction strength [21] and trap geometry [22–24]. In holographic superfluids (studied using the AdS/CFT correspondence), temperature is an additional control parameter [25], and it has been argued that a temperature increase can favour the $m = 4, 5, 6$ instability of the $\ell = 4$ QV [26]. It is also possible to prevent splitting using a Mexican hat-shaped trap [27], inducing a convergent flow through particle depletion [28–33], modulating the interaction strength [34], or adding a secondary particle species inside the vortex core [35]. Further intricacies arise in two-component BECs, including $\ell = 1$ instabilities and splitting-merging revivals [36], $\ell > 1$ ground states [37], temperature-dependent

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dynamical transitions in composite vortices [38], and fractional QVs in spinor condensates [39].

Overall, there is a huge interest in understanding the splitting of large QVs and the ensuing dynamics. In most cases discussed above, the vortex motion following the splitting is captured by Eq. (1) at late times, at least in large systems. The purpose of the work is to illustrate that a new class of vortex states (with no counterpart in the long-range PV theory) can arise for QVs at close-range. In Sec. II, we demonstrate that these states represent a distinct vortex configuration, orbiting a common center more slowly than the typical configuration corresponding to the PV dynamics. Then, in Sec. III, we present a simple model to demonstrate that modulating one of the system's parameters can drive transitions between the different configurations, a prediction which is verified by full numerical simulation in Sec. IV. We conclude in Sec. V by discussing various scenarios (particularly ones involving dense vortex clusters) where these new vortex states might play a role in the dynamics.

II. VORTEX STATES

We consider the Gross-Pitaevskii equation (GPE),

$$i\hbar\partial_t\Psi = \left(-\frac{\hbar^2}{2M}\nabla^2 + V(\mathbf{x}) + g|\Psi|^2 - \mu\right)\Psi, \quad (2)$$

describing the condensed fraction of a cold atomic gas, e.g., ^{23}Na or ^{87}Rb [14,17], near $T = 0$ K. Variants of the GPE describe polariton condensates and quantum fluids of light [8]. We assume a quasi-2D condensate with tight confinement in the vertical direction and work primarily in polar coordinates $\mathbf{x} = (r, \theta)$. g is the interaction strength, μ is the chemical potential, and $V(\mathbf{x})$ is the trapping potential. The system is described by the condensate wavefunction $\Psi = \sqrt{\bar{n}}e^{i\Phi}$, where $\bar{n}$ is the number density of atoms in the condensate, which move with velocity $\mathbf{v} = \frac{\hbar}{M}\nabla\Phi$. The ratio of the nonlinear term to the kinetic term defines the healing length $\xi = \hbar/\sqrt{M\mu}$, and the speed of sound (density waves) in the uniform state $\bar{n} = \mu/g$ is given by $c = \sqrt{\mu/M}$. Later, we will work with a time-dependent interaction parameter and therefore split $g = g_0 + \delta g(t)$, where g_0 is a constant reference value and δg is a small perturbation. In what follows, we work with the following dimensionless quantities,

$$\frac{(ct, \mathbf{x})}{\xi} \rightarrow (t, \mathbf{x}), \quad \frac{V}{\mu} \rightarrow V, \quad \sqrt{\frac{g_0}{\mu}}\Psi \rightarrow \Psi, \quad \frac{g}{g_0} \rightarrow g, \quad (3)$$

and take the trapping potential as

$$V = \frac{V_0}{1 + (V_0 - 1)e^{a(r_B - r)}}, \quad (4)$$

where V_0 , a , and r_B determine the height, smoothness, and radius of the trap wall. We always set $V_0 = a = 10$ but leave r_B as an adjustable parameter.

Equation (2) has stationary vortex solutions given by $\Phi = \ell\theta$, where ℓ is the winding number mentioned in the introduction. We fix $\ell > 0$ without loss of generality and consider a vortex at the center of the circular trapping potential in Eq. (4). The density $\bar{n}(r)$ is determined numerically by evolving (2) in imaginary time $\tau = it$, which is a convenient method for

obtaining the lowest energy state [41]. The solution for $\bar{n}(r)$ vanishes exponentially outside r_B and scales as $\bar{n} \sim r^{2\ell}$ inside the vortex core, which has a characteristic width $\ell\xi$. When the vortex is excited, the full order parameter becomes $\Psi = e^{i\ell\theta}(\sqrt{\bar{n}} + \psi)$. Infinitesimal excitations can be expanded in modes $\psi = \sum_m (u_m e^{im\theta} + v_m^* e^{-im\theta})$ with $m \in (-\infty, \infty)$, where u_m and v_m^* are the positive and negative frequency components of ψ respectively. These obey the linear equation,

$$i\partial_t|\psi_m\rangle = \mathcal{L}|\psi_m\rangle, \quad \mathcal{L} = \begin{pmatrix} \mathcal{D}_+ & g\bar{n} \\ -g\bar{n} & -\mathcal{D}_- \end{pmatrix},$$

$$\mathcal{D}_\pm = -\frac{1}{2}\nabla_r^2 + \frac{(\ell \pm m)^2}{2r^2} + 2g\bar{n} + V - 1, \quad (5)$$

where $|\psi_m\rangle = (u_m, v_m)^T$ [42]. Since the background vortex is stationary, we can write $|\psi_m\rangle = \sum_n a_n e^{-i\omega_n t} |\psi_{mn}\rangle$, where a_n are the mode amplitudes and $n \in [0, \infty)$ indexes the different eigenmodes. These are normalized in the inner product,

$$\langle\psi_\lambda|\psi_{\lambda'}\rangle_{\sigma_3} \equiv \int d^2\mathbf{x} (u_\lambda^* v_{\lambda'}^*) \sigma_3 \begin{pmatrix} u_{\lambda'} \\ v_{\lambda'} \end{pmatrix} = \mathcal{N}_\lambda \delta_{\lambda\lambda'}, \quad (6)$$

where σ_3 is the third Pauli matrix, $\mathcal{N}_\lambda = \pm 1$ is the sign of the norm, and we write $\lambda = (m, n)$ for shorthand. The eigenfrequencies ω_λ are determined by the eigenvalue equation $\omega_\lambda |\psi_\lambda\rangle = \mathcal{L}|\psi_\lambda\rangle$ and fall into two categories. Modes with ω^{vor} are localized in the vortex core (i.e., decay toward $r = r_B$) and encode the vortex dynamics, whereas those which extend outside the vortex core are sound waves (phonons with ω^{ph}). The subset of vortex states with both $\omega_\lambda > 0$ and $\mathcal{N}_\lambda = -1$ correspond to energetic instabilities of the vortex, that is, excitations that grow in the presence of a dissipation mechanism. If ω^{vor} and ω^{ph} become sufficiently close for modes with opposite norms, the two modes hybridize, and the energetic instability gets promoted to a dynamical one with $\mathcal{N}_\lambda = 0$ [42,43], which grows even in the absence of a dissipation mechanism. Physically, the dynamically unstable mode describes the transfer of energy between the central vortex and a surrounding phonon, see e.g. [24,40]. By selecting a small value of r_B , the dynamical instability can be avoided since $\omega^{\text{ph}} \sim r_B^{-1}$ at low frequency while ω^{vor} is roughly r_B independent.

In the following, we will be primarily interested in the dynamics of the negative energy vortex modes, that is, states that lower the total energy of the vortex. Fixing $m > 0$, the different $\mathcal{N}_\lambda = -1$ states with $\omega_\lambda > 0$ correspond to distinct configurations of vortices as the cluster loses energy. Their frequencies can be approximated using the WKB method described in [24,40]. We briefly outline the methodology here, but refer the reader to the original works for the full details.

When wave solutions to Eq. (5) oscillate on a length scale which is shorter than characteristic variations in $\bar{n}$, the solutions look locally like plane waves $|\psi_m\rangle \sim \exp(i \int p(r) dr)$ with a slowly varying radial wavenumber $p(r)$. Replacing $-i\partial_r \rightarrow p$ in Eq. (5) and eliminating the plane wave amplitudes gives a local dispersion relation,

$$\left(\omega - \frac{m\ell}{r^2}\right)^2 = g\bar{n}\tilde{k}^2 + \frac{\tilde{k}^4}{4}, \quad \tilde{k}^2 = p^2 + \frac{\tilde{m}^2}{r^2}, \quad (7)$$

where $\tilde{m}$ is an effective azimuthal number given by $\tilde{m}^2 = m^2 + (r^2 \nabla_r^2 \sqrt{\bar{n}})/\sqrt{\bar{n}}$, which differs from m inside the vor-

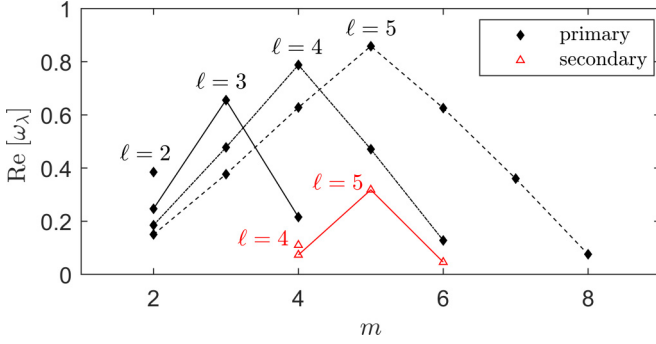


FIG. 1. Frequency (real part) of the vortex excitations computed using the WKB method of [40]. Excitations of the same ℓ -vortex with different m are connected by lines. Vortices with $|\ell| \leq 3$ have only a primary mode. The secondary mode arises for the first time for $\ell = 4$ in the $m = 4$ band, while for $\ell = 5$ there is a secondary mode for $m = 4, 5, 6$. In later sections, we will induce transitions between the $m = 5$ primary and secondary modes of a vortex with $\ell = 5$.

tex core due to the strong density gradient there. Solving Eq. (7) at fixed ω, m for the local value of p , one finds that the vortex core acts as a cavity that traps short wavelength excitations in a certain frequency range. The resonant frequencies of the core are given by $\cos S(\omega_{\lambda}^{\text{vor}}) = 0$, where $S(\omega) = \int_{r_1(\omega)}^{r_2(\omega)} p(r; \omega) dr$ and $r_{1,2}$ are determined by the condition $p(r_{1,2}) = 0$ (note, for $m = \ell$, the lower limit of the phase integral in S is $r = 0$). This is just the requirement that the average wavelength should be a half-integer multiple of the effective length of the cavity, mirroring the particle in a box set-up in elementary quantum mechanics.

Since S increases with the size of the vortex core, there will be an increasing number of states for larger ℓ . In Fig. 1, we plot the zeros of $\cos S(\omega_{\lambda}^{\text{vor}})$ for $\ell = 2, 3, 4, 5$. In general, there are negative energy vortex modes in the band $2 \leq m \leq 2\ell - 2$ [42]. For $\ell = 2$, the only such mode occurs for $m = 2$ and encodes the splitting into two vortices with $\ell = 1$, whereas for $\ell = 3$, there are states for $m = 2, 3, 4$, corresponding to a linear/triangular/square arrangement of $\ell = 1$ vortices, where the latter also includes an $\ell = -1$ vortex at $r = 0$. A new phenomenon occurs for $\ell = 4$, namely, the occurrence of a secondary (lower frequency) mode with $m = 4$, in addition to the primary mode present at a lower ℓ . These secondary modes become more numerous for larger ℓ , e.g., $\ell = 5$ possesses secondary modes for $m = 4, 5, 6$. From here on, we focus on the case $\ell = m = 5$ due to numerical artefacts when working with a fourfold symmetric Cartesian grid—namely a non-physical, stationary $m = 4$ perturbation, which introduces a source of error when modulating the interaction parameter as we do later on.

The secondary modes correspond to a distinct state of orbital motion with no counterpart in the PV dynamics. To demonstrate this, we simulate (2) with an initial condition containing an $\ell = 5$ vortex seeded with a mode amplitude $a_{\lambda_i} = 0.1$ in either the primary (λ_p) or secondary (λ_s) vortex mode. The numerical method is based on the split-step Fourier method described in [24]. For the interaction parameter, we take $g = 1$. To find out what happens to the vortices when the seed mode grows, we introduce phenomenological

dissipation by replacing $i\partial_t \rightarrow (\gamma - i)^{-1}\partial_t$ in Eq. (2) [44], where $\gamma = 2.5 \times 10^{-2}$ is the dissipation parameter. See Appendix A for numerical details.

When the initial condition contains the primary mode, the exponential growth makes the central vortex split into a vortex necklace, whose orbital frequency Ω^{vor} coincides with the PV model at large vortex separation [see Fig. 2(a)]. Since the PV orbital frequency smoothly merges with $\omega_{\lambda_p}^{\text{vor}}/m$, we deduce that the primary mode is the extension of the dynamics described by Eq. (1) when the vortex separation goes to zero (this is also seen for $\ell = m = 2$ in [24]). Conversely, a seed secondary mode does not cause splitting, and the vortex cores remain overlapping [see Fig. 2(b)]. This difference is explained by the different mode functions shown in panels (a2) and (b2). The primary mode has a single extremum at $r = 0$, which causes condensate particles to fill in the center and the vortices to move out into a circular necklace configuration. Conversely, the secondary mode has two extrema, and the zero between them means Ψ retains its original value there, preventing the vortices from separating. Eventually, the primary mode (which grows faster) dominates and the vortex splits. Any attempt to push the secondary mode to larger vortex separations (i.e., increasing the seed amplitude a_{λ_s}) excites the primary mode and leads to splitting. The secondary mode appears, therefore, to be a unique feature of close-range vortex dynamics without a counterpart in the long-range (PV) theory.

III. EXCITING THE SECONDARY MODE

Since the primary mode is more unstable, it is natural to wonder whether there is any realistic scenario where the secondary mode can be observed. We show now that the secondary mode can be accessed starting from the primary by modulating one of the system parameters at a frequency $\Omega = \zeta(\omega_{\lambda_p}^{\text{vor}} - \omega_{\lambda_s}^{\text{vor}})$, where ζ is close to 1, to induce Rabi oscillations between the states. We choose to vary the interaction parameter,

$$g(t) = 1 + \delta g, \quad \delta g = \delta g_0 \sin \Omega t, \quad (8)$$

which could be achieved with a Feshbach resonance [45]. Another option would be to work with a harmonic trapping potential and modulate the trap frequency. The following treatment shares similarities with [46], where a periodic stirring potential was used to drive nonlinear transitions between the ground state and single vortex state.

The first effect of this modulation is to alter the background (symmetric) vortex state. Provided $|\delta g_0| \ll 1$, this shift can be treated as a linear perturbation which obeys,

$$i\partial_t |\psi_0\rangle = \mathcal{L} |\psi_0\rangle + \delta g(t) \bar{n}^{3/2} (1 \quad -1)^T, \quad (9)$$

with $|\psi_0\rangle = (\psi_0 \quad \psi_0^*)^T$ and $\psi_0 = \int_0^{2\pi} \frac{d\theta}{2\pi} (\Psi e^{-i\ell\theta} - \sqrt{\bar{n}})$. This ψ_0 will modulate the parameters in the $m \neq 0$ equation, which to linear order in δg_0 is,

$$i\partial_t |\psi_m\rangle = (\mathcal{L} + \delta \mathcal{L}) |\psi_m\rangle, \quad \delta \mathcal{L} = \begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix},$$

$$A = 2\bar{n}\delta g + 4\sqrt{\bar{n}}\text{Re}[\psi_0], \quad B = \bar{n}\delta g + 2\sqrt{\bar{n}}\psi_0. \quad (10)$$

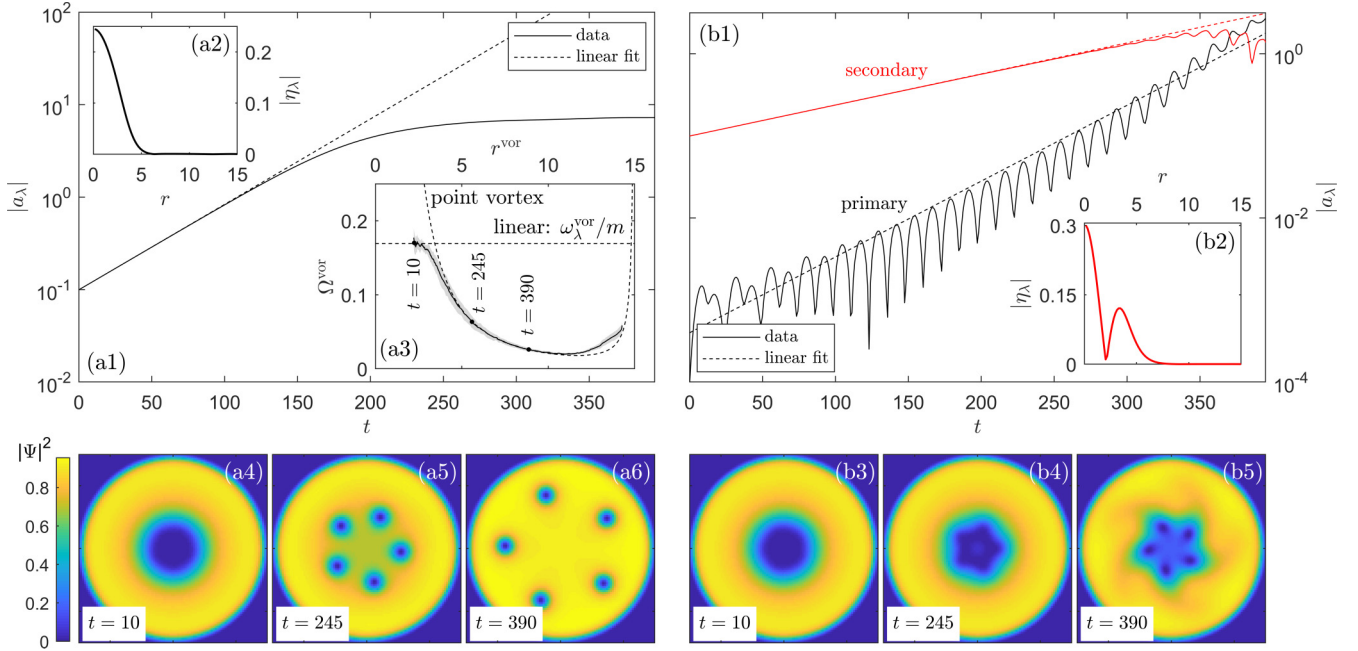


FIG. 2. (a): Evolution of the $m = 5$ primary mode around an $\ell = 5$ vortex at the center of a trap with radius $r_B = 15$. (a1) The primary mode amplitude (solid line) agrees with the linear growth rate (dashed line) at early times. (a2) Representative mode function for the primary mode, where the combination $\eta_\lambda = u_\lambda + v_\lambda$ is proportional to density perturbations [40]. (a3) The orbital frequency Ω^{vor} of the 5 vortices (black line) compared with the linear theory and PV predictions (uncertainty in vortex tracking shown in grey). (a4–a6) $|\Psi|^2$ at the three different times shown as black points on panel (a3). (b): The same for a seed secondary mode. (b1) Primary and secondary mode growth rates (black and red solid lines) agree with the linear growth rates (dashed lines). (b2) Mode function of the secondary mode. (b3–b5) Growth of the secondary mode does not split the vortices but leads to an arrangement with overlapping vortex cores. (b5) The vortices split when the primary mode becomes dominant.

All the ω_λ are assumed real, which occurs provided we don't have a dynamical instability (i.e., for small trap sizes as explained above). Applying basic techniques, the state is projected onto the basis of the linear equations $|\psi_m\rangle = \sum_n a_\lambda(t) e^{-i\omega_\lambda t} |\psi_\lambda\rangle$ with amplitudes a_λ , which vary slowly relative to ω_λ^{-1} (recall that $\lambda = (m, n)$). Since the background is symmetric in θ and Eq. (10) is linear, the different m -modes do not couple. We therefore work with a single mode ($m = 5$ in our case) and drop the index m . This is done so that, for the remainder of this section, all subscripts refer to the level n in the $m = 5$ mode. The equation for each mode amplitude becomes,

$$i\mathcal{N}_n \dot{a}_n = \sum_{n'} \rho_{nn'}(t) e^{i(\omega_n - \omega_{n'})t}, \quad (11)$$

where $\rho_{nn'}(t) = \langle \psi_n | \delta \mathcal{L} | \psi_{n'} \rangle_{\sigma_3}$. We now make the simplifying assumption that the system behaves as a two-level system, which is satisfied in the usual way provided the background modulation Ω is close to the level difference. We take the two levels to be the primary and secondary vortex modes, which for small traps ($r_B \lesssim 10.5$) correspond to $n = 1$ and $n = 0$ respectively. Note that for a trap this small, the $n = 0, 1$ modes have a smaller frequency than the lowest energy phonon in the system, meaning the dynamical instability does not occur, and the effect of phonons on the dynamics can be ignored to a first approximation.

If modulations in g are switched on abruptly, $\delta \mathcal{L}$ will contain multiple frequencies. There will be a dominant component at $\omega = \pm \Omega$ (i.e., the modulation frequency) and

subdominant components at the $m = 0$ eigenfrequencies, provided Ω is not on resonance with these. Focussing on only the dominant contributions, we can approximate $\rho_{nn'} \simeq \tilde{\rho}_{nn'}^+ e^{-i\Omega t} + \tilde{\rho}_{nn'}^- e^{i\Omega t}$, where $\tilde{\rho}_{nn'}^\pm$ are the Fourier components of $\rho_{nn'}$ at $\omega = \pm \Omega$. Writing $a_n(t) = b_n(t) \exp(i \int_0^t \rho_{nn}(t') dt')$, the b_n obey,

$$\begin{aligned} i\dot{b}_0 &= -\tilde{\rho}_{01}^+ e^{-i \int_0^t \Delta^+ dt'} b_1 - \tilde{\rho}_{01}^- e^{-i \int_0^t \Delta^- dt'} b_1, \\ i\dot{b}_1 &= -\tilde{\rho}_{10}^+ e^{i \int_0^t \Delta^- dt'} b_0 - \tilde{\rho}_{10}^- e^{i \int_0^t \Delta^+ dt'} b_0, \end{aligned} \quad (12)$$

with $\Delta^\pm = \omega_1 - \omega_0 - \rho_{11} + \rho_{00} \pm \Omega$. The terms ρ_{00} and ρ_{11} are oscillatory and small compared to the remaining terms, which lead to secular evolution of the phase. Neglecting them, the $\Delta^\pm$ become constants. Furthermore, in the rotating wave approximation (RWA), the terms containing Δ^+ oscillate much more rapidly than the ones with Δ^- and tend to average out. Hence, writing $\tilde{\rho} \equiv \tilde{\rho}_{01}^- = \tilde{\rho}_{10}^{+*}$ and $\Delta \equiv \Delta^-$, the equations become

$$i\dot{b}_0 = -\tilde{\rho} e^{-i\Delta t} b_1, \quad i\dot{b}_1 = -\tilde{\rho}^* e^{i\Delta t} b_0. \quad (13)$$

The solution for an initial amplitude $b_1(0) = \alpha$ in the primary mode and $b_0(0) = 0$ in the secondary is

$$\begin{aligned} b_1(t) &= e^{\frac{i\Delta t}{2}} \alpha \left(\cos \kappa t - \frac{i\Delta}{2\kappa} \sin \kappa t \right), \\ b_0(t) &= e^{-\frac{i\Delta t}{2}} \frac{i\tilde{\rho}\alpha}{\kappa} \sin \kappa t, \quad \kappa = \frac{\sqrt{\Delta^2 + 4|\tilde{\rho}|^2}}{2}. \end{aligned} \quad (14)$$

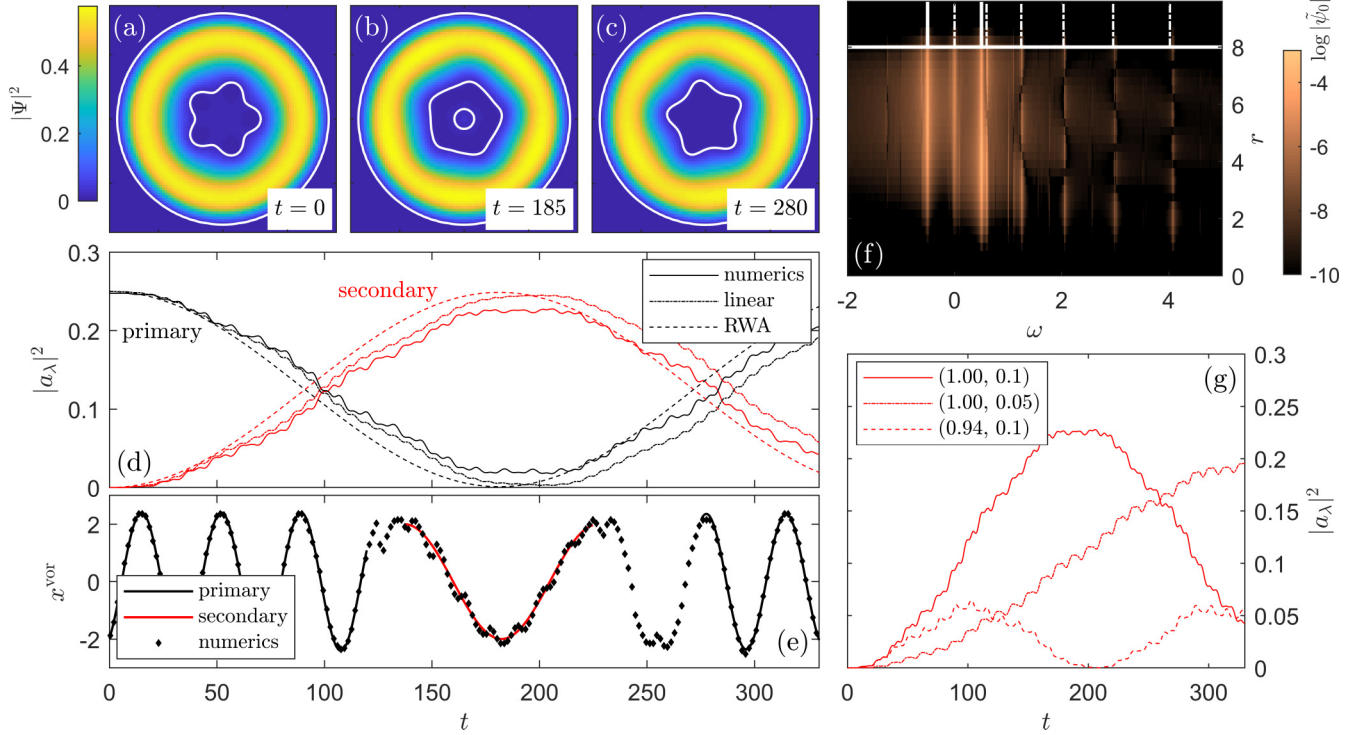


FIG. 3. (a)–(c): $|\Psi|^2$ at three representative times showing the full density profile corresponding to the primary vortex mode (a), the secondary (b), and a mixture of the two (c). (d): The mode amplitudes during the simulation as a function of time, compared against the linear model in Eq. (11) using $n \in [-20, 20]$, and the RWA prediction of Eq. (14). (e): The x coordinate of one of the vortices, fit with the frequency ω_{λ_i}/m for the primary (λ_p) and secondary (λ_s) mode. (f): Fourier transform of the $m = 0$ excitation induced by varying the interaction parameter compared against the modulation frequency Ω (solid vertical white lines) and the $m = 0$ eigenmodes (dash-dotted vertical white lines). (g): Secondary mode amplitudes for the values of $(\zeta, \delta g_0)$ shown in the legend.

These describe out-of-phase oscillations with a period π/κ of the mode occupation numbers. Using $\Omega = \zeta(\omega_1 - \omega_0)$, we have $\Delta = (\zeta - 1)(\omega_1 - \omega_0)$. Hence, when the parameter ζ is close to 1, the primary mode amplitude $|b_1|$ can be brought close to zero at $t = \pi/2\kappa$, where the secondary mode amplitude $|b_0|$ reaches a maximum of around $|\alpha|$.

IV. NUMERICAL SIMULATIONS

To verify the predictions of (14), we perform a numerical simulation of (2) for a trap radius $r_B = 8$. In the initial condition, we seed the stationary $\ell = 5$ vortex with the $m = 5$ primary mode of amplitude $\alpha = 0.5$. For the interaction parameter in (8), we use the values $\delta g_0 = 0.1$ and $\zeta = 1.00$. Snapshots of the evolution are shown in Figs. 3(a)–3(c), with contours of $|\Psi|^2 = 0.02$ shown in white. Panel (a), where the vortex is in the primary state, has a distinct pattern from panel (b), where the secondary mode is excited.

The mode amplitudes a_λ during the simulation are computed by projecting Ψ onto the eigenbasis $|\psi_\lambda\rangle$. We display $|a_\lambda|^2$ for the primary and secondary vortex modes in Fig. 3(d), observing that the minimum (maximum) of the primary (secondary) mode amplitude coincides with the distinct density pattern in panel (b). The amplitudes are in good agreement with the predictions of the linear model in Eq. (11), with differences expected due to the model not accounting for nonlinear mode interactions. The two states also correspond to different vortex orbital motion, which we

verify by tracking the vortices during the simulation. In Fig. 3(e), the x -coordinate of one of the vortices is plotted (black diamonds) and compared with a sinusoidal oscillation Ω^{vor} corresponding to the primary mode (black line) and secondary mode (red line). At the beginning and end of the simulation, the vortices are in the fast orbital state, corresponding to the close-range modification of the PV dynamics in Eq. (1). At intermediate times, the vortices orbit more slowly, which is only possible due to the overlapping vortex cores.

We now compare the numerical results with the linear model of the previous section. In Fig. 3(e), we show the frequency content of the $m = 0$ mode, which is extracted by computing $\tilde{\psi}_0(\omega, r) = \int dt e^{i\omega t} \int_0^{2\pi} \frac{d\theta}{2\pi} (\Psi e^{-i\ell\theta} - \sqrt{n})$. This is compared with the modulation frequency bands at $\pm\Omega$ (solid vertical white lines) and the $m = 0$ eigenfrequencies (dashed-dotted vertical white lines). Noting that the dominant and leading sub-dominant contributions come from $\pm\Omega$ and the lowest phonon frequencies at $\pm\omega_0$ (ignoring the $\omega = 0$ mode), we fit the coupling coefficients $\rho_{nn'}$ to a sum of four sinusoids at $\pm\Omega, \pm\omega_0$ to extract the Fourier amplitudes $\tilde{\rho}_{nn'}^\pm$ used in Eq. (12). The dominance of the contributions at $\pm\Omega$ justifies the approximations leading to the RWA results in Eq. (14). These are also plotted in Fig. 3(d) and show good qualitative agreement with the numerical results. Despite neglecting both the coupling to $m = 5$ phonons and the $m = 0$ eigenfrequencies in $\rho_{nn'}$, the RWA result gives a good estimate of κ .

Finally, in Fig. 3(g), we illustrate the effect of changing the modulation amplitude δg_0 and frequency Ω , showing only the amplitude of the secondary mode. As expected from the RWA result, the secondary mode undershoots its maximum amplitude when Ω is detuned from $\omega_1 - \omega_0$, whilst a smaller value of δg_0 leads to a decrease in κ . Crucially, optimal conversion of the primary into the secondary mode is achieved when Ω matches the level difference $\omega_1 - \omega_0$.

V. DISCUSSION

We have demonstrated the existence of new (secondary) states of vortex motion not captured by the point-vortex model in Eq. (1). It was shown that these states can be excited by modulating the interaction strength to induce Rabi oscillations starting from the usual (primary) states of vortex motion, which are extensions of the dynamics in Eq. (1) to small inter-vortex distances. In atomic BECs, the splitting patterns, which have received most attention, are the ones corresponding to the primary states. By contrast, the secondary states (which have lower frequencies) seem to be a consequence of overlapping vortex cores and do not extend to the regime of large vortex separations. These states arise due to the generic feature of a widened vortex core at high winding number $|\ell|$ and, as such, may be relevant in a variety of systems beyond the simple Gross-Pitaevskii model we have studied here, e.g., thin superfluid ^4He films [47], exciton-polariton condensates [29], superfluid Fermi gases [48], and superconductors [49]. As $|\ell|$ increases, more and more states per m occupy the vortex core, and the competition between these states might, for example, underpin the complicated decay dynamics of giant vortices [50]. More generally, it would seem reasonable to anticipate close-range effects (such as transient secondary modes) in any situation where vortices arrange themselves in a sufficiently compact cluster, where the mean vortex separation is comparable to the core size, e.g., for a large vortex in the initial condition [51] and for quantum fluids with localized heating [47] or particle losses [31–33].

While we have focused on $\ell = 5$ (for numerical purposes), the secondary mode first appears for $\ell = 4$ with $m = 4$ and $\omega \sim 0.1c/\xi$. $\ell = 4$ vortices have been the subject of various experimental studies [17–20], firstly, because they are large enough that the competition of multiple unstable modes leads to rich dynamics and, secondly, since they can be induced by reversing the trap's magnetic field [16]. Secondary vortex states could be generated in these, or similar, experiments using the Rabi oscillation technique studied here, either by modulating the interaction strength with a Feshbach resonance or the trap size. It is feasible that a similar Rabi oscillation technique could also induce transitions between the threefold and fourfold symmetric splitting patterns of the $\ell = 4$ vortex, where the latter is difficult to access due to its slow decay [20,26]. Secondary states may also be relevant for the dynamics of vortex filaments in 3D, for example, in the turbulent cascade induced by the decay of $\ell = 4$ vortices [52].

We finally comment on related statistical studies of point vortices. Vortices can cluster when mechanical stirring is applied to a quantum fluid [53,54] in situations where the inverse cascade (a key feature of 2D turbulence) is the dominant mechanism in energy transport [55], even resulting in

point-like giant vortex structures in the high energy limit [56]. Separate from this, collisions between three or more vortices play an important role for a large number of vortices evolving toward statistical equilibrium [57,58], influencing the asymptotic scaling of the vortex density [59,60]. Although these studies are based on the point-vortex model, where vortex separations are assumed much larger than the core size from the outset, they nonetheless represent interesting physical scenarios in which clustering plays a prominent role in the vortex dynamics. This takes on additional significance given that vortex collisions in 2D [13] and reconnections in 3D [61] are typically modelled using ad hoc rules which take effect at close-range. Therefore, a deeper understanding of close-range effects—like the kind studied in the present work—is essential for developing a complete description of vortex dynamics across length scales.

ACKNOWLEDGMENTS

I am grateful to Zoran Hadzibabic for a question and ensuing discussion during the QSimFP Annual Workshop 2022, which prompted me to analyze close-range vortex states more closely. I would also like to thank Ashton Bradley for insightful feedback on the manuscript. This work was supported by the Science and Technology Facilities Council through the UKRI Quantum Technologies for Fundamental Physics Programme (Grant No. ST/T005858/1) and the Engineering and Physical Sciences Research Council through the Stephen Hawking Postdoctoral Fellowship (Grant No. EP/Z536660/1).

DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible, and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX: NUMERICAL DETAILS

The stationary vortex profile $\Psi = \sqrt{n}e^{i\ell\theta}$ was found by evolving Eq. (2) in imaginary time $\tau = it$ by factoring out the angular phase and simulating only the radial direction. Eigenmodes of the vortex are obtained by diagonalizing $\omega_\lambda|\psi_\lambda\rangle = \mathcal{L}|\psi_\lambda\rangle$, approximating the radial derivatives with 5-point centered finite difference stencils everywhere except $r = 0$ ($r = L$) where we apply a 5-point forward (backward) stencil and a regularity (Dirichlet) boundary condition. The full-time evolution of Eq. (2) was performed on a square grid $x \in [-L, L]$, with $L = r_B + 5$ and $N_x = 128$ grid points (similarly for y). The time step was $\Delta t = L^2/(5N_x^2)$, and the evolution was carried out using a split-step Fourier method, see e.g. [24] for details. The simulations in Figs. 2 and 3 are initialized with $\Psi(t = 0) = e^{i\ell\theta}(\sqrt{n} + a_{\lambda_i}u_{\lambda_i}e^{im\theta} + a_{\lambda_i}^*v_{\lambda_i}^*e^{-im\theta})$, where the mode amplitude a_{λ_i} and index λ_i (denoting either the primary or secondary mode) are specified in the main text. Vortices were tracked by convolving $\Psi/|\Psi|$ from the simulation with $e^{-i\theta}$ then searching for peaks in the convolution matrix at each time step. This procedure returns the vortex po-

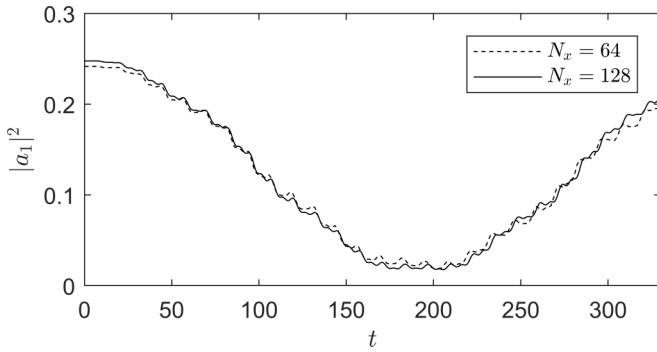


FIG. 4. Reducing the spatial (temporal) resolution by a half (quarter) relative to simulations in Fig. 3(d) results in nearly identical oscillations in the mode occupations. The primary mode occupation is shown here for comparison.

sitions to within pixel accuracy, which suffices for the vortex separations studied in this work. We comment that in Sec. IV, seeding a linear mode in the full nonlinear equation (2) introduces a correction to the vortex density due to backreaction effects. This induces a small shift in the chemical potential μ , which is calculated by measuring the phase drift in Ψ over the duration of the simulation. Using the corrected chemical potential, the normal modes of the vortex are recomputed. This procedure slightly shifts the ω_λ , hence why ζ is quoted to 2 decimal places. Finally, robustness of the numerical method was confirmed by repeating the simulation in Fig. 3 for $N_x = 64$, reducing the spatial (temporal) resolution by a half (quarter). The same parameters were used as those in Fig. 3(d), i.e., $\zeta = 1.00$ and $\delta g_0 = 0.1$. Figure 4 compares the population of the primary ($n = 1$) mode during the simulation. The results are nearly identical, with small differences anticipated due to expected $\mathcal{O}(N_x^{-2})$ numerical errors.

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