

Probing time-reversal-symmetry breaking using a nonlinear superconducting ring resonatorNicolas Dirneger¹, Marie Wesson², Arpit Arora^{1,3,*}, Ioannis Petrides³, Jonathan B. Curtis^{3,4}, Emily M. Been³, Amir Yacoby², and Prineha Narang^{1,3,†}¹*Electrical and Computer Engineering Department, University of California, Los Angeles, California 90095, USA*²*Department of Physics, Harvard University, Massachusetts 02138, USA*³*College of Letters and Science, University of California, Los Angeles, California 90095, USA*⁴*Institute for Theoretical Physics, ETH Zurich, 8093 Zurich, Switzerland*

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Time-reversal-symmetry breaking (TRSB) has been central to detecting exotic phases of matter. Here, we leverage the circuit electrodynamics capabilities of superconducting devices to propose a scheme based on a multimode superconducting ring resonator for the sensitive probing of TRSB in quantum materials. A ring resonator enables nonlinear cross-interactions between the modes which act as built-in amplifiers to be harnessed for enhanced sensing. Using a driven-dissipative model, we explore the nonlinear dynamics of a two-mode superconducting circuit with self- and cross-Kerr nonlinearities under conditions near the bifurcation threshold. By mapping the optimal parameter regimes, we show that even when the photon occupation numbers are subjected to different initial conditions, they can be driven into a symmetric configuration which is broken even with weak TRSB. Through a full quantum analysis we demonstrate that the Kerr-nonlinear interactions up-convert the magnetic effects of the material-resonator hybrid system, enhancing the probing of TRSB. Our findings highlight the utility of superconducting microwave resonators outside of quantum information processing, as a tool for probing exotic states of matter.

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Quantum phases of matter reflect the underlying symmetry of their order parameter. Particularly, time-reversal-symmetry broken (TRSB) orders have been shown to play important roles in many exotic phenomena including topology [1–3], magnetism [4–6], and unconventional superconductivity [7–10]. TRSB manifests in nonreciprocal responses of optoelectronics, superconductors, and collective-mode dynamics, thus playing a crucial role in device operations and uncovering previously inaccessible properties of quantum materials [11–13]. However, TRSB phases can be subtle and sometimes require sensitive probes.

Circuit quantum electrodynamics (cQED) architectures [14,15] are emerging as a universal interface for sensing platforms given their versatility. Research on non-Hermitian sensors [16] and engineered critical dynamics near phase transitions in cQED lattices [17], as well as progress in quantum error correction protocols [18,19], demonstrate that exploiting tunable nonlinearities can substantially boost detection sensitivity and robustness. In this work, we propose a scheme to probe TRSB in quantum materials that leverages the cross-

Kerr nonlinearity in cQED. The role of cross-Kerr interactions has been well recognized in the context of controllable quantum coherence in cQED [20,21], and have recently been used to generate tunable hopping in high-coherence quantum circuit [22] and high-fidelity readout [23,24].

Here, we use nonlinearities of a multimode high- Q superconducting ring resonator. Ring resonators have been extensively investigated in optical [25–27] and silicon platforms [28], and devices based on lithium niobate [29]. The presence of counterpropagating modes arising from higher angular momentum states in the same resonator enable strong nonlinear interactions among the modes, which are otherwise suppressed in traditional multi-resonator systems [30,31]. We show that for nearly degenerate modes of the driven-dissipative resonator, the presence of cross-Kerr interactions can collapse the steady state into symmetric photonic configurations. The symmetry collapse in mode population is lifted even in the presence of a small TRSB perturbation, thus enabling TRSB sensing. This is particularly evident when the system is driven around parametric instabilities where even weak TRSB leads to changes in mode population configurations. As we show below, mode population asymmetry is otherwise suppressed at low and high driving amplitudes outlining the optimal regimes of device operation for high sensitivity. Additionally, we perform the full quantum analysis to examine the role of fluctuations near bifurcation and quantify the signal-to-noise ratio under realistic thermal and dephasing conditions. Exploiting intrinsic nonlinearities in a multimode resonator operating in the GHz regime, we propose an on-chip and cryogenic device for probing hidden correlated order in quantum materials.

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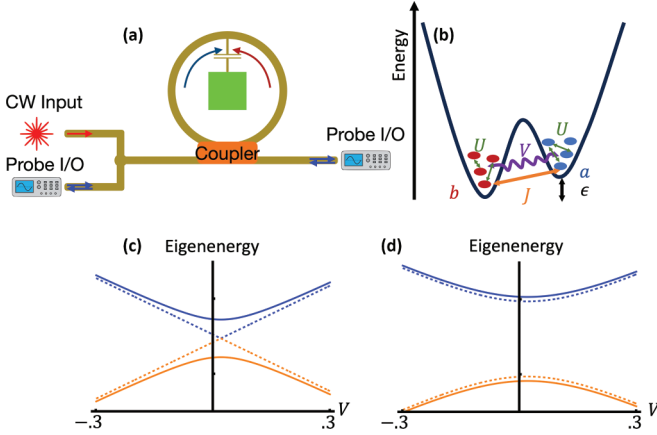


FIG. 1. (a) Schematic of a superconducting ring resonator with counterpropagating modes a and b . The ring resonator is coupled to the continuous wave pump via a coupler (depicted in orange) with an external coupling rate κ , a dissipation rate of modes to the environment γ , and probed from both sides. The sample material to be probed is indicated with the green square. (b) The two modes a and b as two sites of a Hubbard model with self-Kerr U , cross-Kerr V , linear exchange interaction strength J , and natural splitting between the modes, ϵ . (c), (d) Energy eigenvalues as a function of V for standing modes (c) ($\epsilon = 0$) and chiral wave modes (d) ($\epsilon = 1.0$) for the model described in (b) and Eq. (1). Solid lines represent absence of TRSB ($\text{Im}[J] = 0$) while dashed indicate TRSB ($\text{Im}[J] = 0.5$). The remaining parameters were chosen to be $U = 0.6$, $\text{Re}[J] = 0.1$, and $\omega_o = 4.95$.

II. MULTIMODE RING RESONATOR

We consider a superconducting ring resonator [see Fig. 1(a)] that supports two degenerate counterpropagating microwave modes which arise as a consequence of odd angular momentum states, nonlinear kinetic inductance of the superconducting setup [32], or can be generated by bidirectional inputs to the resonator [33,34]. The sample [green box in Fig. 1(a)] is capacitively coupled to the resonator. The TRSB is encoded in the dielectric response of the sample, $E_\mu^* \epsilon_{\mu\nu} E_\nu$. The presence of antisymmetric components, $\epsilon_{\mu\nu} = -\epsilon_{\nu\mu} \neq 0$, e.g., from the Hall response, would alter the transmission spectrum of the system enabling the direct probe of symmetry breaking. For effective manipulation of the modes, a coherent drive is applied to both modes via a transmission line. The driving process involves coupling photons from an external source into the resonator at a controlled rate, allowing the system to reach a steady state with measurable photon occupancy in each mode, as shown in Fig. 1(a). We model the two modes of the ring resonator as two lattice sites of a Hubbard model [see Fig. 1(b), and Supplemental Material [35] for the full derivation]. The full driven-dissipative system can be described by the Hamiltonian

$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}} + \hat{H}_{\text{NL}} + \hat{H}_{\text{drive}}, \quad (1)$$

where $\hat{H}_0 = \omega_a \hat{a}^\dagger \hat{a} + \omega_b \hat{b}^\dagger \hat{b}$ is the intrinsic Hamiltonian, and $\hat{H}_{\text{NL}} = \frac{U}{2} (\hat{a}^{\dagger 2} \hat{a}^2 + \hat{b}^{\dagger 2} \hat{b}^2) + V (\hat{a}^\dagger \hat{a} \hat{b}^\dagger \hat{b})$ describes the nonlinear interaction via the self-Kerr U and cross-Kerr V couplings. The operators $\hat{a}$ and $\hat{b}$ are for clockwise and anticlockwise propagating modes in the ring resonator. The two microwave

modes have frequencies $\omega_a = \omega_o + \epsilon$ and $\omega_b = \omega_o - \epsilon$ where ω_o is the natural frequency of the cavity and ϵ is the natural frequency splitting between the modes. While both self-Kerr and cross-Kerr interactions U, V arise from the nonlinear kinetic inductance, the origin of V can be attributed to a change in frequency of one mode with the population of another mode which is a highly likely situation in a resonator with inherently multiple modes. In a typical cQED setup with single mode resonators, this can be achieved through Josephson junction supported coupling of two resonators [22]. Importantly,

$$\hat{H}_{\text{int}} = (J \hat{a} \hat{b}^\dagger + J^* \hat{a}^\dagger \hat{b}) \quad (2)$$

is the linear exchange interaction between the modes where $\text{Im}[J] \neq 0$ signifies TRSB. According to the coupled-mode theory [36,37], the condition for the presence of TRSB follows directly from the antisymmetric dielectric response as $J \propto \int d\mathbf{r} E_\mu^*(\mathbf{r}) \epsilon_{\mu\nu}(\mathbf{r}) E_\nu(\mathbf{r})$. Finally, $\hat{H}_{\text{drive}} = -i \sum_{j=1}^2 F_j(t) (\sqrt{\kappa_{a,j}} \hat{a}^\dagger + \sqrt{\kappa_{j,b}} \hat{b}^\dagger) + \text{H.c.}$ represents the drive tone applied consisting of a continuous pump and probe tones. $F_j(t) = \sum_m F_{j,m} e^{-i\omega_{j,m} t}$ represents amplitudes and frequencies for both pump and probe tones. The modes are coupled to the resonator with rates κ_a and κ_b ; photon conservation assumed throughout. Note that with $\text{Re}[J] \neq 0$ other nonlinear terms have a negligible impact, previously seen in experiments in systems with linear coupling [30,38].

We characterize our model and highlight the role of V by analyzing the energy eigenvalues of Eq. (1) without a drive [see Fig. 1(b)] under a mean-field approximation, such that $\hat{a}^\dagger \hat{a} = n_a$ and $\hat{b}^\dagger \hat{b} = n_b$ correspond to the population of each mode. In Fig. 1(c), we show the degenerate $\epsilon \rightarrow 0$ case, where $V \neq 0$ enforces complete degeneracy of the modes owing to its competition with $\text{Re}[J]$. However, for modes with $\epsilon \neq 0$, we find that the role V in the system becomes less effective. This demonstrates that large modes with large frequency differences are unfavorable for the device we propose, thus we focus on the degenerate regime.

To understand the dynamics of our system, we define the equations of motions in the Heisenberg picture where we include dissipation coupling to the environment, which can be induced via photon loss by spontaneous decay or photon leakage,

$$\begin{aligned} \dot{\hat{a}} &= \left(-i(\omega_a + U \hat{a}^\dagger \hat{a} + V \hat{b}^\dagger \hat{b} + \epsilon) - \frac{\gamma}{2} \right) \hat{a} + iJ \hat{b} + \sqrt{\kappa} a_{\text{in}}(t), \\ \dot{\hat{b}} &= \left(-i(\omega_b + U \hat{b}^\dagger \hat{b} + V \hat{a}^\dagger \hat{a} - \epsilon) - \frac{\gamma}{2} \right) \hat{b} + iJ^* \hat{a} + \sqrt{\kappa} b_{\text{in}}(t), \end{aligned} \quad (3)$$

where both resonator modes a and b are subject to the total decay given by $\gamma = \kappa_a + \kappa_b$.

III. STEADY-STATE DEVICE OPERATION AND SENSING TRSB

A. Symmetric collapse and steady state

To demonstrate the resonators ability due to $V \neq 0$, we solve for the steady state of the system. In Figs. 2(a) and 2(b) we show the long-time dynamics of $n_{\alpha,\beta}$ for a fixed drive amplitude with $V = 0$ and $V \neq 0$, respectively. Strikingly, the

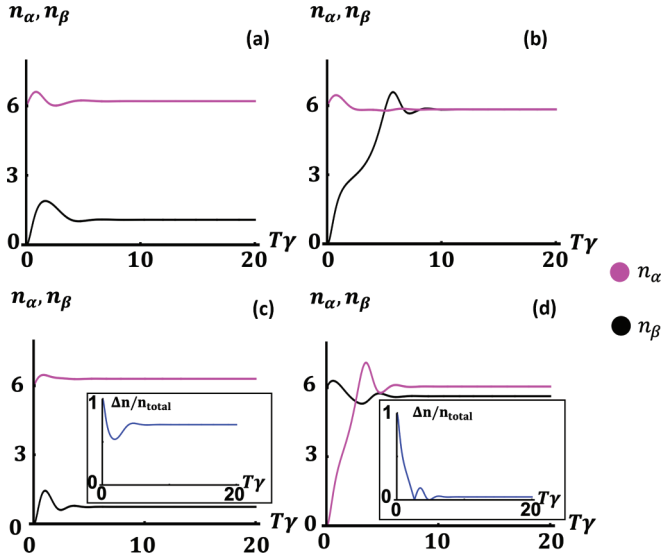


FIG. 2. Long-time dynamics of the driven-dissipative, two-mode resonator obtained from semiclassically solving Eq. (3). (a) Semiclassical steady-state mode populations for $V = 0$ and $V = 0.1$; $n_\alpha = 6.0$ and $n_\beta = 0.0$. (c) The symmetry collapse in populations is strongly lifted when TRSB is introduced into the model, i.e., $\text{Im}[J] \neq 0$ ($\text{Im}[J] = 0.1$). (d) Nonreciprocal behavior of splitting mode populations with respect to initial mode populations, $n_\alpha = 0.0$ and $n_\beta = 6.0$. Other parameters used: $U = 0.6$, $\text{Re}[J] = 0.1$, $\kappa = 1$, $\Delta = -3.5$, and $\gamma = 2$. All parameter values are given in units of dissipation γ .

presence of a cross-Kerr nonlinearity V enables a symmetric collapse of photon populations in the steady state even when they are subjected to different initial conditions. This

overriding of asymmetric initial conditions into a final symmetric state highlights the role of nonlinear cross-mode interactions in enhancing mode correlations. This enables a sensitive probe for TRSB as we note the transition to asymmetric photon populations even for small values of $\text{Im}[J]$ [see Fig. 2(c)]. Interestingly, the splitting of mode population configurations upon TRSB is highly nonreciprocal with respect to the initial mode population, a feature unique to our proposed device, as shown in Fig. 2(d) where the initial mode populations are reversed when compared to Fig. 2(c). We qualitatively quantify the nonreciprocal sensitivity to TRSB by plotting the ratio $\Delta n/n_{\text{total}}$ where $\Delta n = n_\alpha - n_\beta$ and $n_{\text{total}} = n_\alpha + n_\beta$. We note $\Delta n/n_{\text{total}} \approx 70\%$ vs 10% upon switching of the initial conditions [see the inset of Figs. 2(c) and 2(d)].

B. Operating regime and stability analysis

It is useful to operate the device in a regime near the bifurcation threshold where small perturbations can lead to drastic shifts in the steady-state photon occupation numbers. We analyze the variation of steady-state photon populations under variations of drive amplitude F_{in} . The evolution of parametric instabilities with drive amplitude is shown in Figs. 3(a) and 3(b). Stability is determined by linearizing the equations of motion around the mean-field solutions and analyzing the eigenvalues of the resulting stability matrix (see Supplemental Material [35]).

For $V = 0$, the system exhibits the familiar bistable branches of coupled nonlinear resonators, corresponding to low- and high-occupation configurations of the two modes [30]. When $V \neq 0$, these branches hybridize due to the symmetric collapse mechanism described above, producing

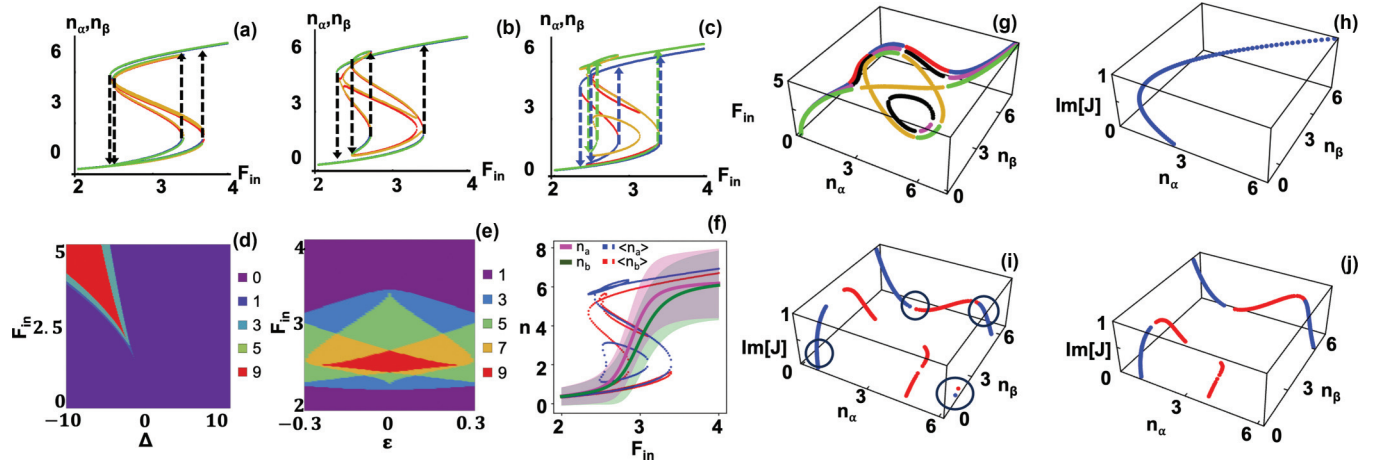


FIG. 3. Parametric instabilities and TRSB sensitivity beyond the bifurcation threshold. (a), (b) Multistable steady-state solutions for populations of each mode when (a) $V = 0$, (b) $V = 0.1$. Here, blue (green) denote stable solutions for mode population n_α (n_β) and red (gold) denote unstable solutions for mode population n_α (n_β). (c) For $\text{Im}[J] = 0.1$, new configurations are obtained, where the two modes undergo low-high transitions at different drives (blue and green dashed arrows). (d), (e) Optimal parameters by determining the number of steady-state solutions in (d) Δ/F_{in} and (e) ϵ/F_{in} phase space, indicating a sensitive device operator for red detuning $\Delta < 0$, and nearly degenerate modes, $\epsilon \rightarrow 0$. (f) Comparison of Semiclassical (red, blue dots) and full quantum (pink, dark-green curves) solutions of photonic population in variation with F_{in} . Shaded regions indicate the instantaneous quantum variance σ_{shot} . (g) 3D visualization of n_α and n_β on F_{in} . The stable (unstable) solutions are shown in green (gold) for $\text{Im}[J] = 0$, magenta (black) for $\text{Im}[J] = 0.1$, and blue (red) for $\text{Im}[J] = 0.2$. When $\text{Im}[J] \neq 0$, the number of stable solutions increases for asymmetric mode configurations. (h), (i) Behavior of photon occupation in variation with $\text{Im}[J]$ for $F_{\text{in}} = 1.8$, $F_{\text{in}} = 2.5$, and $F_{\text{in}} = 3.0$, respectively. Other parameter values are the same as in Fig. 2.

a modified multistable landscape that is highly sensitive to TRSB.

In Fig. 3(c), we show n_α and n_β as a function of F for $\text{Im}[J] > 0$ and find new emergent mode population configurations which demonstrate signatures of TRSB, reminiscent of the system transitioning from symmetric to asymmetric configurations in Fig. 2(c). For $\text{Im}[J] \neq 0$, new asymmetric steady-state branches emerge that are absent in the symmetric case, and the population splitting becomes more pronounced at higher photon numbers, providing a clear operating window for sensing.

Importantly, the difference in mode populations is small when F_{in} is very small or very large, indicating that the region of multistable solutions is the optimal parameter space for device operation. Additionally, here we use the same U for both modes; the steady-state analysis for modes having different U is shown in Supplemental Material [35]. To identify optimal operating conditions, we map the number of steady-state solutions in the Δ/F_{in} and ϵ/F_{in} parameter spaces [Figs. 3(d) and 3(e)]—multistability is maximized for red detuning and nearly degenerate modes.

Next, we visualize in three dimensions (3D) the photon population of each mode. In Fig. 3(g), n_α and n_β are shown as a function of F_{in} . We show stable (unstable) points by green (gold), magenta (black), and blue (red) for $\text{Im}[J] = 0$, $\text{Im}[J] = 0.1$, and $\text{Im}[J] = 0.2$, respectively. Similar to Figs. 3(b) and 3(c), we find drastic changes around the region of parametric instabilities which can be exploited to probe TRSB. Importantly, the 3D visualizations help distinguish the spontaneously symmetry broken states (multistable solutions for $\text{Im}[J] = 0$) from TRSB configurations. Furthermore, any instances of spontaneous symmetry breaking can be averaged out through multiple iterations of measuring the photon population, thus allowing us to distinguish between TRSB and any other form of spontaneous nonlocal interaction. The dependence of photon populations on $\text{Im}[J]$ is shown in Figs. 3(h)–3(j) for different drive amplitudes. For a small drive ($F_{\text{in}} = 1.8$) [Fig. 3(h)], only a single symmetric steady state is present and no measurable TRSB response emerges. Near the bifurcation regime ($F_{\text{in}} = 2.5$) [Fig. 3(i)], multiple steady-state branches appear, and even weak TRSB perturbations induce transitions to asymmetric configurations, providing a sensitive operating point. For larger drive powers ($F_{\text{in}} = 3$) [Fig. 3(j)], the number of accessible configurations decreases and asymmetric states appear only for large $\text{Im}[J]$, reducing the sensitivity of the device. These results confirm that optimized driven-dissipative operation near the bifurcation threshold provides the highest sensitivity to weak TRSB signals in the sample.

IV. QUANTUM ANALYSIS

A. Fluctuation analysis and resolution

While the semiclassical analysis provides a transparent understanding of TRSB sensing using ring resonators, prior studies of driven-dissipative dimers [30] have shown that fluctuations diverge near critical points, potentially destabilizing the steady states. To validate the sensing capability of the ring resonator in the quantum limit, we solve the full Lindblad

Master Equation [39],

$$\dot{\rho} = -\frac{i}{\hbar}[H_{\text{eff}}, \rho] + \sum_i \gamma_i \left(L_i \rho L_i^\dagger - \frac{1}{2} \{L_i^\dagger L_i, \rho\} \right), \quad (4)$$

where the index i sums over the two counterpropagating modes in the ring resonator.

In Fig. 3(f), we show the full quantum solutions of mode populations (pink and dark-green lines) along with semiclassical solutions (red and blue dots) as obtained in Fig. 3(c). The shaded region shows the corresponding quantum variance, $\sigma_{\text{shot}} = \sqrt{\text{Var}(\Delta n)}$ [39]. While we note that fluctuations wash out the multistable states, the conclusion that maximum separation in mode populations for $\text{Im}[J] \neq 0$ occurs beyond the bifurcation regime remains robust even though fluctuations are maximum around the bifurcation regime.

We further analyze the V enabled advantage in ring resonators by simulating multishot measurements which form the basis of schemes based on time-integrated detection. The noise floor for such a measurement is governed by the Standard Error of the Mean (SEM), which scales inversely with the integration time τ such that $\sigma_m(\tau) \approx \sigma_{\text{shot}}/\sqrt{\kappa\tau}$ [39].

In Fig. 4, we analyze the probability of photon occupation $P_i(n) = \text{diag}[\text{Tr}_i \rho]$ and the corresponding probability density function (PDF) (see Supplemental Material [35] for details on the definition of PDF used in this work). For $\text{Im}[J] = 0$ [see Figs. 4(a) and 4(b)] the system resides in a symmetric configuration. Interestingly, we find a stark split in $P_i(n)$ and PDF for $V = 0.1$ [Figs. 4(e) and 4(f)] compared to $V = 0$ [Figs. 4(c) and 4(d)]. This behavior reflects the emergence of interaction-stabilized metastable manifolds: the nonlinear term $V n_\alpha n_\beta$ energetically penalizes simultaneous occupation of both modes, effectively sharpening the potential landscape and suppressing switching between symmetric and asymmetric configurations.

Cross-Kerr initiates a nonlinear feedback mechanism that suppresses fluctuations and improves semiclassical agreement (see Supplemental Material [35] for a comparison of the semiclassical and quantum analysis with corresponding quantum fluctuations). In a setup with multishot measurements, $V \neq 0$ up-converts the magnetic effects in our ring resonator device improving the distinguishability of the photon populations, underpinning the sensing mechanism proposed here.

B. Experimental benchmarks

To assess the robustness of this sensing mechanism, we model the effects of noise within the Lindblad master-equation framework by including photon loss, thermal excitation, and pure dephasing processes. Photon decay at rate κ and thermal occupation n_{th} broaden the photon-number distributions and increase the variance of the population imbalance, while dephasing γ_ϕ introduces additional diffusion in phase space. Typical values of thermal occupations are $n_{\text{th}} \leq 10^{-3}$ – 10^{-2} and pure-dephasing rates around $\gamma_\phi \leq 0.01$ – 0.1κ [40]. Figures 4(g) and 4(h) map the integrated signal-to-noise ratio (SNR) versus thermal occupation n_{th} and dephasing rate γ_ϕ as a function of integration time. In both cases the SNR increases with averaging time, while dephasing and thermal occupation primarily reduce the achievable SNR by

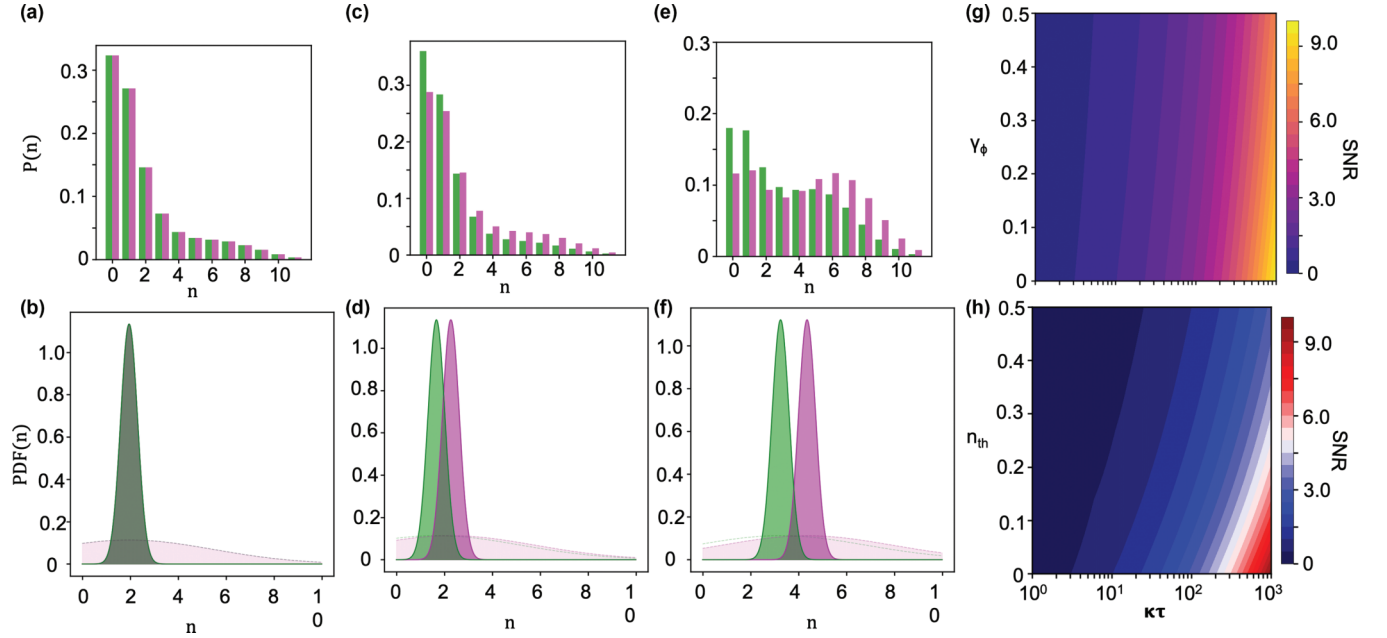


FIG. 4. (a), (c), and (e) Photon number probability $P(n)$ and (b), (d), and (f) probability density function (PDF) obtained from a quantum analysis for Mode A (green) and Mode B (magenta) under varying interaction strengths (V) and TRSB signals ($\text{Im}[J]$); faint dashed lines represent instantaneous single-shot distributions (broadened by quantum fluctuations), while solid lines show the time-integrated distributions ($\kappa\tau = 50$). (a), (b) $\text{Im}[J] = 0$, $V = 0$, (c), (d) $P(n)$ and PDF for $\text{Im}[J] = 0.1$, $V = 0$, and (e), (f) $\text{Im}[J] = 0.1$, $V \neq 0$; $V \neq 0$ resolves the photonic distribution up-converting the magnetic effects in the material-resonator hybrid system. (g), (h) Robustness to Experimental Noise. Contour maps of the integrated SNR as a function of measurement integration time τ and experimental noise sources. (g) Dephasing: Pure dephasing (γ_ϕ) reduces the effective cooperativity and signal contrast. The device demonstrates resilience, maintaining SNR > 5 for dephasing rates up to $\gamma_\phi \approx 0.1\kappa$ for few shot measurements, validating the feasibility of the scheme using current superconducting circuit technology. (h) Thermal Noise: Increasing the bath temperature (n_{th}) broadens the Wigner distribution, requiring longer integration times to resolve the signal. However, even with significant heating ($n_{\text{th}} \approx 0.2$), high-fidelity detection is achievable within standard readout windows ($\kappa\tau \sim 10^2$).

broadening the photon-number distributions, with thermal noise producing the stronger degradation at large noise levels.

The parameter regimes considered here are consistent with experimentally realized superconducting microwave resonators and multimode circuit-QED devices. Typical linewidths lie in the range $\kappa/2\pi \sim 0.1\text{--}5$ MHz, corresponding to quality factors $Q \sim 10^4\text{--}10^6$ depending on geometry and materials. Kerr nonlinearities arising from Josephson elements or engineered nonlinear resonators are routinely observed in the range $U/2\pi \sim 10\text{--}100$ kHz, with comparable or larger effective cross-Kerr couplings achievable in multimode or strongly coupled architectures. Experiments have demonstrated photon-number-dependent shifts and cross-Kerr interactions between microwave modes at the single-photon level, confirming that interaction strengths in this range are realistic for superconducting devices [41–43]. Coupling rates between resonator modes or hybridized excitations typically fall in the range $J/2\pi \sim 0.1\text{--}10$ MHz, while effective non-reciprocal or symmetry-breaking perturbations corresponding to $\text{Im}[J]$ are generally expected to be small compared to the real coupling, often at the level of a few percent or less of J , consistent with perturbative treatments of nonreciprocal or gyrotropic interactions in microwave photonic systems. Drive amplitudes corresponding to intracavity photon numbers in the range $n \sim 1\text{--}10$ are routinely accessible in circuit-QED experiments without inducing unwanted heating or quasiparticle generation, and nonlinear instabilities and bifurcation

phenomena have been observed in this same photon-number regime in Josephson-based resonators [44–46].

Within experimental values of these orders such as detunings of a few linewidths $\Delta/\kappa \sim 1\text{--}10$, near-degenerate mode splittings $\epsilon \ll \kappa$, our simulations indicate that the interaction-enhanced population imbalance remains resolvable over integration times $\kappa\tau \sim 10^1\text{--}10^3$. These values correspond to physical integration times of order $0.1\text{--}10$ μs for typical microwave resonators, well within standard dispersive readout capabilities. Taken together, these estimates demonstrate that the proposed multimode superconducting ring resonator operates in a parameter regime that is not only experimentally accessible but also naturally suited to exploiting cross-Kerr-stabilized metastability as a sensitive probe of weak TRSB signatures in quantum materials.

V. CONCLUSION

In this work, we explore the potential of a multimode superconducting ring resonator within the framework of cQED for detecting TRSB signatures in quantum materials. By leveraging the interplay of nonlinear cross-Kerr interactions, self-Kerr nonlinearities, and driven-dissipative dynamics, we demonstrate the sensitivity of the system to TRSB-induced perturbations through the imaginary component of the exchange coupling parameter, $\text{Im}[J]$, which results in transitions of the system from symmetric to asymmetric photon

configuration states. These symmetry breaking transitions, enabled by the unique nonlinear dynamics of the system, provide a reliable theoretical signature, subject to further experimental validation.

The cross-Kerr term V in a single cavity, which plays a central role in this work, is shown to induce parametric instabilities and multistability regimes, even in the absence of traditional coupled cavity systems. These effects expand the operational versatility of our device and allow for precise control over mode population dynamics. The observed transitions in mode populations underline the practical applicability of the device for experimental detection of TRSB in real-world systems. Our findings establish that the proposed multimode resonator operates effectively in GHz regimes for resonator and drive frequencies, with nonlinearities in the kHz range [47] and interaction strengths in the MHz range [27,48]. These operating regimes ensure compatibility with current experimental paradigms and emphasize the practicality of the approach.

To validate the sensing mechanism beyond the semiclassical approximation, we further carried out a full quantum analysis based on the Lindblad master equation. While quantum fluctuations smooth out idealized bistability in the long-time limit, we find that the TRSB-induced population imbalance remains finite and experimentally resolvable. The cross-Kerr interaction plays a crucial role in this regime by stabilizing metastable configurations, reducing effective noise, and enhancing the distinguishability of photon populations. These results demonstrate that the proposed sensing mechanism remains robust in the quantum regime and does not rely on idealized mean-field behavior.

Unlike traditional Kerr rotation which probes TRSB via polarization rotation of far fields, our approach relies on leveraging intermode coupling and intrinsic nonlinear interactions in superconducting devices. Additionally, given that magneto-optic effects are suppressed at microwave

frequencies, our proposed device emerges as a powerful TRSB tool in the microwave domain. Though recent proposals for microwave Kerr rotations using Sagnac interferometry have been put forward [49], their on-chip compatibility is questionable. On the contrary, our proposed device possesses no such limitations.

More broadly, this work highlights the potential of superconducting resonators not only as elements of quantum information processors but also as precision diagnostic tools for probing exotic phases of matter. By focusing on a single multimode cavity, we extend existing approaches that predominantly rely on coupled resonator arrays and self-Kerr nonlinearities [47,50]. Future research can build on this foundation by integrating such devices into networks of resonators [51,52] and exploring coupling to a wider range of quantum materials. These directions may ultimately enable superconducting resonators to become versatile platforms for precision sensing and quantum-material diagnostics.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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