

Probing strong-field QED via angle-discriminated emissions from electrons traversing colliding laser pulses

Christoffer Olofsson  and Arkady Gonoskov 

Department of Physics, [University of Gothenburg](#), SE-41296 Gothenburg, Sweden



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Future laser-electron colliders will reach quantum parameters χ well in excess of unity, enabling studies of strong-field QED in extreme regimes. However, statistical inference in such experiments requires mitigating premature radiative losses of electrons to enable high- χ QED events, as well as separating the detectable signal of these events from that of lower- χ particles and photons produced by QED cascades. We propose a collider geometry in which electrons traverse the waist of two or four perpendicularly propagating, tightly focused laser pulses. This configuration suppresses both outlined difficulties by leveraging the short interaction length of the waist, rather than relying on the more technically demanding reduction of pulse duration. Moreover, altering the phase and polarization of each pulse causes the electrons to undergo helical motion where the deflection angle is correlated with the field strength, permitting an angle-based discrimination of the signal from high- χ events. Analysis and simulations show that the case of four circularly polarized pulses uniquely permits achieving helical motion throughout the entire focal region, leading to near-perfect high- χ angle-discrimination and thereby high signal-to-noise ratio. These findings support the consideration of the proposed concept as a viable layout for future experiments at PW laser facilities.

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I. INTRODUCTION

With an increasing number of petawatt-class [1–5] and multi-petawatt-class [6–8] laser facilities becoming operational, with several more under development [9–12], the extreme intensities achieved through tight focusing further enable applications such as relativistic particle acceleration [13–17] and high-energy photon generation [18–21]. One ultimate objective can be the observation of Schwinger pair production, as predicted by QED in electromagnetic fields approaching the critical strength $E_{\text{cr}} = m_e^2 c^3 / e \hbar \approx 10^{18} \text{ V m}^{-1}$, where m_e and e are the electron mass and charge, respectively, c is the speed of light, and $\hbar$ is the reduced Planck constant. Producing field strengths of this order remains beyond current capabilities, but may become possible via frequency upshifting through interactions with overdense plasmas [22–26].

Another possibility enabled by current laser technology is observing strong-field QED (SFQED) events using high-energy electrons or photons [27–32]. If a high-energy electron, with Lorentz factor γ , enters a strong electromagnetic field, the observed electric field in the electron's rest frame may exceed E_{cr} . In this case, processes are described by SFQED, quantified by the electron proper acceleration (CGS

units are used throughout)

$$\chi = \gamma E_{\text{cr}}^{-1} \sqrt{(\mathbf{E} + \boldsymbol{\beta} \times \mathbf{B})^2 - (\mathbf{E} \cdot \boldsymbol{\beta})^2}, \quad (1)$$

where $\boldsymbol{\beta}$ is the electron velocity normalized to c and $\mathbf{E}$, $\mathbf{B}$ are the electromagnetic field vectors.

Experimental observations of SFQED in the regime $\chi \lesssim 1$, employing laser-electron collisions, date back to the 1990s, when both nonlinear Compton scattering (NCS) and nonlinear Breit-Wheeler (NBW) pair production were observed at SLAC [33,34]. In an all-optical configuration, NCS was later observed at the Rutherford Appleton Laboratory [35–37] and more recently at the Center for Relativistic Laser Science (CoReLS) [38]. In addition to laser-electron colliders, SFQED processes can also be studied using beam-plasma interactions [39], beam-beam collisions [40], and aligned crystals [41–43].

One direction for future experiments is to explore regimes beyond $\chi \sim 1$ to induce more pronounced nonlinear effects of SFQED, including the observation of QED cascades [44] ($\chi \gtrsim 1$), higher-order processes [45], radiative corrections to the electron mass [40], and the conjectured breakdown of perturbative SFQED at $\chi \sim \alpha_f^{-3/2}$ [46,47], where $\alpha_f \approx 1/137$ is the fine-structure constant. However, the probabilistic nature of SFQED events, combined with spatial field variations along particle trajectories and shot-to-shot fluctuations in experimental conditions, poses significant challenges for extracting quantitative data suitable for comparison with theoretical predictions.

In principle, a Bayesian framework provides an approach for handling probabilistic processes even in the presence of latent variables, such as the impact parameter between the laser and electron beam [37,48]. Nonetheless, the choice of

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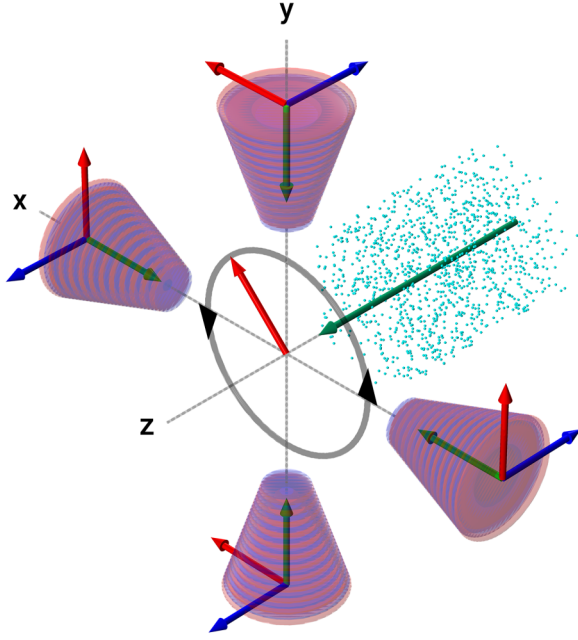


FIG. 1. Four laser pulses focusing in a plane perpendicular to the electron propagation axis. Their phases are set to produce a rotating electric field (red arrow) at the focal point. Green and blue arrows denote propagation directions and the magnetic field, respectively.

collision configuration remains crucial for invoking high- χ events while simultaneously enabling statistical inference on them. Recently, it was shown that, under the assumption of high electron energies, the maximum value of χ for a given total power is achieved using so-called bidipole focusing, while the use of circular polarization enables angle-discriminated extraction of signals from high- χ events [49]. This setup resembles a head-on collision of electrons with an optimally focused laser pulse, while still permitting the use of focusing mirrors with effective f-numbers $f \approx 0.5$ without significant performance loss.

Nevertheless, aside from experimental constraints associated with head-on collisions and tight focusing, this setup becomes inefficient at laser powers exceeding 1 PW, as the onset of QED cascades over typical pulse durations obscures the signal of interest. For electron energies around 20 GeV, this effect largely precludes studies at $\chi \gtrsim 10$.

In this paper, we propose and assess an experimental layout that overcomes the outlined difficulties. This is done as an intermediate step towards the development of future experimental proposals as the interaction geometry can be adjusted in the context of a specific research question or facility. Conceptually, this enables the use of multi-PW laser facilities to conduct quantitative studies of SFQED in the range $\chi \sim 10$ –100. The layout exploits electrons propagating across the waist of tightly focused radiation to reduce the time for cascade development [50], while the angle-discrimination of the signal from high- χ events is achieved by using two or four perpendicularly propagating laser pulses with properly chosen polarization and phase difference (see Fig. 1). This layout permits various modifications, such as the use of a different number of laser pulses to increase the value of χ and the use

of different polarization/phase to control the electromagnetic field observed by electrons.

II. PROBLEM STATEMENT

We consider a geometry that minimizes the interaction time by requiring electrons to traverse the waist of tightly focused laser radiation. Our goal is to identify such a geometry that (i) provides high values of χ and (ii) generates a rotating electromagnetic field that induces helical electron motion which enables angle-based discrimination of signals from high- χ events. To address the first criterion, we analyze achievable χ values using multiple colliding laser pulses (MCLP) as a means to increase the peak electric field strength [51], which has been studied for $n = 2, 4, 8$, and 16 pulses in the context of seeded QED cascades [52–55]. We consider electrons with energy $\varepsilon \approx m_e c^2 \gamma$ and a tightly focused laser pulse of wavelength λ and power P . The size of the focal spot scales as λ^2 ; hence, the peak electric field strength increases with intensity I as $\sqrt{I} \propto \sqrt{P/\lambda^2} = \sqrt{P}/\lambda$. Consequently, χ grows as $\chi \propto \gamma \sqrt{P}/\lambda$ and under the assumptions that $|\beta| \approx 1$ and $\gamma \gg a_{\max}$ its peak value can be expressed as

$$\chi_{\max} = \kappa(\varepsilon/\text{GeV})(\sqrt{P/\text{PW}})(\lambda/\mu\text{m})^{-1}, \quad (2)$$

where $a_{\max}$ is the peak electric field in units of $2\pi m_e c^2 / e\lambda$. Here, κ quantifies how efficiently the laser power is converted into large χ values and its maximum $\kappa \approx 5.25$ occurs for bidipole focusing which implements a combination of electric and magnetic dipole pulses [56,57], as demonstrated in Ref. [49]. Although κ can be significantly lower for electrons traversing the waist of a single focused pulse, the use of n such colliding pulses can partially compensate for this reduction.

This configuration directly addresses point (ii), as rotating electromagnetic fields at the focus can be realized by adjusting each pulse's polarization and phase. As electrons traverse a region where the electromagnetic field vectors rotate around the direction of propagation, their emission angle $\alpha \sim a_{\max}/\gamma$, defined as the angle between the electron's initial propagation axis and its direction at the moment of emission, becomes correlated with χ as both depend on the local field strength. This correlation allows the selection of photons emitted by high- χ electrons based on α . Furthermore, photons of frequency ω that reach the detector with high energy $\delta = \hbar\omega/m_e c^2$ are less likely to have originated from newly formed particles or subsequent emissions, both associated with lower χ values.

In addition to suppressing a QED cascade, the outlined geometry permits the use of longer laser pulses to mitigate the effect of temporal shot-to-shot variations, which cause the overlap between the electrons and the strong-field region to fluctuate. We note that the spatial misalignment can be mitigated by allowing the electron beam to defocus, such that the strong-field region forms within a uniform stream of electrons. While this approach reduces the number of high- χ events, we adopt this configuration as it corresponds to data accumulation over multiple shots.

In the following section we discuss the implementation of the outlined interaction layout and its impact on the value of κ . We also specify how the polarization and phase of each pulse

is chosen to form rotating electromagnetic fields at the focal point to achieve the correlation between χ and α .

III. INTERACTION GEOMETRY

Assuming linear (LP) or circular (CP) polarization for each of the n colliding pulses, all having a central frequency of ω_0 , we examine conditions for generating a rotating electric field at focus. Analogously, a rotating magnetic field can be formed but the impact on κ and the correlation $\alpha \propto \chi$ remain unchanged in the ultrarelativistic limit $|\beta| \approx 1$. The resulting electric field at focus can be expressed as the real part of a complex field vector $e^{-i\omega_0 t} \mathbf{E}^{(n)}$, satisfying $\beta \cdot \mathbf{E}^{(n)} = 0$. In the vicinity of the focal region, $\mathbf{E}^{(n)}$ may be approximated as a superposition of plane waves with frequency ω_0 and wave number k

$$\mathbf{E}^{(n)} = \frac{E_0}{\sqrt{n}} \sum_{j=1}^n e^{ik(\hat{k}_j \cdot \mathbf{r})} \xi_j \hat{e}_j, \quad (3)$$

where $E_0 \propto \sqrt{P}$ correspond to the peak electric field of a single laser pulse, $\hat{k}_j$ is the unit wave vector, and $\hat{e}_j$ are complex unit vectors indicating each pulse's electric field direction and phase. The polarization of each wave is parameterized by $\xi_j = 1$ for LP or $\xi_j = 1/\sqrt{2}$ for CP. The corresponding growth of κ , denoted as $\kappa^{(n)} \propto \chi_{\max}^{(n)}$, is proportional to Eq. (1) at $\mathbf{r} = t = 0$ (assuming all waves are synchronized) and is expressed using Eq. (3) as

$$\kappa^{(n)} \propto \frac{E_0}{\sqrt{n}} \left| \sum_{j=1}^n \xi_j (\hat{e}_j (1 - \beta \cdot \hat{k}_j) + (\beta \cdot \hat{e}_j) \hat{k}_j) \right|. \quad (4)$$

Using the coordinate convention of Fig. 1, the value of κ for the case of electrons colliding head-on with a single pulse, denoted as $\kappa^{(\text{dir})}$, is found by setting $\hat{k}_1 = -\hat{z}$ and $\beta \approx \hat{z}$, so that

$$\kappa^{(\text{dir})} \propto \xi_1 E_0. \quad (5)$$

This geometry gives rise to rotating electromagnetic fields only if the pulse is circularly polarized.

Proceeding to the case of $n = 2$ pulses, the only possible configuration is to employ pulses with linear polarization. Here, a rotating electric field emerges when one pulse arrives from the x axis (see Fig. 1), providing an electric-field component perpendicular to that of the second pulse, arriving from the y axis with a relative phase difference of $\pi/2$. Consequently, the associated magnetic field satisfies $\beta \times \mathbf{B} = 0$ (or $\mathbf{B} = 0$ for a standing wave when the phase difference is zero). The corresponding value of $\kappa^{(2)}$ as compared to Eq. (5) is found from Eq. (4) by setting, e.g., $\hat{k}_1 = \hat{y}$, $\hat{k}_2 = \hat{x}$, $\hat{e}_1 = i\hat{x}$, and $\hat{e}_2 = \hat{y}$

$$\kappa^{(2)} = \frac{\kappa^{(\text{dir})}}{2\sqrt{2}\xi_1}. \quad (6)$$

Notably, this particular setup ensures that in an experiment, neither the electron beam nor the laser pulses impinge on any critical instrumentation positioned along their respective axes. Electrons whose trajectories pass through $x = y = 0$ will experience a circularly rotating electric field, establishing the α - χ correlation described in the preceding section. Around

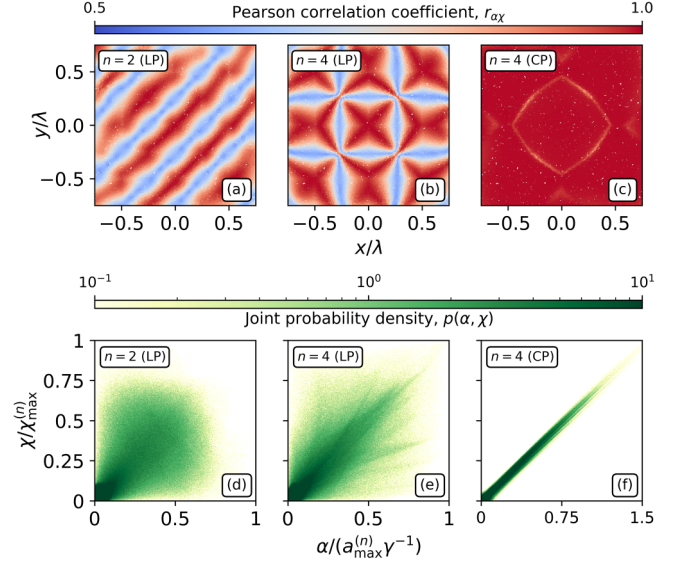


FIG. 2. Correlation strength between χ and α computed from electron trajectories as a function of their initial transverse location (a)–(c) and the corresponding joint probability density for the electron χ and α values (d)–(e).

the strong-field region, the strength of this correlation varies spatially due to interference between the two waves. When $\mathbf{r} \neq 0$, electrons encounter the electric field given by Eq. (3) and is proportional to

$$\propto \hat{x} \sin(ky - \omega_0 t) + \hat{y} \cos(kx - \omega_0 t). \quad (7)$$

The manifest spatial phase difference in Eq. (7) leads to an elliptically rotating or even standing electric field, thereby reducing or eliminating the correlation strength. To quantify this spatial dependence, we introduce the ellipticity

$$\epsilon = 1 - \rho, \quad (8)$$

as a measure of the shape traced by the tip of the electromagnetic field component responsible for transverse electron acceleration. Here,

$$\rho = \frac{F_{\max} - F_{\min}}{F_{\max} + F_{\min}} \quad (9)$$

is a ratio containing the maximum $F_{\max}$ and minimum $F_{\min}$ of the squared field strength over one optical cycle. Circular rotation is demarcated by $F_{\max} = F_{\min}$ whereas a standing wave is characterized by $F_{\min} = 0$, yielding no correlation. Whenever $F_{\max} = F_{\min} = 0$, we define $\epsilon = 0$ as electrons pass through unaffected. Using Eq. (7) as input, the ellipticity for the present configuration becomes

$$\epsilon = 1 - |\sin[k(y - x)]|, \quad (10)$$

and the strongest correlation occurs along lines $y = x + m\frac{\lambda}{2}$, where m is an integer.

This expression is consistent with the spatial structure of the α - χ correlation observed in Fig. 2(a) and with the corresponding electron trajectories shown in Fig. 3. Moving on to the case of $n = 4$ pulses, two possible configurations exist; one for which all beams have LP and the other CP. Furthermore, the pulses need to be paired in the

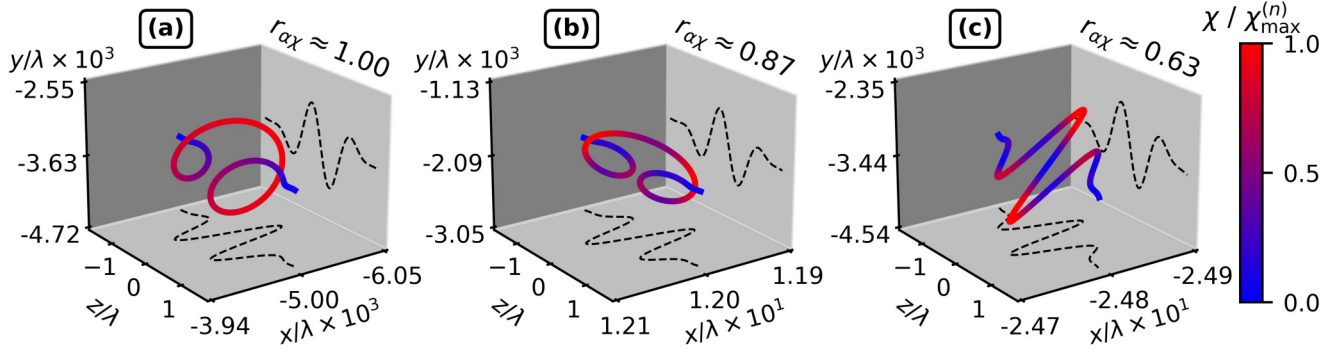


FIG. 3. Simulated electron trajectories in the absence of quantum processes, with initial coordinates $y \approx 0$ and (a) $x \approx 0$, (b) $x \approx 0.12\lambda$, and (c) $x \approx 0.25\lambda$, after traversing the focal field generated by $n = 2$ (LP) colliding laser pulses. Dashed lines show the projections of the trajectory onto the xz and yz planes. The Pearson correlation coefficient $r_{\alpha\chi}$ associated with each trajectory is given in the upper right corner of each plot.

counterpropagating arrangement shown by Fig. 1, where one pair has perpendicular propagation axis to that of the second pair, which has an additional phase difference of $\pi/2$. For LP, the rotating electric field is similar to the previous case except that at focus the electric field contribution is doubled and $\mathbf{B} = 0$. For instance, setting $\hat{k}_1 = -\hat{k}_2 = \hat{x}$, $\hat{k}_3 = -\hat{k}_4 = \hat{y}$ and $\hat{e}_1 = \hat{e}_2 = \hat{y}$, $\hat{e}_3 = \hat{e}_4 = i\hat{x}$ in Eq. (4) we find that $\kappa^{(4)}$ is related to $\kappa^{(\text{dir})}$ by

$$\kappa^{(4)} = \frac{\kappa^{(\text{dir})}}{2\xi_1} \text{ (LP)}, \quad (11)$$

and that the corresponding electric field is proportional to

$$\propto \hat{x} \sin(\omega_0 t) \cos(ky) + \hat{y} \cos(\omega_0 t) \cos(kx). \quad (12)$$

The ellipticity that results from Eq. (12) can be expressed as

$$\epsilon = \frac{2 \cos^2(kx) \cos^2(ky)}{\cos^4(kx) + \cos^4(ky)}, \quad (13)$$

where $\epsilon = 1$ is now achieved when $y = \pm x + m\frac{\lambda}{2}$ and its spatial dependence is consistent with that found in Fig. 2(b).

When all beams are circularly polarized, a rotating electric field arises only if each pair of counterpropagating pulses have opposite helicity. Now, let $\hat{e}_{1,2} = (\hat{y} \pm i\hat{z})$ and $\hat{e}_{3,4} = i(\hat{x} \pm i\hat{z})$ so that the scaling of $\kappa^{(4)}$ relative to $\kappa^{(\text{dir})}$ becomes

$$\kappa^{(4)} = \frac{\kappa^{(\text{dir})}}{\sqrt{2}\xi_1} \text{ (CP)}. \quad (14)$$

Numerical values of κ for several f-numbers are listed in Table I, confirming the predicted scalings. Notably, electrons may also experience a nonzero magnetic field $\mathbf{B}^{(4)}$ such that they are accelerated by the component $\mathbf{E}^{(4)} + \hat{z} \times \mathbf{B}^{(4)}$ which is proportional to

$$\propto [\cos(kx) + \cos(ky)][\hat{x} \sin(\omega_0 t) + \hat{y} \cos(\omega_0 t)] + \hat{z}[\cos(\omega_0 t) \sin(kx) + \sin(\omega_0 t) \sin(ky)]. \quad (15)$$

Evidently, the transverse component of Eq. (15) is circularly rotating everywhere in space. This means that ϵ is always unity [see Fig. 2(c)] except when the electromagnetic field vanishes along $y = \pm x + m\frac{\lambda}{2}$ where now $m \neq 0$. Consequently, the overall correlation strength is the largest for this geometry and becomes weaker for $n = 2$ and $n = 4$ (LP). This is further confirmed by calculating the joint probability

density $p(\alpha, \chi)$ from the simulated electron trajectories within the beam, as shown in the bottom row of Fig. 2. The distribution $p(\alpha, \chi)$ in the case of directly colliding pulses appears in Fig. 2 of Ref. [49].

Generalization beyond $n = 4$ is not treated in detail here, except to estimate an upper bound on the number of pulses. To prevent beam overlap outside the focal region, and assuming each pulse is confined within a cone of aperture angle $\Theta = \arctan \frac{1}{2f}$ the condition $n\Theta \leq \pi$ must be satisfied. For the geometries considered here, where beams propagate along orthogonal axes, this yields a lower bound on the f-number: $f \geq 0.5$.

IV. STABILITY ANALYSIS

To assess the robustness of the proposed geometries to spatiotemporal misalignment and phase imperfections, we first consider the case of $n = 2$ colliding laser pulses. Starting from Eq. (3), we introduce for each wave an additional phase factor $e^{i\delta_j}$, with $\delta_j \in [0, 2\pi)$, and a relative amplitude scaling $0 \leq a_j \leq 1$, so that the total electric field becomes

$$\mathbf{E}^{(n)} = \frac{E_0}{\sqrt{n}} \sum_{j=1}^n a_j e^{i[k(\hat{k}_j \cdot \mathbf{r}) + \delta_j]} \xi_j \hat{e}_j. \quad (16)$$

In addition, we generalize the relative angle between the propagation directions by requiring $\hat{k}_1 \cdot \hat{k}_2 = -\sin \psi$, where $\psi = 0$ corresponds to the original configuration.

Without loss of generality, we fix $a_2 = 1$ and define $a_1 \equiv q$ and let $\hat{k}_2 = \hat{x} \cos \psi - \hat{y} \sin \psi$ with $\hat{e}_2 = \hat{x} \sin \psi + \hat{y} \cos \psi$, while keeping $\hat{k}_1 = \hat{y}$ and $\hat{e}_1 = i\hat{x}$. Under these assumptions, the first wave derived from Eq. (16) becomes proportional to

$$\propto q e^{i(\phi_1 - \omega_0 t + \pi/2)} \hat{x}, \quad (17)$$

where $\phi_1 = ky + \delta_1$. The corresponding electric field for the second wave becomes proportional to

$$\propto e^{i(\phi_2 - \omega_0 t)} (\hat{x} \sin \psi + \hat{y} \cos \psi), \quad (18)$$

where $\phi_2 = k(x \cos \psi - y \sin \psi) + \delta_2$. By adding Eqs. (17) and (18), taking the real part, and analyzing the extrema of the squared magnitude, we obtain $F_{\text{max}} = C_0 + R$ and

TABLE I. Values of κ and $\chi_{\max}$ for various powers, electron energies, f-numbers, and interaction layouts. Bold entries indicate geometries that suppress QED cascades by letting electrons traverse the waist of the intersecting pulses.

f	Geometry	Linear polarization				Circular polarization			
		κ	$\chi_{\max}$	$\chi_{\max}$	$\chi_{\max}$	κ	$\chi_{\max}$	$\chi_{\max}$	$\chi_{\max}$
			1 PW, 10 GeV	3 PW, 20 GeV	10 PW, 100 GeV		1 PW, 10 GeV	3 PW, 20 GeV	10 PW, 100 GeV
1.0	Head-on	2.04	~25	~88	~806	1.44	~18	~62	~570
	$n = 2$	0.72	~9	~31	~285	—	—	—	—
	$n = 4$	1.02	~13	~44	~403	1.44	~18	~62	~570
0.75	Head-on	2.53	~31	~109	~1000	1.79	~22	~77	~707
	$n = 2$	0.89	~11	~38	~351	—	—	—	—
	$n = 4$	1.27	~16	~55	~502	1.79	~22	~77	~707
0.5	Head-on	3.22	~40	~139	~1272	2.28	~28	~98	~900
	$n = 2$	1.14	~14	~49	~450	—	—	—	—
	$n = 4$	1.62	~20	~70	~640	2.28	~28	~98	~900
$\lesssim 0.25$	Bidipole	5.25	~65	~227	~2075	3.71	~46	~160	~1467

$F_{\min} = C_0 - R$ where

$$R = \sqrt{(1 - q^2)^2 - 4q(q \sin \psi - \sin \Delta)(q \sin \Delta - \sin \psi)},$$

$$C_0 = 1 + q^2 - 2q \sin \psi \sin \Delta, \quad (19)$$

where $\Delta = k[y(1 + \sin \psi) - x \cos \psi] + (\delta_1 - \delta_2)$. Finally, the corresponding expression for Eq. (9) is

$$\rho = \frac{\sqrt{(1 - q^2)^2 - 4q(q \sin \psi - \sin \Delta)(q \sin \Delta - \sin \psi)}}{1 + q^2 - 2q \sin \psi \sin \Delta}. \quad (20)$$

From the above expressions, several qualitative features follow. A phase offset simply enters as an additive term in Δ , shifting the spatial location of helical electron motion. Since the electron beam is much wider than the focal region, the effect on the overall correlation is negligible. In contrast, a relative amplitude imbalance reduces the magnitude of ϵ and a nonzero angle ψ tilts and rescales the regions of strong correlation. The latter is confirmed by the upper row of Fig. 4, where the joint probability density $p(\alpha, \chi)$ (bottom row) remains only moderately affected for $\psi \lesssim 30^\circ$. Thus, while the two-pulse geometry is not immune to such imperfections, small misalignments ($\psi \ll 1$) and modest amplitude differences ($q \approx 0.9-0.95$) do not destroy the α - χ correlation.

Further, we perform simulations to assess the effect of applying $q = 0.8$ and $q = 0.5$ to both the two-pulse and the four-pulse geometries. As for the latter, we set $a_1 = a_2 = 1$ and $a_3 = a_4 = q$. The resulting joint probability densities are shown in Fig. 5. Here, a moderate imbalance ($q = 0.8$) maintains the correlation, while a stronger reduction ($q = 0.5$) produces a noticeable distortion.

The only exception is the setup $n = 4$ (CP) which maintains robust correlation, although at a reduced value of $\chi_{\max}$. To understand why this happens, recall that in this situation the electrons experience acceleration due to $\mathbf{F}^{(n)} = \mathbf{E}^{(n)} + \hat{\mathbf{z}} \times \mathbf{B}^{(n)}$. This field combination can be computed from Eq. (16) with the replacement $\hat{\mathbf{e}}_j \rightarrow \hat{\mathbf{e}}_j + (\hat{\mathbf{z}} \cdot \hat{\mathbf{e}}_j)\hat{\mathbf{k}}_j$ to account for the magnetic-field contribution. The computation of $\mathbf{F}^{(4)}$, keeping $\hat{\mathbf{k}}_j$ and $\hat{\mathbf{e}}_j$ as in the previous analysis for $n = 4$ (CP)

but with general relative amplitudes a_j and phase variations δ_j yields the transverse components

$$\hat{\mathbf{x}} \cdot \mathbf{F}^{(4)} \propto [H(x, y) \sin(\omega_0 t) - G(x, y) \cos(\omega_0 t)],$$

$$\hat{\mathbf{y}} \cdot \mathbf{F}^{(4)} \propto [H(x, y) \cos(\omega_0 t) + G(x, y) \sin(\omega_0 t)], \quad (21)$$

where

$$H(x, y) = \sum_{j=1}^4 a_j \cos[k(\hat{\mathbf{k}}_j \cdot \mathbf{r}) + \delta_j],$$

$$G(x, y) = \sum_{j=1}^4 a_j \sin[k(\hat{\mathbf{k}}_j \cdot \mathbf{r}) + \delta_j]. \quad (22)$$

Notably, the magnitude of the transverse projection of $\mathbf{F}^{(4)}$ varies only in space and reveals that additional configurations

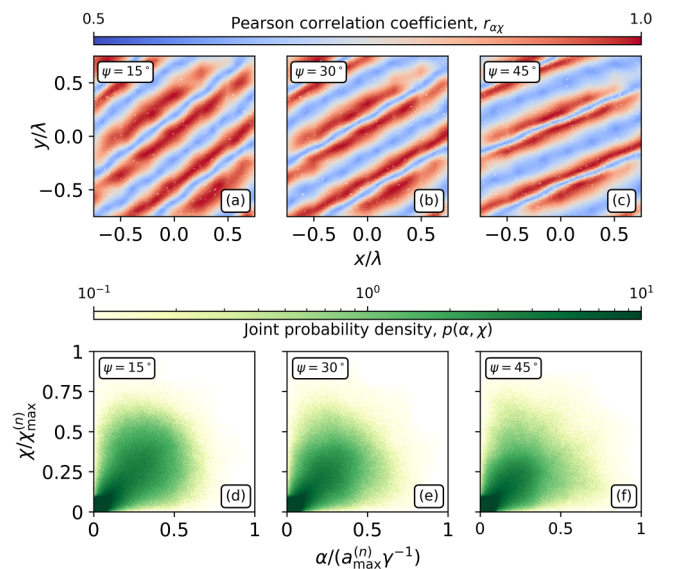


FIG. 4. Correlation strength between χ and α with respect to the electron transverse location (a)–(c) and corresponding joint probability density (d)–(f) for three different angles $\pi/2 + \psi$ between the propagation directions in the configuration $n = 2$.

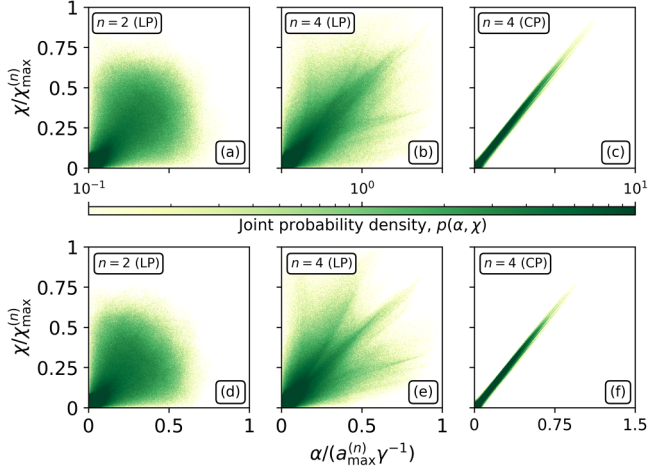


FIG. 5. Joint probability density of electron χ and α values derived from electron trajectories after traversing geometries subject to a relative amplitude scaling of $q = 0.8$ (a)–(c) and $q = 0.5$ (d)–(f).

might be possible, such as the use of a single wave or two counterpropagating waves with opposite helicity at the cost of reducing the value for $\chi_{\max}$. Lastly, though the correlation $\alpha \propto \chi$ holds regardless of varying amplitudes a_j or phase imperfections δ_j one should note that the overall strength of the transverse acceleration still varies across space as suggested by Eq. (22).

In the following section, we compare the effectiveness for geometries generating rotating fields via $n = 2$ (LP) and $n = 4$ (CP) pulses with a single, directly colliding laser pulse with circular polarization where $\kappa^{(4)} = \kappa^{(\text{dir})}$ holds.

V. NUMERICAL ANALYSIS

The effectiveness of each interaction geometry is assessed by simulating the number of signal photons produced as a function of laser power. We introduce the signal-to-noise ratio $\eta = N_{\text{signal}}/N_{\text{total}}$ as outlined in Ref. [49], where N_{total} is the number of detected photons satisfying $\delta \geq 0.5$ and $\alpha \geq c_\alpha a_{\max} \gamma^{-1}$. The parameter c_α represents the effective size of a spatial filter that selects off-axis photons. The number of signal photons, $N_{\text{signal}} \leq N_{\text{total}}$, are those which were also emitted at $\chi \geq \chi_{\max}/2$ by particles retaining at least 99% of their initial energy. To further clarify the distinction between N_{signal} and N_{total} , their corresponding angular distribution are shown in Fig. 6 for the case $n = 4$ (CP) with an input power of $P = 1$ PW and $c_\alpha = 0.6$.

Simulations were performed using the energy-conserving, dispersion-free particle-in-cell code π -PIC [58], extended to include NCS, NBW, QED cascades, and higher-order processes based on the implementation described in Ref. [59]. All laser pulses are assigned a central wavelength of $\lambda = 800$ nm and a pulse duration of $\tau = 30$ fs. Each pulse is assumed to be focused by a mirror with $f = 1$, and is modeled in the far field as occupying a spherical sector of radius R_0 and thickness $\ell = c\tau$. The corresponding spatial profile of the laser field is described by [60]

$$u(\mathbf{r}) = \frac{A}{|\mathbf{r}|} u_{\parallel}(\zeta) u_{\perp}(\theta), \quad (23)$$

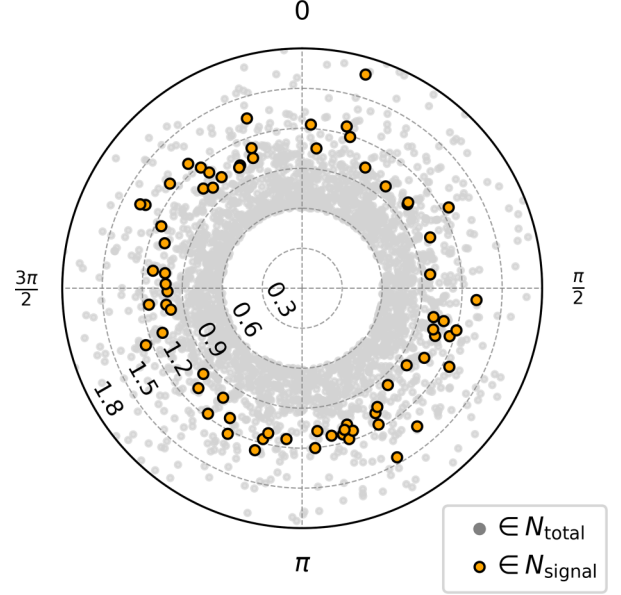


FIG. 6. Angular distribution of photons characterized as N_{signal} (orange) and N_{total} (gray), plotted with respect to $\alpha/(a_{\max}^{(n)} \gamma^{-1})$ (radial direction) and the azimuthal emission angle $\varphi \in [0, 2\pi)$, defined as the angle between the electron's transverse propagation direction and the x axis.

where $A = \sqrt{\frac{4P}{c(1-\cos\Theta)}}$ is the amplitude, $\zeta = |\mathbf{r}| - R_0$ denotes the longitudinal coordinate relative to the pulse center and $\theta = \arcsin(\frac{|\mathbf{r}_{\perp}|}{|\mathbf{r}|})$, with $\mathbf{r}_{\perp}$ denoting the coordinates perpendicular to the direction of propagation. The factorization in Eq. (23) consists of a longitudinal $u_{\parallel}$ and a transverse $u_{\perp}$ part, given by

$$\begin{aligned} u_{\parallel}(\zeta) &= \sin\left(\frac{2\pi\zeta}{\lambda}\right) \cos^2\left(\frac{\pi\zeta}{\ell}\right) \Pi\left(\zeta, -\frac{\ell}{2}, \frac{\ell}{2}\right), \\ u_{\perp}(\theta) &= \cos^2\left(\frac{\pi(\theta - \Theta + \frac{\phi}{2})}{2\phi}\right) \Pi\left(\theta, \Theta - \frac{\phi}{2}, \Theta + \frac{\phi}{2}\right) \\ &\quad + \Pi\left(\theta, 0, \Theta - \frac{\phi}{2}\right), \end{aligned} \quad (24)$$

where $\Pi(\zeta, \zeta_{\min}, \zeta_{\max})$ is the rectangular function defined to equal unity for $\zeta \in [\zeta_{\min}, \zeta_{\max}]$ and zero otherwise. The parameter $\phi = 0.1$ is an angle that allows for a smooth transition around Θ .

The electron beam has a density of $n_e = 10^{19} \text{ cm}^{-3}$ and is represented of N_e macroparticles, uniformly distributed within a cylindrical volume of radius $r_e = 0.9\lambda$ and length $\ell_e = 1.5\lambda$, respectively. The electron energy is set to $\varepsilon = 20$ GeV, which can be achieved either using an all-optical setup based on laser wakefield acceleration [61], or through conventional accelerator technologies [62–64]. Given the short interaction time and the low relativistic electron density, collective effects are negligible. This permits the use of alternative field solvers such as those based on the Stratton-Chu integral representation [65–67]. These methods can improve performance when evolving tightly focused laser beams by avoiding grid interpolation, though in the present case the accuracy gains are likely marginal.

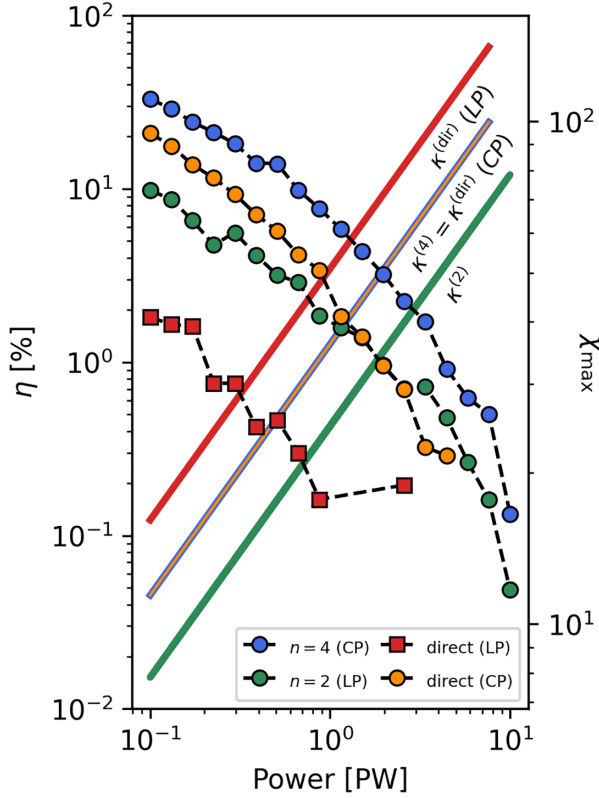


FIG. 7. Signal-to-noise ratio (markers) and $\chi_{\max}$ (solid lines) as a function of laser power. Points where $\eta = 0$ are omitted.

The computational domain is a cubic grid consisting of 128^3 cells with a side length of 32λ . Initially, each pulse is propagated so that it is located $L = 10\lambda$ away from the origin, which also denotes the distance to the center of the electron beam. The simulation is then carried out for a duration of $2L/c$ with a given time step Δt .

We first perform one such simulation per geometry using $N_e = 5 \times 10^4$ macroparticles and a timestep $\Delta t = \frac{\lambda}{12c}$ at a fixed power of $P = 1$ PW, with strong-field QED processes disabled. For each macroparticle, the values of α and χ are recorded at every timestep and subsequently used to compute the Pearson correlation coefficient $r_{\alpha\chi}$ and the joint probability density $p(\alpha, \chi)$ (see Figs. 2, 4, and 5). To justify the omission of quantum processes we emphasize that N_{signal} primarily consists of emissions from electrons that have emitted a single hard photon, effectively inheriting the emission angle and χ value from a parent electron that has not experienced significant recoil.

Next, we perform a series of simulations with $N_e = 2 \times 10^5$ and $\Delta t = \frac{\lambda}{32c}$ for each geometry with varying input power. This time, SFQED processes are enabled and at the end of each simulation, η is evaluated using $c_\alpha = 0.6$, except for the $n = 4$ (CP) case where $c_\alpha = 1.0$. The resulting parameter scan is shown in Fig. 7. When $n = 2$ (LP) pulses are used, the signal-to-noise ratio reaches approximately 1% at an input power of 2 PW, corresponding to $\chi_{\max} \sim 30$. We reiterate that in this configuration, neither the electron beam nor the laser pulses propagate toward sensitive equipment; however, this advantage comes at the cost of lower $\chi_{\max}$ values compared to direct collisions.

For directly colliding pulses with circular polarization, the signal-to-noise ratio is slightly higher at sub-petawatt powers but eventually declines due to the onset of QED cascades which arise from the relatively long interaction time. In contrast, while linearly polarized pulses yield higher $\chi_{\max}$ values, the absence of a strong α - χ correlation significantly reduces η .

Finally, the configuration with $n = 4$ (CP) pulses yields $\chi_{\max}$ values identical to those of the direct collision geometry, as predicted by Eq. (14). Nevertheless, this setup achieves the highest signal-to-noise ratio among all configurations, reaching $\eta \lesssim 1\%$ at $\chi_{\max} \sim 100$. This is a result of the strong correlation inherent to this geometry, which increases the likelihood of identifying a high- χ photon based on its emission angle.

In conclusion, PW-class laser systems combined with multi-GeV electron beams can achieve $\chi_{\max} \sim 10$ –60 for the $n = 2$ and $n = 4$ pulse configurations (see Table I) with the opportunity to mitigate a QED cascade and retrieve a SFQED signal from photons. Assuming a laser power of 10 PW and projected advances toward 100 GeV electron accelerators [68], values of $\chi_{\max} \sim 285$ –570 become accessible. Moreover, by considering the corresponding values of $\kappa^{(n)}$, the use of a plasma converter for the colliding pulses [22] could potentially raise $\chi_{\max}$ to $\sim 3.7 \times 10^4$ for the $n = 2$ (LP) setup and $\sim 7.4 \times 10^4$ for the $n = 4$ (CP) configuration.

Lastly, even if $\eta \ll 1\%$ and spatiotemporal variations are present, it is still possible to test theoretical predictions of SFQED by accumulating data from many repeated collisions and employ Bayesian techniques, such as approximate Bayesian computation [48].

VI. CONCLUSIONS

In this paper we elaborated on and assessed a laser-electron collider geometry where electrons were delivered through the waist of either $n = 2$ or $n = 4$ colliding laser pulses that collectively formed a rotating electromagnetic field at focus. This enabled the separation of photons originating from the strong-field region based on their emission angle, while minimizing the electron interaction time and thereby suppressing the development of significant QED cascades and their associated low- χ contributions. Our results indicate that dividing the available power between two laser pulses yields a signal-to-noise ratio comparable to that of a single head-on circularly polarized pulse, while avoiding irradiation of optical components and the electron source. By considering four such pulses with circular polarization, the overall values of χ and signal-to-noise ratio are increased. This suggests that multi-PW laser systems could access and probe SFQED processes where $\chi \sim 10$ –100. At even higher powers, it may become necessary to increase the electron energy to further prevent the major impact of a QED cascade. Additionally, the initial spin state of the electrons can affect SFQED rates [69], which may, in turn, influence the present results, potentially in a favorable way and could reduce the growth rate of QED cascades [70]. Finally, the polarization of emitted photons may provide an additional criterion for discriminating signal events. Both aspects represent promising directions for future investigation.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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