

# **Erratum: Gravitational decoherence estimation in optomechanical systems** **[Phys. Rev. A **114**, 022442 (2026)]**

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In the original paper, the diffusion term in the equation of motion for  $\langle \hat{b}^2 \rangle$  had an incorrect sign, affecting its particular solution and the time-dependent and stationary terms of the covariance matrix. Here we correct these expressions, while we clarify their physical interpretation. The main results, including the purity and quantum Fisher information (QFI), remain unchanged, upon a consistent shift of the squeezing angle by  $\pi/2$ . Throughout,  $\Lambda = \Lambda_T + \Lambda_g$  denotes the total diffusion rate. Equations from the original paper retain their original numbers, whereas equations introduced here are labeled (E1), (E2), and so forth. The numerical code reproducing the figures with the corrected expressions, including Fig. 1 of this Erratum, is available in Ref. [1].

**Quadrature convention.** The quadratures used in the covariance-matrix and QFI formulas are dimensionless and should be defined as  $\hat{x} = \frac{\hat{b} + \hat{b}^\dagger}{\sqrt{2}}$ ,  $\hat{p} = -\frac{i(\hat{b} - \hat{b}^\dagger)}{\sqrt{2}}$ , so that  $[\hat{x}, \hat{p}] = i$  and  $[\hat{O}_j, \hat{O}_k] = i\Omega_{jk}$  for  $\hat{\mathbf{O}} = (\hat{x}, \hat{p})^T$ . With this convention, the uncertainty relation in Eq. (1), the mean-excitation formula, and the purity formula in Eq. (27) retain their published forms. If  $\hat{X}$  and  $\hat{P}$  denote the dimensional position and momentum, respectively, then  $\hat{X} = \sqrt{\frac{\hbar}{m\omega_m}} \hat{x}$ ,  $\hat{P} = \sqrt{\hbar m\omega_m} \hat{p}$ , and  $\hat{b} = (\hat{x} + i\hat{p})/\sqrt{2}$ .

**Sign of the diffusion contribution.** With  $\hat{X}_b = \hat{b} + \hat{b}^\dagger = \sqrt{2}\hat{x}$ , the diffusion term in Eq. (3) gives

$$\left. \frac{d}{dt} \langle \hat{\mathbf{O}} \rangle \right|_{\Lambda} = -\Lambda \langle [\hat{X}_b, [\hat{X}_b, \hat{\mathbf{O}}]] \rangle. \quad (\text{E1})$$

Since  $[\hat{X}_b, [\hat{X}_b, \hat{b}^2]] = 2$ , Eqs. (8) and (11) should read

$$\frac{d}{dt} \langle \hat{b}^2 \rangle = -(2i\omega_m + \gamma) \langle \hat{b}^2 \rangle - 2\Lambda, \quad (8)$$

and

$$\langle \hat{b}^2 \rangle_t = e^{-(2i\omega_m + \gamma)t} \langle \hat{b}^2 \rangle_0 - \frac{2\Lambda}{2i\omega_m + \gamma} [1 - e^{-(2i\omega_m + \gamma)t}]. \quad (11)$$

In quadrature form, Eq. (1) gives  $\left. \frac{d}{dt} \langle \hat{x}^2 \rangle \right|_{\Lambda} = 0$  and  $\left. \frac{d}{dt} \langle \hat{p}^2 \rangle \right|_{\Lambda} = 4\Lambda$ . Thus, diffusion acts directly on the momentum quadrature. The corresponding diffusion matrix is

$$D_{\text{diff}} = \gamma \left( \bar{n}_{\text{th}} + \frac{1}{2} \right) I_2 + 4\Lambda \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \quad (\text{E2})$$

**Covariance matrix.** For the complex operator vector  $(\hat{b}, \hat{b}^\dagger)^T$ , the covariance matrix transforms by congruence rather than by Hermitian similarity. Consequently, Eq. (13) should read

$$C(t) = D(t)C(0)D^T(t) + C_{\text{noise}}(t). \quad (13)$$

Moreover,  $S(t)$  in Eq. (15) is the linear drift propagator, not a symplectic matrix for  $\gamma > 0$ .

**Time-dependent covariance matrix.** Equation (19) retains its form,

$$\sigma_{\text{noise}}(t) = \begin{pmatrix} N(t) + A(t) & B(t) \\ B(t) & N(t) - A(t) \end{pmatrix}, \quad (19)$$

where  $N(t)$  remains as defined in Eq. (20). The corrected expressions replacing Eqs. (21) and (22) are

$$A(t) = -\frac{2\Lambda}{\gamma^2 + 4\omega_m^2} [\gamma(1 - e^{-\gamma t} \cos(2\omega_m t)) + 2\omega_m e^{-\gamma t} \sin(2\omega_m t)] \quad (21)$$

and

$$B(t) = \frac{2\Lambda}{\gamma^2 + 4\omega_m^2} [2\omega_m(1 - e^{-\gamma t} \cos(2\omega_m t)) - \gamma e^{-\gamma t} \sin(2\omega_m t)]. \quad (22)$$

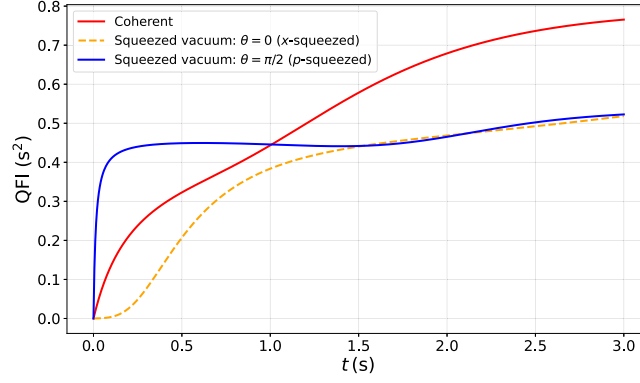


FIG. 1. QFI for estimating  $\Lambda_g$  with the corrected diffusion terms. Equal-energy coherent and squeezed-vacuum probes ( $n_0 = 4$ ) are compared for  $\theta = 0$  ( $x$  squeezed) and  $\theta = \pi/2$  ( $p$  squeezed). Parameters:  $\omega_m = 1$ ,  $\gamma = 0.1$ ,  $\Lambda_T = \bar{n}_{\text{th}} = 1$ ,  $\Lambda_g = 10^{-8}$ ,  $|\alpha_0| = 2$ , and  $r = \text{arcsinh}(2)$ .

*Stationary covariance matrix.* The form of Eq. (23) is retained, while Eq. (24) should be replaced by

$$A_\infty = -\frac{2\Lambda\gamma}{\gamma^2 + 4\omega_m^2}, \quad B_\infty = \frac{4\Lambda\omega_m}{\gamma^2 + 4\omega_m^2}. \quad (24)$$

The ratio in Eq. (25) is unchanged:  $B_\infty/A_\infty = -2\omega_m/\gamma = -2Q$ . The limiting expressions stated below Eq. (25) should instead read

$$\begin{aligned} \gamma \gg \omega_m : A_\infty &\simeq -\frac{2\Lambda}{\gamma}, B_\infty \simeq 0, \\ \omega_m \gg \gamma : A_\infty &\simeq 0, B_\infty \simeq \frac{\Lambda}{\omega_m}. \end{aligned}$$

*Squeezing angle and invariance of the reported results.* For a squeezed-vacuum probe with squeezing magnitude  $r$  and angle  $\theta$ , the initial covariance matrix is

$$\sigma_0(r, \theta) = R(\theta) \frac{1}{2} \begin{pmatrix} e^{-2r} & 0 \\ 0 & e^{2r} \end{pmatrix} R^T(\theta), \quad (E3)$$

where  $R(\theta) = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$ . Since the diffusion acts directly along the momentum quadrature, the favorable short-time orientation is  $\theta = \pi/2$ , for which  $\hat{p}$  is squeezed, rather than  $\theta = 0$ , which was implicitly adopted in the published calculations and for which  $\hat{x}$  is squeezed.

More generally, denoting by  $\sigma_p(t; 0)$  the covariance matrix obtained from the published expressions with an initially  $x$ -squeezed probe, the corrected covariance matrix satisfies

$$\sigma_{\text{corr}}\left(t; \frac{\pi}{2}\right) = R\left(\frac{\pi}{2}\right) \sigma_p(t; 0) R^T\left(\frac{\pi}{2}\right). \quad (E4)$$

The corrected noise covariance is obtained from the published one by a  $\pi/2$  phase-space rotation. Applying this rotation also to the initial covariance gives Eq. (11), since the rotation commutes with  $S(t)$ . This parameter-independent rotation preserves the purity and the QFI for estimating  $\Lambda_g$ . The same rotation argument applies to squeezed thermal probes. Consequently, all the published purity and QFI curves are recovered when the initial squeezing angle is changed from  $\theta = 0$  to  $\theta = \pi/2$ . Thus, the reported short-time advantage corresponds to a momentum-squeezed probe.

Figure 1 illustrates this point directly. It is not necessary to reproduce the figures from the original paper, since their numerical curves are identical under Eq. (11) of this Erratum. Finally, the numerical parameters not explicitly reported in the original discussion are  $\bar{n}_{\text{th}} = 1$  and  $\Lambda_T = 1$  in the units adopted for the simulations, unless otherwise specified. The QFI formula in Eq. (26), the purity formula in Eq. (27), and the principal conclusions of the paper are unchanged.

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*Data availability.* The code and parameters used to reproduce the numerical figures discussed in this Erratum, are available in Zenodo [1].

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- [1] L. A. M. Souza, Code for the erratum to “Gravitational decoherence estimation in optomechanical systems,” [Computer software], Zenodo, 2026, <https://doi.org/10.5281/zenodo.22715661>.