

Generation of the complete Bell basis via Hong-Ou-Mandel interference of vector modes

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Optical vector modes (VMs), characterized by spatially varying polarization distributions, have become essential tools across microscopy, metrology, optical trapping, nanophotonics, and optical communications. Here, we investigate the Hong-Ou-Mandel (HOM) interference of VMs providing a full spatial characterization of the output state. We find that, by carefully selecting the input VMs, it is possible to simultaneously observe all four polarization Bell states and their superpositions in the spatially varying polarization pattern of the two photons. These results represent a significant step in advancing our understanding of HOM interference within structured photons, offering promising avenues for high-dimensional quantum information processing and, in particular, high-dimensional quantum communication, quantum sensing, and advanced photonic technologies reliant on tailored quantum states of light.

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I. INTRODUCTION

Hong-Ou-Mandel (HOM) interference is a two-particle quantum interference effect and plays a central role in many quantum technologies that require interaction between two particles [1,2]. When two indistinguishable photons enter a 50:50 beam splitter from different input ports, the photons destructively interfere and experience “bunching,” in which they exit through the same output port of the beam splitter, resulting in the well-known dip in coincidence counts. If the two input photons are entangled, then a HOM dip in the coincidences will be observed if the two photons’ entangled state is symmetric, and a HOM peak will be observed if the entangled state is antisymmetric. HOM interference is widely used in photonic quantum experiments, such as the implementation of quantum logic gates [3], quantum imaging [4,5], quantum communication [6], and optimal quantum cloning [7–9]. The effect is also used to characterize photon-pair sources [10], optimize quantum teleportation [11], and enable Boson sampling experiments [12,13].

Vector modes (VMs), also known as vector vortex modes, possess a distinctive spatially varying polarization structure

that differentiates them from homogeneously polarized light beams [14]. This characteristic has allowed its application in various fields, including nonlinear optics [15], high-resolution imaging [16], and optical trapping [17,18]. VMs also offer enhanced capabilities, such as sharper focus spots [19,20], and improved performance in quantum key distribution [21,22]. Two primary methods for generating VMs are the intracavity method, which incorporates mode-selection optical elements within a laser resonator [23–25], and the extracavity method, which employs specialized optical components, such as q -plates [26–28] or spatial light modulators within Sagnac interferometers, to convert Gaussian beams into VMs [29]. The entanglement of VMs has also been explored, where polarization-entangled photon pairs are converted into VMs, creating a complex, spatially varying entanglement structure [30–34].

Recent studies on HOM interference with VMs have revealed spatially distributed HOM dips and peaks [35]. However, these studies are restricted to two spatial dimensions, focusing on the spatially dependent polarization structure of photons from only one of the interferometer’s output ports, captured with an event camera, while photons in the other port are detected with a bucket detector. By correlating time-stamped events between the camera and the bucket detector, a two-dimensional map of the final-state structure was constructed. Nevertheless, the complete structure of the VMs after HOM interference is inherently four-dimensional as it depends on the spatial coordinates of photons from both output ports.

Here, we demonstrate a full spatial characterization of HOM interference with VMs, revealing the complete four-dimensional state structure. Remarkably, unlike previous studies suggesting that only the antisymmetric Bell state ψ^- can emerge from HOM interference when postselecting on

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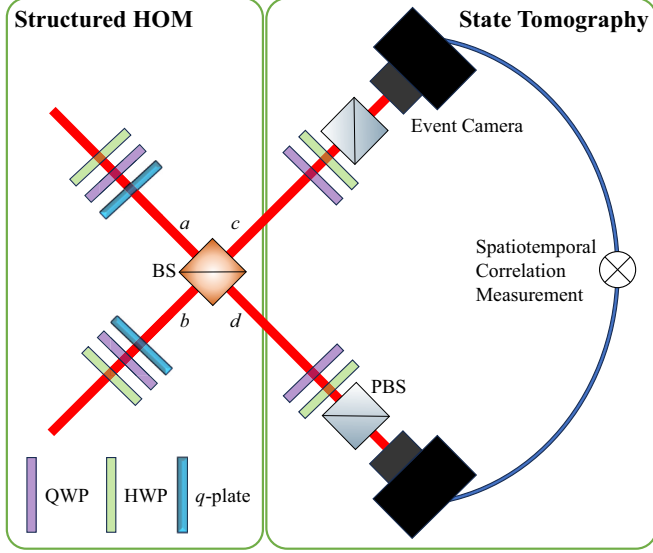


FIG. 1. Space-resolved HOM interference of VMs. Single Gaussian modes are sent to two q -plates to create VMs. These overlap at a 50:50 beam splitter (BS), where HOM interference occurs. Space-resolved polarization state tomography is performed by recording the outputs of two projection stages, consisting of a QWP, a HWP, and a polarizing beam splitter (PBS), onto two event cameras (in the actual experiment, two different regions of a single camera were used instead). By extracting temporal-spatial correlated events between all pixels of the two cameras, a spatially varying density matrix can be retrieved, from which the distribution of the Bell states is obtained.

coincidence events between the two output ports, we find that interference between specific VM combinations can simultaneously yield all four polarization Bell states and their superpositions, depending on the relative transverse spatial

locations of detected photons in the two output ports. We believe that this finding has significant implications for quantum technologies that leverage HOM interference.

II. RESULTS

A simplified experimental scheme for realizing HOM interference of VMs is shown in Fig. 1; the detailed experimental setup can be found in the Supplemental Material [36]. Photon pairs are generated from spontaneous parametric down-conversion (SPDC) in a type-II 5-mm-long periodically poled potassium titanyl phosphate (ppKTP) crystal and coupled to single-mode fibers (not shown in the figure) to select only the Gaussian mode. Two sets of quarter-wave plates (QWPs) and half-wave plates (HWPs) are used to prepare the input polarization states:

$$|\psi_0\rangle = a_{\pi_1}^\dagger b_{\pi_2}^\dagger |0\rangle, \quad (1)$$

where $a_{\pi_1}^\dagger$ ($b_{\pi_2}^\dagger$) is the creation operator for a photon in input port a (b) with polarization $|\pi_1\rangle$ ($|\pi_2\rangle$). The two photons are then sent to two q -plates. In the circular polarization basis, where $|L\rangle$ and $|R\rangle$ are left-handed and right-handed circular polarization states, the action of a q -plate with topological charge q and birefringence δ on an input Gaussian mode is [37,38]

$$\begin{aligned} \hat{U}_q^\delta \begin{bmatrix} |L\rangle \\ |R\rangle \end{bmatrix} &= F_0(r) \cos\left(\frac{\delta}{2}\right) \begin{bmatrix} |L\rangle \\ |R\rangle \end{bmatrix} \\ &+ i \sin\left(\frac{\delta}{2}\right) F_q(r) \begin{bmatrix} e^{-i2q\theta} |R\rangle \\ e^{i2q\theta} |L\rangle \end{bmatrix}, \end{aligned} \quad (2)$$

where r and θ are the radial and azimuthal coordinate in the transverse plane, respectively, and $F_0(r)$ and $F_q(r)$ are hypergeometric-Gauss functions [39], which can be approximated as the Laguerre-Gauss mode with the orbital angular

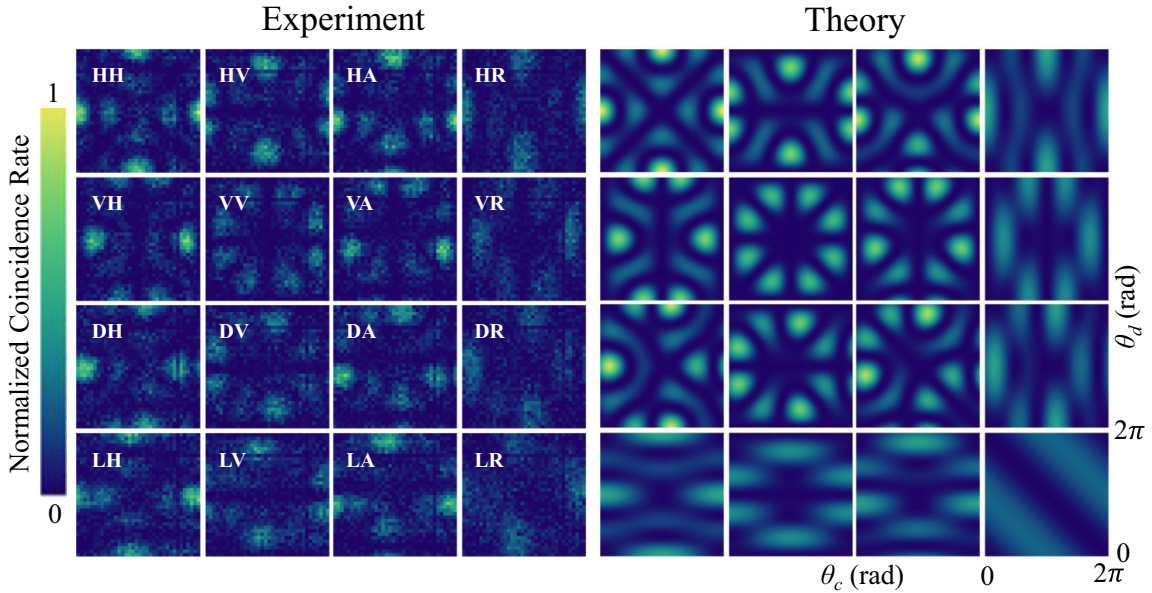


FIG. 2. Space-resolved polarimetry of HOM interference of VMs. Experimental (left panels) and theoretical (right panels) azimuthal correlations of the two-photon state for the 16 projections required for the complete state tomography. The topological charges of the two q -plates are $q_a = 1$ and $q_b = 1/2$. The initial state is $\hat{a}_H^\dagger \hat{b}_H^\dagger |0\rangle$.

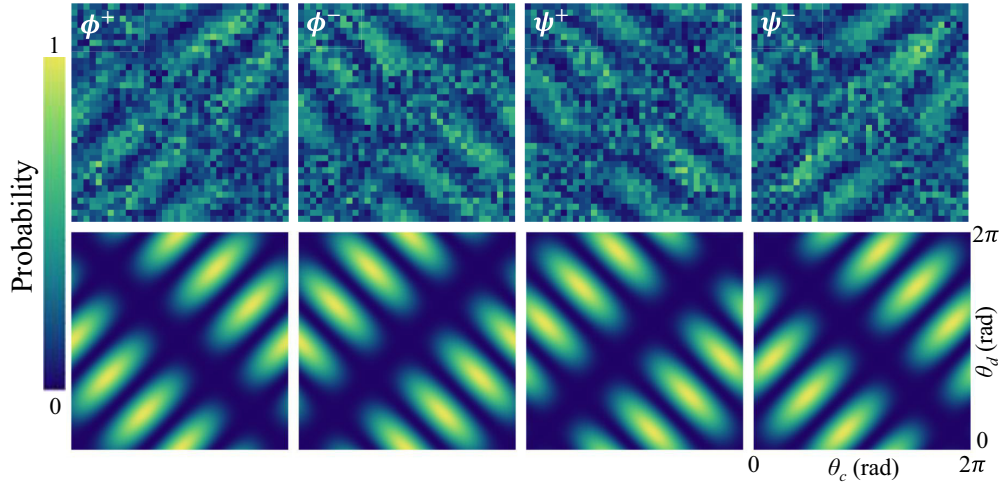


FIG. 3. Probability distributions of the four Bell states. Experimental (top row) and theoretical (bottom row) spatial patterns of the four Bell states obtained via HOM interference of VMs for the case study $q_a = 1$ and $q_b = 1/2$ and the initial state $\hat{a}_H^\dagger \hat{b}_H^\dagger |0\rangle$.

momentum index $\ell = 2q$ and a radial index equal to 0: $F_q(r)e^{\pm i2q\theta} \equiv \text{LG}_{\pm 2q,0}(r, \theta)$.

The birefringence parameter δ is uniform across the plate and can be electrically controlled [40]. In this experiment, we set $\delta = \pi$, which corresponds to a fully tuned q -plate. Interference between VMs takes place within a 50:50 beam splitter (BS), whose action, neglecting a global phase, reads

$$\begin{aligned} |\psi\rangle_a &\xrightarrow{\text{BS}} \frac{1}{\sqrt{2}}(|\psi\rangle_c + |\psi\rangle_d), \\ |\psi\rangle_b &\xrightarrow{\text{BS}} \frac{1}{\sqrt{2}}(|\psi\rangle_c - |\psi\rangle_d), \end{aligned} \quad (3)$$

where c and d are the output ports, and the transformation applies to all degrees of freedom. Assuming two horizontally polarized input photons, $|\psi_0\rangle = a_H^\dagger b_H^\dagger |0\rangle$, with $|H\rangle = (|L\rangle + |R\rangle)/\sqrt{2}$, and after postselecting only on coincidence events between output ports c and d , the resulting state after the BS can be written as (see the Supplemental Material for the detailed derivation [36])

$$|\psi_f\rangle = \mathcal{N}(s_{\phi^+} |\phi^+\rangle + s_{\phi^-} |\phi^-\rangle + s_{\psi^+} |\psi^+\rangle + s_{\psi^-} |\psi^-\rangle), \quad (4)$$

where $\mathcal{N}$ is a normalization constant; $|\phi^\pm\rangle = (|HH\rangle \pm |VV\rangle)/\sqrt{2}$ and $|\psi^\pm\rangle = (|HV\rangle \pm |VH\rangle)/\sqrt{2}$ are the four polarization Bell states; and $s_{\phi^\pm}$ and $s_{\psi^\pm}$ are given by

$$\begin{aligned} s_{\phi^+} &= F_{q_a}(r_d)F_{q_b}(r_c) \cos(2q_a\theta_d - 2q_b\theta_c) \\ &\quad - F_{q_a}(r_c)F_{q_b}(r_d) \cos(2q_a\theta_c - 2q_b\theta_d), \\ s_{\phi^-} &= F_{q_a}(r_d)F_{q_b}(r_c) \cos(2q_a\theta_d + 2q_b\theta_c) \\ &\quad - F_{q_a}(r_c)F_{q_b}(r_d) \cos(2q_a\theta_c + 2q_b\theta_d), \\ s_{\psi^+} &= F_{q_a}(r_d)F_{q_b}(r_c) \sin(2q_a\theta_d + 2q_b\theta_c) \\ &\quad - F_{q_a}(r_c)F_{q_b}(r_d) \sin(2q_a\theta_c + 2q_b\theta_d), \\ s_{\psi^-} &= F_{q_a}(r_d)F_{q_b}(r_c) \sin(2q_a\theta_d - 2q_b\theta_c) \\ &\quad + F_{q_a}(r_c)F_{q_b}(r_d) \sin(2q_a\theta_c - 2q_b\theta_d), \end{aligned} \quad (5)$$

with q_a (q_b) being the topological charge of the q -plate in input port a (b).

To experimentally characterize this state, the output photons are sent to a polarization tomography apparatus consisting of a QWP, a HWP, and a polarizing beam splitter (PBS) in each output arm, and they are then directed to two different regions of an event camera (TPX3CAM) [41,42]. The size of the beam spot on the camera covers approximately 30×30 pixels. Photon pairs are identified using time-correlation measurements between the camera pixels. The space-resolved outcomes of 16 polarimetric measurements are processed with a maximum-likelihood method to retrieve a density matrix, $\rho(r_c, \theta_c, r_d, \theta_d)$, at each pixel [43], from which the probability distributions of the four Bell states are extracted:

$$\begin{aligned} P_{\phi^\pm}(r_c, \theta_c, r_d, \theta_d) &= \langle \phi^\pm | \rho(r_c, \theta_c, r_d, \theta_d) | \phi^\pm \rangle, \\ P_{\psi^\pm}(r_c, \theta_c, r_d, \theta_d) &= \langle \psi^\pm | \rho(r_c, \theta_c, r_d, \theta_d) | \psi^\pm \rangle. \end{aligned} \quad (6)$$

Figure 2 shows the complete set of 16 polarization measurements from the two output ports with the two q -plates having topological charges $q_a = 1$ and $q_b = 1/2$. Due to the cylindrical symmetry of the Laguerre-Gauss modes in which the final state showed minimal variations with respect to the radial coordinates, we only plotted the azimuthal correlations for each polarization projection, obtained as the marginal distributions $P(\theta_c, \theta_d) = \sum_{r_c, r_d} P(r_c, \theta_c, r_d, \theta_d)$. The comparison with theoretical predictions (gray scale), calculated from Eq. (4), shows good agreement in all polarization projections.

From the reconstructed marginal density matrix $\rho(\theta_c, \theta_d)$, we extracted the corresponding space-dependent Bell state decomposition, as shown in Fig. 3. Contrary to the typical HOM effect, where only the ψ^- state is expected to be observed when conditioned on coincidence events between the two output ports, the output states of VM HOM feature contributions from all Bell states, depending on the specific location in the coordinate space. The locations where each Bell state can be

uniquely found require the following conditions be met:

$$\begin{aligned}
 \text{only } P_{\phi^+} > 0 \text{ iff } & (q_a + q_b)(\theta_c - \theta_d) = \frac{(2n+1)\pi}{2} \\
 & \text{and } (q_a - q_b)(\theta_c - \theta_d) = m\pi; \\
 \text{only } P_{\phi^-} > 0 \text{ iff } & (q_a + q_b)(\theta_c + \theta_d) = \frac{(2n+1)\pi}{2} \\
 & \text{and } (q_a - q_b)(\theta_c + \theta_d) = m\pi; \\
 \text{only } P_{\psi^+} > 0 \text{ iff } & (q_a + q_b)(\theta_c + \theta_d) = n\pi \\
 & \text{and } (q_a - q_b)(\theta_c + \theta_d) = m\pi; \\
 \text{only } P_{\psi^-} > 0 \text{ iff } & (q_a + q_b)(\theta_c - \theta_d) = n\pi \\
 & \text{and } (q_a - q_b)(\theta_c - \theta_d) = m\pi,
 \end{aligned} \tag{7}$$

with n and m being integers. From this, we also obtain the following conditions on q_a and q_b :

$$\begin{aligned}
 \text{only } P_{\phi^+} > 0 \text{ iff } & q_a = \frac{(2n+2m+1)}{(2n-2m+1)}q_b; \\
 \text{only } P_{\phi^-} > 0 \text{ iff } & q_a = \frac{(2n+2m+1)}{(2n-2m+1)}q_b; \\
 \text{only } P_{\psi^+} > 0 \text{ iff } & q_a = \frac{(n+m)}{(n-m)}q_b; \\
 \text{only } P_{\psi^-} > 0 \text{ iff } & q_a = \frac{(n+m)}{(n-m)}q_b; \\
 \text{for all cases } & |q_a| \neq |q_b|.
 \end{aligned} \tag{8}$$

In the case where $q_a = 1$ and $q_b = 1/2$, the conditions are satisfied for only $P_{\psi^+} > 0$ if and only if $\theta_c = -\theta_d$ and for only $P_{\psi^-} > 0$ where $\theta_c = \theta_d$; however, the conditions cannot be satisfied for only $P_{\phi^+} > 0$ or only $P_{\phi^-} > 0$. Along the directions $\theta_c = -\theta_d$ and $\theta_c = \theta_d$, average purities of 0.76 ± 0.15 and 0.74 ± 0.14 were recorded, respectively, where the error is given by the standard deviation over all the pixels. The smallest q_a and q_b values where all four Bell states can be uniquely found are $q_a = \pm 3/2$ and $q_b = \mp 1/2$. For more details on the derivation of the conditions in Eq. (7), see the Supplemental Material [36]. Polarization measurements and Bell state decomposition for different q -plate settings and input polarization states can also be found in the Supplemental Material [36].

III. CONCLUSIONS

In conclusion, we investigated the HOM interference of VMs and, by performing a full spatial characterization of the spatially varying polarization structure of the two photons, we found that the resulting spatial distribution is composed of a superposition of all four Bell states and, under certain conditions, all four Bell states can be uniquely found simultaneously within this spatial distribution. We provided theoretical and experimental evidence of this effect in the case of VMs generated by q -plates, which naturally induce an azimuthal structure on the final distribution. However, this functionality can be easily generalized to arbitrary spatial

patterns [44], allowing the distribution of Bell states of the output to be tailored to specific demands. For instance, liquid-crystal polarization gratings (g -plates) [45] could be used to map each Bell state onto a two-dimensional Cartesian grid.

Compared to the conventional approach based on applying local operations to convert each Bell state into any other [46], our method offers the remarkable advantage of multiplexing this operation with the only need of a single event camera, thus avoiding bulky setups requiring multiple beam splitters, wave plates, and single-photon detectors. At the same time, our setup can also address superpositions of Bell states instead of pure Bell states, to be used in entanglement-based communication [47,48], computation [49], and error-correction protocols [50,51]. In the near future, it would be interesting to extend similar concepts to a higher number of input photons, where the controlled generation of multiparticle maximally entangled states remains extremely challenging [52–54].

A major limiting factor in the accuracy of the results, which we estimate to have a 10–30% uncertainty on the number of coincidences detected between the camera pixels for our dataset, is the detection efficiency of the experimental setup, which is only $\sim 1\%$. This low efficiency is mainly due to the quantum efficiency of the event camera, which is around 8% [55], combined with spatially filtering the SPDC using single-mode fibers, and losses in the optical elements. To reduce data-acquisition time and improve photon statistics, we limited the beam size to a resolution of 30×30 pixels on the camera. Although we could use a higher resolution, achieving the same number of coincidences per pixel would require a much longer data-acquisition time. However, these are only technical limitations of the detection system. With more efficient event-based cameras [56], we expect that the accuracy and detection time of our detection method can be significantly improved.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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