

Sensing decoherence by using edge states

Andrey R. Kolovsky 

*Kirensky Institute of Physics, 660036 Krasnoyarsk, Russia;
Siberian Federal University, 660041 Krasnoyarsk, Russia;
and Kimyo International University in Tashkent, 100121 Tashkent, Uzbekistan*



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In the absence of decoherence and disorder the current of noninteracting fermionic particles across a finite lattice connecting two reservoirs (leads) with different chemical potentials is known to be ballistic. It is also known that decoherence suppresses this ballistic current. However, if decoherence is weak, the change in the current can be undetectable. In this work we show that the effect of a weak decoherence can be amplified by orders of magnitude if the lattice has edge states. This allows us to use a lattice with edge states as a sensor for measuring the decoherence rate.

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I. INTRODUCTION

Since the discovery of the topological insulators, the edge states in topologically nontrivial lattices have attracted a lot of attention in many different systems [1]. In particular, we mention fascinating laboratory experiments with photonic crystals, where transport along the edges of a two-dimensional photonic lattice was visualized *in situ* [2,3]. Besides three- and two-dimensional topological lattices there are also quasi-one-dimensional and truly one-dimensional lattices [4] which have exponentially localized states located at the lattice ends. Because of this exponential localization, edge states in one-dimensional lattices do not contribute to quantum transport and, in this sense, there is no principal difference between lattices with and without edge states. However, it appears not to be the case if there are decoherence processes in the system, for example, due to residual interactions between the carriers. As a rule, decoherence suppresses the current across the lattice but it is also known that in some cases it can enhance it—the phenomenon known as the environment-assisted quantum transport [5–8]. Moreover, decoherence can be the main mechanism for inducing the directed current, where biased lattices are the most famous example [9,10]. In this work we analyze the effect of decoherence for fermionic carriers in the topological one-dimensional lattices. We will show below that weak decoherence creates the new conduction window in the gap between Bloch bands where the edge states are located. It is argued that by measuring the stationary current in this window one can measure the decoherence rate in the system with high precision.

In this paper we study the problem where two reservoirs of Fermi particles with slightly different chemical potentials are

connected by the Su-Schrieffer-Heeger (SSH) lattice, Sec. II, and by the flux rhombic lattice, Sec. III. We mention that both lattices attract considerable interest and have been realized in laboratory experiments by using different physical platforms. In particular, the topological invariant of the SSH lattice—the Zak phase—was directly measured by using cold atoms in the double-periodic optical lattice in Ref. [11] and the phenomenon of the Aharonov-Bohm caging in the flux rhombic lattice was directly observed in the photonic crystal in Ref. [12] and in the array of transmons in Ref. [13]. In the present work we use these two lattices as typical representatives of one-dimensional lattices with edge states. While the SSH lattice is simpler for the numerical analysis, the flux rhombic lattice has the unique feature that for the Peierls phase ϕ equal to π all eigenstates of this lattice including the edge states are the compact states and, thus, all Bloch bands are flat [14]. This allows us to obtain analytic estimates for the stationary current induced by decoherence.

II. SU-SCHRIEFFER-HEEGER LATTICE

As the first example we consider the SSH lattice,

$$\hat{H}_s = \delta \sum_{\ell=1}^L |\ell\rangle\langle\ell| - \sum_{\ell=1}^{L-1} \frac{J_{\ell}}{2} (|\ell+1\rangle\langle\ell| + \text{H.c.}), \quad (1)$$

where the hopping matrix elements J_{ℓ} take values J and $\tilde{J} \neq J$ for alternating sites. As for any bipartite lattice, the energy spectrum of the SSH lattice consists of two Bloch bands separated by the energy gap. However, due to the topological nature of the SSH lattice, it can have edge states with energies located in the gap [see Fig. 1(a)]. The number of edge states depends on how the lattice is terminated and, thus, varies between zero and two. In what follows we will focus on the case with two edge states. Let us also mention here that, besides cold atoms in the double-periodic optical lattice, the Hamiltonian (1) also describes electrons in the semiconductor

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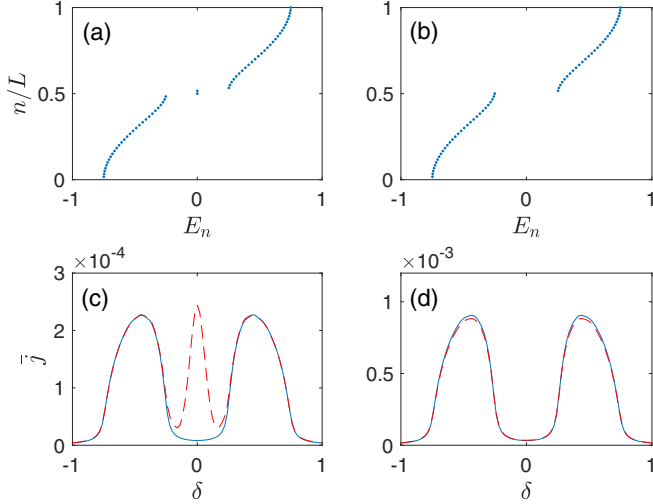


FIG. 1. Upper panels: Energy spectrum of the SSH lattice of the length $L = 60$ with two edge states (a) and without edge states (b). The hopping elements are $J = 1$ and $\tilde{J} = 0.5$. Lower panels: The current across the SSH lattice with (c) and without (d) edge states as the function of the gate voltage δ for $\kappa = 0$ (blue solid lines) and $\kappa = 0.003$ (dashed red lines). Parameters of reservoirs are $\mu_L = \pi/40$, $\mu_R = -\pi/40$, $1/\beta = 0$, and $\gamma = 0.05$. The coupling constant is $\epsilon = 0.2$.

GaAs/AlAs-like superlattices if we change the thickness of every second AlAs layer.

Now we attach two reservoirs of Fermi particles (the leads) to the lattice ends:

$$\hat{H}_{\text{tot}} = \hat{H}_s + \sum_{i=L,R} (\hat{H}_i + \epsilon \hat{H}_{s,i}), \quad (2)$$

where we introduce the coupling constant ϵ . Usually, one models the lead Hamiltonians $\hat{H}_i$ by semi-infinite tight-binding chains. Alternatively, one can model them by the tight-binding rings of size M where M eventually tends to infinity [15,16]. We are interested in the current across the lattice for $\epsilon \neq 0$.

There are two common approaches to the outlined problem. In the first approach one eliminates particle reservoirs and derives a master equation for the reduced density matrix of the carriers in the lattice. Unfortunately, in the general case it is a non-Markovian master equation which is hard to solve. The second approach is based on the Landauer theory which relates the system conductance to the transmission probability $|t|^2$ for the plane waves propagating in the leads. Unfortunately, this approach also has limited validity because it ignores the nonunitary dissipative dynamics of the carriers in reservoirs. In the present work we use the approach which incorporates these two common approaches in the single theory [16]. The only assumption of this third approach is that the dissipative dynamics of the carriers in the leads can be described or mimicked by means of the specific Lindblad operators.

Let us first look at problem (2) from the viewpoint of the Landauer theory whose main ingredient is the energy-dependent transmission probability $|t(E)|^2$. The latter is known to exhibit a number of transmission peaks of the

width $\Gamma_n \sim \epsilon^2$ located at eigenvalues E_n of the Hamiltonian (1). Exclusions are eigenvalues $E_n = 0$ associated with the edge states because, instead of resonant transmission, here we meet the phenomenon of resonant trapping characterized by the Wigner delay or dwell time [17]. Thus, according to the Landauer theory the stationary current as the function of parameter δ should show $L - 2$ transmission peaks. In practice, however, these individual peaks are not resolved because of the peak broadening due to the finite chemical potential difference of the leads, finite temperature, and the dissipative dynamics of the carriers in the leads. All these broadening mechanisms are fully captured by the master equation for the total density matrix $\mathcal{R} = \mathcal{R}(t)$,

$$\frac{d\mathcal{R}}{dt} = -i[\hat{H}_{\text{tot}}, \mathcal{R}] + \sum_{i=L,R} \mathcal{L}_i(\mathcal{R}), \quad (3)$$

where the Lindblad relaxation operators $\mathcal{L}_i(\mathcal{R})$ enforce relaxation of the isolated ($\epsilon = 0$) leads to the thermal equilibrium characterized by the inverse temperature β and the chemical potential μ of the respective lead [16,18–20]. In terms of the reduced density matrix of a given, let us say the left reservoir, $\rho_L(t) = \text{Tr}_{s,R}[\mathcal{R}(t)]$, the explicit form of this operator in the quasimomentum basis $|k\rangle$ is

$$\mathcal{L}_i[\rho_i(t)] = -\gamma[\rho_i(t) - \tilde{\rho}_i], \quad \tilde{\rho}_i = \sum_k n_k |k\rangle \langle k|, \quad (4)$$

where n_k are the occupation numbers of the quasimomentum states given by the Fermi-Dirac distribution,

$$n_k = \frac{1}{e^{\beta(E_k - \mu)} + 1}, \quad E_k = -J \cos\left(\frac{2\pi k}{M}\right). \quad (5)$$

Notice that operators $\mathcal{L}_i(\mathcal{R})$ act only on the leads and, thus, the current across the lattice is ballistic independent of the value of the relaxation constant γ .

Since our aim is the effect of decoherence on the stationary current across the lattice, we include in the master equation (3) an additional relaxation operator:

$$\mathcal{D}(\rho_s) = -\frac{\kappa}{2} \sum_{\ell} (\hat{\ell} \hat{\ell} \rho_s - 2 \hat{\ell} \rho_s \hat{\ell} + \hat{\ell} \hat{\ell} \rho_s), \quad \hat{\ell} = |\ell\rangle \langle \ell|, \quad (6)$$

which acts on the lattice. To some extent, the operator (6) describes the effect of residual interparticle interactions, which in the context of the discussed problem can be viewed as the effect of environment [21]. It is easy to show that for $\epsilon = 0$ this relaxation operator causes exponential decay of off-diagonal elements of the density matrix $\rho_s(t) = \text{Tr}_{L,R}[\mathcal{R}(t)]$ with the rate κ . The operator (6) is known to change the ballistic transport regime into the diffusive regime, so that in the thermodynamic limit $L \rightarrow \infty$ the current decreases as $1/L$ [22]. In what follows, however, we will consider finite lattices where discrimination between different transport regimes is not so obvious.

We proceed with numerical results. First, we discuss the case $\kappa = 0$. The solid line in Fig. 1(c) exemplifies the stationary current $\bar{j}$ across the SSH lattice in the case where the individual resonant peaks are not resolved. The obtained dependence of the current on the gate voltage δ obviously reproduces the band structure of the SSH lattice. Notice that

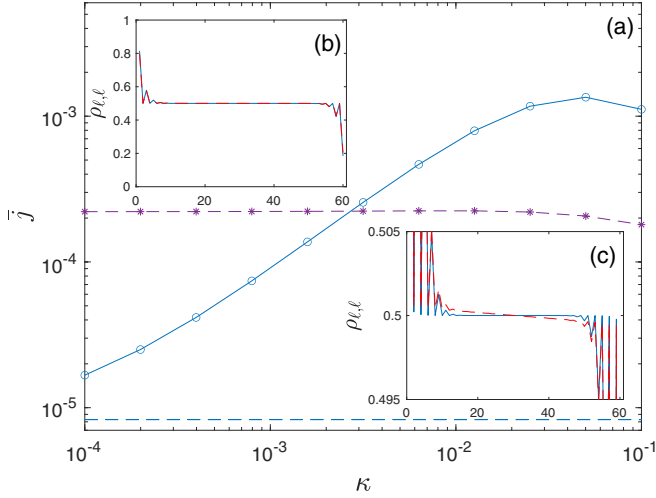


FIG. 2. Main panel: Stationary current as the function of the decoherence rate κ for $\delta = \pm 0.4$ (asterisks) and $\delta = 0$ (open circles). The dashed lines indicate the current magnitude for $\delta = 0$ and $\kappa = 0$. Panel (b): Occupations of the lattice sites in the stationary regime for $\delta = 0$ and $\kappa = 0$ (blue solid line) and $\kappa = 0.003$ (dashed red line). Panel (c) zooms in the central region of panel (b).

the edge states do not participate in the ballistic transport and, thus, cannot be revealed in the dependence $\bar{j} = \bar{j}(\delta)$. Yet, they manifest themselves in the stationary population of the lattice sites. It is seen in Fig. 2(b) that the presence of edge states results in a huge population imbalance between the left and right end sites of the lattice, which is the consequence of the above-mentioned resonant trapping. We stress that this imbalance appears only if the energy interval

$$\mu_R - \gamma/2 < E < \mu_L + \gamma/2 \quad (7)$$

includes energies of the edge states. Going ahead, we notice that this huge imbalance is the underlying mechanism for “amplification” of the decoherence effect.

Let us now set $\kappa \neq 0$ and let us discuss the destructive effect of decoherence in some more detail. As was already mentioned, without taking the thermodynamic limit $L \rightarrow \infty$ the discrimination between different transport regimes is not a trivial task. One of the signatures of the diffusive current is nonzero population gradient $d\rho_{l,l}/dl$ in the *central part* of a finite lattice [23]. Clearly, this population gradient is a continuous function of the decoherence rate and it may vary from $+0$ to the maximally possible value of $1/L$. Thus, it looks reasonable to distinguish between weakly diffusive and strongly diffusive transport regimes depending on the value of the population gradient. In what follows we focus on the limit of small decoherence rate where the destructive effect of decoherence is hard to detect and, according to the population gradient criterion, the current is almost ballistic [see Fig. 2(c)]. However, if the lattice has edge states this negligible destructive effect of decoherence is converted into the constructive effect—the new conduction window in the energy gap which is easy to detect.

The dashed line in Fig. 1(c) shows the dependence of the stationary current $\bar{j}$ on the gate voltage δ for $\kappa = 0.003$. The new transmission peak at $\delta = 0$ is clearly seen. Since the

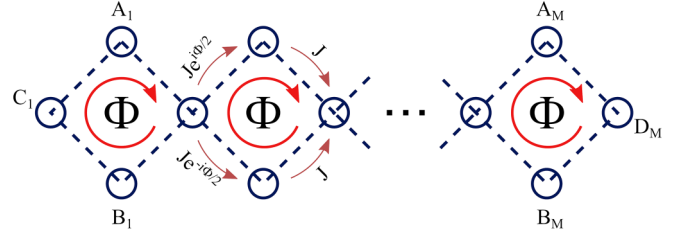


FIG. 3. The flux rhombic lattice.

origin of this peak is the resonant population of the edge states, its width is determined by Eq. (7) which is independent of κ . On the contrary, the height of the peak is mainly determined by the decoherence rate. The dependence of the peak height on κ is depicted by open circles in Fig. 2(a). Additionally, asterisks show the current in the conduction bands at $\delta = \pm 0.4$ and the dashed line indicates the value of the current at $\delta = 0$ and $\kappa = 0$. (In fact, for $\kappa = 0$ this residual current in the band gap can be made arbitrarily small by decreasing γ .) It is seen in Fig. 2(a) that for small κ , where the destructive effect of decoherence is negligible (see asterisks), the resonant peak at $\delta = 0$ grows linearly with κ . Thus, by measuring the height of this peak one can measure the decoherence rate in the system. For the sake of completeness we also depict in Fig. 1(d) the stationary current in the SSH lattice without the edge states; no qualitative difference between the cases $\kappa = 0$ and $\kappa = 0.003$ is observed.

III. FLUX RHOMBIC LATTICE

As the second example we consider the flux rhombic lattice (see Fig. 3). The control parameter of this lattice is the Peierls phase ϕ , which in the case of charged fermions is determined by the value of the magnetic flux through a rhomb. If $\phi \neq \pi$ the spectrum of the flux rhombic lattice consists of one flat band, two dispersive bands, and four exponentially localized edge states [see Fig. 4(a)]. Then, it is not surprising that the transport properties of the rhombic lattice resemble those of the SSH lattice. In particular, for nonzero decoherence rate κ the stationary current across the lattice as the function of the gate voltage δ shows additional transmission peaks at energies of the edge states [see the red dashed line in Fig. 4(b)]. The heights of the peaks associated with Bloch bands and edge states are shown in Fig. 5 by asterisks and open circles, respectively. As compared to Fig. 2, in Fig. 5 we extend for the purpose of future discussion the upper limit of the κ axis up to $\kappa = 100$. Considering for the moment only the region of small $\kappa < 0.1$, the results shown in Fig. 5 are seen to be similar to those obtained for the SSH lattice. Let us also mention that, according to the population gradient criterion the transport regime for $\kappa < 0.1$ and the considered value of the Peierls phase $\phi = \pi - 0.4$ should be classified as weakly diffusive [see the blue solid line in Fig. 5(b)].

As already mentioned in Sec. I, the unique feature of the rhombic lattice is that for $\phi = \pi$ the dispersive bands become flat. Thus, the ballistic transport across the lattice is forbidden and the dependence $\bar{j} = \bar{j}(\delta)$ has only two peaks which are associated with the edge states. Notice that for $\phi = \pi$ these states are the compact states, where only the sites $C_1, A_1,$

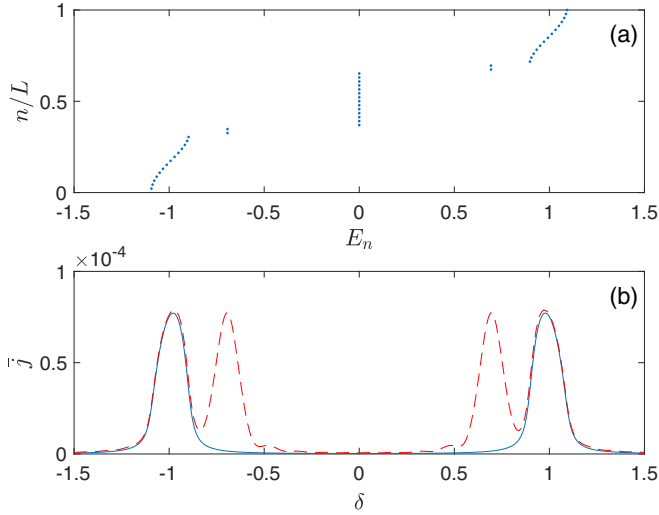


FIG. 4. Upper panel: Energy spectrum of the flux rhombic lattice with $L = 15$ rhombs for $\phi = \pi - 0.4$. The modulus of the hopping elements is $|J| = 1$. Lower panel: The current across the lattice as the function of the gate voltage δ for $\kappa = 0$ (blue solid line) and $\kappa = 0.001$ (red dashed line). Parameters of reservoirs are $\mu_L = \pi/40$, $\mu_R = -\pi/40$, $1/\beta = 0$, and $\gamma = 0.05$. The coupling constant is $\epsilon = 0.2$.

B_1 and the sites A_L, B_L, D_L have nonzero occupation numbers. Remarkably, this qualitative change from the exponentially localized to the compact edge states is reflected in the transition from the weakly diffusive transport regime to the strongly diffusive transport regime [see the red solid line in Fig. 5(b)].

The characteristic feature of the pure diffusive regimes is the Esaki-Tsu-like dependence of the stationary current on the

decoherence rate κ ,

$$\bar{j} \sim \frac{\kappa}{\kappa^2 + \text{const}}. \quad (8)$$

The numerically obtained dependence $\bar{j} = \bar{j}(\kappa)$ in the case $\phi = \pi$ is depicted in Fig. 5(a) by the dash-dotted line and it indeed can be well approximated by the Esaki-Tsu formula [24]. We mention that originally Eq. (8) was derived for electrons in the infinite tilted (biased) lattice but nowadays one finds the Esaki-Tsu-like dependence in many different problems. In this respect, we would like to mention the recent work [26] which addresses the stationary current of fermions across the *finite-size* biased lattice. Surprisingly, this problem appears to be very relevant to the subject of the present work because of the presence of edge states originated by edge Bloch oscillations.

IV. CONCLUSIONS

We studied the effect of weak decoherence on the quantum transport of fermionic particles in the tight-binding lattices with edge states, namely, the SSH lattice and the flux rhombic lattice. For the vanishing decoherence rate the edge states do not contribute to the current because of their finite localization length. However, these states are resonantly populated or depleted when the Fermi energy of the leads coincides with the energy of a given edge state. This creates a huge imbalance in the stationary population of the left and right ends of the lattice, which is a precondition for amplification of the current induced by decoherence.

The simplest from the theoretical viewpoint case is the flux rhombic lattice with the Peierls phase ϕ equal to π , where all Bloch bands are flat and the edge states are compact. Here the induced current is pure diffusive and, as the function of the decoherence rate, obeys the celebrated Esaki-Tsu equation. The general case, which is actually simpler from the experimental viewpoint, is provided by the SSH lattice, where Bloch bands are dispersive and the edge states are only exponentially localized. Here the current in the edge states conduction window is neither pure diffusive nor ballistic. Moreover, its magnitude can exceed the magnitude of the pure ballistic current in the absence of decoherence processes. We argue that by measuring this current one can measure very low decoherence rates which would be undetectable if the lattice had no edge states.

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DATA AVAILABILITY

The data that support the findings of this article are available within the article.

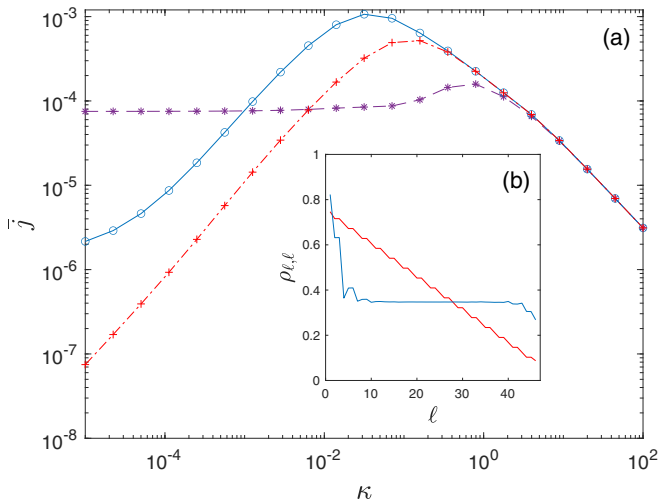


FIG. 5. Stationary current as the function of the decoherence rate κ for $\delta = 1$ (asterisks) and $\delta \approx 0.7$ corresponding to energies of edge states (open circles). The other system parameters are the same as in Fig. 4. The additional red dash-dotted line is the stationary current for $\phi = 0$ where it is pure diffusive. The inset shows the lattice site populations for $\kappa = 0.001$ and $\phi = \pi - 0.4$ (blue line) and $\phi = \pi$ (red line).

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